

## MATCHING IN A NETWORK<sup>1</sup>

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Pigeons were given practice choosing between pairs of alternatives yielding different frequencies of reinforcement. Four individual alternatives were set into four pairwise choices. Averaged over subjects, the distribution of responses in each choice approximated matching. The four individual alternatives were then presented, two by two, in two pairwise choices for which there had been no initial practice. No further reinforcement was given during the tests with the new pairs. Transfer to the two test pairs deviated systematically from matching in most cases by exaggerating the preference for the alternative that had had the higher frequency of reinforcement.

*Key words:* matching law, choice, transitivity of preference, reinforcement, pigeons

Given a set of response alternatives, differing in reinforcement but equivalent in form, choices may be expected to obey the "matching" relation, according to which responses are distributed in the same proportion as reinforcements (Herrnstein, 1970). The purpose of the present experiment is to specify more precisely than before how reinforcement frequency makes contact with a subject. From past work, we can say that responding is proportional to relative frequency of reinforcement. But we do not know whether this states a basic psychological law or is derived from other processes. For example, it is possible that the absolute rates of reinforcement connected with each response must first be discriminated before the relative frequencies can take control of responding. Or it is possible that no response process can be shown to be a function of the absolute frequency of reinforcement, but only functions of relative frequency. Information about absolute frequencies of reinforcement may or may not be retained and transferable as the relative frequencies change. Our approach is to examine how preference transfers when the reinforcement for given alternatives is held constant but the context of choice is changed.

Imagine a pigeon that has practice making choices in a situation composed of more than

two response alternatives, but always between pairs. Imagine also that the reinforcement associated with each response alternative is fixed for each alternative, but different among the alternatives. Suppose, for example, that there are four alternatives reinforced on VI 1-min, VI 2-min, VI 3-min, and VI 4-min, respectively. The pigeon has ample practice with each alternative but has not chosen between all possible pairings. Upon what factors in its prior experience will the pigeon base its choice when first confronted with a choice between two alternatives that it has often encountered before, but never as a pair? From such transfer tests, it might be possible to construct a more valid picture of what goes on to produce matching in asymptotic choice.

### METHOD

#### *Subjects*

Four male homing pigeons, between 1- and 2-yr old, experimentally inexperienced at the start of the experiment, were kept at about 80% of free-feeding weights for the duration of the study.

#### *Apparatus*

A pigeon chamber was equipped with four conventional response keys, each requiring a force of about 0.14 N for operation. The four keys were lined up across one wall of the chamber, 24 cm above a wire-mesh floor and spaced 6.4 cm apart, measuring on centers (see Figure 1). The opening to the feeder, a

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5-cm square, was centered below the keys, 5 cm above the floor. The keys were transilluminated by Christmas tree bulbs, with colors as noted in Figure 1. Each peck on the keys activated a heavy-duty relay in the experimental chamber, giving the subjects auditory feedback for their responses. Outside noises were masked by continuous white noise. The chamber illumination came only from the keylights and the feeder light (two 7-W white bulbs turned on during feedings).

Recording and programming were automatic.

### Procedure

*Training procedures.* The first training procedure started after the pigeons were taught to peck the keys, eat from the feeder during its 2-sec operation, and given preliminary exposure to the four variable-interval schedules (see Figure 1). For about 20 sessions, only one key was operative per session, with a different key selected every day. A key was functional when, and only when, it was illuminated. The schedules of reinforcement were always as shown in Figure 1. Then, for another 25 sessions, each pigeon was exposed to a given pair of keys on concurrent VI VI with a 2-sec COD (explained below). A given pair was used for about six consecutive sessions and then another

pair was used. The four pairs used in this phase of training were the "training pairs", as defined below. Following this, the final training procedure began.

In the final training procedure, keys were illuminated in pairs, and a key's schedule ran only when it was illuminated. As shown in Figure 1, each key scheduled a different variable interval: going from left to right, the schedules were, nominally, VI 1, VI 4, VI 2, and VI 3 (all schedule values in minutes). The final training procedure was thus a series of concurrent VI VI schedules. The changeover delay of 2 sec (COD 2-sec) set the minimum time between a shift from one key to another and a reinforced peck at the new location. Of the six possible concurrent pairs (AB, AC, AD, BC, BD, CD), only four were used in training (AB, AC, BD, CD, using the nomenclature of Figure 1). Note that these four pairs contain each key exactly twice, and that the remaining two pairs (AD, BC) contain all four exactly once. A day's session presented successive 8-min exposures to the training pairs, until ending at the one hundredth reinforcement. Pairs came in irregular order, with the provision that a pair never succeeded itself and that pairs occurred equally often in the long run. The final training procedure lasted 130 sessions and was immediately followed by the transfer test.

*Transfer test.* The main change was the addition of the two remaining key combinations (AD and BC) to the series. The test pairs were presented for 30 sec at a time, instead of 8 min, and responding was never reinforced in their presence. There were seven such tests per session, in an irregular order, but with at least three of each pair. The tests took place between the end of one 8-min training period and the beginning of another. There was no consistent relation between the keys involved in a given test and the training periods that bracketed it. During the test procedure, the schedules of reinforcement and the COD were unchanged for the regular training pairs. Twenty sessions of tests terminated the experiment.

### RESULTS

Figure 2 shows the distribution of responses during the final training procedure, before transfer tests were begun. The ratio of left to right responses is plotted as a function of the

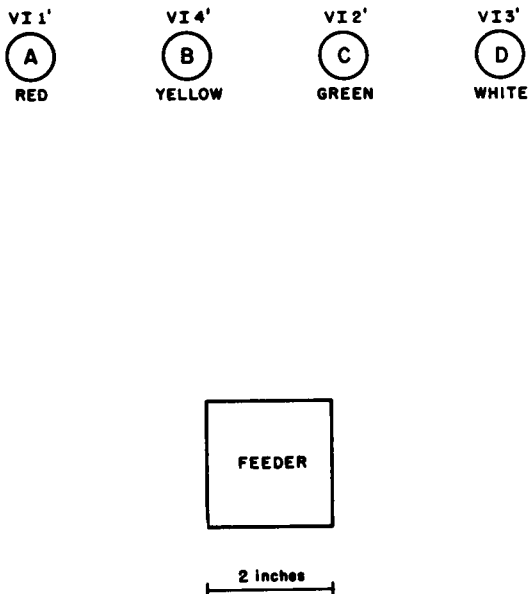


Fig. 1. Interior view of the front wall of the experimental chamber, showing the four response keys and the opening to the feeder.

ratio of left to right reinforcements for each of the four training pairs (*i.e.*, AB, AC, BD, and CD) during the final 10 training sessions. The axes are logarithmic, to show the extent and type of deviation from matching (Baum, 1974). Individual and average data are shown, along with the locus of perfect matching. The averages give the arithmetic means of the ratios. Although the average function closely approximates matching, individual data points deviate quite widely. Two subjects (58, 61) show overmatching (Baum, 1974), but the effect is washed out in the average by the results from the remaining two subjects.

Figure 2 is based on the numerical data in Tables 1 and 2. Table 1 gives absolute rein-

forcement rates during training pairs for the 10 sessions plotted in Figure 2. For each pigeon, the average reinforcement rate in each pairwise choice is given in reinforcements per minute. In most cases, the agreement across conditions and pigeons is close. For example, of the eight values for key A, six fall within the range from 0.93 to 0.97 reinforcements per minute. At the other end of the range of reinforcement frequencies is key B, which spanned the proportionally much larger distance from 0.15 to 0.28 reinforcements per minute. However, this spread is somewhat mitigated by the tallies for individual pigeons: #57 had 0.21 and 0.22 reinforcements per minute for B in the two combinations con-

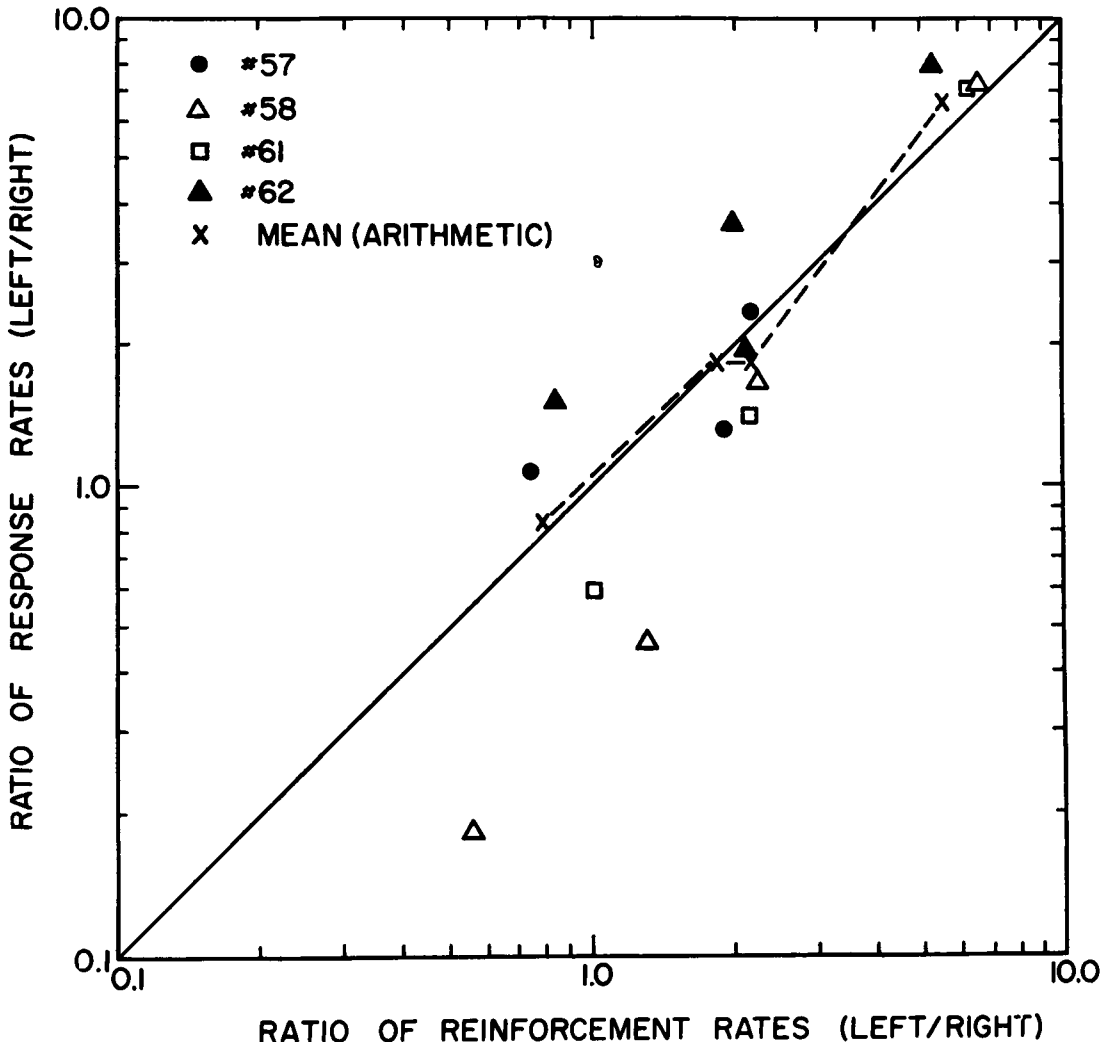


Fig. 2. Ratio of left responses over right responses as a function of the ratio of left reinforcements over right reinforcements for the four training pairs. Individual and average data are given.

Table 1  
Reinforcements per Minute

Subject	Training Pairs							
	AC		AB		BD		CD	
	A (VI 1) <sup>a</sup>	C (VI 2)	A (VI 1)	B (VI 4)	B (VI 4)	D (VI 3)	C (VI 2)	D (VI 3)
57	0.94	0.44	0.97	0.21	0.22	0.30	0.49	0.26
58	0.89	0.40	0.97	0.15	0.15	0.27	0.36	0.28
61	0.93	0.44	0.99	0.16	0.28	0.28	0.48	0.23
62	0.93	0.44	0.93	0.18	0.24	0.29	0.47	0.24
Mean	0.92	0.43	0.97	0.18	0.22	0.29	0.45	0.25

<sup>a</sup>Nominal values in minutes.

taining B; #58 had 0.15 and 0.15; #61 had 0.16 and 0.28 (the worst discrepancy in the table, both relatively and absolutely); and #62 had 0.18 and 0.24.

Figure 2 presented the ratios of responding for the four training pairs. Table 2 shows the corresponding data for absolute rates of responding, for individual subjects and averaged for the group. Ratios based on the averages would not exactly duplicate the averages shown in Figure 2 because of the difference between a ratio of averages and an average of ratios. Nevertheless, Table 2 shows the same general trends as do the points in Figure 2. The rate of responding to any given key is generally a direct function of its own rate of reinforcement, and an inverse function of the rate of reinforcement on the key with which it is being paired. This aspect of the data receives further attention in the Discussion.

The results of the transfer tests are summarized in Figures 3 and 4. In Figure 3, responding to the two test pairs, AD and BC, are shown as relative rates of responding to the left member of each pair (*i.e.*,  $A/A + D$  and  $B/B + C$ ), on the first, fifth, tenth, fifteenth,

and twentieth session of tests, plus that for the very first 30-sec test on the first session. The tests were conducted for 20 sessions, each with a total of seven 30-sec samplings of the two test pairs. The horizontal line designated "reinforcement prediction" shows the extrapolated relative frequencies of reinforcement for A in the AD pair and B in the BC, had the transfer tests not been in extinction. To obtain these proportions, we averaged the actual reinforcement rates on the four keys, as given in Table 1. The other horizontal lines are explained in the Discussion.

If matching had transferred, the points in Figure 3 would have fallen on the reinforcement prediction lines. Clearly, they did not, although several of the points for both AD and BC were in the vicinity. Most of the points for AD deviated sharply toward greater preference than matching would imply. For BC, the wide variability prevents any obvious conclusion except that the preference for B was virtually always lower than that for A. There was no clear trend in the relative frequencies during these 20 sessions of tests, in spite of the use of extinction. In fact, the very first 30-sec

Table 2  
Responses per Minute

Subject	Training Pairs							
	AC		AB		BD		CD	
	A (VI 1) <sup>a</sup>	C (VI 2)	A (VI 1)	B (VI 4)	B (VI 4)	D (VI 3)	C (VI 2)	D (VI 3)
57	88.05	38.28	78.32	17.04	45.30	42.84	61.62	48.00
58	21.71	13.02	28.71	3.95	3.85	20.91	10.37	22.29
61	27.23	19.47	40.64	5.81	13.89	23.26	38.36	20.23
62	46.39	23.78	54.10	6.68	34.23	22.37	52.81	14.36
Mean	45.85	23.64	50.44	8.37	24.32	27.35	40.79	26.22

<sup>a</sup>Nominal values in minutes. See Table 1 for actual values.

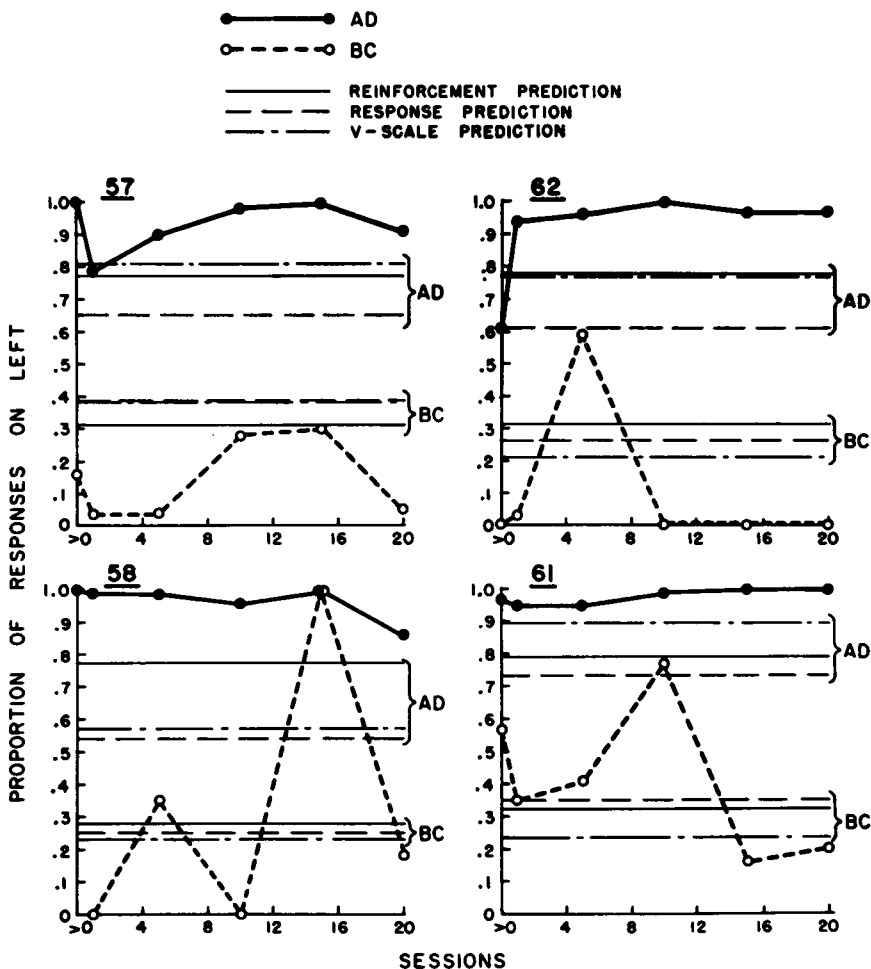


Fig. 3. Proportion of responses to the left key in the two transfer tests (i.e., A of AD and B of BC) for the very first test (at > 0) and for five full sessions of tests. The horizontal lines are explained in the text.

test in the first session showed the same tendency toward exaggerated preference for AD.

Since the test pairs were on extinction, there is probably a practical limit to the number of meaningful replications. Continued probings with the test pairs confound the effects of extinction with whatever else governs responding at first. Consequently, it is the responding in the earlier tests that should be given greatest weight, except for the problem of sample size. The more narrowly we are confined to the beginning of the test phase, the smaller the sample of behavior and the greater the relative error. At least for AD, the dilemma is more hypothetical than real, for Figure 3 leaves little doubt that A was highly preferred over D, nearly exclusively in most instances, and throughout the 20 sessions of tests. The vari-

ability in the data for BC does, however, raise some question of interpretation. Although it is clear that C is generally preferred over B, it is hard to assign a magnitude to the preference, beyond saying that it is at least as great as that implied by transfer of matching, and on occasion greater. It should be noted that the variability for BC is not necessarily out of line with usual findings in concurrent procedures. The data points in Figure 3 plot either 30-sec or 1.5- to 2-min samples of behavior for individual subjects, considerably smaller than commonly used. Figure 4 averages over Sessions 1, 5, 10, 15, and 20 of the test phase for each subject in order to increase the stability of the measure of preference. The abscissa is the reinforcement prediction for each subject, based on the observed rates of

reinforcement during the training phase. The ordinate is the corresponding relative frequency of responding in the tests, to A (upper points) relative to A + D, and to B (lower points) relative to B + C. The diagonal is the predicted distribution of responding in the tests if it had matched the distribution of reinforcements in training. The step function shows a discontinuous change in preference where the relative frequency of reinforcement in training shifts in favor of one alternative or the other.

Figure 5 shows the absolute rates of responding to each key during only test pairings for test Sessions 1, 5, 10, 15, and 20. Data from the four subjects have been averaged. Some effect of extinction seems evident in the falling rates on C and D, but is questionable for the non-monotonic functions describing A and B.

The testing procedure presupposes that a 30-sec sample of responding is directly comparable to results obtained during training, when pairs were presented for 8-min periods. To check this assumption, responses to the training pairs were tallied for the first 30 sec and compared with those for the entire 8 min. It was found that responding during the first 30 sec does not differ systematically from responding in general and the two are therefore comparable. As expected, however, there is greater variability for the shorter sample.

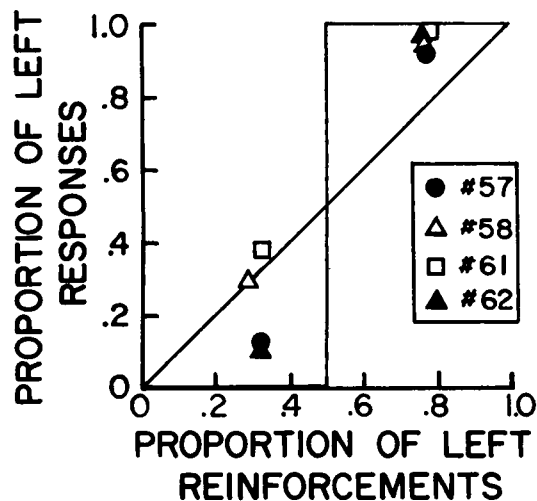


Fig. 4. Proportion of left responses during transfer tests by individual subjects averaged over the first, fifth, tenth, fifteenth, and twentieth transfer test sessions. The x-axis is the proportion of left reinforcements obtained during training.

## DISCUSSION

The final training procedure is of interest in its own right. Although Reynolds (1963) and Miller and Loveland (1974) have shown approximate matching in pigeons choosing among, respectively, three and five alternatives, the present experiment seems to be the first published demonstration of matching for pairwise samples from a larger network of alternatives. The matching in Figure 2 is more variable than typical two-choice data perhaps because a larger network of choices poses a harder discrimination problem. The results in the tests may shed some light on what could be making it harder, as noted below.

The analysis of absolute rates of responding under the final training procedure is summarized in Figure 6. We have used Herrnstein's (1970) general equation for reinforced responding:

$$P_1 = \frac{kR_1}{\Sigma R_1} \quad (1)$$

Applied to the present experiment,  $P_1$  is the rate of pecking at a key;  $R_1$  is the associated rate of reinforcement;  $k$  is a parameter extracted from the data, and  $\Sigma R_1$  represents the total rate of reinforcement. At any given moment, the denominator for the present procedure comprises three terms of which two are known independent variables. Since the procedure consists of a series of concurrent schedules, the denominator reduces to:

$$\Sigma R_1 = R_1 + R_2 + R_3 \quad (2)$$

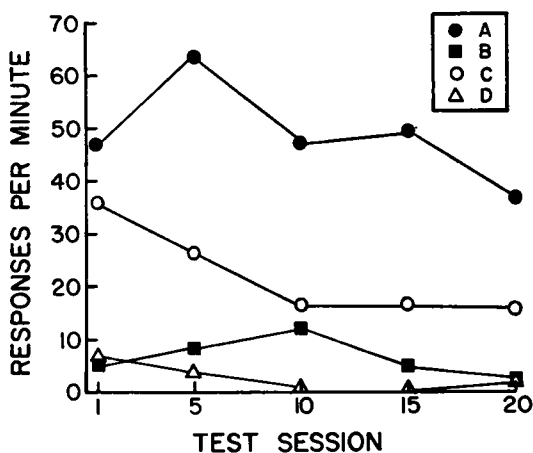


Fig. 5. Rate of responding to each key during tests for sample test sessions. Data are averaged over subjects.

$R_1$  is the rate of reinforcement associated with  $P_1$ ;  $R_2$  is the rate of reinforcement associated with the other key; and  $R_e$  is the residual, unscheduled reinforcement, in effect a second parameter of equation 1, to be estimated from the obtained data.

We treat each pairing of keys in the final training procedure as a distinct and free-standing concurrent schedule. That is to say, equation 1 is applied separately to the rate of pecking at each key in the following pairings (see Figure 1): AB, AC, BD, and CD. There are therefore two separate predictions for rate of responding for each key, depending on what other key it is being paired with. This analysis ignores sequential interactions, even though the procedure is actually a multiple schedule of the general type that produces such interactions (see Herrnstein, 1970; Herrn-

stein and Loveland, 1974). However, with 8-min components following each other randomly, the error caused by this omission is likely to be negligible.

Figure 6 shows the outcome of this analysis applied to the same data that were used in Figure 2: 10 sessions averaged over the four subjects. The ordinate is the rate of pecking at each key; the abscissa is the value of the ratio  $R_1/\sum R_i$ , using the obtained rates of reinforcement given in Table 1. We fitted single values of  $k$  and  $R_e$  to the entire function (*i.e.*, for training pairs only): 85 responses per minute and 0.35 reinforcements per minute, respectively. The value of  $k$  is in the familiar range for pigeons under comparable conditions, but  $R_e$  may seem a bit large (correcting for units, see de Villiers and Herrnstein, *in press*). However, theory predicts a relatively

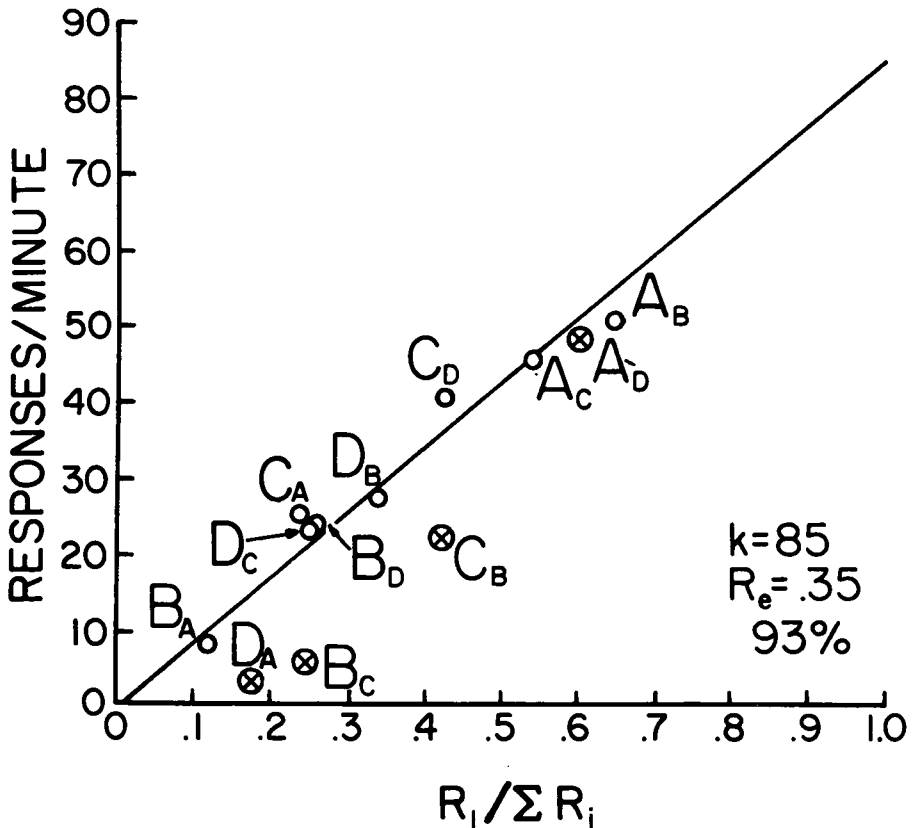


Fig. 6. Open circles give rates of responding to individual keys during the final training procedure. The larger letter near each data point identifies the key whose rate is plotted; the smaller letter, the key with which it was being paired at that time. Parameters for equation 1, and data variance accounted for, are shown. The diagonal line plots equation 1 for the given parameters, and the x-axis gives the relative frequency of reinforcement as defined in the text. Circles containing x's show the average rates of responding during tests.

large value for  $R_e$ , since the feeder cycle was only 2 sec, rather than the more common 3 to 5 sec (see Herrnstein, 1974, for discussion).

The diagonal line in Figure 6 plots equation 1, given the parameters. The letters identify the key whose rate is plotted, as well as the key opposing it at that time. The data suggest that the main controlling factor for rate of pecking is the relative, not the absolute, rate of reinforcement, as implied by equation 1. Thus, pecking to keys B, C, and D is at various times almost the same, even though they were on nominal VI 4, VI 2, and VI 3 (see Table 1 for actual values), but only when the relative rates of reinforcement were nearly the same. Moreover, the rate of pecking to B increased almost threefold, even though the absolute rate of reinforcement remained essentially constant (see Table 1) when the other key changed from VI 1 to VI 3. There is, in short, only about 7% of the variance in the average training data not accounted for by equation 1. Of course, Figure 6 is for averaged data and the individual functions (available from Tables 1 and 2) would have shown considerably more variability unaccounted for by equation 1.

The results of the transfer test suggest that when matching does occur, as it did on the average during training (see Figure 2), it must to some degree depend on the ongoing interaction of responding and reinforcement. This may explain why the individual functions in training (see Figure 2) scattered so widely around the matching line, though averaging quite close to it. Four pairwise choices among four alternatives may require more complex, or, at least, different interactions than a straight choice among four alternatives. Before this interaction can push responding toward matching, the responding tends to overmatch, as shown for relative responding in Figures 3 and 4. The circled crosses in Figure 6 show absolute rates of responding for the tests, averaging over subjects and over Sessions 1, 5, 10, 15, and 20 during the test phase. These points would have approximated the theoretical curve, given both matching and transfer. Except for pecking at A during test AD, the rates of responding to the tests are depressed below the predicted level. The pattern of absolute rates of responding is necessarily mirrored in the relative measures of transfer presented in Figures 3 and 4. It also includes whatever effects of extinction there may have

been, as presented in Figure 5. The exaggerated preferences in the transfer tests may be related to the accentuation during extinction of rate differences in a multiple VI 3 VI 9 (Gollub and Urban, 1958). However, Figure 3 shows sharpened preferences in unreinforced transfer tests before extinction had produced measurable reductions in response rate.

The subjects seem to have learned something about the four alternatives as individual occasions for responding, for the test choices are far from random, even though they were being paired only in extinction. The rates of responding are a monotonically increasing function of the abscissa values in Figure 6. But they also come close to being a monotonic function of the absolute rates of reinforcement in training, which were 0.943, 0.440, 0.269, and 0.261 reinforcements per minute for A, C, D, and B, respectively. The minor reversal between responding to D and B is self-evidently insignificant statistically (see Tables 1 and 2 for the range of rates of responding under given rates of reinforcement).

The horizontal lines in Figure 3 explore several obvious quantitative bases for transfer in the tests. The "reinforcement prediction", which assumes that the subject obeys the matching law without prior exposure to the context of reinforcement, has already been discounted. The "v-scale prediction" derives from Luce (1959). According to Luce, the ratios of relative frequencies of response define a scale identified as the "v-scale". For example, the following relationship can be shown to follow from the presuppositions of his model:

$$\frac{v(A)}{v(C)} = \frac{\frac{a}{a+c}}{\frac{c}{a+c}} = \frac{a}{c} \quad (3)$$

where the lower-case letters refer to the simple rates of responding to the alternatives identified by the upper-case letters. The v-scale is said to define a characteristic of response strength such that it should predict the distribution of responses in any pair of alternatives. To predict choices in the test pairs, the model would assume that the v-scale values for AD will follow from those for AC and CD, or, equivalently, from AB and BD. Those for BC would depend on BA and AC or BD and DC. The actual computation of the v-scale predictions uses the relative rates of responding to



the relevant alternatives during the training, as follows:

$$\frac{vA}{vD} = \frac{vA}{vC} \cdot \frac{vC}{vD} = \frac{\frac{a}{a+c}}{\frac{c}{a+c}} \cdot \frac{\frac{c}{c+d}}{\frac{d}{c+d}} = \frac{a}{d};$$

or

$$\frac{vA}{vB} \cdot \frac{vB}{vD} = \frac{\frac{a}{a+b}}{\frac{b}{a+b}} \cdot \frac{\frac{b}{b+d}}{\frac{d}{b+d}} = \frac{a}{d}$$

(4)

$$\frac{vB}{vC} = \frac{vB}{vA} \cdot \frac{vA}{vC} = \frac{\frac{b}{a+b}}{\frac{a}{a+b}} \cdot \frac{\frac{a}{a+c}}{\frac{c}{a+c}} = \frac{b}{c};$$

or

$$\frac{vB}{vD} \cdot \frac{vD}{vC} = \frac{\frac{b}{b+d}}{\frac{b}{b+d}} \cdot \frac{\frac{d}{d+c}}{\frac{c}{d+c}} = \frac{b}{c}$$

Each test pair has two predictions based on two combinations of training pairs, as shown. The imperfect internal consistency of responding assures that the two estimates will be at least somewhat different.

These quantities have been calculated for each subject and are shown as the "v-scale prediction" in Figure 3. The two ways of deriving each prediction (see equation 4) have been averaged in Figure 3, but the individual predictions would have done about as poorly.

The final horizontal line—"response prediction"—uses the average rates of responding during training to predict relative frequencies of responding for AD and BC. For example, for #57, the average rate of responding to A during training was 83.19 responses per minute (see Table 2); for D it was 45.42 responses per minute. Accordingly, the "response prediction" for A in test pair AD is  $83.19/(83.19 + 45.42)$  or, 0.65, which is the value plotted in Figure 3. The other predictions were similarly calculated. In general, the response predictions are the worst of the three, since they predicted smaller-than-matching preferences, while the actual tests showed greater-than-matching preferences in most cases.

The failure of the v-scale may seem to contradict Greeno (1968, pp. 46-54), who presented evidence in favor of Luce's choice axiom from transfer experiments on animals. He

showed that Young's (1947) data on food preferences are explained by Luce's value (v) scale. Rats were given pair-wise tests with sucrose *versus* wheat powder and with sucrose *versus* casein. The preferences were noted. Then, the rats were tested with wheat powder *versus* casein. The v-scale predictions were reasonably well confirmed. Similarly, Greeno showed that one of the monkeys in Premack (1963) was also transferring according to the v-scale predictions. The monkey was first given access, one by one, to three playthings—a handle, a door, and a plunger. Time spent with each was noted and then used to predict pair-wise choices. Again, the v-scale predictions were closely approximated. Meyer, LoPopolo, and Singh's (1966) results with monkeys were comparable. But none of these tests for transfer were in extinction. They were done with reinforcement continuing and interacting with behavior. Luce's theory says that preferences are transitive, and is usually tested and confirmed in situations that rearrange contexts of choice while continuing to reinforce the alternatives being presented. The transitivity of choice in such experiments cannot be attributed to values established in an earlier context and then transferred to the new pairs. For that purpose, extinction tests seem appropriate.

One experiment that did test for transfer in extinction was Estes's (1966) on college students picking between pairs of nonsense syllables with which arbitrary numbers of points had been associated. The subjects were instructed to pick the nonsense syllable worth more points in each pair. As in the present experiment, all the alternatives were first made familiar and then tested in novel choices in extinction. Like our pigeons, Estes's human subjects managed, on the average, to pick the "better" alternatives in the transfer tests. However, since his training pairs gave almost exclusive preference, it is not possible to say whether he would have found a departure from transitivity toward exaggerated preference, as we did in six of eight cases (see Figure 4), if there had been room for it.

In a review of data on human subjects in noncontingent, probability-learning procedures, Estes (1976) concluded that subjects transfer past experience with given events to novel pairings on the basis of a scale expressing the relative frequencies of the events, but only after selective attention is taken account

of. While there may be some question whether we should expect pigeons in concurrent schedules to behave like humans in outcome-prediction experiments, it seems notable that in a large number of transfer tests, Estes found no indication of the kind of choice enhancement displayed here by all subjects for AD and two of the four subjects for BC. Since his data show nothing like it, it is not surprising that Estes's model neither predicts nor accounts for the choice enhancement observed here.

Herrnstein and Loveland (1975) have shown that on concurrent ratio schedules, pigeons tend to maximize reinforcement frequency by choosing exclusively the alternative with the higher reinforcement probability. They further showed that this finding is not only consistent with the matching principle but is the only outcome consistent with it, given the properties of ratio schedules. The tendency toward exclusiveness shown in the present transfer test may therefore indicate that novel pairings within a network are treated as if they were on ratio schedules until proven otherwise by an ongoing interaction between reinforcement and responding. Since the tests were in extinction, the interaction fails to produce the patterns of responding observed with concurrent interval schedules.

In Figure 7 are shown the relative frequencies of A in AD and B in BC, the same transfer data already plotted in Figure 4. However, the x-axis has been redefined as might be appropriate for the foregoing hypothesis. The obtained probabilities of reinforcement for A, B, C, and D were calculated from the training phase by dividing numbers of reinforcements by numbers of pecks for each key (taking subjects individually). These values were then expressed relatively: for the AD test, the relevant abscissa value is:

$$\frac{p_r(A)}{p_r(A) + p_r(D)}; \quad (5)$$

for the BC test, it is:

$$\frac{p_r(B)}{p_r(B) + p_r(C)}.$$

These relative probabilities of reinforcement are conceptually analogous to the relative probability values examined in Herrnstein and Loveland (1975), except that they are *independent* variables with ratio schedules and *dependent* variables with interval schedules. With

the probabilities as independent variables, the distribution of responding approximated the step function in Figure 7, rather than the diagonal tracing probability matching (Herrnstein and Loveland, 1975). With the probabilities as dependent variables, as in the present case, the points for individual subjects in Figure 7 also approximate the step function, although the data are insufficient to be considered definitive.

It may be noteworthy that AD showed less variability than BC, since BC falls near the region of the step function where variability can be expected to be maximal. Moreover, the two subjects with the higher relative probabilities for B showed the higher rates on B. In these respects, Figure 7 supports the hypothesis. On the other hand, the hypothesis would be disconfirmed if abscissa values above 0.5 reliably yielded preferences below 0.5 (as is the case for #58 and #61).

There are further constraints on the hypothesis arising in the relationship between the extent of matching in the original training and the variable plotted along the x-axis in Figure 7, the relative probability of reinforcement. Equation 1 can be rearranged to give the probability of reinforcement for a response as follows, taking key A as an example:

$$p_r(A) = \frac{R_A}{P_A} = \frac{\sum R_i}{k} \quad (6).$$

Insofar as matching holds, probabilities of re-

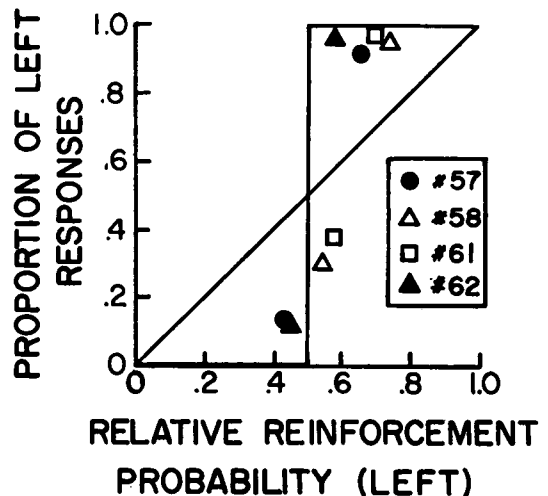


Fig. 7. The ordinate of Figure 4 plotted against relative probability of reinforcement, as explained in text.

inforcement for the alternatives in each of the training pairs thus varies directly with the rate of reinforcement from any source. For a pair of concurrent alternatives, e.g., AC, matching implies,

$$p_r(A) = p_r(C) = \frac{R_A}{P_A} = \frac{R_C}{P_C} = \frac{\Sigma R_i}{k} \quad (7)$$

Since  $\Sigma R_i$  varies with the context of reinforcement, the probabilities of reinforcement will equalize around different values for different training pairs, given matching. The data in Figure 7 suggest that this aspect of reinforcement may be relevant to transfer, but it would take substantially broader sampling of contexts and transfer tests to assess the hypothesis adequately. Given the present results, we conclude only that matching requires an ongoing interaction with the conditions of reinforcement and that what is learned about individual alternatives bears an as-yet-unspecified relationship to frequency or probability of reinforcement.

#### REFERENCES

- Baum, W. M. On two types of deviation from the matching law: bias and undermatching. *Journal of the Experimental Analysis of Behavior*, 1974, 22, 231-242.
- de Villiers, P. A. and Herrnstein, R. J. Toward a law of response strength. *Psychological Bulletin*, (in press).
- Estes, W. K. Transfer of verbal discriminations based on differential reward magnitudes. *Journal of Experimental Psychology*, 1966, 72, 276-283.
- Estes, W. K. The cognitive side of probability learning. *Psychological Review*, 1976, 83, 37-64.
- Gollub, L. R. and Urban, J. T. The accentuation of a rate difference during extinction. *Journal of the Experimental Analysis of Behavior*, 1958, 1, 365-369.
- Greeno, J. *Elementary theoretical psychology*. Reading, Massachusetts: Addison-Wesley, 1968.
- Herrnstein, R. J. On the law of effect. *Journal of the Experimental Analysis of Behavior*, 1970, 13, 243-266.
- Herrnstein, R. J. Formal properties of the matching law. *Journal of the Experimental Analysis of Behavior*, 1974, 21, 159-164.
- Herrnstein, R. J. and Loveland, D. H. Hunger and contrast in a multiple schedule. *Journal of the Experimental Analysis of Behavior*, 1974, 21, 511-517.
- Herrnstein, R. J. and Loveland, D. H. Maximizing and matching on concurrent ratio schedules. *Journal of the Experimental Analysis of Behavior*, 1975, 24, 107-116.
- Luce, R. D. *Individual choice behavior: a theoretical analysis*. New York: Wiley, 1959.
- Meyer, D. R., LoPopolo, M. H., and Singh, D. Learning and transfer in the monkey as a function of differential levels of incentive. *Journal of Experimental Psychology*, 1966, 72, 284-286.
- Miller, H. L., Jr. and Loveland, D. H. Matching when the number of response alternatives is large. *Animal Learning and Behavior*, 1974, 2, 106-110.
- Premack, D. Rate differential reinforcement in monkey manipulation. *Journal of the Experimental Analysis of Behavior*, 1963, 6, 81-89.
- Reynolds, G. S. On some determinants of choice in pigeons. *Journal of the Experimental Analysis of Behavior*, 1963, 6, 53-59.
- Young, P. T. Studies of food preference, appetite, and dietary habit: VII. Palatability in relation to learning and performance. *Journal of Comparative and Physiological Psychology*, 1947, 40, 37-72.

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