

MATCHING SINCE BAUM (1979)

J. H. WEARDEN AND I. S. BURGESS

MANCHESTER UNIVERSITY AND MANCHESTER POLYTECHNIC

Data from recent studies employing concurrent variable-interval schedules are reviewed. Subject species employed in different experiments have included rats, pigeons, and humans, and reinforcers have varied from food and shock avoidance to points exchangeable for money. Undermatching (a greater preference for the schedule of the concurrent pair that delivers the lower rate of reinforcement than the Matching Law predicts) has been preponderant in recent studies, irrespective of whether behavior has been measured in terms of response ratios or time allocation, with the possible exception of data produced by human subjects. Little difference in the degree of undermatching exhibited by response and time measures has been found, except in the results from a single laboratory, in which time-allocation measures have tended to undermatch less than response measures. Procedural features, such as type of manipulandum used and changeover delay, seem to have little effect on the degree of undermatching exhibited, but asymmetrical response manipulanda (such as lever and key) for the different concurrent schedules, or other asymmetries in the experimental situation, show up clearly in bias measures, in a manner consistent with previous analyses.

Key words: matching, undermatching, procedural variables, choice

Much recent work in the experimental analysis of behavior has been focused upon quantitative description of performance (see Bradshaw, Szabadi, & Lowe [1981] for examples). Particular impetus to this approach was given by the discovery in the early 1960's that, when exposed to concurrent variable-interval (VI) schedules of reinforcement with food, organisms distributed their responses between the two concurrent-response alternatives according to a simple rule, now known as the Matching Law (Herrnstein, 1961; see also Catania, 1963, 1966). This states that

$$\frac{R_1}{R_2} = \frac{r_1}{r_2}, \quad (1)$$

where R_1 and R_2 are the numbers of responses on the two alternatives and r_1 and r_2 the rates of reinforcement the concurrent schedules deliver. In cases where the amount of time spent under each of the VI alternatives can be measured (e.g., Baum & Rachlin, 1969), an analogous relation for time matching is

$$\frac{T_1}{T_2} = \frac{r_1}{r_2}, \quad (2)$$

where T_1 and T_2 are the lengths of time spent in responding on alternatives 1 and 2.

The relation between behavior distribution and reinforcement ratio embodied in Equations 1 and 2 has attracted interest not only because of its simplicity, but also because it may be consistent with some type of optimal response strategy, such as the maximizing of momentary (Shimp, 1969; Staddon, Hinson, & Kram, 1981) or overall (Houston & McNamara, 1981; Staddon & Motheral, 1978) rate or probability of reinforcement. The relation between the Matching Law and an organism's foraging behavior in the wild has been another area of interest (see Lea [1981] and Staddon [1980] for discussions).

Reviews of results from studies employing concurrent VI schedules (Baum, 1979; Myers & Myers, 1977) have suggested, however, that the simple Matching Law may not always be adequate, and that it should be replaced by the more complex Equations 3 and 4 below:

$$\frac{R_1}{R_2} = k_1 \left(\frac{r_1}{r_2} \right)^{a_1} \quad (3)$$

$$\frac{T_1}{T_2} = k_2 \left(\frac{r_1}{r_2} \right)^{a_2}. \quad (4)$$

Here, terms are as in Equations 1 and 2 except for two exponents (a_1 and a_2) and two

Reprints may be obtained from J. H. Wearden, Department of Psychology, The University, Manchester M13 9PL United Kingdom.

scalar multipliers (k_1 and k_2). In an influential article, Baum (1974) proposed that the scalar multipliers represented unknown sources of variation in the experimental situation, describable as a bias towards one response alternative rather than another regardless of the rate of reinforcement it delivered. Among possible sources of bias cited by Baum were different amounts of the reinforcer for responding on the different alternatives and asymmetric preference for either the position or color of the response manipulanda. The exponents were said to represent undermatching if they were less than 1.0, or overmatching if greater than 1.0. Undermatching is the tendency to respond more (or to spend more time) on the alternative delivering the lower rate of reinforcement than the Matching Law would predict, whereas overmatching, the opposite tendency, is an exaggerated preference for higher rates of reinforcement. Bias and undermatching (or overmatching) can be easily measured by rewriting Equations 3 and 4 as

$$\log \left(\frac{R_1}{R_2} \right) = a_1 \log \left(\frac{r_1}{r_2} \right) + \log k_1 \quad (5)$$

$$\log \left(\frac{T_1}{T_2} \right) = a_2 \log \left(\frac{r_1}{r_2} \right) + \log k_2. \quad (6)$$

These equations yield straight lines with slope a_1 and intercept $\log k_1$ or slope a_2 and intercept $\log k_2$, when log response ratios (or time ratios) are plotted against log reinforcement ratios. For simplicity below, a_1 will be referred to as the response slope, a_2 as the time slope.

Data from concurrent schedules have previously been reviewed by Catania (1966), by de Villiers (1977), and most recently by Baum (1979). One aim of the present article is to provide an account of the results of more recent experiments, since more than 120 sets of data from experiments using concurrent VI schedules or similar procedures have been published since 1976, when Baum (1979) finished collecting data for his article. A second aim is to use recent data to substantiate or, where necessary, to reevaluate the conclusions drawn by previous commentators. Of particular interest here is the possibility, discussed below, that response and time slopes might differ systematically, and the implication of Baum's (1974) dis-

cussion of bias that certain types of procedure might reliably generate biases towards one of the concurrent alternatives.

RESPONSE AND TIME SLOPES

The examination of the results of earlier workers by Myers and Myers (1977) was perhaps the first clear indication that undermatching might be the normal result of experiments employing concurrent VI schedules. Baum (1979) generally concurred with this conclusion and reported that undermatching was more marked when behavior was measured in terms of numbers of responses than time allocation (but see Mullins, Agunwamba, & Donohoe, 1982). For example, Baum (1979) reported that data in his survey (109 sets) produced a median response slope of .83 (the mode was even lower, at .79). Time slopes also systematically undermatched (median .92), although they showed less undermatching than response slopes (having, for example, a modal value of 1.0).

Such a result might have various interpretations. Suppose, for example, that response slopes tend to have lower values than time slopes, as Baum (1979) reported. There are two possibilities that should be clearly distinguished. The first is that matching in terms of time allocation might reflect more accurately underlying conformity to the type of processes that might generate matching than do response measures, which are possibly contaminated by fluctuations in local rates of responding (de Villiers, 1977). A time-allocation measure might therefore provide a more appropriate measure of an organism's adjustment to concurrent VI contingencies than response distributions and might be a more apt subject for the construction of theoretical models (e.g., Houston & McNamara, 1981). On the other hand, time slopes may be closer to matching in Baum's survey simply because they were generally higher than response slopes. In the case where subjects overmatched in response slopes, time slopes might deviate even further away from matching. In this case, the more accurate conformity of time slopes to the simple matching law might be partly coincidental. The data analysis presented below permits these two possibilities to be distinguished.

CAUSES OF UNDERMATCHING

Another tendency reported by Baum (1979) was for data from different laboratories to differ systematically in degree of deviation from matching. Baum reported, for example, that although his own data exhibited slopes close to matching, those from Davison's laboratory (e.g., Beutrais & Davison, 1977; Davison & Hunter, 1976; Lobb & Davison, 1975) showed consistent undermatching. Baum (1979) attributed these differences to unspecified procedural differences between laboratories, thereby implicitly characterizing undermatching as a result that can be affected substantially by procedural changes. There is, in fact, some evidence that within-subject manipulation of changeover delays (Shull & Pliskoff, 1967) or of behavior required to change over between the response alternatives (Baum, 1981) can alter the observed degree of correspondence with the matching law. If the procedural account of undermatching is generally correct, however, data from different sorts of experiments might be expected to differ substantially in degree of undermatching. If, on the other hand, undermatching results principally from some organismic process (Wearden, Note 1), it should be a more general phenomenon, occurring under a wide variety of procedures. Recent concurrent VI studies have employed a greater diversity of procedures, reinforcers, and subject types than previously, and their data might permit some evaluation of this possibility.

SELECTION OF DATA

The criteria for inclusion in the present article were (1) that the study had not previously been reviewed by Baum (1979); (2) that individual organism, rather than mean, data were provided; (3) that at least four different pairs of concurrent VI schedules had been used; and (4) that when a changeover response was required, the response requirement on the changeover manipulandum was a single response rather than a schedule. Studies that required some pattern of behavior other than a direct movement to change over between two continuously available manipulanda were also excluded (e.g., Baum, 1981).

All studies known to the present authors that met these criteria were included, with the

exception of the results of Pierce, Epling, and Greer (1981). These workers, who studied human performance in a communication task, reported negative time and response slopes from some subjects, a result very much at variance with the bulk of data from other experiments. Some experiments that included phases other than simple concurrent VI were included, (e.g., Bradshaw, Szabadi, & Bevan, 1979), but only data from the concurrent VI phase were used. A few experiments that did not, strictly, meet all of the above requirements were also included. For example, Rodewald (1978) employed concurrent random-interval schedules, and both Lobb and Davison (1977) and Leigland (1979) contrasted a VI schedule with some sort of multiple schedule. Data from these studies did not substantially alter the conclusions derived from the analysis of simple concurrent VI data.

The principal results of interest for this review were the values of the constants (a_1 , a_2 , k_1 , k_2) provided by fits of Equations 5 and 6 to data. In one case (Poling, 1978) the straight line was fitted by the present authors; in all other cases the results were calculated by the original experimenters. No account was taken of goodness of fit of Equations 5 and 6 to data, except to exclude the "shocks received" analysis of Logue and de Villiers (1978) since their "shocks avoided" analysis clearly accounted for more of the variance for both subjects used.

Following Baum (1979), data from Davison's experiments were analyzed separately from that of other workers.

About 120 fits of Equations 5 and 6 to individual subject data are available from recent studies. Values of slopes and intercepts for both response-distribution and time-allocation measures are shown in Table 1.

AGGREGATE AND DISTRIBUTIONAL MEASURES OF SLOPES

The left panels of Figure 1 show relative frequencies of response and time slopes from data other than those produced by Davison and his colleagues. Undermatching (slope less than 1.0) was predominant in both response-distribution and time-allocation measures. Mean response slope was .87, the median .86, and the modal class ranged from .9 to 1.0. Time slopes also had a mean of .87, their median was .87,

Table 1

Individual data included in the aggregate figures. Species: P = pigeon, R = rat, H = human. Slopes and intercepts come from fits of Equations 5 and 6.

Study	Subject	Species	Responses		Time	
			Slope	Intercept	Slope	Intercept
Bradshaw, Szabadi, and Bevan (1979)	BB	H	1.2	-.16	1.04	-.16
	LK	H	1.36	.45	1.10	.31
	MS	H	1.36	.24	1.11	.05
Bradshaw, Szabadi, Bevan, and Ruddle (1979)	AD	H	1.15	.16	1.17	.13
	MW	H	.90	.32	.96	.3
	SW	H	1.13	.09	1.10	.1
Davison and Ferguson (1978)	161	P	1.07	.43	1.15	.14
	162	P	1.16	-.55	1.04	-.14
	164	P	.63	-.17	.72	-.03
	165	P	.87	-.89	.93	-.44
	166	P	.64	-1.2	.52	-.86
Hunter and Davison (1978)	H1	P	.76	.02	.85	.02
	H2	P	.93	.03	.89	.01
	H3	P	.82	-.10	.92	.02
	H4	P	.93	.06	1.02	.06
	H5	P	.91	-.21	.91	.05
	H6	P	.93	.03	.79	.04
Hutton, Gardner, and Lewis (1978)	M1	P	.69	.13	.66	.27
	M3	P	.78	-.07	.85	-.06
	M4	P	.60	-.10	.56	-.41
Leigland (1979)	S253	P	.85	.02	.84	-.04
	S254	P	.91	-.06	.83	-.01
	S255	P	.83	-.02	.80	0
	S259	P	.74	0	.70	-.02
	S260	P	.82	-.09	.96	-.04
	S261	P	.84	-.04	.92	-.02
	S256	P	.74	0	.73	-.03
	S265	P	.64	-.02	.80	.02
	S258	P	.92	-.19	.91	-.04
	S262	P	.49	-.06	.68	-.02
	S266	P	.47	.08	.66	.04
	S264	P	.81	.03	.88	.01
	Lobb and Davison (1977)	21	P	.77	.20	
22		P	.76	.30		
23		P	.82	.37		
24		P	.74	.26		
25		P	.72	.19		
26		P	.84	.13		
21		P	.69	.24		
22		P	.85	.23		
23		P	.93	.35		
24		P	.79	.24		
25		P	.91	.06		
26		P	.77	.26		
Logue and de Villiers (1978)		R7	R	.92	.28	1.32
	R9	R	.82	-.23	1.22	.02
Norman and McSweeney (1978)	R1	R	.92	.09	.96	-.09
	R2	R	1.17	-.04	1.08	.10
	R3	R	.78	-.18	.72	-.13
	R4	R	.90	0	1.01	.01
	R5	R	.98	.06	1.01	.01
Poling (1978)	R7	R			.96	.20
	R8	R			.92	.47
	R9	R			.75	0

Table 1 continued

Study	Subject	Species	Responses		Time	
			Slope	Intercept	Slope	Intercept
Rodewald (1978).	7018	P	.89	-.10		
	8236	P	.74	-.16		
	7036	P	.79	.01		
	8271	P	.61	.11		
	8122	P	.70	.15		
	8062	P	1.18	-.05		
	7804	P	1.16	.07		
	7024	P	1.24	.06		
	7063	P	.52	-.13		
	7760	P	1.20	-.30		
	8306	P	.94	-.07		
	7582	P	.74	.11		
	7722	P	.93	.10		
	7635	P	.90	-.04		
	7595	P	.58	.19		
Ruddle, Bradshaw, Szabadi, and Bevan (1979)	JJ	H	1.01	.13		
	MD	H	.68	.15		
	PD	H	1.30	.37		
	LD	H	.54	.22		
	MH	H	1.26	.18		
	GB	H	1.14	.04		
Wheatley and Engberg (1978)	KT1	P	.86	.52	.79	.02
	KT2	P	.70	.47	.64	-.01
	KT3	P	.53	.50	.45	.08
	TT1	P	.27	.04	.36	.05
	TT2	P	.95	.11	.73	.05

and the modal class was again from .9 to 1.0. It did not appear, therefore, that undermatching was less marked in time-allocation measures than in response ratios.

The right-hand panels of Figure 1 show data from Davison's laboratory. Once again, undermatching in both time and response slopes was the norm. There was, however, a suggestion that response slopes tended to be lower than time slopes, as supported by summary statistics. The mean response slope was .84, the me-

dian .82, and the modal class from .7 to .8. The mean time slope was .89, median .91, and the modal class from .9 to 1.0. Thus, although Davison's data are roughly comparable to those from other laboratories in exhibiting undermatching in both response and time slopes, his data tend to differ from those of others in that response-ratio undermatching tends to be more pronounced than time-ratio undermatching.

The method of presentation of results in Figure 1 may, however, be misleading on this and other points. First, not all the included data come from studies that measured both response and time slopes. Second, even when both types of data are available, Figure 1 does not permit the response and time slopes of individual subjects to be related and does not therefore permit the assessment of differing interpretations, noted above, of differences between response and time slopes.

Figure 2 shows data from only those experiments that have taken response ratio and time allocation measures from the same subjects. Once again, data from Davison's laboratories were analyzed separately.

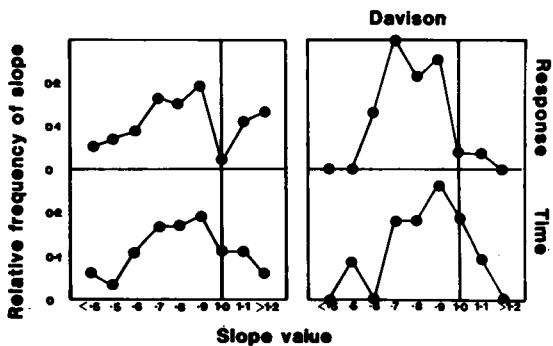


Fig. 1. Relative frequencies of slopes of response-ratio and time-allocation measures. The right-hand panels show data from Davison's laboratories; the left-hand panels show data from other studies.

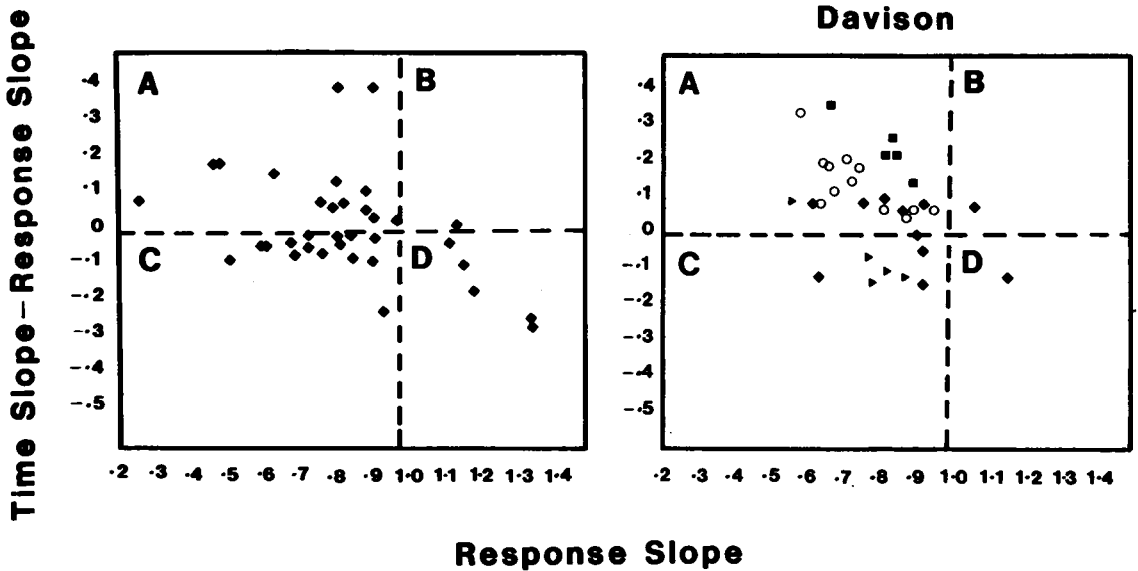


Fig. 2. Individual data points from studies in which both response and time measures were taken from individual subjects. The difference between time and response slopes is plotted against the response slope. Right-hand panels show data from Davison's laboratory. Points from studies included in Table 1 are filled diamonds; also shown are data from Beaudrais and Davison (1977) by triangles, Davison and Hunter (1976) by unfilled circles, and Lobb and Davison (1975) by filled squares. Left-hand panels show data from other laboratories.

Each half of Figure 2 divides into four quadrants: Data points in Quadrants B and D are instances of overmatching, whereas those in Quadrants A and C are instances of undermatching. The method of presentation also permits a more sophisticated analysis of the relation between response and time slopes for individual subjects than Figure 1 allowed. Suppose, for example, that time-allocation measures simply tended to have higher slopes than response measures. In that case, data points should fall predominantly into Quadrants A and B. If, on the other hand, time slopes tend to be generally closer to matching than response slopes (regardless of the response slope value), then data points should fall into A and D. If there is no systematic relation between response and time slopes, data points should be distributed more or less equally between A and B, on one hand, and C and D, on the other.

Considering first data other than from Davison's work, the left half of Figure 2 shows an overwhelming predominance of points in Quadrants A and C relative to Quadrants B and D. Response undermatching was therefore the norm in the present data sample. When response slopes overmatched, however, time slopes generally tended to be closer to match-

ing (Quadrant D). When response slopes undermatched, on the other hand, time slopes were as likely to be farther away from matching as nearer (i.e., points were distributed about equally in Quadrants A and C). The analysis in Figure 2 therefore supports the assertion made above that there is, generally, little systematic relation between the value of the response and time slopes in recent concurrent VI studies, except that both have a very strong tendency to undermatch. Figure 2 does, however, suggest that when overmatching in response measures occurs (as it does only very rarely), time measures tend to be closer to matching.

The filled diamonds in the right-hand panels of Figure 2 show recent data from Davison's laboratory. Most subjects undermatched, and of these, five showed time slopes that were closer to matching than response slopes, and three exhibited slopes that were farther away. Baum (1979) also included data from three of Davison's studies that had derived both response and time measures from individual subjects (Beaudrais & Davison, 1977; Davison & Hunter, 1976; Lobb & Davison, 1975); if these are included in Figure 2 (as filled triangles, unfilled circles, and squares, respectively) it becomes clear that data from Davison's labora-

tory do seem to support the view that response and time slopes differ in the degree of undermatching exhibited. Of the data points shown in the right-hand part of Figure 2 that undermatch, 23 have time slopes closer to matching than response slopes, whereas only eight are further away. The hypothesis that time slopes are equally likely to be greater than or smaller than response slopes can be rejected statistically ($p < .006$ by sign test, Siegel, 1956).

Baum (1979) included two other studies in which response and time slopes came from individual subjects. One of these (Catania, 1966) produced data points very similar to the general trend of Davison's results, and the other (Pliskoff & Brown, 1976) yielded results like those exhibited by recent studies other than Davison's, showing a preponderance of response-ratio undermatching with no obvious relation between the values of response and time slopes.

It would appear, therefore, that the conclusion that response and time slopes tend to differ systematically (particularly in the direction of time slopes being closer to matching) depends very heavily on data from Davison's laboratories. On the other hand, Davison's data are not in any other obvious way atypical. Although Baum (1979) suggested that Davidson's results differed from his own norm of matching, they are quite consistent, in the degree of undermatching exhibited, with the overwhelming majority of recent findings, which also exhibit undermatching.

PROCEDURES AND PREFERENCE

The finding of recent studies that response and time measures systematically undermatch comes from experiments employing a wide variety of procedures, subject species, and reinforcers. Undermatching occurs when pigeons pecking conventional response keys are reinforced by delivery of food (Rodewald, 1978) or by avoidance of shock (Hutton, Gardner, & Lewis, 1978). Rat subjects also show undermatching when their lever pressing is reinforced by delivery of food (Norman & McSweeney, 1978) or by avoidance of shock (Logue & de Villiers, 1978), or when their allocation of time to different regions of an apparatus can avoid shock (Poling, 1978). When, in experiments with pigeons, the response manipulanda are clearly distinguished by being different

colors (Leigland, 1979; Lobb & Davison, 1977), or of different type (Davison & Ferguson, 1978; Wheatley & Engberg, 1978), undermatching is once again found. Perhaps the only hint of an exception to the preponderance of undermatching comes from studies suggesting that humans might tend to overmatch. For example, all three subjects studied by Bradshaw, Szabadi, and Bevan (1979) overmatched in both response and time measures, as did two of three subjects studied by Bradshaw, Szabadi, Bevan, and Ruddle (1979). When response measures alone were recorded in experiments with humans (Ruddle, Bradshaw, Szabadi, & Bevan, 1979) overmatching was found in four of six subjects.

Apart from the possibility that human performance may not be exactly like that of animals, the bulk of recent findings suggests that undermatching is not the result of some procedural oddity, since it occurs under such a wide variety of conditions. One aspect of a concurrent VI experiment that has received some previous discussion as a cause of undermatching is the changeover delay (COD) (e.g., see Baum, 1979). Although there is evidence that within-subject manipulations of COD can alter the degree of under- or overmatching (Baum, 1981; Catania & Cutts, 1963; Scown, Foster, & Temple, 1981), such effects are difficult to observe in between-experiment comparisons. CODs in recent studies have tended typically to be in the range of 1 to 3 sec, but undermatching occurs with CODs outside this range—for example, 0 sec when pigeons' key pecking is negatively reinforced (Hutton et al., 1978) or 5 sec when rats' responses are reinforced with food (Norman & McSweeney, 1978). The experiments with human subjects by Bradshaw and his colleagues, which tended to find overmatching, employed a 0-sec COD. It appears unlikely from these results that a long COD is necessary to avoid undermatching. However, it is possible that comparisons of the within-subject and between-experiment effects of COD may be invalid and that the contradiction implied above between the two sorts of effects is more apparent than real.

The main effect of procedural variations between experiments seems to be upon bias rather than undermatching. Bias has received little attention either in reviews (Baum, 1979) or in theoretical accounts of deviations from matching (e.g., Wearden, 1980) since Baum

(1974) advanced his influential analysis of the phenomenon. Essentially, Baum argued that bias resulted from asymmetries in the experimental situation, such as color or position preference, responses that differed in effort required, or differences in the amount of reinforcement produced by responses to the various concurrent alternatives. In many cases, such asymmetries in the experimental situation may arise because of lack of experimental control or subjects' idiosyncracies, but Baum's account also implies that programmed asymmetries in the experimental situation should also produce clear bias effects. Recent data offer strong support for this suggestion. When the concurrent response procedure differs substantially from the traditional presentation of two identical response manipulanda delivering reinforcement on a simple VI schedule, the result is often a clear bias. For example, when a VI schedule was pitted against a multiple VI schedule (Lobb & Davison, 1977), all subjects exhibited a bias towards the multiple schedule. Similarly, when pigeons were confronted with VI reinforcement provided by either a lever or key, subjects were strongly biased towards the key (Davison & Ferguson, 1978), particularly when preference was measured in terms of response distributions rather than time allocation (Wheatley & Engberg, 1978), a result consistent with the asymmetrical response requirement. Bias effects can also be more subtle. For example, Ruddle et al. (1979) exposed humans to various concurrent VI schedules of points (where the points were later exchangeable for money). The VI schedules were programmed on spatially separated levers, only one of which could be operated at once. All subjects exhibited a bias towards the right-hand lever, possibly because humans tend to be right-handed (although Ruddle et al. provided no data on the handedness of their subjects).

Such effects are consistent with the biases that occur when schedules of different type are concurrently arranged (e.g., Herrnstein & Heyman, 1979; Rider, 1981). Davison (1982) has also noted the development of biases when VI and fixed-ratio schedules are concurrently available but has argued that the generalized Matching Law (Equations 3 and 4) may need extension to handle data from asymmetrical schedule conditions. For example, subjects may exhibit different sensitivities to variation in the reinforcement rate under schedules of

different types and, in addition, the extent of a bias observed may not always be independent of the degree of under- (or over-) matching.

Recent data (Table 1) suggest, however, that when apparently symmetrical concurrent VI schedules are arranged, biases are generally of small magnitude and of variable sign between subjects. This is consistent with Baum's (1974) suggestion that in many cases bias arises because of unmeasured influences present in the experimental situation.

CONCLUSIONS

Data from recent experiments employing concurrent VI schedules support several conclusions. First, undermatching is the normal outcome of such experiments, whether preference is measured in terms of ratios of numbers of responses on the concurrent alternatives or in terms of time allocation. Second, the degree of undermatching exhibited appears little affected by procedural variations between experiments and occurs in studies using different species, reinforcers, response manipulanda, and values of changeover delay. Third, procedural differences between experiments can markedly affect the degree of bias observed, particularly when procedures arrange some type of asymmetry in the experimental situation, for example the use of different response manipulanda in the different VI components.

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