

V.I. Arnold V.V. Kozlov
A.I. Neishtadt

Mathematical Aspects of Classical and Celestial Mechanics

Second Edition

With 81 Figures



Springer

Mathematical Aspects of Classical and Celestial Mechanics

V.I. Arnold V.V. Kozlov A.I. Neishtadt

Translated from the Russian
by A. Iacob

Contents

Chapter 1. Basic Principles of Classical Mechanics	1
§ 1. Newtonian Mechanics	1
1.1. Space, Time, Motion	1
1.2. The Newton-Laplace Principle of Determinacy	2
1.3. The Principle of Relativity	4
1.4. Basic Dynamical Quantities. Conservation Laws	6
§ 2. Lagrangian Mechanics	9
2.1. Preliminary Remarks	9
2.2. Variations and Extremals	10
2.3. Lagrange's Equations	12
2.4. Poincaré's Equations	13
2.5. Constrained Motion	16
§ 3. Hamiltonian Mechanics	20
3.1. Symplectic Structures and Hamilton's Equations	20
3.2. Generating Functions	22
3.3. Symplectic Structure of the Cotangent Bundle	23
3.4. The Problem of n Point Vortices	24
3.5. The Action Functional in Phase Space	26
3.6. Integral Invariants	27
3.7. Applications to the Dynamics of Ideal Fluids	29
3.8. Principle of Stationary Isoenergetic Action	30
§ 4. Vakonomic Mechanics	31
4.1. Lagrange's Problem	32
4.2. Vakonomic Mechanics	33

4.3. The Principle of Determinacy	36
4.4. Hamilton's Equations in Redundant Coordinates	37
§ 5. Hamiltonian Formalism with Constraints	38
5.1. Dirac's Problem	38
5.2. Duality	40
§ 6. Realization of Constraints	40
6.1. Various Methods of Realizing Constraints	40
6.2. Holonomic Constraints	41
6.3. Anisotropic Friction	42
6.4. Adjoining Masses	43
6.5. Adjoining Masses and Anisotropic Friction	46
6.6. Small Masses	47
Chapter 2. The n -Body Problem	49
§ 1. The Two-Body Problem	49
1.1. Orbits	49
1.2. Anomalies	53
1.3. Collisions and Regularization	55
1.4. Geometry of the Kepler Problem	57
§ 2. Collisions and Regularization	58
2.1. Necessary Conditions for Stability	58
2.2. Simultaneous Collisions	59
2.3. Binary Collisions	60
2.4. Singularities of Solutions in the n -Body Problem	62
§ 3. Particular Solutions	64
3.1. Central Configurations	65
3.2. Homographic Solutions	65
3.3. The Amended Potential and Relative Equilibria	66
§ 4. Final Motions in the Three-Body Problem	67
4.1. Classification of Final Motions According to Chazy	67
4.2. Symmetry of Past and Future	68
§ 5. The Restricted Three-Body Problem	69
5.1. Equations of Motion. The Jacobi Integral	69
5.2. Relative Equilibria and the Hill Region	71
5.3. Hill's Problem	72
§ 6. Ergodic Theorems in Celestial Mechanics	75
6.1. Stability in the Sense of Poisson	75
6.2. Probability of Capture	76
Chapter 3. Symmetry Groups and Reduction (Lowering the Order)	78
§ 1. Symmetries and Linear First Integrals	78
1.1. E. Noether's Theorem	78
1.2. Symmetries in Nonholonomic Mechanics	82

1.3. Symmetries in Vakonomic Mechanics	84
1.4. Symmetries in Hamiltonian Mechanics	84
§ 2. Reduction of Systems with Symmetry	86
2.1. Lowering the Order (the Lagrangian Aspect)	86
2.2. Lowering the Order (the Hamiltonian Aspect)	91
2.3. Examples: Free Motion of a Rigid Body and the Three-Body Problem	96
§ 3. Relative Equilibria and Bifurcations of Invariant Manifolds	101
3.1. Relative Equilibria and the Amended Potential	101
3.2. Invariant Manifolds, Regions of Possible Motions, and Bifurcation Sets	102
3.3. The Bifurcation Set in the Planar Three-Body Problem	104
3.4. Bifurcation Sets and Invariant Manifolds in the Motion of a Heavy Rigid Body with a Fixed Point	105
Chapter 4. Integrable Systems and Integration Methods	107
§ 1. Brief Survey of Various Approaches to the Integrability of Hamiltonian Systems	107
1.1. Quadratures	107
1.2. Complete Integrability	109
1.3. Normal Forms	111
§ 2. Completely Integrable Systems	114
2.1. Action-Angle Variables	114
2.2. Noncommutative Sets of First Integrals	118
2.3. Examples of Completely Integrable Systems	119
§ 3. Some Methods of Integrating Hamiltonian Systems	124
3.1. Method of Separation of Variables	124
3.2. Method of L - A (Lax) Pairs	129
§ 4. Nonholonomic Integrable Systems	131
4.1. Differential Equations with Invariant Measure	131
4.2. Some Solved Problems of Nonholonomic Mechanics	134
Chapter 5. Perturbation Theory for Integrable Systems	138
§ 1. Averaging of Perturbations	138
1.1. The Averaging Principle	138
1.2. Procedure for Eliminating Fast Variables in the Absence of Resonances	142
1.3. Procedure for Eliminating Fast Variables in the Presence of Resonances	145
1.4. Averaging in Single-Frequency Systems	146
1.5. Averaging in Systems with Constant Frequencies	153
1.6. Averaging in Nonresonant Domains	155
1.7. The Effect of a Single Resonance	156
1.8. Averaging in Two-Frequency Systems	161

1.9. Averaging in Multi-Frequency Systems	165
§ 2. Averaging in Hamiltonian Systems	167
2.1. Application of the Averaging Principle	167
2.2. Procedures for Eliminating Fast Variables	175
§ 3. The KAM Theory	182
3.1. Unperturbed Motion. Nondegeneracy Conditions	182
3.2. Invariant Tori of the Perturbed System	183
3.3. Systems with Two Degrees of Freedom	186
3.4. Diffusion of Slow Variables in Higher-Dimensional Systems, and its Exponential Estimate	189
3.5. Variants of the Theorem on Invariant Tori	191
3.6. A Variational Principle for Invariant Tori. Cantori	194
3.7. Applications of the KAM Theory	197
§ 4. Adiabatic Invariants	200
4.1. Adiabatic Invariance of the Action Variable in Single- Frequency Systems	200
4.2. Adiabatic Invariants of Multi-Frequency Hamiltonian Systems	205
4.3. Procedure for Eliminating Fast Variables. Conservation Time of Adiabatic Invariants	207
4.4. Accuracy of the Conservation of Adiabatic Invariants	208
4.5. Perpetual Conservation of Adiabatic Invariants	210
 Chapter 6. Nonintegrable Systems	 212
§ 1. Near-Integrable Hamiltonian Systems	212
1.1. Poincaré's Methods	213
1.2. Creation of Isolated Periodic Solutions is an Obstruction to Integrability	215
1.3. Applications of Poincaré's Method	218
§ 2. Splitting of Asymptotic Surfaces	220
2.1. Conditions for Splitting	221
2.2. Splitting of Asymptotic Surfaces is an Obstruction to Integrability	224
2.3. Applications	227
§ 3. Quasi-Random Oscillations	231
3.1. The Poincaré Map	232
3.2. Symbolic Dynamics	235
3.3. Nonexistence of Analytic First Integrals	237
§ 4. Nonintegrability in the Neighborhood of an Equilibrium Position (Siegel's Method)	238
§ 5. Branching of Solutions and Nonexistence of Single-Valued First Integrals	241
5.1. Branching of Solutions is an Obstruction to Integrability	241
5.2. Monodromy Groups of Hamiltonian Systems with Single- Valued First Integrals	244

§ 6. Topological and Geometrical Obstructions to Complete Integrability of Natural Systems with Two Degrees of Freedom	248
6.1. Topology of the Configuration Space of Integrable Systems	248
6.2. Geometrical Obstructions to Integrability	250
Chapter 7. Theory of Small Oscillations	251
§ 1. Linearization	251
§ 2. Normal Forms of Linear Oscillations	252
2.1. Normal Form of Linear Natural Lagrangian Systems	252
2.2. The Rayleigh-Fischer-Courant Theorems on the Behavior of Characteristic Frequencies under an Increase in Rigidity and under Imposition of Constraints	253
2.3. Normal Forms of Quadratic Hamiltonians	253
§ 3. Normal Forms of Hamiltonian Systems Near Equilibria	255
3.1. Reduction to Normal Form	255
3.2. Phase Portraits of Systems with Two Degrees of Freedom in the Neighborhood of an Equilibrium Position under Resonance	258
3.3. Stability of Equilibria in Systems with Two Degrees of Freedom under Resonance	264
§ 4. Normal Forms of Hamiltonian Systems Near Closed Trajectories	266
4.1. Reduction to the Equilibrium of a System with Periodic Coefficients	266
4.2. Reduction of Systems with Periodic Coefficients to Normal Form	267
4.3. Phase Portraits of Systems with two Degrees of Freedom Near a Closed Trajectory under Resonance	267
§ 5. Stability of Equilibria in Conservative Fields	271
Comments on the Bibliography	274
Recommended Reading	276
Bibliography	278
Index	286