MATHEMATICAL ASPECTS OF CONSCIOUSNESS THEORY

Matti Pitkänen

Köydenpunojankatu D 11, 10900, Hanko, Finland

Contents

List of Figures

Introduction

M. Pitkänen[1], February 1, 2006

[1] Department of Physical Sciences, High Energy Physics Division,
PL 64, FIN-00014, University of Helsinki, Finland.
matpitka@rock.helsinki.fi, http://www.physics.helsinki.fi/∼matpitka/.
Recent address: Puutarhurinkatu 10,10960, Hanko, Finland.

Contents

0.1 Background

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [?]. The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last twenty-three years for the realization of this dream and this has resulted in seven online books [?, ?, ?, ?, ?, ?, ?] about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology [?, ?, ?, ?, ?, ?, ?, ?].

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

For few yeas ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be.

The fifth thread came with the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and certainly possible in TGD framework. The identification of hierarchy of Planck constants whose values TGD "predicts" in terms of dark matter hierarchy would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The seven online books [?, ?, ?, ?, ?, ?, ?] about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology [?, ?, ?, ?, ?, ?, ?, ?] are warmly recommended to the interested reader.

0.2 Basic Ideas of TGD

The basic physical picture behind TGD was formed as a fusion of two rather disparate approaches: namely TGD is as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model.

0.2.1 TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_+ \times CP_2$, where M^4_+ denotes the interior of the future light cone of the Minkowski space (to be referred as light cone in the sequel) and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [?, ?, ?, ?]. The identification of the space-time as a submanifold [?, ?] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity [Misner-Thorne-Wheeler, Logunov *et al*]. The actual choice $H = M^4_+ \times CP_2$ implies the breaking of the Poincare invariance in the cosmological scales but only at the quantum level. It soon however turned out that submanifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

0.2.2 TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

0.2.3 Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there is "vapor phase" that is a "gas" of particle like 3-surfaces (counterpart of the "baby universies" of GRT) and the nonconservation of energy in GRT corresponds to the transfer of energy between the topological condensate and vapor phase.

0.3 The five threads in the development of quantum TGD

The development of TGD has involved four strongly interacting threads: physics as infinite-dimensional geometry; p-adic physics; TGD inspired theory of consciousness and TGD as a generalized number theory. In the following these five threads are briefly described.

0.3.1 Quantum TGD as configuration space spinor geometry

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and are the following ones:

a) Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [**?**, **?**, **?**]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B + C$. Classically this corresponds to a path of configuration space leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

b) Configuration space is endowed with the metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory.

0.3.2 p-Adic TGD

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzless and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

There were many interpretational and technical questions crying for a definite answer. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *The Physics*? Should one perform p-adicization also at the level of the configuration space of 3-surfaces? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question.

Despite these frustrating uncertainties, the number of the applications of the poorly defined p-adic physics growed steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

0.3.3 TGD as a generalization of physics to a theory consciousness

General coordinate invariance forces the identification of quantum jump as quantum jump between entire deterministic quantum histories rather than time=constant snapshots of single history. The new view about quantum jump forces a generalization of quantum measurement theory such that observer becomes part of the physical system. Thus a general theory of consciousness is unavoidable outcome. This theory is developed in detail in the books [?, ?, ?, ?, ?, ?, ?, ?].

Quantum jump as a moment of consciousness

The identification of quantum jump between deterministic quantum histories (configuration space spinor fields) as a moment of consciousness defines microscopic theory of consciousness. Quantum jump involves the steps

$$\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f \quad ,$$

where U is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution of quantum mechanics. U is however only formally analogous to Schrödinger time evolution of infinite duration although there is *no* real time evolution involved. It is not however clear whether one should regard U-matrix and S-matrix as two different things or not: U-matrix is a completely universal object characterizing the dynamics of evolution by self-organization whereas S-matrix is a highly context dependent concept in wave mechanics and in quantum field theories where it at least formally represents unitary time translation operator at the limit of an infinitely long interaction time. The S-matrix understood in the spirit of superstring models is however something very different and could correspond to U-matrix.

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parameterize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes now a part of quantum theory. Together these requirements imply that the final states of quantum jump correspond to quantum superpositions of space-time surfaces which are macroscopically equivalent. Hence the world of conscious experience looks classical. At least formally quantum jump can be interpreted also as a quantum computation in which matrix U represents unitary quantum computation which is however not identifiable as unitary translation in time direction and cannot be 'engineered'.

The notion of self

The concept of self is absolutely essential for the understanding of the macroscopic and macro-temporal aspects of consciousness. Self corresponds to a subsystem able to remain un-entangled under the sequential informational 'time evolutions' U. Exactly vanishing entanglement is practically impossible in ordinary quantum mechanics and it might be that 'vanishing entanglement' in the condition for self-property should be replaced with 'subcritical entanglement'. On the other hand, if space-time decomposes into p-adic and real regions, and if entanglement between regions representing physics in different number fields vanishes, space-time indeed decomposes into selves in a natural manner.

It is assumed that the experiences of the self after the last 'wake-up' sum up to single average experience. This means that subjective memory is identifiable as conscious, immediate short term memory. Selves form an infinite hierarchy with the entire Universe at the top. Self can be also interpreted as mental images: our mental images are selves having mental images and also we represent mental images of a higher level self. A natural hypothesis is that self S experiences the experiences of its subselves as kind of abstracted experience: the experiences of subselves S_i are not experienced as such but represent kind of averages $\langle S_{ij}\rangle$ of sub-subselves S_{ij}. Entanglement between selves, most naturally realized by the formation of join along boundaries bonds between cognitive or material space-time sheets, provides a possible a mechanism for the fusion of selves to larger selves (for instance, the fusion of the mental images representing separate right and left visual fields to single visual field) and forms wholes from parts at the level of mental images.

Relationship to quantum measurement theory

The third basic element relates TGD inspired theory of consciousness to quantum measurement theory. The assumption that localization occurs in zero modes in each quantum jump implies that the world of conscious experience looks classical. It also implies the state function reduction of the standard quantum measurement theory as the following arguments demonstrate (it took incredibly long time to realize this almost obvious fact!).

a) The standard quantum measurement theory a la von Neumann involves the interaction of brain with the measurement apparatus. If this interaction corresponds to entanglement between microscopic degrees of freedom m with the macroscopic effectively classical degrees of freedom M characterizing the reading of the measurement apparatus coded to brain state, then the reduction of this entanglement in quantum jump reproduces standard quantum measurement theory provide the unitary time evolution operator U acts as flow in zero mode degrees of freedom and correlates completely some orthonormal basis of configuration space spinor fields in non-zero modes with the values of the zero modes. The flow property guarantees that the localization is consistent with unitarity: it also means 1-1 mapping of quantum state basis to classical variables (say, spin direction of the electron to its orbit in the external magnetic field).

b) Since zero modes represent classical information about the geometry of space-time surface (shape, size, classical Kähler field,...), they have interpretation as effectively classical degrees of freedom and are the TGD counterpart of the degrees of freedom M representing the reading of the measurement apparatus. The entanglement between quantum fluctuating non-zero modes and zero modes is the TGD counterpart for the $m - M$ entanglement. Therefore the localization in zero modes is equivalent with a quantum jump leading to a final state where the measurement apparatus gives a definite reading.

This simple prediction is of utmost theoretical importance since the black box of the quantum measurement theory is reduced to a fundamental quantum theory. This reduction is implied by the replacement of the notion of a point like particle with particle as a 3-surface. Also the infinite-dimensionality of the zero mode sector of the configuration space of 3-surfaces is absolutely essential. Therefore the reduction is a triumph for quantum TGD and favors TGD against string models.

Standard quantum measurement theory involves also the notion of state preparation which reduces to the notion of self measurement. Each localization in zero modes is followed by a cascade of self measurements leading to a product state. This process is obviously equivalent with the state preparation process. Self measurement is governed by the so called Negentropy Maximization Principle (NMP) stating that the information content of conscious experience is maximized. In the self measurement the density matrix of some subsystem of a given self localized in zero modes (after ordinary quantum measurement) is measured. The self measurement takes place for that subsystem of self for which the reduction of the entanglement entropy is maximal in the measurement. In p-adic context NMP can be regarded as the variational principle defining the dynamics of cognition. In real context self measurement could be seen as a repair mechanism allowing the system to fight against quantum thermalization by reducing the entanglement for the subsystem for which it is largest (fill the largest hole first in a leaking boat).

Selves self-organize

The fourth basic element is quantum theory of self-organization based on the identification of quantum jump as the basic step of self-organization [**?**]. Quantum entanglement gives rise to the generation of long range order and the emergence of longer p-adic length scales corresponds to the emergence of larger and larger coherent dynamical units and generation of a slaving hierarchy. Energy (and quantum entanglement) feed implying entropy feed is a necessary prerequisite for quantum self-organization. Zero modes represent fundamental order parameters and localization in zero modes implies that the sequence of quantum jumps can be regarded as hopping in the zero modes so that Haken's classical theory of self organization applies almost as such. Spin glass analogy is a further important element: self-organization of self leads to some characteristic pattern selected by dissipation as some valley of the "energy" landscape.

Dissipation can be regarded as the ultimate Darwinian selector of both memes and genes. The mathematically ugly irreversible dissipative dynamics obtained by adding phenomenological dissipation terms to the reversible fundamental dynamical equations derivable from an action principle can be understood as a phenomenological description replacing in a well defined sense the series of reversible quantum histories with its envelope.

Classical non-determinism of Kähler action

The fifth basic element are the concepts of association sequence and cognitive space-time sheet. The huge vacuum degeneracy of the Kähler action suggests strongly that the absolute minimum space-time is not always unique. For instance, a sequence of bifurcations can occur so that a given space-time branch can be fixed only by selecting a finite number of 3-surfaces with time like(!) separations on the orbit of 3-surface. Quantum classical correspondence suggest an alternative formulation. Space-time surface decomposes into maximal deterministic regions and their temporal sequences have interpretation a space-time correlate for a sequence of quantum states defined by the initial (or final) states of quantum jumps. This is consistent with the fact that the variational principle selects preferred extremals of Kähler action as generalized Bohr orbits.

In the case that non-determinism is located to a finite time interval and is microscopic, this sequence of 3-surfaces has interpretation as a simulation of a classical history, a geometric correlate for contents of consciousness. When non-determinism has long lasting and macroscopic effect one can identify it as volitional non-determinism associated with our choices. Association sequences relate closely with the cognitive space-time sheets defined as space-time sheets having finite time duration and psychological time can be identified as a temporal center of mass coordinate of the cognitive space-time sheet. The gradual drift of the cognitive space-time sheets to the direction of future force by the geometry of the future light cone explains the arrow of psychological time.

p-Adic physics as physics of cognition and intentionality

The sixth basic element adds a physical theory of cognition to this vision. TGD space-time decomposes into regions obeying real and p-adic topologies labelled by primes $p = 2, 3, 5, ...$ p-Adic regions obey the same field equations as the real regions but are characterized by p-adic non-determinism since the functions having vanishing p-adic derivative are pseudo constants which are piecewise constant functions. Pseudo constants depend on a finite number of positive pinary digits of arguments just like numerical predictions of any theory always involve decimal cutoff. This means that p-adic space-time regions are obtained by gluing together regions for which integration constants are genuine constants. The natural interpretation of the p-adic regions is as cognitive representations of real physics. The freedom of imagination is due to the p-adic non-determinism. p-Adic regions perform mimicry and make possible for the Universe to form cognitive representations about itself. p-Adic physics space-time sheets serve also as correlates for intentional action.

A more more precise formulation of this vision requires a generalization of the number concept obtained by fusing reals and p-adic number fields along common rationals (in the case of algebraic extensions among common algebraic numbers). This picture is discussed in [?]. The application this notion at the level of the imbedding space implies that imbedding space has a book like structure with various variants of the imbedding space glued together along common rationals (algebraics). The implication is that genuinely p-adic numbers (non-rationals) are strictly infinite as real numbers so that most points of p-adic space-time sheets are at real infinity, outside the cosmos, and that the projection to the real imbedding space is discrete set of rationals (algebraics). Hence cognition and intentionality are almost completely outside the real cosmos and touch it at a discrete set of points only.

This view implies also that purely local p-adic physics codes for the p-adic fractality characterizing long range real physics and provides an explanation for p-adic length scale hypothesis stating that the primes $p \simeq 2^k$, k integer are especially interesting. It also explains the long range correlations and short term chaos characterizing intentional behavior and explains why the physical realizations of cognition are always discrete (say in the case of numerical computations). Furthermore, a concrete quantum model for how intentions are transformed to actions emerges.

The discrete real projections of p-adic space-time sheets serve also space-time correlate for a logical thought. It is very natural to assign to p-adic pinary digits a p-valued logic but as such this kind of logic does not have any reasonable identification. p-Adic length scale hypothesis suggest that the $p = 2^k - n$ pinary digits represent a Boolean logic B^k with k elementary statements (the points of the k-element set in the set theoretic realization) with n taboos which are constrained to be identically true.

0.3.4 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. For few yeas ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. It relies on the notion of number theoretic compactifiction stating that space-time surfaces can be regarded either as hyper-quaternionic, and thus maximally associative, 4-surfaces in M^8 identifiable as space of hyper-octonions or as surfaces in $M^4 \times CP_2$ [?].

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory. The work with Riemann hypothesis led to further ideas.

After the realization that infinite primes can be mapped to polynomials representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially satisfying is that p-adic and real regions of the space-time surface could emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably extremely brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

0.3.5 Dynamical quantized Planck constant and dark matter hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale [?] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [?].

Already before learning about Nottale's paper I had proposed the possibility that Planck constant is quantized [?] and the spectrum is given in terms of logarithms of Beraha numbers: the lowest Beraha number B_3 is completely exceptional in that it predicts infinite value of Planck constant. The inverse of the gravitational Planck constant could correspond a gravitational perturbation of this as $1/\hbar_{gr} = v_0/GMm$. The general philosophy would be that when the quantum system would become non-perturbative, a phase transition increasing the value of $\hbar$ occurs to preserve the perturbative character and at the transition $n = 4 \rightarrow 3$ only the small perturbative correction to $1/\hbar(3) = 0$ remains. This would apply to QCD and to atoms with $Z > 137$ as well.

TGD predicts correctly the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n^2-fold: much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. Also a model for the formation of planetary system as a condensation of ordinary matter around quantum coherent dark matter emerges [?].

Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics. An attractive solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

p-Adic and dark matter hierarchies and hierarchy of moments of consciousness

Dark matter hierarchy assigned to a spectrum of Planck constant having arbitrarily large values brings additional elements to the TGD inspired theory of consciousness.

a) Macroscopic quantum coherence can be understood since a particle with a given mass can in principle appear as arbitrarily large scaled up copies (Compton length scales as $\hbar$). The phase transition to this kind of phase implies that space-time sheets of particles overlap and this makes possible macroscopic quantum coherence.

b) The space-time sheets with large Planck constant can be in thermal equilibrium with ordinary ones without the loss of quantum coherence. For instance, the cyclotron energy scale associated with EEG turns out to be above thermal energy at room temperature for the level of dark matter hierarchy corresponding to magnetic flux quanta of the Earth's magnetic field with the size scale of Earth and a successful quantitative model for EEG results [?].

Dark matter hierarchy leads to detailed quantitative view about quantum biology with several testable predictions [?]. The applications to living matter suggests that the basic hierarchy corresponds to a hierarchy of Planck constants coming as $\hbar(k) = \lambda^k(p)\hbar_0$, $\lambda \simeq 2^{11}$ for $p = 2^{127-1}$, $k = 0, 1, 2, ...$ [?]. Also integer valued sub-harmonics and integer valued sub-harmonics of λ might be possible. Each p-adic length scale corresponds to this kind of hierarchy and number theoretical arguments suggest a general formula for the allowed values of Planck constant λ depending logarithmically on p-adic prime [?]. Also the value of $\hbar_0$ has spectrum characterized by Beraha numbers $B_n = 4cos^2(\pi/n)$, $n \geq 3$, varying by a factor in the range $n > 3$ [?]. It must be however emphasized that the relation of this picture to the model of quantized gravitational Planck constant h_{gr} appearing in Nottale's model is not yet completely understood.

The general prediction is that Universe is a kind of inverted Mandelbrot fractal for which each bird's eye of view reveals new structures in long length and time scales representing scaled down copies of standard physics and their dark variants. These structures would correspond to higher levels in self hierarchy. This prediction is consistent with the belief that 75 per cent of matter in the universe is dark.

1. Living matter and dark matter

Living matter as ordinary matter quantum controlled by the dark matter hierarchy has turned out to be a particularly successful idea. The hypothesis has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of EEG [?]. Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [?, ?]. A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [?].

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of $\hbar$ at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

2. Dark matter hierarchy and the notion of self

The vision about dark matter hierarchy leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [**?**, **?**]. The larger the value of Planck constant, the longer the subjectively experienced duration and the average geometric duration $T(k) \propto \lambda^k$ of the quantum jump.

Quantum jumps form also a hierarchy with respect to p-adic and dark hierarchies and the geometric durations of quantum jumps scale like $\hbar$. Dark matter hierarchy suggests also a slight modification of the notion of self. Each self involves a hierarchy of dark matter levels, and one is led to ask whether the highest level in this hierarchy corresponds to single quantum jump rather than a sequence of quantum jumps. The averaging of conscious experience over quantum jumps would occur only for sub-selves at lower levels of dark matter hierarchy and these mental images would be ordered, and single moment of consciousness would be experienced as a history of events. The quantum parallel dissipation at the lower levels would give rise to the experience of flow of time. For instance, hadron as a macro-temporal quantum system in the characteristic time scale of hadron is a dissipating system at quark and gluon level corresponding to shorter p-adic time scales. One can ask whether even entire life cycle could be regarded as a single quantum jump at the highest level so that consciousness would not be completely lost even during deep sleep. This would allow to understand why we seem to know directly that this biological body of mine existed yesterday.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self corresponds at the highest dark matter level to single moment of consciousness. Self would experience the average over the sequence of moments of consciousness associated with each sub-self but there would be no averaging over the separate mental images of this kind, be their parallel or serial. These mental images correspond to sub-selves having shorter wake-up periods than self and would be experienced as being time ordered. Hence the digits in the phone number are experienced as separate mental images and ordered with respect to experienced time.

3. The time span of long term memories as signature for the level of dark matter hierarchy

The simplest dimensional estimate gives for the average increment τ of geometric time in quantum jump $\tau \sim 10^4\ CP_2$ times so that $2^{127}-1 \sim 10^{38}$ quantum jumps are experienced during secondary p-adic time scale $T_2(k=127) \simeq 0.1$ seconds which is the duration of physiological moment and predicted to be fundamental time scale of human consciousness [**?**]. A more refined guess is that $\tau_p = \sqrt{p}\tau$ gives the dependence of the duration of quantum jump on p-adic prime p. By multi-p-fractality predicted by TGD and explaining p-adic length scale hypothesis, one expects that at least $p=2$-adic level is also always present. For the higher levels of dark matter hierarchy τ_p is scaled up by $\hbar/\hbar_0$. One can understand evolutionary leaps as the emergence of higher levels at the level of individual organism making possible intentionality and memory in the time scale defined τ [**?**].

Higher levels of dark matter hierarchy provide a neat quantitative view about self hierarchy and its evolution. For instance, EEG time scales corresponds to $k=4$ level of hierarchy and a time scale of .1 seconds [**?**], and EEG frequencies correspond at this level dark photon energies above the thermal threshold so that thermal noise is not a problem anymore. Various levels of dark matter hierarchy would naturally correspond to higher levels in the hierarchy of consciousness and the typical duration of life cycle would give an idea about the level in question.

The level would determine also the time span of long term memories as discussed in [**?**]. $k=7$ would correspond to a duration of moment of conscious of order human lifetime which suggests that $k=7$ corresponds to the highest dark matter level relevant to our consciousness whereas higher levels would in general correspond to transpersonal consciousness. $k=5$ would correspond to time scale of short term memories measured in minutes and $k=6$ to a time scale of memories measured in days.

The emergence of these levels must have meant evolutionary leap since long term memory is also accompanied by ability to anticipate future in the same time scale. This picture would suggest that the basic difference between us and our cousins is not at the level of genome as it is usually understood but at the level of the hierarchy of magnetic bodies [**?**, **?**]. In fact, higher levels of dark matter hierarchy motivate the introduction of the notions of super-genome and hyper-genome. The genomes of entire organ can join to form super-genome expressing genes coherently. Hyper-genomes would result from the fusion of genomes of different organisms and collective levels of consciousness would express themselves via hyper-genome and make possible social rules and moral.

0.4 Bird's eye of view about the topics of the book

The topics of this book are general mathematical ideas, many of them inspired by TGD inspired theory of consciousness.

The topics of this book are general mathematical ideas behind TGD inspired theory of consciousness and quantum TGD itself, many of them inspired by TGD inspired theory of consciousness.

1. p-Adic physics and consciousness

Physics as a generalized number theory vision received a strong boost from TGD inspired theory of consciousness. There are good reasons for considering p-adic physics as a good candidate for physics of cognition and intention and this leads to a concrete quantum model for how intentions are transformed to actions in quantum jumps transforming p-adic space-time sheets to real ones [**?**]. The logical aspects of cognition would naturally be represented by the discrete projections of p-adic space-time sheets to real imbedding space as space-time correlates.

Most points of p-adic space-time sheets are at infinity in real sense so that cognition literally views the real cosmos from outside. The projection to real imbedding space is discrete and this could reflect that fact that the part of our cognition which is representable at the level of real physics is bound to be always discrete and actually finite (consider only numerical computations).

2. Von Neumann algebras and consciousness

The von Neumann algebras known as hyper-finite factors of type II_1 appear naturally in TGD framework: as a matter fact configuration space spinors associated with single point of configuration space ("world of classical worlds") decomposes to a direct integral of these algebras. This alone leads to amazingly strong physical predictions and implies deep connections with conformal field theories, knot-, braid- and quantum groups, and topological quantum computation.

The braids formed by magnetic flux tubes seem to be ideal for the realization of topological quantum computations by coding quantum computations to the braidings of the flux tubes. Living system is populated of molecular structures which form braids so that in TGD Universe bio-systems are basic candidates for topological quantum computers. The possibility to communicate with the geometric past using negative energy bosons suggests that the constraints posed by the non-polynomial computation time on ordinary quantum computations might be circumvented by using time loops, and living matter might utilize this mechanism routinely. The vision about DNA as topological quantum computer is a concrete proposal about how this might be achieved.

3. Dark matter hierarchy and consciousness

The most recent piece in the big picture is the vision about dark matter as a hierarchy of phases of matter having no local interactions (vertices of Feynman graphs) with other levels. The level of dark matter hierarchy is characterized partially by the value of Planck constant labeling the pages of the book like structure formed by singular covering spaces of the imbedding space $M^4 \times CP_2$ glued together along a four-dimensional back. Particles at different pages are dark relative to each other since purely local interactions defined in terms of the vertices of Feynman diagram involve only particles at the same page. The phase transitions changing the value of Planck constants having interpretation as tunneling between different pages of the book like structure would induce the phase transitions of gel phases abundant in living matter.

4. Infinite primes and consciousness

The notion of infinite prime was the first mathematical invention inspired by TGD inspired theory of consciousness. The construction of infinite primes is very much analogous to a repeated second quantization of a super-symmetric arithmetic quantum field theory (with analogs of bound states included). One can also interpret space-time surfaces as representations of infinite primes, integers, and rationals analogous to representations of polynomials as surfaces of their zeros.

Infinite primes form an infinite hierarchy and the realization of this hierarchy at the level of physics would mean that the hierarchy of consciousness is also infinite and that we represent only single level in this hierarchy looking infinitesimal from the point of view of higher levels. The notion of infinite rational predicts also an infinite number of real units with infinitely rich number theoretical anatomy so that single space-time point becomes a Platonia able to represent every quantum state of the entire Universe in its structure. This means kind of Brahman=Atman identity at the level of mathematics.

5. Categories and consciousness

One can consider the possibility that category theory reflects the basic structures of conscious thought. Although my understanding of category theory is rather meager, I cannot avoid the temptation to discuss the possible applications of category theory to TGD and TGD inspired theory of consciousness. The comparison of the generalized inherent logics associated with categories to the Boolean logic naturally associated with the configuration space spinor fields is also of interest. Category theoretical ideas find a more concrete application in quantum TGD proper in characterization of Feynman diagrams.

6. Organization of the book

The contents of the book are organized as follows.

1. The first three chapters of the book are devoted to hyper-finite factors of type II_1 and to the physical vision about dark matter provided by the hierarchy of Planck constants realized in terms of generalization of the imbedding space concept.

2. Second part is devoted to the idea that a process analogous to topological quantum computation take place at fundamental level in TGD Universe and for the model for how DNA and cell membrane could act as topological quantum computer. The braiding of magnetic flux tubes connecting nucleotides and lipids of the cell membrane is the key element of the model and the model leads to a plethora of ideas about catalyst action and bio-control.

3. The third part of the book contains a chapter about infinite primes and category theoretical ideas. Both are written for more than decade ago and the vision about the role of categories has been updated since then.

The seven online books about TGD [**?**, **?**, **?**, **?**, **?**, **?**, **?**] and eight online books about TGD inspired theory of consciousness and quantum biology [**?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**] are warmly recommended for the reader willing to get overall view about what is involved.

0.5 The contents of the book

0.5.1 New Physics and Mathematics Involved with TGD

Was von Neumann right after all?

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors of type II_1 could provide the mathematics needed to develop a more explicit view about the construction of S-matrix. This has turned out to be the case to the extend that a general master formula for S-matrix with interactions described as a deformation of ordinary tensor product to Connes tensor products emerges.

1. Some background

It has been for few years clear that TGD could emerge from the mere infinite-dimensionality of the Clifford algebra of infinite-dimensional "world of classical worlds" and from number theoretical vision in which classical number fields play a key role and determine imbedding space and space-time dimensions. This would fix completely the "world of classical worlds".

Infinite-dimensional Clifford algebra is a standard representation for von Neumann algebra known as a hyper-finite factor of type II_1. In TGD framework the infinite tensor power of C(8), Clifford algebra of 8-D space would be the natural representation of this algebra.

2. How to localize infinite-dimensional Clifford algebra?

The basic new idea is to make this algebra *local*: local Clifford algebra as a generalization of gamma field of string models.

1. Represent Minkowski coordinate of M^d as linear combination of gamma matrices of D-dimensional space. This is the first guess. One fascinating finding is that this notion can be quantized and classical M^d is genuine quantum M^d with coordinate values eigenvalues of quantal commuting

Hermitian operators built from matrix elements. Euclidian space is not obtained in this manner. Minkowski signature is something quantal and the standard quantum group $Gl_(2,q)(C)$ with (non-Hermitian matrix elements) gives M^4.

2. Form power series of the M^d coordinate represented as linear combination of gamma matrices with coefficients in corresponding infinite-D Clifford algebra. You would get tensor product of two algebra.

3. There is however a problem: one cannot distinguish the tensor product from the original infinite-D Clifford algebra. $D = 8$ is however an exception! You can replace gammas in the expansion of M^8 coordinate by hyper-octonionic units which are non-associative (or octonionic units in quantum complexified-octonionic case). Now you cannot anymore absorb the tensor factor to the Clifford algebra and you get genuine M^8-localized factor of type II_1. Everything is determined by infinite-dimensional gamma matrix fields analogous to conformal super fields with z replaced by hyperoctonion.

4. Octonionic non-associativity actually reproduces whole classical and quantum TGD: space-time surface must be associative sub-manifolds hence hyper-quaternionic surfaces of M^8. Representability as surfaces in $M^4 \times CP_2$ follows naturally, the notion of configuration space of 3-surfaces, etc....

3. Connes tensor product for free fields as a universal definition of interaction quantum field theory

This picture has profound implications. Consider first the construction of S-matrix.

1. A non-perturbative construction of S-matrix emerges. The deep principle is simple. The canonical outer automorphism for von Neumann algebras defines a natural candidate unitary transformation giving rise to propagator. This outer automorphism is trivial for II_1 factors meaning that all lines appearing in Feynman diagrams must be on mass shell states satisfying Super Virasoro conditions. You can allow all possible diagrams: all on mass shell loop corrections vanish by unitarity and what remains are diagrams with single N-vertex.

2. At 2-surface representing N-vertex space-time sheets representing generalized Bohr orbits of incoming and outgoing particles meet. This vertex involves von Neumann trace (finite!) of localized gamma matrices expressible in terms of fermionic oscillator operators and defining free fields satisfying Super Virasoro conditions.

3. For free fields ordinary tensor product would not give interacting theory. What makes S-matrix non-trivial is that *Connes tensor product* is used instead of the ordinary one. This tensor product is a universal description for interactions and we can forget perturbation theory! Interactions result as a deformation of tensor product. Unitarity of resulting S-matrix is unproven but I dare believe that it holds true.

4. The subfactor $\mathcal{N}$ defining the Connes tensor product has interpretation in terms of the interaction between experimenter and measured system and each interaction type defines its own Connes tensor product. Basically $\mathcal{N}$ represents the limitations of the experimenter. For instance, IR and UV cutoffs could be seen as primitive manners to describe what $\mathcal{N}$ describes much more elegantly. At the limit when $\mathcal{N}$ contains only single element, theory would become free field theory but this is ideal situation never achievable.

5. Large $\hbar$ phases provide good hopes of realizing topological quantum computation. There is an additional new element. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and decoherence is not a problem as long as it does not induce this transition.

Does TGD predict the spectrum of Planck constants?

The quantization of Planck constant has been the basic them of TGD since 2005. The basic idea was stimulated by the finding of Nottale that planetary orbits could be seen as Bohr orbits with enormous value of Planck constant given by $\hbar_{gr} = GM_1M_2/v_0$, $v_0 \simeq 2^{-11}$ for the inner planets. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales. The second crucial empirical input were the anomalies associated with living matter. The revised version of the chapter represents the vision about quantization of Planck constants from a perspective given by almost five years work with the idea. A very concise summary about the situation is as follows.

1. The hierarchy of Planck constants cannot be realized without generalizing the notions of imbedding space $M^4 \times CP_2$ and space-time since particles with different values of Planck constant cannot appear in the same interaction vertex. This suggests some kind of book like structure for both M^4 and CP_2 factors of the generalized imbedding space is suggestive.

2. Schrödinger equation suggests that Planck constant corresponds to a scaling factor of M^4 metric whose value labels different pages of the book. The scaling of M^4 coordinate so that original metric results in M^4 factor is possible so that the scaling of $\hbar$ corresponds to the scaling of the size of causal diamond CD defined as the intersection of future and past directed light-cones. The light-like 3-surfaces having their 2-D and light-boundaries of CD are in a key role in the realization of zero energy states. The infinite-D spaces formed by these 3-surfaces define the fundamental sectors of the configuration space (world of classical worlds). Since the scaling of CD does not simply scale space-time surfaces, the coding of radiative corrections to the geometry of space-time sheets becomes possible and Kähler action can be seen as expansion in powers of $\hbar/\hbar_0$.

3. Quantum criticality of TGD Universe is one of the key postulates of quantum TGD. The most important implication is that Kähler coupling strength is analogous to critical temperature. The exact realization of quantum criticality would be in terms of critical sub-manifolds of M^4 and CP_2 common to all sectors of the generalized imbedding space. Quantum criticality would mean that the two kinds of number theoretic braids assignable to M^4 and CP_2 projections of the partonic 2-surface belong by the definition of number theoretic braids to these critical sub-manifolds. At the boundaries of CD associated with positive and negative energy parts of zero energy state in given time scale partonic two-surfaces belong to a fixed page of the Big Book whereas light-like 3-surface decomposes into regions corresponding to different values of Planck constant much like matter decomposes to several phases at thermodynamical criticality.

4. The connection with Jones inclusions was originally a purely heuristic guess based on the observation that the finite groups characterizing Jones inclusion characterize also pages of the Big Book. The key observation is that Jones inclusions are characterized by a finite subgroup $G \subset SU(2)$ and that this group also characterizes the singular covering or factor spaces associated with CD or CP_2 so that the pages of generalized imbedding space could indeed serve as correlates for Jones inclusions. The elements of the included algebra $\mathcal{M}$ are invariant under the action of G and $\mathcal{M}$ takes the role of complex numbers in the resulting non-commutative quantum theory.

5. The understanding of quantum TGD at parton level led to the realization that the dynamics of Kähler action realizes finite measurement resolution in terms of finite number of modes of the induced spinor field. This automatically implies cutoffs to the representations of various super-conformal algebras typical for the representations of quantum groups closely associated with Jones inclusions. The Clifford algebra spanned by the fermionic oscillator operators would provide a realization for the factor space $\mathcal{N}/\mathcal{M}$ of hyper-finite factors of type II_1 identified as the infinite-dimensional Clifford algebra $\mathcal{N}$ of the configuration space and included algebra $\mathcal{M}$ determining the finite measurement resolution. The resulting quantum Clifford algebra has anti-commutation relations dictated by the fractionization of fermion number so that its unit becomes $r = \hbar/\hbar_0$. $SU(2)$ Lie algebra transforms to its quantum variant corresponding to the quantum phase $q = exp(i2\pi/r)$.

6. Jones inclusions appear as two variants corresponding to $\mathcal{N} : \mathcal{M} < 4$ and $\mathcal{N} : \mathcal{M} = 4$. The tentative interpretation is in terms of singular G-factor spaces and G-coverings of M^4 or CP_2 in some sense. The alternative interpretation in terms of two geodesic spheres of CP_2 would mean asymmetry between M^4 and CP_2 degrees of freedom.

7. Number theoretic Universality suggests an answer why the hierarchy of Planck constants is necessary. One must be able to define the notion of angle -or at least the notion of phase and of trigonometric functions- also in p-adic context. All that one can achieve naturally is the notion of phase defined as root of unity and introduced by allowing algebraic extension of p-adic number field by introducing the phase if needed. In the framework of TGD inspired theory of consciousness this inspires a vision about cognitive evolution as the gradual emergence of increasingly complex algebraic extensions of p-adic numbers and involving also the emergence of improved angle resolution expressible in terms of phases $exp(i2\pi/n)$ up to some maximum value of n. The coverings and factor spaces would realize these phases geometrically and quantum phases q naturally assignable to Jones inclusions would realize them algebraically. Besides p-adic coupling constant evolution based on hierarchy of p-adic length scales there would be coupling constant evolution with respect to $\hbar$ and associated with angular resolution.

Quantum Hall effect and Hierarchy of Planck Constants

I have already earlier proposed the explanation of FQHE, anyons, and fractionization of quantum numbers in terms of hierarchy of Planck constants realized as a generalization of the imbedding space $H = M^4 \times CP_2$ to a book like structure. The book like structure applies separately to CP_2 and to causal diamonds ($CD \subset M^4$) defined as intersections of future and past directed light-cones. The pages of the Big Book correspond to singular coverings and factor spaces of CD (CP_2) glued along 2-D subspace of CD (CP_2) and are labeled by the values of Planck constants assignable to CD and CP_2 and appearing in Lie algebra commutation relations. The observed Planck constant $\hbar$, whose square defines the scale of M^4 metric corresponds to the ratio of these Planck constants. The key observation is that fractional filling factor results if $\hbar$ is scaled up by a rational number.

In this chapter I try to formulate more precisely this idea. The outcome is a rather detailed view about anyons on one hand, and about the Kähler structure of the generalized imbedding space on the other hand.

1. Fundamental role is played by the assumption that the Kähler gauge potential of CP_2 contains a gauge part with no physical implications in the context of gauge theories but contributing to physics in TGD framework since $U(1)$ gauge transformations are representations of symplectic transformations of CP_2. Also in the case of CD it makes also sense to speak about Kähler gauge potential. The gauge part codes for Planck constants of CD and CP_2 and leads to the identification of anyons as states associated with partonic 2-surfaces surrounding the tip of CD and fractionization of quantum numbers. Explicit formulas relating fractionized charges to the coefficients characterizing the gauge parts of Kähler gauge potentials of CD and CP_2 are proposed based on some empirical input.

2. One important implication is that Poincare and Lorentz invariance are broken inside given CD although they remain exact symmetries at the level of the geometry of world of classical worlds (WCW). The interpretation is as a breaking of symmetries forced by the selection of quantization axis.

3. Anyons would basically correspond to matter at 2-dimensional "partonic" surfaces of macroscopic size surrounding the tip of the light-cone boundary of CD and could be regarded as gigantic elementary particle states with very large quantum numbers and by charge fractionization confined around the tip of CD. Charge fractionization and anyons would be basic characteristic of dark matter (dark only in relative sense). Hence it is not surprising that anyons would have applications going far beyond condensed matter physics. Anyonic dark matter concentrated at 2-dimensional surfaces would play key key role in the the physics of stars and black holes, and also in the formation of planetary system via the condensation of the ordinary matter around dark matter. This assumption was the basic starting point leading to the discovery of the hierarchy of Planck constants. In living matter membrane like structures would represent a

key example of anyonic systems as the model of DNA as topological quantum computer indeed assumes.

4. One of the basic questions has been whether TGD forces the hierarchy of Planck constants realized in terms of generalized imbedding space or not. The condition that the choice of quantization axes has a geometric correlate at the imbedding space level motivated by quantum classical correspondence of course forces the hierarchy: this has been clear from the beginning. It is now clear that first principle description of anyons requires the hierarchy in TGD Universe. The hierarchy reveals also new light to the huge vacuum degeneracy of TGD and reduces it dramatically at pages for which CD corresponds to a non-trivial covering or factor space, which suggests that mathematical existence of the theory necessitates the hierarchy of Planck constants. Also the proposed manifestation of Equivalence Principle at the level of symplectic fusion algebras as a duality between descriptions relying on the symplectic structures of CD and CP_2 forces the hierarchy of Planck constants.

0.5.2 Quantum Computation in TGD Universe

Topological Quantum Computation in TGD Universe

Topological quantum computation (TQC) is one of the most promising approaches to quantum computation. The coding of logical qubits to the entanglement of topological quantum numbers promises to solve the de-coherence problem whereas the S-matrices of topological field theories (modular functors) providing unitary representations for braids provide a realization of quantum computer programs with gates represented as simple braiding operations. Because of their effective 2-dimensionality anyon systems are the best candidates for realizing the representations of braid groups.

TGD allows several new insights related to quantum computation. TGD predicts new information measures as number theoretical negative valued entanglement entropies defined for systems having extended rational entanglement and characterizes bound state entanglement as bound state entanglement. Negentropy Maximization Principle and p-adic length scale hierarchy of space-time sheets encourage to believe that Universe itself might do its best to resolve the de-coherence problem. The new view about quantum jump suggests strongly the notion of quantum parallel dissipation so that thermalization in shorter length scales would guarantee coherence in longer length scales. The possibility of negative energies and communications to geometric future in turn might trivialize the problems caused by long computation times: computation could be iterated again and again by turning the computer on in the geometric past and TGD inspired theory of consciousness predicts that something like this occurs routinely in living matter.

The absolute minimization of Kähler action is the basic variational principle of classical TGD and predicts extremely complex but non-chaotic magnetic flux tube structures, which can get knotted and linked. The dimension of CP_2 projection for these structures is $D = 3$. These structures are the corner stone of TGD inspired theory of living matter and provide the braid structures needed by TQC.

Anyons are the key actors of TQC and TGD leads to detailed model of anyons as systems consisting of track of a periodically moving charged particle realized as a flux tube containing the particle inside it. This track would be a space-time correlate for the outcome of dissipative processes producing the asymptotic self-organization pattern. These tracks in general carry vacuum Kähler charge which is topologized when the CP_2 projection of space-time sheet is $D = 3$. This explains charge fractionization predicted to occur also for other charged particles. When a system approaches chaos periodic orbits become slightly aperiodic and the correlate is flux tube which rotates N times before closing. This gives rise to Z_N valued topological quantum number crucial for TQC using anyons ($N = 4$ holds true in this case). Non-Abelian anyons are needed by TQC, and the existence of long range classical electro-weak fields predicted by TGD is an essential prerequisite of non-Abelianity.

Negative energies and zero energy states are of crucial importance of TQC in TGD. The possibility of phase conjugation for fermions would resolve the puzzle of matter-antimatter asymmetry in an elegant manner. Anti-fermions would be present but have negative energies. Quite generally, it is possible to interpret scattering as a creation of pair of positive and negative energy states, the latter representing the final state. One can characterize precisely the deviations of this Eastern world view with respect to the Western world view assuming an objective reality with a positive definite energy and understand why the Western illusion apparently works. In the case of TQC the initial *resp.* final state of braided anyon system would correspond to positive *resp.* negative energy state.

The light-like boundaries of magnetic flux tubes are ideal for TQC. The point is that 3-dimensional light-like quantum states can be interpreted as representations for the time evolution of a two-dimensional system and thus represented self-reflective states being "about something". The light-likeness (no geometric time flow) is a space-time correlate for the ceasing of subjective time flow during macro-temporal quantum coherence. The S-matrices of TQC can be coded to these light-like states such that each elementary braid operation corresponds to positive energy anyons near the boundary of the magnetic flux tube A and negative energy anyons with opposite topological charges residing near the boundary of flux tube B and connected by braided threads representing the quantum gate. Light-like boundaries also force Chern-Simons action as the only possible general coordinate invariant action since the vanishing of the metric determinant does not allow any other candidate. Chern-Simons action indeed defines the modular functor for braid coding for a TQC program.

DNA as Topological Quantum Computer

This chapter represents an overall view about gradual evolution of ideas about how DNA might act as a topological quantum computer. The first idea was that the braids formed by DNA or RNA could be involved but it turned out soon that this is probably not a realistic option. The reason is simply that DNA braiding is completely rigid and the number of braids is only 2. Three is the minimal number. This might put bells ringing.

The natural question was whether hydrogen bonds might be involved with short braids connecting DNA strand and its conjugate. Later it became clear that the braids could be quite long and would allow more complex braiding patterns.

The emergence of number theoretical braids as fundamental structures in quantum TGD led to more realistic visions. DNA strands would naturally define the linear structures from which braid strands emerge transversally. Dynamical braiding (recall the dance metaphor) is fundamental for tqc and would be naturally carried out by lipids at the cell membrane which as a liquid crystal is 2-D liquid.

The sections of this chapter summarize the still going development of ideas. It must be emphasized sections are not completely internally consistent since they indeed represent evolution of ideas in which working hypothesis are studied and rejected. One example of a rejected idea is that introns could be specialized to tqc. The idea that conjugate DNA is involved with tqc and ordinary DNA takes care of printing function defined in very general sense won this idea in the fight for memetic survival.

The model which looks the most plausible one relies on two specific ideas.

1. Sharing of labor means conjugate DNA would do tqc and DNA would "print" the outcome of tqc in terms of RNA yielding aminoacids in case of exons. RNA could result in the case of introns. The experience about computers and the general vision provided by TGD suggests that introns could express the outcome of tqc also electromagnetically in terms of standardized field patterns. Also speech would be a form of gene expression. The quantum states braid would entangle with characteristic gene expressions.

2. The manipulation of braid strands transversal to DNA must take place at 2-D surface. The ends of the space-like braid are dancers whose dancing pattern defines the time-like braid, the running of classical tqc program. Space-like braid represents memory storage and tqc program is automatically written to memory during the tqc. The inner membrane of the nuclear envelope and cell membrane with entire endoplasmic reticulum included are good candidates for dancing halls. The 2-surfaces containing the ends of the hydrophobic ends of lipids could be the parquets and lipids the dancers. This picture seems to make sense.

0.5.3 Part III: Categories, Number Theory and Consciousness

Infinite primes and consciousness

Infinite primes are besides p-adicization and the representation of space-time surface as a hyper-quaternionic sub-manifold of hyper-octonionic space, basic pillars of the vision about TGD as a generalized number theory and will be discussed in the third part of the multi-chapter devoted to the attempt to articulate this vision as clearly as possible.

1. *Why infinite primes are unavoidable*

Suppose that 3-surfaces could be characterized by p-adic primes characterizing their effective p-adic topology. p-Adic unitarity implies that each quantum jump involves unitarity evolution U followed by a quantum jump. Simple arguments show that the p-adic prime characterizing the 3-surface representing the entire universe increases in a statistical sense. This leads to a peculiar paradox: if the number of quantum jumps already occurred is infinite, this prime is most naturally infinite. On the other hand, if one assumes that only finite number of quantum jumps have occurred, one encounters the problem of understanding why the initial quantum history was what it was. Furthermore, since the size of the 3-surface representing the entire Universe is infinite, p-adic length scale hypothesis suggest also that the p-adic prime associated with the entire universe is infinite.

These arguments motivate the attempt to construct a theory of infinite primes and to extend quantum TGD so that also infinite primes are possible. Rather surprisingly, one can construct what might be called generating infinite primes by repeating a procedure analogous to a quantization of a super symmetric quantum field theory. At given level of hierarchy one can identify the decomposition of space-time surface to p-adic regions with the corresponding decomposition of the infinite prime to primes at a lower level of infinity: at the basic level are finite primes for which one cannot find any formula.

2. Two views about the role of infinite primes and physics in TGD Universe

Two different views about how infinite primes, integers, and rationals might be relevant in TGD Universe have emerged.

1. The first view is based on the idea that infinite primes characterize quantum states of the entire Universe. 8-D hyper-octonions make this correspondence very concrete since 8-D hyper-octonions have interpretation as 8-momenta. By quantum-classical correspondence also the decomposition of space-time surfaces to p-adic space-time sheets should be coded by infinite hyper-octonionic primes. Infinite primes could even have a representation as hyper-quaternionic 4-surfaces of 8-D hyper-octonionic imbedding space.

2. The second view is based on the idea that infinitely structured space-time points define space-time correlates of mathematical cognition. The mathematical analog of Brahman=Atman identity would however suggest that both views deserve to be taken seriously.

3. Infinite primes and infinite hierarchy of second quantizations

The discovery of infinite primes suggested strongly the possibility to reduce physics to number theory. The construction of infinite primes can be regarded as a repeated second quantization of a super-symmetric arithmetic quantum field theory. Later it became clear that the process generalizes so that it applies in the case of quaternionic and octonionic primes and their hyper counterparts. This hierarchy of second quantizations means an enormous generalization of physics to what might be regarded a physical counterpart for a hierarchy of abstractions about abstractions about.... The ordinary second quantized quantum physics corresponds only to the lowest level infinite primes. This hierarchy can be identified with the corresponding hierarchy of space-time sheets of the many-sheeted space-time.

One can even try to understand the quantum numbers of physical particles in terms of infinite primes. In particular, the hyper-quaternionic primes correspond four-momenta and mass squared is prime valued for them. The properties of 8-D hyper-octonionic primes motivate the attempt to identify the quantum numbers associated with CP_2 degrees of freedom in terms of these primes. The representations of color group $SU(3)$ are indeed labelled by two integers and the states inside given representation by color hyper-charge and iso-spin.

It turns out that associativity constraint allows only rational infinite primes. One can however decompose rational infinite primes to hyper-octonionic infinite primes at lower level of the hierarchy. Physically this would mean that the number theoretic 8-momenta have only time-component. This decomposition is completely analogous to the decomposition of hadrons to its colored constituents and might be even interpreted in terms of color confinement. The interpretation of the decomposition of rational primes to primes in the algebraic extensions of rationals, hyper-quaternions, and hyper-octonions would have an interpretation as an increase of number theoretical resolution and the principle of number theoretic confinement could be seen as a fundamental physical principle implied by associativity condition.

4. Infinite primes as a bridge between quantum and classical

An important stimulus came from the observation stimulated by algebraic number theory. Infinite primes can be mapped to polynomial primes and this observation allows to identify completely generally the spectrum of infinite primes whereas hitherto it was possible to construct explicitly only what might be called generating infinite primes.

This in turn led to the idea that it might be possible represent infinite primes (integers) geometrically as surfaces defined by the polynomials associated with infinite primes (integers).

Obviously, infinite primes would serve as a bridge between Fock-space descriptions and geometric descriptions of physics: quantum and classical. Geometric objects could be seen as concrete representations of infinite numbers providing amplification of infinitesimals to macroscopic deformations of space-time surface. We see the infinitesimals as concrete geometric shapes!

5. Conjecture about various equivalent characterizations of space-times as surfaces

One can imagine several number-theoretic characterizations of the space-time surface.

1. The approach based on octonions and quaternions suggests that space-time surfaces correspond to associative, or equivalently, hyper-quaternionic surfaces of hyper-octonionic imbedding space HO. Also co-associative, or equivalently, co-hyper-quaternionic surfaces are possible. These foliations can be mapped in a natural manner to the foliations of $H = M^4 \times CP_2$ by space-time surfaces which are identified as preferred extremals of the Kähler action (absolute minima or maxima for regions of space-time surface in which action density has definite sign). These views are consistent if hyper-quaternionic space-time surfaces correspond to so called Kähler calibrations [**?**].

2. Hyper-octonion real-analytic surfaces define foliations of the imbedding space to hyper-quaternionic 4-surfaces and their duals to co-hyper-quaternionic 4-surfaces representing space-time surfaces.

3. Rational infinite primes can be mapped to rational functions of n arguments. For hyper-octonionic arguments non-associativity makes these functions poorly defined unless one assumes that arguments are related by hyper-octonion real-analytic maps so that only single independent variable remains. These hyper-octonion real-analytic functions define foliations of HO to space-time surfaces if 2) holds true.

The challenge of optimist is to prove that these characterizations are equivalent.

6. The representation of infinite hyper-octonionic primes as 4-surfaces

The difficulties caused by the Euclidian metric signature of the number theoretical norm forced to give up the idea that space-time surfaces could be regarded as quaternionic sub-manifolds of octonionic space, and to introduce complexified octonions and quaternions resulting by extending quaternionic and octonionic algebra by adding imaginary units multiplied with $\sqrt{-1}$. This spoils the number field property but the notion of prime is not lost. The sub-space of hyper-quaternions *resp.* -octonions is obtained from the algebra of ordinary quaternions and octonions by multiplying the imaginary part with $\sqrt{-1}$. The transition is the number theoretical counterpart for the transition from Riemannian to pseudo-Riemannin geometry performed already in Special Relativity.

The commutative $\sqrt{-1}$ relates naturally to the algebraic extension of rationals generalized to an algebraic extension of rational quaternions and octonions and conforms with the vision about how quantum TGD could emerge from infinite dimensional Clifford algebra identifiable as a hyper-finite factor of type II_1 [**?**, **?**].

The notions of hyper-quaternionic and octonionic manifold make sense but it is implausible that $H = M^4 \times CP_2$ could be endowed with a hyper-octonionic manifold structure. Indeed, space-time surfaces can be assumed to be hyper-quaternionic or co-hyper-quaternionic 4-surfaces of 8-dimensional Minkowski space M^8 identifiable as the hyper-octonionic space HO. Since the hyper-quaternionic sub-spaces of HO with a locally fixed complex structure (preferred imaginary unit contained by tangent space at each point of HO) are labelled by CP_2, each (co)-hyper-quaternionic four-surface of HO defines a 4-surface of $M^4 \times CP_2$. One can say that the number-theoretic analog of spontaneous compactification occurs.

Any hyper-octonion analytic function $HO \rightarrow HO$ defines a function $g : OH \rightarrow SU(3)$ acting as the group of octonion automorphisms leaving a preferred imaginary unit invariant, and g in turn defines a foliation of HO and $H = M^4 \times CP_2$ by space-time surfaces. The selection can be local which means that G_2 appears as a local gauge group.

Since the notion of prime makes sense for the complexified octonions, it makes sense also for the hyper-octonions. It is possible to assign to infinite prime of this kind a hyper-octonion analytic polynomial $P : HO \rightarrow HO$ and hence also a foliation of HO and $H = M^4 \times CP_2$ by 4-surfaces. Therefore space-time surface can be seen as a geometric counterpart of a Fock state. The assignment is not unique but determined only up to an element of the local octonionic automorphism group G_2 acting in HO and fixing the local choices of the preferred imaginary unit of the hyper-octonionic tangent plane. In fact, a map $HO \rightarrow S^6$ characterizes the choice since $SO(6)$ acts effectively as a local gauge group.

The construction generalizes to all levels of the hierarchy of infinite primes if one poses the associativity requirement implying that hyper-octonionic variables are related by hyper-octonion real-analytic maps, and produces also representations for integers and rationals associated with hyper-octonionic numbers as space-time surfaces. A close relationship with algebraic geometry results and the polynomials define a natural hierarchical structure in the space of 3-surfaces. By the effective 2-dimensionality naturally associated with infinite primes represented by real polynomials 4-surfaces are determined by data given at partonic 2-surfaces defined by the intersections of 3-D and 7-D light-like causal determinants. In particular, the notions of genus and degree serve as classifiers of the algebraic geometry of the 4-surfaces. The great dream is to prove that this construction yields the solutions to the absolute minimization of Kähler action.

7. Generalization of ordinary number fields: infinite primes and cognition

Both fermions and p-adic space-time sheets are identified as correlates of cognition in TGD Universe. The attempt to relate these two identifications leads to a rather concrete model for how bosonic generators of super-algebras correspond to either real or p-adic space-time sheets (actions and intentions) and fermionic generators to pairs of real space-time sheets and their p-adic variants obtained by algebraic continuation (note the analogy with fermion hole pairs).

The introduction of infinite primes, integers, and rationals leads also to a generalization of real numbers since an infinite algebra of real units defined by finite ratios of infinite rationals multiplied by ordinary rationals which are their inverses becomes possible. These units are not units in the p-adic sense and have a finite p-adic norm which can be differ from one. This construction generalizes also to the case of hyper- quaternions and -octonions although non-commutativity and in case of octonions also non-associativity pose technical problems to which the reduction to ordinary rational is simplest cure which would however allow interpretation as decomposition of infinite prime to hyper-octonionic lower level constituents. Obviously this approach differs from the standard introduction of infinitesimals in the sense that sum is replaced by multiplication meaning that the set of real units becomes infinitely degenerate.

Infinite primes form an infinite hierarchy so that the points of space-time and imbedding space can be seen as infinitely structured and able to represent all imaginable algebraic structures. Certainly counter-intuitively, single space-time point is even capable of representing the quantum state of the entire physical Universe in its structure. For instance, in the real sense surfaces in the space of units correspond to the same real number 1, and single point, which is structure-less in the real sense could represent arbitrarily high-dimensional spaces as unions of real units.

One might argue that for the real physics this structure is completely invisible and is relevant only for the physics of cognition. On the other hand, one can consider the possibility of mapping the configuration space and configuration space spinor fields to the number theoretical anatomies of a single point of imbedding space so that the structure of this point would code for the world of classical worlds and for the quantum states of the Universe. Quantum jumps would induce changes of configuration space spinor fields interpreted as wave functions in the set of number theoretical anatomies of single point of imbedding space in the ordinary sense of the word, and evolution would reduce to the evolution of the structure of a typical space-time point in the system. Physics would reduce to space-time level but in a generalized sense. Universe would be an algebraic hologram, and there is an obvious connection both with Brahman=Atman identity of Eastern philosophies and Leibniz's notion of monad.

Category theory, quantum TGD and TGD inspired theory of consciousness

Category theory has been proposed as a new approach to the deep problems of modern physics, in particular quantization of General Relativity. Category theory might provide the desired systematic approach to fuse together the bundles of general ideas related to the construction of quantum TGD proper. Category theory might also have natural applications in the general theory of consciousness and the theory of cognitive representations.

1. The ontology of quantum TGD and TGD inspired theory of consciousness based on the trinity of geometric, objective and subjective existences could be expressed elegantly using the language of the category theory. Quantum classical correspondence might allow a mathematical formulation in terms of structure respecting functors mapping the categories associated with the three kinds of existences to each other.

2. Cognition is categorizing and category theory suggests itself as a tool for understanding cognition and self hierarchies and the abstraction processes involved with conscious experience.

3. Categories possess inherent generalized logic based on set theoretic inclusion which in TGD framework is naturally replaced with topological condensation: the outcome is quantum variants for the notions of sieve, topos, and logic. This suggests the possibility of geometrizing the logic of both geometric, objective and subjective existences and perhaps understand why ordinary consciousness experiences the world through Boolean logic and Zen consciousness experiences universe through three-valued logic. Also the right-wrong logic of moral rules and beautiful-ugly logic of aesthetics seem to be too naive and might be replaced with a more general quantum logic.

Bibliography

Online books about TGD

[1] M. Pitkänen (2006), *Topological Geometrodynamics: Overview*.
http://tgd.wippiespace.com/public_html/tgdview/tgdview.html.

[2] M. Pitkänen (2006), *Quantum Physics as Infinite-Dimensional Geometry*.
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html.

[3] M. Pitkänen (2006), *Physics in Many-Sheeted Space-Time*.
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html.

[4] M. Pitkänen (2006), *Quantum TGD*.
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html.

[5] M. Pitkänen (2006), *TGD as a Generalized Number Theory*.
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html.

[6] M. Pitkänen (2006), *p-Adic length Scale Hypothesis and Dark Matter Hierarchy*.
http://tgd.wippiespace.com/public_html/paddark/paddark.html.

[7] M. Pitkänen (2006), *TGD and Fringe Physics*.
http://tgd.wippiespace.com/public_html/freenergy/freenergy.html.

Online books about TGD inspired theory of consciousness and quantum biology

[8] M. Pitkänen (2006), *Bio-Systems as Self-Organizing Quantum Systems*.
http://tgd.wippiespace.com/public_html/bioselforg/bioselforg.html.

[9] M. Pitkänen (2006), *Quantum Hardware of Living Matter*.
http://tgd.wippiespace.com/public_html/bioware/bioware.html.

[10] M. Pitkänen (2006), *TGD Inspired Theory of Consciousness*.
http://tgd.wippiespace.com/public_html/tgdconsc/tgdconsc.html.

[11] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*.
http://tgd.wippiespace.com/public_html/genememe/genememe.html.

[12] M. Pitkänen (2006), *TGD and EEG*.
http://tgd.wippiespace.com/public_html/tgdeeg/tgdeeg/tgdeeg.html.

[13] M. Pitkänen (2006), *Bio-Systems as Conscious Holograms*.
http://tgd.wippiespace.com/public_html/hologram/hologram.html.

[14] M. Pitkänen (2006), *Magnetospheric Consciousness*.
http://tgd.wippiespace.com/public_html/magnconsc/magnconsc.html.

[15] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*.
http://tgd.wippiespace.com/public_html/magnconsc/mathconsc.html.

References to the chapters of books

[C6] The chapter *Was von Neumann Right After All* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#vNeumann.

[C10] The chapter *Does TGD Predict the Spectrum of Planck Constants?* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#Planck.

[D6] The chapter *TGD and Astrophysics* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#astro.

[E1] The chapter *TGD as a Generalized Number Theory: p-Adicization Program* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visiona.

[E2] The chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visionb.

[E3] The chapter *TGD as a Generalized Number Theory: Infinite Primes* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visionc.

[E9] The chapter *Topological Quantum Computation in TGD Universe* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#tqc.

[I1] The chapter *Quantum Theory of Self-Organization* of [?].
http://tgd.wippiespace.com/public_html/bioselforg/bioselforg.html#selforgac.

[J6] The chapter *Coherent Dark Matter and Bio-Systems as Macroscopic Quantum Systems* of [?].
http://tgd.wippiespace.com/public_html/bioware/bioware.html#darkbio.

[L1] The chapter *Genes and Memes* of [?].
http://tgd.wippiespace.com/public_html/genememe/genememe.html#genememec.

[L2] The chapter *Many-Sheeted DNA* of [?].
http://tgd.wippiespace.com/public_html/genememe/genememe.html#genecodec.

[M3] The chapter *Dark Matter Hierarchy and Hierarchy of EEGs* of [?].
http://tgd.wippiespace.com/public_html/tgdeeg/tgdeeg/tgdeeg.html#eegdark.

Articles related to TGD

[16] Pitkänen, M. (1983) International Journal of Theor. Phys. ,22, 575.

[17] M. Pitkänen (2006), *Topological Geometrodynamics: an Overall View*,
http://tgd.wippiespace.com/public_html/TGDbrief.pdf.

[18] M. Pitkänen (2006), *TGD Inspired Theory of Consciousness*,
http://tgd.wippiespace.com/public_html/tgdconsc.pdf.

[19] M. Pitkänen (2006), *TGD Inspired Quantum Model of Living Matter*,
http://tgd.wippiespace.com/public_html/quantumbio.pdf.

[20] M. Pitkïen (2007), *About Correspondence Between Infinite Primes, Space-time Surfaces, and Configuration Space Spinor Fields.*
http://tgd.wippiespace.com/public_html/brahma.pdf.

Mathematics related references

[21] Eguchi, T., Gilkey, B., Hanson, J. (1980): Phys. Rep. 66, 6.

[22] Hawking, S.,W. and Pope, C., N. (1978): *Generalized Spin Structures in Quantum Gravity.* Physics Letters Vol 73 B, no 1.

[23] Gibbons, G., W., Pope, C., N. (1977): *CP_2 as gravitational instanton.* Commun. Math. Phys. 55, 53.

[24] Pope, C., N. (1980): *Eigenfunctions and $Spin^c$ Structures on CP_2*. D.A.M.T.P. preprint.

[25] Eisenhart (1964): *Riemannian Geometry.* Princeton University Press.

[26] Spivak, M. (1970): *Differential Geometry I,II,III,IV.* Publish or Perish. Boston.

[27] Milnor, J. (1965): *Topology form Differential Point of View.* The University Press of Virginia.

[28] E. C. Zeeman (ed.)(1977), *Catastrophe Theory*, Addison-Wessley Publishing Company.

[29] Wallace (1968): *Differential Topology.* W. A. Benjamin, New York.

References related to physics anomalies

[30] D. Da Roacha and L. Nottale (2003), *Gravitational Structure Formation in Scale Relativity*, astro-ph/0310036.

Part I

NEW PHYSICS AND MATHEMATICS INVOLVED WITH TGD

Chapter 1

Was von Neumann Right After All?

1.1 Introduction

The work with TGD inspired model [?] for topological quantum computation [?] led to the realization that von Neumann algebras [?, ?, ?, ?], in particular so called hyper-finite factors of type II_1 [?], seem to provide the mathematics needed to develop a more explicit view about the construction of S-matrix. In this chapter I will discuss various aspects of type II_1 factors and their physical interpretation in TGD framework. The lecture notes of R. Longo [?] give a concise and readable summary about the basic definitions and results related to von Neumann algebras and I have used this material freely in this chapter.

1.1.1 Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [?].

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless.

1.1.2 Von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac [?] based on the notion of delta function, plus the emergence of s [?], the possibility to formulate the notion of delta function rigorously in terms of distributions [?, ?], and the emergence of path integral approach [?] meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [?, ?] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [?, ?] relate closely to type II_1 factors. In topological quantum computation [?] based on braid groups [?] modular S-matrices they play an especially important role.

In algebraic quantum field theory [?] defined in Minkowski space the algebras of observables associated with bounded space-time regions correspond quite generally to the type III_1 hyper-finite factor [?, ?].

1.1.3 Factors of type II_1 and quantum TGD

For me personally the realization that TGD Universe is tailored for topological quantum computation [?] led also to the realization that hyper-finite (ideal for numerical approximations) von Neumann algebras of type II_1 have a direct relevance for TGD.

The basic facts about hyper-finite von Neumann factors of type II_1 suggest a more concrete view about the general mathematical framework needed.

1. The effective 2-dimensionality of the construction of quantum states and configuration space geometry in quantum TGD framework makes hyper-finite factors of type II_1 very natural as operator algebras of the state space. Indeed, the generators of conformal algebras, the gamma matrices of the configuration space, and the modes of the induced spinor fields are labeled by discrete labels. Hence the tangent space of the configuration space is a separable Hilbert space and its Clifford algebra is a hyper-finite type II_1 factor. Super-symmetry requires that the bosonic algebra generated by configuration space Hamiltonians and the Clifford algebra of configuration space both correspond to hyper-finite type II_1 factors.

2. Four-momenta relate to the positions of tips of future and past directed light cones appearing naturally in the construction of S-matrix. In fact, configuration space of 3-surfaces can be regarded as union of big-bang/big crunch type configuration spaces obtained as a union of light-cones parameterized by the positions of their tips. The algebras of observables associated with bounded regions of M^4 are hyper-finite and of type III_1 in algebraic quantum field theory [?]. The algebras of observables in the space spanned by the tips of these light-cones are not needed in the construction of S-matrix so that there are good hopes of avoiding infinities coming from infinite traces.

3. Many-sheeted space-time concept forces to refine the notion of sub-system. Jones inclusions $\mathcal{N} \subset \mathcal{M}$ for factors of type II_1 define in a generic manner to imbed interacting sub-systems to a universal II_1 factor which now naturally corresponds to the infinite Clifford algebra of the tangent space of configuration space of 3-surfaces and contains interaction as $\mathcal{M} : \mathcal{N}$-dimensional analog of tensor factor. Topological condensation of space-time sheet to a larger space-time sheet, the formation of bound states by the generation of join along boundaries bonds, interaction vertices in which space-time surface branches like a line of Feynman diagram: all these situations might be described by Jones inclusion [?, ?] characterized by the Jones index $\mathcal{M} : \mathcal{N}$ assigning to the inclusion also a minimal conformal field theory and quantum group in case of $\mathcal{M} : \mathcal{N} < 4$ and conformal theory with $k = 1$ Kac Moody for $\mathcal{M} : \mathcal{N} = 4$ [?].

4. Von Neumann's somewhat artificial idea about identical a priori probabilities for states could replaced with the finiteness requirement of quantum theory. Indeed, it is traces which produce the infinities of quantum field theories. That $\mathcal{M} : \mathcal{N} = 4$ option is not realized physically as quantum field theory (it would rather correspond to string model type theory characterized by a Kac-Moody algebra instead of quantum group), could correspond to the fact that dimensional

regularization works only in $D = 4 - \epsilon$. Dimensional regularization with space-time dimension $D = 4 - \epsilon \to 4$ could be interpreted as the limit $\mathcal{M} : \mathcal{N} \to 4$. $\mathcal{M}$ as an $\mathcal{M} : \mathcal{N}$-dimensional $\mathcal{N}$-module would provide a concrete model for a quantum space with non-integral dimension as well as its Clifford algebra. An entire sequence of regularized theories corresponding to the allowed values of $\mathcal{M} : \mathcal{N}$ would be predicted.

1.1.4 How to localize infinite-dimensional Clifford algebra?

The basic new idea is to make this algebra *local*: local Clifford algebra as a generalization of gamma field of string models.

1. Represent Minkowski coordinate of M^d as linear combination of gamma matrices of D-dimensional space. This is the first guess. One fascinating finding is that this notion can be quantized and classical M^d is genuine quantum M^d with coordinate values eigenvalues of quantal commuting Hermitian operators built from matrix elements. Euclidian space is not obtained in this manner. Minkowski signature is something quantal and the standard quantum group $Gl_(2,q)(C)$ with (non-Hermitian matrix elements) gives M^4.

2. Form power series of the M^d coordinate represented as linear combination of gamma matrices with coefficients in corresponding infinite-D Clifford algebra. You would get tensor product of two algebra.

3. There is however a problem: one cannot distinguish the tensor product from the original infinite-D Clifford algebra. $D = 8$ is however an exception! You can replace gammas in the expansion of M^8 coordinate by hyper-octonionic units which are non-associative (or octonionic units in quantum complexified-octonionic case). Now one cannot anymore absorb the tensor factor to the Clifford algebra and one obatins a genuine M^8-localized factor of type II_1. Everything is determined by infinite-dimensional gamma matrix fields analogous to conformal super fields with z replaced by hyperoctonion.

4. Octonionic non-associativity actually reproduces whole classical and quantum TGD: space-time surface must be associative sub-manifolds hence hyper-quaternionic surfaces of M^8. Representability as surfaces in $M^4 \times CP_2$ follows naturally, the notion of configuration space of 3-surfaces, etc....

1.1.5 Non-trivial S-matrix from the Connes tensor product for free fields

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors of type II_1 could provide the mathematics needed to develop a more explicit view about the construction of S-matrix. This has turned out to be the case to the extent that a general master formula for S-matrix with interactions described as a deformation of ordinary tensor product to Connes tensor products emerges.

It has been for few years clear that TGD could emerge from the mere infinite-dimensionality of the Clifford algebra of infinite-dimensional "world of classical worlds" and from number theoretical vision in which classical number fields play a key role and determine imbedding space and space-time dimensions. This would fix completely the "world of classical worlds".

Infinite-dimensional Clifford algebra is a standard representation for von Neumann algebra known as a hyper-finite factor of type II_1. In TGD framework the infinite tensor power of C(8), Clifford algebra of 8-D space would be the natural representation of this algebra.

This picture has profound implications. Consider first the construction of S-matrix.

1. A non-perturbative construction of S-matrix emerges. The deep principle is simple. The canonical outer automorphism for von Neumann algebras defines a natural candidate unitary transformation giving rise to propagator. This outer automorphism is however trivial for II_1 factors meaning that all lines appearing in Feynman diagrams must be on mass shell states satisfying Super Virasoro conditions. One can allow all possible diagrams: all on mass shell loop corrections vanish by unitarity and what remains are diagrams with single N-vertex.

2. At 2-surface representing N-vertex space-time sheets representing generalized Bohr orbits of incoming and outgoing particles meet. This vertex involves von Neumann trace (finite!) of localized gamma matrices expressible in terms of fermionic oscillator operators and defining free fields satisfying Super Virasoro conditions.

3. For free fields ordinary tensor product would not give interacting theory. What makes S-matrix non-trivial is that *Connes tensor product* is used instead of the ordinary one. This tensor product is a universal non-pertrubative description for interactions! Interactions result as a deformation of tensor product. Unitarity of resulting S-matrix is unproven but I dare believe that it holds true.

4. The subfactor $\mathcal{N}$ defining the Connes tensor product has interpretation in terms of the interaction between experimenter and measured system and each interaction type defines its own Connes tensor product. Basically $\mathcal{N}$ represents the limitations of the experimenter. For instance, IR and UV cutoffs could be seen as primitive manners to describe what $\mathcal{N}$ describes much more elegantly. At the limit when $\mathcal{N}$ contains only single element, theory would become free field theory but this is ideal situation never achievable.

1.1.6 Cognitive consciousness, quantum computations, and Jones inclusions

Large $\hbar$ phases provide good hopes of realizing topological quantum computation. There is an additional new element. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and decoherence is not a problem as long as it does not induce this transition.

1.2 Von Neumann algebras

In this section basic facts about von Neumann algebras are summarized using as a background material the concise summary given in the lecture notes of Longo [**?**].

1.2.1 Basic definitions

A formal definition of von Neumann algebra [**?**, **?**, **?**] is as a *-subalgebra of the set of bounded operators $\mathcal{B}(\mathcal{H})$ on a Hilbert space $\mathcal{H}$ closed under weak operator topology, stable under the conjugation $J =^*$: $x \rightarrow x^*$, and containing identity operator Id. This definition allows also von Neumann algebras for which the trace of the unit operator is not finite.

Identity operator is the only operator commuting with a simple von Neumann algebra. A general von Neumann algebra allows a decomposition as a direct integral of simple algebras, which von Neumann called factors. Classification of von Neumann algebras reduces to that for factors.

$\mathcal{B}(\mathcal{H})$ has involution * and is thus a *-algebra. $\mathcal{B}(\mathcal{H})$ has order order structure $A \geq 0 : (Ax, x) \geq 0$. This is equivalent to $A = BB^*$ so that order structure is determined by algebraic structure. $\mathcal{B}(\mathcal{H})$ has metric structure in the sense that norm defined as supremum of $||Ax||$, $||x|| \leq 1$ defines the notion of continuity. $||A||^2 = inf\{\lambda > 0 : AA^* \leq \lambda I\}$ so that algebraic structure determines metric structure.

There are also other topologies for $\mathcal{B}(\mathcal{H})$ besides norm topology.

1. $A_i \rightarrow A$ strongly if $||Ax - A_ix|| \rightarrow 0$ for all x. This topology defines the topology of C^* algebra. $\mathcal{B}(\mathcal{H})$ is a Banach algebra that is $||AB|| \leq ||A|| \times ||B||$ (inner product is not necessary) and also C^* algebra that is $||AA^*|| = ||A||^2$.

2. $A_i \rightarrow A$ weakly if $(A_ix, y) \rightarrow (Ax, y)$ for all pairs (x, y) (inner product is necessary). This topology defines the topology of von Neumann algebra as a sub-algebra of $\mathcal{B}(\mathcal{H})$.

Denote by M' the commutant of $\mathcal{M}$ which is also algebra. Von Neumann's bicommutant theorem says that $\mathcal{M}$ equals to its own bi-commutant. Depending on whether the identity operator has a finite trace or not, one distinguishes between algebras of type II_1 and type II_∞. II_1 factor allow trace with properties $tr(Id) = 1$, $tr(xy) = tr(yx)$, and $tr(x^*x) > 0$, for all $x \neq 0$. Let $L^2(\mathcal{M})$ be the Hilbert space obtained by completing $\mathcal{M}$ respect to the inner product defined $\langle x|y\rangle = tr(x^*y)$ defines inner product in $\mathcal{M}$ interpreted as Hilbert space. The normalized trace induces a trace in M', natural trace $Tr_{M'}$, which is however not necessarily normalized. JxJ defines an element of M': if $\mathcal{H} = L^2(\mathcal{M})$, the natural trace is given by $Tr_{M'}(JxJ) = tr_M(x)$ for all $x \in M$ and bounded.

1.2.2 Basic classification of von Neumann algebras

Consider first some definitions. First of all, Hermitian operators with positive trace expressible as products xx^* are of special interest since their sums with positive coefficients are also positive.

In quantum mechanics Hermitian operators can be expressed in terms of projectors to the eigen states. There is a natural partial order in the set of isomorphism classes of projectors by inclusion: $E < F$ if the image of $\mathcal{H}$ by E is contained to the image of $\mathcal{H}$ by a suitable isomorph of F. Projectors are said to be metrically equivalent if there exist a partial isometry which maps the images $\mathcal{H}$ by them to each other. In the finite-dimensional case metric equivalence means that isomorphism classes are identical $E = F$.

The algebras possessing a minimal projection E_0 satisfying $E_0 \leq F$ for any F are called type I algebras. Bounded operators of n-dimensional Hilbert space define algebras I_n whereas the bounded operators of infinite-dimensional separable Hilbert space define the algebra I_∞. I_n and I_∞ correspond to the operator algebras of quantum mechanics. The states of harmonic oscillator correspond to a factor of type I.

The projection F is said to be finite if $F < E$ and $F \equiv E$ implies $F = E$. Hence metric equivalence means identity. Simple von Neumann algebras possessing finite projections but no minimal projections so that any projection E can be further decomposed as $E = F + G$, are called factors of type II.

Hyper-finiteness means that any finite set of elements can be approximated arbitrary well with the elements of a finite-dimensional sub-algebra. The hyper-finite II_∞ algebra can be regarded as a tensor product of hyper-finite II_1 and I_∞ algebras. Hyper-finite II_1 algebra can be regarded as a Clifford algebra of an infinite-dimensional separable Hilbert space sub-algebra of I_∞.

Hyper-finite II_1 algebra can be constructed using Clifford algebras $C(2n)$ of $2n$-dimensional spaces and identifying the element x of $2^n \times 2^n$ dimensional $C(n)$ as the element $diag(x,x)/2$ of $2^{n+1} \times 2^{n+1}$-dimensional $C(n+1)$. The union of algebras $C(n)$ is formed and completed in the weak operator topology to give a hyper-finite II_1 factor. This algebra defines the Clifford algebra of infinite-dimensional separable Hilbert space and is thus a sub-algebra of I_∞ so that hyper-finite II_1 algebra is more regular than I_∞.

von Neumann algebras possessing no finite projections (all traces are infinite or zero) are called algebras of type III. It was later shown by Connes [?] that these algebras are labeled by a parameter varying in the range $[0,1]$, and referred to as algebras of type III_x. III_1 category contains a unique hyper-finite algebra. It has been found that the algebras of observables associated with bounded regions of 4-dimensional Minkowski space in quantum field theories correspond to hyper-finite factors of type III_1 [?]. Also statistical systems at finite temperature correspond to factors of type III and temperature parameterizes one-parameter set of automorphisms of this algebra [?]. Zero temperature limit correspond to I_∞ factor and infinite temperature limit to II_1 factor.

1.2.3 Non-commutative measure theory and non-commutative topologies and geometries

von Neumann algebras and C^* algebras give rise to non-commutative generalizations of ordinary measure theory (integration), topology, and geometry. It must be emphasized that these structures are completely natural aspects of quantum theory. In particular, for the hyper-finite type II_1 factors quantum groups and Kac Moody algebras [?] emerge quite naturally without any need for ad hoc modifications such as making space-time coordinates non-commutative. The effective 2-dimensionality of quantum TGD (partonic or stringy 2-surfaces code for states) means that these structures appear completely naturally in TGD framework.

Non-commutative measure theory

von Neumann algebras define what might be a non-commutative generalization of measure theory and probability theory [?].

1. Consider first the commutative case. Measure theory is something more general than topology since the existence of measure (integral) does not necessitate topology. Any measurable function f in the space $L^\infty(X,\mu)$ in measure space (X,μ) defines a bounded operator M_f in the space $\mathcal{B}(L^2(X,\mu))$ of bounded operators in the space $L^2(X,\mu)$ of square integrable functions with action of M_f defined as $M_f g = fg$.

2. Integral over $\mathcal{M}$ is very much like trace of an operator $f_{x,y} = f(x)\delta(x,y)$. Thus trace is a natural non-commutative generalization of integral (measure) to the non-commutative case and defined for von Neumann algebras. In particular, generalization of probability measure results if the case $tr(Id) = 1$ and algebras of type I_n and II_1 are thus very natural from the point of view of non-commutative probability theory.

The trace can be expressed in terms of a cyclic vector Ω or vacuum/ground state in physicist's terminology. Ω is said to be cyclic if the completion $\overline{M\Omega} = H$ and separating if $x\Omega$ vanishes only for $x = 0$. Ω is cyclic for $\mathcal{M}$ if and only if it is separating for M'. The expression for the trace given by

$$Tr(ab) = \left(\frac{(ab+ba)}{2}, \Omega\right) \tag{1.2.1}$$

is symmetric and allows to defined also inner product as $(a,b) = Tr(a^*b)$ in $\mathcal{M}$. If Ω has unit norm $(\Omega,\Omega) = 1$, unit operator has unit norm and the algebra is of type II_1. Fermionic oscillator operator algebra with discrete index labeling the oscillators defines II_1 factor. Group algebra is second example of II_1 factor.

The notion of probability measure can be abstracted using the notion of state. State ω on a C^* algebra with unit is a positive linear functional on $\mathcal{U}$, $\omega(1) = 1$. By so called KMS construction [?] any state ω in C^* algebra $\mathcal{U}$ can be expressed as $\omega(x) = (\pi(x)\Omega, \Omega)$ for some cyclic vector Ω and π is a homomorphism $\mathcal{U} \to \mathcal{B}(\mathcal{H})$.

Non-commutative topology and geometry

C^* algebras generalize in a well-defined sense ordinary topology to non-commutative topology.

1. In the Abelian case Gelfand Naimark theorem [?] states that there exists a contravariant functor F from the category of unital abelian C^* algebras and category of compact topological spaces. The inverse of this functor assigns to space X the continuous functions f on X with norm defined by the maximum of f. The functor assigns to these functions having interpretation as eigen states of mutually commuting observables defined by the function algebra. These eigen states are delta functions localized at single point of X. The points of X label the eigenfunctions and thus define the spectrum and obviously span X. The connection with topology comes from the fact that continuous map $Y \to X$ corresponds to homomorphism $C(X) \to C(Y)$.

2. In non-commutative topology the function algebra $C(X)$ is replaced with a general C^* algebra. Spectrum is identified as labels of simultaneous eigen states of the Cartan algebra of C^* and defines what can be observed about non-commutative space X.

3. Non-commutative geometry can be very roughly said to correspond to *-subalgebras of C^* algebras plus additional structure such as symmetries. The non-commutative geometry of Connes [?] is a basic example here.

1.2.4 Modular automorphisms

von Neumann algebras allow a canonical unitary evolution associated with any state ω fixed by the selection of the vacuum state Ω [?]. This unitary evolution is an automorphism fixed apart form unitary automorphisms $A \to UAU^*$ related with the choice of Ω.

Let ω be a normal faithful state: $\omega(x^*x) > 0$ for any x. One can map $\mathcal{M}$ to $L^2(\mathcal{M})$ defined as a completion of $\mathcal{M}$ by $x \rightarrow x\Omega$. The conjugation * in $\mathcal{M}$ has image at Hilbert space level as a map $S_0 : x\Omega \rightarrow x^*\Omega$. The closure of S_0 is an anti-linear operator and has polar decomposition $S = J\Delta^{1/2}$, $\Delta = SS^*$. Δ is positive self-adjoint operator and J anti-unitary involution. The following conditions are satisfied

$$\begin{aligned} \Delta^{it}\mathcal{M}\Delta^{-it} &= \mathcal{M} \ , \\ J\mathcal{M}J &= \mathcal{M}' \ . \end{aligned} \tag{1.2.2}$$

Δ^{it} is obviously analogous to the time evolution induced by positive definite Hamiltonian and induces also the evolution of the expectation ω as $\pi \rightarrow \Delta^{it}\pi\Delta^{-it}$.

1.2.5 Joint modular structure and sectors

Let $\mathcal{N} \subset \mathcal{M}$ be an inclusion. The unitary operator $\gamma = J_N J_M$ defines a canonical endomorphisms $M \rightarrow N$ in the sense that it depends only up to inner automorphism on $\mathcal{N}$, γ defines a sector of $\mathcal{M}$. The sectors of $\mathcal{M}$ are defined as $Sect(\mathcal{M}) = End(\mathcal{M})/Inn(\mathcal{M})$ and form a semi-ring with respected to direct sum and composition by the usual operator product. It allows also conjugation.

$L^2(\mathcal{M})$ is a normal bi-module in the sense that it allows commuting left and right multiplications. For $a, b \in M$ and $x \in L^2(\mathcal{M})$ these multiplications are defined as $axb = aJb^*Jx$ and it is easy to verify the commutativity using the factor $Jy^*J \in \mathcal{M}'$. Connes [?] has shown that all normal bi-modules arise in this way up to unitary equivalence so that representation concepts make sense. It is possible to assign to any endomorphism ρ index $Ind(\rho) \equiv M : \rho(\mathcal{M})$. This means that the sectors are in 1-1 correspondence with inclusions. For instance, in the case of hyper-finite II_1 they are labeled by Jones index. Furthermore, the objects with non-integral dimension $\sqrt{[\mathcal{M} : \rho(\mathcal{M})]}$ can be identified as quantum groups, loop groups, infinite-dimensional Lie algebras, etc...

1.2.6 Basic facts about hyper-finite factors of type III

Hyper-finite factors of type II_1, II_∞ and III_1, III_0, III_λ, $\lambda \in (0, 1)$, allow by definition hierarchy of finite approximations and are unique as von Neumann algebras. Also hyper-finite factors of type II_∞ and type III could be relevant for the formulation of TGD. HFFs of type II_∞ and III could appear at the level operator algebra but that at the level of quantum states one would obtain HFFs of type II_1. These extended factors inspire highly non-trivial conjectures about quantum TGD. The book of Connes [?] provides a detailed view about von Neumann algebras in general.

Basic definitions and facts

A highly non-trivial result is that HFFs of type II_∞ are expressible as tensor products $II_\infty = II_1 \otimes I_\infty$, where II_1 is hyper-finite [?].

1. The existence of one-parameter family of outer automorphisms

The unique feature of factors of type III is the existence of one-parameter unitary group of outer automorphisms. The automorphism group originates in the following manner.

1. Introduce the notion of linear functional in the algebra as a map $\omega : \mathcal{M} \rightarrow C$. ω is said to be hermitian it respects conjugation in $\mathcal{M}$; positive if it is consistent with the notion of positivity for elements of $\mathcal{M}$ in which case it is called weight; state if it is positive and normalized meaning that $\omega(1) = 1$, faithful if $\omega(A) > 0$ for all positive A; a trace if $\omega(AB) = \omega(BA)$, a vector state if $\omega(A)$ is "vacuum expectation" $\omega_\Omega(A) = (\Omega, \omega(A)\Omega)$ for a non-degenerate representation $(\mathcal{H}, \pi)$ of $\mathcal{M}$ and some vector $\Omega \in \mathcal{H}$ with $||\Omega|| = 1$.

2. The existence of trace is essential for hyper-finite factors of type II_1. Trace does not exist for factors of type III and is replaced with the weaker notion of state. State defines inner product via the formula $(x, y) = \phi(y^*x)$ and * is isometry of the inner product. *-operator has property known as pre-closedness implying polar decomposition $S = J\Delta^{1/2}$ of its closure. Δ is positive definite unbounded operator and J is isometry which restores the symmetry between $\mathcal{M}$ and its commutant $\mathcal{M}'$ in the Hilbert space $\mathcal{H}_\phi$, where $\mathcal{M}$ acts via left multiplication: $\mathcal{M}' = J\mathcal{M}J$.

3. The basic result of Tomita-Takesaki theory is that Δ defines a one-parameter group $\sigma_\phi^t(x) = \Delta^{it} x \Delta^{-it}$ of automorphisms of $\mathcal{M}$ since one has $\Delta^{it}\mathcal{M}\Delta^{-it} = \mathcal{M}$. This unitary evolution is an automorphism fixed apart from unitary automorphism $A \rightarrow UAU^*$ related with the choice of ϕ. For factors of type I and II this automorphism reduces to inner automorphism so that the group of outer automorphisms is trivial as is also the outer automorphism associated with ω. For factors of type III the group of these automorphisms divided by inner automorphisms gives a one-parameter group of $Out(\mathcal{M})$ of outer automorphisms, which does not depend at all on the choice of the state ϕ.

More precisely, let ω be a normal faithful state: $\omega(x^*x) > 0$ for any x. One can map $\mathcal{M}$ to $L^2(\mathcal{M})$ defined as a completion of $\mathcal{M}$ by $x \rightarrow x\Omega$. The conjugation * in $\mathcal{M}$ has image at Hilbert space level as a map $S_0 : x\Omega \rightarrow x^*\Omega$. The closure of S_0 is an anti-linear operator and has polar decomposition $S = J\Delta^{1/2}$, $\Delta = SS^*$. Δ is positive self-adjoint operator and J anti-unitary involution. The following conditions are satisfied

$$\begin{aligned} \Delta^{it}\mathcal{M}\Delta^{-it} &= \mathcal{M} \ , \\ J\mathcal{M}J &= \mathcal{M}' \ . \end{aligned} \tag{1.2.3}$$

Δ^{it} is obviously analogous to the time evolution induced by positive definite Hamiltonian and induces also the evolution of the expectation ω as $\pi \rightarrow \Delta^{it}\pi\Delta^{-it}$. What makes this result thought provoking is that it might mean a universal quantum dynamics apart from inner automorphisms and thus a realization of general coordinate invariance and gauge invariance at the level of Hilbert space.

2. Classification of HFFs of type III

Connes achieved an almost complete classification of hyper-finite factors of type III completed later by others. He demonstrated that they are labeled by single parameter $0 \leq \lambda \leq 1]$ and that factors of type III_λ, $0 \leq \lambda < 1$ are unique. Haagerup showed the uniqueness for $\lambda = 1$. The idea was that the the group has an invariant, the kernel $T(M)$ of the map from time like R to $Out(M)$, consisting of those values of the parameter t for which σ_ϕ^t reduces to an inner automorphism and to unity as outer automorphism. Connes also discovered also an invariant, which he called spectrum $S(\mathcal{M})$ of $\mathcal{M}$ identified as the intersection of spectra of $\Delta_\phi \backslash \{0\}$, which is closed multiplicative subgroup of R^+.

Connes showed that there are three cases according to whether $S(\mathcal{M})$ is

1. R^+, type III_1
2. $\{\lambda^n, n \in Z\}$, type III_λ.
3. $\{1\}$, type III_0.

   The value range of λ is this by convention. For the reversal of the automorphism it would be that associated with $1/\lambda$.

Connes constructed also an explicit representation of the factors $0 < \lambda < 1$ as crossed product II_∞ factor $\mathcal{N}$ and group Z represented as powers of automorphism of II_∞ factor inducing the scaling of trace by λ. The classification of HFFs of type III reduced thus to the classification of automorphisms of $\mathcal{N} \otimes \mathcal{B}(\mathcal{H}$. In this sense the theory of HFFs of type III was reduced to that for HFFs of type II_∞ or even II_1. The representation of Connes might be also physically interesting.

Probabilistic view about factors of type III

Second very concise representation of HFFs relies on thermodynamical thinking and realizes factors as infinite tensor product of finite-dimensional matrix algebras acting on state spaces of finite state systems with a varying and finite dimension n such that one assigns to each factor a density matrix characterized by its eigen values. Intuitively one can think the finite matrix factors as associated with n-state system characterized by its energies with density matrix ρ defining a thermodynamics. The logarithm of the ρ defines the single particle quantum Hamiltonian as $H = log(\rho)$ and $\Delta = \rho = exp(H)$ defines the automorphism σ_ϕ for each finite tensor factor as $exp(iHt)$. Obviously free field representation is in question.

Depending on the asymptotic behavior of the eigenvalue spectrum one obtains different factors [**?**].

1. Factor of type I corresponds to ordinary thermodynamics for which the density matrix as a function of matrix factor approaches sufficiently fast that for a system for which only ground state has non-vanishing Boltzmann weight.

2. Factor of type II_1 results if the density matrix approaches to identity matrix sufficiently fast. This means that the states are completely degenerate which for ordinary thermodynamics results only at the limit of infinite temperature. Spin glass could be a counterpart for this kind of situation.

3. Factor of type III results if one of the eigenvalues is above some lower bound for all tensor factors in such a manner that neither factor of type I or II_1 results but thermodynamics for systems having infinite number of degrees of freedom could yield this kind of situation.

This construction demonstrates how varied representations factors can have, a fact which might look frustrating for a novice in the field. In particular, the infinite tensor power of $M(2,C)$ with state defined as an infinite tensor power of $M(2,C)$ state assigning to the matrix A the complex number $(\lambda^{1/2}A_{11} + \lambda^{-1/2}\ \phi(A) = A_{22})/(\lambda^{1/2} + \lambda^{-1/2})$ defines HFF III_λ [?, ?]. Formally the same algebra which for $\lambda = 1$ gives ordinary trace and HFF of type II_1, gives III factor only by replacing trace with state. This simple model was discovered by Powers in 1967.

It is indeed the notion of state or thermodynamics is what distinguishes between factors. This looks somewhat weird unless one realizes that the Hilbert space inner product is defined by the "thermodynamical" state ϕ and thus probability distribution for operators and for their thermal expectation values. Inner product in turn defines the notion of norm and thus of continuity and it is this notion which differs dramatically for $\lambda = 1$ and $\lambda < 1$ so that the completions of the algebra differ dramatically.

In particular, there is no sign about I_∞ tensor factor or crossed product with Z represented as automorphisms inducing the scaling of trace by λ. By taking tensor product of I_∞ factor represented as tensor power with induces running from $-\infty$ to 0 and II_1 HFF with indices running from 1 to ∞ one can make explicit the representation of the automorphism of II_∞ factor inducing scaling of trace by λ and transforming matrix factors possessing trace given by square root of index $\mathcal{M} : \mathcal{N}$ to those with trace 2.

1.3 Braid group, von Neumann algebras, quantum TGD, and formation of bound states

The article of Vaughan Jones in [?] discusses the relation between knot theory, statistical physics, and von Neumann algebras. The intriguing results represented stimulate concrete ideas about how to understand the formation of bound states quantitatively using the notion of join along boundaries bond. All mathematical results represented in the following discussion can be found in [?] and in the references cited therein so that I will not bother to refer repeatedly to this article in the sequel.

1.3.1 Factors of von Neumann algebras

Von Neumann algebras M are algebras of bounded linear operators acting in Hilbert space. These algebras contain identity, are closed with respect to Hermitian conjugation, and are topologically complete. Finite-dimensional von Neuman algebras decompose into a direct sum of algebras M_n, which act essentially as matrix algebras in Hilbert spaces $\mathcal{H}_{nm}$, which are tensor products $C^n \otimes \mathcal{H}_m$. Here $\mathcal{H}_m$ is an m-dimensional Hilbert space in which M_n acts trivially. m is called the multiplicity of M_n.

A factor of von Neumann algebra is a von Neumann algebra whose center is just the scalar multiples of identity. The algebra of bounded operators in an infinite-dimensional Hilbert space is certainly a factor. This algebra decomposes into "atoms" represented by one-dimensional projection operators. This kind of von Neumann algebras are called type I factors.

The so called type II_1 factors and type III factors came as a surprise even for Murray and von Neumann. II_1 factors are infinite-dimensional and analogs of the matrix algebra factors M_n. They allow a trace making possible to define an inner product in the algebra. The trace defines a generalized dimension for any subspace as the trace of the corresponding projection operator. This dimension is however continuous and in the range $[0, 1]$: the finite-dimensional analog would be the dimension of the sub-space divided by the dimension of $\mathcal{H}_n$ and having values $(0, 1/n, 2/n, ..., 1)$. II_1 factors are isomorphic and there exists a minimal "hyper-finite" II_1 factor is contained by every other II_1 factor.

Just as in the finite-dimensional case, one can to assign a multiplicity to the Hilbert spaces where II_1 factors act on. This multiplicity, call it $dim_M(\mathcal{H})$ is analogous to the dimension of the Hilbert space tensor factor $\mathcal{H}_m$, in which II_1 factor acts trivially. This multiplicity can have all positive real values. Quite generally, von Neumann factors of type I and II_1 are in many respects analogous to the coefficient field of a vector space.

1.3.2 Sub-factors

Sub-factors $N \subset M$, where N and M are of type II_1 and have same identity, can be also defined. The observation that M is analogous to an algebraic extension of N motivates the introduction of index $|M : N|$, which is essentially the dimension of M with respect to N. This dimension is an analog for the complex dimension of CP_2 equal to 2 or for the algebraic dimension of the extension of p-adic numbers.

The following highly non-trivial results about the dimensions of the tensor factors hold true.

1. If $N \subset M$ are II_1 factors and $|M : N| < 4$, there is an integer $n \geq 3$ such $|M : N| = r = 4cos^2(\pi/n)$, $n \geq 3$.

2. For each number $r = 4cos^2(\pi/n)$ and for all $r \geq 4$ there is a sub-factor $R_r \subset R$ with $|R : R_r| = r$.

   One can say that M effectively decomposes to a tensor product of N with a space, whose dimension is quantized to a certain algebraic number r. The values of r corresponding to $n = 3, 4, 5, 6...$ are $r = 1, 2, 1 + \Phi \simeq 2.61, 3, ...$ and approach to the limiting value $r = 4$. For $r \geq 4$ the dimension becomes continuous.

An even more intriguing result is that by starting from $N \subset M$ with a projection e_N: $M \rightarrow N$ one can extend M to a larger II_1 algebra $\langle M, e_N\rangle$ such that one has

$$\begin{aligned} |\langle M, e_N\rangle : M| &= |M : N| \ , \\ tr(xe_N) &= |M : N|^{-1} tr(x) \ , \ x \in M \ . \end{aligned} \tag{1.3.1}$$

One can continue this process and the outcome is a tower of II_1 factors $M_i \subset M_{i+1}$ defined by $M_1 = N$, $M_2 = M$, $M_{i+1} = \langle M_i, e_{M_{i-1}}\rangle$. Furthermore, the projection operators $e_{M_i} \equiv e_i$ define a Temperley-Lieb representation of the braid algebra via the formulas

$$\begin{aligned} e_i^2 &= e_i \ , \\ e_i e_{i\pm1} e_i &= \tau e_i \ , \ \tau = 1/|M : N| \\ e_i e_j &= e_j e_i \ , \ |i - j| \geq 2 \ . \end{aligned} \tag{1.3.2}$$

Temperley Lieb algebra will be discussed in more detail later. Obviously the addition of a tensor factor of dimension r is analogous with the addition of a strand to a braid.

The hyper-finite algebra R is generated by the set of braid generators $\{e_1, e_2, ...\}$ in the braid representation corresponding to r. Sub-factor R_1 is obtained simply by dropping the lowest generator e_1, R_2 by dropping e_1 and e_2, etc..

1.3.3 II_1 factors and the spinor structure of infinite-dimensional configuration space of 3-surfaces

The following observations serve as very suggestive guidelines for how one could interpret the above described results in TGD framework.

1. The discrete spectrum of dimensions $1, 2, 1 + \Phi, 3, ..$ below $r < 4$ brings in mind the discrete energy spectrum for bound states whereas the for $r \geq 4$ the spectrum of dimensions is analogous to a continuum of unbound states. The fact that r is an algebraic number for $r < 4$ conforms

with the vision that bound state entanglement corresponds to entanglement probabilities in an extension of rationals defining a finite-dimensional extension of p-adic numbers for every prime p.

2. The discrete values of r correspond precisely to the angles ϕ allowed by the unitarity of Temperley-Lieb representations of the braid algebra with $d = -\sqrt{r}$. For $r \geq 4$ Temperley-Lieb representation is not unitary since $cos^2(\pi/n)$ becomes formally larger than one (n would become imaginary and continuous). This could mean that $r \geq 4$, which in the generic case is a transcendental number, represents unbound entanglement, which in TGD Universe is not stable against state preparation and state function reduction processes.

3. The formula $tr(xe_N) = |M : N|^{-1} tr(x)$ is completely analogous to the formula characterizing the normalization of the link invariant induced by the second Markov move in which a new strand is added to a braid such that it braids only with the leftmost strand and therefore does not change the knot resulting as a link closure. Hence the addition of a single strand seems to correspond to an introduction of an r-dimensional sub-factor to II_1 factor.

In TGD framework the generation of bound state has the formation of (possibly braided join along boundaries bonds as a space-time correlate and this encourages a rather concrete interpretation of these findings. Also the I_1 factors themselves have a nice interpretation in terms of the configuration space spinor structure.

1. The interpretation of II_1 factors in terms of Clifford algebra of configuration space

The Clifford algebra of an infinite-dimensional Hilbert space defines a II_1 factor. The counterparts for e_i would naturally correspond to the analogs of projection operators $(1+\sigma_i)/2$ and thus to operators of form $(1 + \Sigma_{ij})/2$, defined by a subset of sigma matrices. The first guess is that the index pairs are $(i, j) = (1, 2), (2, 3), (3, 4), ...$ The dimension of the Clifford algebra is 2^N for N-dimensional space so that $\Delta N = 1$ would correspond to $r = 2$ in the classical case and to one qubit. The problem with this interpretation is $r > 2$ has no physical interpretation: the formation of bound states is expected to reduce the value of r from its classical value rather than increase it.

One can however consider also the sequence $(i, j) = (1, 1 + k), (1 + k, 1 + 2k), (1 + 2k, 1 + 3k), ...$ For $k = 2$ the reduction of r from $r = 4$ would be due to the loss of degrees of freedom due to the formation of a bound state and $(r = 4, \Delta N = 2)$ would correspond to the classical limit resulting at the limit of weak binding. The effective elimination of the projection operators from the braid algebra would reflect this loss of degrees of freedom. This interpretation could at least be an appropriate starting point in TGD framework.

In TGD Universe physical states correspond to configuration space spinor fields, whose gamma matrix algebra is constructed in terms of second quantized free induced spinor fields defined at space-time sheets. The original motivation was the idea that the quantum states of the Universe correspond to the modes of purely classical free spinor fields in the infinite-dimensional configuration space of 3-surfaces (the world of classical worlds) possessing general coordinate invariant (in 4-dimensional sense!) Kähler geometry. Quantum information-theoretical motivation could have come from the requirement that these fields must be able to code information about the properties of the point (3-surface, and corresponding space-time sheet). Scalar fields would treat the 3-surfaces as points and are thus not enough. Induced spinor fields allow however an infinite number of modes: according to the naive Fourier analyst's intuition these modes are in one-one correspondence with the points of the 3-surface. Second quantization gives much more. Also non-local information about the induced geometry and topology must be coded, and here quantum entanglement for states generated by the fermionic oscillator operators coding information about the geometry of 3-surface provides enormous information storage capacity.

In algebraic geometry also the algebra of the imbedding space of algebraic variety divided by the ideal formed by functions vanishing on the surface codes information about the surface: for instance, the maximal ideals of this algebra code for the points of the surface (functions of imbedding space vanishing at a particular point). The function algebra of the imbedding space indeed plays a key role in the construction of the configuration space-geometry besides second quantized fermions.

The Clifford algebra generated by the configuration space gamma matrices at a given point (3-surface) of the configuration space of 3-surfaces could be regarded as a II_1-factor associated with the local tangent space endowed with Hilbert space structure (configuration space Kähler metric). The counterparts for e_i would naturally correspond to the analogs of projection operators $(1+\sigma_i)/2$ and thus operators of form $(G_{\overline{A}B} \times 1 + \Sigma_{\overline{A}B})$ formed as linear combinations of components of the Kähler metric and of the sigma matrices defined by gamma matrices and contracted with the generators of the isometries of the configuration space. The addition of single complex degree of freedom corresponds to $\Delta N = 2$ and $r = 4$ and the classical limit and would correspond to the addition of single braid. $(r < 4, \Delta N < 2)$ would be due to the binding effects.

$r = 1$ corresponds to $\Delta N = 0$. The first interpretation is in terms of strong binding so that the addition of particle does not increase the number of degrees of freedom. In TGD framework $r = 1$ might also correspond to the addition of zero modes which do not contribute to the configuration space metric and spinor structure but have a deep physical significance. $(r = 2, \Delta N = 1)$ would correspond to strong binding reducing the spinor and space-time degrees of freedom by a factor of half. $r = \Phi^2$ $(n = 5)$ *resp.* $r = 3$ $(n = 6)$ corresponds to $\Delta N_r \simeq 1.3885$ *resp.* $\Delta N_r = 1.585$. Using the terminology of quantum field theories, one might say that in the infinite-dimensional context a given complex bound state degree of freedom possesses anomalous real dimension $r < 2$. $r \geq 4$ would correspond to a unbound entanglement and increasingly classical behavior.

1.3.4 About possible space-time correlates for the hierarchy of II_1 subfactors

By quantum classical correspondence the infinite-dimensional physics at the configuration space level should have definite space-time correlates. In particular, the dimension r should have some fractal dimension as a space-time correlate.

1. *Quantum classical correspondence*

Join along boundaries bonds serve as correlates for bound state formation. The presence of join along boundaries bonds would lead to a generation of bound states just by reducing the degrees of freedom to those of connected 3-surface. The bonds would constrain the two 3-surfaces to single space-like section of imbedding space.

This picture would allow to understand the difficulties related to Bethe-Salpeter equations for bound states based on the assumption that particles are points moving in M^4. The restriction of particles to time=constant section leads to a successful theory which is however non-relativistic. The basic binding energy would relate to the entanglement of the states associated with the bonded 3-surfaces. Since the classical energy associated with the bonds is positive, the binding energy tends to be reduced as r increases.

By spin glass degeneracy join along boundaries bonds have an infinite number of degrees of freedom in the ordinary sense. Since the system is infinite-dimensional and quantum critical, one expects that the number r of degrees freedom associated with a single join along boundaries bond is universal. Since join along boundaries bonds correspond to the strands of a braid and are correlates for the bound state formation, the natural guess is that $r = 4cos^2(\pi/n)$, $n = 3, 4, 5, ...$ holds true. $r < 4$ should characterize both binding energy and the dimension of the effective tensor factor introduced by a new join along boundaries bond.

The assignment of 2 "bare" and $\Delta N \leq 2$ renormalized real dimensions to single join along boundaries bond is consistent with the effective two-dimensionality of anyon systems and with the very notion of the braid group. The picture conforms also with the fact that the degrees of freedom in question are associated with metrically 2-dimensional light-like boundaries (of say magnetic flux tubes) acting as causal determinants. Also vibrational degrees of freedom described by Kac-Moody algebra are present and the effective 2-dimensionality means that these degrees of freedom are not excited and only topological degrees of freedom coded by the position of the puncture remain.

$(r \geq 4, \Delta N \geq 2)$, if possible at all, would mean that the tensor factor associated with the join along boundaries bond is effectively more than 4-dimensional due to the excitation of the vibrational Kac-Moody degrees of freedom. The finite value of r would mean that most of theme are eliminated also now but that their number is so large that bound state entanglement is not possible anymore.

The introduction of non-integer dimension could be seen as an effective description of an infinite-dimensional system as a finite-dimensional system in the spirit of renormalization group philosophy. The non-unitarity of $r \geq 4$ Temperley-Lieb representations could mean that they correspond to unbound entanglement unstable against state function reduction and preparation processes. Since this kind of entanglement does not survive in quantum jump it is not representable in terms of braid groups.

2. Does r define a fractal dimension of CP_2 projection of partonic 2-surface?

On basis of the quantum classical correspondence one expects that r should define some fractal dimension at the space-time level. Since r varies in the range $1,..,4$ and corresponds to the fractal dimension of 2-D Clifford algebra the corresponding spinors would have dimension $d = \sqrt{r}$. There are two options.

1. $D = r/2$ is suggested on basis of the construction of quantum version of M^d.
2. $D = log_2(r)$ is natural on basis of the dimension $d = 2^{D/2}$ of spinors in D-dimensional space.

r can be assigned with CP_2 degrees of freedom in the model for the quantization of Planck constant based on the explicit identification of Josephson inclusions in terms of finite subgroups of $SU(2) \subset SU(3)$. Hence D should relate to the CP_2 projection of the partonic 2-surface and one could have $D = D(X^2)$, the latter being the average dimension of the CP_2 projection of the partonic 2-surface for the preferred extremals of Kähler action.

Since a strongly interacting non-perturbative phase should be in question, the dimension for the CP_2 projection of the space-time surface must be at least $D(X^4) = 2$ to guarantee that non-vacuum extremals are in question. This is true for $D(X^2) = r/2 \geq 1$. The logarithmic formula $D(X^2) = log_2(r) \geq 0$ gives $D(X^2) = 0$ for $n = 3$ meaning that partonic 2-surfaces are vacua: space-time surface can still be a non-vacuum extremal.

As n increases, the number of CP_2 points covering a given M^4 point and related by the finite subgroup of $G \subset SU(2) \subset SU(3)$ defining the inclusion increases so that the fractal dimension of the CP_2 projection is expected to increase also. $D(X^2) = 2$ would correspond to the space-time surfaces for which partons have topological magnetic charge forcing them to have a 2-dimensional CP_2 projection. There are reasons to believe that the projection must be homologically non-trivial geodesic sphere of CP_2.

1.3.5 Could binding energy spectra reflect the hierarchy of effective tensor factor dimensions?

If one takes completely seriously the idea that join along boundaries bonds are a correlate of binding then the spectrum of binding energies might reveal the hierarchy of the fractal dimensions $r(n)$. Hydrogen atom and harmonic oscillator have become symbols for bound state systems. Hence it is of interest to find whether the binding energy spectrum of these systems might be expressed in terms of the "binding dimension" $x(n) = 4 - r(n)$ characterizing the deviation of dimension from that at the limit of a vanishing binding energy. The binding energies of hydrogen atom are in a good approximation given by $E(n)/E(1) = 1/n^2$ whereas in the case of harmonic oscillator one has $E(n)/E_0 = 2n+1$. The constraint $n \geq 3$ implies that the principal quantum number must correspond $n-2$ in the case of hydrogen atom and to $n-3$ in the case of harmonic oscillator.

Before continuing one must face an obvious objection. By previous arguments different values of r correspond to different values of $\hbar$. The value of $\hbar$ cannot however differ for the states of hydrogen atom. This is certainly true. The objection however leaves open the possibility that the states of the light-like boundaries of join along boundaries bonds correspond to reflective level and represent some aspects of the physics of, say, hydrogen atom.

In the general case the energy spectrum satisfies the condition

$$\frac{E_B(n)}{E_B(3)} = \frac{f(4-r(n))}{f(3)} , \tag{1.3.3}$$

where f is some function. The simplest assumption is that the spectrum of binding energies $E_B(n) = E(n) - E(\infty)$ is a linear function of $r(n) - 4$:

$$\frac{E_B(n)}{E_B(3)} = \frac{4-r(n)}{3} = \frac{4}{3}sin^2(\frac{\pi}{n}) \rightarrow \frac{4\pi^2}{3} \times \frac{1}{n^2} . \tag{1.3.4}$$

In the linear approximation the ratio $E(n+1)/E(n)$ approaches $(n/n+1)^2$ as in the case of hydrogen atom but for small values the linear approximation fails badly. An exact correspondence results for

$$\frac{E(n)}{E(1)} = \frac{1}{n^2} \ ,$$

$$n = \frac{1}{\pi \arcsin\left(\sqrt{1-r(n+2)/4}\right)} - 2 \ .$$

Also the ionized states with $r \geq 4$ would correspond to bound states in the sense that two particle would be constrained to move in the same space-like section of space-time surface and should be distinguished from genuinely free states when particles correspond to disjoint space-time sheets.

For the harmonic oscillator one express $E(n) - E(0)$ instead of $E(n) - E(\infty)$ as a function of $x = 4 - r$ and one would have

$$\frac{E(n)}{E(0)} = 2n + 1 \ ,$$

$$n = \frac{1}{\pi \arcsin\left(\sqrt{1-r(n+3)/4}\right)} - 3 \ .$$

In this case ionized states would not be possible due to the infinite depth of the harmonic oscillator potential well.

1.3.6 Four-color problem, II_1 factors, and anyons

The so called four-color problem can be phrased as a question whether it is possible to color the regions of a plane map using only four colors in such a manner that no adjacent regions have the same color (for an enjoyable discussion of the problem see [?]). One might call this kind of coloring complete. There is no loss of generality in assuming that the map can be represented as a graph with regions represented as triangle shaped faces of the graph. For the dual graph the coloring of faces becomes coloring of vertices and the question becomes whether the coloring is possible in such a manner that no vertices at the ends of the same edge have same color. The problem can be generalized by replacing planar maps with maps defined on any two-dimensional surface with or without boundary and arbitrary topology. The four-color problem has been solved with an extensive use of computer [?] but it would be nice to understand why the complete coloring with four colors is indeed possible.

There is a mysterious looking connection between four-color problem and the dimensions $r(n) = 4cos^2(\pi/n)$, which are in fact known as Beraha numbers in honor of the discoverer of this connection [?]. Consider a more general problem of coloring two-dimensional map using m colors. One can construct a polynomial $P(m)$, so called chromatic polynomial, which tells the number of colorings satisfying the condition that no neighboring vertices have the same color. The vanishing of the chromatic polynomial for an integer value of m tells that the complete coloring using m colors is not possible.

$P(m)$ has also other than integer valued real roots. The strange discovery due to Beraha is that the numbers $B(n)$ appear as approximate roots of the chromatic polynomial in many situations. For instance, the four non-integral real roots of the chromatic polynomial of the truncated icosahedron are very close to $B(5)$, $B(7)$, $B(8)$ and $B(9)$. These findings led Beraha to formulate the following conjecture. Let P_i be a sequence of chromatic polynomials for a graph for which the number of vertices approaches infinity. If r_i is a root of the polynomial approaching a well-defined value at the limit $i \to \infty$, then the limiting value of $r(i)$ is Beraha number.

A physicist's proof for Beraha's conjecture based on quantum groups and conformal theory has been proposed [?]. It is interesting to look for the a possible physical interpretation of 4-color problem and Beraha's conjecture in TGD framework.

1. In TGD framework $B(n)$ corresponds to a renormalized dimension for a 2-spin system consisting of two qubits, which corresponds to 4 different colors. For $B(n) = 4$ two spin 1/2 fermions obeying Fermi statistics are in question. Since the system is 2-dimensional, the general case corresponds to two anyons with fractional spin $B(n)/4$ giving rise to $B(n) < 4$ colors and obeying fractional statistics instead of Fermi statistics. One can replace coloring problem with the problem whether an ideal antiferro-magnetic lattice using anyons with fractional spin $B(n)/4$

is possible energetically. In other words, does this system form a quantum mechanical bound state even at the limit when the lengths of the edges approach to zero.

2. The failure of coloring means that there are at least two neighboring vertices in the lattice with the property that the spins at the ends of the same edge are in the same direction. Lattice defect would be in question. At the limit of an infinitesimally short edge length the failure of coloring is certainly not an energetically favored option for fermionic spins ($m = 4$) but is allowed by anyonic statistics for $m = B(n) < 4$. Thus one has reasons to expect that when anyonic spin is $B(n)/4$ the formation of a purely 2-anyon bound states becomes possible and they form at the limit of an infinitesimal edge length a kind of topological macroscopic quantum phase with a non-vanishing binding energy. That $B(n)$ are roots of the chromatic polynomial at the continuum limit would have a clear physical interpretation.

3. Only $B(n) < 4$ defines a sub-factor of von Neumann algebra allowing unitary Temperley-Lieb representations. This is consistent with the fact that for $m = 4$ complete coloring must exists. The physical argument is that otherwise a macroscopic quantum phase with non-vanishing binding energy could result at the continuum limit and the upper bound for r from unitarity would be larger than 4. For $m = 4$ the completely anti-ferromagnetic state would represent the ground state and the absence of anyon-pair condensate would mean a vanishing binding energy.

1.4 Inclusions of II_1 and III_1 factors

Inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras have physical interpretation as a mathematical description for sub-system-system relation. For type I algebras the inclusions are trivial and tensor product description applies as such. For factors of II_1 and III the inclusions are highly non-trivial. The inclusion of type II_1 factors were understood by Vaughan Jones [?]and those of factors of type III by Alain Connes [?].

Sub-factor $\mathcal{N}$ of $\mathcal{M}$ is defined as a closed *-stable C-subalgebra of $\mathcal{M}$. Let $\mathcal{N}$ be a sub-factor of type II_1 factor $\mathcal{M}$. Jones index $\mathcal{M} : \mathcal{N}$ for the inclusion $\mathcal{N} \subset \mathcal{M}$ can be defined as $\mathcal{M} : \mathcal{N} = dim_N(L^2(\mathcal{M})) = Tr_{N'}(id_{L^2(\mathcal{M})})$. One can say that the dimension of completion of $\mathcal{M}$ as $\mathcal{N}$ module is in question.

1.4.1 Basic findings about inclusions

What makes the inclusions non-trivial is that the position of $\mathcal{N}$ in $\mathcal{M}$ matters. This position is characterized in case of hyper-finite II_1 factors by index $\mathcal{M} : \mathcal{N}$ which can be said to the dimension of $\mathcal{M}$ as $\mathcal{N}$ module and also as the inverse of the dimension defined by the trace of the projector from $\mathcal{M}$ to $\mathcal{N}$. It is important to notice that $\mathcal{M} : \mathcal{N}$ does not characterize either $\mathcal{M}$ or $\mathcal{M}$, only the imbedding.

The basic facts proved by Jones are following [?].

1. For pairs $\mathcal{N} \subset \mathcal{M}$ with a finite principal graph the values of $\mathcal{M} : \mathcal{N}$ are given by

$$\begin{aligned} &a)\ \ \mathcal{M} : \mathcal{N} = 4cos^2(\pi/h) \ \ , \quad h \geq 3 \ \ , \\ &b)\ \ \mathcal{M} : \mathcal{N} \geq 4 \ \ . \end{aligned} \tag{1.4.1}$$

the numbers at right hand side are known as Beraha numbers [?]. The comments below give a rough idea about what finiteness of principal graph means.

2. As explained in [?], for $\mathcal{M} : \mathcal{N} < 4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h = (dimg(g) - r)/r$. The Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. For $\mathcal{M} : \mathcal{N} = 4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of SU(2) and the interpretation proposed in [?] is

following. The ADE diagrams are associated with the $n = \infty$ case having $\mathcal{M} : \mathcal{N} \geq 4$. There are diagrams corresponding to infinite subgroups: SU(2) itself, circle group U(1), and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection. The diagrams corresponding to finite subgroups are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with n=6,7,8 for tedrahedron, cube, dodecahedron. For $\mathcal{M} : \mathcal{N} < 4$ ordinary Dynkin graphs of D_{2n} and E_6, E_8 are allowed.

The interpretation of [?] is that the subfactors correspond to inclusions $\mathcal{N} \subset \mathcal{M}$ defined in the following manner.

1. Let G be a finite subgroup of SU(2). Denote by R the infinite-dimensional Clifford algebras resulting from infinite-dimensional tensor power of $M_2(C)$ and by R_0 its subalgebra obtained by restricting $M_2(C)$ element of the first factor to be unit matrix. Let G act by automorphisms in each tensor factor. G leaves R_0 invariant. Denote by R_0^G and R^G the sub-algebras which remain element wise invariant under the action of G. The resulting Jones inclusions $R_0^G \subset R^G$ are consistent with the ADE correspondence.

2. The argument suggests the existence of quantum versions of subgroups of SU(2) for which representations are truncations of those for ordinary subgroups. The results have been generalized to other Lie groups.

3. Also $SL(2, C)$ acts as automorphisms of $M_2(C)$. An interesting question is what happens if one allows G to be any discrete subgroups of SL(2,C). Could this give inclusions with $\mathcal{M} : \mathcal{N} > 4$?. The strong analogy of the spectrum of indices with spectrum of energies with hydrogen atom would encourage this interpretation: the subgroup SL(2,C) not reducing to those of SU(2) would correspond to the possibility for the particle to move with respect to each other with constant velocity.

1.4.2 The fundamental construction and Temperley-Lieb algebras

It was shown by Jones [?] that for a given Jones inclusion with $\beta = \mathcal{M} : \mathcal{N} < \infty$ there exists a tower of finite II_1 factors $\mathcal{M}_k$ for $k = 0, 1, 2, ...$ such that

1. $\mathcal{M}_0 = \mathcal{N}$, $\mathcal{M}_1 = \mathcal{M}$,

2. $\mathcal{M}_{k+1} = End_{\mathcal{M}_{k-1}}\mathcal{M}_k$ is the von Neumann algebra of operators on $L^2(\mathcal{M}_k)$ generated by $\mathcal{M}_k$ and an orthogonal projection $e_k : L^2(\mathcal{M}_k) \rightarrow L^2(\mathcal{M}_{k-1})$ for $k \geq 1$, where $\mathcal{M}_k$ is regarded as a subalgebra of $\mathcal{M}_{k+1}$ under right multiplication.

It can be shown that $\mathcal{M}_{k+1}$ is a finite factor. The sequence of projections on $\mathcal{M}_\infty = \cup_{k\geq 0}\mathcal{M}_k$ satisfies the relations

$$\begin{array}{ll} e_i^2 = e_i \ , & e_i^{=} e_i \ , \\ e_i = \beta e_i e_j e_i & \text{for } \ |i-j| = 1 \ , \\ e_i e_j = e_j e_i & \text{for } \ |i-j| \geq 2 \ . \end{array} \tag{1.4.2}$$

The construction of hyper-finite II_1 factor using Clifford algebra $C(2)$ represented by 2×2 matrices allows to understand the theorem in $\beta = 4$ case in a straightforward manner. In particular, the second formula involving β follows from the identification of x at $(k-1)^{th}$ level with $(1/\beta)diag(x, x)$ at k^{th} level.

By replacing 2×2 matrices with $\sqrt{\beta} \times \sqrt{\beta}$ matrices one can understand heuristically what is involved in the more general case. $\mathcal{M}_k$ is $\mathcal{M}_{k-1}$ module with dimension $\sqrt{\beta}$ and $\mathcal{M}_{k+1}$ is the space of $\sqrt{\beta} \times \sqrt{\beta}$ matrices $\mathcal{M}_{k-1}$ valued entries acting in $\mathcal{M}_k$. The transition from $\mathcal{M}_k$ to $\mathcal{M}_{k-1}$ linear maps of $\mathcal{M}_k$ happens in the transition to the next level. x at $(k-1)^{th}$ level is identified as $(x/\beta) \times Id_{\sqrt{\beta}\times\sqrt{\beta}}$ at the next level. The projection e_k picks up the projection of the matrix with $\mathcal{M}_{k-1}$ valued entries in the direction of the $Id_{\sqrt{\beta}\times\sqrt{\beta}}$.

The union of algebras $A_{\beta,k}$ generated by $1, e_1, ..., e_k$ defines Temperley-Lieb algebra A_β [?]. This algebra is naturally associated with braids. Addition of one strand to a braid adds one generator to this algebra and the representations of the Temperley Lieb algebra provide link, knot, and 3-manifold invariants [?]. There is also a connection with systems of statistical physics and with Yang-Baxter algebras [?].

A further interesting fact about the inclusion hierarchy is that the elements in $\mathcal{M}_i$ belonging to the commutator $\mathcal{N}'$ of $\mathcal{N}$ form finite-dimensional spaces. Presumably the dimension approaches infinity for $n \to \infty$.

1.4.3 Connection with Dynkin diagrams

The possibility to assign Dynkin diagrams ($\beta < 4$) and extended Dynkin diagrams ($\beta = 4$ to Jones inclusions can be understood heuristically by considering a characterization of so called bipartite graphs [?, ?] by the norm of the adjacency matrix of the graph.

Bipartite graphs Γ is a finite, connected graph with multiple edges and black and white vertices such that any edge connects white and black vertex and starts from a white one. Denote by $w(\Gamma)$ ($b(\Gamma)$) the number of white (black) vertices. Define the adjacency matrix $\Lambda = \Lambda(\Gamma)$ of size $b(\Gamma) \times w(\Gamma)$ by

$$w_{b,w} = \left\{ \begin{array}{ll} m(e) & \text{if there exists } e \text{ such that } \delta e = b - w \ , \\ 0 & \text{otherwise} \ . \end{array} \right. \tag{1.4.3}$$

Here $m(e)$ is the multiplicity of the edge e.

Define norm $||\Gamma||$ as

$$\begin{aligned} ||X|| &= max\{||X||; \ ||x|| \leq 1\} \ , \\ ||\Gamma|| &= ||\Lambda(\Gamma)|| = \left|\left| \begin{array}{ll} 0 & \Lambda(\Gamma) \\ \Lambda(\Gamma)^t & 0 \end{array} \right|\right| \ . \end{aligned} \tag{1.4.4}$$

Note that the matrix appearing in the formula is $(m+n)\times(m+n)$ symmetric square matrix so that the norm is the eigenvalue with largest absolute value.

Suppose that Γ is a connected finite graph with multiple edges (sequences of edges are regarded as edges). Then

1. If $||\Gamma|| \leq 2$ and if Γ has a multiple edge, $||\Gamma|| = 2$ and $\Gamma = \tilde{A}_1$, the extended Dynkin diagram for $SU(2)$ Kac Moody algebra.

2. $||\Gamma|| < 2$ if and only Γ is one of the Dynkin diagrams of A,D,E. In this case $||\Gamma|| = 2cos(\pi/h)$, where h is the Coxeter number of Γ.

3. $||\Gamma|| = 2$ if and only if Γ is one of the extended Dynkin diagrams $\tilde{A}, \tilde{D}, \tilde{E}$.

This result suggests that one can indeed assign to the Jones inclusions Dynkin diagrams. To really understand how the inclusions can be characterized in terms bipartite diagrams would require a deeper understanding of von Neumann algebras. The following argument only demonstrates that bipartite graphs naturally describe inclusions of algebras.

1. Consider a bipartite graph. Assign to each white vertex linear space $W(w)$ and to each edge of a linear space $W(b,w)$. Assign to a given black vertex the vector space $\oplus_{\delta e = b-w} W(b,w) \otimes W(w)$ where (b,w) corresponds to an edge ending to b.

2. Define $\mathcal{N}$ as the direct sum of algebras $End(W(w))$ associated with white vertices and $\mathcal{M}$ as direct sum of algebras $\oplus_{\delta e = b-w} End(W(b,w)) \otimes End(W(w))$ associated with black vertices.

3. There is homomorphism $N \to M$ defined by imbedding direct sum of white endomorphisms x to direct sum of tensor products x with the identity endomorphisms associated with the edges starting from x.

It is possible to show that Jones inclusions correspond to the Dynkin diagrams of A_n, D_{2n}, and E_6, E_8 and extended Dynkin diagrams of ADE type. In particular, the dual of the bi-partite graph associated with $\mathcal{M}_{n-1} \subset \mathcal{M}_n$ obtained by exchanging the roles of white and black vertices describes the inclusion $\mathcal{M}_n \subset \mathcal{M}_{n+1}$ so that two subsequent Jones inclusions might define something fundamental (the corresponding space-time dimension is $2 \times log_2(\mathcal{M} : \mathcal{N}) \leq 4$.

1.4.4 Indices for the inclusions of type III_1 factors

Type III_1 factors appear in relativistic quantum field theory defined in 4-dimensional Minkowski space [?]. An overall summary of basic results discovered in algebraic quantum field theory is described in the lectures of Longo [?]. In this case the inclusions for algebras of observables are induced by the inclusions for bounded regions of M^4 in axiomatic quantum field theory. Tomita's theory of modular Hilbert algebras [?, ?] forms the mathematical corner stone of the theory.

The basic notion is Haag-Kastler net [?] consisting of bounded regions of M^4. Double cone serves as a representative example. The von Neumann algebra $\mathcal{A}(O)$ is generated by observables localized in bounded region O. The net satisfies the conditions implied by local causality:

1. Isotony: $O_1 \subset O_2$ implies $\mathcal{A}(O_1) \subset \mathcal{A}(O_2)$.
2. Locality: $O_1 \subset O_2'$ implies $\mathcal{A}(O_1) \subset \mathcal{A}(O_2)'$ with O' defined as $\{x : \langle x, y\rangle < 0 \ \text{ for all } \ y \in O\}$.
3. Haag duality $\mathcal{A}(O')' = \mathcal{A}(O)$.

   Besides this Poincare covariance, positive energy condition, and the existence of vacuum state is assumed.

DHR (Doplicher-Haag-Roberts) [?] theory allows to deduce the values of Jones index and they are squares of integers in dimensions $D > 2$ so that the situation is rather trivial. The 2-dimensional case is distinguished from higher dimensional situations in that braid group replaces permutation group since the paths representing the flows permuting identical particles can be linked in $X^2 \times T$ and anyonic statistics [?, ?] becomes possible. In the case of 2-D Minkowski space M^2 Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ plus a set of discrete values of $\mathcal{M} : \mathcal{N}$ in the range $(4, 6)$ are possible. In [?] some values are given ($\mathcal{M} : \mathcal{N} = 5, 5.5049..., 5.236...., 5.828...$).

At least intersections of future and past light cones seem to appear naturally in TGD framework such that the boundaries of future/past directed light cones serve as seats for incoming/outgoing states defined as intersections of space-time surface with these light cones. III_1 sectors cannot thus be excluded as factors in TGD framework. On the other hand, the construction of S-matrix at space-time level is reduced to II_1 case by effective 2-dimensionality.

1.5 TGD and hyper-finite factors of type II_1: ideas and questions

By effective 2-dimensionality of the construction of quantum states the hyper-finite factors of type II_1 fit naturally to TGD framework. In particular, infinite dimensional spinors define a canonical representations of this kind of factor. The basic question is whether only hyper-finite factors of type II_1 appear in TGD framework. Affirmative answer would allow to interpret physical M-matrix as time like entanglement coefficients.

1.5.1 What kind of hyper-finite factors one can imagine in TGD?

The working hypothesis has been that only hyper-finite factors of type II_1 appear in TGD. The basic motivation has been that they allow a new view about M-matrix as an operator representable as time-like entanglement coefficients of zero energy states so that physical states would represent laws of physics in their structure. They allow also the introduction of the notion of measurement resolution directly to the definition of reaction probabilities by using Jones inclusion and the replacement of state space with a finite-dimensional state space defined by quantum spinors. This hypothesis is of course just an attractive working hypothesis and deserves to be challenged.

Configuration space spinors

For configuration space spinors the HFF II_1 property is very natural because of the properties of infinite-dimensional Clifford algebra and the inner product defined by the configuration space geometry does not allow other factors than this. A good guess is that the values of conformal weights label the factors appearing in the tensor power defining configuration space spinors. Because of the non-degeneracy and super-symplectic symmetries the density matrix representing metric must be essentially unit matrix for each conformal weight which would be the defining characteristic of hyper-finite factor of type II_1.

Bosonic degrees of freedom

The bosonic part of the super-symplectic algebra consists of Hamiltonians of CH in one-one correspondence with those of $\delta M^4_\pm \times CP_2$. Also the Kac-Moody algebra acting leaving the light-likeness of the partonic 3-surfaces intact contributes to the bosonic degrees of freedom. The commutator of these algebras annihilates physical states and there are also Virasoro conditions associated with ordinary conformal symmetries of partonic 2-surface [?]. The labels of Hamiltonians of configuration space and spin indices contribute to bosonic degrees of freedom.

Hyper-finite factors of type II_1 result naturally if the system is an infinite tensor product finite-dimensional matrix algebra associated with finite dimensional systems [?]. Unfortunately, neither Virasoro, symplectic nor Kac-Moody algebras do have decomposition into this kind of infinite tensor product. If bosonic degrees for super-symplectic and super-Kac Moody algebra indeed give I_∞ factor one has HFF if type II_∞. This looks the most natural option but threatens to spoil the beautiful idea about M-matrix as time-like entanglement coefficients between positive and negative energy parts of zero energy state.

The resolution of the problem is surprisingly simple and trivial after one has discovered it. The requirement that state is normalizable forces to project M-matrix to a finite-dimensional sub-space in bosonic degrees of freedom so that the reduction $I_\infty \rightarrow I_n$ occurs and one has the reduction $II_\infty \rightarrow II_1 \times I_n = II_1$ to the desired HFF.

One can consider also the possibility of taking the limit $n \rightarrow \infty$. One could indeed say that since I_∞ factor can be mapped to an infinite tensor power of $M(2, C)$ characterized by a state which is not trace, it is possible to map this representation to HFF by replacing state with trace [?]. The question is whether the forcing the bosonic foot to fermionic shoe is physically natural. One could also regard the II_1 type notion of probability as fundamental and also argue that it is required by full super-symmetry realized also at the level of many-particle states rather than mere single particle states.

How the bosonic cutoff is realized?

Normalizability of state requires that projection to a finite-dimensional bosonic sub-space is carried out for the bosonic part of the M-matrix. This requires a cutoff in quantum numbers of super-conformal algebras. The cutoff for the values of conformal weight could be formulated by replacing integers with Z_n or with some finite field $G(p, 1)$. The cutoff for the labels associated with Hamiltonians defined as an upper bound for the dimension of the representation looks also natural.

Number theoretical braids which are discrete and finite structures would define space-time correlate for this cutoff. p-Adic length scale $p \simeq 2^k$ hypothesis could be interpreted as stating the fact that only powers of p up to p^k are significant in p-adic thermodynamics which would correspond to finite field $G(k, 1)$ if k is prime. This has no consequences for p-adic mass calculations since already the first two terms give practically exact results for the large primes associated with elementary particles [?].

Finite number of strands for the theoretical braids would serve as a correlate for the reduction of the representation of Galois group S_∞ of rationals to an infinite produce of diagonal copies of finite-dimensional Galois group so that same braid would repeat itself like a unit cell of lattice i condensed matter [?].

HFF of type III for field operators and HFF of type II_1 for states?

One could also argue that the Hamiltonians with fixed conformal weight are included in fermionic II_1 factor and bosonic factor I_∞ factor, and that the inclusion of conformal weights leads to a factor of type III. Conformal weight could relate to the integer appearing in the crossed product representation $III = Z \times_{cr} II_\infty$ of HFF of type III [?].

The value of conformal weight is non-negative for physical states which suggests that Z reduces to semigroup N so that a factor of type III would reduce to a factor of type II_∞ since trace would become finite. If unitary process corresponds to an automorphism for II_∞ factor, the action of automorphisms affecting scaling must be uni-directional. Also thermodynamical irreversibility suggests the same. The assumption that state function reduction for positive energy part of state implies unitary process for negative energy state and vice versa would only mean that the shifts for positive and negative energy parts of state are opposite so that $Z \rightarrow N$ reduction would still hold true.

HFF of type II_1 for the maxima of Kähler function?

Probabilistic interpretation allows to gain heuristic insights about whether and how hyper-finite factors of type type II_1 might be associated with configuration space degrees of freedom. They can appear both in quantum fluctuating degrees of freedom associated with a given maximum of Kähler function and in the discrete space of maxima of Kähler function.

Spin glass degeneracy is the basic prediction of classical TGD and means that instead of a single maximum of Kähler function analogous to single free energy minimum of a thermodynamical system there is a fractal spin glass energy landscape with valleys inside valleys. The discretization of the configuration space in terms of the maxima of Kähler function crucial for the p-adicization problem, leads to the analog of spin glass energy landscape and hyper-finite factor of type II_1 might be the appropriate description of the situation.

The presence of the tensor product structure is a powerful additional constraint and something analogous to this should emerge in configuration space degrees of freedom. Fractality of the many-sheeted space-time is a natural candidate here since the decomposition of the original geometric structure to parts and replacing them with the scaled down variant of original structure is the geometric analog of forming a tensor power of the original structure.

1.5.2 Direct sum of HFFs of type II_1 as a minimal option

HFF II_1 property for the Clifford algebra of the configuration space means a definite distinction from the ordinary Clifford algebra defined by the fermionic oscillator operators since the trace of the unit matrix of the Clifford algebra is normalized to one. This does not affect the anti-commutation relations at the basic level and delta functions can appear in them at space-time level. At the level of momentum space I_∞ property requires discrete basis and anti-commutators involve only Kronecker deltas. This conforms with the fact that HFF of type II_1 can be identified as the Clifford algebra associated with a separable Hilbert space.

II_∞ factor or direct sum of HFFs of type II_1?

The expectation is that super-symplectic algebra is a direct sum over HFFs of type II_1 labeled by the radial conformal weight. In the same manner the algebra defined by fermionic anti-commutation relations at partonic 2-surface would decompose to a direct sum of algebras labeled by the conformal weight associated with the light-like coordinate of X_l^3. Super-conformal symmetry suggests that also the configuration space degrees of freedom correspond to a direct sum of HFFs of type II_1.

One can of course ask why not $II_\infty = I_\infty \times II_1$ structures so that one would have single factor rather than a direct sum of factors.

1. The physical motivation is that the direct sum property allow to decompose M-matrix to direct summands associated with various sectors with weights whose moduli squared have an interpretation in terms of the density matrix. This is also consistent with p-adic thermodynamics where conformal weights take the place of energy eigen values.

2. II_∞ property would predict automorphisms scaling the trace by an arbitrary positive real number $\lambda \in R_+$. These automorphisms would require the scaling of the trace of the projectors of Clifford algebra having values in the range $[0,1]$ and it is difficult to imagine how these automorphisms could be realized geometrically.

How HFF property reflects itself in the construction of geometry of WCW?

The interesting question is what HFF property and finite measurement resolution realizing itself as the use of projection operators means concretely at the level of the configuration space geometry.

Super-Hamiltonians define the Clifford algebra of the configuration space. Super-conformal symmetry suggests that the unavoidable restriction to projection operators instead of complex rays is realized also configuration space degrees of freedom. Of course, infinite precision in the determination of the shape of 3-surface would be physically a completely unrealistic idea.

In the fermionic situation the anti-commutators for the gamma matrices associated with configuration space individual Hamiltonians in 3-D sense are replaced with anti-commutators where Hamiltonians are replaced with projectors to subspaces of the space spanned by Hamiltonians. This projection is realized by restricting the anti-commutator to partonic 2-surfaces so that the anti-commutator depends only the restriction of the Hamiltonian to those surfaces.

What is interesting that the measurement resolution has a concrete particle physical meaning since the parton content of the system characterizes the projection. The larger the number of partons, the better the resolution about configuration space degrees of freedom is. The degeneracy of configuration space metric would be interpreted in terms of finite measurement resolution inherent to HFFs of type II_1, which is not due to Jones inclusions but due to the fact that one can project only to infinite-D subspaces rather than complex rays.

Effective 2-dimensionality in the sense that configuration space Hamiltonians reduce to functionals of the partonic 2-surfaces of X^3_l rather than functionals of X^3_l could be interpreted in this manner. For a wide class of Hamiltonians actually effective 1-dimensionality holds true in accordance with conformal invariance.

The generalization of configuration space Hamiltonians and super-Hamiltonians by allowing integrals over the 2-D boundaries of the patches of X^3_l would be natural and is suggested by the requirement of discretized 3-dimensionality at the level of configuration space.

By quantum classical correspondence the inclusions of HFFs related to the measurement resolution should also have a geometric description. Measurement resolution corresponds to braids in given time scale and as already explained there is a hierarchy of braids in time scales coming as negative powers of two corresponding to the addition of zero energy components to positive/negative energy state. Note however that particle reactions understood as decays and fusions of braid strands could also lead to a notion of measurement resolution.

1.5.3 Bott periodicity, its generalization, and dimension $D = 8$ as an inherent property of the hyper-finite II_1 factor

Hyper-finite II_1 factor can be constructed as infinite-dimensional tensor power of the Clifford algebra $M_2(C) = C(2)$ in dimension $D = 2$. More precisely, one forms the union of the Clifford algebras $C(2n) = C(2)^{\otimes n}$ of $2n$-dimensional spaces by identifying the element $x \in C(2n)$ as block diagonal elements $diag(x,x)$ of $C(2(n+1))$. The union of these algebras is completed in weak operator topology and can be regarded as a Clifford algebra of real infinite-dimensional separable Hilbert space and thus as sub-algebra of I_∞. Also generalizations obtained by replacing complex numbers by quaternions and octions are possible.

1. The dimension 8 is an inherent property of the hyper-finite II_1 factor since Bott periodicity theorem states $C(n+8) = C_n(16)$. In other words, the Clifford algebra $C(n+8)$ is equivalent with the algebra of 16×16 matrices with entries in $C(n)$. Or articulating it still differently: $C(n+8)$ can be regarded as 16×16 dimensional module with $C(n)$ valued coefficients. Hence the elements in the union defining the canonical representation of hyper-finite II_1 factor are $16^n \times 16^n$ matrices having $C(0)$, $C(2)$, $C(4)$ or $C(6)$ valued valued elements.

2. The idea about a local variant of the infinite-dimensional Clifford algebra defined by power series of space-time coordinate with Taylor coefficients which are Clifford algebra elements fixes the interpretation. The representation as a linear combination of the generators of Clifford algebra of the finite-dimensional space allows quantum generalization only in the case of Minkowski spaces. However, if Clifford algebra generators are representable as gamma *matrices*, the powers of coordinate can be absorbed to the Clifford algebra and the local algebra is lost. Only if the generators are represented as quantum versions of octonions allowing no matrix representation because of their non-associativity, the local algebra makes sense. From this it is easy to deduce both quantum and classical TGD.

1.5.4 The interpretation of Jones inclusions in TGD framework

By the basic self-referential property of von Neumann algebras one can consider several interpretations of Jones inclusions consistent with sub-system-system relationship, and it is better to start by considering the options that one can imagine.

How Jones inclusions relate to the new view about sub-system?

Jones inclusion characterizes the imbedding of sub-system $\mathcal{N}$ to $\mathcal{M}$and $\mathcal{M}$ as a finite-dimensional $\mathcal{N}$-module is the counterpart for the tensor product in finite-dimensional context. The possibility to express $\mathcal{M}$ as $\mathcal{N}$ module $\mathcal{M}/\mathcal{N}$ states fractality and can be regarded as a kind of self-referential "Brahman=Atman identity" at the level of infinite-dimensional systems.

Also the mysterious looking almost identity $CH^2 = CH$ for the configuration space of 3-surfaces would fit nicely with the identity $M \oplus M = M$. $M \otimes M \subset M$ in configuration space Clifford algebra degrees of freedom is also implied and the construction of $\mathcal{M}$ as a union of tensor powers of $C(2)$ suggests that $M \otimes M$ allows $\mathcal{M} : \mathcal{N} = 4$ inclusion to $\mathcal{M}$. This paradoxical result conforms with the strange self-referential property of factors of II_1.

The notion of many-sheeted space-time forces a considerable generalization of the notion of sub-system and simple tensor product description is not enough. Topological picture based on the length scale resolution suggests even the possibility of entanglement between sub-systems of un-entangled sub-systems. The possibility that hyper-finite II_1-factors describe the physics of TGD also in bosonic degrees of freedom is suggested by configuration space super-symmetry. On the other hand, bosonic degrees could naturally correspond to I_∞ factor so that hyper-finite II_∞ would be the net result.

The most general view is that Jones inclusion describes all kinds of sub-system-system inclusions. The possibility to assign conformal field theory to the inclusion gives hopes of rather detailed view about dynamics of inclusion.

1. The topological condensation of space-time sheet to a larger space-time sheet mediated by wormhole contacts could be regarded as Jones inclusion. $\mathcal{N}$ would correspond to the condensing space-time sheet, $\mathcal{M}$ to the system consisting of both space-time sheets, and $\sqrt{\mathcal{M} : \mathcal{N}}$ would characterize the number of quantum spinorial degrees of freedom associated with the interaction between space-time sheets. Note that by general results $\mathcal{M} : \mathcal{N}$ characterizes the fractal dimension of quantum group ($\mathcal{M} : \mathcal{N} < 4$) or Kac-Moody algebra ($\mathcal{M} : \mathcal{N} = 4$) [**?**].

2. The branchings of space-time sheets (space-time surface is thus homologically like branching like of Feynman diagram) correspond naturally to n-particle vertices in TGD framework. What is nice is that vertices are nice 2-dimensional surfaces rather than surfaces having typically pinch singularities. Jones inclusion would naturally appear as inclusion of operator spaces $\mathcal{N}_i$ (essentially Fock spaces for fermionic oscillator operators) creating states at various lines as sub-spaces $N_i \subset M$ of operators creating states in common von Neumann factor $\mathcal{M}$. This would allow to construct vertices and vertices in natural manner using quantum groups or Kac-Moody algebras.

   The fundamental $\mathcal{N} \subset \mathcal{M} \subset \mathcal{M} \otimes_N \mathcal{M}$ inclusion suggests a concrete representation based on the identification $N_i = M$, where M is the universal Clifford algebra associated with incoming line and $\mathcal{N}$ is defined by the condition that $\mathcal{M}/\mathcal{N}$ is the quantum variant of Clifford algebra of H. N-particle vertices could be defined as traces of Connes products of the operators creating incoming and outgoing states. It will be found that this leads to a master formula for S-matrix if the generalization of the old-fashioned string model duality implying that all generalized Feynman diagrams reduce to diagrams involving only single vertex is accepted.

3. If 4-surfaces can branch as the construction of vertices requires, it is difficult to argue that 3-surfaces and partonic/stringy 2-surfaces could not do the same. As a matter fact, the master formula for S-matrix to be discussed later explains the branching of 4-surfaces as an apparent affect. Despite this one can consider the possibility that this kind of joins are possible so that a new kind of mechanism of topological condensation would become possible. 3-space-sheets and partonic 2-surfaces whose p-adic fractality is characterized by different p-adic primes could be connected by "joins" representing branchings of 2-surfaces. The structures formed by soap film foam provide a very concrete illustration about what would happen. In the TGD based model

of hadrons [?] it has been assumed that join along boundaries bonds (JABs) connect quark space-time space-time sheets to the hadronic space-time sheet. The problem is that, at least for identical primes, the formation of join along boundaries bond fuses two systems to single bound state. If JABs are replaced joins, this objection is circumvented.

4. The space-time correlate for the formation of bound states is the formation of JABs. Standard intuition tells that the number of degrees of freedom associated with the bound state is smaller than the number of degrees of freedom associated with the pair of free systems. Hence the inclusion of the bound state to the tensor product could be regarded as Jones inclusion. On the other hand, one could argue that the JABs carry additional vibrational degrees of freedom so that the idea about reduction of degrees of freedom might be wrong: free system could be regarded as sub-system of bound state by Jones inclusion. The self-referential holographic properties of von Neumann algebras allow both interpretations: any system can be regarded as sub-system of any system in accordance with the bootstrap idea.

5. Maximal deterministic regions inside given space-time sheet bounded by light-like causal determinants define also sub-systems in a natural manner and also their inclusions would naturally correspond to Jones inclusions.

6. The TGD inspired model for topological quantum computation involves the magnetic flux tubes defined by join along boundaries bonds connecting space-time sheets having light-like boundaries. These tubes condensed to background 3-space can become linked and knotted and code for quantum computations in this manner. In this case the addition of new strand to the system corresponds to Jones inclusion in the hierarchy associated with inclusion $\mathcal{N} \subset \mathcal{M}$. The anyon states associated with strands would be represented by a finite tensor product of quantum spinors assignable to $\mathcal{M}/\mathcal{N}$ and representing quantum counterpart of H-spinors.

One can regard $\mathcal{M}:\mathcal{N}$ degrees of freedom correspond to quantum group or Kac-Moody degrees of freedom. Quantum group degrees of freedom relate closely to the conformal and topological degrees of freedom as the connection of II_1 factors with topological quantum field theories and braid matrices suggests itself. For the canonical inclusion this factorization would correspond to factorization of quantum H-spinor from configuration space spinor.

A more detailed study of canonical inclusions to be carried out later demonstrates what this factorization corresponds at the space-time level to a formation of space-time sheets which can be regarded as multiple coverings of M^4 and CP_2 with invariance group $G = G_a \times G_b \subset SL(2,C) \times SU(2)$, $SU(2) \subset SU(3)$. The unexpected outcome is that Planck constants assignable to M^4 and CP_2 degrees of freedom depend on the canonical inclusions. The existence of macroscopic quantum phases with arbitrarily large Planck constants is predicted.

It would seem possible to assign the $\mathcal{M}:\mathcal{N}$ degrees quantum spinorial degrees of freedom to the interface between subsystems represented by $\mathcal{N}$ and $\mathcal{M}$. The interface could correspond to the wormhole contacts, joins, JABs, or light-like causal determinants serving as boundary between maximal deterministic regions, etc... In terms of the bipartite diagrams representing the inclusions, joins (say) would correspond to the edges connecting white vertices representing sub-system (the entire system without the joins) to black vertices (entire system).

About the interpretation of $\mathcal{M}:\mathcal{N}$ degrees of freedom

The Clifford algebra $\mathcal{N}$ associated with a system formed by two space-time sheet can be regarded as $1 \leq \mathcal{M}:\mathcal{N} \leq 4$-dimensional module having $\mathcal{N}$ as its coefficients. It is possible to imagine several interpretations the degrees of freedom labeled by β.

1. The $\beta = \mathcal{M}:\mathcal{N}$ degrees of freedom could relate to the interaction of the space-time sheets. Beraha numbers appear in the construction of S-matrices of topological quantum field theories and an interpretation in terms of braids is possible. This would suggest that the interaction between space-time sheets can be described in terms of conformal quantum field theory and the S-matrices associated with braids describe this interaction. Jones inclusions would characterize the effective number of active conformal degrees of freedom. At $n = 3$ limit these degrees of freedom disappear completely since the conformal field theory defined by the Chern-Simons action describing this interaction would become trivial ($c = 0$ as will be found).

2. The interpretation in terms of imbedding space Clifford algebra would suggest that β-dimensional Clifford algebra of $\sqrt{\beta}$-dimensional spinor space is in question. For $\beta = 4$ the algebra would be the Clifford algebra of 2-dimensional space. $\mathcal{M}/\mathcal{N}$ would have interpretation as complex quantum spinors with components satisfying $z_1z_2 = qz_2z_1$ and its conjugate and having fractal complex dimension $\sqrt{\beta}$. This would conform with the effective 2-dimensionality of TGD. For $\beta < 4$ the fractal dimension of partonic quantum spinors defining the basic conformal fields would be reduced and become $d = 1$ for $n = 3$: the interpretation is in terms of strong correlations caused by the non-commutativity of the components of quantum spinor. For number theoretical generalizations of infinite-dimensional Clifford algebras $Cl(C)$ obtained by replacing C with Abelian complexification of quaternions or octonions one would obtain higher-dimensional spinors.

1.5.5 Configuration space, space-time, and imbedding space and hyper-finite type II_1 factors

The preceding considerations have by-passed the question about the relationship of the configuration space tangent space to its Clifford algebra. Also the relationship between space-time and imbedding space and their quantum variants could be better. In particular, one should understand how effective 2-dimensionality can be consistent with the 4-dimensionality of space-time.

Super-conformal symmetry and configuration space Poisson algebra as hyper-finite type II_1 factor

It would be highly desirable to achieve also a description of the configuration space degrees of freedom using von Neumann algebras. Super-conformal symmetry relating fermionic degrees of freedom and configuration space degrees of freedom suggests that this might be the case. Super-symplectic algebra has as its generators configuration space Hamiltonians and their super-counterparts identifiable as CH gamma matrices. Super-symmetry requires that the Clifford algebra of CH and the Hamiltonian vector fields of CH with symplectic central extension both define hyper-finite II_1 factors. By super-symmetry Poisson bracket corresponds to an anti-commutator for gamma matrices. The ordinary quantized version of Poisson bracket is obtained as $\{P_i, Q_j\} \rightarrow [P_i, Q_j] = J_{ij}Id$. Finite trace version results by assuming that Id corresponds to the projector CH Clifford algebra having unit norm. The presence of zero modes means direct integral over these factors.

Configuration space gamma matrices anti-commuting to identity operator with unit norm corresponds to the tangent space $T(CH)$ of CH. Thus it would be not be surprising if $T(CH)$ could be imbedded in the sigma matrix algebra as a sub-space of operators defined by the gamma matrices generating this algebra. At least for $\beta = 4$ construction of hyper-finite II_1 factor this definitely makes sense.

The dimension of the configuration space defined as the trace of the projection operator to the sub-space spanned by gamma matrices is obviously zero. Thus configuration space has in this sense the dimensionality of single space-time point. This sounds perhaps absurd but the generalization of the number concept implied by infinite primes indeed leads to the view that single space-time point is infinitely structured in the number theoretical sense although in the real sense all states of the point are equivalen. The reason is that there is infinitely many numbers expressible as ratios of infinite integers having unit real norm in the real sense but having different p-adic norms.

How to understand the dimensions of space-time and imbedding space?

One should be able to understand the dimensions of 3-space, space-time and imbedding space in a convincing matter in the proposed framework. There is also the question whether space-time and imbedding space emerge uniquely from the mathematics of von Neumann algebras alone.

1. The dimensions of space-time and imbedding space

Two sub-sequent inclusions dual to each other define a special kind of inclusion giving rise to a quantum counterpart of $D = 4$ naturally. This would mean that space-time is something which emerges at the level of cognitive states.

The special role of classical division algebras in the construction of quantum TGD [?], $D = 8$ Bott periodicity generalized to quantum context, plus self-referential property of type II_1 factors might explain why 8-dimensional imbedding space is the only possibility.

State space has naturally quantum dimension $D \leq 8$ as the following simple argument shows. The space of quantum states has quark and lepton sectors which both are super-symmetric implying $D \leq 4$ for each. Since these sectors correspond to different Hamiltonian algebras (triality one for quarks and triality zero for leptonic sector), the state space has quantum dimension $D \leq 8$.

2. How the lacking two space-time dimensions emerge?

3-surface is the basic dynamical unit in TGD framework. This seems to be in conflict with the effective 2-dimensionality [?] meaning that partonic 2-surface code for quantum states, and with the fact that hyper-finite II_1 factors have intrinsic quantum dimension 2.

A possible resolution of the problem is that the foliation of 3-surface by partonic two-surfaces defines a one-dimensional direct integral of isomorphic hyper-finite type II_1 factors, and the zero mode labeling the 2-surfaces in the foliation serves as the third spatial coordinate. For a given 3-surface the contribution to the configuration space metric can come only from 2-D partonic surfaces defined as intersections of 3-D light-like CDs with $X^7_\pm$ [?, ?]. Hence the direct integral should somehow relate to the classical non-determinism of Kähler action.

1. The one-parameter family of intersections of light-like CD with $X^7_\pm$ inside $X^4 \cap X^7_\pm$ could indeed be basically due to the classical non-determinism of Kähler action. The contribution to the metric from the normal light-like direction to $X^3 = X^4 \cap X^7_\pm$ can cause the vanishing of the metric determinant $\sqrt{g_4}$ of the space-time metric at $X^2 \subset X^3$ under some conditions on X^2. This would mean that the space-time surface $X^4(X^3)$ is not uniquely determined by the minimization principle defining the value of the Kähler action, and the complete dynamical specification of X^3 requires the specification of partonic 2-surfaces X^2_i with $\sqrt{g_4} = 0$.

2. The known solutions of field equations [?] define a double foliation of the space-time surface defined by Hamilton-Jacobi coordinates consisting of complex transversal coordinate and two light-like coordinates for M^4 (rather than space-time surface). Number theoretical considerations inspire the hypothesis that this foliation exists always [?]. Hence a natural hypothesis is that the allowed partonic 2-surfaces correspond to the 2-surfaces in the restriction of the double foliation of the space-time surface by partonic 2-surfaces to X^3, and are thus locally parameterized by single parameter defining the third spatial coordinate.

3. There is however also a second light-like coordinate involved and one might ask whether both light-like coordinates appear in the direct sum decomposition of II_1 factors defining $T(CH)$. The presence of two kinds of light-like CDs would provide the lacking two space-time coordinates and quantum dimension $D = 4$ would emerge at the limit of full non-determinism. Note that the duality of space-like partonic and light-like stringy 2-surfaces conforms with this interpretation since it corresponds to a selection of partonic/stringy 2-surface inside given 3-D CD whereas the dual pairs correspond to different CDs.

4. That the quantum dimension would be $2D_q = \beta < 4$ above CP_2 length scale conforms with the fact that non-determinism is only partial and time direction is dynamically frozen to a high degree. For vacuum extremals there is strong non-determinism but in this case there is no real dynamics. For CP_2 type extremals, which are not vacuum extremals as far action and small perturbations are considered, and which correspond to $\beta = 4$ there is a complete non-determinism in time direction since the M^4 projection of the extremal is a light-like random curve and there is full 4-D dynamics. Light-likeness gives rise to conformal symmetry consistent with the emergence of Kac Moody algebra [?].

3. Time and cognition

In a completely deterministic physics time dimension is strictly speaking redundant since the information about physical states is coded by the initial values at 3-dimensional slice of space-time. Hence the notion of time should emerge at the level of cognitive representations possible by to the non-determinism of the classical dynamics of TGD.

Since Jones inclusion means the emergence of cognitive representation, the space-time view about physics should correspond to cognitive representations provided by Feynman diagram states with zero energy with entanglement defined by a two-sided projection of the lowest level S-matrix. These states would represent the "laws of quantum physics" cognitively. Also space-time surface serves as a classical correlate for the evolution by quantum jumps with maximal deterministic regions serving as correlates of quantum states. Thus the classical non-determinism making possible cognitive representations would bring in time. The fact that quantum dimension of space-time is smaller than $D = 4$ would reflect the fact that the loss of determinism is not complete.

4. Do space-time and imbedding space emerge from the theory of von Neumann algebras and number theory?

The considerations above force to ask whether the notions of space-time and imbedding space emerge from von Neumann algebras as predictions rather than input. The fact that it seems possible to formulate the S-matrix and its generalization in terms of inherent properties of infinite-dimensional Clifford algebras suggest that this might be the case.

Inner automorphisms as universal gauge symmetries?

The continuous outer automorphisms Δ^{it} of HFFs of type III are not completely unique and one can worry about the interpretation of the inner automorphisms. A possible resolution of the worries is that inner automorphisms act as universal gauge symmetries containing various super-conformal symmetries as a special case. For hyper-finite factors of type II_1 in the representation as an infinite tensor power of $M_2(C)$ this would mean that the transformations non-trivial in a finite number of tensor factors only act as analogs of local gauge symmetries. In the representation as a group algebra of S_∞ all unitary transformations acting on a finite number of braid strands act as gauge transformations whereas the infinite powers $P \times P \times ...$, $P \in S_n$, would act as counterparts of global gauge transformations. In particular, the Galois group of the closure of rationals would act as local gauge transformations but diagonally represented finite Galois groups would act like global gauge transformations and periodicity would make possible to have finite braids as space-time correlates without a loss of information.

Do unitary isomorphisms between tensor powers of II_1 define vertices?

What would be left would be the construction of unitary isomorphisms between the tensor products of the HFFs of type $II_1 \otimes I_n = II_1$ at the partonic 2-surfaces defining the vertices. This would be the only new element added to the construction of braiding M-matrices.

As a matter fact, this element is actually not completely new since it generalizes the fusion rules of conformal field theories, about which standard example is the fusion rule $\phi_i = c_i^{\ jk}\phi_j\phi_k$ for primary fields. These fusion rules would tell how a state of incoming HFF decomposes to the states of tensor product of two outgoing HFFs.

These rules indeed have interpretation in terms of Connes tensor products $\mathcal{M}\otimes_{\mathcal{N}}...\otimes_{\mathcal{N}}\mathcal{M}$ for which the sub-factor $\mathcal{N}$ takes the role of complex numbers [?] so that one has $\mathcal{M}$ becomes $\mathcal{N}$ bimodule and "quantum quantum states" have $\mathcal{N}$ as coefficients instead of complex numbers. In TGD framework this has interpretation as quantum measurement resolution characterized by $\mathcal{N}$ (the group G characterizing leaving the elements of $\mathcal{N}$ invariant defines the measured quantum numbers).

1.5.6 Quaternions, octonions, and hyper-finite type II_1 factors

Quaternions and octonions as well as their hyper counterparts obtained by multiplying imaginary units by commuting $\sqrt{-1}$ and forming a sub-space of complexified division algebra, are in in a central role in the number theoretical vision about quantum TGD [?]. Therefore the question arises whether complexified quaternions and perhaps even octonions could be somehow inherent properties of von Neumann algebras. One can also wonder whether the quantum counterparts of quaternions and octonions could emerge naturally from von Neumann algebras. The following considerations allow to get grasp of the problem.

Quantum quaternions and quantum octonions

Quantum quaternions have been constructed as deformation of quaternions [?]. The key observation that the Glebsch Gordan coefficients for the tensor product $3\otimes 3 = 5\oplus\oplus 3\oplus 1$ of spin 1 representation of $SU(2)$ with itself gives the anti-commutative part of quaternionic product as spin 1 part in the decomposition whereas the commutative part giving spin 0 representation is identifiable as the scalar product of the imaginary parts. By combining spin 0 and spin 1 representations, quaternionic product can be expressed in terms of Glebsh-Gordan coefficients. By replacing GGC:s by their quantum group versions for group $sl(2)_q$, one obtains quantum quaternions.

There are two different proposals for the construction of quantum octonions [?, ?]. Also now the idea is to express quaternionic and octonionic multiplication in terms of Glebsch-Gordan coefficients and replace them with their quantum versions.

1. The first proposal [?] relies on the observation that for the tensor product of $j = 3$ representations of $SU(2)$ the Glebsch-Gordan coefficients for $7\otimes 7\rightarrow 7$ in $7\otimes 7 = 9\oplus 7\oplus 5\oplus 3\oplus 1$ defines a product, which is equivalent with the antisymmetric part of the product of octonionic imaginary units. As a matter fact, the antisymmetry defines 7-dimensional Malcev algebra defined by the anticommutator of octonion units and satisfying b definition the identity

$$[[x,y,z]\,,x] = [x,y,[x,z]] \quad , \quad [x,y,z] \equiv [x,[y,z]] + [y,[z,x]] + [z,[x,y]] \quad . \tag{1.5.1}$$

   7-element Malcev algebra defining derivations of octonionic algebra is the only complex Malcev algebra not reducing to a Lie algebra. The $j = 0$ part of the product corresponds also now to scalar product for imaginary units. Octonions are constructed as sums of $j = 0$ and $j = 3$ parts and quantum Glebsch-Gordan coefficients define the octonionic product.

2. In the second proposal [?] the quantum group associated with $SO(8)$ is used. This representation does not allow unit but produces a quantum version of octonionic triality assigning to three octonions a real number.

Quaternionic or octonionic quantum mechanics?

There have been numerous attempts to introduce quaternions and octonions to quantum theory. Quaternionic or octonionic quantum mechanics, which means the replacement of the complex numbers as coefficient field of Hilbert space with quaternions or octonions, is the most obvious approach (for example and references to the literature see for instance [?].

In both cases non-commutativity poses serious interpretational problems. In the octonionic case the non-associativity causes even more serious obstacles [?, ?].

1. Assuming that an orthonormalized state basis with respect to an octonion valued inner product has been found, the multiplication of any basis with octonion spoils the orthonormality. The proposal to circumvent this difficulty discussed in [?] eliminates non-associativity by assuming that octonions multiply states one by one (rather than multiplying each other before multiplying the state). Effectively this means that octonions are replaced with 8×8-matrices.

2. The definition of the tensor product leads also to difficulties since associativity is lost (recall that Yang-Baxter equation codes for associativity in case of braid statistics [?, ?]).

3. The notion of hermitian conjugation is problematic and forces a selection of a preferred imaginary unit, which does not look nice. Note however that the local selection of a preferred imaginary unit is in a key role in the proposed construction of space-time surfaces as hyper-quaternionic or co-hyper-quaternionic surfaces and allows to interpret space-time surfaces either as surfaces in 8-D Minkowski space M^8 of hyper-octonions or in $M^4\times CP_2$. This selection turns out to have quite different interpretation in the proposed framework.

Hyper-finite factor II_1 has a natural Hyper-Kähler structure

In the case of hyper-finite factors of type II_1 quaternions a more natural approach is based on the generalization of the Hyper-Kähler structure rather than quaternionic quantum mechanics. The reason is that also configuration space tangent space should and is expected to have this structure [?]. The Hilbert space remains a complex Hilbert space but the quaternionic units are represented as operators in Hilbert space. The selection of the preferred unit is necessary and natural. The identity operator representing quaternionic real unit has trace equal to one, is expected to give rise to the series of quantum quaternion algebras in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ having interpretation as N-modules.

The representation of the quaternion units is rather explicit in the structure of hyper-finite II_1 factor. The $\mathcal{M} : \mathcal{N} \equiv \beta = 4$ hierarchical construction can be regarded as Connes tensor product of infinite number of 4-D Clifford algebras of Euclidian plane with Euclidian signature of metric ($diag(-1,-1)$). This algebra is nothing but the quaternionic algebra in the representation of quaternionic imaginary units by Pauli spin matrices multiplied by i.

The imaginary unit of the underlying complex Hilbert space must be chosen and there is whole sphere S^2 of choices and in every point of configuration space the choice can be made differently. The space-time correlate for this local choice of preferred hyper-octonionic unit [?]. At the level of configuration space geometry the quaternion structure of the tangent space means the existence of Hyper-Kähler structure guaranteing that configuration space has a vanishing Einstein tensor. It it would not vanish, curvature scalar would be infinite by symmetric space property (as in case of loop spaces) and induce a divergence in the functional integral over 3-surfaces from the expansion of $\sqrt{g}$ [?].

The quaternionic units for the II_1 factor, are simply limiting case for the direct sums of 2×2 units normalized to one. Generalizing from $\beta = 4$ to $\beta < 4$, the natural expectation is that the representation of the algebra as $\beta = \mathcal{M} : \mathcal{N}$-dimensional $\mathcal{N}$-module gives rise to quantum quaternions with quaternion units defined as infinite sums of $\sqrt{\beta} \times \sqrt{\beta}$ matrices.

At Hilbert space level one has an infinite Connes tensor product of 2-component spinor spaces on which quaternionic matrices have a natural action. The tensor product of Clifford algebras gives the algebra of 2×2 quaternionic matrices acting on 2-component quaternionic spinors (complex 4-component spinors). Thus double inclusion could correspond to (hyper-)quaternionic structure at space-time level. Note however that the correspondence is not complete since hyper-quaternions appear at space-time level and quaternions at Hilbert space level.

Von Neumann algebras and octonions

The octonionic generalization of the Hyper-Kähler manifold does not make sense as such since octonionic units are not representable as linear operators. The allowance of anti-linear operators inherently present in von Neumann algebras could however save the situation. Indeed, the Cayley-Dickson construction for the division algebras (for a nice explanation see [?]), which allows to extend any * algebra, and thus also any von Neumann algebra, by adding an imaginary unit it and identified as *, comes in rescue.

The basic idea of the Cayley-Dickson construction is following. The * operator, call it J, representing a conjugation defines an *anti-linear* operator in the original algebra A. One can extend A by adding this operator as a new element to the algebra. The conditions satisfied by J are

$$a(Jb) = J(a^*b) \ , \quad (aJ)b = (ab^*)J \ , \quad (Ja)(bJ^{-1}) = (ab)^* \ . \tag{1.5.2}$$

In the associative case the conditions are equivalent to the first condition.

It is intuitively clear that this addition extends the hyper-Kähler structure to an octonionic structure at the level of the operator algebra. The quantum version of the octonionic algebra is fixed by the quantum quaternion algebra uniquely and is consistent with the Cayley-Dickson construction. It is not clear whether the construction is equivalent with either of the earlier proposals [?, ?]. It would however seem that the proposal is simpler.

Physical interpretation of quantum octonion structure

Without further restrictions the extension by J would mean that vertices contain operators, which are superpositions of linear and anti-linear operators. This would give superpositions of states and their time-reversals and mean that state could be a superposition of states with opposite values of say fermion numbers. The problem disappears if either the linear operators A or anti-linear operators JA can be used to construct physical states from vacuum. The fact, that space-time surfaces are either hyper-quaternionic or co-hyper-quaternionic, is a space-time correlate for this restriction.

The $HQ - coHQ$ duality discussed in [?] states that the descriptions based on hyper-quaternionic and co-hyper-quaternionic surfaces are dual to each other. The duality can have two meanings.

1. The vacuum is invariant under J so that one can use either complexified quaternionic operators A or their co-counterparts of form JA to create physical states from vacuum.

2. The vacuum is not invariant under J. This could relate to the breaking of CP and T invariance known to occur in meson-antimeson systems. In TGD framework two kinds of vacua are predicted corresponding intuitively to vacua in which either the product of all positive or negative energy fermionic oscillator operators defines the vacuum state, and these two vacua could correspond to a vacuum and its J conjugate, and thus to positive and negative energy states. In this case the two state spaces would not be equivalent although the physics associated with them would be equivalent.

The considerations of [?] related to the detailed dynamics of $HQ-coHQ$ duality demonstrate that the variational principles defining the dynamics of hyper-quaternionic and co-hyper-quaternionic space-time surfaces are antagonistic and correspond to world as seen by a conscientous book-keeper on one hand and an imaginative artist on the other hand. HQ case is conservative: differences measured by the magnitude of Kähler action tend to be minimized, the dynamics is highly predictive, and minimizes the classical energy of the initial state. $coHQ$ case is radical: differences are maximized (this is what the construction of sensory representations would require). The interpretation proposed in [?] was that the two space-time dynamics are just different predictions for what would happen (has happened) if no quantum jumps would occur (had occurred). A stronger assumption is that these two views are associated with systems related by time reversal symmetry.

What comes in mind first is that this antagonism follows from the assumption that these dynamics are actually time-reversals of each other with respect to M^4 time (the rapid elimination of differences in the first dynamics would correspond to their rapid enhancement in the second dynamics). This is not the case so that T and CP symmetries are predicted to be broken in accordance with the CP breaking in meson-antimeson systems [?] and cosmological matter-antimatter asymmetry [?].

1.5.7 Does the hierarchy of infinite primes relate to the hierarchy of II_1 factors?

The hierarchy of Feynman diagrams accompanying the hierarchy defined by Jones inclusions $\mathcal{M}_0 \subset \mathcal{M}_1 \subset ...$ gives a concrete representation for the hierarchy of cognitive dynamics providing a representation for the material world at the lowest level of the hierarchy. This hierarchy seems to relate directly to the hierarchy of space-time sheets.

Also the construction of infinite primes [?] leads to an infinite hierarchy. Infinite primes at the lowest level correspond to polynomials of single variable x_1 with rational coefficients, next level to polynomials x_1 for which coefficients are rational functions of variable x_2, etc... so that a natural ordering of the variables is involved.

If the variables x_i are hyper-octonions (subs-space of complexified octonions for which elements are of form $x + \sqrt{-1}y$, where x is real number and y imaginary octonion and $\sqrt{-1}$ is commuting imaginary unit, this hierarchy of states could provide a realistic representation of physical states as far as quantum numbers related to imbedding space degrees of freedom are considered in M^8 picture dual to $M^4 \times CP_2$ picture [?]. Infinite primes are mapped to space-time surfaces in a manner analogous to the mapping of polynomials to the loci of their zeros so that infinite primes, integers, and rationals become concrete geometrical objects.

Infinite primes are also obtained by a repeated second quantization of a super-symmetric arithmetic quantum field theory. Infinite rational numbers correspond in this description to pairs of positive energy and negative energy states of opposite energies having interpretation as pairs of initial and final states so that higher level states indeed represent transitions between the states. For these reasons this hierarchy has been interpreted as a correlate for a cognitive hierarchy coding information about quantum dynamics at lower levels. This hierarchy has also been assigned with the hierarchy of space-time sheets. Just as the hierarchy of generalized Feynman diagrams provides self representations of the lowest matter level and is coded by it, finite primes code the hierarchy of infinite primes.

Infinite primes, integers, and rationals have finite p-adic norms equal to 1, and one can wonder whether a Hilbert space like structure with dimension given by an infinite prime or integer makes sense, and whether it has anything to do with the Hilbert space for which dimension is infinite in the sense of the limiting value for a dimension of sub-space. The Hilbert spaces with dimension equal to infinite prime would define primes for the tensor product of these spaces. The dimension of this kind of space defined as any p-adic norm would be equal to one.

One cannot exclude the possibility that infinite primes could express the infinite dimensions of hyper-finite III_1 factors, which cannot be excluded and correspond to that part of quantum TGD which relates to the imbedding space rather than space-time surface. Indeed, infinite primes code naturally for the quantum numbers associated with the imbedding space. Secondly, the appearance of 7-D light-like causal determinants $X^7_\pm = M^4_\pm \times CP_2$ forming nested structures in the construction of S-matrix brings in mind similar nested structures of algebraic quantum field theory [?]. If this is were the case, the hierarchy of Beraha numbers possibly associated with the phase resolution could correspond to hyper-finite factors of type II_1, and the decomposition of space-time surface to regions labeled by p-adic primes and characterized by infinite primes could correspond to hyper-finite factors of type III_1 and represent imbedding space degrees of freedom.

The state space would in this picture correspond to the tensor products of hyper-finite factors of type II_1 and III_1 (of course, also factors I_n and I_∞ are also possible). III_1 factors could be assigned to the sub-configuration spaces defined by 3-surfaces in regions of M^4 expressible in terms of unions and intersections of $X^7_\pm = M^4_\pm \times CP_2$. By conservation of four-momentum, bounded regions of this kind are possible only for the states of zero net energy appearing at the higher levels of hierarchy. These sub-configuration spaces would be characterized by the positions of the tips of light cones $M^4_\pm \subset M^4$ involved. This indeed brings in continuous spectrum of four-momenta forcing to introduce non-separable Hilbert spaces for momentum eigen states and necessitating III_1 factors. Infinities would be avoided since the dynamics proper would occur at the level of space-time surfaces and involve only II_1 factors.

1.6 Could HFFs of type III have application in TGD framework?

One can imagine several manners for how HFFs of type III could emerge in TGD although the proposed view about M-matrix in zero energy ontology suggests that HFFs of type III_1 should be only an auxiliary tool at best. Same is suggested with interpretational problems associated with them. Both TGD inspired quantum measurement theory, the idea about a variant of HFF of type II_1 analogous to a local gauge algebra, and some other arguments, suggest that HFFs of type III could be seen as a useful idealization allowing to make non-trivial conjectures both about quantum TGD and about HFFs of type III. Quantum fields would correspond to HFFs of type III and II_∞ whereas physical states (M-matrix) would correspond to HFF of type II_1. I have summarized first the problems of III_1 factors so that reader can decide whether the further reading is worth of it.

1.6.1 Problems associated with the physical interpretation of III_1 factors

Algebraic quantum field theory approach [?, ?] has led to a considerable understanding of relativistic quantum field theories in terms of hyper-finite III_1 factors. There are however several reasons to suspect that the resulting picture is in conflict with physical intuition. Also the infinities of non-trivial relativistic QFTs suggest that something goes wrong.

Are the infinities of quantum field theories due the wrong type of von Neumann algebra?

The infinities of quantum field theories involve basically infinite traces and it is now known that the algebras of observables for relativistic quantum field theories for bounded regions of Minkowski space correspond to hyper-finite III_1 algebras, for which non-trivial traces are always infinite. This might be the basic cause of the divergence problems of relativistic quantum field theory.

On basis of this observations there is some temptation to think that the finite traces of hyper-finite II_1 algebras might provide a resolution to the problems but not necessarily in QFT context. One can play with the thought that the subtraction of infinities might be actually a process in which III_1 algebra is transformed to II_1 algebra. A more plausible idea suggested by dimensional regularization is that the elimination of infinities actually gives rise to II_1 inclusion at the limit $\mathcal{M}:\mathcal{N}\rightarrow 4$. It is indeed known that the dimensional regularization procedure of quantum field theories can be formulated in terms of bi-algebras assignable to Feynman diagrams and [?, ?] and the emergence of bi-algebras suggests that a connection with II_1 factors and critical role of dimension $D=4$ might exist.

Continuum of inequivalent representations of commutation relations

There is also a second difficulty related to type III algebras. There is a continuum of inequivalent representations for canonical commutation relations [?]. In thermodynamics this is blessing since temperature parameterizes these representations. In quantum field theory context situation is however different and this problem has been usually put under the rug.

Entanglement and von Neumann algebras

In quantum field theories where 4-D regions of space-time are assigned to observables. In this case hyper-finite type III_1 von Neumann factors appear. Also now inclusions make sense and has been studied: in fact, the parameters characterizing Jones inclusions appear also now and this due to the very general properties of the inclusions.

The algebras of type III_1 have rather counter-intuitive properties from the point of view of entanglement. For instance, product states between systems having space-like separation are not possible at all so that one can speak of intrinsic entanglement [?]. What looks worse is that the decomposition of entangled state to product states is highly non-unique.

Mimicking the steps of von Neumann one could ask what the notion of observables could mean in TGD framework. Effective 2-dimensionality states that quantum states can be constructed using the data given at partonic or stringy 2-surfaces. This data includes also information about normal derivatives so that 3-dimensionality actually lurks in. In any case this would mean that observables are assignable to 2-D surfaces. This would suggest that hyper-finite II_1 factors appear in quantum TGD at least as the contribution of single space-time surface to S-matrix is considered. The contributions for configuration space degrees of freedom meaning functional (not path-) integral over 3-surfaces could of course change the situation.

Also in case of II_1 factors, entanglement shows completely new features which need not however be in conflict with TGD inspired view about entanglement. The eigen values of density matrices are infinitely degenerate and quantum measurement can remove this degeneracy only partially. TGD inspired theory of consciousness has led to the identification of rational (more generally algebraic entanglement) as bound state entanglement stable in state function reduction. When an infinite number of states are entangled, the entanglement would correspond to rational (algebraic number) valued traces for the projections to the eigen states of the density matrix. The symplectic transformations of CP_2 are almost $U(1)$ gauge symmetries broken only by classical gravitation. They imply a gigantic spin glass degeneracy which could be behind the infinite degeneracies of eigen states of density matrices in case of II_1 factors.

1.6.2 Quantum measurement theory and HFFs of type III

The attempt to interpret the HFFs of type III in terms of quantum measurement theory based on Jones inclusions leads to highly non-trivial conjectures about these factors.

Could the scalings of trace relate to quantum measurements?

What should be understood is the physical meaning of the automorphism inducing the scaling of trace. In the representation based of factors based on infinite tensor powers the action of g should transform single $n\times n$ matrix factor with density matrix Id/n to a density matrix e_{11} of a pure state.

Obviously the number of degrees of freedom is affected and this can be interpreted in terms of appearance or disappearance of correlations. Quantization and emergence of non-commutativity indeed implies the emergence of correlations and effective reduction of degrees of freedom. In particular, the fundamental quantum Clifford algebra has reduced dimension $\mathcal{M}:\mathcal{N}=r\leq 4$ instead of $r=4$ since the replacement of complex valued matrix elements with $\mathcal{N}$ valued ones implies non-commutativity and correlations.

The transformation would be induced by the shift of finite-dimensional state to right or left so that the number of matrix factors overlapping with I_∞ part increases or is reduced. Could it have interpretation in terms of quantum measurement for a quantum Clifford factor? Could quantum measurement for $\mathcal{M}/\mathcal{N}$ degrees of freedom reducing the state in these degrees of freedom to a pure state be interpreted as a transformation of single finite-dimensional matrix factor to a type I factor inducing the scaling of the trace and could the scalings associated with automorphisms of HFFs of type III also be interpreted in terms of quantum measurement?

This interpretation does not as such say anything about HFF factors of type III since only a decomposition of II_1 factor to I_2^k factor and II_1 factor with a reduced trace of projector to the latter. However, one can ask whether the scaling of trace for HFFs of type III could correspond to a situation in which infinite number of finite-dimensional factors have been quantum measured. This would correspond to the inclusion $\mathcal{N}\subset\mathcal{M}_\infty=\cup_n\mathcal{M}_n$ where $\mathcal{N}\subset\mathcal{M}\subset ...\mathcal{M}_n...$ defines the canonical inclusion sequence. Physicist can of course ask whether the presence of infinite number of I_2-, or more generally, I_n-factors is at all relevant to quantum measurement and it has already become clear that situation at the level of M-matrix reduces to I_n.

Could the theory of HHFs of type III relate to the theory of Jones inclusions?

The idea about a connection of HFFs of type III and quantum measurement theory seems to be consistent with the basic facts about inclusions and HFFs of type III_1.

1. Quantum measurement would scale the trace by a factor $2^k/\sqrt{\mathcal{M}:\mathcal{N}}$ since the trace would become a product for the trace of the projector to the newly born $M(2,C)^{\otimes k}$ factor and the trace for the projection to $\mathcal{N}$ given by $1/\sqrt{\mathcal{M}:\mathcal{N}}$. The continuous range of values $\mathcal{M}:\mathcal{N}\geq 4$ gives good hopes that all values of λ are realized. The prediction would be that $2^k\sqrt{\mathcal{M}:\mathcal{N}}\geq 1$ holds always true.

2. The values $\mathcal{M}:\mathcal{N}\in\{r_n=4cos^2(\pi/n)\}$ for which the single $M(2,C)$ factor emerges in state function reduction would define preferred values of the inverse of $\lambda=\sqrt{\mathcal{M}:\mathcal{N}/4}$ parameterizing factors III_λ. These preferred values vary in the range $[1/2,1]$.

3. $\lambda=1$ at the end of continuum would correspond to HFF III_1 and to Jones inclusions defined by infinite cyclic subgroups dense in $U(1)\subset SU(2)$ and this group combined with reflection. These groups correspond to the Dynkin diagrams A_∞ and D_∞. Also the classical values of $\mathcal{M}:\mathcal{N}=n^2$ characterizing the dimension of the quantum Clifford $\mathcal{M}:\mathcal{N}$ are possible. In this case the scaling of trace would be trivial since the factor n to the trace would be compensated by the factor $1/n$ due to the disappearance of $\mathcal{M}/\mathcal{N}$ factor III_1 factor.

4. Inclusions with $\mathcal{M}:\mathcal{N}=\infty$ are also possible and they would correspond to $\lambda=0$ so that also III_0 factor would also have a natural identification in this framework. These factors correspond to ergodic systems and one might perhaps argue that quantum measurement in this case would give infinite amount of information.

5. This picture makes sense also physically. p-Adic thermodynamics for the representations of super-conformal algebra could be formulated in terms of factors of type I_∞ and in excellent approximation using factors I_n. The generation of arbitrary number of type II_1 factors in quantum measurement allow this possibility.

The end points of spectrum of preferred values of λ are physically special

The fact that the end points of the spectrum of preferred values of λ are physically special, supports the hopes that this picture might have something to do with reality.

1. The Jones inclusion with $q=exp(i\pi/n)$, $n=3$ (with principal diagram reducing to a Dynkin diagram of group SU(3)) corresponds to $\lambda=1/2$, which corresponds to HFF III_1 differing in

essential manner from factors III_λ, $\lambda < 1$. On the other hand, $SU(3)$ corresponds to color group which appears as an isometry group and important subgroup of automorphisms of octonions thus differs physically from the ADE gauge groups predicted to be realized dynamically by the TGD based view about McKay correspondence [**?**].

2. For $r = 4$ SU(2) inclusion parameterized by extended ADE diagrams $M(2,C)^{\otimes 2}$ would be created in the state function reduction and also this would give $\lambda = 1/2$ and scaling by a factor of 2. Hence the end points of the range of discrete spectrum would correspond to the same scaling factor and same HFF of type III. SU(2) could be interpreted either as electro-weak gauge group, group of rotations of th geodesic sphere of $\delta M^4_\pm$, or a subgroup of SU(3). In TGD interpretation for McKay correspondence a phase transition replacing gauge symmetry with Kac-Moody symmetry.

3. The scalings of trace by factor 2 seem to be preferred physically which should be contrasted with the fact that primes near prime powers of 2 and with the fact that quantum phases $q = exp(i\pi/n)$ with n equal to Fermat integer proportional to power of 2 and product of the Fermat primes (the known ones are 5, 17, 257, and $2^{16}+1$) are in a special role in TGD Universe.

1.6.3 What could one say about II_1 automorphism associated with the II_∞ automorphism defining factor of type III?

An interesting question relates to the interpretation of the automorphisms of II_∞ factor inducing the scaling of trace.

1. If the automorphism for Jones inclusion involves the generator of cyclic automorphism sub-group Z_n of II_1 factor then it would seem that for other values of λ this group cannot be cyclic. SU(2) has discrete subgroups generated by arbitrary phase q and these are dense in $U(1) \subset SU(2)$ sub-group. If the interpretation in terms of Jones inclusion makes sense then the identification $\lambda = \sqrt{\mathcal{M} : \mathcal{N}}/2^k$ makes sense.

2. If HFF of type II_1 is realized as group algebra of infinite symmetric group [**?**], the outer automorphism induced by the diagonally imbedded finite Galois groups can induce only integer values of n and Z_n would correspond to cyclic subgroups. This interpretation conforms with the fact that the automorphisms in the completion of inner automorphisms of HFF of type II_1 induce trivial scalings. Therefore only automorphisms which do not belong to this completion can define HFFs of type III.

1.6.4 What could be the physical interpretation of two kinds of invariants associated with HFFs type III?

TGD predicts two kinds of counterparts for S-matric: M-matrix and U-matrix. Both are expected to be more or less universal. There are also *two* kinds of invariants and automorphisms associated with HFFs of type III.

1. The first invariant corresponds to the scaling $\lambda \in]0,1[$ of the trace associated with the automorphism of factor of II_∞. Also the end points of the interval make sense. The inverse of this scaling accompanies the inverse of this automorphism.

2. Second invariant corresponds to the time scales $t = T_0$ for which the outer automorphism σ_t reduces to inner automorphism. It turns out that T_0 and λ are related by the formula $\lambda^{iT_0} = 1$, which gives the allowed values of T_0 as $T_0 = n2\pi/log(\lambda)$ [**?**]. This formula can be understood intuitively by realizing that λ corresponds to the eigenvalue of the density matrix $\Delta = e^H$ in the simplest possible realization of the state ϕ.

The presence of two automorphisms and invariants brings in mind U matrix characterizing the unitary process occurring in quantum jump and M-matrix characterizing time like entanglement.

1. If one accepts the vision based on quantum measurement theory then λ corresponds to the scaling of the trace resulting when quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ reduces to a tensor power of

$M(2,C)$ factor in the state function reduction. The proposed interpretation for U process would be as the inverse of state function reduction transforming this factor back to $\mathcal{M}/\mathcal{N}$. Thus U process and state function reduction would correspond naturally to the scaling and its inverse. This picture might apply not only in single particle case but also for zero energy states which can be seen as states associated the a tensor power of HFFs of type II_1 associated with partons.

2. The implication is that U process can occur only in the direction in which trace is reduced. This would suggest that the full III_1 factor is not a physical notion and that one must restrict the group Z in the crossed product $Z \times_{cr} II_\infty$ to the group N of non-negative integers. In this kind of situation the trace is well defined since the traces for the terms in the crossed product comes as powers λ^{-n} so that the net result is finite. This would mean a reduction to II_∞ factor.

3. Since time t is a natural parameter in elementary particle physics experiment, one could argue that σ_t could define naturally M-matrix. Time parameter would most naturally correspond to a parameter of scaling affecting all $M^4_\pm$ coordinates rather than linear time. This conforms also with the fundamental role of conformal transformations and scalings in TGD framework.

The identification of the full M-matrix in terms of σ does not seem to make sense generally. It would however make sense for incoming and outgoing number theoretic braids so that σ could define universal braiding M-matrices. Inner automorphisms would bring in the dependence on experimental situation. The reduction of the braiding matrix to an inner automorphism for critical values of t which could be interpreted in terms of scaling by power of p. This trivialization would be a counterpart for the elimination of propagator legs from M-matrix element. Vertex itself could be interpreted as unitary isomorphism between tensor product of incoming and outgoing HFFs of type II_1 would code all what is relevant about the particle reaction.

1.6.5 Does the time parameter t represent time translation or scaling?

The connection $T_n = n2\pi/log(\lambda)$ would give a relationship between the scaling of trace and value of time parameter for which the outer automorphism represented by σ reduces to inner automorphism. It must be emphasized that the time parameter t appearing in σ need not have anything to do with time translation. The alternative interpretation is in terms of $M^4_\pm$ scaling (implying also time scaling) but one cannot exclude even preferred Lorentz boosts in the direction of quantization axis of angular momentum.

Could the time parameter correspond to scaling?

The central role of conformal invariance in quantum TGD suggests that t parameterizes scaling rather than translation. In this case scalings would correspond to powers of $(K\lambda)^n$. The numerical factor K which cannot be excluded a priori, seems to reduce to $K = 1$.

1. The scalings by powers of p have a simple realization in terms of the representation of HFF of type II_∞ as infinite tensor power of $M(p,C)$ with suitably chosen densities matrices in factors to get product of I_∞ and II_1 factor. These matrix algebras have the remarkable property of defining prime tensor power factors of finite matrix algebras. Thus p-adic fractality would reflect directly basic properties of matrix algebras as suggested already earlier. That scalings by powers of p would correspond to automorphism reducing to inner automorphisms would conform with p-adic fractality.

2. Also scalings by powers $[\sqrt{\mathcal{M}:\mathcal{N}}/2^k]^n$ would be physically preferred if one takes previous arguments about Jones inclusions seriously and if also in this case scalings are involved. For $q = exp(i\pi/n)$, $n = 5$ the minimal value of n allowing universal topological quantum computation would correspond to a scaling by Golden Mean and these fractal scalings indeed play a key role in living matter. In particular, Golden Mean makes it visible in the geometry of DNA.

Could the time parameter correspond to time translation?

One can consider also the interpretation of σ_t as time translation. TGD predicts a hierarchy of Planck constants parameterized by rational numbers such that integer multiples are favored. In particular, integers defining ruler and compass polygons are predicted to be in a very special role physically. Since the geometric time span associated with zero energy state should scale as Planck constant one expects that preferred values of time t associated with σ are quantized as rational multiples of some fundamental time scales, say the basic time scale defined by CP_2 length or p-adic time scales.

1. For $\lambda = 1/p$, p prime, the time scale would be $T_n = nT_1$, $T_1 = T_0 = 2\pi/log(p)$ which is not what p-adic length scale hypothesis would suggest.

2. For Jones inclusions one would have $T_n/T_0 = n2\pi/log(2^{2k}/\mathcal{M} : \mathcal{N})$. In the limit when λ becomes very small (the number k of reduced $M(2,C)$ factors is large one obtains $T_n = (n/k)t_1$, $T_1 = T_0\pi/log(2)$. Approximate rational multiples of the basic length scale would be obtained as also predicted by the general quantization of Planck constant.

p-Adic thermodynamics from first principles

Quantum field theory at non-zero temperature can be formulated in the functional integral formalism by replacing the time parameter associated with the unitary time evolution operator $U(t)$ with a complexified time containing as imaginary part the inverse of the temperature: $t \rightarrow t + i\hbar/T$. In the framework of standard quantum field theory this is a mere computational trick but the time parameter associated with the automorphisms σ_t of HFF of type III is a temperature like parameter from the beginning, and its complexification would naturally lead to the analog of thermal QFT.

Thus thermal equilibrium state would be a genuine quantum state rather than fictive but useful auxiliary notion. Thermal equilibrium is defined separately for each incoming parton braid and perhaps even braid (partons can have arbitrarily large size). At elementary particle level p-adic thermodynamics could be in question so that particle massivation would have first principle description. p-Adic thermodynamics is under relatively mild conditions equivalent with its real counterpart obtained by the replacement of p^{L_0} interpreted as a p-adic number with p^{-L_0} interpreted as a real number.

1.6.6 Could HFFs of type III be associated with the dynamics in $M^4_\pm$ degrees of freedom?

HFFs of type III could be also assigned with the poorly understood dynamics in $M^4_\pm$ degrees of freedom which should have a lot of to do with four-dimensional quantum field theory. Hyper-finite factors of type III_1 might emerge when one extends II_1 to a local algebra by multiplying it with hyper-octonions replaced as analog of matrix factor and considers hyper-quaternionic subalgebra. The resulting algebra would be the analog of local gauge algebra and the elements of algebra would be analogous to conformal fields with complex argument replaced with hyper-octonionic, -quaternionic, or -complex one. Since quantum field theory in M^4 gives rise to hyper-finite III_1 factors one might guess that the hyper-quaternionic restriction indeed gives these factors.

The expansion of the local HFF II_∞ element as $O(m) = \sum_n m^n O_n$, where M^4 coordinate m is interpreted as hyper-quaternion, could have interpretation as expansion in which O_n belongs to $\mathcal{N}g^n$ in the crossed product $\mathcal{N} \times_{cr} \{g^n, n \in Z\}$. The analogy with conformal fields suggests that the power g^n inducing λ^n fold scaling of trace increases the conformal weight by n.

One can ask whether the scaling of trace by powers of λ defines an inclusion hierarchy of sub-algebras of conformal sub-algebras as suggested by previous arguments. One such hierarchy would be the hierarchy of sub-algebras containing only the generators O_m with conformal weight $m \geq n$, $n \in Z$.

It has been suggested that the automorphism Δ could correspond to scaling inside light-cone. This interpretation would fit nicely with Lorentz invariance and TGD in general. The factors III_λ with λ generating semi-subgroups of integers (in particular powers of primes) could be of special physical importance in TGD framework. The values of t for which automorphism reduces to inner automorphism should be of special physical importance in TGD framework. These automorphisms correspond to scalings identifiable in terms of powers of p-adic prime p so that p-adic fractality would find an explanation at the fundamental level.

If the above mentioned expansion in powers of m^n of $M^4_\pm$ coordinate makes sense then the action of σ^t representing a scaling by p^n would leave the elements O invariant or induce a mere inner automorphism. Conformal weight n corresponds naturally to n-ary p-adic length scale by uncertainty principle in p-adic mass calculations.

The basic question is the physical interpretation of the automorphism inducing the scaling of trace by λ and its detailed action in HFF. This scaling could relate to a scaling in M^4 and to the appearance in the trace of an integral over M^4 or subspace of it defining the trace. Fractal structures suggests itself strongly here. At the level of construction of physical states one always selects some minimum non-positive conformal weight defining the tachyonic ground state and physical states have non-negative conformal weights. The interpretation would be as a reduction to HHF of type II_∞ or even II_1.

1.6.7 Could the continuation of braidings to homotopies involve Δ^{it} automorphisms

The representation of braidings as special case of homotopies might lead from discrete automorphisms for HFFs type II_1 to continuous outer automorphisms for HFFs of type III_1. The question is whether the periodic automorphism of II_1 represented as a discrete sub-group of $U(1)$ would be continued to $U(1)$ in the transition.

The automorphism of II_∞ HFF associated with a given value of the scaling factor λ is unique. If Jones inclusions defined by the preferred values of λ as $\lambda = \sqrt{\mathcal{M} : \mathcal{N}}/2^k$ (see the previous considerations), then this automorphism could involve a periodic automorphism of II_1 factor defined by the generator of cyclic subgroup Z_n for $\mathcal{M} : \mathcal{N} < 4$ besides additional shift transforming II_1 factor to I_∞ factor and inducing the scaling.

1.6.8 HFFs of type III as super-structures providing additional uniqueness?

If the braiding M-matrices are as such highly unique. One could however consider the possibility that they are induced from the automorphisms σ_t for the HFFs of type III restricted to HFFs of type II_∞. If a reduction to inner automorphism in HFF of type III implies same with respect to HFF of type II_∞ and even II_1, they could be trivial for special values of time scaling t assignable to the partons and identifiable as a power of prime p characterizing the parton. This would allow to eliminate incoming and outgoing legs. This elimination would be the counterpart of the division of propagator legs in quantum field theories. Particle masses would however play no role in this process now although the power of padic prime would fix the mass scale of the particle.

1.7 Space-time as surface of $M^4 \times CP_2$ and inclusions of hyperfinite type II_1 factors

Double Jones inclusion plays a pivotal role in the theory of von Neumann algebras. Double inclusion $\mathcal{N} \subset \mathcal{M} \subset \mathcal{M}_1$, where $\mathcal{M}_1 = \mathcal{M} \otimes_N \mathcal{M}$ is Connes tensor product obtained by identifying the elements $nm_1 \otimes m_2$ and $m_1 \otimes m_2 n$ so that multiplication by $\mathcal{N}$ from left is equivalent to that from right. This double inclusion extends to an infinite hierarchy of inclusions.

These two inclusions are dual and the values of $\mathcal{M} : \mathcal{N}$ are same. One can write M_1 as N module of $\mathcal{M}/\mathcal{N} \otimes \mathcal{M}/\mathcal{N}$ so that one has quantal counterpart of $2d$-spinors associated with 4-D manifolds. This suggests an interpretation of the Jones inclusion $\mathcal{M} \subset \mathcal{M}_1$ as a quantal representation for an imbedding of 2-D manifold into 4-D manifold at the level of spinors regarded having $\mathcal{N}$-valued components.

This is however not quite enough: in TGD one has imbeddings of 4-D space-times to 8-D imbedding space, and one can ask whether also these imbeddings could be represented using Jones inclusions. Quaternionic and octonionic matrix algebras might provide as solution to the problem.

This encourages to ask whether fundamental physics could somehow reduce to Jones inclusions of infinite-dimensional Clifford algebras induced by the inclusion sequence $C \subset H \subset O$ for the classical number fields and by its hyper-counterpart as suggested by number theoretic vision [?]. In other words, does the notion of space-time as a surface of $M^4 \times CP_2$ emerge automatically from number theoretical infinite-dimensional Clifford algebras? The following arguments try to demonstrate that the generalization of the number theoretical infinite-dimensional Clifford algebras by making them local algebras with respect to hyper-F ($F = C, H, O$) could realize the dream.

1.7.1 Jones inclusion as a representation for the imbedding $X^4 \subset M^4 \times CP_2$?

The obvious guess is that the complex matrix algebra $M_2(C)$ as a building block of an infinite-dimensional Clifford algebra must be replaced with the quaternionic matrix algebra $M_2(H)$. This is possible since $M_2(H) \otimes_C M_2(H)$ can be defined using Connes tensor product to guarantee that the multiplication by complex numbers from left in the tensor product is equivalent with the multiplication from right. For octonions this construction fails unless one allows non-associativity which is not a problem mathematically but poses interpretational problems.

In this case the quantum spinors associated with $\mathcal{M}/\mathcal{N}$ would have 4 complex components and correspond to 4-D space-time. The quantum spinors associated with $\mathcal{M}_1/\mathcal{N}$ would have 16 complex components and correspond to 8-D space-time. This suggests that $\mathcal{M} \subset \mathcal{M}_1$ represents the imbedding of the space-time surface to the imbedding space: the problems with signature are avoided since spinors are used. $\mathcal{N} \subset \mathcal{M}$ could in turn be interpreted as a representation of the imbedding of 2-dimensional partonic surfaces obtained as a cross section of light-like causal determined into X^4. $\mathcal{M}$ ($\mathcal{M}_1$) could in turn be interpreted in terms of $\mathcal{N}$-valued quantization of space-time (imbedding space) spinors. Any quaternionic double imbedding with $\mathcal{N} : \mathcal{M} \leq 4$ would have the same interpretation so that the space-time and imbedding space dimension would be universal.

This picture is however not yet quite satisfactory. Number theoretic vision suggests that also octonions are important. This suggests that the double inclusion $N \subset M \subset M_1$ should be replaced with a purely number theoretic Jones inclusions induced by the inclusion $C \subset H \subset O$.

1.7.2 Why $X^4 \subset M^4 \times CP_2$?

The number theoretic vision [?] interprets space-time surfaces as hyper-quaternionic sub-manifolds of hyper-octonionic imbedding space. The previous considerations would in turn suggests that physical Clifford algebra could be seen as a quaternionic sub-algebra of octonionic algebra by restricting the octonionic coefficients of the Clifford basis to be quaternionic. Thus it would seem that complexified quaternions and octonions are needed and that Clifford algebra degrees should correspond to quaternions-/octonions and bosonic center of mass degrees of freedom to hyper-quaternions/-octonions. In the following an attempt to complete these observations to a coherent picture is made.

CP_2 parameterizes quaternionic sub-factors

Formally the notion of Connes tensor product generalizes to the octonionic context. Strictly speaking, the non-associativity of the matrix multiplication means leaving the framework of von Neumann algebras and could lead to interpretational difficulties although the products as such are completely well defined.

In accordance with the number theoretical vision one might hope that the basic laws of physics laws would result as a resolution of these interpretational difficulties. Quaternionic Clifford algebra indeed emerges naturally as a subalgebra obtained by restricting the octonionic matrices to have quaternion-valued elements. Quaternion-octonion inclusion could thus define the fundamental Jones inclusion at the level of configuration space Clifford algebra. This inclusion could be completed to a double inclusion corresponding to $C \subset H \subset O$.

Remarkably, the set of these subalgebras is parameterized by $CP_2 = SU(3)/U(2)$ since $SU(3)$ is the group of (hyper)-octonionic automorphisms and $U(2)$ leaves a given quaternionic plane invariant. The necessity to select of this sub-algebra would follow from the associativity requirement [?, ?]. This also conforms with the fact that complex CP_2 coordinates behave locally like $U(2)$ spinor.

M^4 parameterizes the tips of hyper-quaternionic light cones

The next question is how to obtain M^4 factor and how the identification space-time surfaces as hyper-quaternionic sub-manifolds of hyper-octonionic imbedding space [?] could emerge from this picture.

Consider first what can be regarded as understood.

1. Future and past light-cones appear naturally in the construction of the geometry of the world of the classical worlds (configuration space). Configuration space can be regarded as a union of configuration spaces associated with the future and past light-cones. Therefore the points m of M^4 would appear as moduli labeling the sub-configuration spaces $CH_{+-}(m)$ consisting of

surfaces in the union $M^4_{+-} = M^4_+ \cup M^4_-$ in the union $\cup_m CH_{+-}(m)$ defining the configuration space.

2. There is an analogy with conformal field theories. Configuration space spinor fields depend on configuration space coordinates, in particular moduli characterizing the position of light cone cosmology CH_{+-} in decomposition $\cup_m CH_{+-}(m)$. The dependence on m is completely analogous to the dependence of conformal fields on complex coordinate z in conformal fields and induces corresponding dependence on quantum states created by the CH spinor fields. Once this dependence is known, it is possible to calculate products of configuration space spinors fields associated with different light cones M^4_{+-} using operator product expansions in complete analogy with conformal field theories. The M^4 dependence of n-point functions could thus be calculated.

What one should show?

The ambitious goal is to show that $CH = \cup_m CH_{+-}(m)$, the notion of space-time as a 4-surface $X^4 \subset H$, and partonic picture (effective 2-dimensionality) summarized as $X^2 \subset X^4 \subset H$ emerges from the number theoretic sub-factor double induced by $C \subset H \subset O$ automatically. The notion of imbedding space and identification of space-time as surface of imbedding space would thus emerge from number theory alone.

1. The Cayley-Dickson construction giving a series of algebras obtained by adding the * operation of existing algebra as a new imaginary unit to existing algebra indeed defines a von Neumann algebra and one would have infinite series of inclusions. The resulting imaginary unit obviously does not commute with existing imaginary units so that the commuting hyper-octonionic imaginary unit would not result in this manner. One should show that the commuting imaginary unit emerges naturally.

2. Bott periodicity states that Clifford algebras are in well defined sense equivalent *mod* 8. Hence one can ask could the further continuation would give $C \subset H \subset O \subset O \subset ...$ at the level of von Neumann algebras also in accordance with the non-existence of further number fields.

3. More concretely, one should demonstrate that the configuration space spinor fields satisfying the Super Virasoro conditions form a family parameterized by hyper-quaternionic coordinate m appearing as an expansion parameter of field and mass squared formula characterizes the dependence on m. If one can start from the parametrization of CH spinors (as opposed to spinor fields) as Laurent series of hyper-quaternion m without any a priori space-time interpretation there are hopes for this.

   This is the case if configuration space spinors expressible as a Laurent series in hyper-quaternionic/-octonionic parameter having operator coefficients in H-/O-Clifford algebra are a natural notion. These series would corresponds to a generalization of corresponding expansions in complex coordinate for Kac-Moody and super-conformal algebra elements and conformal fields in general. This suggests the notion of local von Neumann algebras and their inclusions as a key concept. Note that this does not yet imply the notion of configuration space spinor field.

Clues

The construction of the representations of super symplectic and super Moody algebras involves the choice of a fixed future or past light cone since super-symplectic algebra is defined at its boundary. Since four momentum labels the states of these representations, also the translational degrees of freedom associated with the tip of the light cone brings in the desired moduli. If one can show that super-conformal symmetries accompany naturally the inclusions of von Neumann algebras, one could also understand the emergence of hyper-quaternionic M^4 coordinate as moduli. The allowed values of the index $\mathcal{M} : \mathcal{N}$ indeed label minimal conformal field theories and quantum groups associated with conformal field theories. Therefore there are reasons to optimism.

The hyper-quaternionic inverse fails to exist when the hyper-quaternion is light-like. M^4_{+-} would thus define converge region of the Laurent series, and explain the restriction of space-time surfaces inside $M^4_\pm$ (classical causality) implying light cone cosmology. This would bring in the desired super-symplectic conformal invariance in $\delta M^4_+ \times CP_2$ and four-momentum and hyper-quaternionic parameter space would have the desired interpretation as M^4.

Two options for defining infinite-dimensional Clifford algebras

There are two options concerning the definition of infinite-dimensional Clifford algebra.

1. One could define them using matrix algebra $C(k)$, $k = 2,4,6,8$ as basic building brick. Bott periodicity implies that $C(8)\otimes\mathcal{M}$ and $\mathcal{M}$ are isomorphic as infinite-dimensional Clifford algebras.

2. The elements of matrix algebras belong naturally to some number field and in the matrix algebras $M_2(F)$, 2×2 matrices having $F = C, H$, and possibly even O valued matrix elements, could be considered as the building block of configuration space Clifford algebra.

The algebras defined by $M_2(C)$ and $M_2(H)$ certainly exists with latter having real coefficients as Abelian coefficient ring. $M_2(C)$ matrices can be also regarded as complexified quaternions H_C with Abelian complexification. Hence it seems that $M_2(H_C) = M_2(C) \otimes H = H_C \otimes H = M_2(C) \otimes M_2(C)$ as a building block gives subalgebra of the algebra generated by $M_2(C)$ having only $C(4)$ elements with coefficients in the ring defined by the infinite tensor product of 16×16 matrices.

The matrix algebra defined by $M_2(O_C) = M_2(C)\otimes O$ where C commutes with O is non-associative and one can question the idea that it could define elegantly an infinite-dimensional spinor algebra. Hence it would seem safest to consider four options: Cl_2 generated by $M_2(C)$ plus the algebras Cl_k generated by $C(k)$, $k = 4,6,8$. In TGD framework $Cl_8 \equiv Cl$ is natural since configuration space Clifford algebra can be naturally regarded as an infinite tensor power of Clifford algebra of the imbedding space and since space time spinors are induced from H-spinors.

The replacement of infinite-dimensional Clifford algebras with their local variants

Hyper-quaternions/-octonions should naturally relate to a local algebra defined by the von Neumann algebra analogous to Kac Moody, Virasoro, and conformal algebras. One can indeed consider local hyper-F variants for all algebras defined by the sequence of Jones inclusions.

There is however a very delicate point involved. The power series in hyper-F coordinate variable with Taylor coefficients in infinite-dimensional Clifford algebra Cl belong to the tensor product $F_C\otimes Cl$. The coordinate of hyper-F is representable as a linear combination of Clifford algebra generators of M^k, $k = 2,4,8$.

If these generators have matrix representations as gamma matrices, the resulting algebra can be absorbed to Cl (defined most naturally as an infinite tensor power of $C(8)$) as a finite-dimensional tensor factor so that no local gauge algebra results. Only for 8-dimensional case in which Clifford algebra generators can be represented as octonionic units situation changes since non-associativity does not allow this absorption. From the power series of hyper-octonion one obtains by a restriction to a maximal associative subspace power series in the quaternionic coordinate.

One can still wonder why just hyper-octonions and hyper-quaternions. The construction of the quantum variants of complexified quaternions and octonions by replacing the coefficients by non-Hermitian operators provides the answer to this question. Complexified M^D, call it M^D_C, can be represented as a space spanned by Clifford algebra generators and its quantum counterpart is obtained by replacing the coefficients with non-Hermitian operators. The points of the real Minkowski space are represented as eigenstates of the hermitian operators $m^k_C + (m^k_C)^{dagger}$. These coordinate operators indeed commute so that their spectra define ordinary M^8 and its sub-manifolds M^4 as genuine quantal concepts. For Euclidian sub-space of maximal dimension the commutativity fails. This point is discussed in detail in the section devoted to the quantization of Planck constant.

This picture leads to a generalization of Jones inclusion to the Jones inclusion of local von Neumann algebras. It would not be surprising if local von Neumann algebras could be regarded as direct integral of factors. One might hope that the local variants of number theoretical Clifford algebras could be regarded as maximal extensions of them analogous to local number fields.

Since configuration space gamma matrices associated with CH_+ generate the Clifford algebra, the generalization boils down to an extension of these gamma matrices to hyper-quaternion valued fields expressible as power expansion in hyper-quaterionic coordinate. These fields are very much analogous to the corresponding fields appearing in super-string model. The difference is that these fields are non-hermitian, do not satisfy Majorana property, and carry well defined lepton or quark numbers [?]. Anticommutation relations for these fields should fix the anticommutation relations for the operator coefficients for powers of hyper-quaternion coordinate. The outcome is admittedly very stringy and TGD can be interpreted as a generalization of super string model.

Similar relations are obtained for gamma matrices at the partonic boundary components corresponding and $C \subset H$ inclusion relates them to the gamma matrices for $X^3 \subset X^4(X^3)$ whereas $X^4 \subset H$ relates these gamma matrices to the quantized versions of H gamma matrices. As a matter fact, induction procedure for octonionic quantum gamma matrices of H should give the gamma matrices at hyper-quaternionic space-time surfaces and further induction procedure at hyper-complex sections of partonic orbits. Therefore also induction procedure fits nicely TGD view.

Only the quantum variants of M^4 and M^8 emerge from local hyper-finite II_1 factors

Super-symmetry suggests that the representations of CH Clifford algebra $\mathcal{M}$ as $\mathcal{N}$ module $\mathcal{M}/\mathcal{N}$ should have bosonic counterpart in the sense that the coordinate for M^8 representable as a particular $M^2(Q)$ element should have quantum counterpart. Same would apply to M^4 coordinate representable as $M^2(C)$ element. Quantum matrix representation of $\mathcal{M}/\mathcal{N}$ as $SL_q(2,F)$ matrix, $F = C, H$ is the natural candidate for this representation. As a matter fact, this guess is not quite correct. It is the interpretation of $M_2(C)$ as a quaternionic quantum algebra whose generalization to the octonionic quantum algebra works.

Quantum variants of M^D exist for all dimensions but only spaces M^4 and M^8 and their linear sub-spaces emerge from hyper-finite factors of type II_1. This is due to the non-associativity of the octonionic representation of the gamma matrices making it impossible to absorb the powers of the octonionic coordinate to the Clifford algebra element so that the local algebra character would disappear. Even more: quantum coordinates for these spaces are commutative operators so that their spectra define ordinary M^4 and M^8 which are thus already quantal concepts.

Consider first hyper-quaternions and the emergence of M^4.

1. The commutation relations for $M_{2,q}(C)$ matrices

$$\left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \quad , \tag{1.7.1}$$

read as

$$\begin{array}{lll} ab = qba \ , & ac = qac \ , & bd = qdb \ , cd = qdc \ , \\ [a,d] = (q-q^{-1})bc \ , & bc = cb \ . & \end{array} \tag{1.7.2}$$

2. These relations can be extended by postulating complex conjugates of these relations for complex conjugates $a^\dagger, b^\dagger, c^\dagger, d^\dagger$ plus the following non-vanishing commutators of type $[x, y^\dagger]$:

$$[a,a^\dagger] = [b,b^\dagger] = [c,c^\dagger] = [d,d^\dagger] = 1 \ . \tag{1.7.3}$$

3. The matrices representing M^4 point must be expressible as sums of Pauli spin matrices. This can be represented as following conditions on physical states

$$\begin{array}{rcl} O|phys\rangle & = & 0 \ , \\ O & \in & \{a - a^\dagger, d - d^\dagger, b - c^\dagger, c - b^\dagger\} \ . \end{array} \tag{1.7.4}$$

For instance, the first two conditions follow from the reality of Pauli sigma matrices $\sigma_x, \sigma_y, \sigma_z$. These conditions are compatible only if the operators O commute.

4. What is essential is that the operators of O are of form $A - A^\dagger$ and their commutators are also of the same form that the commutativity conditions reduce the condition that the Lie-algebra like structure generated by these operators annihilates the physical state. Hence it is possible to define quantum states for which M^4 coordinates have well-defined eigenvalues so that ordinary M^4 emerges purely quantally from quaternions whose real coefficients are made non-Hermitian operators to obtain operator complexificiation of quaternions. Also the quantum states in which M^4 coordinates are emerge naturally.

5. $M_{2,q}(C)$ matrices define the quantum analog of C^4 and one can wonder whether also other linear sub-spaces can be defined consistently or whether M^4_q and thus Minkowski signature is unique. This seems to be not the case. For instance, the replacement $a - \overline{a} \rightarrow a + \overline{a}$ making also time variable Euclidian is impossible since $[a + \overline{a}, d - \overline{d}] = 2(q - q^{-1})(bc + b^\dagger c^\dagger$ is not proportional to a difference of operator and its hermitian conjugate and one does not obtain closed algebra.

What about M^8: does it have analogous description in terms of physical states annihilated by the Lie algebra generated by the differences $a_i - a_i^\dagger$, $i = 0, ..7$?

1. The representation of M^4 point as $M_2(C)$ matrix can be interpreted a combination of 4-D gamma matrices defining hyper-quaternionic units. Hyper-octonionic units indeed have anticommutation relations of gamma matrices of M^8 and would give classical representation of M^8. The counterpart of $M_{2,q}(C)$ would thus be obtained by replacing the coefficients of hyper-octonionic units with operators satisfying the generalization of $M_{2,q}(C)$ commutation relations. One should identify the reality conditions and find whether they are mutually consistent.

2. In quaternionic case basis for matrix algebra is formed by the sigma matrices and M^4 point is represented by a hermitian matrix expressible as linear combination of hermitian sigma matrices with coefficients which act on physical states like hermitian operators. In the hyper-octonionic case would expect that real octonion unit and octonionic imaginary units multiplied by commuting imaginary unit to define the counterparts of sigma matrices and that the physically representable sub-space of complex quantum octonions corresponds to operator valued coordinates which act like hermitian matrices. The restriction to complex quaternionic sub-space must give hyper-quaternions and M^4 so that the only sensible generalization is that M^8 holds quite generally. This is also required by SO^7 invariance allowing to choose the sub-space M^4 freely. Again the key point should be that the conditions giving rise to real eigenvalues give rise to a Lie-algebra which must annihilate the physical state. For other signatures one would not obtain Lie algebra.

3. One can also make guess for the concrete realization of the algebra. Introduce the coefficients of E^4 gamma matrices having interpretation as quaternionic units as

$$\begin{array}{ll} a_0 = ix(a+d) \ , & a_3 = x(a-d) \ , \\ a_1 = x(ib+c) \ , & a_2 = x(ib-c) \ , \\ x = \frac{1}{\sqrt{2}} \ , & \end{array}$$

and write the commutations relations for them to see how the generalization should be performed.

4. The selections of complex and quaternionic sub-algebras of octonions are fundamental for TGD and quantum octonionic algebra should reflect these selections in its structure. In the case of hyper-quaternions the selection of commutative sub-algebra implies the breaking of 4-D Lorentz symmetry. In the case of hyper-octonions the selection of hyper-quaternion sub-algebra should induce the breaking of 8-D Lorentz symmetry. Hyper-quaternionic sub-algebra obeys the commutations of $M_q(2,C)$ whereas the coefficients in the complement commute mutually and quantum commute with the complex sub-algebra. This nails down the commutation relations completely:

$$\begin{aligned}
[a_0, a_3] &= \frac{i}{2}(q-q^{-1})(a_1^2-a_2^2) \ , \\
[a_i, a_j] &= 0 \ , \ i,j \neq 0,3 \ , \\
a_0 a_i &= q a_i a_0 \ , \ i \neq 0,3 \ , \\
a_3 a_i &= q a_i a_3 \ , \ i \neq 0,3 \ .
\end{aligned} \tag{1.7.5}$$

Note that there is symmetry breaking in the sense that the commutation relations for sub-algebras relating to both M^4 and M^2 are in distinguished role.

Dimensions $D = 4$ and $D = 8$ are indeed unique if one takes this argument seriously.

1. For dimensions other than $D = 4$ and $D = 8$ a representation of the point of M^D as element of Clifford algebra of M^D is needed. The coefficients should be real for the signatures and this requires that the elements of Clifford algebra are Hermitian. Gamma matrices are the only natural candidates and when Majorana conditions can be satisfied one would obtain quantum representation of M^D. 10-D Minkowski space of super-string models would represent one example of this kind of situation.

2. For other dimensions $D \geq 8$ but now octonionic units must be replaced by gamma matrices and an explicit matrix representation can be introduced. These gamma matrices can be included as a tensor factor to the infinite-dimensional Clifford algebra so that the local Clifford algebra reduces to a mere Clifford algebra. The units of quantum octonions which are just ordinary octonion units do not however allow matrix representation so that this reduction is not possible and imbedding space and space-time indeed emerge genuinely. The non-associativity of octonions would determine the laws of physics in TGD Universe!

Clearly, the special role of classical number fields and uniqueness of space-time and imbedding space dimensions becomes really manifest only when a quantal deformation of the quaternionic and octonionic matrix algebras is performed. Also the quantum variants of the space-time surface and quite generally, manifolds obtained from linear spaces by geometric constructions become possible by restriction to operator sub-spaces for which spectra defines hyper-quaternionic sub-manifolds.

The emergence of space-time as a four-surface and $HO-H$ duality

HO-H duality [?] states that space-time surface can be equivalently regarded as surface in hyper-octonionic imbedding space and $M^4 \times CP_2$. This duality can be understood in the proposed framework.

1. The analog of Jones inclusion for the local Clifford algebra involves the restriction of hyper-O (-H) coordinate to hyper-H (-C) coordinate to guarantee associativity (commutativity) in calculation of S-matrix elements. The most general inclusion of this kind gives rise to a hyper-quaternionic sub-manifold X^4 of HO.

2. The identification of X^4 as a 4-surface in $M^4 \times CP_2$ results from the local selection of the hyper-quaternionic Clifford algebra as subalgebra assigning to a point of X^4 also point of M^4 besides point of both M^4.

3. The notion of configuration space spinor field follows by allowing quantum superpositions of configuration space spinors $\Psi(X^4 \subset M^4_\pm(m) \times CP_2)$ over X^4. The super-symplectic algebra associated with the light-cone boundary leads to the existing construction of super-symplectic and Kac-Moody representations. Super-symplectic representations can be assigned with $H \subset O$ inclusion and super Kac-Moody representations with $C \subset O$ inclusion.

4. It deserves to be noticed that the construction does not require introduction of more general structures than future and past light cones at the basic level. This simplifies dramatically the construction of configuration space geometry.

Configuration space gamma matrices as hyper-octonionic conformal fields having values in HFF?

The fantastic properties of HFFs of type II_1 inspire the idea that a localized hyper-octonionic version of Clifford algebra of configuration space might allow to see space-time, embedding space, and configuration space as emergent structures. Surprisingly, commutativity and associativity imply most of the speculative "must-be-true's" of quantum TGD.

Configuration space gamma matrices act only in vibrational degrees of freedom of 3-surface. One must also include center of mass degrees of freedom which appear as zero modes. The natural idea is that the resulting local gamma matrices define a local version of HFF of type II_1 as a generalization of conformal field of gamma matrices appearing super string models obtained by replacing complex numbers with hyper-octonions identified as a subspace of complexified octonions. As a matter fact, one can generalize octonions to quantum octonions for which quantum commutativity means restriction to a hyper-octonionic subspace of quantum octonions. Non-associativity is essential for obtaining something non-trivial: otherwise this algebra reduces to HFF of type II_1 since matrix algebra as a tensor factor would give an algebra isomorphic with the original one. The octonionic variant of conformal invariance fixes the dependence of local gamma matrix field on the coordinate of HO. The coefficients of Laurent expansion of this field must commute with octonions.

The world of classical worlds has been identified as a union of configuration spaces associated with $M^4_\pm$ labeled by points of H or equivalently HO. The choice of quantization axes certainly fixes a point of H (HO) as a point remaining fixed under $SO(1,3) \times U(2)$ ($SO(1,3) \times SO(4)$). The condition that hyper-quaternionic inverses of $M^4 \subset HO$ points exist suggest a restriction of arguments of the n-point function to the interior of $M^4_\pm$.

Associativity condition for the n-point functions forces to restrict the arguments to a hyper-quaternionic plane $HQ = M^4$ of HO. One can also consider the commutativity condition by requiring that arguments belong to a preferred commutative sub-space HC of HO. Fixing preferred real and imaginary units means a choice of $M^2 = HC$ interpreted as a partial choice of quantization axes. This has quite strong implications.

1. The hyper-quaternionic planes with a fixed choice of M^2 are labeled by points of CP_2. If the condition $M^2 \subset T^4$ characterizes the tangent planes of all points of $X^4 \subset HO$ it is possible to map $X^4 \subset HO$ to $X^4 \subset H$ so that $HO - H$ duality ("number theoretic compactification") emerges. $X^4 \subset H$ should correspond to a preferred extremal of Kähler action. The physical interpretation would be as a global fixing of the plane of non-physical polarizations in M^8: it is not quite clear whether this choice of polarization need not have direct counterpart for $X^4 \subset H$. Standard model symmetries emerge naturally. The resulting surface in $X^4 \subset H$ would be analogous to a warped plane in E^3. This new result suggests rather direct connection with super string models. In super string models one can choose the polarization plane freely and one expects also now that the generalized choice $M^2 \subset M^4 \subset M^8$ of polarization plane can be made freely without losing Poincare invariance with reasonable assumption about zero energy states.

2. One would like to fix local tangent planes T^4 of X^4 at 3-D light-like surfaces X^3_l fixing the preferred extremal of Kähler action defining the Bohr orbit. An additional direction t should be added to the tangent plane T^3 of X^3_l to give T^4. This might be achieved if t belongs to M^2 and perhaps corresponds to a light-like vector in M^2.

3. Assume that partonic 2-surfaces X belong to $\delta M^4_\pm \subset HO$ defining ends of the causal diamond. This is obviously an additional boundary condition. Hence the points of partonic 2-surfaces are associative and can appear as arguments of n-point functions. One thus finds an explanation for the special role of partonic 2-surfaces and a reason why for the role of light-cone boundary. Note that only the ends of lightlike 3-surfaces need intersect $M^4_\pm \subset HO$. A stronger condition is that the pre-images of light-like 3-surfaces in H belong to $M^4_\pm \subset HO$.

4. Commutativity condition is satisfied if the arguments of the n-point function belong to an intersection $X^2 \cap M^2 \subset HQ$ and this gives a discrete set of points as intersection of light-like radial geodesic and X^2 perhaps identifiable in terms of points in the intersection of number theoretic braids with $\delta H_\pm$. One should show that this set of points consists of rational or at most algebraic points. Here the possibility to choose X^2 to some degree could be essential.

As a matter fact, any radial light ray from the tip of light-cone allows commutativity and one can consider the possibility of integrating over n-point functions with arguments at light ray to obtain maximal information. For the pre-images of light-like 3-surfaces commutativity would allow one-dimensional curves having interpretation as braid strands. These curves would be contained in plane M^2 and it is not clear whether a unique interpretation as braid strands is possible (how to tell whether the strand crossing another one is infinitesimally above or below it?). The alternative assumption consistent with virtual parton interpretation is that light-like geodesics of X^3 are in question.

To sum up, this picture implies HO-H duality with a choice of a preferred imaginary unit fixing the plane of non-physical polarizations globally, standard model symmetries, and number theoretic braids. The introduction of hyper-octonions could be however criticized: could octonions and quaternions be enough after all? Could HO-H duality be replaced with O-H duality and be interpreted as the analog of Wick rotation? This would mean that quaternionic 4-surfaces in E^8 containing global polarization plane E^2 in their tangent spaces would be mapped by essentially by the same map to their counterparts in $M^4 \times CP_2$,and the time coordinate in E^8 would be identified as the real coordinate. Also light-cones in E^8 would make sense as the inverse images of $M^4_\pm$.

1.7.3 Quantal Brahman=Atman identity

The hierarchy of infinite primes (and of integers and rationals) [?] was the first mathematical notion stimulated by TGD inspired theory of consciousness. The construction recipe is equivalent with a repeated second quantization of super-symmetric arithmetic quantum field theory with bosons and fermions labeled by primes such that the many particle states of previous level become the elementary particles of new level. The hierarchy of space-time sheets with many particle states of space-time sheet becoming elementary particles at the next level of hierarchy and also the hierarchy of n:th order logics are also possible correlates for this hierarchy. For instance, the description of proton as an elementary fermion would be in a well defined sense exact in TGD Universe.

This construction leads also to a number theoretic generalization of space-time point since given real number has infinitely rich number theoretical structure not visible at the level of the real norm of the number a due to the existence of real units expressible in terms of ratios of infinite integers. This number theoretical anatomy suggest kind of number theoretical Brahman=Atman principle stating that the set consisting of number theoretic variants of single point of the imbedding space (equivalent in real sense) is able to represent the points of the world of classical worlds or even quantum states of the Universe. Also a formulation in terms of number theoretic holography is possible.

Just for fun and to test these ideas one can consider a model for the representation of the configuration space spinor fields in terms of algebraic holography. I have considered guesses for this kind of map earlier [?] and it is interesting to find whether additional constraints coming from zero energy ontology and finite measurement resolution might give. The identification of quantum corrections as insertion of zero energy states in time scale below measurement resolution to positive or negative energy part of zero energy state and the identification of number theoretic braid as a space-time correlate for the finite measurement resolution give considerable additional constraints.

1. The fundamental representation space consists of wave functions in the Cartesian power U^8 of space U of real units associated with any point of H. That there are 8 real units rather than one is somewhat disturbing: this point will be discussed below. Real units are ratios of infinite integers having interpretation as positive and negative energy states of a super-symmetric arithmetic QFT at some level of hierarchy of second quantizations. Real units have vanishing net quantum numbers so that only zero energy states defining the basis for configuration space spinor fields should be mapped to them. In the general case quantum superpositions of these basis states should be mapped to the quantum superpositions of real units. The first guess is that real units represent a basis for configuration space spinor fields constructed by applying bosonic and fermionic generators of super-symplectic and super Kac-Moody type algebras to the vacuum state.

2. What can one say about this map bringing in mind Gödel numbering? Each pair of bosonic and corresponding fermionic generator at the lowest level must be mapped to its own finite prime. If this map is specified, the map is fixed at the higher levels of the hierarchy. There

exists an infinite number of this kind of correspondences. To achieve some uniqueness, one should have some natural ordering which one might hope to reflect real physics. The irreps of the (non-simple) Lie group involved can be ordered almost uniquely. For simple group this ordering would be with respect to the sum $N = N_F + N_{F,c}$ of the numbers N_F *resp.* $N_{F,c}$ of the fundamental representation *resp.* its conjugate appearing in the minimal tensor product giving the irrep. The generalization to non-simple case should use the sum of the integers N_i for different factors for factor groups. Groups themselves could be ordered by some criterion, say dimension. The states of a given representation could be mapped to subsequent finite primes in an order respecting some natural ordering of the states by the values of quantum numbers from negative to positive (say spin for $SU(2)$ and color isospin and hypercharge for $SU(3)$). This would require the ordering of the Cartesian factors of non-simple group, ordering of quantum numbers for each simple group, and ordering of values of each quantum number from positive to negative.

The presence of conformal weights brings in an additional complication. One cannot use conformal as a primary orderer since the number of $SO(3) \times SU(3)$ irreps in the super-symplectic sector is infinite. The requirement that the probabilities predicted by p-adic thermodynamics are rational numbers or equivalently that there is a length scale cutoff, implies a cutoff in conformal weight. The vision about M-matrix forces to conclude that different values of the total conformal weight n for the quantum state correspond to summands in a direct sum of HFFs. If so, the introduction of the conformal weight would mean for a given summand only the assignment n conformal weights to a given Lie-algebra generator. For each representation of the Lie group one would have n copies ordered with respect to the value of n and mapped to primes in this order.

3. Cognitive representations associated with the points in a subset, call it P, of the discrete intersection of p-adic and real space-time sheets, defining number theoretic braids, would be in question. Large number of partonic surfaces can be involved and only few of them need to contribute to P in the measurement resolution used. The fixing of P means measurement of N positions of H and each point carries fermion or anti-fermion numbers. A more general situation corresponds to plane wave type state obtained as superposition of these states. The condition of rationality or at least algebraicity means that discrete variants of plane waves are in question.

4. By the finiteness of the measurement resolution configuration space spinor field decomposes into a product of two parts or in more general case, to their superposition. The part Ψ_+, which is above measurement resolution, is representable using the information contained by P, coded by the product of second quantized induced spinor field at points of P, and provided by physical experiments. Configuration space "orbital" degrees of freedom should not contribute since these points are fixed in H.

5. The second part of the configuration space spinor field, call it Ψ_-, corresponds to the information below the measurement resolution and assignable with the complement of P and mappable to the structure of real units associated with the points of P. This part has vanishing net quantum numbers and is a superposition over the elements of the basis of CH spinor fields and mapped to a quantum superposition of real units. The representation of Ψ_- as a Schrödinger amplitude in the space of real units could be highly unique. Algebraic holography principle would state that the information below measurement resolution is mapped to a Schrödinger amplitude in space of real units associated with the points of P.

6. This would be also a representation for perceiver-external world duality. The correlation function in which P appears would code for the information appearing in M-matrix representing the laws of physics as seen by conscious entity about external world as an outsider. The quantum superposition of real units would represent the purely subjective information about the part of universe below measurement resolution.

There is an objection against this picture. One obtains an 8-plet of arithmetic zero energy states rather than one state only. What this strange 8-fold way could mean?

1. The crucial observation is that hyper-finite factor of type II_1 (HFF) creates states for which center of mass degrees of freedom of 3-surface in H are fixed. One should somehow generalize

the operators creating local HFF states to fields in H, and an octonionic generalization of conformal field suggests itself. I have indeed proposed a quantum octonionic generalization of HFF extending to an HFF valued field Ψ in 8-D quantum octonionic space with the property that maximal quantum commutative sub-space corresponds to hyper-octonions. This construction raises $X^4 \subset M^8$ and by number theoretic compactification also $X^4 \subset H$ in a unique position since non-associativity of hyper-octonions does not allow to identify the algebra of HFF valued fields in M^8 with HFF itself.

2. The value of Ψ in the space of quantum octonions restricted to a maximal commutative subspace can be expressed in terms of 8 HFF valued coefficients of hyper-octonion units. By the hyper-octonionic generalization of conformal invariance all these 8 coefficients must represent zero energy HFF states. The restriction of Ψ to a given point of P would give a state, which has 8 HFF valued components and Brahman=Atman identity would map these components to U^8 associated with P. One might perhaps say that 8 zero energy states are needed in order to code the information about the H positions of points P. The condition that Ψ represents a state with vanishing quantum numbers gives additional constraints. The interpretation inspired by finite measurement resolution is that the coordinate h associated with Ψ corresponds to a zero energy insertion to a positive or negative energy state localizable to a causal diamond inside the upper or lower half of the causal diamond of observer. Below measurement resolution for imbedding space coordinates $\Psi(h)$ would correspond to a nonlocal operator creating a zero energy state. This would mean that Brahman=Atman would apply to the mini-worlds below the measurement resolution rather than to entire Universe but by algebraic fractality of HFFs this would would not be a dramatic loss.

1.7.4 One element field, quantum measurement theory and its q-variant, and the Galois fields associated with infinite primes

John Baez talked in This Weeks Finds (Week 259) [?] about one-element field - a notion inspired by the $q = exp(i2\pi/n) \rightarrow 1$ limit for quantum groups. This limit suggests that the notion of one-element field for which 0=1 - a kind of mathematical phantom for which multiplication and sum should be identical operations - could make sense. Physicist might not be attracted by this kind of identification.

In the following I want to articulate some comments from the point of view of quantum measurement theory and its generalization to q-measurement theory which I proposed for some years ago and which is represented above.

I also consider and alternative interpretation in terms of Galois fields assignable to infinite primes which form an infinite hierarchy. These Galois fields have infinite number of elements but the map to the real world effectively reduces the number of elements to 2: 0 and 1 remain different.

$q \rightarrow 1$ limit as transition from quantum physics to effectively classical physics?

The $q \rightarrow 1$ limit of quantum groups at q-integers become ordinary integers and n-D vector spaces reduce to n-element sets. For quantum logic the reduction would mean that 2^N-D spinor space becomes 2^N-element set. N qubits are replaced with N bits. This brings in mind what happens in the transition from wave mechanism to classical mechanics. This might relate in interesting manner to quantum measurement theory.

Strictly speaking, $q \rightarrow 1$ limit corresponds to the limit $q = exp(i2\pi/n)$, $n \rightarrow \infty$ since only roots of unity are considered. This also correspond to Jones inclusions at the limit when the discrete group Z_n or or its extension-both subgroups of $SO(3)$- to contain reflection has infinite elements. Therefore this limit where field with one element appears might have concrete physical meaning. Does the system at this limit behave very classically?

In TGD framework this limit can correspond to either infinite or vanishing Planck constant depending on whether one consider orbifolds or coverings. For the vanishing Planck constant one should have classicality: at least naively! In perturbative gauge theory higher order corrections come as powers of $g^2/4\pi\hbar$ so that also these corrections vanish and one has same predictions as given by classical field theory.

Q-measurement theory and $q \rightarrow 1$ limit

Q-measurement theory differs from quantum measurement theory in that the coordinates of the state space, say spinor space, are non-commuting. Consider in the sequel q-spinors for simplicity.

Since the components of quantum spinor do not commute, one cannot perform state function reduction. One can however measure the modulus squared of both spinor components which indeed commute as operators and have interpretation as probabilities for spin up or down. They have a universal spectrum of eigen values. The interpretation would be in terms of fuzzy probabilities and finite measurement resolution but may be in different sense as in case of HFF:s. Probability would become the observable instead of spin for q not equal to 1.

At $q \rightarrow 1$ limit quantum measurement becomes possible in the standard sense of the word and one obtains spin down or up. This in turn means that the projective ray representing quantum states is replaced with one of n possible projective rays defining the points of n-element set. For HFF:s of type II_1 it would be N-rays which would become points, N the included algebra. One might also say that state function reduction is forced by this mapping to *single* object at $q \rightarrow 1$ limit.

On might say that the set of orthogonal coordinate axis replaces the state space in quantum measurement. We do this replacement of space with coordinate axis all the time when at blackboard. Quantum consciousness theorist inside me adds that this means a creation of symbolic representations and that the function of quantum classical correspondences is to build symbolic representations for quantum reality at space-time level.

$q \rightarrow 1$ limit should have space-time correlates by quantum classical correspondence. A TGD inspired geometro-topological interpretation for the projection postulate might be that quantum measurement at $q \rightarrow 1$ limit corresponds to a leakage of 3-surface to a dark sector of imbedding space with $q \rightarrow 1$ (Planck constant near to 0 or ∞ depending on whether one has $n \rightarrow \infty$ covering or division of M^4 or CP_2 by a subgroup of $SU(2)$ becoming infinite cyclic - very roughly!) and Hilbert space is indeed effectively replaced with n rays. For $q \neq 1$ one would have only probabilities for different outcomes since things would be fuzzy.

In this picture classical physics and classical logic would be the physical counterpart for the shadow world of mathematics and would result only as an asymptotic notion.

Could 1-element fields actually correspond to Galois fields associated with infinite primes?

Finite field G_p corresponds to integers modulo p and product and sum are taken only modulo p. An alternative representation is in terms of phases $exp(ik2\pi/p)$, $k = 0, ..., p-1$ with sum and product performed in the exponent. The question is whether could one define these fields also for infinite primes [?] by identifying the elements of this field as phases $exp(ik2\pi/\Pi)$ with k taken to be finite integer and Π an infinite prime (recall that they form infinite hierarchy). Formally this makes sense. 1-element field would be replaced with infinite hierarchy of Galois fields with infinite number of elements!

The probabilities defined by components of quantum spinor make sense only as real numbers and one can indeed map them to real numbers by interpreting q as an ordinary complex number. This would give same results as $q \rightarrow 1$ limit and one would have effectively 1-element field but actually a Galois field with infinite number of elements.

If one allows k to be also infinite integer but not larger than than Π in the real sense, the phases $exp(ik2\pi/\Pi)$ would be well defined as real numbers and could differ from 1. All real numbers in the range $[-1, 1]$ would be obtained as values of $cos(k2\pi/\Pi)$ so that this limit would effectively give real numbers.

This relates also interestingly to the question whether the notion of p-adic field makes sense for infinite primes. The p-adic norm of any infinite-p p-adic number would be power of π either infinite, zero, or 1. Excluding infinite normed numbers one would have effectively only p-adic integers in the range $1, ...\Pi - 1$ and thus only the Galois field $G < sub > \Pi < /sub >$ representable also as quantum phases.

I conclude with a nice string of text from John'z page:

What's a mathematical phantom? According to Wraith, it's an object that doesn't exist within a given mathematical framework, but nonetheless "obtrudes its effects so convincingly that one is forced to concede a broader notion of existence".

and unashamedly propose that perhaps Galois fields associated with infinite primes might provide this broader notion of existence! In equally unashamed tone I ask whether there exists also hierarchy of conscious entities at $q=1$ levels in real sense and whether we might identify ourselves as this kind of entities? Note that if cognition corresponds to p-adic space-time sheets, our cognitive bodies have literally infinite geometric size in real sense.

One-element field realized in terms of real units with number theoretic anatomy

One-element field looks rather self-contradictory notion since 1 and 0 should be represented by same element. The real units expressible as ratios of infinite rationals could however provide a well-defined realization of this notion.

1. The condition that same element represents the neutral element of both sum and product gives strong constraint on one-element field. Consider an algebra formed by reals with sum and product defined in the following manner. Sum, call it $\oplus$, corresponds to the ordinary product $x \times y$ for reals whereas product, call it $\otimes$, is identified as the non-commutative product $x \otimes y = x^y$. $x=1$ represents both the neutral element (0) of $\oplus$ and the unit of $\otimes$. The sub-algebras generated by 1 and multiple powers $P_n(x) = P_{n-1}(x) \otimes x = x \otimes ... \otimes x$ form commutative sub-algebras of this algebra. When one restricts the consideration to $x=1$ one obtains one-element field as sub-field which is however trivial since $\oplus$ and $\otimes$ are identical operations in this subset.

2. One can get over this difficulty by keeping the operations $\oplus$ and $\otimes$, by assuming one-element property only with respect to the real and various p-adic norms, and by replacing ordinary real unit 1 with the algebra of real units formed from infinite primes by requiring that the real and various p-adic norms of the resulting numbers are equal to one. As far as real and various p-adic norms are considered, one has commutative one-element field. When number theoretic anatomy is taken into account, the algebra contains infinite number of elements and is non-commutative with respect to the product since the number theoretic anatomies of x^y and y^x are different.

1.7.5 Relation to other ideas

Category formed by Clifford algebras as a basic structure

A proper mathematical framework seems to be a category having as objects the number theoretical Clifford algebras listed below and probably many others while Jones inclusion would define the fundamental arrow.

One can imagine huge variety of natural inclusions of Clifford algebras in TGD framework.

1. Each space-time sheet can be regarded as 3-surface belonging to a single particle sector of configuration space and the corresponding Clifford algebra can be included to various Clifford algebras associated with collections of space-time sheets containing this space-time sheet as a topologically condensed space-time sheet. This inclusion is certainly a fundamental one. Universe as a computer idea would encourage to think that Universe is utilizing this kind of inclusions to mimic itself.

2. The $R_0^G \subset R^G$ type inclusions seem to be associated with non-perturbative phases with modified values of Planck constants and scaling factors of metrics in M^4 and CP_2 degrees of freedom. Groups $G = G_a \times G_b \subset SL(2,C) \times SU(2)$ characterize these inclusions and modular subgroups of SL(2,Z) and their complexificiations which are unit matrices modulo p^k, are of special interest concerning p-adicization by algebraic continuation. The group theoretical discretization using an extension of rationals for F in $SL(2,F)$ is in accordance with the decomposition $CH = \cup_a CH_a$, a the dip of light cone.

3. Number theoretical inclusion sequence defines a canonical inclusion sequence. For this inclusion N is very small as compared to M, which suggest that the index is infinite. A concrete matrix representation would suggest an interpretation as an infinite tensor power of the standard $R_0^G \subset R^G$ inclusion so that the index would be an infinite power of $\mathcal{M}:\mathcal{N}$. For instance, $CL(O)$ would be $CL(H)_\infty$ in some sense. The index would be an infinite power of $M:N$ and remain finite only for $n=3$ and corresponding groups A_2 (Z_2 and color group) and E_6 (tedrahedral group and E_6) would be special.

4. Also sub-algebras generated by $M_2(F)$ containing only matrices for which the ratios of elements belong to some algebraic extension of rationals are possible. Also this gives rise to inclusion hierarchies expected to be of special importance for p-adicization.

Dualities and number theoretic Jones inclusions

Dualities define a relatively new and speculative element in TGD. Jones inclusions provide also new insights and support to various proposed dualities of TGD.

1. 7-3 duality duality states roughly that TGD could be described either in terms of 3-dimensional light like causal determinants of space-time surfaces or in terms of the light cones $\delta M^4_+ \times CP_2$ or equivalently 7-D lightcones of HO. A concrete implication is the generalization of coset mechanism for superconformal invariance in which differences of super-symplectic and super Kac-Moody generators annihilate physical states. A possible correlate for this duality would be the canonical pair of Jones inclusions $\mathcal{N} \subset \mathcal{M}$ and $\mathcal{M} \subset \mathcal{M} \otimes_{\mathcal{N}} \mathcal{M}$, which are dual.

   If Cl is generated by an infinite tensor power of $C(8)$, the inclusion sequence would be restricted to the tensor factor O_C in $O_C \otimes Cl$ and be represented as a sequence of $C_C \subset H_C \subset O_C$ of finite-dimensional inclusions. The quantum counterpart of C_C is quantum plane. In quantum case the identification of the index for this inclusion would be very naturally $\sqrt{\mathcal{M} : \mathcal{N}}$ reproducing correctly quantum variants of the dimensions in question.

2. HO-H duality follow naturally from the basic picture. In H description electro-weak quantum numbers are spinlike and color in CP_2 partial waves and HO description color is is spinlike and electro-weak quantum numbers correspond to E^4 partial waves.

3. Hyperquaterionic-co-hyperquaternionic duality follows from the possibility of identifying space-time surfaces as associative or co-associative sub-manifolds of HO. This duality means that space-time surfaces in HO become to certain extend a relative notion.

1.8 HFFs and M-matrix

In this section- written years before the preceding chapter reflecting much more detailed understanding of quantum TGD - a general master formula for the construction of S-matrix in terms of Connes tensor product is proposed. M-matrix elements would be obtained by as Connes tensor product for Clifford algebra elements creating positive and negative energy parts of the zero hierarchy state so that a hierarchy of M-matrices parameterized by Jones inclusions characterizing measurement resolution would result. M-matrix for them gives rise to thermal S-matrix as a kind of statistical average over undetected degrees of freedom. The explicit construction of M-matrix in terms of Connes tensor product seems a remote possibility -at least with my minor rudimentary knowledge about this topic, and the basic point is that Connes tensor product implies that M-matrix is essentially unique apart from the square root of density matrix. Therefore this section can be seen as complementary to the preceding one.

1.8.1 Von Neumann algebras and TGD

The realization that so called hyper-finite factors of type II_1 are an inherent property of quantum TGD meant a breakthrough in the understanding of the mathematical structure of quantum TGD.

HFFs, configuration space spinor structure, and finite measurement resolution

Configuration space Clifford algebra - configuration space is understood here as that associated with fixed CD and thus identifiable as the space of light-like 3-surfaces connecting the light-like boundaries of CD- provides the basic instance of hyperfinite factor of type II_1 (or possibly direct integral of them). It is generated by the fermionic oscillator operators and since the number of on mass shell oscillator operators is finite, one obtains a finite-dimensional algebra. The interpretation is most naturally in terms of quantum variant of HFF obtained as coset space $\mathcal{M}/\mathcal{N}$. If also virtual states labeled by non-vanishing conformal weights identifiable as stringy excitations of fermions are allowed, infinite-D

Clifford algebra is obtained and would correspond to $\mathcal{M}$. Measurement resolution in quantum sense would mean here impossibility to observe off mass shell states.

The inclusion of fermionic oscillator operators with higher conformal weight allow to realize anti-commutation relations in a large set of points and the nodes of the symplectic triangulation are good candidates in this respect. According to the proposal discussed, the vertices of the symplectic triangulation would be determined by the measurement of the maxima for the magnitude of Kähler magnetic field. This would bring in additional classical information which could be claimed to represent off mass shell data since classical fields in general are off mass shell.

Could TGD emerge as a non-trivial local variant of infinite-dimensional Clifford algebra?

An interesting possibility is that TGD could emerge from a local version of hyper-finite factor of type II_1 represented as an infinite-dimensional Clifford algebra must exist (as analog of say local gauge groups). This implies a connection with the classical number fields. Quantum version of complexified octonions defining the coordinate with respect to which one localizes is unique by its non-associativity allowing to uniquely separate the powers of octonionic coordinate from the associative infinite-dimensional Clifford algebra elements appearing as Taylor coefficients in the expansion of Clifford algebra valued field.

Associativity condition implies the classical and quantum dynamics of TGD. Space-time surfaces are hyper-quaternionic of co-hyper-quatenionic sub-manifolds of hyper-octonionic imbedding space HO. Also the interpretation as a four-surface in $H = M^4 \times CP_2$ emerges and implies $HO - H$ duality. What is also nice that Minkowski spaces correspond to the spectra for the eigenvalues of maximal set of commuting quantum coordinates of suitably defined quantum spaces. Thus Minkowski signature has quantal explanation.

Quantization of Planck constants

The geometric and topological interpretation of Jones inclusions led to the understanding of the quantization of Planck constants assignable to M^4 and CP_2 degrees of freedom (identical in "ground state"). The Planck constants are scaled up by the integer n defining the quantum phase $q = exp(i\pi/n)$ characterizing the Jones inclusion, which in turn corresponds to subgroup G of $SL(2,C) \times SU(2)_L \times U(1)$ in the simplest situation. The quantum phase can be assigned also to q=1 inclusions in which case second quantum phase can be associated with the monodromies of the corresponding conformal field theory.

The scaling of $M^4_\pm$ Planck constant by n_a means scaling of CP_2 metric by n_a, where n_a is the order of the maximal cyclic subgroup of G_a. And vice versa. The observed Planck constant must correspond to $\hbar_{eff}/\hbar_0 = n_a/n_b$ from the fact that only the ratio $\hbar(M^4_\pm)/\hbar(CP_2)$ appears in Kähler action. CP_2 can therefore have arbitrarily large size: hyper space travel might not be unrealistic after all! For infinite subgroups such as $G = SU(2) \subset SU(3)$ the situation is somewhat different. The variants of imbedding space can meet each other if either M^4 or CP_2 factors have same value of Planck constant so that a fan (or rather tree-) like structure results. Analogous picture emerged already earlier from the gluing of the p-adic variants of imbedding space along common rationals (or algebraics in more general case). The phase transitions changing the Planck constant have purely topological description.

An important outcome was the interpretation of McKay correspondence: one can assign to the ADE diagram of $q \neq 1$ Jones inclusion the corresponding gauge group. The $n(G)$-fold covering of $M^4_\pm$ points by a finite number of CP_2 points makes possible to realize the multiplets of gauge group purely geometrically in terms of G group algebra. In the case of extended ADE diagrams assignable to $q = 1$ Jones inclusions the group is Kac-Moody group. This picture applies both in M^4 and CP_2 degrees of freedom.

1. In CP_2 degrees of freedom this framework allows to understand anyonic charge fractionization and raises the question whether fractional Hall effect corresponds to the integer valued quantum Hall effect with scaled up Planck constant and whether free quarks could be integer charged and have fractional charges only inside hadrons.

2. In $M^4_\pm$ degrees of freedom this picture has fascinating cosmological consequences and leads to a possible explanation for the quantization of cosmic recession velocities in terms of lattice

like structures (tesselations) of lightcone proper time constant hyperboloid defined by infinite subgroups of Lorentz group and consisting of dark matter in macroscopically quantum coherent phase.

1.8.2 Finite measurement resolution: from S-matrix to M-matrix

The finite resolution of quantum measurement leads in TGD framework naturally to the notion of quantum S-matrix for which elements have values in sub-factor of HFF rather than being complex numbers. It is still possible to satisfy generalized unitarity condition but one can also consider the possibility that only probabilities are conserved.

Jones inclusion as characterizer of finite measurement resolution at the level of S-matrix

Jones inclusion $\mathcal{N} \subset \mathcal{M}$ characterizes naturally finite measurement resolution. This means following things.

1. Complex rays of state space resulting usually in an ideal state function reduction are replaced by $\mathcal{N}$-rays since $\mathcal{N}$ defines the measurement resolution and takes the role of complex numbers in ordinary quantum theory so that non-commutative quantum theory results. Non-commutativity corresponds to a finite measurement resolution rather than something exotic occurring in Planck length scales. The quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ creates physical states modulo resolution. The fact that $\mathcal{N}$ takes the role of gauge algebra suggests that it might be necessary to fix a gauge by assigning to each element of $\mathcal{M}/\mathcal{N}$ a unique element of $\mathcal{M}$. Quantum Clifford algebra with fractal dimension $\beta = \mathcal{M} : \mathcal{N}$ creates physical states having interpretation as quantum spinors of fractal dimension $d = \sqrt{\beta}$. Hence direct connection with quantum groups emerges.

2. The notions of unitarity, hermiticity, and eigenvalue generalized. The elements of unitary and hermitian matrices and $\mathcal{N}$-valued. Eigenvalues are Hermitian elements of $\mathcal{N}$ and thus correspond entire spectra of Hermitian operators. The mutual non-commutativity of eigenvalues guarantees that it is possible to speak about state function reduction for quantum spinors. In the simplest case of a 2-component quantum spinor this means that second component of quantum spinor vanishes in the sense that second component of spinor annihilates physical state and second acts as element of $\mathcal{N}$ on it. The non-commutativity of spinor components implies correlations between then and thus fractal dimension is smaller than 2.

3. The intuition about ordinary tensor products suggests that one can decompose Tr in $\mathcal{M}$ as

$$Tr_{\mathcal{M}}(X) = Tr_{\mathcal{M}/\mathcal{N}}(Tr_{\mathcal{N}}(X)) \ . \tag{1.8.1}$$

Suppose one has fixed gauge by selecting basis $|r_k\rangle$ for $\mathcal{M}/\mathcal{N}$. In this case one expects that operator in $\mathcal{M}$ defines an operator in $\mathcal{M}/\mathcal{N}$ by a projection to the preferred elements of $\mathcal{M}$.

$$\langle r_1|X|r_2\rangle = \langle r_1|Tr_{\mathcal{N}}(X)|r_2\rangle \ . \tag{1.8.2}$$

4. Scattering probabilities in the resolution defined by $\mathcal{N}$ are obtained in the following manner. The scattering probability between states $|r_1\rangle$ and $|r_2\rangle$ is obtained by summing over the final states obtained by the action of $\mathcal{N}$ from $|r_2\rangle$ and taking the analog of spin average over the states created in the similar from $|r_1\rangle$. $\mathcal{N}$ average requires a division by $Tr(P_{\mathcal{N}}) = 1/\mathcal{M} : \mathcal{N}$ defining fractal dimension of $\mathcal{N}$. This gives

$$p(r_1 \rightarrow r_2) = \mathcal{M} : \mathcal{N} \times \langle r_1|Tr_{\mathcal{N}}(SP_{\mathcal{N}}S^{\dagger})|r_2\rangle \ . \tag{1.8.3}$$

This formula is consistent with probability conservation since one has

$$\sum_{r_2} p(r_1 \rightarrow r_2) \quad = \quad \mathcal{M} : \mathcal{N} \times Tr_N(SS^\dagger) = \mathcal{M} : \mathcal{N} \times Tr(P_N) = 1 \ . \tag{1.8.4}$$

5. Unitary at the level of $\mathcal{M}/\mathcal{N}$ is obtained if the unit operator Id for $\mathcal{M}$ can be decomposed into an analog of tensor product for the unit operators of $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$.

Quantum M-matrix

The description of finite measurement resolution in terms of Jones inclusion $\mathcal{N} \subset \mathcal{M}$ seems to boil down to a simple rule. Replace ordinary quantum mechanics in complex number field C with that in $\mathcal{N}$. This means that the notions of unitarity, hermiticity, Hilbert space ray, etc.. are replaced with their $\mathcal{N}$ counterparts.

The full M-matrix in $\mathcal{M}$ should be reducible to a finite-dimensional quantum M-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ which can be regarded as a finite-dimensional matrix algebra with non-commuting $\mathcal{N}$-valued matrix elements. This suggests that full M-matrix can be expressed as M-matrix with $\mathcal{N}$-valued elements satisfying $\mathcal{N}$-unitarity conditions.

Physical intuition also suggests that the transition probabilities defined by quantum S-matrix must be commuting hermitian $\mathcal{N}$-valued operators inside every row and column. The traces of these operators give $\mathcal{N}$-averaged transition probabilities. The eigenvalue spectrum of these Hermitian gives more detailed information about details below experimental resolution. $\mathcal{N}$-hermicity and commutativity pose powerful additional restrictions on the M-matrix.

Quantum M-matrix defines $\mathcal{N}$-valued entanglement coefficients between quantum states with $\mathcal{N}$-valued coefficients. How this affects the situation? The non-commutativity of quantum spinors has a natural interpretation in terms of fuzzy state function reduction meaning that quantum spinor corresponds effectively to a statistical ensemble which cannot correspond to pure state. Does this mean that predictions for transition probabilities must be averaged over the ensemble defined by "quantum quantum states"?

Quantum fluctuations and Jones inclusions

Jones inclusions $\mathcal{N} \subset \mathcal{M}$ provide also a first principle description of quantum fluctuations since quantum fluctuations are by definition quantum dynamics below measurement resolution. This gives hopes for articulating precisely what the important phrase "long range quantum fluctuations around quantum criticality" really means mathematically.

1. Phase transitions involve a change of symmetry. One might hope that the change of the symmetry group $G_a \times G_b$ could universally code this aspect of phase transitions. This need not always mean a change of Planck constant but it means always a leakage between sectors of imbedding space. At quantum criticality 3-surfaces would have regions belonging to at least two sectors of H.

2. The long range of quantum fluctuations would naturally relate to a partial or total leakage of the 3-surface to a sector of imbedding space with larger Planck constant meaning zooming up of various quantal lengths.

3. For M-matrix in $\mathcal{M}/\mathcal{N}$ quantum criticality would mean a special kind of eigen state for the transition probability operator defined by the M-matrix. The properties of the number theoretic braids contributing to the S-matrix should characterize this state. The strands of the critical braids would correspond to fixed points for $G_a \times G_b$ or its subgroup.

4. Accepting number theoretical vision, quantum criticality would mean that super-canonical conformal weights and/or generalized eigenvalues of the modified Dirac operator correspond to zeros of Riemann ζ so that the points of the number theoretic braids would be mapped to fixed points of G_a and G_b at geodesic spheres of $\delta M^4_+ = S^2 \times R_+$ and CP_2. Also weaker critical points which are fixed points of only subgroup of G_a or G_b can be considered.

1.8.3 Does Connes tensor product fix the allowed M-matrices?

Hyperfinite factors of type II_1 and the inclusion $\mathcal{N} \subset \mathcal{M}$ inclusions have been proposed to define quantum measurement theory with a finite measurement resolution characterized by $\mathcal{N}$ and with complex rays of state space replaced with $\mathcal{N}$ rays. What this really means is far from clear.

1. Naively one expects that matrices whose elements are elements of $\mathcal{N}$ give a representation for M. Now however unit operator has unit trace and one cannot visualize the situation in terms of matrices in case of $\mathcal{M}$ and $\mathcal{N}$.

2. The state space with $\mathcal{N}$ resolution would be formally $\mathcal{M}/\mathcal{N}$ consisting of $\mathcal{N}$ rays. For $\mathcal{M}/\mathcal{N}$ one has finite-D matrices with non-commuting elements of $\mathcal{N}$. In this case quantum matrix elements should be multiplets of selected elements of $\mathcal{N}$, **not all** possible elements of $\mathcal{N}$. One cannot therefore think in terms of the tensor product of $\mathcal{N}$ with $\mathcal{M}/\mathcal{N}$ regarded as a finite-D matrix algebra.

3. What does this mean? Obviously one must pose a condition implying that $\mathcal{N}$ action commutes with matrix action just like C: this poses conditions on the matrices that one can allow. Connes tensor product [?] does just this. Note I have proposed already earlier the reduction of interactions to Connes tensor product (see the section *"Could Connes tensor product...."* later in this chapter) but without reference to zero energy ontology as a fundamental manner to define measurement resolution with respect time and assuming unitarity.

The argument demonstrating almost uniqueness of M-matrix

The starting point is the Jones inclusion sequence

$$\mathcal{N} \subset \mathcal{M} \subset \mathcal{M} \otimes_N \mathcal{M} ...$$

Here $\mathcal{M} \otimes_N \mathcal{M}$ is Connes tensor product which can be seen as elements of the ordinary tensor product commuting with $\mathcal{N}$ action so that $\mathcal{N}$ indeed acts like complex numbers in $\mathcal{M}$. $\mathcal{M}/\mathcal{N}$ is in this picture represented with $\mathcal{M}$ in which operators defined by Connes tensor products of elements of $\mathcal{M}$. The replacement $\mathcal{M} \to \mathcal{M}/\mathcal{N}$ corresponds to the replacement of the tensor product of elements of $\mathcal{M}$ defining matrices with Connes tensor product.

One can try to generalize this picture to zero energy ontology.

1. $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M}$ would be generalized by $\mathcal{M}_+ \otimes_{\mathcal{N}} \mathcal{M}_-$. Here $\mathcal{M}_+$ would create positive energy states and $\mathcal{M}_-$ negative energy states and $\mathcal{N}$ would create zero energy states in some shorter time scale resolution: this would be the precise meaning of finite measurement resolution.

2. Connes entanglement with respect to $\mathcal{N}$ would define a non-trivial and unique recipe for constructing M-matrices as a generalization of M-matrices expressible as products of square root of density matrix and unitary S-matrix but it is not how clear how many M-matrices this allows. In any case M-matrices would depend on the triplet $(\mathcal{N}, \mathcal{M}_+, \mathcal{M}_-)$ and this would correspond to p-adic length scale evolution giving replacing coupling constant evolution in TGD framework. Thermodynamics would enter the fundamental quantum theory via the square root of density matrix.

3. The defining condition for the variant of the Connes tensor product proposed here has the following equivalent forms

$$MN = N^*M \quad , \quad N = M^{-1}N^*M \quad , \quad N^* = MNM^{-1} \quad . \tag{1.8.5}$$

If M_1 and M_2 are two M-matrices satisfying the conditions then the matrix $M_{12} = M_1M_2^{-1}$ satisfies the following equivalent conditions

$$N = M_{12}NM_{12}^{-1} \quad , \quad [N, M_{12}] = 0 \quad . \tag{1.8.6}$$

Jones inclusions with $\mathcal{M} : \mathcal{N} \leq 4$ are irreducible which means that the operators commuting with $\mathcal{N}$ consist of complex multiples of identity. Hence one must have $M_{12} = 1$ so that M-matrix is unique in this case. For $\mathcal{M} : \mathcal{N} > 4$ the complex dimension of commutator algebra of $\mathcal{N}$ is 2 so that M-matrix depends should depend on single complex parameter. The dimension of the commutator algebra associated with the inclusion gives the number of parameters appearing in the M-matrix in the general case.

When the commutator has complex dimension $d > 1$, the representation of $\mathcal{N}$ in $\mathcal{M}$ is reducible: the matrix analogy is the representation of elements of $\mathcal{N}$ as direct sums of d representation matrices. M-matrix is a direct sum of form $M = a_1M_1 \oplus a_2M_2 \oplus ...$, where M_i are unique. The condition $\sum_i ; |a_i|^2 = 1$ is satisfied and *-commutativity holds in each summand separately.

There are several questions. Could M_i define unique universal unitary S-matrices in their own blocks? Could the direct sum define a counterpart of a statistical ensemble? Could irreducible inclusions correspond to pure states and reducible inclusions to mixed states? Could different values of energy in thermodynamics and of the scaling generator L_0 in p-adic thermodynamics define direct summands of the inclusion? The values of conserved quantum numbers for the positive energy part of the state indeed naturally define this kind of direct direct summands.

It must be of course noticed that reducibility and thermodynamics emerge naturally also in another sense since a direct sum of HFFs of type II_1 is what one expects. The radial conformal weights associated light-cone boundary and X_l^3 would indeed naturally label the factors in the direct sum.

4. Zero energy ontology is a key element of this picture and the most compelling argument for zero energy ontology is the possibility of describing coherent states of Cooper pairs without giving up fermion number, charge, etc. conservation and automatic emerges of length scale dependent notion of quantum numbers (quantum numbers identified as those associated with positive energy factor).

To sum up, interactions would be an outcome of a finite measurement resolution and at the never-achievable limit of infinite measurement resolution the theory would be free: this would be the counterpart of asymptotic freedom.

How to define the inclusion of $\mathcal{N}$ physically?

The overall picture looks beautiful but it is not clear how one could define the inclusion $\mathcal{N} \subset \mathcal{M}$ precisely. One must distinguish between two cases corresponding to the unitary U-matrix representing unitary process associated with the quantum jump and defined between zero energy states and M-matrix defining the time-like entanglement between positive and negative energy states.

1. In the case of U-matrix both $\mathcal{N}$ and $\mathcal{M}$ corresponds to zero energy states. The time scale of the zero energy state created by $\mathcal{N}$ should be shorter than that for the state defined naturally as the temporal distance t_{+-} between the tips of the light-cones $M^4_\pm$ associated with the state and defining diamond like structure.

2. In the case of M-matrix one has zero energy subalgebra of algebra creating positive or negative energy states in time scale t_{+-}. In this case the time scale for zero energy states is smaller than $t_{+-}/2$. The defining conditions for the Connes tensor product are analogous to crossing symmetry but with the restriction that the crossed operators create zero energy states.

Quantum classical correspondence requires a precise formulation for the action of $\mathcal{N}$ at space-time level and this is a valuable guideline in attempts to understand what is involved. Consider now the definition of the action of $\mathcal{N}$ in the case of M-matrix.

1. In standard QFT picture the action of the element of $\mathcal{N}$ multiplies the positive or negative energy parts of the state with an operator creating a zero energy state.

2. At the space-time level one can assign positive/negative energy states to the incoming/outcoing 3-D lines of generalized Feynman diagrams (recall that in vertices the 3-D light-like surfaces meet along their ends). At the parton level the addition of a zero energy state would be simply

addition of a collection of light-like partonic 3-surfaces describing a zero energy state in a time scale shorter than that associated with incoming/outgoing positive/negative energy space-time sheet. The points of the discretized number theoretic braid would naturally contain the insertions of the second quantized induced spinor field in the description of M-matrix element in terms of N-point function.

3. At first look this operation looks completely trivial but this is not the case. The point is that the 3-D lines of zero energy diagram and those of the original positive/negative energy diagram must be assigned to *single connected* 4-D space-time surface. Note that even the minima of the λ are not same as for the original positive energy state and free zero energy state since the minimization is affected by the constraint that the resulting space-time sheet is connected.

4. What happens if one allows several disconnected space-time sheets in the initial state? Could/should one assign the zero energy state to a particular incoming space-time sheet? If so, what space-time sheet of the final state should one attach the *-conjugate of this zero energy state? Or should one allow a non-unique assignment and interpret the result in terms of different phases? If one generalizes the connectedness condition to the connectedness of the entire space-time surface characterizing zero energy state one would bet rid of the question but can still wonder how unique the assignment of the 4-D space-time surface to a given collection of light-like 3-surfaces is.

How to define Hermitian conjugation physically?

Second problem relates to the realization of Hermitian conjugation $\mathcal{N} \rightarrow \mathcal{N}^*$ at the space-time level. Intuitively it seems clear that the conjugation must involve M^4 time reflection with respect to some origin of M^4 time mapping partonic 3-surfaces to their time mirror images and performing T-operation for induced spinor fields acting at the points of discretized number theoretic braids.

Suppose that incoming and outgoing states correspond to light-cones M^4_+ and M^4_- with tips at points $m^0 = 0$ and $m^0 = t_{+-}$. This does not require that the preferred sub-manifolds M^2 and S^2_{II} are same for positive/negative energy states and inserted zero energy states. In this case the point $(m^0 = t_{+-}/2, m^k = 0)$ would be the natural reflection point and the operation mapping the action of $\mathcal{N}$ to the action of $\mathcal{N}^*$ would be unique.

Can one allow several light cones in the initial and final states or should one restrict M-matrix to single diamond like structure defined by the two light-cones? The most reasonable option seems to be an assignment of a diamond shape pair of light-cones to each zero energy component of the state. The temporal distance t_{+-} between the tips of the light-cones would assign a precise time scale assigned to the zero state. The zero energy states inserted to a state characterized by a time scale t_{+-} would correspond to time scales $t < t_{+-}/2$ so that a hierarchy in powers of 2 would emerge naturally. Note that the choice of quantization axes (manifolds M^2 and S^2_{II}) could be different at different levels of hierarchy.

This picture would apply naturally also in the case of U-matrix and make the cutoff hierarchy discrete in accordance with p-adic length scale hypothesis bringing in also quantization of the time scales t_{+-}. In the case of U-matrix $\mathcal{N}$ would contain besides the zero energy algebra of M-matrix also the subalgebra for which the positive and negative energy parts reside at different sides of the center of the diamond.

How to generalize the notion of observable?

The almost-uniqueness of M-matrix seems too good to be true and in this kind of situation it is best to try to find an argument killing the hypothesis. The first test is whether the ordinary quantum measurement theory with Hermitian operators identified as observables generalizes.

The basic implication is that M should commute with Hermitian operators of $\mathcal{N}$ assuming that they exist in some sense. All Hermitian elements of $\mathcal{N}$ could be regarded not only as observables but also as conserved charges defining symmetries of M which would be thus maximal. The geometric counterpart for this would be the fact that configuration space is a union of symmetric spaces having maximal isometry group. Super-conformal symmetries of M-matrix would be in question.

The task is to define what Hermiticity means in this kind of situation. The super-positions $N + N^*$ and products N^*N defined in an appropriate sense should Hermitian operators. One can define what the products MN^* and NM mean. There are also two Hermitian conjugations involved: $\mathcal{M}$ conjugation and $\mathcal{N}$ conjugation.

1. Consider first Hermitian conjugation in $\mathcal{M}$. The operators of $\mathcal{N}$ creating zero energy states on the positive energy side and $\mathcal{N}^*$ acting on the negative energy side are not Hermitian in the hermitian conjugation of $\mathcal{M}$. If one defines $MN^* \equiv N^*M$ and $NM \equiv MN$, the operators $N+N^*$ and N^*N indeed commute with M by the basic condition. One could label the states created by $\mathcal{M}$ by eigenvalues of a maximally commuting sub-algebra of $\mathcal{N}$. Clearly, the operators acting on positive and negative energy state spaces should be interpreted in terms of a polarization $\mathcal{N} = \mathcal{N}_+ + \mathcal{N}_-$ such that $\mathcal{N}_{+/-}$ acts on positive/negative energy states.

2. In the Hermitian conjugation of $\mathcal{N}$ which does not move the operator from positive energy state to negative energy state there certainly exist Hermitian operators and they correspond to zero energy states invariant under exchange of the incoming and outgoing states but in time scale $t_{+-}/2$. These operators are not Hermitian in $\mathcal{M}$. The commutativity of M with these operators follows also from the basic conditions.

Fractal hierarchy of state function reductions

In accordance with fractality, the conditions for the Connes tensor product at a given time scale imply the conditions at shorter time scales. On the other hand, in shorter time scales the inclusion would be deeper and would give rise to a larger reducibility of the representation of $\mathcal{N}$ in $\mathcal{M}$. Formally, as $\mathcal{N}$ approaches to a trivial algebra, one would have a square root of density matrix and trivial S-matrix in accordance with the idea about asymptotic freedom.

M-matrix would give rise to a matrix of probabilities via the expression $P(P_+ \to P_-) = Tr[P_+M^\dagger P_-M]$, where P_+ and P_- are projectors to positive and negative energy energy $\mathcal{N}$-rays. The projectors give rise to the averaging over the initial and final states inside $\mathcal{N}$ ray. The reduction could continue step by step to shorter length scales so that one would obtain a sequence of inclusions. If the U-process of the next quantum jump can return the M-matrix associated with $\mathcal{M}$ or some larger HFF, U process would be kind of reversal for state function reduction.

Analytic thinking proceeding from vision to details; human life cycle proceeding from dreams and wild actions to the age when most decisions relate to the routine daily activities; the progress of science from macroscopic to microscopic scales; even biological decay processes: all these have an intriguing resemblance to the fractal state function reduction process proceeding to to shorter and shorter time scales. Since this means increasing thermality of M-matrix, U process as a reversal of state function reduction might break the second law of thermodynamics.

The conservative option would be that only the transformation of intentions to action by U process giving rise to new zero energy states can bring in something new and is responsible for evolution. The non-conservative option is that the biological death is the U-process of the next quantum jump leading to a new life cycle. Breathing would become a universal metaphor for what happens in quantum Universe. The 4-D body would be lived again and again.

How quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or an action describing the interaction of the induced gauge field with the charges associated with the braid strand. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography. The difference between Chern-Simons action characterizing quantum state and the fundamental Chern-Simons type factor associated with the Kähler form would be that the latter emerges as the phase of the Dirac determinant.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

Some further comments about Connes tensor product

Below some further comments related to Connes tensor product.

1. M-matrix as an anti-unitary operator

The proposed form of Connes tensor product cannot be correct if M is a linear operator. The point is that if the conditions hold true for operator N and M then they cannot hold true for iN and M. One could restrict Connes tensor product to only Hermitian operators of $\mathcal{N}$ so that M-matrix would have $\mathcal{N}$ as symmetries. Another equivalent way to cope with the difficulty is to assume that M is anti-unitary operator. This assumption is natural since negative energy states are identified as hermitian conjugates of positive energy states so that entanglement matrix is interpreted as matrix multiplication plus conjugation acting on (say) negative energy states.

In both cases the interpretation is that the Hermitian operators of $\mathcal{N}$ act as symmetries of M-matrix. Quite generally, the interpretation would be in terms of symmetries of $U(n)$ or its subgroup. This conforms with the earlier view that finite measurement resolution allows to have any compact group as the group of dynamical symmetries. This might also relate to the connection between between Jones inclusions and Dynkin diagrams of ADE groups.

2. Connes tensor product in finite-D case

In the finite-dimensional case Connes tensor product of Hilbert spaces has a rather simple interpretation. If the matrix algebra N of $n \times n$ matrices acts on V from right, V can be regarded as a space formed by $m \times n$ matrices for some value of m. If N acts from left on W, W can be regarded as space of $n \times r$ matrices.

1. In the first representation the Connes tensor product of spaces V and W consists of $m \times r$ matrices and Connes tensor product is represented as the product VW of matrices as $(VW)_{mr}e^{mr}$. In this representation the information about N disappears completely as the interpretation in terms of measurement resolution suggests. The sum over intermediate states defined by N brings in mind path integral.

2. An alternative and more physical representation is as a state

$$\sum_n V_{mn}W_{nr}e^{mn} \otimes e^{nr}$$

   in the tensor product $V \otimes W$.

3. One can also consider two spaces V and W in which N acts from right and define Connes tensor product for $A^\dagger \otimes_N B$ or its tensor product counterpart. This case corresponds to the modification of the Connes tensor product of positive and negative energy states. Since Hermitian conjugation is involved, matrix product does not define the Connes tensor product now. For $m = r$ case entanglement coefficients should define a unitary matrix commuting with the action of the

Hermitian matrices of N and interpretation would be in terms of symmetry. HFF property would encourage to think that this representation has an analog in the case of HFFs of type II_1.

4. Also type I_n factors are possible and for them Connes tensor product makes sense if one can assign the inclusion of finite-D matrix algebras to a measurement resolution.

3. Connes tensor product in positive/negative energy sector

Ordinary linear Connes tensor product makes sense also in positive/negative energy sector and also now it makes sense to speak about measurement resolution. Hence one can ask whether Connes tensor product should be posed as a constraint on space-like entanglement. The interpretation could be in terms of the formation of bound states. The reducibility of HFFs and inclusions means that the tensor product is not uniquely fixed and ordinary entanglement could correspond to this kind of entanglement. Also the counterpart of p-adic coupling constant evolution would makes sense. The interpretation of Connes tensor product would be as the variance of the states with respect to some subgroup of $U(n)$ associated with the measurement resolution: the analog of color confinement would be in question.

4. The Lie-algebra of symmetries of M-matrix defines also Jordan algebra

Hermitian operators of $\mathcal{N} \subset \mathcal{M}$ act as maximal symmetries of M-matrix. The linear combinations of Hermitian operators with real coefficients are Hermitian and define an algebra under the product $A \circ B = (AB + BA)/2$, which is commutative and non-associative but satisfies the weaker associativity condition $(xy)(xx) = x(y(xx))$. A so called Hermitian Jordan algebra - introduced originally as a formalization of the algebra of observables - is in question [?]. Now Jordan algebra would characterize measurement resolution.

There exists four infinite families of Jordan algebras plus one exceptional Jordan algebra. The finite-dimensional real, complex, and quaternionic matrix algebras with product defined as above are Jordan algebras. Also the Euclidian gamma matrix algebra defined by Euclidian inner product and with real coefficients is Jordan algebra and known as spin factor: now the commutativity is not put in by hand. The exceptional Jordan algebra consists of real linear space of Hermitian 3×3 matrices with octonionic coefficients and with symmetrized product.

The maximal symmetries of M-matrix mean that the Hermitian generators of the algebra define a generalization of finite-dimensional Jordan algebra. The condition that all Hermitian operators involved are finite-dimensional brings in mind the definition of the permutation group S_∞ as consisting of finite permutations only and also the definition of infinite-dimensional Clifford algebra. Thus the natural interpretation of the algebra in question would be as maximal possible dynamical gauge symmetry implied by the finite measurement resolution. The active symmetries would be analogous to global gauge transformations and act non-trivially on all tensor factors in tensor product representation as a tensor product of 2×2 Clifford algebras.

Quaternionic Jordan algebra is natural in TGD framework since 2×2 Clifford algebra reduces to complexified quaternions and contains as sub-algebras real and complex Jordan algebras. Also Clifford algebra of world of classical worlds is a generalized Jordan algebra.

What about the octonionic Jordan algebra? There are intriguing hints that octonions might be important for TGD.

1. U(1), SU(2), and SU(3) are the factors of standard model gauge group and also the natural symmetries of minimal Jordan algebras relying on complex numbers, quaternions, and octonions. These symmetry groups relate also naturally to the geometry of CP_2.

2. 8-D Clifford algebra allows also octonionic representation.

3. The idea that one could make HFF of type II_1 a genuine local algebra analogous to gauge algebra can be realized only if the coordinate is non-associative since otherwise the coordinate can be represented as a tensor factor represented by a matrix algebra. Octonionic coordinate means an exception and would make 8-D imbedding space unique in that it would allow local version of HFF of type II_1.

4. These observations partially motivate a nebulous concept that I have christened HO-H duality [?]- admittedly a rather speculative idea - stating that TGD can be formulated alternatively using

hyper-octonions (subspace of complexified octonions with Minkowskian signature of metric) as the imbedding space and assuming that the dynamics is determined by the condition that space-time surfaces are hyper-quaternionic or co-hyper-quaternionic (and thus associative or co-associative). Associativity condition would determine the dynamics.

The question is therefore whether also 3×3 octonionic Jordan algebra might have some role in TGD framework.

1. Suppose for a moment that the above interpretation for the Hermitian operators as elements of a sub-factor $\mathcal{N}$ defining the measurement resolution generalizes also to the case of octonionic state space and operators represented as octonionic matrices. Also the direct sums of octonion valued matrices belonging to the octonionic Jordan algebra define a Jordan algebra and included algebras would now correspond to direct sums for copies of this Jordan algebra. One could perhaps say that the gauge symmetries associated with octonionic $\mathcal{N}$ would reduce to the power $SU(3)_o^n = SU(3)_o \times SU(3)_o \times ...$ of the octonionic $SU(3)$ acting on the fundamental triplet representation.

2. Triplet character is obviously problematic and one way out could be projectivization leading to the octonionic counterpart of CP_2. Octonionic scalings should not affect the physical state so that physical states as octonionic rays would correspond to octonionic CP_n. It is not however possible to realize the linear superposition of quantum states in CP_n. The octonionic (quaternionic) counterpart of CP_2 would be 2×8-dimensional and $U(2)_o$ would act as a matrix multiplication in this space. Realizing associativity (commutativity) condition for 2×8 spinors defined by octonionic CP_2 by replacing octonions with quaternions (complex numbers) would give 2×4-dimensional (2×2-dimensional) space.

3. This gives rise to two questions. The first question is whether CP_2 as a factor of imbedding space could somehow relate to the octonionic Jordan algebra. Could one think that this factor relates to the configuration space degrees of freedom assignable to CP_2 rather than Clifford algebra degrees of freedom? That color does not define spin like quantum numbers in TGD would conform with this. Note that the partial waves associated S^2 associated with light-cone boundary would correspond naturally to SU(2) and quaternionic algebra.

   Second question is whether the HFF of type II_1 could result from its possibly existing octonionic generalization by these two steps and whether the reduction of the octonionic symmetries to complex situation would give $SU(3) \times SU(3)...$ reducing to $U(2) \times U(2) \times ....$

1.8.4 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge from quantum TGD proper?

What p-adic coupling constant evolution really means has remained for a long time more or less open. The progress made in the understanding of the M-matrix of theory has however changed the situation dramatically.

M-matrix and coupling constant evolution

The final breakthrough in the understanding of p-adic coupling constant evolution came through the understanding of the M-matrix defining entanglement coefficients between positive and negative energy parts of zero energy states in zero energy ontology. M-matrix has interpretation as a "complex square root" of density matrix and thus provides a unification of thermodynamics and quantum theory. S-matrix is analogous to the phase of Schrödinger amplitude multiplying positive and real square root of density matrix analogous to modulus of Schrödinger amplitude.

The notion of finite measurement resolution realized in terms of inclusions of von Neumann algebras allows to demonstrate that the irreducible components of M-matrix are unique and possesses huge symmetries in the sense that the hermitian elements of included factor $\mathcal{N} \subset \mathcal{M}$ defining the measurement resolution act as symmetries of M-matrix, which suggests a connection with integrable quantum field theories.

It is also possible to understand coupling constant evolution as a discretized evolution associated with time scales T_n, if they come as octaves of a fundamental time scale: $T_n = 2^n T_0$. A weaker condition would be $T_p = pT_0$, p prime, and would assign all p-adic time scales to the size scale hierarchy of CDs. Number theoretic universality requires that renormalized coupling constants are rational or at most algebraic numbers and this is achieved by this discretization since the logarithms of discretized mass scale appearing in the expressions of renormalized coupling constants reduce to the form $log(2^n) = nlog(2)$ and with a proper choice of the coefficient of logarithm log(2) dependence disappears so that rational number results.

p-Adic coupling constant evolution

$T_n = 2^n T_0$ as the hierarchy of scale scales of CDs is suggestive on basis of their inherent geometry. A weaker condition would be $T_p = pT_0$, p prime, and would assign all p-adic time scales to the size scale hierarchy of CDs.

One can wonder how this picture relates to the earlier hypothesis that p-adic length coupling constant evolution. Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ (or $T_p = pT_0$) induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks like an attractive idea but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3. For $T_p = pT_0$ the above argument is not enough for p-adic length scale hypothesis and p-adic length scale hypothesis might be seen as an outcome of a process analogous to natural selection. Resonance like effect favoring octaves of a fundamental frequency might be in question. In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of configuration space.

1.8.5 Planar algebras and generalized Feynman diagrams

Planar algebras [?] are a very general notion due to Vaughan Jones and a special class of them is known to characterize inclusion sequences of hyper-finite factors of type II_1 [?]. In the following an argument is developed that planar algebras might have interpretation in terms of planar projections of generalized Feynman diagrams (these structures are metrically 2-D by presence of one light-like direction so that 2-D representation is especially natural). In [?] the role of planar algebras and their generalizations is also discussed.

Planar algebra very briefly

First a brief definition of planar algebra.

1. One starts from planar k-tangles obtained by putting disks inside a big disk. Inner disks are empty. Big disk contains $2k$ braid strands starting from its boundary and returning back or ending to the boundaries of small empty disks in the interior containing also even number of incoming lines. It is possible to have also loops. Disk boundaries and braid strands connecting them are different objects. A black-white coloring of the disjoint regions of k-tangle is assumed and there are two possible options (photo and its negative). Equivalence of planar tangles under diffeomorphisms is assumed.

2. One can define a product of k-tangles by identifying k-tangle along its outer boundary with some inner disk of another k-tangle. Obviously the product is not unique when the number of inner disks is larger than one. In the product one deletes the inner disk boundary but if one interprets this disk as a vertex-parton, it would be better to keep the boundary.

3. One assigns to the planar k-tangle a vector space V_k and a linear map from the tensor product of spaces V_{k_i} associated with the inner disks such that this map is consistent with the decomposition k-tangles. Under certain additional conditions the resulting algebra gives rise to an algebra characterizing multi-step inclusion of HFFs of type II_1.

4. It is possible to bring in additional structure and in TGD framework it seems necessary to assign to each line of tangle an arrow telling whether it corresponds to a strand of a braid associated with positive or negative energy parton. One can also wonder whether disks could be replaced with closed 2-D surfaces characterized by genus if braids are defined on partonic surfaces of genus g. In this case there is no topological distinction between big disk and small disks. One can also ask why not allow the strands to get linked (as suggested by the interpretation as planar projections of generalized Feynman diagrams) in which case one would not have a planar tangle anymore.

General arguments favoring the assignment of a planar algebra to a generalized Feynman diagram

There are some general arguments in favor of the assignment of planar algebra to generalized Feynman diagrams.

1. Planar diagrams describe sequences of inclusions of HFF:s and assign to them a multi-parameter algebra corresponding indices of inclusions. They describe also Connes tensor powers in the simplest situation corresponding to Jones inclusion sequence. Suppose that also general Connes tensor product has a description in terms of planar diagrams. This might be trivial.

2. Generalized vertices identified geometrically as partonic 2-surfaces indeed contain Connes tensor products. The smallest sub-factor N would play the role of complex numbers meaning that due to a finite measurement resolution one can speak only about N-rays of state space and the situation becomes effectively finite-dimensional but non-commutative.

3. The product of planar diagrams could be seen as a projection of 3-D Feynman diagram to plane or to one of the partonic vertices. It would contain a set of 2-D partonic 2-surfaces. Some of them would correspond vertices and the rest to partonic 2-surfaces at future and past directed light-cones corresponding to the incoming and outgoing particles.

4. The question is how to distinguish between vertex-partons and incoming and outgoing partons. If one does not delete the disk boundary of inner disk in the product, the fact that lines arrive at it from both sides could distinguish it as a vertex-parton whereas outgoing partons would correspond to empty disks. The direction of the arrows associated with the lines of planar diagram would allow to distinguish between positive and negative energy partons (note however line returning back).

5. One could worry about preferred role of the big disk identifiable as incoming or outgoing parton but this role is only apparent since by compactifying to say S^2 the big disk exterior becomes an interior of a small disk.

A more detailed view

The basic fact about planar algebras is that in the product of planar diagrams one glues two disks with identical boundary data together. One should understand the counterpart of this in more detail.

1. The boundaries of disks would correspond to 1-D closed space-like stringy curves at partonic 2-surfaces along which fermionic anti-commutators vanish.

2. The lines connecting the boundaries of disks to each other would correspond to the strands of number theoretic braids and thus to braidy time evolutions. The intersection points of lines with disk boundaries would correspond to the intersection points of strands of number theoretic braids meeting at the generalized vertex.

   [Number theoretic braid belongs to an algebraic intersection of a real parton 3-surface and its p-adic counterpart obeying same algebraic equations: of course, in time direction algebraicity allows only a sequence of snapshots about braid evolution].

3. Planar diagrams contain lines, which begin and return to the same disk boundary. Also "vacuum bubbles" are possible. Braid strands would disappear or appear in pairwise manner since they correspond to zeros of a polynomial and can transform from complex to real and vice versa under rather stringent algebraic conditions.

4. Planar diagrams contain also lines connecting any pair of disk boundaries. Stringy decay of partonic 2-surfaces with some strands of braid taken by the first and some strands by the second parton might bring in the lines connecting boundaries of any given pair of disks (if really possible!).

5. There is also something to worry about. The number of lines associated with disks is even in the case of k-tangles. In TGD framework incoming and outgoing tangles could have odd number of strands whereas partonic vertices would contain even number of k-tangles from fermion number conservation. One can wonder whether the replacement of boson lines with fermion lines could imply naturally the notion of half-k-tangle or whether one could assign half-k-tangles to the spinors of the configuration space ("world of classical worlds") whereas corresponding Clifford algebra defining HFF of type II_1 would correspond to k-tangles.

1.8.6 Miscellaneous

The following considerations are somewhat out-of-date: hence the title 'Miscellaneous'.

Connes tensor product and fusion rules

One should demonstrate that Connes tensor product indeed produces an M-matrix with physically acceptable properties.

The reduction of the construction of vertices to that for n-point functions of a conformal field theory suggest that Connes tensor product is essentially equivalent with the fusion rules for conformal fields defined by the Clifford algebra elements of $CH(CD)$ (4-surfaces associated with 3-surfaces at the boundary of causal diamond CD in M^4), extended to local fields in M^4 with gamma matrices acting on configuration space spinors assignable to the partonic boundary components.

Jones speculates that the fusion rules of conformal field theories can be understood in terms of Connes tensor product [?] and refers to the work of Wassermann about the fusion of loop group representations as a demonstration of the possibility to formula the fusion rules in terms of Connes tensor product [?].

Fusion rules are indeed something more intricate that the naive product of free fields expanded using oscillator operators. By its very definition Connes tensor product means a dramatic reduction of degrees of freedom and this indeed happens also in conformal field theories.

1. For non-vanishing n-point functions the tensor product of representations of Kac Moody group associated with the conformal fields must give singlet representation.

2. The ordinary tensor product of Kac Moody representations characterized by given value of central extension parameter k is not possible since k would be additive.

3. A much stronger restriction comes from the fact that the allowed representations must define integrable representations of Kac-Moody group [?]. For instance, in case of $SU(2)_k$ Kac Moody algebra only spins $j \leq k/2$ are allowed. In this case the quantum phase corresponds to $n = k+2$. $SU(2)$ is indeed very natural in TGD framework since it corresponds to both electro-weak $SU(2)_L$ and isotropy group of particle at rest.

Fusion rules for localized Clifford algebra elements representing operators creating physical states would replace naive tensor product with something more intricate. The naivest approach would start from M^4 local variants of gamma matrices since gamma matrices generate the Clifford algebra Cl associated with $CH(CD)$. This is certainly too naive an approach. The next step would be the localization of more general products of Clifford algebra elements elements of Kac Moody algebras creating physical states and defining free on mass shell quantum fields. In standard quantum field theory the next step would be the introduction of purely local interaction vertices leading to divergence difficulties. In the recent case one transfers the partonic states assignable to the light-cone boundaries $\delta M^4_\pm(m_i) \times CP_2$ to the common partonic 2-surfaces X^2_V along $X^3_{L,i}$ so that the products of field operators at the same space-time point do not appear and one avoids infinities.

The remaining problem would be the construction an explicit realization of Connes tensor product. The formal definition states that left and right $\mathcal{N}$ actions in the Connes tensor product $\mathcal{M} \otimes_\mathcal{N} \mathcal{M}$ are identical so that the elements $nm_1 \otimes m_2$ and $m_1 \otimes m_2 n$ are identified. This implies a reduction of degrees of freedom so that free tensor product is not in question. One might hope that at least in the simplest choices for $\mathcal{N}$ characterizing the limitations of quantum measurement this reduction is equivalent with the reduction of degrees of freedom caused by the integrability constraints for Kac-Moody representations and dropping away of higher spins from the ordinary tensor product for the representations of quantum groups. If fusion rules are equivalent with Connes tensor product, each type of quantum measurement would be characterized by its own conformal field theory.

In practice it seems safest to utilize as much as possible the physical intuition provided by quantum field theories. In [?] a rather precise vision about generalized Feynman diagrams is developed and the challenge is to relate this vision to Connes tensor product.

Connection with topological quantum field theories defined by Chern-Simons action

There is also connection with topological quantum field theories (TQFTs) defined by Chern- Simons action [?].

1. The light-like 3-surfaces X^3_l defining propagators can contain unitary matrix characterizing the braiding of the lines connecting fermions at the ends of the propagator line. Therefore the modular S-matrix representing the braiding would become part of propagator line. Also incoming particle lines can contain similar S-matrices but they should not be visible in the M-matrix. Also entanglement between different partonic boundary components of a given incoming 3-surface by a modular S-matrix is possible.

2. Besides CP_2 type extremals MEs with light-like momenta can appear as brehmstrahlung like exchanges always accompanied by exchanges of CP_2 type extremals making possible momentum conservation. Also light-like boundaries of magnetic flux tubes having macroscopic size could carry light-like momenta and represent similar brehmstrahlung like exchanges. In this case the modular S-matrix could make possible topological quantum computations in $q \neq 1$ phase [?]. Notice the somewhat counter intuitive implication that magnetic flux tubes of macroscopic size would represent change in quantum jump rather than quantum state. These quantum jumps can have an arbitrary long geometric duration in macroscopic quantum phases with large Planck constant [?].

There is also a connection with topological QFT defined by Chern-Simons action allowing to assign topological invariants to the 3-manifolds [?]. If the light-like CDs $X^3_{L,i}$ are boundary components, the 3-surfaces associated with particles are glued together somewhat like they are glued in the process allowing to construct 3-manifold by gluing them together along boundaries. All 3-manifold topologies can be constructed by using only torus like boundary components.

This would suggest a connection with 2+1-dimensional topological quantum field theory defined by Chern-Simons action allowing to define invariants for knots, links, and braids and 3-manifolds using surgery along links in terms of Wilson lines. In these theories one consider gluing of two 3-manifolds, say three-spheres S^3 along a link to obtain a topologically non-trivial 3-manifold. The replacement of link with Wilson lines in $S^3\#S^3 = S^3$ reduces the calculation of link invariants defined in this manner to Chern-Simons theory in S^3.

In the recent situation more general structures are possible since arbitrary number of 3-manifolds are glued together along link so that a singular 3-manifolds with a book like structure are possible. The allowance of CDs which are not boundaries, typically 3-D light-like throats of wormhole contacts at which induced metric transforms from Minkowskian to Euclidian, brings in additional richness of structure. If the scaling factor of CP_2 metric can be arbitrary large as the quantization of Planck constant predicts, this kind of structure could be macroscopic and could be also linked and knotted. In fact, topological condensation could be seen as a process in which two 4-manifolds are glued together by drilling light-like CD:s and connected by a piece of CP_2 type extremal.

1.9 Jones inclusions and cognitive consciousness

Configuration space spinors have a natural interpretation in terms of a quantum version of Boolean algebra. Beliefs of various kinds are the basic element of cognition and obviously involve a representation of the external world or part of it as states of the system defining the believer. Jones inclusions mediating unitary mappings between the spaces of configuration spaces spinors of two systems are excellent candidates for these maps, and it is interesting to find what one kind of model for beliefs this picture leads to.

The resulting quantum model for beliefs provides a cognitive interpretation for quantum groups and predicts a universal spectrum for the probabilities that a given belief is true. This spectrum depends only on the integer n characterizing the quantum phase $q = exp(i2\pi/n)$ characterizing the Jones inclusion. For $n \neq \infty$ the logic is inherently fuzzy so that absolute knowledge is impossible. $q = 1$ gives ordinary quantum logic with qbits having precise truth values after state function reduction.

1.9.1 Does one have a hierarchy of U- and M-matrices?

U-matrix describes scattering of zero energy states and since zero energy states can be illustrated in terms of Feynman diagrams one can say that scattering of Feynman diagrams is in question. The initial and final states of the scattering are superpositions of Feynman diagrams characterizing the corresponding M-matrices which contain also the positive square root of density matrix as a factor.

The hypothesis that U-matrix is the tensor product of S-matrix part of M-matrix and its Hermitian conjugate would make U-matrix an object deducible by physical measurements. One cannot of course exclude that something totally new emerges. For instance, the description of quantum jumps creating zero energy state from vacuum might require that U-matrix does not reduce in this manner. One can assign to the U-matrix a square like structure with S-matrix and its Hermitian conjugate assigned with the opposite sides of a square.

One can imagine of constructing higher level physical states as composites of zero energy states by replacing the S-matrix with M-matrix in the square like structure. These states would provide a physical representation of U-matrix. One could define U-matrix for these states in a similar manner. This kind of hierarchy could be continued indefinitely and the hierarchy of higher level U and M-matrices would be labeled by a hierarchy of n-cubes, $n = 1, 2,...$ TGD inspired theory of consciousness suggests that this hierarchy can be interpreted as a hierarchy of abstractions represented in terms of physical states. This hierarchy brings strongly in mind also the hierarchies of n-algebras and n-groups and this forces to consider the possibility that something genuinely new emerges at each step of the hierarchy. A connection with the hierarchies of infinite primes [?] and Jones inclusions are suggestive.

1.9.2 Feynman diagrams as higher level particles and their scattering as dynamics of self consciousness

The hierarchy of inclusions of hyper-finite factors of II_1 as counterpart for many-sheeted space-time lead inevitably to the idea that this hierarchy corresponds to a hierarchy of generalized Feynman diagrams for which Feynman diagrams at a given level become particles at the next level. Accepting this idea, one is led to ask what kind of quantum states these Feynman diagrams correspond, how one could describe interactions of these higher level particles, what is the interpretation for these higher level states, and whether they can be detected.

Jones inclusions as analogs of space-time surfaces

The idea about space-time as a 4-surface replicates itself at the level of operator algebra and state space in the sense that Jones inclusion can be seen as a representation of the operator algebra $\mathcal{N}$ as infinite-dimensional linear sub-space (surface) of the operator algebra $\mathcal{M}$. This encourages to think that generalized Feynman diagrams could correspond to image surfaces in II_1 factor having identification as kind of quantum space-time surfaces.

Suppose that the modular S-matrices are representable as the inner automorphisms $\Delta(\mathcal{M}_k^{it}$ assigned to the external lines of Feynman diagrams. This would mean that $\mathcal{N} \subset \mathcal{M}_k$ moves inside $calM_k$ along a geodesic line determined by the inner automorphism. At the vertex the factors $calM_k$ to fuse along $\mathcal{N}$ to form a Connes tensor product. Hence the copies of $\mathcal{N}$ move inside $\mathcal{M}_k$ like incoming 3-surfaces in H and fuse together at the vertex. Since all $\mathcal{M}_k$ are isomorphic to a universal factor $\mathcal{M}$, many-sheeted space-time would have a kind of quantum image inside II_1 factor consisting of pieces which are $d = \mathcal{M} : \mathcal{N}/2$-dimensional quantum spaces according to the identification of the quantum space as subspace of quantum group to be discussed later. In the case of partonic Clifford algebras the dimension would be indeed $d \leq 2$.

The hierarchy of Jones inclusions defines a hierarchy of S-matrices

It is possible to assign to a given Jones inclusion $\mathcal{N} \subset \mathcal{M}$ an entire hierarchy of Jones inclusions $\mathcal{M}_0 \subset \mathcal{M}_1 \subset \mathcal{M}_2 ...$, $\mathcal{M}_0 = N$, $\mathcal{M}_1 = M$. A possible interpretation for these inclusions would be as a sequence of topological condensations.

This sequence also defines a hierarchy of Feynman diagrams inside Feynman diagrams. The factor $\mathcal{M}$ containing the Feynman diagram having as its lines the unitary orbits of $\mathcal{N}$ under $\Delta_{\mathcal{M}}$ becomes a parton in $\mathcal{M}_1$ and its unitary orbits under $\Delta_{\mathcal{M}_1}$ define lines of Feynman diagrams in M_1. The concrete representation for M-matrix or projection of it to some subspace as entanglement coefficients of partons at the ends of a braid assignable to the space-like 3-surface representing a vertex of a higher level Feynman diagram. In this manner quantum dynamics would be coded and simulated by quantum states.

The outcome can be said to be a hierarchy of Feynman diagrams within Feynman diagrams, a fractal structure for which many particle scattering events at a given level become particles at the next level. The particles at the next level represent dynamics at the lower level: they have the property of "being about" representing perhaps the most crucial element of conscious experience. Since net conserved quantum numbers can vanish for a system in TGD Universe, this kind of hierarchy indeed allows a realization as zero energy states. Crossing symmetry can be understood in terms of this picture and has been applied to construct a model for M-matrix at high energy limit [?].

One might perhaps say that quantum space-time corresponds to a double inclusion and that further inclusions bring in N-parameter families of space-time surfaces.

Higher level Feynman diagrams

The lines of Feynman diagram in $\mathcal{M}_{n+1}$ are geodesic lines representing orbits of $\mathcal{M}_n$ and this kind of lines meet at vertex and scatter. The evolution along lines is determined by $\Delta_{\mathcal{M}_{n+1}}$. These lines contain within themselves $\mathcal{M}_n$ Feynman diagrams with similar structure and the hierarchy continues down to the lowest level at which ordinary elementary particles are encountered.

For instance, the generalized Feynman diagrams at the second level are ribbon diagrams obtained by thickening the ordinary diagrams in the new time direction. The interpretation as ribbon diagrams crucial for topological quantum computation and suggested to be realizable in terms of zero energy states in [?] is natural. At each level a new time parameter is introduced so that the dimension of the diagram can be arbitrarily high. The dynamics is not that of ordinary surfaces but the dynamics induced by the $\Delta_{\mathcal{M}_n}$.

Quantum states defined by higher level Feynman diagrams

The intuitive picture is that higher level quantum states corresponds to the self reflective aspect of existence and must provide representations for the quantum dynamics of lower levels in their own structure. This dynamics is characterized by M-matrix whose elements have representation in terms of Feynman diagrams.

1. These states correspond to zero energy states in which initial states have "positive energies" and final states have "negative energies". The net conserved quantum numbers of initial and final state partons compensate each other. Gravitational energies, and more generally gravitational quantum numbers defined as absolute values of the net quantum numbers of initial and final states do not vanish. One can say that thoughts have gravitational mass but no inertial mass.

2. States in sub-spaces of positive and negative energy states are entangled with entanglement coefficients given by M-matrix at the level below.

To make this more concrete, consider first the simplest non-trivial case. In this case the particles can be characterized as ordinary Feynman diagrams, or more precisely as scattering events so that the state is characterized by $\hat{S} = P_{in}SP_{out}$, where S is S-matrix and P_{in} *resp.* P_{out} is the projection to a subspace of initial *resp.* final states. An entangled state with the projection of S-matrix giving the entanglement coefficients is in question.

The larger the domains of projectors P_{in} and P_{out}, the higher the representative capacity of the state. The norm of the non-normalized state $\hat{S}$ is $Tr(\hat{S}\hat{S}^\dagger) \leq 1$ for II_1 factors, and at the limit $\hat{S} = S$ the norm equals to 1. Hence, by II_1 property, the state always entangles infinite number of states, and can in principle code the entire S-matrix to entanglement coefficients.

The states in which positive and negative energy states are entangled by a projection of S-matrix might define only a particular instance of states for which conserved quantum numbers vanish. The model for the interaction of Feynman diagrams discussed below applies also to these more general states.

The interaction of $\mathcal{M}_n$ Feynman diagrams at the second level of hierarchy

What constraints can one pose to the higher level reactions? How Feynman diagrams interact? Consider first the scattering at the second level of hierarchy ($\mathcal{M}_1$), the first level $\mathcal{M}_0$ being assigned to the interactions of the ordinary matter.

1. Conservation laws pose constraints on the scattering at level $\mathcal{M}_1$. The Feynman diagrams can transform to new Feynman diagrams only in such a manner that the net quantum numbers are conserved separately for the initial positive energy states and final negative energy states of the diagram. The simplest assumption is that positive energy matter and negative energy matter know nothing about each other and effectively live in separate worlds. The scattering matrix form Feynman diagram like states would thus be simply the tensor product $S \otimes S^\dagger$, where S is the S-matrix characterizing the lowest level interactions and identifiable as unitary factor of M-matrix for zero energy states. Reductionism would be realized in the sense that, apart from the new elements brought in by $\Delta_{\mathcal{M}_n}$ defining single particle free dynamics, the lowest level would determine in principle everything occurring at the higher level providing representations about representations about... for what occurs at the basic level. The lowest level would represent the physical world and higher levels the theory about it.

2. The description of hadronic reactions in terms of partons serves as a guide line when one tries to understand higher level Feynman diagrams. The fusion of hadronic space-time sheets corresponds to the vertices $\mathcal{M}_1$. In the vertex the analog of parton plasma is formed by a process known as parton fragmentation. This means that the partonic Feynman diagrams belonging to disjoint copies of $\mathcal{M}_0$ find themselves inside the same copy of $\mathcal{M}_0$. The standard description would apply to the scattering of the initial *resp.* final state partons.

3. After the scattering of partons hadronization takes place. The analog of hadronization in the recent case is the organization of the initial and final state partons to groups I_i and F_i such that the net conserved quantum numbers are same for I_i and F_i. These conditions can be satisfied

if the interactions in the plasma phase occur only between particles belonging to the clusters labeled by the index i. Otherwise only single particle states in $\mathcal{M}_1$ would be produced in the reactions in the generic case. The cluster decomposition of S-matrix to a direct sum of terms corresponding to partitions of the initial state particles to clusters which do not interact with each other obviously corresponds to the "hadronization". Therefore no new dynamics need to be introduced.

4. One cannot avoid the question whether the parton picture about hadrons indeed corresponds to a higher level physics of this kind. This would require that hadronic space-time sheets carry the net quantum numbers of hadrons. The net quantum numbers associated with the initial state partons would be naturally identical with the net quantum numbers of hadron. Partons and they negative energy conjugates would provide in this picture a representation of hadron about hadron. This kind of interpretation of partons would make understandable why they cannot be observed directly. A possible objection is that the net gravitational mass of hadron would be three times the gravitational mass deduced from the inertial mass of hadron if partons feed their gravitational fluxes to the space-time sheet carrying Earth's gravitational field.

5. This picture could also relate to the suggested duality between string and parton pictures [?]. In parton picture hadron is formed from partons represented by space-like 2-surfaces X_i^2 connected by join along boundaries bonds. In string picture partonic 2-surfaces are replaced with string orbits. If one puts positive and negative energy particles at the ends of string diagram one indeed obtains a higher level representation of hadron. If these pictures are dual then also in parton picture positive and negative energies should compensate each other. Interestingly, light-like 3-D causal determinants identified as orbits of partons could be interpreted as orbits of light like string word sheets with "time" coordinate varying in space-like direction.

Scattering of Feynman diagrams at the higher levels of hierarchy

This picture generalizes to the description of higher level Feynman diagrams.

1. Assume that higher level vertices have recursive structure allowing to reduce the Feynman diagrams to ordinary Feynman diagrams by a procedure consisting of finite steps.

2. The lines of diagrams are classified as incoming or outgoing lines according to whether the time orientation of the line is positive or negative. The time orientation is associated with the time parameter t_n characterizing the automorphism $\Delta_{\mathcal{M}_\backslash}^{it_n}$. The incoming and outgoing net quantum numbers compensate each other. These quantum numbers are basically the quantum numbers of the state at the lowest level of the hierarchy.

3. In the vertices the $\mathcal{M}_{n+1}$ particles fuse and $\mathcal{M}_n$ particles form the analog of quark gluon plasma. The initial and final state particles of $\mathcal{M}_n$ Feynman diagram scatter independently and the S-matrix S_{n+1} describing the process is tensor product $S_n \otimes S_n^\dagger$. By the clustering property of S-matrix, this scattering occurs only for groups formed by partons formed by the incoming and outgoing particles $\mathcal{M}_n$ particles and each outgoing $\mathcal{M}_{n+1}$ line contains and irreducible $\mathcal{M}_n$ diagram. By continuing the recursion one finally ends down with ordinary Feynman diagrams.

1.9.3 Logic, beliefs, and spinor fields in the world of classical worlds

Beliefs can be characterized as Boolean value maps $\beta_i(p)$ telling whether i believes in proposition p or not. Additional structure is brought in by introducing the map $\lambda_i(p)$ telling whether p is true or not in the environment of i. The task is to find quantum counterpart for this model.

Configuration space spinors as logic statements

In TGD framework the infinite-dimensional configuration space (CH) spinor fields defined in CH, the "world of classical worlds", describe quantum states of the Universe [?]. CH spinor field can be regarded as a state in infinite-dimensional Fock space and are labeled by a collection of various two valued indices like spin and weak isospin. The interpretation is as a collection of truth values of logic statements one for each fermionic oscillator operator in the state. For instance, spin up and down would correspond to two possible truth values of a proposition characterized by other quantum numbers of the mode.

The hierarchy of space-time sheet could define a physical correlate for the hierarchy of higher order logics (statements about statements about...). The space-time sheet containing N fermions topologically condensed at a larger space-time sheet behaves as a fermion or boson depending on whether N is odd or even. This hierarchy has also a number theoretic counterpart: the construction of infinite primes [?] corresponds to a repeated second quantization of a super-symmetric quantum field theory.

Quantal description of beliefs

The question is whether TGD inspired theory of consciousness allows a fundamental description of beliefs.

1. Beliefs define a model about some subsystem of universe constructed by the believer. This model can be understood as some kind of representation of real word in the state space representing the beliefs.

2. One can wonder what is the difference between real and p-adic variants of CH spinor fields and whether they could represent reality and beliefs about reality. CH spinors (as opposed to spinor fields) are constructible in terms of fermionic oscillator operators and seem to be universal in the sense that one cannot speak about p-adic and real CH spinors as different objects. Real/ p-adic spinor fields however have real/p-adic space-time sheets as arguments. This would suggest that there is no fundamental difference between the logic statements represented by p-adic and real CH spinors.

These observations suggest a more concrete view about how beliefs emerge physically.

The idea that p-adic CH spinor fields could serve as representations of beliefs and real CH spinor fields as representations of reality looks very nice but the fact that the outcomes of p-adic-to-real phase transition and its reversal are highly non-predictable does not support it as such.

Quantum statistical determinism could however come into rescue. Belief could be represented as an ensemble of p-adic mental images resulting in transitions of real mental images representing reality to p-adic states. p-Adic ensemble average would represent the belief.

It is not at all clear whether real-to-padic transitions can occur at high enough rate since p-adic-to-real transition are expected to be highly irreversible. The real initial states much have nearly vanishing quantum numbers emitted in the transition to p-adic state to guarantee conservation laws (p-adic conservation laws hold true only piecewise since conserved quantities are pseudo constants). The system defined by an ensemble of real Boolean mental images representing reality would automatically generate a p-adic variant representing a belief about reality.

p-Adic CH spinors can also represent the cognitive aspects of intention whereas p-adic space-time sheets would represent its geometric aspects reflected in sensory experience.p-Adic space-time sheet could also serve only as a space-time correlate for the fundamental representation of intention in terms of p-adic CH spinor field. This view is consistent with the proposed identification of beliefs since the transitions associated with intentions *resp.* beliefs would be p-adic-to-real *resp.* real-to-padic.

1.9.4 Jones inclusions for hyperfinite factors of type II_1 as a model for symbolic and cognitive representations

Consider next a more detailed model for how cognitive representations and beliefs are realized at quantum level. This model generalizes trivially to symbolic representations.

The Clifford algebra of gamma matrices associated with CH spinor fields corresponds to a von Neumann algebra known as hyper-finite factor of type II_1. The mathematics of these algebras is extremely beautiful and reproduces basic mathematical structures of modern physics (conformal field theories, quantum groups, knot and braid groups,....) from the mere assumption that the world of classical worlds possesses infinite-dimensional Kähler geometry and allows spinor structure.

The almost defining feature is that the infinite-dimensional unit matrix of the Clifford algebra in question has by definition unit trace. Type II_1 factors allow also what are known as Jones inclusions of Clifford algebras $\mathcal{N} \subset \mathcal{M}$. What is special to II_1 factors is that the induced unitary mappings between spinor spaces are genuine inclusions rather than 1-1 maps.

The S-matrix associated with the real-to-p-adic quantum transition inducing belief from reality would naturally define Jones inclusion of CH Clifford algebra $\mathcal{N}$ associated with the real space-time sheet to the Clifford algebra $\mathcal{M}$ associated with the p-adic space-time sheet. The moduli squared of S-matrix elements would define probabilities for pairs or real and belief states.

In Jones inclusion $\mathcal{N} \subset \mathcal{M}$ the factor $\mathcal{N}$ is included in factor $\mathcal{M}$ such that $\mathcal{M}$ can be expressed as $\mathcal{N}$-module over quantum space $\mathcal{M}/\mathcal{N}$ which has fractal dimension given by Jones index $\mathcal{M} : \mathcal{N} = 4cos^2(\pi/n) \leq 4$, $n = 3, 4, ...$ varying in the range $[1, 4]$. The interpretation is as the fractal dimension corresponding to a dimension of Clifford algebra acting in $d = \sqrt{\mathcal{M} : \mathcal{N}}$-dimensional spinor space: d varies in the range $[1, 2]$. The interpretation in terms of a quantal variant of logic is natural.

Probabilistic beliefs

For $\mathcal{M} : \mathcal{N} = 4$ ($n = \infty$) the dimension of spinor space is $d = 2$ and one can speak about ordinary 2-component spinors with $\mathcal{N}$-valued coefficients representing generalizations of qubits. Hence the inclusion of a given $\mathcal{N}$-spinor as M-spinor can be regarded as a belief on the proposition and for the decomposition to a spinor in N-module $\mathcal{M}/\mathcal{N}$ involves for each index a choice $\mathcal{M}/\mathcal{N}$ spinor component selecting super-position of up and down spins. Hence one has a superposition of truth values in general and one can speak only about probabilistic beliefs. It is not clear whether one can choose the basis in such a manner that $\mathcal{M}/\mathcal{N}$ spinor corresponds always to truth value 1. Since CH spinor field is in question and even if this choice might be possible for a single 3-surface, it need not be possible for deformations of it so that at quantum level one can only speak about probabilistic beliefs.

Fractal probabilistic beliefs

For $d < 2$ the spinor space associated with $\mathcal{M}/\mathcal{N}$ can be regarded as quantum plane having complex quantum dimension d with two non-commuting complex coordinates z^1 and z^2 satisfying $z^1z^2 = qz^2z^1$ and $\overline{z^1z^2} = \overline{q}\overline{z^2z^1}$. These relations are consistent with hermiticity of the real and imaginary parts of z^1 and z^2 which define ordinary quantum planes. Hermiticity also implies that one can identify the complex conjugates of z^i as Hermitian conjugates.

The further commutation relations $[z^1, \overline{z^2}] = [z^2, \overline{z^1}] = 0$ and $[z^1, \overline{z^1}] = [z^2, \overline{z^2}] = r$ give a closed algebra satisfying Jacobi identities. One could argue that $r \geq 0$ should be a function $r(n)$ of the quantum phase $q = exp(i2\pi/n)$ vanishing at the limit $n \to \infty$ to guarantee that the algebra becomes commutative at this limit and truth values can be chosen to be non-fuzzy. $r = sin(\pi/n)$ would be the simplest choice. As will be found, the choice of $r(n)$ does not however affect at all the spectrum for the probabilities of the truth values. $n = \infty$ case corresponding to non-fuzzy quantum logic is also possible and must be treated separately: it corresponds to Kac Moody algebra instead of quantum groups.

The non-commutativity of complex spinor components means that z^1 and z^2 are not independent coordinates: this explains the reduction of the number of the effective number of truth values to $d < 2$. The maximal reduction occurs to $d = 1$ for $n = 3$ so that there is effectively only single truth value and one could perhaps speak about taboo or dogma or complete disappearance of the notions of truth and false (this brings in mind reports about meditative states: in fact $n = 3$ corresponds to a phase in which Planck constant becomes infinite so that the system is maximally quantal).

As non-commuting operators the components of d-spinor are not simultaneously measurable for $d < 2$. It is however possible to measure simultaneously the operators describing the probabilities $z^1\overline{z^1}$ and $z^2\overline{z^2}$ for truth values since these operators commute. An inherently fuzzy Boolean logic would be in question with the additional feature that the spinorial counterparts of statement and its negation cannot be regarded as independent observables although the corresponding probabilities satisfy the defining conditions for commuting observables.

If one can speak of a measurement of probabilities for $d < 2$, it differs from the ordinary quantum measurement in the sense that it cannot involve a state function reduction to a pure qubit meaning irreducible quantal fuzziness. One could speak of fuzzy qbits or fqbits (or quantum qbits) instead of qbits. This picture would provide the long sought interpretation for quantum groups.

The previous picture applies to all representations $M_1 \subset M_2$, where M_1 and M_2 denote either real or p-adic Clifford algebras for some prime p. For instance, real-real Jones inclusion could be interpreted as symbolic representations assignable to a unitary mapping of the states of a subsystem M_1 of the external world to the state space M_2 of another real subsystem. $p_1 \rightarrow p_2$ unitary inclusions would in turn map cognitive representations to cognitive representations. There is a strong temptation to assume that these Jones inclusions define unitary maps realizing universe as a universal quantum computer mimicking itself at all levels utilizing cognitive and symbolic representations. Subsystem-system inclusion would naturally define one example of Jones inclusion.

The spectrum of probabilities of truth values is universal

It is actually possible to calculate the spectrum of the probabilities of truth values with rather mild additional assumptions.

1. Since the Hermitian operators $X_1 = (z^1\overline{z^1}+\overline{z^1}z^1)/2$ and $X_2 = (z^2\overline{z^2}+\overline{z^2}z^2)/2$ commute, physical states can be chosen to be eigen states of these operators and it is possible to assign to the truth values probabilities given by $p_1 = X_1/R^2$ and $p_2 = X_2/R^2$, $R^2 = X_1 + X_2$.

2. By introducing the analog of the harmonic oscillator vacuum as a state $|0\rangle$ satisfying $z^1|0\rangle = z^2|0\rangle = 0$, one obtains eigen states of X_1 and X_2 as states $|n_1, n_2\rangle = \overline{z^1}^{n_1}\overline{z^2}^{n_2}|0\rangle$, $n_1 \geq 0, n_2 \geq 0$. The eigenvalues of X_1 and X_2 are given by a modified harmonic oscillator spectrum as $(1/2 + n_1 q^{n_2})r$ and $(1/2 + n_2 q^{n_1})r$. The reality of eigenvalues (hermiticity) is guaranteed if one has $n_1 = N_1 n$ and $n_1 = N_2 n$ and implies that the spectrum of eigen states gets increasingly thinner for $n \rightarrow \infty$. This must somehow reflect the fractal dimension. The fact that large values of oscillator quantum numbers n_1 and n_2 correspond to the classical limit suggests that modulo condition guarantees approximate classicality of the logic for $n \rightarrow \infty$.

3. The probabilities p_1 and p_2 for the truth values given by $(p_1, p_2) = (1/2 + N_1 n, 1/2 + N_2 n)/[1 + (N_1 + N_2)n]$ are rational and allow an interpretation as both real and p-adic numbers. All states are are inherently fuzzy and only at the limits $N_1 \gg N_2$ and $N_2 \gg N_1$ non-fuzzy states result. As noticed, $n = \infty$ must be treated separately and corresponds to an ordinary non-fuzzy qbit logic. At $n \rightarrow \infty$ limit one has $(p_1, p_2) = (N_1, N_2)/(N_1, N_2)$: at this limit $N_1 = 0$ or $N_2 = 0$ states are non-fuzzy.

How to define variants of belief quantum mechanically?

Probabilities of true and false for Jones inclusion characterize the plausibility of the belief and one can ask whether this description is enough to characterize states such as knowledge, misbelief, doubt, delusion, and ignorance. The truth value of $\beta_i(p)$ is determined by the measurement of probability assignable to Jones inclusion on the p-adic side. The truth value of $\lambda_i(p)$ is determined by a similar measurement on the real side. β and λ appear completely symmetrically and one can consider all kinds of triplets $\mathcal{M}_1 \subset \mathcal{M}_2 \subset \mathcal{M}_3$ assuming that there exist unitary S-matrix like maps mediating a sequence $\mathcal{M}_1 \subset \mathcal{M}_2 \subset \mathcal{M}_3$ of Jones inclusions. Interestingly, the hierarchies of Jones inclusions are a key concept in the theory of hyper-finite factors of type II_1 and pair of inclusions plays a fundamental role.

Let us restrict the consideration to the situation when $\mathcal{M}_1$ corresponds to a real subsystem of the external world, $\mathcal{M}_2$ its real representation by a real subsystem, and $\mathcal{M}_3$ to p-adic cognitive representation of M_3. Assume that both real and p-adic sides involve a preferred state basis for qubits representing truth and false.

Assume first that both $\mathcal{M}_1 \subset \mathcal{M}_2$ and $\mathcal{M}_2 \subset \mathcal{M}_3$ correspond to $d = 2$ case for which ordinary quantum measurement or truth value is possible giving outcome true or false. Assume further that the truth values have been measured in both M_2 and M_3.

1. Knowledge corresponds to the proposition $\beta_i(p) \wedge \lambda_i(p)$.

2. Misbelief to the proposition $\beta_i(p) \wedge \neq \lambda_i(p)$.
   Knowledge and misbelief would involve both the measurement of real and p-adic probabilities .

3. Assume next that one has $d < 2$ form $\mathcal{M}_2 \subset \mathcal{M}_3$. Doubt can be regarded neither belief or disbelief: $\beta_i(p) \wedge \neq \beta_i(\neq p)$: belief is inherently fuzzy although proposition can be non-fuzzy.

   Assume next that truth values in $\mathcal{M}_1 \subset \mathcal{M}_2$ inclusion corresponds to $d < 2$ so that the basic propositions are inherently fuzzy.

4. Delusion is a belief which cannot be justified: $\beta_i(p) \wedge \lambda_i(p) \wedge \neq \lambda(\neq p))$. This case is possible if $d = 2$ holds true for $\mathcal{M}_2 \subset \mathcal{M}_3$. Note that also misbelief that cannot be shown wrong is possible.
   In this case truth values cannot be quantum measured for $\mathcal{M}_1 \subset \mathcal{M}_2$ but can be measured for $\mathcal{M}_2 \subset \mathcal{M}_3$. Hence the states are products of pure $\mathcal{M}_3$ states with fuzzy $\mathcal{M}_2$ states.

5. Ignorance corresponds to the proposition $\beta_i(p) \wedge \neq \beta_i(\neq p) \wedge \lambda_i(p) \wedge \neq \lambda(\neq p))$. Both real representational states and belief states are inherently fuzzy.

Quite generally, only for $d_1 = d_2 = 2$ ideal knowledge and ideal misbelief are possible. Fuzzy beliefs and logics approach to ordinary one at the limit $n \to \infty$, which according to the proposal of [?] corresponds to the ordinary value of Planck constant. For other cases these notions are only approximate and quantal approach allows to characterize the goodness of the approximation. A new kind of inherent quantum uncertainty of knowledge is in question and one could speak about a Uncertainty Principle for cognition and symbolic representations. Also the unification of symbolic and various kinds of cognitive representations deserves to be mentioned.

1.9.5 Intentional comparison of beliefs by topological quantum computation?

Intentional comparison would mean that for a given initial state also the final state of the quantum jump is fixed. This requires the ability to engineer S-matrix so that it leads from a given state to single state only. Any S-matrix representing permutation of the initial states fulfills these conditions. This condition is perhaps unnecessarily strong.

Quantum computation is basically the engineering of S-matrix so that it represents a superposition of parallel computations. In TGD framework topological quantum computation based on the braiding of magnetic flux tubes would be represented as an evolution characterized by braid [?]. The dynamical evolution would be associated with light-like boundaries of braids. This evolution has dual interpretations either as a limit of time evolution of quantum state (program running) or a quantum state satisfying conformal invariance constraints (program code).

The dual interpretation would mean that conformally invariant states are equivalent with engineered time evolutions and topological computation realized as braiding connecting the quantum states to be compared (beliefs represented as many-fermion states at the boundaries of magnetic flux tubes) could give rise to conscious computational comparison of beliefs. The complexity of braiding would give a measure for how much the states to be compared differ.

Note that quantum computation is defined by a unitary map which could also be interpreted as symbolic representation of states of system M_1 as states of system M_2 mediated by the braid of join along boundaries bonds connecting the two space-time sheets in question and having light-like boundaries. These considerations suggest that the idea about S-matrix of the Universe should be generalized so that the dynamics of the Universe is dynamics of mimicry described by an infinite collection of fermionic S-matrices representable in terms of Jones inclusions.

1.9.6 The stability of fuzzy qbits and quantum computation

The stability of fqbits against state function reduction might have deep implications for quantum computation since quantum spinors would be stable against state function reduction induced by the perturbations inducing de-coherence in the normal situation. If this is really true, and if the only dangerous perturbations are those inducing the phase transition to qbits, the implications for quantum computation could be dramatic. Of course, the rigidity of qbits could be just another way to say that topological quantum computations are stable against thermal perturbations not destroying anyons [?].

The stability of fqbits could also be another manner to state the stability of rational, or more generally algebraic, bound state entanglement against state function reduction, which is one of the basic hypothesis of TGD inspired theory of consciousness [?]. For sequences of Jones inclusions or equivalently, for multiple Connes tensor products, one would obtain tensor products of quantum spinors making possible arbitrary complex configurations of fqbits. Anyonic braids in topological quantum computation would have interpretation as representations for this kind of tensor products.

1.9.7 Fuzzy quantum logic and possible anomalies in the experimental data for the EPR-Bohm experiment

The experimental data for EPR-Bohm experiment [?] excluding hidden variable interpretations of quantum theory. What is less known that the experimental data indicates about possibility of an anomaly challenging quantum mechanics [?]. The obvious question is whether this anomaly might provide a test for the notion of fuzzy quantum logic inspired by the TGD based quantum measurement theory with finite measurement resolution.

The anomaly

The experimental situation involves emission of two photons from spin zero system so that photons have opposite spins. What is measured are polarizations of the two photons with respect to polarization axes which differ from standard choice of this axis by rotations around the axis of photon momentum characterized by angles α and β. The probabilities for observing polarizations (i,j), where i,j is taken Z_2 valued variable for a convenience of notation are $P_{ij}(\alpha,\beta)$, are predicted to be $P_{00} = P_{11} = cos^2(\alpha-\beta)/2$ and $P_{01} = P_{10} = sin^2(\alpha-\beta)/2$.

Consider now the discrepancies.

1. One has four identities $P_{i,i} + P_{i,i+1} = P_{ii} + P_{i+1,i} = 1/2$ having interpretation in terms of probability conservation. Experimental data of [?] are not consistent with this prediction [?] and this is identified as the anomaly.

2. The QM prediction $E(\alpha,\beta) = \sum_i (P_{i,i} - P_{i,i+1}) = cos(2(\alpha-\beta)$ is not satisfied neither: the maxima for the magnitude of E are scaled down by a factor $\simeq .9$. This deviation is not discussed in [?].

Both these findings raise the possibility that QM might not be consistent with the data. It turns out that fuzzy quantum logic predicted by TGD and implying that the predictions for the probabilities and correlation must be replaced by ensemble averages, can explain anomaly b) but not anomaly a). A "mundane" explanation for anomaly a) is proposed.

Predictions of fuzzy quantum logic for the probabilities and correlations

1. The description of fuzzy quantum logic in terms statistical ensemble

The fuzzy quantum logic implies that the predictions $P_{i,j}$ for the probabilities should be replaced with ensemble averages over the ensembles defined by fuzzy quantum logic. In practice this means that following replacements should be carried out:

$$\begin{aligned} P_{i,j} &\rightarrow P^2 P_{i,j} + (1-P)^2 P_{i+1,j+1} \\ &+ P(1-P)\left[P_{i,j+1} + P_{i+1,j}\right] \ . \end{aligned} \tag{1.9.1}$$

Here P is one of the state dependent universal probabilities/fuzzy truth values for some value of n characterizing the measurement situation. The concrete predictions would be following

$$\begin{aligned} P_{0,0} = P_{1,1} &\rightarrow A\frac{cos^2(\alpha-\beta)}{2} + B\frac{sin^2(\alpha-\beta)}{2} \\ &= (A-B)\frac{cos^2(\alpha-\beta)}{2} + \frac{B}{2} \ , \\ P_{0,1} = P_{1,0} &\rightarrow A\frac{sin^2(\alpha-\beta)}{2} + B\frac{cos^2(\alpha-\beta)}{2} \\ &= (A-B)\frac{sin^2(\alpha-\beta)}{2} + \frac{B}{2} \ , \\ A &= P^2 + (1-P)^2 \ , \ B = 2P(1-P) \ . \end{aligned} \tag{1.9.2}$$

The prediction is that the graphs of probabilities as a function as function of the angle $\alpha-\beta$ are scaled by a factor $1-4P(1-P)$ and shifted upwards by $P(1-P)$. The value of P, and one might hope even the value of n labeling Jones inclusion and the integer m labeling the quantum state might be deducible from the experimental data as the upward shift. The basic prediction is that the maxima of curves measuring probabilities $P_(i,j)$ have minimum at $B/2 = P(1-P)$ and maximum is scaled down to $(A-B)/2 = 1/2 - 2P(1-P)$.

If the P is same for all pairs i,j, the correlation $E = \sum_i (P_{ii} - P_{i,i+1})$ transforms as

$$E(\alpha,\beta) \quad \rightarrow \quad [1-4P(1-P)]\, E(\alpha,\beta) \ . \tag{1.9.3}$$

Only the normalization of $E(\alpha,\beta)$ as a function of $\alpha-\beta$ reducing the magnitude of E occurs. In particular the maximum/minimum of E are scaled down from $E=\pm 1$ to $E = \pm(1-4P(1-P))$.

From the figure 1b) of [**?**] the scaling down indeed occurs for magnitudes of E with same amount for minimum and maximum. Writing $P = 1-\epsilon$ one has $A - B \simeq 1-4\epsilon$ and $B \simeq 2\epsilon$ so that the maximum is in the first approximation predicted to be at $1-4\epsilon$. The graph would give $1-P \simeq \epsilon \simeq .025$. Thus the model explains the reduction of the magnitude for the maximum and minimum of E which was not however considered to be an anomaly in [**?**, **?**].

A further prediction is that the identities $P(i,i) + P(i+1,i) = 1/2$ should still hold true since one has $P_{i,i} + P_{i,i+1} = (A-B)/2 + B = 1$. This is implied also by probability conservation. The four curves corresponding to these identities do not however co-incide as the figure 6 of [**?**] demonstrates. This is regarded as the basic anomaly in [**?**, **?**]. From the same figure it is also clear that below $\alpha - \beta < 10$ degrees $P_{++} = P_{--}$ $\Delta P_{+-} = -\Delta P_{-+}$ holds true in a reasonable approximation. After that one has also non-vanishing ΔP_{ii} satisfying $\Delta P_{++} = -\Delta P_{--}$. This kind of splittings guarantee the identity $\sum_{ij} P_{ij} = 1$. These splittings are not visible in E.

Since probability conservation requires $P_{ii} + P_{ii+1} = 1$, a mundane explanation for the discrepancy could be that the failure of the conditions $P_{i,i} + P_{ii+1} = 1$ means that the measurement efficiency is too low for P_{+-} and yields too low values of $P_{+-} + P_{--}$ and $P_{+-} + P_{++}$. The constraint $\sum_{ij} P_{ij} = 1$ would then yield too high value for P_{-+}. Similar reduction of measurement efficiency for P_{++} could explain the splitting for $\alpha - \beta > 10$ degrees.

Clearly asymmetry with respect to exchange of photons or of detectors is in question.

1. The asymmetry of two photon state with respect to the exchange of photons could be considered as a source of asymmetry. This would mean that the photons are not maximally entangled. This could be seen as an alternative "mundane" explanation.

2. The assumption that the parameter P is different for the detectors does not change the situation as is easy to check.

3. One manner to achieve splittings which resemble observed splittings is to assume that the value of the probability parameter P depends on the *polarization pair*: $P = P(i,j)$ so that one has $(P(-,+), P(+,-)) = (P+\Delta, P-\Delta)$ and $(P(-,-), P(+,+)) = (P+\Delta_1, P-\Delta_1)$. $\Delta \simeq .025$ and $\Delta_1 \simeq \Delta/2$ could produce the observed splittings qualitatively. One would however always have $P(i,i) + P(i,i+1) \geq 1/2$. Only if the procedure extracting the correlations uses the constraint $\sum_{i,j} P_{ij} = 1$ effectively inducing a constant shift of P_{ij} downwards an asymmetry of observed kind can result. A further objection is that there are no special reason for the values of $P(i,j)$ to satisfy the constraints.

2. Is it possible to say anything about the value of n in the case of EPR-Bohm experiment?

To explain the reduction of the maximum magnitudes of the correlation E from 1 to $\sim .9$ in the experiment discussed above one should have $p_1 \simeq .9$. It is interesting to look whether this allows to deduce any information about the valued of n. At the limit of large values of $N_i n$ one would have $(N_1 - N_2)/(N_1 + N_2) \simeq .4$ so that one cannot say anything about n in this case. $(N_1, N_2) = (3,1)$ satisfies the condition exactly. For $n = 3$, the smallest possible value of n, this would give $p_1 \simeq .88$ and for $n = 4$ $p_1 = .41$. With high enough precision it might be possible to select between $n = 3$ and $n = 4$ options if small values of N_i are accepted.

1.9.8 Category theoretic formulation for quantum measurement theory with finite measurement resolution?

I have been trying to understand whether category theory might provide some deeper understanding about quantum TGD, not just as a powerful organizer of fuzzy thoughts but also as a tool providing genuine physical insights. Marni Dee Sheppeard (or Kea in her blog Arcadian Functor at http://kea-monad.blogspot.com/) is also interested in categories but in much more technical sense. Her dream is to find a category theoretical formulation of M-theory as something, which is not the 11-D something making me rather unhappy as a physicist with second foot still deep in the muds of low energy phenomenology.

Locales, frames, Sierpinski topologies and Sierpinski space

The ideas below popped up when Kea mentioned in M-theory lesson 51 the notions of locale and frame [?]. In Wikipedia I learned that complete Heyting algebras, which are fundamental to category theory, are objects of three categories with differing arrows. CHey, Loc and its opposite category Frm (arrows reversed). Complete Heyting algebras are partially ordered sets which are complete lattices. Besides the basic logical operations there is also algebra multiplication (I have considered the possible role of categories and Heyting algebras in TGD in [?]). From Wikipedia I also learned that locales and the dual notion of frames form the foundation of pointless topology [?]. These topologies are important in topos theory which does not assume axiom of choice.

The so called particular point topology [?] assumes a selection of single point but I have the physicist's feeling that it is otherwise rather near to pointless topology. Sierpinski topology [?] is this kind of topology. Sierpinski topology is defined in a simple manner: the set is open only if it contains a given preferred point p. The dual of this topology defined in the obvious sense exists also. Sierpinski space consisting of just two points 0 and 1 is the universal building block of these topologies in the sense that a map of an arbitrary space to Sierpinski space provides it with Sierpinski topology as the induced topology. In category theoretical terms Sierpinski space is the initial object in the category of frames and terminal object in the dual category of locales. This category theoretic reductionism looks highly attractive.

Particular point topologies, their generalization, and number theoretical braids

Pointless, or rather particular point topologies might be very interesting from physicist's point of view. After all, every classical physical measurement has a finite space-time resolution. In TGD framework discretization by number theoretic braids replaces partonic 2-surface with a discrete set consisting of algebraic points in some extension of rationals: this brings in mind something which might be called a topology with a set of particular algebraic points. Could this preferred set belongs to any open set in the particular point topology appropriate in this situation?

Perhaps the physical variant for the axiom of choice could be restricted so that only sets of algebraic points in some extension of rationals can be chosen freely and the choices is defined by the intersection of p-adic and real partonic 2-surfaces and in the framework of TGD inspired theory of consciousness would thus involve the interaction of cognition and intentionality with the material world. The extension would depend on the position of the physical system in the algebraic evolutionary hierarchy defining also a cognitive hierarchy. Certainly this would fit very nicely to the formulation of quantum TGD unifying real and p-adic physics by gluing real and p-adic number fields to single super-structure via common algebraic points.

Analogs of particular point topologies at the level of state space: finite measurement resolution

There is also a finite measurement resolution in Hilbert space sense not taken into account in the standard quantum measurement theory based on factors of type I. In TGD framework one indeed introduces quantum measurement theory with a finite measurement resolution so that complex rays become included hyper-finite factors of type II_1 (HFFs).

1. Could topology with particular algebraic points have a generalization allowing a category theoretic formulation of the quantum measurement theory without states identified as complex rays?

2. How to achieve this? In the transition of ordinary Boolean logic to quantum logic in the old fashioned sense (von Neuman again!) the set of subsets is replaced with the set of subspaces of Hilbert space. Perhaps this transition has a counterpart as a transition from Sierpinski topology to a structure in which sub-spaces of Hilbert space are quantum sub-spaces with complex rays replaced with the orbits of subalgebra defining the measurement resolution. Sierpinski space {0,1} would in this generalization be replaced with the quantum counterpart of the space of 2-spinors. Perhaps one should also introduce q-category theory with Heyting algebra being replaced with q-quantum logic.

Fuzzy quantum logic as counterpart for Sierpinksi space

The program formulated above might indeed make sense. The lucky association induced by Kea's blog was to the ideas about fuzzy quantum logic realized in terms of quantum 2-spinor that I had developed a couple of years ago. Fuzzy quantum logic would reflect the finite measurement resolution. I just list the pieces of the argument.

Spinors and qbits: Spinors define a quantal variant of Boolean statements, qbits. One can however go further and define the notion of quantum qbit, qqbit. I indeed did this for couple of years ago (the last section of this chapter).

Q-spinors and qqbits: For q-spinors the two components a and b are not commuting numbers but non-Hermitian operators: $ab = qba$, q a root of unity. This means that one cannot measure both a and b simultaneously, only either of them. $aa^\dagger$ and $bb^\dagger$ however commute so that probabilities for bits 1 and 0 can be measured simultaneously. State function reduction is not possible to a state in which a or b gives zero. The interpretation is that one has q-logic is inherently fuzzy: there are no absolute truths or falsehoods. One can actually predict the spectrum of eigenvalues of probabilities for say 1. Obviously quantum spinors would be state space counterparts of Sierpinski space and for $q \neq 1$ the choice of preferred spinor component is very natural. Perhaps this fuzzy quantum logic replaces the logic defined by the Heyting algebra.

Q-locale: Could one think of generalizing the notion of locale to quantum locale by using the idea that sets are replaced by sub-spaces of Hilbert space in the conventional quantum logic. Q-openness would be defined by identifying quantum spinors as the initial object, q-Sierpinski space. a (resp. b for the dual category) would define q-open set in this space. Q-open sets for other quantum spaces would be defined as inverse images of a (resp. b) for morphisms to this space. Only for q=1 one could have the q-counterpart of rather uninteresting topology in which all sets are open and every map is continuous.

Q-locale and HFFs: The q-Sierpinski character of q-spinors would conform with the very special role of Clifford algebra in the theory of HFFs, in particular, the special role of Jones inclusions to which one can assign spinor representations of $SU(2)$. The Clifford algebra and spinors of the world of classical worlds identifiable as Fock space of quark and lepton spinors is the fundamental example in which 2-spinors and corresponding Clifford algebra serves as basic building brick although tensor powers of any matrix algebra provides a representation of HFF.

Q-measurement theory: Finite measurement resolution (q-quantum measurement theory) means that complex rays are replaced by sub-algebra rays. This would force the Jones inclusions associated with $SU(2)$ spinor representation and would be characterized by quantum phase q and bring in the q-topology and q-spinors. Fuzzyness of qqbits of course correlates with the finite measurement resolution.

Q-n-logos: For other q-representations of SU(2) and for representations of compact groups (Appendix) one would obtain something which might have something to do with quantum n-logos, quantum generalization of n-valued logic. All of these would be however less fundamental and induced by q-morphisms to the fundamental representation in terms of spinors of the world of classical worlds. What would be however very nice that if these q-morphisms are constructible explicitly it would become possible to build up q-representations of various groups using the fundamental physical realization - and as I have conjectured [**?**] - McKay correspondence and huge variety of its generalizations would emerge in this manner.

The analogs of Sierpinski spaces: The discrete subgroups of $SU(2)$, and quite generally, the groups Z_n associated with Jones inclusions and leaving the choice of quantization axes invariant, bring in mind the n-point analogs of Sierpinski space with unit element defining the particular point. Note however that $n \geq 3$ holds true always so that one does not obtain Sierpinski space itself. If all these n preferred points belong to any open set it would not be possible to decompose this preferred set to two subsets belonging to disjoint open sets. Recall that the generalized imbedding space related to the quantization of Planck constant is obtained by gluing together coverings $M^4 \times CP_2 \rightarrow M^4 \times CP_2/G_a \times G_b$ along their common points of base spaces. The topology in question would mean that if some point in the covering belongs to an open set, all of them do so. The interpretation would be that the points of fiber form a single inseparable quantal unit.

Number theoretical braids identified as as subsets of the intersection of real and p-adic variants of algebraic partonic 2-surface define a second candidate for the generalized Sierpinski space with a set of preferred points.

1.10 Appendix: Inclusions of hyper-finite factors of type II_1

Many names have been assigned to inclusions: Jones, Wenzl, Ocneacnu, Pimsner-Popa, Wasserman [?]. It would seem to me that the notion Jones inclusion includes them all so that various names would correspond to different concrete realizations of the inclusions conjugate under outer automorphisms.

1. According to [?] for inclusions with $\mathcal{M} : \mathcal{N} \leq 4$ (with $A_1^{(1)}$ excluded) there exists a countable infinity of sub-factors with are pairwise non inner conjugate but conjugate to $\mathcal{N}$.

2. Also for any finite group G and its outer action there exists uncountably many sub-factors which are pairwise non inner conjugate but conjugate to the fixed point algebra of G [?]. For any amenable group G the the inclusion is also unique apart from outer automorphism [?].

Thus it seems that not only Jones inclusions but also more general inclusions are unique apart from outer automorphism.

Any *-endomorphism σ, which is unit preserving, faithful, and weakly continuous, defines a sub-factor of type II_1 factor [?]. The construction of Jones leads to a atandard inclusion sequence $\mathcal{N} \subset \mathcal{M} \subset \mathcal{M}^1 \subset ....$ This sequence means addition of projectors e_i, $i < 0$, having visualization as an addition of braid strand in braid picture. This hierarchy exists for all factors of type II. At the limit $\mathcal{M}^\infty = \cup_i \mathcal{M}^i$ the braid sequence extends from $-\infty$ to ∞. Inclusion hierarchy can be understood as a hierarchy of Connes tensor powers $\mathcal{M} \otimes_\mathcal{N} \mathcal{M} .... \otimes_\mathcal{N} \mathcal{M}$. Also the ordinary tensor powers of hyper-finite factors of type II_1 (HFF) as well as their tensor products with finite-dimensional matrix algebras are isomorphic to the original HFF so that these objects share the magic of fractals.

Under certain assumptions the hierarchy can be continued also in opposite direction. For a finite index an infinite inclusion hierarchy of factors results with the same value of index. σ is said to be basic if it can be extended to *-endomorphisms from $\mathcal{M}^1$ to $\mathcal{M}$. This means that the hierarchy of inclusions can be continued in the opposite direction: this means elimination of strands in the braid picture. For finite factors (as opposed to hyper-finite ones) there are no basic *-endomorphisms of $\mathcal{M}$ having fixed point algebra of non-abelian G as a sub-factor [?].

1.10.1 Jones inclusions

For hyper-finite factors of type II_1 Jones inclusions allow basic *-endomorphism. They exist for all values of $\mathcal{M} : \mathcal{N} = r$ with $r \in \{4cos^2(\pi/n) | n \geq 3\} \cap [4, \infty)$ [?]. They are defined for an algebra defined by projectors e_i, $i \geq 1$. All but nearest neighbor projectors commute. $\lambda = 1/r$ appears in the relations for the generators of the algebra given by $e_i e_j e_i = \lambda e_i$, $|i - j| = 1$. $\mathcal{N} \subset \mathcal{M}$ is identified as the double commutator of algebra generated by e_i, $i \geq 2$.

This means that principal graph and its dual are equivalent and the braid defined by projectors can be continued not only to $-\infty$ but that also the dropping of arbitrary number of strands is possible [?]. It would seem that ADE property of the principal graph meaning single root length codes for the duality in the case of $r \leq 4$ inclusions.

Irreducibility holds true for $r < 4$ in the sense that the intersection of $Q' \cap P = P' \cap P = C$. For $r \geq 4$ one has $dim(Q' \cap P) = 2$. The operators commuting with Q contain besides identify operator of Q also the identify operator of P. Q would contain a single finite-dimensional matrix factor less than P in this case. Basic *-endomorphisms with $\sigma(P) = Q$ is $\sigma(e_i) = e_{i+1}$. The difference between genuine symmetries of quantum TGD and symmetries which can be mimicked by TGD could relate to the irreducibility for $r < 4$ and raise these inclusions in a unique position. This difference could partially justify the hypothesis [?] that only the groups $G_a \times G_b \subset SU(2) \times SU(2) \subset SL(2,C) \times SU(3)$ define orbifold coverings of $H_\pm = M^4_\pm \times CP_2 \rightarrow H_\pm/G_a \times G_b$.

1.10.2 Wasserman's inclusion

Wasserman's construction of $r = 4$ factors clarifies the role of the subgroup of $G \subset SU(2)$ for these inclusions. Also now $r = 4$ inclusion is characterized by a discrete subgroup $G \subset SU(2)$ and is given by $(1 \otimes \mathcal{M})^G \subset (M_2(C) \times \mathcal{M})^G$. According to [?] Jones inclusions are irreducible also for $r = 4$. The definition of Wasserman inclusion for $r = 4$ seems however to imply that the identity matrices of both $\mathcal{M}^G$ and $(M(2,C) \otimes \mathcal{M})^G$ commute with $\mathcal{M}^G$ so that the inclusion should be reducible for $r = 4$.

Note that G leaves both the elements of $\mathcal{N}$ and $\mathcal{M}$ invariant whereas $SU(2)$ leaves the elements of $\mathcal{N}$ invariant. $M(2,C)$ is effectively replaced with the orbifold $M(2,C)/G$, with G acting as automoprhisms. The space of these orbits has complex dimension $d = 4$ for finite G.

For $r < 4$ inclusion is defined as $M^G \subset M$. The representation of G as outer automorphism must change step by step in the inclusion sequence $... \subset \mathcal{N} \subset \mathcal{M} \subset ...$ since otherwise G would act trivially as one proceeds in the inclusion sequence. This is true since each step brings in additional finite-dimensional tensor factor in which G acts as automorphisms so that although $\mathcal{M}$ can be invariant under $G_\mathcal{M}$ it is not invariant under $G_\mathcal{N}$.

These two inclusions might accompany each other in TGD based physics. One could consider $r < 4$ inclusion $\mathcal{N} = \mathcal{M}^G \subset \mathcal{M}$ with G acting non-trivially in $\mathcal{M}/\mathcal{N}$ quantum Clifford algebra. $\mathcal{N}$ would decompose by $r = 4$ inclusion to $\mathcal{N}_1 \subset \mathcal{N}$ with $SU(2)$ taking the role of G. $\mathcal{N}/\mathcal{N}_1$ quantum Clifford algebra would transform non-trivially under $SU(2)$ but would be G singlet.

In TGD framework the G-invariance for $SU(2)$ representations means a reduction of S^2 to the orbifold S^2/G. The coverings $H_\pm \rightarrow H_\pm/G_a \times G_b$ should relate to these double inclusions and $SU(2)$ inclusion could mean Kac-Moody type gauge symmetry for $\mathcal{N}$. Note that the presence of the factor containing only unit matrix should relate directly to the generator d in the generator set of affine algebra in the McKay construction [?]. The physical interpretation of the fact that almost all ADE type extended diagrams ($D_n^{(1)}$ must have $n \geq 4$) are allowed for $r = 4$ inclusions whereas D_{2n+1} and E_6 are not allowed for $r < 4$, remains open.

1.10.3 Generalization from $SU(2)$ to arbitrary compact group

The inclusions with index $\mathcal{M} : \mathcal{N} < 4$ have one-dimensional relative commutant $\mathcal{N}' \cup \mathcal{M}$. The most obvious conjecture that $\mathcal{M} : \mathcal{N} \geq 4$ corresponds to a non-trivial relative commutant is wrong. The index for Jones inclusion is identifiable as the square of quantum dimension of the fundamental representation of $SU(2)$. This identification generalizes to an arbitrary representation of arbitrary compact Lie group.

In his thesis Wenzl [?] studied the representations of Hecke algebras $H_n(q)$ of type A_n obtained from the defining relations of symmetric group by the replacement $e_i^2 = (q-1)e_i + q$. H_n is isomorphic to complex group algebra of S_n if q is not a root of unity and for $q = 1$ the irreducible representations of $H_n(q)$ reduce trivially to Young's representations of symmetric groups. For primitive roots of unity $q = exp(i2\pi/l)$, $l = 4, 5...$, the representations of $H_n(\infty)$ give rise to inclusions for which index corresponds to a quantum dimension of any irreducible representation of $SU(k)$, $k \geq 2$. For $SU(2)$ also the value $l = 3$ is allowed for spin 1/2 representation.

The inclusions are obtained by dropping the first m generators e_k from $H_\infty(q)$ and taking double commutant of both H_∞ and the resulting algebra. The relative commutant corresponds to $H_m(q)$. By reducing by the minimal projection to relative commutant one obtains an inclusion with a trivial relative commutant. These inclusions are analogous to a discrete states superposed in continuum. Thus the results of Jones generalize from the fundamental representation of $SU(2)$ to all representations of all groups $SU(k)$, and in fact to those of general compact groups as it turns out.

The generalization of the formula for index to square of quantum dimension of an irreducible representation of $SU(k)$ reads as

$$\mathcal{M} : \mathcal{N} = \prod_{1 \leq r < s \leq k} \frac{sin^2\left((\lambda_r - \lambda_s + s - r)\pi/l\right)}{sin^2\left((s-r)n/l\right)} \quad . \tag{1.10.1}$$

Here λ_r is the number of boxes in the r^{th} row of the Yang diagram with n boxes characterizing the representations and the condition $1 \leq k \leq l-1$ holds true. Only Young diagrams satisfying the condition $l-k = \lambda_1 - \lambda_{r_{max}}$ are allowed.

The result would allow to restrict the generalization of the imbedding space in such a manner that only cyclic group Z_n appears in the covering of $M^4 \rightarrow M^4/G_a$ or $CP_2 \rightarrow CP_2/G_b$ factor. Be as it may, it seems that quantum representations of any compact Lie group can be realized using the generalization of the imbedding space. In the case of SU(2) the interpretation of higher-dimensional quantum representations in terms of Connes tensor products of 2-dimensional fundamental representations is highly suggestive.

The groups $SO(3,1) \times SU(3)$ and $SL(2,C) \times U(2)_{ew}$ have a distinguished position both in physics and quantum TGD and the vision about physics as a generalized number theory implies them. Also the general pattern for inclusions selects these groups, and one can say that the condition that all possible statistics are realized is guaranteed by the choice $M^4 \times CP_2$.

1. $n > 2$ for the quantum counterparts of the fundamental representation of $SU(2)$ means that braid statistics for Jones inclusions cannot give the usual fermionic statistics. That Fermi statistics cannot "emerge" conforms with the role of infinite-D Clifford algebra as a canonical representation of HFF of type II_1. $SO(3,1)$ as isometries of H gives Z_2 statistics via the action on spinors of M^4 and $U(2)$ holonomies for CP_2 realize Z_2 statistics in CP_2 degrees of freedom.

2. $n > 3$ for more general inclusions in turn excludes Z_3 statistics as braid statistics in the general case. $SU(3)$ as isometries induces a non-trivial Z_3 action on quark spinors but trivial action at the imbedding space level so that Z_3 statistics would be in question.

Bibliography

Online books about TGD

[1] M. Pitkänen (2006), *Topological Geometrodynamics: Overview.* http://tgd.wippiespace.com/public_html/tgdview/tgdview.html.

[2] M. Pitkänen (2006), *Quantum Physics as Infinite-Dimensional Geometry.* http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html.

[3] M. Pitkänen (2006), *Physics in Many-Sheeted Space-Time.* http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html.

[4] M. Pitkänen (2006), *Quantum TGD.* http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html.

[5] M. Pitkänen (2006), *TGD as a Generalized Number Theory.* http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html.

[6] M. Pitkänen (2006), *p-Adic length Scale Hypothesis and Dark Matter Hierarchy.* http://tgd.wippiespace.com/public_html/paddark/paddark.html.

[7] M. Pitkänen (2006), *TGD and Fringe Physics.* http://tgd.wippiespace.com/public_html/freenergy/freenergy.html.

Online books about TGD inspired theory of consciousness and quantum biology

[8] M. Pitkänen (2006), *Bio-Systems as Self-Organizing Quantum Systems.* http://tgd.wippiespace.com/public_html/bioselforg/bioselforg.html.

[9] M. Pitkänen (2006), *Quantum Hardware of Living Matter.* http://tgd.wippiespace.com/public_html/bioware/bioware.html.

[10] M. Pitkänen (2006), *TGD Inspired Theory of Consciousness.* http://tgd.wippiespace.com/public_html/tgdconsc/tgdconsc.html.

[11] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory.* http://tgd.wippiespace.com/public_html/genememe/genememe.html.

[12] M. Pitkänen (2006), *TGD and EEG.* http://tgd.wippiespace.com/public_html/tgdeeg/tgdeeg/tgdeeg.html.

[13] M. Pitkänen (2006), *Bio-Systems as Conscious Holograms.* http://tgd.wippiespace.com/public_html/hologram/hologram.html.

[14] M. Pitkänen (2006), *Magnetospheric Consciousness.* http://tgd.wippiespace.com/public_html/magnconsc/magnconsc.html.

[15] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory.* http://tgd.wippiespace.com/public_html/magnconsc/mathconsc.html.

References to the chapters of books

[A1] The chapter *An Overview about the Evolution of Quantum TGD* of [?].
http://tgd.wippiespace.com/public_html/tgdview/tgdview.html#evoI.

[A2] The chapter *An Overview about Quantum TGD* of [?].
http://tgd.wippiespace.com/public_html/tgdview/tgdview.html#evoII.

[B2] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part I* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#compl1.

[B3] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part II* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#compl2.

[B4] The chapter *Configuration Space Spinor Structure* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#cspin.

[C2] The chapter *Construction of Quantum Theory: Symmetries* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#quthe.

[C3] The chapter *Construction of Quantum Theory: S-matrix* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#towards.

[C4] The chapter *Category Theory and Quantum TGD* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#categorynew.

[C5] The chapter *Earlier Attempts to Construct S-matrix* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#smatrix.

[C9] The chapter *Does TGD Predict the Spectrum of Planck Constants?* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#Planck.

[D1] The chapter *Basic Extremals of Kähler Action* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#class.

[D3] The chapter *The Relationship Between TGD and GRT* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#tgdgrt.

[D4] The chapter *Cosmic Strings* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#cstrings.

[D5] The chapter *TGD and Cosmology* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#cosmo.

[D6] The chapter *TGD and Astrophysics* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#astro.

[E1] The chapter *TGD as a Generalized Number Theory: p-Adicization Program* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visiona.

[E2] The chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visionb.

[E3] The chapter *TGD as a Generalized Number Theory: Infinite Primes* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visionc.

[E7] The chapter *Category Theory, Quantum TGD, and TGD Inspired Theory of Consciousness* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#categoryc.

[E8] The chapter *Riemann Hypothesis and Physics* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#riema.

[E9] The chapter *Topological Quantum Computation in TGD Universe* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#tqc.

[F2] The chapter *Massless States and Particle Massivation* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#mless.

[F3] The chapter *p-Adic Particle Massivation: Hadron Masses* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#padmass2.

[F4] The chapter *p-Adic Particle Massivation: Hadron Masses* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#padmass3.

[F5] The chapter *p-Adic Particle Massivation: New Physics* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#padmass4.

[F9] The chapter *Dark Nuclear Physics and Living Matter* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#exonuclear.

[H1] The chapter *Matter, Mind, Quantum* of [?].
http://tgd.wippiespace.com/public_html/tgdconsc/tgdconsc.html#conscic.

[H8] The chapter *p-Adic Physics as Physics of Cognition and Intention* of [?].
http://tgd.wippiespace.com/public_html/tgdconsc/tgdconsc.html#cognic.

[J1] The chapter *Bio-Systems as Super-Conductors: part I* of [?].
http://tgd.wippiespace.com/public_html/bioware/bioware.html#superc1.

[J2] The chapter *Bio-Systems as Super-Conductors: part II* of [?].
http://tgd.wippiespace.com/public_html/bioware/bioware.html#superc2.

[J3] The chapter *Bio-Systems as Super-Conductors: part III* of [?].
http://tgd.wippiespace.com/public_html/bioware/bioware.html#superc3.

[J6] The chapter *Coherent Dark Matter and Bio-Systems as Macroscopic Quantum Systems* of [?].
http://tgd.wippiespace.com/public_html/bioware/bioware.html#darkbio.

[K1] The chapter *Time, Spacetime and Consciousness* of [?].
http://tgd.wippiespace.com/public_html/hologram/hologram.html#time.

[M3] The chapter *Dark Matter Hierarchy and Hierarchy of EEGs* of [?].
http://tgd.wippiespace.com/public_html/tgdeeg/tgdeeg/tgdeeg.html#eegdark.

Mathematics related references

[16] F. J. Murray, J. von Neumann (1936), *On Rings of Operators*, Ann. Math. 37,116-229.
F. J. Murray, J. von Neumann (1937), *On Rings of Operators II*, Trans. Am. Math. Soc. 41, 208-248.
F. J. Murray, J. von Neumann (1940), *On Rings of Operators III*, Ann. Math. 41,94-161.
F. J. Murray, J. von Neumann (1943), *On Rings of Operators IV*, Ann. Math. 44,716-808.

[17] J. von Neumann (1937), *Quantum Mechanics of Infinite Systems*, first published in (Redei and Stöltzner 2001, 249-268). [A mimeographed version of a lecture given at Pauli's seminar held at the Institute for Advanced Study in 1937, John von Neumann Archive, Library of Congress, Washington, D.C.]

[18] M. Redei and M. Stöltzner (eds) (2001), *John von Neumann and the Foundations of Quantum Physics*, Vol. 8, Dordrecht: Kluwer Academic Publishers. V. S. Sunder (1987), *An Invitation to von Neumann Algebras*, New York: Springer-Verlag.

[19] J. Dixmier (1981), *Von Neumann Algebras*, Amsterdam: North-Holland Publishing Company. [First published in French in 1957: Les Algebres d'Operateurs dans l'Espace Hilbertien, Paris: Gauthier-Villars.]

[20] R. Longo (2004), *Operators algebras and Index Theorems in Quantum Field Theory*, Andrejevski lectures in Göttingen 2004 (lecture notes).

[21] V. F. R. Jones (1983), *Braid groups, Hecke algebras and type II_1 factors*, Geometric methods in operator algebras, Proc. of the US-Japan Seminar, Kyoto, July 1983.

[22] V. F. Jones(1980), *Actions of finite groups on hyperfinite factor type II_1*. Memoirs of the American Mathematical Society, no 237, volume 28.

[23] A. Wassermann (1998), *Operator algebras and conformal field theory. III. Fusion of positive energy representations of LSU(N) using bounded operators*. Invent. Math. 133, no. 3, 467-538.

[24] P. A. M. Dirac (1939), *A New Notation for Quantum Mechanics*, Proceedings of the Cambridge Philosophical Society, 35: 416-418.
J. E. Roberts (1966), *The Dirac Bra and Ket Formalism*, Journal of Mathematical Physics, 7: 1097-1104. Halvorson, Hans and Clifton, Rob (2001),

[25] L. Schwartz (1945),*Generalisation de la Notion de Fonction, de Derivation, de Transformation de Fourier et Applications Mathematiques et Physiques*, Annales de l'Universite de Grenoble, 21: 57-74. 1951-1952,
L. Scwartz (1951-1952), *Theorie des Distributions*, Publications de l'Institut de Mathematique de l'Universite de Strasbourg, Vols 9-10, Paris: Hermann.

[26] I. Gelfand and M. Neumark (1943), *On the Imbedding of Normed Rings into the Ring of Operators in Hilbert Space*, Recueil Mathematique [Matematicheskii Sbornik] Nouvelle Serie, 12 [54]: 197-213. [Reprinted in C^*-algebras: 1943-1993, in the series Contemporary Mathematics, 167, Providence, R.I. : American Mathematical Society, 1994.]
I. Gelfand and N. Ya. Vilenkin (1964), *Generalized Functions*, Volume 4, New York: Academic Press. [First published in Russian in 1961.]

[27] R. P. Feynman (1948), *Space-Time Approach to Non-Relativistic Quantum Mechanics*, Reviews of Modern Physics, 20: 367387. [It is reprinted in (Schwinger 1958).]

[28] R. J. Rivers (1987), *Path Integral Methods in Quantum Field Theory*, Cambridge: Cambridge University Press.

[29] C. N. Yang, M. L. Ge (1989), *Braid Group, Knot Theory, and Statistical Mechanics*, World Scientific.
V. F. R. Jones, *Hecke algebra representations of braid groups and link polynomial*, Ann. Math., 126(1987), 335-388.

[30] C. Kassel (1995), *Quantum Groups*, Springer Verlag.

[31] C. Gomez, M. Ruiz-Altaba, G. Sierra (1996), *Quantum Groups and Two-Dimensional Physics*, Cambridge University Press.

[32] E. Witten 1989), *Quantum field theory and the Jones polynomial*, Comm. Math. Phys. 121 , 351-399.

[33] S. Sawin (1995), *Links, Quantum Groups, and TQFT's*, q-alg/9506002.

[34] H. Saleur (1990), *Zeroes of chromatic polynomials: a new approach to the Beraha conjecture using quantum groups*, Comm. Math. Phys. 132, 657.

[35] M. Takesaki (1970), *Tomita's Theory of Modular Hilbert Algebras and Its Applications*,Lecture Notes in Mathematics 128, Springer, Berlin.

[36] R. Haag and D. Kastler (1964), *An Algebraic Approach to Quantum Field Theory*, Journal of Mathematical Physics, 5: 848-861.

[37] B. Schroer (2001), *Lectures on Algebraic Quantum Field Theory and Operator Algebras*, `http://xxx.lanl.gov/abs/math-ph/0102018`.

[38] H. J. Borchers (2000), *On Revolutionizing QFT with Tomita's Modular Theory* `http://www.lqp.uni-goettingen.de/papers/99/04/99042900.html`. J. Math. Phys. 41 (2000) 3604-3673.

[39] A. Connes (1973), *Une classification des facteurs de type III*, Ann. Sci. Ecole Norm. Sup. 6, 133-252.

[40] A. Connes (1994), *Non-commutative Geometry*, San Diego: Academic Press.

[41] Y. Ito and I. Nakamura(1996) *Hilbert schemes and simple singularities*, Proc. of EuroConference on Algebraic Geometry, Warwick 1996, ed. by K. Hulek et al., CUP, (1999),pp. 151–233, `http://www.math.sci.hokudai.ac.jp/~nakamura/ADEHilb.pdf`.

[42] V. F. R. Jones (1983), *Index for Subfactors*, Invent. Math. (72),1-25.

[43] F. M. Goodman, P. de la Harpe and V. F. R. Jones (1989), *Coxeter graphs and towers of algebras*, Springer-Verlag.

[44] V. F. R. Jones(2000), *The planar algebra of a bipartite graph*, in "Knots in Hellas′98", World Sci. Publishing (2000), 94-117.

[45] N. H. V. Temperley and E. H. Lieb (1971), *Relations between the percolation and colouring problem and other graph-theoretical problems associated with regular planar lattices:some exact results for the percolation problem*, Proc. Roy. Soc. London 322 (1971), 251-280.

[46] O. Bratteli and D. W. Robinson (1979-1981), *Operator Algebras and Quantum Statistical Mechanics*, Volumes 1-2, New York: Springer-Verlag.

[47] V. Jones (2003), *In and around the origin of quantum groups*, arXiv:math.OA/0309199.

[48] F. Wilzek (1990), *Fractional Statistics and Anyon Super-Conductivity*, World Scientific.
R. B. Laughlin (1990), Phys. Rev. Lett. 50, 1395.

[49] R. B. Laughlin (1990), Phys. Rev. Lett. 50, 1395.

[50] A. Connes and D. Kreimer (1998), *Hofp algebras, renormalization, and non-commutative geometry*, Quantum Field Theory; Perspective and Prospective (C. DeWitt-Morette and J.-B.- Zueber, eds.), Kluwer, Dortrecth, 1999, 59-109. CMP 2000:05, Commun. Math. Phys. 199, 203.242(1998). MR 99h:81137.
Ibid (1999), *Renormalization in quantum field theory and the Riemann-Hilbert problem I: the Hopf algebra structure of graphs and the main theorem*, arXiv:hep-th/9912092.
Ibid (2000), *Renormalization in quantum field theory and the Riemann-Hilbert problem II: the β function, diffeomorphisms and the renormalization group*, arXiv:hep-th/0003188.

[51] A. Connes (2005), *Quantum Fields and Motives*, hep-th/0504085.

[52] C. N. Yang, M. L. Ge (1989), *Braid Group, Knot Theory, and Statistical Mechanics*, World Scientific.

[53] K. Appel and W.Haken (1989), *Every Planar Map is Four Colorable*, Contemporary Mathematics (American Mathematical Society), v. 98.

[54] R. Haag (1992), *Local Quantum Physics*, Springer, Berlin

[55] M. Rainer (2000), *Algebraic Quantum Field Theory on Manifolds: A Haag-Kastler Setting for Quantum Geometry*, gr-qc/9911076.

[56] S. Doplicher, R. Haag, J. E. Roberts (1971-1974), *Local Observables and Particle Statistics I and II.* Commun. Math. Phys. 23 and 35.
S. Doplicher, J. E. Roberts (1989), Ann. Math. 130, 75-119.
S. Doplicher, J. E. Roberts (1989), Invent. Math. 98, 157-218.

[57] H. P. Halvorson (2001), *Locality, Localization, and the Particle Concept: Topics in the Foundations of Quantum Field Theory*, Ph. D Dissertation. `http://philsci-archive.pitt.edu/archive/00000346/00/main-new.pdf`.

[58] M. Chown (2004), *Quantum Rebel*, popular article about the experiment of Shahriar Afshar. New Scientist, Vol 183, No 2457.

[59] L.P. Horwitz (1996), *Hypercomplex quantum mechanics*, quant-ph/9602001.

[60] S. De Leo and K. Abdel-Khalek (1996), *Octonionic quantum mechanics and complex geometry*, Prog.Theor.Phys. 96 (1996) 823-832, hep-th/9609032.

[61] S. Okubo (1997), *Angular momentum, quaternion, octonion, and Lie-super algebra* $Osp(1,2)$, ph/9710038.

[62] M. Brenner (1999), *Quantum octonions*, Communications in Algebra 27 (1999), 2809-2831. `http://math.usask.ca/~bremner/research/publications/qo.pdf`.

[63] G. Benkart, J. M. Perez-Izquierdo, *A quantum octonion algebra*, math.QA/9801141.

[64] John C. Baez (2001), *The Octonions*, Bull. Amer. Math. Soc. 39 (2002), 145-205. `http://math.ucr.edu/home/baez/Octonions/octonions.html`.

[65] *The 15 most famous transcendental numbers*, `http://sprott.physics.wisc.edu/pickover/trans.html`.

[66] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, A. Zeilinger (1998), Phys. Rev. Lett., 81, 23, p. 5039.

[67] D. Evans and Y. Kawahigashi(1998), *Quantum symmetries on operator algebras*, Oxford University Press, New York. The review of the book can be found in Bulletin of the American Mathematical Society, Vol. 28, Number 3, Pages 369-377. `http://www.ams.org/bull/2001-38-03/S0273-0979-01-00906-5/S0273-0979-01-00906-5.pdf`.

[68] M. Choda (1996), *Conjugate but non inner conjugate subfactors*, Proceedings of the American Mathematical Society, Vol. 124, Number 1, January. `http://links.jstor.org/sici?sici=0002-9939(199601)124%3A1%3C147%3ACBNICS%3E2.0.CO%3B2-0`.

[69] H. Wenzl (1988), *Hecke algebra of type* A_n *and subfactors*, Invent. math. 92, 349-383.

[70] V. Chari and A. Pressley (1994), *A Guide to Quantum Groups*, Cambridge University Press.

[71] *Planar algebra*, `http://en.wikipedia.org/wiki/Planar_algebra`.

[72] D. Bisch (2003), *Subfactors and Planar Algebras*, arXiv:math: OA/0304340.

[73] *Langlands program*,`http://en.wikipedia.org/wiki/Langlands_program`.

[74] *Jordan algebra*, `http://en.wikipedia.org/wiki/Jordan_algebra`.

[75] *Frames and locales*, `http://en.wikipedia.org/wiki/Frames_and_locales`

[76] *Pointless topology*, `http://en.wikipedia.org/wiki/Pointless_topology`.

[77] *Particular point topology*, `http://en.wikipedia.org/wiki/Particular_point_topology`.

[78] *Sierpinski space*, `http://en.wikipedia.org/wiki/Sierpinski_space`

Quantum computation

[79] M. Freedman, H. Larsen, and Z. Wang (2002), *A modular functor which is universal for quantum computation*, Found. Comput. Math. 1, no 2, 183-204. Comm. Math. Phys. 227, no 3, 605-622. quant-ph/0001108.
M. H. Freedman (2001), *Quantum Computation and the localization of Modular Functors*, Found. Comput. Math. 1, no 2, 183-204.
M. H. Freedman (1998), *P/NP, and the quantum field computer*, Proc. Natl. Acad. Sci. USA 95, no. 1, 98-101.
A. Kitaev (1997), Annals of Physics, vol 303, p.2. See also *Fault tolerant quantum computation by anyons*, quant-ph/9707021.
L. H. Kauffmann and S, J. Lomonaco Jr. (2004), *Braiding operations are universal quantum gates*, arxiv.org/quant-ph/0401090.
Paul Parsons (2004) , *Dancing the Quantum Dream*, New Scientist 24. January. www.newscientist.com/hottopics.

[80] John Baez (2007), *This Weeks Finds (Week 259)*, `http://math.ucr.edu/home/baez/week259.html`.

Physics related references

[81] J. K. Webb *et al* (2001), *Further Evidence for Cosmological Evolution of the Fine Structure Constant*, arXiv:astro-ph/0012539.
T. Chiva and K. Kohri (2001), *Quintessence Cosmology and Varying* α, arXiv:hep-ph/0111086.

[82] J. K. Webb, M. T. Murphy, V. V. Flambaum, V. A. Dzuba, J. D. Barrow, C. W. Churchill, J. X. Prochaska and A. M. Wolfe, *Further Evidence for Cosmological Evolution of the Fine Structure Constant*, Phys. Rev. Lett., Vol. 87, No. 9, Paper No. 091301; August 27, 2001. arxiv.org/abs/astro-ph/0012539.

[83] J. D. Barrow and J. K. Webb (2005), *Inconstant Constants: Do the inner workings of nature change with time?*. Scientific American, June issue. `http://www.sciam.com/article.cfm?chanID=sa006&colID=1&articleID=0005BFE6-2965-128A-A96583414B7F0000`.

[84] R. Srianand, H. Chand, P. Petitjean and B. Aracil (2004), *Limits on the Time Variation of the Electromagnetic Fine-Structure Constant in the Low Energy Limit from Absorption Lines in the Spectra of Distant Quasars*, Phys. Rev. Lett., Vol. 92, Paper No. 121302; March 26, 2004. astro-ph/0402177.

[85] T. Ludham and L. McLerran (2003), *What Have We Learned From the Relativistic Heavy Ion Collider?*, Physics Today, October issue. `http://www.physicstoday.org/vol-56/iss-10/p48.html`.
E. S. Reich (2005), *Black hole like phenomenon created by collider*, New Scientist 19, issue 2491.

[86] A. Helmi(2000), *Halo streams as relicts from the formation of the Milky Way*, arXiv.org/astro-ph/0008086.

[87] *Weinberg angle*, `http://en.wikipedia.org/wiki/Weinberg_angle`.

References related to anomalies

[88] D. Da Roacha and L. Nottale (2003), *Gravitational Structure Formation in Scale Relativity*, astro-ph/0310036.

[89] G. Adenier and A. Khrennikov (2006), *Anomalies in experimental data for EPR-Bohm experiment: Are both classical and quantum mechanis wrong?*, arXiv: quantum-ph/0607172.

[90] G. Adenier and A. Khrennikov (2006), *Is the Fair Sampling Assumption Supported by EPR Experiments?*, arXiv: quantum-ph/0606122.

Chapter 2

Does TGD Predict a Spectrum of Planck Constants?

2.1 Introduction

The quantization of Planck constant has been the basic them of TGD since 2005 and the perspective in the earlier version of this chapter reflected the situation for about year and one half after the basic idea stimulated by the finding of Nottale [?] that planetary orbits could be seen as Bohr orbits with enormous value of Planck constant given by $\hbar_{gr} = GM_1M_2/v_0$, $v_0 \simeq 2^{-11}$ for the inner planets. The general form of $\hbar_{gr}$ is dictated by Equivalence Principle. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales.

The second crucial empirical input were the anomalies associated with living matter. Mention only the effects of ELF radiation at EEG frequencies on vertebrate brain and anomalous behavior of the ionic currents through cell membrane. If the value of Planck constant is large, the energy of EEG photons is above thermal energy and one can understand the effects on both physiology and behavior. If ionic currents through cell membrane have large Planck constant the scale of quantum coherence is large and one can understand the observed low dissipation in terms of quantum coherence.

2.1.1 The evolution of mathematical ideas

From the beginning the basic challenge -besides the need to deduce a general formula for the quantized Planck constant- was to understand how the quantization of Planck constant is mathematically possible. From the beginning it was clear that since particles with different values of Planck constant cannot appear in the same vertex, a generalization of space-time concept is needed to achieve this.

During last five years or so many deep ideas -both physical and mathematical- related to the construction of quantum TGD have emerged and this has led to a profound change of perspective in this and also other chapters. The overall view about TGD is described briefly in [?].

1. For more than five years ago I realized that von Neumann algebras known as hyperfinite factors of type II_1 (HFFs) are highly relevant for quantum TGD since the Clifford algebra of configuration space ("world of classical worlds", WCW) is direct sum over HFFs. Jones inclusions are particular class of inclusions of HFFs and quantum groups are closely related to them. This led to a conviction that Jones inclusions can provide a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with M^4 and CP_2 degrees of freedom (later I replaced M^4 by its light cone $M^4_\pm$ and finally with the causal diamond CD defined as intersection of future and past light-cones of M^4).

2. The notion of zero energy ontology replaces physical states with zero energy states consisting of pairs of positive and negative energy states at the light-like boundaries $\delta M^4_\pm \times CP_2$ of CDs forming a fractal hierarchy containing CDs within CDs. In standard ontology zero energy state corresponds to a physical event, say particle reaction. This led to the generalization of S-matrix to M-matrix identified as Connes tensor product characterizing time like entanglement between

positive and negative energy states. M-matrix is product of square root of density matrix and unitary S-matrix just like Schrödinger amplitude is product of modulus and phase, which means that thermodynamics becomes part of quantum theory and thermodynamical ensembles are realized as single particle quantum states. This led also to a solution of long standing problem of understanding how geometric time of the physicist is related to the experienced time identified as a sequence of quantum jumps interpreted as moments of consciousness [?] in TGD inspired theory of consciousness which can be also seen as a generalization of quantum measurement theory [?].

3. Another closely related idea was the emergence of measurement resolution as the basic element of quantum theory. Measurement resolution is characterized by inclusion $\mathcal{M} \subset \mathcal{N}$ of HFFs with $\mathcal{M}$ characterizing the measurement resolution in the sense that the action of $\mathcal{M}$ creates states which cannot be distinguished from each other within measurement resolution used. Hence complex rays of state space are replaced with $\mathcal{M}$ rays. One of the basic challenges is to define the nebulous factor space $\mathcal{N}/\mathcal{M}$ having finite fractional dimension $\mathcal{N} : \mathcal{M}$ given by the index of inclusion. It was clear that this space should correspond to quantum counterpart of Clifford algebra of world of classical worlds reduced to a finite-quantum dimensional algebra by the finite measurement resolution [?].

4. The realization that light-like 3-surfaces at which the signature of induced metric of space-time surface changes from Minkowskian to Euclidian are ideal candidates for basic dynamical objects besides light-like boundaries of space-time surface was a further decisive step or progress. This led to vision that quantum TGD is almost topological quantum field theory ("almost" because light-likeness brings in induced metric) characterized by Chern-Simons action for induced Kähler gauge potential of CP_2. Together with zero energy ontology this led to the generalization of the notion of Feynman diagram to a light-like 3-surface for which lines correspond to light-like 3-surfaces and vertices to 2-D partonic surface at which these 3-D surface meet. This means a strong departure from string model picture. The interaction vertices should be given by N-point functions of a conformal field theory with second quantized induced spinor fields defining the basic fields in terms of which also the gamma matrices of world of classical worlds could be constructed as super generators of super conformal symmetries [?].

5. By quantum classical correspondence finite measurement resolution should have a space-time correlate. The obvious guess was that this correlate is discretization at the level of construction of M-matrix. In almost-TQFT context the effective replacement of light-like 3-surface with braids defining basic objects of TQFTs is the obvious guess. Also number theoretic universality necessary for the p-adicization of quantum TGD by a process analogous to the completion of rationals to reals and various p-adic number fields requires discretization since only rational and possibly some algebraic points of the imbedding space (in suitable preferred coordinates) allow interpretation both as real and p-adic points. It was clear that the construction of M-matrix boils to the precise understanding of number theoretic braids [?].

6. The interaction with M-theory dualities [?] led to a handful of speculations about dualities possible in TGD framework, and one of these dualities- $M^8 - M^4 \times CP_2$ duality - eventually led to a unique identification of number theoretic braids. The dimensions of partonic 2-surface, space-time, and imbedding space strongly suggest that classical number fields, or more precisely their complexifications might help to understand quantum TGD. If the choice of imbedding space is unique because of uniqueness of infinite-dimensional Kähler geometric existence of world of classical worlds then standard model symmetries coded by $M^4 \times CP_2$ should have some deeper meaning and the most obvious guess is that $M^4 \times CP_2$ can be understood geometrically. $SU(3)$ belongs to the automorphism group of octonions as well as hyper-octonions M^8 identified by subspace of complexified octonions with Minkowskian signature of induced metric. This led to the discovery that hyper-quaternionic 4-surfaces in M^8 can be mapped to $M^4 \times CP_2$ provided their tangent space contains preferred $M^2 \subset M^4 \subset M^4 \times E^4$. Years later I realized that the map generalizes so that M^2 can depend on the point of X^4. The interpretation of $M^2(x)$ is both as a preferred hyper-complex (commutative) sub-space of M^8 and as a local plane of non-physical polarizations so that a purely number theoretic interpretation of gauge conditions emerges in TGD framework. This led to a rapid progress in the construction of the quantum TGD. In

particular, the challenge of identifying the preferred extremal of Kähler action associated with a given light-like 3-surface X_l^3 could be solved and the precise relation between M^8 and $M^4 \times CP_2$ descriptions was understood [?].

7. Also the challenge of reducing quantum TGD to the physics of second quantized induced spinor fields found a resolution recently [?]. For years ago it became clear that the vacuum functional of the theory must be the Dirac determinant associated with the induced spinor fields so that the theory would predict all coupling parameters from quantum criticality. Even more, the vacuum functional should correspond to the exponent of Kähler action for a preferred extremal. The problem was that the generalized eigenmodes of Chern-Simons Dirac operator allow a generalized eigenvalues to be arbitrary functions of two coordinates in the directions transversal to the light-like direction of X_l^3. The progress in the understanding of number theoretic compactification however allowed to understand how the information about the preferred extremal of Kähler action is coded to the spectrum of eigen modes.

   The basic idea is simple and I actually discovered it for more than half decade ago but forgot! The generalized eigen modes of 3-D Chern-Simons Dirac operator D_{C-S} correspond to the zero modes of a 4-D modified Dirac operator defined by Kähler action localized to X_l^3 so that induced spinor fields can be seen as 4-D spinorial shock waves. The led to a concrete interpretation of the eigenvalues as analogous to cyclotron energies of fermion in classical electro-weak magnetic fields defined by the induced spinor connection and a connection with anyon physics emerges by 2-dimensionality of the evolving system. Also it was possible to identify the boundary conditions for the preferred extremal of Kähler action -analog of Bohr orbit- at X_l^3 and also to the vision about how general coordinate invariance allows to use any light-like 3-surface $X^3 \subset X^4(X_l^3)$ instead of using only wormhole throat to second quantize induced spinor field.

8. It became as a total surprise that due to the huge vacuum degeneracy of induced spinor fields the number of generalized eigenmodes identified in this manner was finite. The good news was that the theory is manifestly finite and zeta function regularization is not needed to define the Dirac determinant. The manifest finiteness had been actually must-be-true from the beginning. The apparently bad news was that the Clifford algebra of WCW world constructed from the oscillator operators is bound to be finite-dimensional. The resolution of the paradox comes from the realization that this algebra represents the somewhat mysterious coset space $\mathcal{N}/\mathcal{M}$ so that finite measurement resolution and the notion inclusion are coded by the vacuum degeneracy of Kähler action and the maximally economical description in terms of inclusions emerges automatically.

9. A unique identification of number theoretic braids became also possible and relates to the construction of the generalized imbedding space by gluing together singular coverings and factor spaces of $CD\backslash M^2$ and $CP_2\backslash S_I^2$ to form a book like structure. Here M^2 is preferred plane of M^4 defining quantization axis of energy and angular momentum and S_I^2 is one of the two geodesic sphere of CP_2. The interpretation of the selection of these sub-manifolds is as a geometric correlate for the selection of quantization axes and CD defining basic sector of world of classical worlds is replaced by a union corresponding to these choices. Number theoretic braids come in too variants dual to each other, and correspond to the intersection of M^2 and M^4 projection of X_l^3 on one hand and S_I^2 and CP_2 projection of X_l^3 on the other hand. This is simplest option and would mean that the points of number theoretic braid belong to M^2 (S_I^2) and are thus quantum critical although entire X^2 at the boundaries of CD belongs to a fixed page of the Big Book. This means solution of a long standing problem of understanding in what sense TGD Universe is quantum critical. The phase transitions changing Planck constant correspond to tunneling represented geometrically by a leakage of partonic 2-surface from a page of Big Book to another one.

10. Many other steps of progress have occurred during the last years. Much earlier it had become clear that the basic difference between TGD and string models is that in TGD framework the super algebra generators are non-hermitian and carry quark or lepton number [?]. Superspace concept is un-necessary because super generators anticommute to Hamiltonians of bosonic symmetries rather than corresponding vector fields. This allows to avoid the Majorana condition of super string models fixing space-time dimension to 10 or 11. During last years a much more precise understanding of super-symplectic and super Kac-Moody symmetries has emerged. The

generalized coset representation for these two Super Virasoro algebras generalizes Equivalence Principle and predicts as a special case the equivalence of gravitational and inertial masses. Coset construction also provides justification for p-adic thermodynamics in apparent conflict with super-conformal invariance. The construction of the fusion rules of symplectic QFT as analog of conformal QFT led to the notion of number theoretic braid and to an explicit construction of a hierarchy of algebras realizing symplectic fusion rules and the notion of finite measurement resolution [?]. This approach led to the formulation of generalized Feynman diagrams and coupling constant evolution in terms of operads Taylor made for a mathematical realization of the notion of coupling constant evolution. One of the future challenges is to combine symplectic fusion algebras with the realization for the hierarchy of Planck constants.

2.1.2 The evolution of physical ideas

The evolution of physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The basic idea was that ordinary matter condenses around dark matter which is a phase of matter characterized by non-standard value of Planck constant.

2. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase [?]. If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [?] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with minimum size of order Schwarstchild radius r_S of order scaled up Planck length: $r_S \sim \sqrt{\hbar G}$. Black hole entropy being inversely proportional to $\hbar$ is predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

3. Darkness is a relative concept and due to the fact that particles at different pages of book cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to different page of book are however possible and change Planck constant so that particle exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. This allows to conclude that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [?, ?].

4. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and aminoacids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially shocking outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [?, ?].

2.1.3 Brief summary about the generalization of the imbedding space concept

A brief summary of the basic vision in order might help reader to assimilate the more detailed representation about the generalization of imbedding space.

1. The hierarchy of Planck constants cannot be realized without generalizing the notions of imbedding space and space-time since particles with different values of Planck constant cannot appear in the same interaction vertex. This suggests some kind of book like structure for both M^4 and CP_2 factors of the generalized imbedding space is suggestive.

2. Schrödinger equation suggests that Planck constant corresponds to a scaling factor of M^4 metric whose value labels different pages of the book. The scaling of M^4 coordinate so that original metric results in M^4 factor is possible so that the scaling of $\hbar$ corresponds to the scaling of the size of causal diamond CD defined as the intersection of future and past directed light-cones. The light-like 3-surfaces having their 2-D and light-boundaries of CD are in a key role in the realization of zero energy states. The infinite-D spaces formed by these 3-surfaces define the fundamental sectors of the configuration space (world of classical worlds). Since the scaling of CD does not simply scale space-time surfaces, the coding of radiative corrections to the geometry of space-time sheets becomes possible and Kähler action can be seen as expansion in powers of $\hbar/\hbar_0$.

3. Quantum criticality of TGD Universe is one of the key postulates of quantum TGD. The most important implication is that Kähler coupling strength is analogous to critical temperature. The exact realization of quantum criticality would be in terms of critical sub-manifolds of M^4 and CP_2 common to all sectors of the generalized imbedding space. Quantum criticality would mean that the two kinds of number theoretic braids assignable to M^4 and CP_2 projections of the partonic 2-surface belong by the definition of number theoretic braids to these critical sub-manifolds. At the boundaries of CD associated with positive and negative energy parts of zero energy state in given time scale partonic two-surfaces belong to a fixed page of the Big Book whereas light-like 3-surface decomposes into regions corresponding to different values of Planck constant much like matter decomposes to several phases at thermodynamical criticality.

4. The connection with Jones inclusions was originally a purely heuristic guess based on the observation that the finite groups characterizing Jones inclusion characterize also pages of the Big Book. The key observation is that Jones inclusions are characterized by a finite subgroup $G \subset SU(2)$ and that this group also characterizes the singular covering or factor spaces associated with CD or CP_2 so that the pages of generalized imbedding space could indeed serve as correlates for Jones inclusions. The elements of the included algebra $\mathcal{M}$ are invariant under the action of G and $\mathcal{M}$ takes the role of complex numbers in the resulting non-commutative quantum theory.

5. The understanding of quantum TGD at parton level led to the realization that the dynamics of Kähler action realizes finite measurement resolution in terms of finite number of modes of the induced spinor field. This automatically implies cutoffs to the representations of various super-conformal algebras typical for the representations of quantum groups closely associated with Jones inclusions [?]. The Clifford algebra spanned by the fermionic oscillator operators would provide a realization for the factor space $\mathcal{N}/\mathcal{M}$ of hyper-finite factors of type II_1 identified as the infinite-dimensional Clifford algebra $\mathcal{N}$ of the configuration space and included algebra $\mathcal{M}$ determining the finite measurement resolution. The resulting quantum Clifford algebra has anti-commutation relations dictated by the fractionization of fermion number so that its unit becomes $r = \hbar/\hbar_0$. $SU(2)$ Lie algebra transforms to its quantum variant corresponding to the quantum phase $q = exp(i2\pi/r)$.

6. Jones inclusions appear as two variants corresponding to $\mathcal{N} : \mathcal{M} < 4$ and $\mathcal{N} : \mathcal{M} = 4$. The tentative interpretation is in terms of singular G-factor spaces and G-coverings of M^4 or CP_2 in some sense. The alternative interpretation in terms of two geodesic spheres of CP_2 would mean asymmetry between M^4 and CP_2 degrees of freedom.

7. Number theoretic Universality suggests an answer why the hierarchy of Planck constants is necessary. One must be able to define the notion of angle -or at least the notion of phase and of trigonometric functions- also in p-adic context. All that one can achieve naturally is the notion of phase defined as root of unity and introduced by allowing algebraic extension of p-adic number field by introducing the phase if needed. In the framework of TGD inspired theory of consciousness this inspires a vision about cognitive evolution as the gradual emergence of

increasingly complex algebraic extensions of p-adic numbers and involving also the emergence of improved angle resolution expressible in terms of phases $exp(i2\pi/n)$ up to some maximum value of n. The coverings and factor spaces would realize these phases geometrically and quantum phases q naturally assignable to Jones inclusions would realize them algebraically. Besides p-adic coupling constant evolution based on hierarchy of p-adic length scales there would be coupling constant evolution with respect to $\hbar$ and associated with angular resolution.

2.2 Experimental input

In this section basic experimental inputs suggesting a hierarchy of Planck constants and the identification of dark matter as phases with non-standard value of Planck constant are discussed.

2.2.1 Hints for the existence of large $\hbar$ phases

Quantum classical correspondence suggests the identification of space-time sheets identifiable as quantum coherence regions. Since they can have arbitrarily large sizes, phases with arbitrarily large quantum coherence lengths and arbitrarily long de-coherence times seem to be possible in TGD Universe. In standard physics context this seems highly implausible. If Planck constant can have arbitrarily large values, the situation changes since Compton lengths and other quantum scales are proportional to $\hbar$. Dark matter is excellent candidate for large $\hbar$ phases.

The expression for $\hbar_{gr}$ in the model explaining the Bohr orbits for planets is of form $\hbar_{gr} = GM_1M_2/v_0$ [?]. This suggests that the interaction is associated with some kind of interface between the systems, perhaps join along boundaries connecting the space-time sheets associated with systems possessing gravitational masses M_1 and M_2. Also a large space-time sheet carrying the mutual classical gravitational field could be in question. This argument generalizes to the case $\hbar/\hbar_0 = Q_1Q_2\alpha/v_0$ in case of generic phase transition to a strongly interacting phase with α describing gauge coupling strength.

There exist indeed some experimental indications for the existence of phases with a large $\hbar$.

1. With inspiration coming from the finding of Nottale [?] I have proposed an explanation of dark matter as a macroscopic quantum phase with a large value of $\hbar$ [?]. Any interaction, if sufficiently strong, can lead to this kind of phase. The increase of $\hbar$ would make the fine structure constant α in question small and guarantee the convergence of perturbation series.

2. Living matter could represent a basic example of large $\hbar$ phase [?, ?]. Even ordinary condensed matter could be "partially dark" in many-sheeted space-time[?]. In fact, the realization of hierarchy of Planck constants leads to a considerably weaker notion of darkness stating that only the interaction vertices involving particles with different values of Planck constant are impossible and that the notion of darkness is relative notion. For instance, classical interactions and photon exchanges involving a phase transition changing the value of $\hbar$ of photon are possible in this framework.

3. There is claim about a detection in RHIC (Relativistic Heavy Ion Collider in Brookhaven) of states behaving in some respects like mini black holes [?]. These states could have explanation as color flux tubes at Hagedorn temperature forming a highly tangled state and identifiable as stringy black holes of strong gravitation. The strings would carry a quantum coherent color glass condensate, and would be characterized by a large value of $\hbar$ naturally resulting in confinement phase with a large value of α_s [?]. The progress in hadronic mass calculations led to a concrete model of color glass condensate of single hadron as many-particle state of super-symplectic gluons [?, ?]- something completely new from the point of QCD - responsible for non-perturbative aspects of hadron physics. In RHIC events these color glass condensate would fuse to single large condensate. This condensate would be present also in ordinary black-holes and the blackness of black-hole would be darkness.

4. I have also discussed a model for cold fusion based on the assumption that nucleons can be in large $\hbar$ phase. In this case the relevant strong interaction strength is $Q_1Q_2\alpha_{em}$ for two nucleon clusters inside nucleus which can increase $\hbar$ so large that the Compton length of protons becomes of order atomic size and nuclear protons form a macroscopic quantum phase [?].

2.2.2 Quantum coherent dark matter and $\hbar$

The argument based on gigantic value of $\hbar_{gr}$ explaining darkness of dark mater is attractive but one should be very cautious.

Consider first ordinary QED: $e = \sqrt{\alpha 4\pi\hbar}$ appears in vertices so that perturbation expansion in powers of $\sqrt{\hbar}$ basically. This would suggest that large $\hbar$ leads to large effects. All predictions are however in powers of alpha and large $\hbar$ means small higher order corrections. What happens can be understood on basis of dimensional analysis. For instance, cross sections are proportional to $(\hbar/m)^2$, where m is the relevant mass and the remaining factor depends on $\alpha = e^2/(4\pi\hbar)$ only. In the more general case tree amplitudes with n vertices are proportional to e^n and thus to $\hbar^{n/2}$ and loop corrections give only powers of α which get smaller when $\hbar$ increases. This must relate to the powers of $1/\hbar$ from the integration measure associated with the momentum loop integrals affected by the change of α.

Consider now the effects of the scaling of $\hbar$. The scaling of Compton lengths and other quantum kinematical parameters is the most obvious effect. An obvious effect is due to the change of $\hbar$ in the commutation relations and in the change of unit of various quantum numbers. In particular, the right hand side of oscillator operator commutation and anti-commutation relations is scaled. A further effect is due to the scaling of the eigenvalues of the modified Dirac operator $\hbar\Gamma^\alpha D_\alpha$.

The exponent $exp(K)$ of Kähler function K defining perturbation series in the configuration space degrees of freedom is proportional to $1/g_K^2$ and does not depend on $\hbar$ at all if there is only single Planck constant. The propagator is proportional to g_K^2. This can be achieved also in QED by absorbing e from vertices to e^2 in photon propagator. Hence it would seem that the dependence on α_K (and $\hbar$) must come from vertices which indeed involve Jones inclusions of the II_1 factors of the incoming and outgoing lines.

This however suggests that the dependence of the scattering amplitudes on $\hbar$ is purely kinematical so that all higher radiative corrections would be absent. This seems to leave only one option: the scale factors of covariant CD and CP_2 metrics can vary and might have discrete spectrum of values.

1. The invariance of Kähler action with respect to overall scaling of metric however allows to keep CP_2 metric fixed and consider only a spectrum for the scale factors of M^4 metric.

2. The first guess motivated by Schrödinger equation is that the scaling factor of covariant CD metric corresponds the ratio $r^2 = (\hbar/\hbar_0)^2$. This would mean that the value of Kähler action depends on r^2. The scaling of M^4 coordinate by r the metric reduces to the standard form but if causal diamonds with quantized temporal distance between their tips are the basic building blocks of the configuration space geometry as zero energy ontology requires, this scaling of $\hbar$ scales the size of CD by r so that genuine effect results since M^4 scalings are not symmetries of Kähler action.

3. In this picture r would code for radiative corrections to Kähler function and thus space-time physics. Even in the case that the radiative corrections to the configuration space functional integral vanish, as suggested by quantum criticality, they would be actually taken into account.

This kind of dynamics is not consistent with the original view about imbedding space and forces to generalize the notion of imbedding spaces since it is clear that particles with different Planck constants cannot appear in the same vertex of Feynman diagram. Somehow different values of Planck constant must be analogous to different pages of book having almost copies of imbedding space as pages. A possible resolution of the problem cames from the realization that the fundamental structure might be the inclusion hierarchy of number theoretical Clifford algebras from which entire TGD could emerge including generalization of the imbedding space concept.

2.2.3 The phase transition changing the value of Planck constant as a transition to non-perturbative phase

A phase transition increasing $\hbar$ as a transition guaranteeing the convergence of perturbation theory

The general vision is that a phase transition increasing $\hbar$ occurs when perturbation theory ceases to converge. Very roughly, this would occur when the parameter $x = Q_1Q_2\alpha$ becomes larger than one. The net quantum numbers for "spontaneously magnetized" regions provide new natural units for quantum numbers. The assumption that standard quantization rules prevail poses very strong restrictions on allowed physical states and selects a subspace of the original configuration space. One can of course, consider the possibility of giving up these rules at least partially in which case a spectrum of fractionally charged anyon like states would result with confinement guaranteed by the fractionization of charges.

The necessity of large $\hbar$ phases has been actually highly suggestive since the first days of quantum mechanics. The classical looking behavior of macroscopic quantum systems remains still a poorly understood problem and large $\hbar$ phases provide a natural solution of the problem.

In TGD framework quantum coherence regions correspond to space-time sheets. Since their sizes are arbitrarily large the conclusion is that macroscopic and macro-temporal quantum coherence are possible in all scales. Standard quantum theory definitely fails to predict this and the conclusion is that large $\hbar$ phases for which quantum length and time scales are proportional to $\hbar$ and long are needed.

Somewhat paradoxically, large $\hbar$ phases explain the effective classical behavior in long length and time scales. Quantum perturbation theory is an expansion in terms of gauge coupling strengths inversely proportional to $\hbar$ and thus at the limit of large $\hbar$ classical approximation becomes exact. Also the Coulombic contribution to the binding energies of atoms vanishes at this limit. The fact that we experience world as a classical only tells that large $\hbar$ phase is essential for our sensory perception. Of course, this is not the whole story and the full explanation requires a detailed anatomy of quantum jump.

The criterion for the occurrence of the phase transition increasing the value of $\hbar$

In the case of planetary orbits the large value of $\hbar_{gr} = 2GM/v_0$ makes possible to apply Bohr quantization to planetary orbits. This leads to a more general idea that the phase transition increasing $\hbar$ occurs when the system consisting of interacting units with charges Q_i becomes non-perturbative in the sense that the perturbation series in the coupling strength $\alpha Q_i Q_j$, where α is the appropriate coupling strength and $Q_i Q_j$ represents the maximum value for products of gauge charges, ceases to converge. Thus Mother Nature would resolve the problems of theoretician. A primitive formulation for this criterion is the condition $\alpha Q_i Q_j \geq 1$.

The first working hypothesis was the existence of dark matter hierarchies with $\hbar = \lambda^k \hbar_0$, $k = 0, 1, ...$, $\lambda = n/v_0$ or $\lambda = 1/nv_0$, $v_0 \simeq 2^{-11}$. This rule turned out to be quite too specific. The mathematically plausible formulation predicts that in principle any rational value for $r = \hbar(M^4)/\hbar(CP_2)$ is possible but there are certain number theoretically preferred values of r such as those coming powers of 2.

2.3 A generalization of the notion of imbedding space as a realization of the hierarchy of Planck constants

In the following the basic ideas concerning the realization of the hierarchy of Planck constants are summarized and after that a summary about generalization of the imbedding space is given. In [?] the important delicacies associated with the Kähler structure of generalized imbedding space are discussed. The background for the recent vision is quite different from that for half decade ago. Zero energy ontology and the notion of causal diamond, number theoretic compactication leading to the precise identification of number theoretic braids, the realization of number theoretic universality, and the understanding of the quantum dynamics at the level of modified Dirac action fix to a high degree the vision about generalized imbedding space.

2.3.1 Basic ideas

The first key idea in the geometric realization of the hierarchy of Planck constants emerges from the study of Schrödinger equation and states that Planck constant appears a scaling factor of M^4 metric. Second key idea is the connection with Jones inclusions inspiring an explicit formula for Planck constants. For a long time this idea remained heuristic must-be-true feeling but the recent view about quantum TGD provide a justification for it.

Scaling of Planck constant and scalings of CD and CP_2 metrics

The key property of Schrödinger equation is that kinetic energy term depends on $\hbar$ whereas the potential energy term has no dependence on it. This makes the scaling of $\hbar$ a non-trivial transformation. If the contravariant metric scales as $r = \hbar/\hbar_0$ the effect of scaling of Planck constant is realized at the level of imbedding space geometry provided it is such that it is possible to compare the regions of generalized imbedding space having different value of Planck constant.

In the case of Dirac equation same conclusion applies and corresponds to the minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$. Consider next the situation in TGD framework.

1. The minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$ does not make sense in the case of CP_2 Dirac operator since, by the non-triviality of spinor connection, one cannot choose the value of $\hbar$ freely. In fact, spinor connection of CP_2 is defined in such a manner that spinor connection corresponds to the quantity $\hbar eQA$, where denotes A gauge potential, and there is no natural manner to separate $\hbar e$ from it.

2. The contravariant CD metric scales like $\hbar^2$. In the case of Dirac operator in $M^4 \times CP_2$ one can assign separate Planck constants to Poincare and color algebras and the scalings of CD and CP_2 metrics induce scalings of corresponding values of $\hbar^2$. As far as Kähler action is considered, CP_2 metric could be always thought of being scaled to its standard form.

3. Dirac equation gives the eigenvalues of wave vector squared $k^2 = k^i k_i$ rather than four-momentum squared $p^2 = p^i p_i$ in CD degrees of freedom and its analog in CP_2 degrees of freedom. The values of k^2 are proportional to $1/r^2$ so that p^2 does not depend on it for $p^i = \hbar k^i$: analogous conclusion applies in CP_2 degrees of freedom. This gives rise to the invariance of mass squared and the desired scaling of wave vector when $\hbar$ changes.

This consideration generalizes to the case of the induced gamma matrices and induced metric in X^4, modified Dirac operator, and Kähler action which carry dynamical information about the ratio $r = \hbar_{eff}/\hbar_0$.

Kähler function codes for a perturbative expansion in powers of $\hbar(CD)/\hbar(CP_2)$

Suppose that one accepts that the spectrum of CD *resp.* CP_2 Planck constants is accompanied by a hierarchy of overall scalings of covariant CD (causal diamond) metric by $(\hbar(M^4)/\hbar_0)^2$ and CP_2 metric by $(\hbar(CP_2)/\hbar_0)^2$ followed by overall scaling by $r^2 = (\hbar_0/\hbar(CP_2))^2$ so that CP_2 metric suffers no scaling and difficulties with isometric gluing procedure of sectors are avoided.

The first implication of this picture is that the modified Dirac operator determined by the induced metric and spinor structure depends on r in a highly nonlinear manner but there is no dependence on the overall scaling of the H metric. This in turn implies that the fermionic oscillator algebra used to define configuration space spinor structure and metric depends on the value of r. Same is true also for Kähler action and configuration space Kähler function. Hence Kähler function is analogous to an effective action expressible as infinite series in powers of r.

This interpretation allows to overcome the paradox caused by the hypothesis that loop corrections to the functional integral over configuration space defined by the exponent of Kähler function serving as vacuum functional vanish so that tree approximation is exact. This would imply that all higher order corrections usually interpreted in terms of perturbative series in powers of $1/\hbar$ vanish. The paradox would result from the fact that scattering amplitudes would not receive higher order corrections and classical approximation would be exact.

The dependence of both states created by Super Kac-Moody algebra and the Kähler function and corresponding propagator identifiable as contravariant configuration space metric would mean that the expressions for scattering amplitudes indeed allow an expression in powers of r. What is so remarkable is that the TGD approach would be non-perturbative from the beginning and "semiclassical" approximation, which might be actually exact, automatically would give a full expansion in powers of r. This is in a sharp contrast to the usual quantization approach.

Jones inclusions and hierarchy of Planck constants

From the beginning it was clear that Jones inclusions of hyper-finite factors of type II_1 are somehow related to the hierarchy of Planck constants. The basic motivation for this belief has been that configuration space Clifford algebra provides a canonical example of hyper-finite factor of type II_1 and that Jones inclusion of these Clifford algebras is excellent candidate for a first principle description of finite measurement resolution.

Consider the inclusion $\mathcal{N} \subset \mathcal{M}$ of hyper-finite factors of type II_1 [?]. A deep result is that one can express $\mathcal{M}$ as $\mathcal{N} : \mathcal{M}$-dimensional module over N with fractal dimension $\mathcal{N} : \mathcal{M} = B_n$. $\sqrt{B_n}$ represents the dimension of a space of spinor space renormalized from the value 2 corresponding to $n = \infty$ down to $\sqrt{B_n} = 2cos(\pi/n)$ varying thus in the range $[1, 2]$. B_n in turn would represent the dimension of the corresponding Clifford algebra. The interpretation is that finite measurement resolution introduces correlations between components of quantum spinor implying effective reduction of the dimension of quantum spinors providing a description of the factor space $\mathcal{N}/\mathcal{M}$.

This would suggest that somehow the hierarchy of Planck constants must represent finite measurement resolution and since phase factors coming as roots of unity are naturally associated with Jones inclusions the natural guess was that angular resolution and coupling constant evolution associated with it is in question. This picture would suggest that the realization of the hierarchy of Planck constant in terms of a book like structure of generalized imbedding space provides also a geometric realization for a hierarchy of Jones inclusions.

The notion of number theoretic braid and realization that the modified Dirac operator has only finite number of generalized eigenmodes -thanks to the vacuum degeneracy of Kähler action- finally led to the understanding how the notion of finite measurement resolution is coded to the Kähler action and the realized in practice by second quantization of induced spinor fields and how these spinor fields endowed with q-anticommutation relations give rise to a representations of finite-quantum dimensional factor spaces $\mathcal{N}/\mathcal{M}$ associated with the hierarchy of Jones inclusions having generalized imbedding space as space-time correlate. This means enormous simplification since infinite-dimensional spinor fields in infinite-dimensional world of classical worlds are replaced with finite-quantum-dimensional spinor fields in discrete points sets provided by number theoretic braids.

The study of a concrete model for Jones inclusions in terms of finite subgroups G of $SU(2)$ defining sub-algebras of infinite-dimensional Clifford algebra as fixed point sub-algebras leads to what looks like a correct track concerning the understanding of quantization of Planck constants.

The ADE diagrams of A_n and D_{2n} characterize cyclic and dihedral groups whereas those of E_6 and E_8 characterize tedrahedral and icosahedral groups. This approach leads to the hypothesis that the scaling factor of Planck constant assignable to Poincare (color) algebra corresponds to the order of the maximal cyclic subgroup of $G_b \subset SU(2)$ ($G_a \subset SL(2, C)$) acting as symmetry of space-time sheet in CP_2 (CD) degrees of freedom. It predicts arbitrarily large CD and CP_2 Planck constants in the case of A_n and D_{2n} under rather general assumptions.

There are two manners for how G_a and G_b can act as symmetries corresponding to G_i coverings and factors spaces. These coverings and factor spaces are singular and associated with spaces $\hat{CD}\backslash M^2$ and $CP_2\backslash S^2_I$, where S^2_I is homologically trivial geodesic sphere of CP_2. The physical interpretation is that M^2 and S^2_I fix preferred quantization axes for energy and angular moment and color quantum numbers so that also a connection with quantum measurement theory emerges.

2.3.2 The vision

A brief summary of the basic vision behind the generalization of the imbedding space concept needed to realize the hierarchy of Planck constants is in order before going to the detailed representation.

1. The hierarchy of Planck constants cannot be realized without generalizing the notions of imbedding space and space-time because particles with different values of Planck constant cannot appear in the same interaction vertex. Some kind of book like structure for the generalized imbedding space forced also by p-adicization but in different sense is suggestive. Both M^4 and CP_2 factors would have the book like structure so that a Cartesian product of books would be in question.

2. The study of Schrödinger equation suggests that Planck constant corresponds to a scaling factor of CD metric whose value labels different pages of the book. The scaling of M^4 coordinate so that original metric results in CD factor is possible so that the interpretation for scaled up value of $\hbar$ is as scaling of the size of causal diamond CD.

3. The light-like 3-surfaces having their 2-D and light-boundaries of CD are in a key role in the realization of zero energy states, and the infinite-D spaces of light-like 3-surfaces inside scaled variants of CD define the fundamental building brick of the configuration space (world of classical worlds). Since the scaling of CD does not simply scale space-time surfaces the effect of scaling on classical and quantum dynamics is non-trivial and a coupling constant evolution results and the coding of radiative corrections to the geometry of space-time sheets becomes possible. The basic geometry of CD suggests that the allowed sizes of CD come in the basic sector $\hbar = \hbar_0$ as powers of two. This predicts p-adic length scale hypothesis and lead to number theoretically universal discretized p-adic coupling constant evolution. Since the scaling is accompanied by a formation of singular coverings and factor spaces, different scales are distinguished at the level of topology. p-Adic length scale hierarchy affords similar characterization of length scales in terms of effective topology.

4. The idea that TGD Universe is quantum critical in some sense is one of the key postulates of quantum TGD. The basic ensuing prediction is that Kähler coupling strength is analogous to critical temperature. Quantum criticality in principle fixes the p-adic evolution of various coupling constants also the value of gravitational constant. The exact realization of quantum criticality would be in terms of critical sub-manifolds of M^4 and CP_2 common to all sectors of the generalized imbedding space. Quantum criticality of TGD Universe means that the two kinds of number theoretic braids assignable to M^4 and CP_2 projections of the partonic 2-surface belong by the very definition of number theoretic braids to these critical sub-manifolds. At the boundaries of CD associated with positive and negative energy parts of zero energy state in a given time scale partonic two-surfaces belong to a fixed page of the Big Book whereas light-like 3-surface decomposes to regions corresponding to different values of Planck constant much like matter decomposes to several phases at criticality.

The connection with Jones inclusions was originally a purely heuristic guess, and it took half decade to really understand why and how they are involved. The notion of measurement resolution is the key concept.

1. The key observation is that Jones inclusions are characterized by a finite subgroup $G \subset SU(2)$ and the this group also characterizes the singular covering or factor spaces associated with CD or CP_2 so that the pages of generalized imbedding space could indeed serve as correlates for Jones inclusions.

2. The dynamics of Kähler action realizes finite measurement resolution in terms of finite number of modes of the induced spinor field automatically implying cutoffs to the representations of various super-conformal algebras typical for the representations of quantum groups associated with Jones inclusions. The interpretation of the Clifford algebra spanned by the fermionic oscillator operators is as a realization for the concept of the factor space $\mathcal{N}/\mathcal{M}$ of hyper-finite factors of type II_1 identified as the infinite-dimensional Clifford algebra $\mathcal{N}$ of the configuration space and included algebra $\mathcal{M}$ determining the finite measurement resolution for angle measurement in the sense that the action of this algebra on zero energy state has no detectable physical effects. $\mathcal{M}$ takes the role of complex numbers in quantum theory and makes physics non-commutative. The resulting quantum Clifford algebra has anti-commutation relations dictated by the fractionization of fermion number so that unit becomes $r = \hbar/\hbar_0$. $SU(2)$ Lie algebra transforms to its quantum variant corresponding to the quantum phase $q = exp(i2\pi/r)$.

3. G invariance for the elements of the included algebra can be interpreted in terms of finite measurement resolution in the sense that action by G invariant Clifford algebra element has no detectable effects. Quantum groups realize this view about measurement resolution for angle measurement. The G-invariance of the physical states created by fermionic oscillator operators which by definition are not G invariant guarantees that quantum states as a whole have non-fractional quantum numbers so that the leakage between different pages is possible in principle. This hypothesis is consistent with the TGD inspired model of quantum Hall effect [**?**].

4. Concerning the formula for Planck constant in terms of the integers n_a and n_b characterizing orders of the maximal cyclic subgroups of groups G_a and G_b defining coverings and factor spaces associated with CD and CP_2 the basic constraint is that the overall scaling of H metric has no

effect on physics. What matters is the ratio of Planck constants $r = \hbar(M^4)/\hbar(CP_2)$ appearing as a scaling factor of M^4 metric. This leaves two options if one requires that the Planck constant defines a homomorphism. The model for dark gravitons suggests a unique choice between these two options but one must keep still mind open for the alternative.

5. Jones inclusions appear as two variants corresponding to $\mathcal{N} : \mathcal{M} < 4$ and $\mathcal{N} : \mathcal{M} = 4$. The tentative interpretation is in terms of singular G-factor spaces and G-coverings of M^4 and CP_2 in some sense. The alternative interpretation assigning the inclusions to the two different geodesic spheres of CP_2 would mean asymmetry between M^4 and CP_2 degrees of freedom and is therefore not convincing.

6. The natural question is why the hierarchy of Planck constants is needed. Is it really necessary? Number theoretic Universality suggests that this is the case. One must be able to define the notion of angle -or at least the notion of phase and of trigonometric functions- also in the p-adic context. All that one can achieve naturally is the notion of phase defined as a root of unity and introduced by allowing algebraic extension of p-adic number field by introducing the phase. In the framework of TGD inspired theory of consciousness this inspires a vision about cognitive evolution as the gradual emergence of increasingly complex algebraic extensions of p-adic numbers and involving also the emergence of improved angle resolution expressible in terms of phases $exp(i2\pi/n)$ up to some maximum value of n. The coverings and factor spaces would realize these phases purely geometrically and quantum phases q assignable to Jones inclusions would realize them algebraically. Besides p-adic coupling constant evolution based on the hierarchy of p-adic length scales there would be coupling constant evolution with respect to $\hbar$ and associated with angular resolution.

Both covering spaces and factor spaces are possible

The most general option replaces CD and CP_2 with the factor spaces and singular coverings defined by groups $G \subset SU(2)$ associated with Jones inclusions.

1. Singular coverings and factgors spaces are not possible for M^4, CP_2, or H since their fundamental groups are trivial. On the other hand, the fixing of quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is a geodesic sphere of CP_2. $\hat{CD} = CD\backslash M^2$ and $\hat{CP}_2 = CP_2\backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. H_4 represents a straight cosmic string. Quantum field theory phase corresponds to Jones inclusions with Jones index $\mathcal{M} : \mathcal{N} < 4$. Stringy phase would by previous arguments correspond to $\mathcal{M} : \mathcal{N} = 4$. Also these Jones inclusions are labeled by finite subgroups of $SO(3)$ and thus by Z_n identified as a maximal Abelian subgroup.

   One can argue that cosmic strings are not allowed in QFT phase. This would encourage the replacement $\hat{CD} \times \hat{CP}_2$ implying that surfaces in $M^4 \times S^2$ and $M^2 \times CP_2$ are not allowed. In particular, cosmic strings and CP_2 type extremals with CD projection in M^2 and thus light-like geodesic without zitterwebegung essential for massivation are forbidden. This brings in mind instability of Higgs=0 phase.

3. The covering spaces in question would correspond to the Cartesian products of the covering spaces of $\hat{CD}$ and $\hat{CP}_2$ by Z_{n_a} and Z_{n_b} with fundamental group is $Z_{n_a} \times Z_{n_b}$. One can also consider extension by replacing M^2 and S^2 with its orbit under G_a (say tedrahedral, octahedral, or icosahedral group). The resulting space will be denoted by $\hat{CD}\hat{\times}G_a$ *resp.* $\hat{CP}_2\hat{\times}G_b$.

4. One expects the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would

be replaced by the set of quantization axes going through the vertices of tedrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

5. Also the orbifolds $\hat{CD}/G_a \times \hat{CP_2}/G_b$ can be allowed as also the spaces $\hat{CD}/G_a \times (\hat{CP_2}\hat{\times}G_b)$ and $(\hat{CD}\hat{\times}G_a) \times \hat{CP_2}/G_b$. Hence the previous framework would generalize considerably by the allowance of both coset spaces and covering spaces.

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. This is consistent with the identical vanishing of Chern-Simons action in $M^2 \times S^2$.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunneling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) and classical light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

What is the correct formula for Planck constant?

The question how the Planck constants associated with factors and coverings relate to each other is far from trivial and I have considered several options.

1. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of metric allowed by Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains $r^2 \equiv \hbar^2/\hbar_0^2\hbar^2(M^4)/\hbar^2(CP_2)$. This puts CD and CP_2 in a very symmetric role and allows much more flexibility in the identification of symmetries associated with large Planck constant phases.

2. Algebraist would argue that Planck constant must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa. This gives two options.

3. Option I: $r(X) = n$ for covering and $r(X) = 1/n$ for factor space gives $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$. This gives $r = n_a/n_b$ for $\hat{H}/G_a \times G_b$ option and $r = n_b/n_a$ for $\hat{H}\hat{times}(G_a \times G_b)$ option with obvious formulas for hybrid cases.

4. Option II: $r(X) = 1/n$ for covering and $r(X) = n$ for factor space gives $r = r(CP_2)/r(M^4)$. This gives $r = n_b/n_a$ for $\hat{H}/G_a \times G_b$ option and $r = n_a/n_b$ for $\hat{H}\hat{times}(G_a \times G_b)$ option with obvious formulas for the hybrid cases.

5. At quantum level the fractionization would come from the modification of fermionic anti-commutation (bosonic commutation) relations involving $\hbar$ at the right hand side so that particle number becomes a multiple of $1/n$ or n. If one postulates that the total number states is invariant in the transition, the increase in the number of sheets is compensated by the increase of the fundamental phase space volume proportional to $\hbar$. This would give $r(X) \rightarrow r(X)/n$ for factor space and $r(X) \rightarrow nr(X)$ for the covering space to compensate the n-fold reduction/increase of states. This would favor Option II.

6. The second manner to distinguish between these two options is to apply the theory to concrete physical situations. Since G_a and G_b act as symmetries in CD and CP_2 degrees of freedom, one might of being able to distinguish between the two options if it is possible to distinguish between the action of G as symmetry of quantum states associated with covering and factor space. Also the quantization of the orbital spin quantum number at single particle level as multiples of n can be distinguished from that in multiples of $1/n$.

Quantum classical correspondence suggests that Planck constants appear at the level of the classical dynamics. The first thing to notice is that Kähler action does not depend on the overall scaling of H metric. If also CP_2 metric is scaled up the classical physics as defined by the extremals of Kähler action depends only on the ratio $r = \hbar(M^4)/\hbar(CP_2)$ so that would have symmetry $(\hbar(M^4), \hbar(CP_2)) \rightarrow k(\hbar(M^4), \hbar(CP_2))$. This is a non-trivial prediction since bundle coverings are different. Note however that the distinction between coverings related by scaling is visible in the dynamics involving other levels of dark matter hierarchy. The dependence of Kähler action on r could be interpreted in terms of radiative corrections coded to the Kähler function so that the vanishing of higher order radiative corrections on the functional integral over 3-surfaces around maxima of Kähler function does not lead to a conflict with the fact that radiative corrections must be non-vanishing.

It is important to notice that by introducing same M^4 coordinate for various pages of the book the basic distinction between corresponding CDs is that the temporal distances between the tips of CDs are proportional to $\hbar$ and that the emergence of coverings and factor spaces allows to distinguish between different scales in the scale hierarchy.

2.3.3 Realization of quantum criticality in terms of number theoretic braids

The long standing question has been how to define formally the vision that TGD Universe is quantum critical. The notion of number theoretical braid combined with hierarchy of Planck constant provide a solution to this question and also fixes the precise realization of the generalized imbedding space.

The notion of number theoretical braid

The notion of number theoretic braid is essential for the view about quantum TGD as almost topological quantum field theory. It also realization discretization as a space-time correlate for the finite measurement resolution. Number theoretical universality leads to this notion also and requires that the points in the intersection of the number theoretic braid with partonic 2-surface correspond to rational or at most algebraic points of H in preferred coordinates fixed by symmetry considerations. The challenge has been to find a unique identification of the number theoretic braid. Number theoretic vision indeed makes this possible.

The core element of number theoretic vision is that the laws of physics could be reduced to associativity conditions. One realization for associativity conditions is the level of M^8 endowed with hyper-octonionic structure as a condition that the points sets possible as arguments of N-point function in X^4 are associative and thus belong to hyper-quaternionic subspace $M^4 \subset M^8$. This decomposition must be consistent with the $M^4 \times E^4$ decomposition implied by $M^4 \times CP_2$ decomposition of H. What comes first in mind is that partonic 2-surfaces X^2 belong to $\delta M^4_\pm \subset M^8$ defining the ends of the causal diamond and are thus associative. This boundary condition however freezes E^4 degrees of freedom completely so that M^8 configuration space geometry trivializes.

1. Are the points of number theoretic braid commutative?

One can also consider the commutativity condition by requiring that arguments belong to a preferred commutative hyper-complex sub-space M^2 of M^8 which can be regarded as a curve in complex plane. Fixing preferred real and imaginary units means a choice of M^2 interpreted as a partial choice of quantization axes at the level of M^8. One must distinguish this choice from the hyper-quaternionicity of space-time surfaces and from the condition that each tangent space of X^4 contains $M^2(x) \subset M^4$ in its tangent space or normal space. Commutativity condition indeed implies the notion of number theoretic braid and fixes it uniquely once a global selection of $M^2 \subset M^8$ is made. There is also an alternative identification of number theoretic braid based on the assumption that braids are light-like curves with tangent vector in $M^2(x)$.

1. The strong form of commutativity condition would require that the arguments of the n-point function at partonic 2-surface belong to the intersection $X^2 \cap M_{\pm}$. This however allows quite too few points since an intersection of 2-D and 1-D objects in 7-D space would be in question. Associativity condition would reduce cure the problem but would trivialize configuration space geometry.

2. The weaker condition that only δCD projections for the points of X^2 commute is however sensible since the intersection of 1-D and 2-D surfaces of 3-D space results. This condition is also invariant under number theoretical duality. In the generic case this gives a discrete set of points as intersection of light-like radial geodesic and the projection $P_{\delta M^4_{\pm}}(X^2)$. This set is naturally identifiable in terms of points in the intersection of number theoretic braids with $\delta CD \times E^4$. One should show that this set of points consists of rational or at most algebraic points. Here the possibility to choose X^2 to some degree could be essential. Any radial light ray from the tip of light-cone allows commutativity and one can consider the possibility of integrating over n-point functions with arguments at light ray to obtain maximal information.

3. For the pre-images of light-like 3-surfaces commutativity of the points in $\delta M^4_{\pm}$ projection would allow the projections to be one-dimensional curves of M^2 having thus interpretation as braid strands. M^2 would play exactly the same role as the plane into which braid strands are projected in the construction of braid invariants. Therefore the plane of non-physical polarizations in gauge theories corresponds to the plane to which braids and knots are projected in braid and knot theories. A further constraint is that the braid strand connects algebraic points of M^8 to algebraic points of M^8. It seems that this can be guaranteed only by posing some additional conditions to the light-like 3-surfaces themselves which is of course possible since they are in the role of fundamental dynamical objects.

2. Are number theoretic braids light-like curves with tangent in $M^2(x)$?

There are reasons why the identification of the number theoretic braid strand as a curve having hyper-complex light-like tangent looks more attractive.

1. An alternative identification of the number theoretic braid would give up commutativity condition for CD projection and assume braid strand to be as a light-like curve having light-like tangent belonging to the local hyper-complex tangent sub-space $M^2(x)$ at point x. This definition would apply both in $X^3 \subset \delta M^4_{\pm} \times CP_2$ and in X^3_l. Also now one would have a continuous distribution of number theoretic braids, with one braid assignable to each light-like curve with tangent $\delta M^4_+ \supset M_+(x) \subset M^2(x)$. In this case each light-like curve at δM^4_+ with tangent in $M_+(x)$ would define a number theoretic braid so that the only difference would be the replacement of light-like ray with a more general light-like curve.

2. The preferred plane $M^2(x)$ can be interpreted as the local plane of non-physical polarizations so that the interpretation as a number theoretic analog of gauge conditions posed in both quantum field theories and string models is possible. In TGD framework this would mean that super-conformal degrees of freedom are restricted to the orthogonal complement of $M^2(x)$ and $M^2(x)$ does not contribute to the configuration space metric. In Hamilton-Jacobi coordinates the pairs of light-like curves associated with coordinate lines can be interpreted as curved light rays. Hence the partonic planes $M^2(x_i)$ associated with the points of the number theoretic braid could be also regarded as carriers four-momenta of fermions associated with the braid strands so that the standard gauge conditions $\epsilon \cdot p = 0$ for polarization vector and four-momentum would be realized geometrically. The possibility of M^2 to depend on point of X^3_l would be essential to have non-collinear momenta and for a classical description of interactions between braid strands.

3. One could also define analogs of string world sheets as sub-manifolds of $P_{M^4_+}(X^4)$ having $M^2(x) \subset M^4$ as their tangent space or being assignable to their tangent containing $M_+(x)$ in the case that the distribution defined by the planes $M^2(x)$ exists and is integrable. It must be emphasized that in the case of massless extremals one can assign only $M_+(x) \subset M^4$ to $T(X^4(x))$ so that only a foliation of X^4 by light-like curves in CD is possible. For $P_{M^4_+}(X^4)$ however a foliation by 2-D stringy surfaces is obtained. Integrability of this distribution and thus the duality with stringy description has been suggested to be a basic feature of the preferred extremals and is equivalent with the existence of Hamilton-Jacobi coordinates for a large class of extremals of Kähler action [?].

4. The possibility of dual descriptions based on integrable distribution of planes $M^2(x)$ allowing identification as 2-dimensional stringy sub-manifolds of $X^4(X^3)$ and the flexibility provided by the hyper-complex conformal invariance raise the hopes of achieving the lifting of super-symplectic algebra SS and super Kac-Moody algebra SKM to H. At the light-cone boundary the light-like radial coordinate could be lifted to a hyper-complex coordinate defining coordinate for M^2. At X^3_l one could fix the light-like coordinate varying along the braid strands and it can can be lifted to a light-like hyper-complex coordinate in CD by requiring that the tangent to the coordinate curve is light-like line of $M^2(x)$ at point x. The total four-momenta and color quantum numbers assignable to SS and SKM degrees of freedom are naturally identical since they can be identified as the four-momentum of the partonic 2-surface $X^2 \subset X^3 \cap \delta M^4_\pm \times CP_2$. Equivalence Principle would emerge as an identity.

3. Are also CP_2 duals of number theoretic braids possible?

This picture is probably not enough. From the beginning the idea that also the CP_2 projections of points of X^2 define number theoretic braids has been present. The dual role of the braids defined by M^2 and CP_2 projections of X^2 is suggested both by the construction of the symplectic fusion algebras [?] and by the model of anyons [?]. M^2 and the geodesic sphere $S^2_i \subset CP_2$, where one has either $i = I$ or $i = II$, where $i = I/II$ corresponds to homologically trivial/non-trivial geodesic sphere, are in a key role in the geometric realization of the hierarchy of Planck constants in terms of the book like structure of the generalized imbedding space. The fact that S^2_I corresponds to vacuum extremals would suggest that only the intersection $S^2_{II} \cap P_{CP_2}(X^2)$ can define CP_2 counterpart of the number theoretic braid. CD braid could be the proper description in the associative case (Minkowskian signature of induced metric) and CP_2 braid in the co-associative case (Euclidian signature of induced metric). The duality of these descriptions would be reflected also by the fact that the physical Planck constant is given by $\hbar = r\hbar_0$, $r = \hbar(M^4)/\hbar(CP_2)$, so that only the ratio of the two Planck constants matters in commutation relations.

Number theoretic braids are quantum critical

The elegant manner to realized quantum criticality is as quantum criticality of number theoretic braids.

1. All particles, be their elementary particles or anyonic systems of astrophysical size, have light-like partonic 3-surfaces as space-time correlates. If the corresponding partonic 2-surfaces X^2 are as a whole quantum critical their M^2 projection belongs to to $M_+ \subset \delta M^4_\pm$ or CP_2 projection to S^2_i, $i = I$ or II corresponding to homological trivial and non-trivial geodesic spheres. If X^2 is fully quantum critical, it belongs $M^2 \times S^2$ defining the intersection of all pages of the big book. This definition is quite too stringent. The geometric picture about phase transition implies that only a 1-D curves of partonic 2-surface correspond to exact quantum criticality so that 2-surface could consist of several parts having different values of Planck constant during transition. This would realize at the level of partonic 2-surfaces the geometric view about criticality as decomposition to regions consisting of different phases.

2. The construction of M-matrix utilizes only the data from the discrete number theoretic braids defined by the intersections of with $M_\pm \cap P_{\delta M^4_\pm}(X^2)$ where one has $M_\pm \subset M^2 \cap \delta M^4 \pm$ where $M_\pm = M^2 \cap \delta M^4 \pm$ is light-like ray. The proposed dual description uses number theoretic braids defined by CP_2 projection $S^2_i \cap P_{CP_2}(X^2)$, $i = I$ or II. Quantum criticality of TGD Universe

would mean that M^2 and S_i^2 defining the quantum measurement axis and quantum critical manifolds also define number theoretical braids.

Which geodesic sphere?

There are two geodesic spheres in CP_2. Which one should choose or are both possible? It seems that it is not possible to make final conclusion yet.

1. For the homologically non-trivial geodesic sphere S_{II}^2 corresponding to the simplest cosmic strings, the isometry group is $U(2) \subset SU(3)$. The homologically trivial geodesic sphere S_I^2 has vanishing induced Kähler form and in maximal quantum criticality would correspond to vacuum extremals. It has isometry group $SO(3) \subset SU(3)$. One could argue that $U(1)$ factor excludes S_{II}^2 but there is actually no reason for restring the consideration to $SU(2)$. For $SU(2)$ color isospin would be the quantum number involved and G would act as phase rotations for complex coordinates of CP_2. For $SO(3)$ $SO(2)$ would mix complex coordinates like real coordinates of plane and $SO(2)$ quantum number would not allow interpretation in terms of color.

2. One could also argue that for a quantum critical partonic 2-surface containing regions with different values of Planck constant these regions must correspond to different regions inside which induced Kähler form is non-vanishing since second quantization in this case would be done for a fixed value of Planck constant. This is not guaranteed in the case S_{II}^2 but in the case of S_I^2 there exists a 1-D curve separating the regions with different Planck constant from each other. Again one can however simply restrict the partonic two-surfaces at $\delta CD \times CP_2$ to belong to single page of generalized imbedding space so that quantum criticality as decomposition to regions belonging to different pages would hold true only for X_l^3 and quantum criticality would be purely dynamical phenomenon analogous to quantum tunneling. In particular, there would be no vertex characterizing the phase transition changing the page.

3. Quantum criticality corresponds to long range fluctuations and non-deterministic behavior and there is a strong temptation to assign with with the vacuum degeneracy of Kähler action implying also 4-D spin glass like character of TGD Universe. This would favor S_I^2. If S_I^2 is chosen one can ask whether all surfaces $M^2 \times Y^2$, Y^2 Lagrangian sub-manifold of CP_2 defining vacuum sectors of the theory should be allowed. The answer seems to be "No" since in the generic case $SO(3)$ does not act as H-isometries of Y^2. If one allows these sub-manifolds or even sub-manifolds of form $M^4 \times Y^2$ to appear as intersection of fractally scaled up variants, one must replace Cartan algebra as algebra associated with $SO(3)$ subgroup of symplectic transformations of CP_2 mapping Y^2 to itself (if this kind of algebra exists).

4. If partonic 2-surfaces belong to single page of the Big book, there is no reason to exclude the option allowing both S_I^2 and S_{II}^2. This raises the question whether the two kinds of Jones inclusions corresponding to $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$ correspond to the two geodesic spheres. S_I^2 would correspond to a phase obtained as small perturbations of vacuum extremals and S_{II}^2 to string like objects (Kähler magnetic flux tubes) obtained as perturbations of cosmic strings. The objection is that there would be asymmetry between CD and CP_2 degrees of freedom since for CD only M^2 appears and would correspond to either $\mathcal{M} : \mathcal{N} < 4$ or $\mathcal{M} : \mathcal{N} = 4$. If singular coverings and factor spaces correspond to the two kinds of Jones inclusions, the factors are in completely symmetric position.

5. An argument favoring S_I^2 is that situation should be symmetric with respect to CD and CP_2. Since M^2 corresponds to non-physical polarizations and S_I^2 to vacuum extremals, one might argue that these are the correct choices. S_I^2 corresponds also to a non-holomorphic sub-manifold of CP_2 being given by $\xi^2 = \overline{\xi}^2$ in Eguchi-Hanson coordinates. Symmetry argument suggests also that $M^2(x) \oplus E^2(x)$ decomposition should have CP_2 counter part and correspond to a slicing of CP_2 to geodesic spheres. This slicing would be analogous to the slicing of sphere by geodesic circles labeled by the value of the coordinate θ and intersecting at two diametrically opposite points at equator. The variation range θ defines half-geodesic circle orthogonal to the slices. The spherical CP_2 coordinate pairs (r, Ψ) and (r, Φ) would label the slices of S_I^2 and S_{II}^2 slicings with three intersection points. (r, Ψ) and the complex coordinate for S_{II}^2 would define

the analog of Hamilton-Jacobi coordinates for CP_2 and the analogy with sphere would suggest that the variation ranges of (r, Ψ) *resp.* (r, Φ) define pieces of S^2_{II} *resp.* S^2_I. The dual geodesic spheres are not orthogonal to the geodesic spheres which they label as the expression of CP_2 metric in Eguchi-Hanson coordinates shows [**?**]. In fact, orthogonality fails of longitudinal and transversal planes fails for Hamilton-Jacobi coordinates too.

What are the correct anti-commutation relations for fermionic oscillator operators?

Quantum commutation relations make sense only for quantum version of the $SU(2)$ Lie algebra. The reason is that the quantum commutation relations of form $a^\dagger a + qaa^\dagger = \hbar$ are consistent with the hermiticity of the right hand side only for real values of q.

The vision about fractionization of various quantum numbers as being due to the fractionization of fermion number suggests that the correct anti-commutation relations are of form

$$\begin{aligned} a^\dagger a + aa^\dagger &= \hbar = r\hbar_0 \ , \\ r &= \frac{\hbar(CD)}{\hbar(CP_2)} \ . \end{aligned} \tag{2.3.1}$$

One cannot transform these relations to the standard form for all pages of the Big Book simultaneously by scaling the oscillator operators by $1/\sqrt{r}$. The unit of fermion number becomes r so that single fermion leakage between sectors with different value of r is not possible.

One can consider also a more delicate modification is based on the replacement of r with its q-counterpart r_q.

$$a^\dagger a + aa^\dagger = \hbar_q = r_q\hbar_0 \ . \tag{2.3.2}$$

It is however not completely clear what one means with q-rational. The counterpart n_q of integer n is $n_q = (q^n - q^{-n})/(q - q^{-1})$. The two alternative definitions of r_q for $r = n_1/n_2$ would be

$$r_q = \frac{(q^r - q^{-r})}{q - q^{-1}}$$

and

$$r_q = \frac{m_q}{n_q} \ .$$

$q = exp(i2\pi/r)$ however implies that r_q vanishes in both cases so that the modification does not make sense.

A good guess for the commutation relations of quantum $SU(2)$ is as

$$\begin{aligned} [J_z, J_\pm] &= \pm J_\pm \ , \\ [J_+, J_-] &= r\hbar_0 \times \frac{q^{\frac{J_z}{r\hbar_0}} - q^{-r\frac{J_z}{r\hbar_0}}}{q - q^{-1}} \ , \\ q &= exp(i\frac{2\pi}{r}) \ . \end{aligned} \tag{2.3.3}$$

For the eigenstates of J_z with $J_z = mr\hbar_0$ the right hand side gives $exp(im2\pi/r) - exp(-im2\pi/r)$ just as for the standard quantum group for which one has $r = n$.

One could criticize the appearance of $\hbar(CP_2)$ in the denominator of r as something having no concrete interpretation: why the covering of CP_2 should reduce the unit of fermion number? The interpretation in terms of the scaling of metric does not leave however leave any other option and one should find some elegant interpretation for the formula before one can accept it. A possible interpretation in the case of CD braids is that fermion number $r = n_a/n_b$ for coverings is associated with single CP_2 page so that the n_b pages would carry fermion number n_a. If CP_2 corresponds to factor space and CD to covering, single pages would carry fermion number n_a but only $1/n_B$ CP_2 pages would be present so that again the unit of fermion number would be n_a.

2.4 Jones inclusions and generalization of the imbedding space

The original motivation for the generalization of the imbedding space was the idea that the pages of the Big Book would provide correlates for Jones inclusions. In the following an attempt to formulate this vision more precisely is carried out.

2.4.1 Basic facts about Jones inclusions

Here only basic facts about Jones inclusions are discussed. Appendix contains a more detailed discussion of inclusions of HFFs.

Jones inclusions defined by subgroups of $SL(2,C) \times SU(2)$

Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ have representation as $R_0^G \subset R^G$ with G a discrete subgroup of $SU(2)$. $SO(3)$ or $SU(2)$ can be interpreted as acting in CP_2 as rotations. On quantum spinors the action corresponds to double cover of G.

A more general choice for G would be as a discrete subgroup $G_a \times G_b \subset SL(2,C) \times SU(2) \times SU(2)$. Poincare invariance suggests that the subgroup of $SL(2,C)$ reduces either to a discrete subgroup of $SU(2)$ and in the case that the rotation are genuinely 3-dimensional (E^6, E^8), the only possible interpretation would be as isotropy group of a particle at rest. When the group acts on plane as in case of A_n and D_{2n}, it could be also assigned to a massless particle.

If the group involves boosts it contains an infinite number of elements and it is not clear whether this kind of situation is physically sensible. In this case Jones inclusion could be interpreted as an inclusion for the tensor product of G invariant algebras associated with CD and CP_2 degrees of freedom and one would have $\mathcal{M} : \mathcal{N} = \mathcal{M} : \mathcal{N}(G_a) \times \mathcal{M} : \mathcal{N}(G_b)$. Since the index increases as the order of G increases one has reasons to expect that in the case of $G_a = SL(2,C)$ $N_a = \infty$ implies larger $\mathcal{M} : \mathcal{N}(G_a) > 4$.

A possible interpretation is that the values $\mathcal{M} : \mathcal{N} \leq 4$ are analogous to bound state energies so that a discrete rotation group acting in the relative rotational degrees of freedom can act as a symmetry group whereas the values $\mathcal{M} : \mathcal{N} > 4$ are analogous to ionized states for which particles are almost freely moving with respect to each other with a constant velocity.

When one restricts the coefficients to G-invariant elements of Clifford algebra the Clifford field is G-invariant under the natural action of G. This allows two interpretations. Either the Clifford field is G invariant or that the Clifford field is defined in orbifold $CD/G_a \times CP_2/G_b$. CD/G_a is obtained by replacing hyperboloid H_a $(t^2 - x^2 - y^2 - z^2 = a^2)$ with H_a/G_a. These spaces have been considered as cosmological models having 3-space with finite volume [?] (also a lattice like structure could be in question).

The quantum phases associated with sub-groups of $SU(2)$

It is natural to identify quantum phase as that defined by the maximal cyclic subgroup for finite subgroups of $SU(2)$ and infinite subgroups of $SL(2,C)$. Before continuing a brief summary about quantum phases associated with finite subgroups of $SU(2)$ is in order. E_6 corresponds to $N = 24$ and $n = 3$ and E_8 to icosahedron with $N = 120$, $n = 5$ and Golden mean and the minimal value of n making possible universal topological quantum computer [?].

D_n and A_n have orders $2n$ and $n+1$ and act as symmetry groups of n-polygon and have n-element cyclic group as a maximal cyclic subgroup. For double covers the orders are twice this. Thus A_n *resp.* D_{2n} correspond to $q = exp(i\pi/n)$ *resp.* $q = exp(i\pi/2n)$. Note that the restriction $n \geq 3$ means geometrically that only non-trivial polygons are allowed.

2.4.2 Jones inclusions and the hierarchy of Planck constants

The anyonic arguments for the quantization of Planck constant suggest that one can assign separate scalings of Planck constant to CD and CP_2 degrees of freedom and that these scalings in turn reflect as scalings of $M^4\pm$ and CP_2 metrics. This is definitely not in accordance with the original TGD vision based on uniqueness of imbedding space but makes sense if space-time and imbedding space are emergent concepts as the hierarchy of number theoretical von Neumann algebra inclusions indeed suggests. Indeed, the scaling factors of CD and CP_2 metric remain non-fixed by the general uniqueness arguments since Cartesian product is in question.

Hierarchy of Planck constants and choice of quantization axis

Jones inclusions seem to relate in a natural manner to the selection of quantization axis.

1. In the case of CD the orbifold singularity is for all groups G_a except E_6 and E_8 the time-like plane M^2 corresponding to a radial ray through origin defining the quantization axis of angular momentum and intersecting light-cone boundary along a preferred light-like ray. For E_6 and E_8 (tedrahedral and icosahedral symmetries) the singularity consists of planes M^2 related by symmetries of G sharing time-like line M^1 and in this case there are several alternative identifications of the quantization axes as axis around which the maximal cyclic subgroup acts as rotations.

2. From this it should be obvious that Jones inclusions represented in this manner would relate very closely to the selection of quantization axes and provide a geometric representation for this selection at the level of space-time and configuration space. The existence of the preferred direction of quantization at a given level of dark matter level should have observable consequences. For instance, in cosmology this could mean a breaking of perfect rotational symmetry at dark matter space-time sheets. The interpretation would be as a quantum effect in cosmological length scales. An interesting question is whether the observed asymmetry of cosmic microwave background could have interpretation as a quantum effect in cosmological length and time scales.

Do factor spaces and coverings correspond to the two kinds of Jones inclusions?

What could be the interpretation of the singular coverings and and factor spaces? If both geodesic spheres of CP_2 are allowed $\mathcal{M}:\mathcal{N}=4$ could correspond to the allowance of cosmic strings and other analogous objects. This option is however asymmetric with respect to CD and CP_2 and the more plausible option is that the two kinds of Jones inclusions correspond to singular factor spaces and coverings.

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M}:\mathcal{N}<4$ and $\mathcal{M}:\mathcal{N}=4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M}:\mathcal{N}<4$ and it naturally corresponds to the coset spaces. For $\mathcal{M}:\mathcal{N}=4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.

2. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$. For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{CD}\hat{\times}G_a$ and $\hat{CP}_2\hat{\times}G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane. This picture is also consistent with the G singlets of the quantum states despite the fact that fermionic oscillator operators belong to non-trivial irreps of G.

Coverings and factors spaces form an algebra like structure

It is easy to see that coverings and factor spaces defining the pages of the Big Book form an algebra like structure.

1. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M}:\mathcal{N}<4$ and $\mathcal{M}:\mathcal{N}=4$, which both are labeled by a subset of discrete subgroups of $SU(2)$.

2. The discrete subgroups of $SU(2)$ with fixed quantization axis possess a well defined multiplication with product defined as the group generated by forming all possible products of group elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tedrahedral, octahedral, and icosahedral groups are the generators of this algebra.

By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2 generated by reflection, and tedrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized configuration space spinor fields in the configuration space labeled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

Connection between Jones inclusions, hierarchy of Planck constants, and finite number of spinor modes

The original generalization of the imbedding space to accommodate the hierarchy of Planck constants was based on the idea that the singular coverings and factor spaces associated with the causal diamond CD and CP_2, which appears as factors of $CD \times CP_2$ correspond somehow to Jones inclusions, and that the integers n_a and n_b characterizing the orders of maximal cyclic groups of groups G_a and G_b associated with the two Cartesian factors correspond to quantum phases $q = exp(i2\pi/n_i)$ in such a manner that singular factor spaces correspond to Jones inclusions with index $\mathcal{M} : \mathcal{N} < 4$ and coverings to those with index $\mathcal{M} : \mathcal{N} = 4$.

Since Jones inclusions are interpreted in terms of finite measurement resolution, the mathematical realization of this heuristic picture should rely on the same concept realized also by the fact that the number of non-zero modes for induced spinor fields is finite. This allows to consider two possible interpretations.

1. The finite number of modes defines an approximation to the hyper-finite factor of type II_1 defined by configuration space Clifford algebra.

2. The Clifford algebra spanned by fermionic oscillator operators is quantum Clifford algebra and corresponds to the somewhat nebulous object $\mathcal{N}/\mathcal{M}$ associated with the inclusion $\mathcal{M} \subset \mathcal{N}$ and coding the finite measurement resolution to a finite quantum dimension of the Clifford algebra. The fact that quantum dimension is smaller than the actual dimension would reflect correlations between spinor components so that they are not completely independent.

If the latter interpretation is correct then second quantized induced spinor fields should obey quantum variant of anticommutation relations reducing to ordinary anticommutation relations only for $n_a = n_b = 0$ (no singular coverings nor factor spaces). This would give the desired connection between inclusions and hierarchy of Planck constants. It is possible to have infinite number of quantum group like structure for $\hbar = \hbar_0$.

There are two quantum phases q and one should understand what is the phase that appears in the quantum variant of anti-commutation relations. A possible resolution of the problem relies on the observation that there are two kinds of number theoretic braids. The first kind of number theoretic braid is defined as the intersection of M_+ (or light-like curve of δM^4_+ in more general case) and of δM^4_+ projection of X^2. Second of braid is defined as the intersection of CP_2 projection of X^2 of homologically non-trivial sphere S^2_{II} of CP_2. The intuitive expectation is that these dual descriptions apply for light-like 3-surfaces associated *resp.* co-associative regions of space-time surface and that both descriptions apply at wormhole throats. The duality of these descriptions is guaranteed also at wormhole throats if physical Planck constant is given by $\hbar = r\hbar_0$, $r = \hbar(M^4)/\hbar(CP_2)$, so that only the ratio of the two Planck constants matters in commutation relations. This would suggest that it is $q = exp(i2\pi/r)$, which appears in quantum variant of anti-commutation relations of the induced spinor fields.

The action of $G_a \times G_b$ on configuration space spinors and spinor fields

The first question is what kind of measurement resolution is in question. In zero energy ontology the included states would typically correspond to insertion of zero energy states to the positive or negative part of the physical state in time scale below the time resolution defined by the time scale assignable to the smallest CD present in the zero energy state. Does the description in terms of G invariance apply in this case or does it relate only to time and length scale resolution whereas hierarchy of Planck constants would relate to angle resolution? Assume that this is the case.

The second question is how the idea about $\mathcal{M}$ as an included algebra defining finite measurement resolution and G invariance as a symmetry defining $\mathcal{M}$ as the included algebra relate to each other.

1. One cannot say that G creates states, which cannot be distinguished from each other. Rather G-invariant elements of $\mathcal{M}$ create states whose presence in the state cannot be detected.

2. For covering space option $\mathcal{M}$ represents states which are invariant under discrete subgroup of $SU(2)$ acting in the covering. States with integer spin would be below measurement resolution and only factional spins of form j/n would be observable. For factor space option $\mathcal{M}$ would represents states which are invariant under discrete subgroup of $SU(2)$ acting in H-say states with spin. States with spin which is multiple of n would be below measurement resolution. The situation would be very similar to each other. Number theoretic considerations and the fact that the number of fermionic oscillator operators is finite suggest that that for coverings the condition $L_z < 1$ and for factor spaces the condition $L_z < n$ is satisfied by the generators of Clifford algebra regarded as irreducible representation of G. For factor spaces the interpretation could be in terms of finite angular resolution $\Delta\phi \leq 2\pi/n$ excluding angular momenta $L_z \geq n$. For coverings the resolution would be related to rotations (or rather, braidings) as multiples of 2π: multiples $m2\pi$ $m \geq n$ cannot be distinguished from $m \ mod \ n$ multiples.

3. The minimal assumption is that integer orbital angular momenta are excluded for coverings and n-multiples are excluded for factors spaces. The stronger assumption would be that there is angular momentum cutoff. This point is however very delicate. The states with $j > n$ can be obtained as tensor products of representations with $j = m$. If entanglement is present one cannot anymore express the state as a product of $\mathcal{M}$ element and $\mathcal{N}$ element so that the states $j > n$ created in this manner would not be equivalent with those with $j \ mod \ n$. The replacement of the ordinary tensor product with Connes tensor product would indeed generate automatically entangled states and one could interpret Connes tensor product as a manner to create only the allowed states.

4. For quantum groups allow only finite number of representations up to some maximum spin determined by the integer n characterizing quantum phase q. This would mean angular momentum cutoff leaving only a finite number of representations of quantum group [?]. This fits nicely with what one obtains in the case of factor spaces. For coverings the new element is that the unit of spin becomes $1/n$: otherwise the situation seems to be similar. Quantum group like structure is obtained if the fermionic oscillator operators satisfy the quantum version of anti-commutation relations. The algebra would be very similar except that the orbital angular momentum labeling oscillator operators has different unit. Oscillator operators are naturally in irreducible representations of G and only the non-trivial representations of G are allowed.

5. Besides Jones inclusions corresponding to $\mathcal{M} : \mathcal{N} < 4$ there are inclusions with $\mathcal{M} : \mathcal{N} = 4$ to which one can also assign quantum phases. It would be natural to assign covering spaces and factor spaces to these two kinds of inclusions. For the minimal option excluding only the orbital angular momentum which are integers or multiples of n the fraction of excluded states is very small for coverings so that $\mathcal{M} : \mathcal{N} = 4$ is natural for this option. $\mathcal{M} : \mathcal{N} < 4$ would in turn correspond naturally to factor spaces.

6. Since the two kinds of number theoretic braids correspond to points which belong to M^2 or S^2, one might argue that several quantum anticommutation relations must be satisfied simultaneously. This is not the case since the eigen modes of D_{C-S} and hence also oscillator operators code information about partonic surface X^2 itself and also about $X^4(X^3_l)$ rather than being purely local objects. In the case of covering space the oscillator operators can be arranged to irreducible representations of G and in the case of factor space the oscillator operators are G-invariant.

One must distinguish between G invariance for configuration space spinors and spinor fields.

1. In the case of factor spaces 3-surface are G invariant so that there is no difference between spinors and spinor fields as far as G is considered. Irreducible representations of G would correspond to the superpositions of G-transforms of oscillator operators for a fixed G-invariant X^3_l.

2. For covering space option G-invariance would mean that 3-surface is a mere G-fold copy of single 3-surface. There is no obvious reason to assume this. Hence one cannot separate spinorial degrees of freedom from configuration space degrees of freedom since G affects both the spin degrees of freedom and the 3-surface. Irreducible representations of G would correspond to genuine configuration space spinor fields involving a superposition of G-transforms of also X^3_l. The presence of both orbital and spin degrees of freedom could provide alternative explanation for why $\mathcal{M}:\mathcal{N}=4$ holds true for covering space option.

If the fermionic oscillator algebra is interpreted as a representation for $\mathcal{N}/\mathcal{M}$, allowed fermionic oscillator operators belong to non-trivial irreps of G. One can however ask whether the many-fermion states created by these operators are G-invariant for some physical reason so that one would have kind of G-confinement forcing the states to be many-fermion states with standard unit of quantum numbers for coverings and integer multiples of n for factor spaces. This would conform with the ideas that anyonicity is a microscopic property not visible at the level of entire state and that many-fermion systems in the anyonic state resulting in strong coupling limit for ordinary value of $\hbar$ are in question. The processes changing the value of Planck constant would be phase transitions involving all fermions of the G-invariant state and would be slow for this reason. This would also contribute to the invisibility of dark matter.

2.4.3 Questions

What is the role of dimensions?

Could the dimensions of CD and CP_2 and the dimensions of spaces defined by the choice of the quantization axes play a fundamental role in the construction from the constraint that the fundamental group is non-trivial?

1. Suppose that the sub-manifold in question is geodesic sub-manifold containing the orbits of its points under Cartan subgroup defining quantization axes. A stronger assumption would be that the orbit of maximal compact subgroup is in question.

2. For M^{2n} Cartan group contains translations in time direction with orbit M^1 and Cartan subgroup of $SO(2n-1)$ and would be M^n so that $\hat{M^{2n}}$ would have a trivial fundamental group for $n>2$. Same result applies in massless case for which one has $SO(1,1)\times SO(2n-2)$ acts as Cartan subgroup. The orbit under maximal compact subgroup would not be in question.

3. For CP_2 homologically non-trivial geodesic sphere CP_1 contains orbits of the Cartan subgroup. For $CP_n = SU(n+1)/SU(n)\times U(1)$ having real dimension $2n$ the sub-manifold CP_{n-1} contains orbits of the Cartan subgroup and defines a sub-manifold with codimension 2 so that the dimensional restriction does not appear.

4. For spheres $S^{n-1}=SO(n)/SO(n-1)$ the dimension is $n-1$ and orbit of $SO(n-1)$ of point left fixed by Cartan subgroup $SO(2)\times ..$ would for $n=2$ consist of two points and S_{n-2} in more general case. Again co-dimension 2 condition would be satisfied.

What about holes of the configuration space?

One can raise analogous questions at the level of configuration space geometry. Vacuum extremals correspond to Lagrangian sub-manifolds $Y^2 \subset CP_2$ with vanishing induced Kähler form. They correspond to singularities of the configuration space ("world of classical worlds") and configuration space spinor fields should vanish for the vacuum extremals. Effectively this would mean a hole in configuration space, and the question is whether this hole could also naturally lead to the introduction of covering spaces and factor spaces of the configuration spaces. How much information about the general structure of the theory just this kind of decomposition might allow to deduce? This kind of singularities are infinite-dimensional variants of those discussed in catastrophe theory and this suggests that their understanding might be crucial.

Are more general inclusions of HFFs possible?

The proposed scenario could be criticized because discrete subgroups of $SU(2)$ are in a preferred position. The Jones inclusions considered correspond to quantum spinor representations of various quantum groups $SU(2)_q$, $q = exp(i2\pi/n)$. This explains the result $\mathcal{M}:\mathcal{N} \leq 4$. These representations are certainly in preferred role as far as configuration space spinor fields are considered but it is possible to assign a hierarchy of inclusions of HFFs labeled by quantum phase q with arbitrary representation of an arbitrary compact Lie group. These inclusions would be analogous to discrete states in the continuum $\mathcal{M}:\mathcal{N} > 4$.

Since the inclusions are characterized by single quantum phase $q = exp(i2\pi/n)$ in the case of compact Lie groups (Appendix), one can ask whether more general discrete groups than subgroups of $SU(2)$ should be allowed. The inclusions of HFFs associated with higher dimensional Lie groups have $\mathcal{M}:\mathcal{N} > 4$ and are analogous to bound states in continuum (Appendix). In the case of CP_2 this would allow to consider much more general sub-groups.

The question is therefore whether some principle selects subgroups of $SU(2)$. There are indeed good arguments supporting the hypothesis that only discrete Abelian subgroups of $SU(2)$ are possible.

1. The notion of number theoretic braid allows only the only subgroups of rotation group leaving M^2 invariant and sub-groups of $SU(3)$ leaving geodesic sphere S^2_i invariant. This would drop groups having genuinely 3-D action. In the case of $SU(3)$ discrete subgroups of $SO(3)$ or $U(2)$ remain under consideration. The geodesic sphere of type II is however analogous to North/South pole of S^2 and second phase factor associated with the coordinates (ξ^1, ξ^2) becomes redundant since $(|\xi^1|^2 + |\xi^2|^2)^{1/2}$ becomes infinite at S^2_{II} so that ξ^1/ξ^2 becomes appropriate coordinate. Hence action of $U(2)$ reduces to that of $SU(2)$ since ξ^1 and ξ^2 correspond to same value of color hyper charge associated with $U(1)$.

2. A physically attractive possibility is that $G_a \times G_b$ leaves the choice of quantization axes invariant. This condition makes sense also for coverings. This would leave only Abelian groups into consideration and drop D_{2n}, E_6, and E_8. It is quite possible that only these groups define sectors of the generalized imbedding space. This means that $G_b = Z_{n_1} \times Z_{n_2} \subset U(1)_I \times U(1)_Y \subset SU(2) \times U(1)_Y$ and even more general subgroups of $SU(3)$ (if non-commutativity is allowed) are a priori possible. Again the first argument reduces the list to cyclic subgroups of $SU(2)$.

3. The products of groups Z_n are also number theoretically in a very special position since they relate naturally to the finite cyclic extensions and also to the maximal Abelian extension of rationals. With this restriction on $G_a \times G_b$ one can consider the hypothesis that elementary particles correspond are maximally quantum critical systems left invariant by all groups $G_a \times G_b$ respecting a given choice of quantization axis and implying that darkness is associated only to field bodies and Planck constant becomes characterizer of interactions rather than elementary particles themselves.

2.4.4 How does the hierarchy of Planck constants affect the modified Dirac equation?

It is not quite obvious how $\hbar/\hbar_0 = \hbar(M^4)/\hbar(CP_2)$ and $\hbar(M^4)$ and $\hbar(CP_2)$ make themselves visible in the dynamics of the theory. To see what is involved some simple dimensional considerations are needed.

General view about the role of $\hbar$

The usual convention of putting $\hbar = 1$ simplifies things tremendously but when $\hbar$ is assumed to have a spectrum, one is forced to check how $\hbar$ appears in the theory.

1. $\hbar$ appears in the anticommutators of the induced fermion fields restricted to the points of the number theoretic braids. Standard canonical anticommutation rule states that the anticommutator $\{\overline{\Psi}\hat{\Gamma}^0(x_m), \Psi(x_n)\}$ equals to $\hbar\delta_{m,n}$. This is due to the dimension $\sqrt{\hbar}/L^{3/2}$ of the induced spinor field forcing the modified Dirac action to be proportional to $1/\hbar$. The overall scaling of the action does not matter at all since it implies only an overall scaling of the eigenvalue spectrum giving an additive constant to Kähler action. The scaling of M^4 and CP_2 metrics by the same factor induces the scaling of the modified gamma matrices $\hat{\Gamma}^\alpha$ by same factor which does not affect the value Kähler function apart from additive constant. This conforms with the Weyl invariance of Kähler action. One can therefore identify $\hbar$ as a scaling factor of M^4 metric when CP_2 metric is scaled down to its standard form. By introducing a scaled up M^4 coordinate, the standard form of metric is obtained but the size of causal diamond CD is scaled up so that various values of $\hbar$ label different sizes of CD. Therefore radiative corrections reflect breaking of the scaling invariance with respect to M^4 scalings for the preferred extremals.

2. General Coordinate Invariance forces to conclude that Kähler gauge potential is dimensionless so that Kähler action must be proportional to $\hbar_0/g_K^2$ or $\hbar/g_K^2$. The latter option would lead to non-sensible results since the action of CP_2 type vacuum extremals would be scaled up and gravitational constant would be extremely small for dark matter with large Planck constant. The first option is also consistent with the fact that the scaling of $1/\hbar$-factor in the modified Dirac action affects Kähler function only by an addition of a constant term.

3. It is important to notice the difference between Kähler gauge potential, which is classical field, and gauge potential of QFTs which is quantum field. By General Coordinate Invariance Kähler gauge potential is dimensionless whereas quantal gauge potential has different dimension $[\hbar/g]/L = [\sqrt{\hbar}]/L$. The instanton term in Kähler action is also dimensionless and the invariance under the overall scaling of $\hbar$ requires that the scaling factor equals to $k/4\pi$. Instanton term therefore brings in no dependence on $\hbar$ at the classical level and the character of CP breaking depends on $\hbar$ only via the breaking of scale invariance.

The book like structure of M^4 and CP_2 means that if the M^4 *resp.* CP_2 projection partonic surface corresponds to a light-like ray of M^2 or preferred geodesic sphere S_i^2 of CP_2, problems might be encountered since the value of $\hbar$ is non-unique. S_I^2 produces no problems since it corresponds to vanishing induced Kähler field so that vertices vanish. Partonic 2-surfaces with S_{II}^2 projection need not correspond to preferred extremals. Problems are avoided if these kind of 2-surfaces are not allowed as vertices. Since M-matrix characterizes zero energy state, this kind of condition can be posed at least formally.

An interesting question is what happens in exact quantum criticality (with respect to the change of Planck constant). Could one have a topological field theory in S_i^2 or factorizing QFT in M^2 as one might expect on basis of the observation that at criticality the theory reduces to a pure topological QFT or something analogous to it? For factorizing QFT in M^2 particle scattering is elastic [**?**]: particles just pass by and at most permute their momenta. The S-matrix reduces to a braiding S-matrix at the limits $v \rightarrow 0$ and $v \rightarrow c$ for particle velocities. The S-matrix of factorizing QFTs does not depend on $\hbar$ as is clear from the fact that it depends only on the rapidity differences of the incoming and outgoing particles: this can be seen also from exact some exact solutions to the defining relations of Zamolodchikov algebra [**?**]. Also the N-point functions of topological QFTs are independent of $\hbar$ since there is no coupling constant strength with dimensions of $\hbar$ (such as g^2 in gauge theory) and no fundamental mass parameter so that $\hbar\omega/m$ for some characteristic frequency could introduce $\hbar$ dependence.

Fusion rules suggest that also the N-point functions of conformal field theories can be made independent of Planck constant by a suitable scaling of conformal fields. This is suggested also by the fact that the commutation relations of conformal algebras (energy momentum tensor and Kac-Moody currents) allow indeed elimination of $\hbar$ completely by an appropriate scaling of generators. All this conforms with the notion that radiative corrections correspond to the breaking of scale invariance in M^4 degrees of freedom.

Do anyonic phases make $\hbar(M^4)$ and $\hbar(CP_2)$ separately visible?

There are good reasons to expect that something in the modified Dirac equation differentiates between different pages of book like structure associated with CD (causal diamond of M^4) and CP_2 realizing the hierarchy of phases with different Planck constants. The ratio $\hbar(CD)/\hbar(CP_2)$ is visible via radiative corrections and reflects the breaking of scale invariance associated with scalings of CD making itself manifest at the level of preferred extremals. One could however ask whether also information about $\hbar(CD)$ and $\hbar(CP_2)$ rather than only their ratio could be coded to the modified Dirac action.

The intuitive view is that non-standard values of Planck constant correspond to anyonic or at least potentially anyonic phases of matter with fractionization of quantum numbers. This suggests that the Kähler gauge potential of CP_2 extends to that in $CD \times CP_2$ and possess what gauge theorist would call a pure gauge part. This part could be present also in CD. Since Kähler gauge potential does not relate to a genuine gauge field ($U(1)$ gauge transformations correspond to symplectic transformations of CP_2 and are symmetries only for vacuum extremals), even the addition of a pure gauge might change physics. A more conservative assumption is that the pure gauge part cannot be eliminated by a non-singular global gauge transformation as it typically is in topological gauge theories defined by Chern-Simons action.

The modification of Kähler gauge potential in CD

The singular pure gauge part is visible in the modified Dirac equation in the covariant derivative D_α, which receives ΔA_α term: $D_\alpha \rightarrow D_\alpha + \Delta A_\alpha$.

Let us consider first what happens in the case of CD.

1. Let $M^2 \times E^2$ denote the decomposition of M^4 implied by the hierarchy of Planck constants and introduce cylindrical coordinates (t, z, ρ, ϕ).

2. The presence of M^2 singularity means that there z-axis represents an infinitely thin cylindrical hole in E^3 and homology group is thus non-trivial. Therefore one can one introduce a modification of Kähler gauge potential not allowed in E^3. The modification is $\Delta A_\phi = \Delta = k/n$. As a consequence, the phases $exp(im\phi)$ associated with the eigenstates of orbital angular momentum are transformed to $exp(i(m - k/n)\phi)$ and become many-valued unless one replaces CD with its n-fold covering, which is just what has been done.

3. That spin fractionization takes place becomes clear by studying the expression of the angular momentum current J_z for modified Dirac operator. If the first variation of D_K vanishes in absence of ΔA, there is no contribution to conserved charges from D_α term and only the change of Ψ under the symmetry transformation contributes. ΔA_ϕ however brings to the current associated with J_z an additional term which reduces to fermion current multiplied by ΔA when ΔA_ϕ constant. Genuine charge fractionization results but only if one cannot eliminate ΔA_ϕ by a gauge transformation, which is well defined for the entire partonic 2-surface. This is the case if X^2 encloses the tip of CD so that homological triviality of X^2 in $CD\backslash M^2$ can be seen as a necessary condition for anyonization.

4. ΔA modifies conserved currents associated with all those symmetries which affect the value of ϕ, in particular translations in the plane E^2. If one accepts the angular momentum fractionization in this manner, then only the projection of four-momentum to the plane M^2 is good quantum number. Angular momentum and square of transversal momentum would be additional good quantum numbers. That this is the case is suggested also by p-adic mass calculations and parton model of hadrons.

The modification of Kähler gauge potential in CP_2

In CP_2 degrees of freedom the homologically trivial geodesic sphere S^2_I and its homologically non-trivial counterpart S^2_{II} define candidates for the two backs of the CP_2 book.

1. First a summary of some basic facts about CP_2 is in order.

   (a) The Eguchi-Hanson coordinates (ξ^1, ξ^2) of CP_2 to which $U(2)$ by definition acts linearly are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi+\Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi-\Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \quad (2.4.1)$$

The ranges of the variables r, Θ, Φ, Ψ are $[0,\infty], [0,\pi], [0,4\pi], [0,2\pi]$ respectively. The different choices of quantization of I_3 and Y_A are related by $SU(3)$ rotations of these coordinates.

(b) In these coordinates Kähler gauge potential is given by

$$B_K = \frac{r(d\Psi + cos\Theta d\Phi)}{2F} \ , \ F = 1 + r^2 \ . \quad (2.4.2)$$

Kähler gauge potentials for different choices of quantization axes are related by $U(1)$ gauge transformation induced by the $SU(3)$ rotation in question.

(c) The standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \ \text{or equivalently} \ (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \ \text{or equivalently} \ (\Theta = \pi/2, \Phi = 0) \ .$$

2. The natural identification of the quantization axes corresponds to directions which remain invariant under $SO(2) \subset SO(3) \subset SU(3)$ for S^2_I. For S^2_{II} the action of $U(2)$ reduces to that of $SU(2)$ since and $U(1)$ inducing same phase rotation of both complex coordinates (ξ^1, ξ^2) is the natural identification defined the invariant manifolds which induce slicing of S^2 to circles parallel to equator. The circles to which ΔA can be assigned are circles "going around" S^2_I.

3. In the case of S^2_I different values of angle Φ correspond to the same point of S^2_I and rotations "around" S^2_I correspond to rotations $\Psi \rightarrow \Psi + \delta\Psi$ so that ΔA_Ψ would be naturally non-vanishing in this case. For S^2_{II} the roles of Φ and Ψ are changed.

4. ΔA would be thus of form $\Delta A_\Phi + \Delta A_\Psi$ if one accepts CP_2 book with two backs.

ΔA is necessary for charge fractionization

The introduction of ΔA explicitly into the modified Dirac equation means an explicit breaking of Lorentz invariance and color symmetry. The interpretation is as an imbedding space correlate for the choice of quantization axes. Lorentz symmetry is something sacred and this motivates the question whether the instanton term in modified Dirac action could be enough. It however seems impossible to understand charge fractionization without this term. A further justification for ΔA comes from the requirement that it makes both $\hbar(CD)$ and $\hbar(CP_2)$ visible in the fundamental physics rather than only their ratio $\hbar/\hbar_0 = \hbar(CD)/\hbar(CP_2)$. ΔA indeed implies charge fractionization as the following argument shows in more detail.

1. The values of ΔA relate to the values of integers n_a and n_b appearing in the integers n_a and n_b appearing in the expression of Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ and characterizing the orders of maximal cyclic subgroups associated with the covering. For singular coverings $\Delta A(CD) = 1/n_a$ ($\Delta A(CP_2) = 1/n_b$) would hold true naturally. For singular factor spaces one would have $\Delta A(CD) = n_a$ ($\Delta A(XP_2) = n_b$).

2. The full Kähler gauge potential would be of form $A + \Delta A(M^4) + \Delta A(CP_2)$. As consequence, the Chern-Simons term appearing at X^3_l is of form $A \wedge J + \Delta A(M^4) \wedge J + \Delta A(M^4) \wedge J$ and contains anomalous parts. $\Delta A(M^4)$ implies that $C - S$ action is non-vanishing also for 2-dimensional CP_2 projection at least when X^2 has homologically non-trivial CP_2 projection.

(a) As already explained, ΔA gives an anomalous contribution to spin and color hypercharge and isopin. The contribution is of form $\overline{\Psi}\hat{\Gamma}^{\alpha}\Delta A_{\phi}\Psi$ and boils down to a shift $\Delta S_z = \Delta A_{\phi}$ for single fermion state. Shift is in question whereas non-standard values of $\hbar$ imply scaling of the basic charge unit.

(b) What happens in the case of electro-weak charges is not quite obvious. Electro-weak gauge charges can be identified as fermionic Noether charges associated with D_K. Noether currents are of form $\overline{\Psi}\{\hat{\Gamma}^{\alpha}, Q\}\Psi$. The charge matrices associated with the couplings of photon and Z^0 are covariantly constant being combinations of matrices $P_{\pm} = (1 \pm \Gamma_9)/2$ (coupling to Kähler gauge potential), matrix $P_{\pm}J$, $J = J^{kl}\Sigma_{kl}$(coupling to the vectorial part of spinor curvature), $P_{\pm}(1-\gamma_5)J^{kl}\Sigma_{kl}$ (left handed coupling of Z^0). $\{\hat{\Gamma}^{\alpha}, Q\}$ does not contribute to the divergence of Noether current so that these Noether currents are conserved. W boson charge matrices are not covariantly constant. The non-conservation of corresponding Noether currents obviously reflects electro-weak symmetry breaking. There is no anomalous contribution to electro-weak gauge charges.

(c) An old idea of TGD - motivated by the fact that $U(2) \subset SU(3)$ can be identified with the holonomy group of spinor connection identifiable as $U(2)_{ew}$ - is that electro-weak gauge charges can be identified as color gauge charges assignable to color Noether currents whereas quark and gluon color resides in CP_2 partial waves for center of mass degrees of freedom of X^3 and thus in configuration space degrees of freedom. In fact, the vanishing of the first variation of D_K implies that these charges are vanishing so that the idea as such does not work and by previous observation need not to do so. The anomalous contribution is however non-vanishing since its first variation does not vanish and gives to the electro weak currents an anomalous contribution of form $\overline{\Psi}\hat{\Gamma}^{\alpha}(a\Delta A_{\Phi} + b\Delta A_{\Psi})\Psi$, where a and b depend on the detailed correspondence between electro-weak and color hyper-charge and isospin. Anomalous contributions to isospins are identifiable as such and anomalous Kähler contribution must be proportional to the anomalous color hyper-charge. Leptons (quarks) couple to $n = -3$ ($n = 1$) multiple of Kähler gauge potential so that the contribution to anomalous Kähler charge is of form $nk\Delta A_{\Psi}$, where $k = 1$ is the simplest guess. This predicts the shifts $\Delta I_{3,ew} = \Delta A_{\Phi}$ and $\Delta Q_K = n\Delta A_{\Psi}$ allowing to deduce the shifts of em and Z^0 charges.

3. ΔA affects the transversal part $D_K(X^2)$ of the modified Dirac operator via covariant derivative, which means that the preferred extremal is modified since the moduli of the eigenvalues are modified. Charge fractionization should also make itself visible as a selection of the preferred extremal of Kähler action through stationary phase approximation. The phase factor $exp(i\int Tr(QA\mu)dx^{\mu})$, where Q is the charge matrix characterizing particle assigned to the strands of number theoretic braid generates a correlation between the properties of preferred extremal and quantum numbers associated with X^3_l. If Q is replaced with the fractionized charge the desired correlation results but not otherwise if one believes following argument. Stationary phase approximation gives field equations in which charges at partonic strands play the role of sources: ΔA is not visible in the source nor in the motion of sources nor in the equations of the extremals of $C-S$ action. Thus it seems that extremum is not affected by the modification of Chern-Simons action unless very delicate effects are involved.

2.5 Vision about dark matter as phases with non-standard value of Planck constant

2.5.1 Dark rules

It is useful to summarize the basic phenomenological view about dark matter.

The notion of relative darkness

The essential difference between TGD and more conventional models of dark matter is that darkness is only relative concept.

1. Generalized imbedding space forms a book like structure and particles at different pages of the book are dark relative to each other since they cannot appear in the same vertex identified as the partonic 2-surface along which light-like 3-surfaces representing the lines of generalized Feynman diagram meet.

2. Particles at different space-time sheets act via classical gauge field and gravitational field and can also exchange gauge bosons and gravitons (as also fermions) provided these particles can leak from page to another. This means that dark matter can be even photographed [?]. This interpretation is crucial for the model of living matter based on the assumption that dark matter at magnetic body controls matter visible to us. Dark matter can also suffer a phase transition to visible matter by leaking between the pages of the Big Book.

3. The notion of standard value $\hbar_0$ of $\hbar$ is not a relative concept in the sense that it corresponds to rational $r = 1$. In particular, the situation in which both CD and CP_2 correspond to trivial coverings and factor spaces would naturally correspond to standard physics.

Is dark matter anyonic?

In [?] a detailed model for the Kähler structure of the generalized imbedding space is constructed. What makes this model non-trivial is the possibility that CP_2 Kähler form can have gauge parts which would be excluded in full imbedding space but are allowed because of singular covering/factor-space property. The model leads to the conclusion that dark matter is anyonic if the partonic 2-surface, which can have macroscopic or even astrophysical size, encloses the tip of CD within it. Therefore the partonic 2-surface is homologically non-trivial when the tip is regarded as a puncture. Fractional charges for anyonic elementary particles imply confinement to the partonic 2-surface and the particles can escape the two surface only via reactions transforming them to ordinary particles. This would mean that the leakage between different pages of the big book is a rare phenomenon. This could partially explain why dark matter is so difficult to observe.

Field body as carrier of dark matter

The notion of "field body" implied by topological field quantization is essential. There would be em, Z^0, W, gluonic, and gravitonic field bodies, each characterized by its one prime. The motivation for considering the possibility of separate field bodies seriously is that the notion of induced gauge field means that all induced gauge fields are expressible in terms of four CP_2 coordinates so that only single component of a gauge potential allows a representation as and independent field quantity. Perhaps also separate magnetic and electric field bodies for each interaction and identifiable as flux quanta must be considered. This kind of separation requires that the fermionic content of the flux quantum (say fermion and anti-fermion at the ends of color flux tube) is such that it conforms with the quantum numbers of the corresponding boson.

What is interesting that the conceptual separation of interactions to various types would have a direct correlate at the level of space-time topology. From a different perspective inspired by the general vision that many-sheeted space-time provides symbolic representations of quantum physics, the very fact that we make this conceptual separation of fundamental interactions could reflect the topological separation at space-time level.

p-Adic mass calculations for quarks encourage to think that the p-adic length scale characterizing the mass of particle is associated with its electromagnetic body and in the case of neutrinos with its Z^0 body. Z^0 body can contribute also to the mass of charged particles but the contribution would be small. It is also possible that these field bodies are purely magnetic for color and weak interactions. Color flux tubes would have exotic fermion and anti-fermion at their ends and define colored variants of pions. This would apply not only in the case of nuclear strings but also to molecules and larger structures so that scaled variants of elementary particles and standard model would appear in all length scales as indeed implied by the fact that classical electro-weak and color fields are unavoidable in TGD framework.

One can also go further and distinguish between magnetic field body of free particle for which flux quanta start and return to the particle and "relative field" bodies associated with pairs of particles. Very complex structures emerge and should be essential for the understanding the space-time correlates of various interactions. In a well-defined sense they would define space-time correlate for the conceptual analysis of the interactions into separate parts. In order to minimize confusion it should be emphasized that the notion of field body used in this chapter relates to those space-time correlates of interactions, which are more or less *static* and related to the formation of *bound states*.

2.5.2 Phase transitions changing Planck constant

The general picture is that p-adic length scale hierarchy corresponds to p-adic coupling constant evolution and hierarchy of Planck constants to the coupling constant evolution related to phase resolution. Both evolutions imply a book like structure of the generalized imbedding space.

Transition to large $\hbar$ phase and failure of perturbation theory

One of the first ideas was that the transition to large $\hbar$ phase occurs when perturbation theory based on the expansion in terms of gauge coupling constant ceases to converge: Mother Nature would take care of the problems of theoretician. The transition to large $\hbar$ phase obviously reduces the value of gauge coupling strength $\alpha \propto 1/\hbar$ so that higher orders in perturbation theory are reduced whereas the lowest order "classical" predictions remain unchanged. A possible quantitative formulation of the criterion is that maximal 2-particle gauge interaction strength parameterized as $Q_1Q_2\alpha$ satisfies the condition $Q_1Q_2\alpha \simeq 1$.

A justification for this picture would be that in non-perturbative phase large quantum fluctuations are present (as functional integral formalism suggests). At space-time level this could mean that space-time sheet is near to a non-deterministic vacuum extremal -at least if homologically trivial geodesic sphere defines the number theoretic braids. At certain critical value of coupling constant strength one expects that the transition amplitude for phase transition becomes very large. The resulting phase would be of course different from the original since typically charge fractionization would occur.

One should understand why the failure of the perturbation theory (expected to occur for $\alpha Q_1Q_2 > 1$) induces the reduction of Clifford algebra, scaling down of CP_2 metric, and whether the G-symmetry is exact or only approximate. A partial understanding already exists. The discrete G symmetry and the reduction of the dimension of Clifford algebra would have interpretation in terms of a loss of degrees of freedom as a strongly bound state is formed. The multiple covering of $M^4_\pm$ accompanying strong binding can be understood as an automatic consequence of G-invariance. A concrete realization for the binding could be charge fractionization which would not allow the particles bound on large light-like 3-surface to escape without transformation to ordinary particles.

Two examples perhaps provide more concrete view about this idea.

1. The proposed scenario can reproduce the huge value of the gravitational Planck constant. One should however develop a convincing argument why non-perturbative phase for the gravitating dark matter leads to a formation of $G_a\times$ covering of $CD\backslash M^2 \times CP_2\backslash S^2_I$ with the huge value of $\hbar_{eff} = n_a/n_b \simeq GM_1M_2/v_0$. The basic argument is that the dimensionless parameter $\alpha_{gr} = GM_1M_2/4\pi\hbar$ should be so small that perturbation theory works. This gives $\hbar_{gr} \geq GM_1M_2/4\pi$ so that order of magnitude is predicted correctly.

2. Color confinement represents the simplest example of a transition to a non-perturbative phase. In this case A_2 and $n = 3$ would be the natural option. The value of Planck constant would be 3 times higher than its value in perturbative QCD. Hadronic space-time sheets would be 3-fold coverings of $M^4_\pm$ and baryonic quarks of different color would reside on 3 separate sheets of the covering. This would resolve the color statistics paradox suggested by the fact that induced spinor fields do not possess color as spin like quantum number and by the facts that for orbifolds different quarks cannot move in independent CP_2 partial waves assignable to CP_2 cm degrees of freedom as in perturbative phase.

The mechanism of phase transition and selection rules

The mechanism of phase transition is at classical level similar to that for ordinary phase transitions. The partonic 2-surface decomposes to regions corresponding to difference values of $\hbar$ at quantum criticality in such a manner that regions in which induced Kähler form is non-vanishing are contained within single page of imbedding space. It might be necessary to assume that only a region corresponding to single value of $\hbar$ is possible for partonic 2-surfaces and $\delta CD \times CP_2$ so that quantum criticality would be associated with the intermediate state described by the light-like 3-surface. One could also see the phase transition as a leakage of X^2 from given page to another: this is like going through a closed door through a narrow slit between door and floor. By quantum criticality the points of number theoretic braid are already in the slit.

As in the case of ordinary phase transitions the allowed phase transitions must be consistent with the symmetries involved. This means that if the state is invariant under the maximal cyclic subgroups G_a and G_b then also the final state must satisfy this condition. This gives constraints to the orders of maximal cyclic subgroups Z_a and Z_b for initial and final state: $n(Z_{a_i})$ *resp.* $n(Z_{b_i}))$ must divide $n(Z_{a_f})$ *resp.* $n(Z_{b_f}$ or vice versa in the case that factors of Z_i do not leave invariant the states. If this is the case similar condition must hold true for apppropriate subgroups. In particular, powers of prime Z_{p^n}, $n = 1, 2, ...$ define hierarchies of allowed phase transitions.

2.5.3 Coupling constant evolution and hierarchy of Planck constants

If the overall vision is correct, quantum TGD would be characterized by two kinds of couplings constant evolutions. p-Adic coupling constant evolution would correspond to length scale resolution and the evolution with respect to Planck constant to phase resolution. Both evolution would have number theoretic interpretation.

Evolution with respect to phase resolution

The coupling constant evolution in phase resolution in p-adic degrees of freedom corresponds to emergence of algebraic extensions allowing increasing variety of phases $exp(i2\pi/n)$ expressible p-adically. This evolution can be assigned to the emergence of increasingly complex quantum phases and the increase of Planck constant.

One expects that quantum phases $q = exp(i\pi/n)$ which are expressible using only iterated square root operation are number theoretically very special since they correspond to algebraic extensions of p-adic numbers obtained by an iterated square root operation, which should emerge first. Therefore systems involving these values of q should be especially abundant in Nature. That arbitrarily high square roots are involved as becomes clear by studying the case $n = 2^k$: $cos(\pi/2^k) = \sqrt{[1 + cos(\pi/2^{k-1})]/2}$.

These polygons are obtained by ruler and compass construction and Gauss showed that these polygons, which could be called Fermat polygons, have $n_F = 2^k \prod_s F_{n_s}$ sides/vertices: all Fermat primes F_{n_s} in this expression must be different. The analog of the p-adic length scale hypothesis emerges since larger Fermat primes are near a power of 2. The known Fermat primes $F_n = 2^{2^n} + 1$ correspond to $n = 0, 1, 2, 3, 4$ with $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65537$. It is not known whether there are higher Fermat primes. $n = 3, 5, 15$-multiples of p-adic length scales clearly distinguishable from them are also predicted and this prediction is testable in living matter. I have already earlier considered the possibility that Fermat polygons could be of special importance for cognition and for biological information processing [**?**].

This condition could be interpreted as a kind of resonance condition guaranteing that scaled up sizes for space-time sheets have sizes given by p-adic length scales. The numbers n_F could take the same role in the evolution of Planck constant assignable with the phase resolution as Mersenne primes have in the evolution assignable to the p-adic length scale resolution.

The Dynkin diagrams of exceptional Lie groups E_6 and E_8 are exceptional as subgroups of rotation group in the sense that they cannot be reduced to symmetry transformations of plane. They correspond to the symmetry group $S_4 \times Z_2$ of tedrahedron and $A_5 \times Z_2$ of dodecahedron or its dual polytope icosahedron (A_5 is 60-element subgroup of S_5 consisting of even permutations). Maximal cyclic subgroups are Z_4 and Z_5 and and thus their orders correspond to Fermat polygons. Interestingly, $n = 5$ corresponds to minimum value of n making possible topological quantum computation using braids and also to Golden Mean.

Is there a correlation between the values of p-adic prime and Planck constant?

The obvious question is whether there is a correlation between p-adic length scale and the value of Planck constant. One-to-one correspondence is certainly excluded but loose correlation seems to exist.

1. In [**?**] the information about the number theoretic anatomy of Kähler coupling strength is combined with input from p-adic mass calculations predicting α_K to be the value of fine structure

constant at the p-adic length scale associated with electron. One can also develop an explicit expression for gravitational constant assuming its renormalization group invariance on basis of dimensional considerations and this model leads to a model for the fraction of volume of the wormhole contact (piece of CP_2 type extremal) from the volume of CP_2 characterizing gauge boson and for similar volume fraction for the piece of the CP_2 type vacuum extremal associated with fermion.

2. The requirement that gravitational constant is renormalization group invariant implies that the volume fraction depends logarithmically on p-adic length scale and Planck constant (characterizing quantum scale). The requirement that this fraction in the range $(0,1)$ poses a correlation between the rational characterizing Planck constant and p-adic length scale. In particular, for space-time sheets mediating gravitational interaction Planck constant must be larger than $\hbar_0$ above length scale which is about .1 Angstrom. Also an upper bound for $\hbar$ for given p-adic length scale results but is very large. This means that quantum gravitational effects should become important above atomic length scale [?].

2.6 Some applications

Below some applications of the hierarchy of Planck constants as a model of dark matter are briefly discussed. The range of applications varying from elementary particle physics to cosmology and I hope that this will convince the reader that the idea has strong physical motivations.

2.6.1 A simple model of fractional quantum Hall effect

The generalization of the imbedding space suggests that it could possible to understand fractional quantum Hall effect [?] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \tag{2.6.1}$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15..., 2/3, 3/5, 4/7, 5/9, 6/11, 7/13...,$ $5/3, 8/5, 11/7, 14/9...4/3, 7/5, 10/7, 13/9..., 1/5, 2/9, 3/13..., 2/7, 3/11..., 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [?].

The model of Laughlin [?] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [?]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of imbedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are $2 \times 2 = 4$ combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this one can consider two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. In the following a model based on option II for which the number of states is conserved in the phase transition changing $\hbar$.

1. The easiest manner to understand the observed fractions is by assuming that both CD and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

2. With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

3. Both $\nu = 1/2$ and $\nu = 5/2$ state has been observed [**?**, **?**]. The fractionized charge is $e/4$ in the latter case [**?**, **?**]. Since $n_i > 3$ holds true if coverings and factor spaces are correlates for Jones inclusions, this requires $n_a = 4$ and $n_b = 8$ for $\nu = 1/2$ and $n_b = 4$ and $n_a = 10$ for $\nu = 5/2$. Correct fractionization of charge is predicted. For $n_b = 2$ also Z_2 would appear as the fundamental group of the covering space. Filling fraction 1/2 corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [**?**]. $n_b = 2$ is inconsistent with the observed fractionization of electric charge for $\nu = 5/2$ and with the vision inspired by Jones inclusions.

4. A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

5. Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e\int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F \int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

6. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6 \times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the assumption about charge fractionization -although consistent with fractionization for $\nu = 5/2$, is rather adhoc. Therefore the model can be taken as a warm-up exercise only. In [**?**], where the delicacies of Kähler structure of generalized imbedding space are discussed, also a more detailed of QHE is discussed.

2.6.2 Gravitational Bohr orbitology

The basic question concerns justification for gravitational Bohr orbitology [**?**]. The basic vision is that visible matter identified as matter with $\hbar = \hbar_0$ ($n_a = n_b = 1$) concentrates around dark matter at Bohr orbits for dark matter particles. The question is what these Bohr orbits really mean. Should one in improved approximation relate Bohr orbits to 3-D wave functions for dark matter as ordinary Bohr rules would suggest or do the Bohr orbits have some deeper meaning different from that in wave mechanics. Anyonic variants of partonic 2-surfaces with astrophysical size are a natural guess for the generalization of Bohr orbits.

Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive [**?**].

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation -or at least Bohr rules with appropriate interpretation - would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

Prediction for the parameter v_0

One of the key questions relate to the value of the parameter v_0. Before the introduction of the hierarchy of Planck constants I proposed that the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n-fold: much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. The planetary mass ratios can be produced with an accuracy better than 10 per cent assuming ruler and compass phases.

Further predictions

The study of inclinations (tilt angles with respect to the Earth's orbital plane) leads to a concrete model for the quantum evolution of the planetary system. Only a stepwise breaking of the rotational symmetry and angular momentum Bohr rules plus Newton's equation (or geodesic equation) are needed, and gravitational Shrödinger equation holds true only inside flux quanta for the dark matter.

1. During pre-planetary period dark matter formed a quantum coherent state on the (Z^0) magnetic flux quanta (spherical cells or flux tubes). This made the flux quantum effectively a single rigid body with rotational degrees of freedom corresponding to a sphere or circle (full SO(3) or SO(2) symmetry).

2. In the case of spherical shells associated with inner planets the $SO(3) \rightarrow SO(2)$ symmetry breaking led to the generation of a flux tube with the inclination determined by m and j and a further symmetry breaking, kind of an astral traffic jam inside the flux tube, generated a planet moving inside flux tube. The semiclassical interpretation of the angular momentum algebra predicts the inclinations of the inner planets. The predicted (real) inclinations are 6 (7) resp. 2.6 (3.4) degrees for Mercury resp. Venus). The predicted (real) inclination of the Earth's spin axis is 24 (23.5) degrees.

3. The $v_0 \rightarrow v_0/5$ transition allowing to understand the radii of the outer planets in the model of Da Rocha and Nottale can be understood as resulting from the splitting of (Z^0) magnetic flux tube to five flux tubes representing Earth and outer planets except Pluto, whose orbital parameters indeed differ dramatically from those of other planets. The flux tube has a shape of a disk with a hole glued to the Earth's spherical flux shell.

   It is important to notice that effectively a multiplication $n \rightarrow 5n$ of the principal quantum number is in question. This allows to consider also alternative explanations. Perhaps external gravitational perturbations have kicked dark matter from the orbit or Earth to $n = 5k$, $k = 2, 3, ..., 7$ orbits: the fact that the tilt angles for Earth and all outer planets except Pluto are nearly the same, supports this explanation. Or perhaps there exist at least small amounts of dark matter at all orbits but visible matter is concentrated only around orbits containing some critical amount of dark matter and these orbits satisfy $n \ mod \ 5 = 0$ for some reason.

4. A remnant of the dark matter is still in a macroscopic quantum state at the flux quanta. It couples to photons as a quantum coherent state but the coupling is extremely small due to the gigantic value of $\hbar_{gr}$ scaling alpha by $\hbar/\hbar_{gr}$: hence the darkness.

The rather amazing coincidences between basic bio-rhythms and the periods associated with the states of orbits in solar system suggest that the frequencies defined by the energy levels of the gravitational Schrödinger equation might entrain with various biological frequencies such as the cyclotron frequencies associated with the magnetic flux tubes. For instance, the period associated with $n = 1$ orbit in the case of Sun is 24 hours within experimental accuracy for v_0.

Comparison with Bohr quantization of planetary orbits

The predictions of the generalization of the p-adic length scale hypothesis are consistent with the TGD based model for the Bohr quantization of planetary orbits and some new non-trivial predictions follow.

1. The model can explain the enormous values of gravitational Planck constant $\hbar_{gr}/\hbar_0 =\simeq GMm/v_0) = n_a/n_b$. The favored values of this parameter should correspond to n_{F_a}/n_{F_b} so that the mass ratios $m_1/m_2 = n_{F_{a,1}}n_{F_{b,2}}/n_{F_{b,1}}n_{F_{a,2}}$ for planetary masses should be preferred. The general prediction $GMm/v_0 = n_a/n_b$ is of course not testable.

2. Nottale [?] has suggested that also the harmonics and sub-harmonics of $\hbar_{gr}$ are possible and in fact required by the model for planetary Bohr orbits (in TGD framework this is not absolutely necessary [?]). The prediction is that favored values of n should be of form $n_F = 2^k \prod F_i$ such that F_i appears at most once. In Nottale's model for planetary orbits as Bohr orbits in solar system [?] $n = 5$ harmonics appear and are consistent with either $n_{F,a} \rightarrow F_1 n_{F_a}$ or with $n_{F,b} \rightarrow n_{F_b}/F_1$ if possible.

The prediction for the ratios of planetary masses can be tested. In the table below are the experimental mass ratios $r_{exp} = m(pl)/m(E)$, the best choice of $r_R = [n_{F,a}/n_{F,b}] * X$, X common factor for all planets, and the ratios $r_{pred}/r_{exp} = n_{F,a}(planet)n_{F,b}(Earth)/n_{F,a}(Earth)n_{F,b}(planet)$. The deviations are at most 2 per cent.

$planet$	Me	V	E	M	J
y	$\frac{2^{13}\times 5}{17}$	$2^{11}\times 17$	$2^9\times 5\times 17$	$2^8\times 17$	$\frac{2^{23}\times 5}{7}$
y/x	1.01	.98	1.00	.98	1.01
$planet$	S	U	N	P	
y	$2^{14}\times 3\times 5\times 17$	$\frac{2^{21}\times 5}{17}$	$\frac{2^{17}\times 17}{3}$	$\frac{2^{4}\times 17}{3}$	
y/x	1.01	.98	.99	.99	

Table 1. The table compares the ratios $x = m(pl)/(m(E)$ of planetary mass to the mass of Earth to prediction for these ratios in terms of integers n_F associated with Fermat polygons. y gives the best fit for the allowed factors of the known part y of the rational $n_{F,a}/n_{F,b} = yX$ characterizing planet, and the ratios y/x. Errors are at most 2 per cent.

A stronger prediction comes from the requirement that GMm/v_0 equals to $n = n_{F_a}/n_{F,b}$ $n_F = 2^k \prod_k F_{n_k}$, where $F_i = 2^{2^i} + 1$, $i = 0, 1, 2, 3, 4$ is Fibonacci prime. The fit using solar mass and Earth mass gives $n_F = 2^{254} \times 5 \times 17$ for $1/v_0 = 2044$, which within the experimental accuracy equals to the value $2^{11} = 2048$ whose powers appear as scaling factors of Planck constant in the model for living matter [?]. For $v_0 = 4.6 \times 10^{-4}$ reported by Nottale the prediction is by a factor 16/17.01 too small (6 per cent discrepancy).

A possible solution of the discrepancy is that the empirical estimate for the factor GMm/v_0 is too large since m contains also the the visible mass not actually contributing to the gravitational force between dark matter objects whereas M is known correctly. The assumption that the dark mass is a fraction $1/(1 + \epsilon)$ of the total mass for Earth gives

$$1 + \epsilon = \frac{17}{16} \tag{2.6.2}$$

in an excellent approximation. This gives for the fraction of the visible matter the estimate $\epsilon = 1/16 \simeq 6$ per cent. The estimate for the fraction of visible matter in cosmos is about 4 per cent so that estimate is reasonable and would mean that most of planetary and solar mass would be also dark (as a matter dark energy would be in question).

That $v_0(eff) = v_0/(1-\epsilon) \simeq 4.6 \times 10^{-4}$ equals with $v_0(eff) = 1/(2^7 \times F_2) = 4.5956 \times 10^{-4}$ within the experimental accuracy suggests a number theoretical explanation for the visible-to-dark fraction.

The original unconsciously performed identification of the gravitational and inertial Planck constants leads to some confusing conclusions but it seems that the new view about the quantization of Planck constants resolves these problems and allows to see $\hbar_{gr}$ as a special case of $\hbar_I$.

1. $\hbar_{gr}$ is proportional to the product of masses of interacting systems and not a universal constant like $\hbar$. One can however express the gravitational Bohr conditions as a quantization of circulation $\oint v \cdot dl = n(GM/v_0)\hbar_0$ so that the dependence on the planet mass disappears as required by Equivalence Principle. This would suggest that gravitational Bohr rules relate to velocity rather than inertial momentum as is indeed natural. The quantization of circulation is consistent with the basic prediction that space-time surfaces are analogous to Bohr orbits.

2. $\hbar_{gr}$ seems to characterize a relationship between planet and central mass and quite generally between two systems with the property that smaller system is topologically condensed at the space-time sheet of the larger system. Thus it would seem that $\hbar_{gr}$ is not a universal constant and cannot correspond to a special value of ordinary Planck constant. Certainly this would be the case if $\hbar_I$ is quantized as λ^k-multiplet of ordinary Planck constant with $\lambda \simeq 2^{11}$.

The recent view about the quantization of Planck constant in terms of coverings of CD seems to resolve these problems.

1. The integer quantization of Planck constants is consistent with the huge values of gravitational Planck constant within experimental resolution and the killer test for $\hbar = \hbar_{gr}$ emerges if one takes seriously the stronger prediction $\hbar_{gr} = n_{F,a}/n_{F,b}$.

2. One can also regard $\hbar_{gr}$ as ordinary Planck constant $\hbar_{eff}$ associated with the space-time sheet along which the masses interact provided each pair (M, m_i) of masses is characterized by its own sheets. These sheets could correspond to flux tube like structures carrying the gravitational flux of dark matter. If these sheets corresponds to n_{F_a}-fold covering of CD, one can understand $\hbar_{gr}$ as a particular instance of the $\hbar_{eff}$.

Quantum Hall effect and dark anyonic systems in astrophysical scales

Bohr orbitology could be understood if dark matter concentrates on 2-dimensional partonic surfaces usually assigned with elementary particles and having size of order CP_2 radius. The interpretation is in terms of wormhole throats assignable to topologically condensed CP_2 type extremals (fermions) and pairs of them assignable to wormhole contacts (gauge bosons). Wormhole throat defines the light-like 3-surface at which the signature of metric of space-time surface changes from Minkowskian to Euclidian.

Large value of Planck constant would allow partons with astrophysical size. Since anyonic systems are 2-dimensional, the natural idea is that dark matter corresponds to systems carrying large fermion number residing at partonic 2-surfaces of astrophysical size and that visible matter condenses around these. Not only black holes but also ordinary stars, planetary systems, and planets could correspond at the level of dark matter to atom like structures consisting of anyonic 2-surfaces which can have complex topology (flux tubes associated with planetary orbits connected by radial flux tubes to the central spherical anyonic surface) . Charge and spin fractionization are key features of anyonic systems and Jones inclusions inspiring the generalization of imbedding space indeed involve quantum groups central in the modeling of anyonic systems. Hence one has could hopes that a coherent theoretical picture could emerge along these lines.

This seems to be the case. Anyons and charge and spin fractionization are discussed in detail [**?**] and leads to a precise identification of the delicacies involved with the Kähler gauge potential of CP_2 Kähler form in the sectors of the generalized imbedding space corresponding to various pages of boook like structures assignable to CD and CP_2. The basic outcome is that anyons correspond geometrically to partonic 2-surfaces at the light-like boundaries of CD containing the tip of CD inside them. This is what gives rise to charge fractionization and also to confinement like effects since elementary particles in anyonic states cannot as such leak to the other pages of the generalized imbedding space. G_a and G_b invariance of the states imply that fractionization occurs only at single particle level and total charge is integer valued.

This picture is much more flexible that that based on G_a symmetries of CD orbifold since partonic 2-surfaces do not possess any orbifold symmetries in CD sector anymore. In this framework various astrophysical structures such as spokes and circles would be parts of anyonic 2-surfaces with complex topology representing quantum geometrically quantum coherence in the scale of say solar system. Planets would have formed by the condensation of ordinary matter in the vicinity of the anyonic matter. This would predict stars, planetary system, and even planets to have onion-like structure consisting of shells at the level of dark matter. Similar conclusion is suggested also by purely classical model for the final state of star predicting that matter is strongly concentrated at the surface of the star [?].

Anyonic view about blackholes

A new element to the model of black hole comes from the vision that black hole horizon as a light-like 3-surface corresponds to a light-like orbit of light-like partonic 2-surface. This allows two kinds of black holes. Fermion like black hole would correspond to a deformed CP_2 type extremal which Euclidian signature of metric and topologically condensed at a space-time sheet with a Minkowskian signature. Boson like black hole would correspond to a wormhole contact connecting two space-time sheets with Minkowskian signature. Wormhole contact would be a piece deformed CP_2 type extremal possessing two light-like throats defining two black hole horizons very near to each other. It does not seem absolutely necessary to assume that the interior metric of the black-hole is realized in another space-time sheet with Minkowskian signature.

Second new element relates to the value of Planck constant. For $\hbar_{gr} = 4GM^2$ the Planck length $L_P(\hbar) = \sqrt{\hbar G}$ equals to Schwartschild radius and Planck mass equals to $M_P(\hbar) = \sqrt{\hbar/G} = 2M$. If the mass of the system is below the ordinary Planck mass: $M \leq m_P(\hbar_0)/2 = \sqrt{\hbar_0/4G}$, gravitational Planck constant is smaller than the ordinary Planck constant.

Black hole surface contains ultra dense matter so that perturbation theory is not expected to converge for the standard value of Planck constant but do so for gravitational Planck constant. If the phase transition increasing Planck constant is a friendly gesture of Nature making perturbation theory convergent, one expects that only the black holes for which Planck constant is such that $GM^2/4\pi\hbar < 1$ holds true are formed. Black hole entropy -being proportional to $1/\hbar$- is of order unity so that TGD black holes are not very entropic.

If the partonic 2-surface surrounds the tip of causal diamond CD, the matter at its surface is in anyonic state with fractional charges. Anyonic black hole can be seen as single gigantic elementary particle stabilized by fractional quantum numbers of the constituents preventing them from escaping from the system and transforming to ordinary visible matter. A huge number of different black holes are possible for given value of $\hbar$ since there is infinite variety of pairs (n_a, n_b) of integers giving rise to same value of $\hbar$.

One can imagine that the partonic surface is not exact sphere except for ideal black holes but contains large number of magnetic flux tubes giving rise to handles. Also a pair of spheres with different radii can be considered with surfaces of spheres connected by braided flux tubes. The braiding of these handles can represent information and one can even consider the possibility that black hole can act as a topological quantum computer. There would be no sharp difference between the dark parts of black holes and those of ordinary stars. Only the volume containing the complex flux tube structures associated with the orbits of planets and various objects around star would become very small for black hole so that the black hole might code for the topological information of the matter collapsed into it.

2.6.3 Accelerating periods of cosmic expansion as phase transitions increasing the value of Planck constant

There are several pieces of evidence for accelerated expansion, which need not mean cosmological constant, although this is the interpretation adopted in [?]. Quantum cosmology predicts that astrophysical objects do not follow cosmic expansion except in jerk-wise quantum leaps increasing the value of the gravitational Planck constant. This assumption provides explanation for the apparent cosmological constant. Also planets are predicted to expand in this manner. This provides a new version of Expanding Earth theory originally postulated to explain the intriguing findings suggesting that continents have once formed a connected continent covering the entire surface of Earth but with radius which was one half of the recent one.

The four pieces of evidence for accelerated expansion

1. Supernovas of type Ia

Supernovas of type Ia define standard candles since their luminosity varies in an oscillatory manner and the period is proportional to the luminosity. The period gives luminosity and from this the distance can be deduced by using Hubble's law: $d = cz/H_0$, H_0 Hubble's constant. The observation was that the farther the supernova was the more dimmer it was as it should have been. In other words, Hubble's constant increased with distance and the cosmic expansion was accelerating rather than decelerating as predicted by the standard matter dominated and radiation dominated cosmologies.

2. Mass density is critical and 3-space is flat

It is known that the contribution of ordinary and dark matter explaining the constant velocity of distance stars rotating around galaxy is about 25 per cent from the critical density. Could it be that total mass density is critical?

From the anisotropy of cosmic microwave background one can deduce that this is the case. What criticality means geometrically is that 3-space defined as surface with constant value of cosmic time is flat. This reflects in the spectrum of microwave radiation. The spots representing small anisotropies in the microwave background temperature is 1 degree and this correspond to flat 3-space. If one had dark matter instead of dark energy the size of spot would be .5 degrees!

Thus in a cosmology based on general relativity cosmological constant remains the only viable option. The situation is different in TGD based quantum cosmology based on sub-manifold gravity and hierarchy of gravitational Planck constants.

3. The energy density of vacuum is constant in the size scale of big voids

It was observed that the density of dark energy would be constant in the scale of 10^8 light years. This length scale corresponds to the size of big voids containing galaxies at their boundaries.

4. Integrated Sachs-Wolf effect

Also so called integrated Integrated Sachs-Wolf effect supports accelerated expansion. Very slow variations of mass density are considered. These correspond to gravitational potentials. Cosmic expansion tends to flatten them but mass accretion to form structures compensates this effect so that gravitational potentials are unaffected and there is no effect of CMB. Situation changes if dark matter is replaced with dark energy the accelerated expansion flattening the gravitational potentials wins the tendency of mass accretion to make them deeper. Hence if photon passes by an over-dense region, it receives a little energy. Similarly, photon loses energy when passing by an under-dense region. This effect has been observed.

Accelerated expansion in classical TGD

The minimum TGD based explanation for accelerated expansion involves only the fact that the imbeddings of critical cosmologies correspond to accelerated expansion. A more detailed model allows to understand why the critical cosmology appears during some periods.

The first observation is that critical cosmologies (flat 3-space) imbeddable to 8-D imbedding space H correspond to negative pressure cosmologies and thus to accelerating expansion. The negativity of the counterpart of pressure in Einstein tensor is due to the fact that space-time sheet is forced to be a 4-D surface in 8-D imbedding space. This condition is analogous to a force forcing a particle at the surface of 2-sphere and gives rise to what could be called constraint force. Gravitation in TGD is sub-manifold gravitation whereas in GRT it is manifold gravitation. This would be minimum interpretation involving no assumptions about what mechanism gives rise to the critical periods.

Accelerated expansion and hierarchy of Planck constants

One can go one step further and introduce the hierarchy of Planck constants. The basic difference between TGD and GRT based cosmologies is that TGD cosmology is quantum cosmology. Smooth cosmic expansion is replaced by an expansion occurring in discrete jerks corresponding to the increase of gravitational Planck constant. At space-time level this means the replacement of 8-D imbedding space H with a book like structure containing almost-copies of H with various values of Planck constant as pages glued together along critical manifold through which space-time sheet can leak between sectors with different values of $\hbar$. This process is the geometric correlate for the the phase transition changing the value of Planck constant.

During these phase transition periods critical cosmology applies and predicts automatically accelerated expansion. Neither genuine negative pressure due to "quintessence" nor cosmological constant is needed. Note that quantum criticality replaces inflationary cosmology and predicts a unique cosmology apart from single parameter. Criticality also explains the fluctuations in microwave temperature as long range fluctuations characterizing criticality.

Accelerated expansion and flatness of 3-cosmology

Observations 1) and 2) about super-novae and critical cosmology (flat 3-space) are consistent with this cosmology. In TGD dark energy must be replaced with dark matter because the mass density is critical during the phase transition. This does not lead to wrong sized spots since it is the increase of Planck constant which induces the accelerated expansion understandable also as a constraint force due to imbedding to H.

The size of large voids is the characteristic scale

The TGD based model in its simplest form model assigns the critical periods of expansion to large voids of size 10^8 ly. Also larger and smaller regions can express similar periods and dark space-time sheets are expected to obey same universal "cosmology" apart from a parameter characterizing the duration of the phase transition. Observation 3) that just this length scale defines the scale below which dark energy density is constant is consistent with TGD based model.

The basic prediction is jerkwise cosmic expansion with jerks analogous to quantum transitions between states of atom increasing the size of atom. The discovery of large voids with size of order 10^8 ly but age much longer than the age of galactic large voids conforms with this prediction. One the other hand, it is known that the size of galactic clusters has not remained constant in very long time scale so that jerkwise expansion indeed seems to occur.

Do cosmic strings with negative gravitational mass cause the phase transition inducing accelerated expansion

Quantum classical correspondence is the basic principle of quantum TGD and suggest that the effective antigravity manifested by accelerated expansion might have some kind of concrete space-time correlate. A possible correlate is super heavy cosmic string like objects at the center of large voids which have negative gravitational mass under very general assumptions. The repulsive gravitational force created by these objects would drive galaxies to the boundaries of large voids. At some state the pressure of galaxies would become too strong and induce a quantum phase transition forcing the increase of gravitational Planck constant and expansion of the void taking place much faster than the outward drift of the galaxies. This process would repeat itself. In the average sense the cosmic expansion would not be accelerating.

2.6.4 Phase transition changing Planck constant and expanding Earth theory

TGD predicts that cosmic expansion at the level of individual astrophysical systems does not take place continuously as in classical gravitation but through discrete quantum phase transitions increasing gravitational Planck constant and thus various quantum length and time scales. The reason would be that stationary quantum states for dark matter in astrophysical length scales cannot expand. One would have the analog of atomic physics in cosmic scales. Increases of $\hbar$ by a power of two are favored in these transitions but also other scalings are possible.

This has quite far reaching implications.

1. These periods have a highly unique description in terms of a critical cosmology for the expanding space-time sheet. The expansion is accelerating. The accelerating cosmic expansion can be assigned to this kind of phase transition in some length scale (TGD Universe is fractal). There is no need to introduce cosmological constant and dark energy would be actually dark matter.

2. The recently observed void which has same size of about 10^8 light years as large voids having galaxies near their boundaries but having an age which is much higher than that of the large voids, would represent one example of jerk-wise expansion.

3. This picture applies also to solar system and planets might be perhaps seen as having once been parts of a more or less connected system, the primordial Sun. The Bohr orbits for inner and outer planets correspond to gravitational Planck constant which is 5 times larger for outer planets. This suggests that the space-time sheet of outer planets has suffered a phase transition increasing the size scale by a factor of 5. Earth can be regarded either as n=1 orbit for Planck constant associated with outer planets or n= 5 orbit for inner planetary system. This might have something to do with the very special position of Earth in planetary system. One could even consider the possibility that both orbits are present as dark matter structures. The phase transition would also explain why n=1 and n=2 Bohr orbits are absent and one only n=3,4, and 5 are present.

4. Also planets should have experienced this kind of phase transitions increasing the radius: the increase by a factor two would be the simplest situation.

The obvious question - that I did not ask - is whether this kind of phase transition might have occurred for Earth and led from a completely granite covered Earth - Pangeia without seas - to the recent Earth. Neither it did not occur to me to check whether there is any support for a rapid expansion of Earth during some period of its history.

Situation changed when my son visited me last Saturday and told me about a Youtube video [?] by Neal Adams, an American comic book and commercial artist who has also produced animations for geologists. We looked the amazing video a couple of times and I looked it again yesterday. The video is very impressive artwork but in the lack of references skeptic probably cannot avoid the feeling that Neal Adams might use his highly developed animation skills to cheat you. I found also a polemic article [?] of Adams but again the references were lacking. Perhaps the reason of polemic tone was that the concrete animation models make the expanding Earth hypothesis very convincing but geologists refuse to consider seriously arguments by a layman without a formal academic background.

The claims of Adams

The basic claims of Adams were following.

1. The radius of Earth has increased during last 185 million years (dinosaurs [?] appeared for about 230 million years ago) by about factor 2. If this is assumed all continents have formed at that time a single super-continent, Pangeia, filling the entire Earth surface rather than only 1/4 of it since the total area would have grown by a factor of 4. The basic argument was that it is very difficult to imagine Earth with 1/4 of surface containing granite and 3/4 covered by basalt. If the initial situation was covering by mere granite -as would look natural- it is very difficult for a believer in thermodynamics to imagine how the granite would have gathered to a single connected continent.

2. Adams claims that Earth has grown by keeping its density constant, rather than expanded, so that the mass of Earth has grown linearly with radius. Gravitational acceleration would have thus doubled and could provide a partial explanation for the disappearance of dinosaurs: it is difficult to cope in evolving environment when you get slower all the time.

3. Most of the sea floor is very young and the areas covered by the youngest basalt are the largest ones. This Adams interprets this by saying that the expansion of Earth is accelerating. The alternative interpretation is that the flow rate of the magma slows down as it recedes from the ridge where it erupts. The upper bound of 185 million years for the age of sea floor requires that the expansion period - if it is already over - lasted about 185 million years after which the flow increasing the area of the sea floor transformed to a convective flow with subduction so that the area is not increasing anymore.

4. The fact that the continents fit together - not only at the Atlantic side - but also at the Pacific side gives strong support for the idea that the entire planet was once covered by the super-continent. After the emergence of subduction theory this evidence as been dismissed.

5. I am not sure whether Adams mentions the following objections [?]. Subduction only occurs on the other side of the subduction zone so that the other side should show evidence of being much older in the case that oceanic subduction zones are in question. This is definitely not the case. This is explained in plate tectonics as a change of the subduction direction. My explanation would be that by the symmetry of the situation both oceanic plates bend down so that this would represent new type of boundary not assumed in the tectonic plate theory.

6. As a master visualizer Adams notices that Africa and South-America do not actually fit together in absence of expansion unless one assumes that these continents have suffered a deformation. Continents are not easily deformable stuff. The assumption of expansion implies a perfect fit of *all* continents without deformation.

Knowing that the devil is in the details, I must admit that these arguments look rather convincing to me and what I learned from Wikipedia articles supports this picture.

The critic of Adams of the subduction mechanism

The prevailing tectonic plate theory [?] has been compared to the Copernican revolution in geology. The theory explains the young age of the seafloor in terms of the decomposition of the litosphere to tectonic plates and the convective flow of magma to which oceanic tectonic plates participate. The magma emerges from the crests of the mid ocean ridges representing a boundary of two plates and leads to the expansion of sea floor. The variations of the polarity of Earth's magnetic field coded in sea floor provide a strong support for the hypothesis that magma emerges from the crests.

The flow back to would take place at so called oceanic trenches [?] near continents which represent the deepest parts of ocean. This process is known as subduction. In subduction oceanic tectonic plate bends and penetrates below the continental tectonic plate, the material in the oceanic plate gets denser and sinks into the magma. In this manner the oceanic tectonic plate suffers a metamorphosis returning back to the magma: everything which comes from Earth's interior returns back. Subduction mechanism explains elegantly formation of mountains [?] (orogeny), earth quake zones, and associated zones of volcanic activity [?].

Adams is very polemic about the notion of subduction, in particular about the assumption that it generates steady convective cycle. The basic objections of Adams against subduction are following.

1. There are not enough subduction zones to allow a steady situation. According to Adams, the situation resembles that for a flow in a tube which becomes narrower. In a steady situation the flow should accelerate as it approaches subduction zones rather than slow down. Subduction zones should be surrounded by large areas of sea floor with constant age. Just the opposite is suggested by the fact that the youngest portion of sea-floor near the ridges is largest. The presence of zones at which both ocean plates bend down could improve the situation. Also jamming of the flow could occur so that the thickness of oceanic plate increases with the distance from the eruption ridge. Jamming could increase also the density of the oceanic plate and thus the effectiveness of subduction.

2. There is no clear evidence that subduction has occurred at other planets. The usual defense is that the presence of sea is essential for the subduction mechanism.

3. One can also wonder what is the mechanism that led to the formation of single super continent Pangeia covering 1/4 of Earth's surface. How probable the gathering of all separate continents to form single cluster is? The later events would suggest that just the opposite should have occurred from the beginning.

Expanding Earth theories are not new

After I had decided to check the claims of Adams, the first thing that I learned is that Expanding Earth theory [**?**], whose existence Adams actually mentions, is by no means new. There are actually many of them.

The general reason why these theories were rejected by the main stream community was the absence of a convincing physical mechanism of expansion or of growth in which the density of Earth remains constant.

1. 1888 Yarkovski postulated some sort of aether absorbed by Earth and transforming to chemical elements (TGD version of aether could be dark matter). 1909 Mantovani postulated thermal expansion but no growth of the Earth's mass.

2. Paul Dirac's idea about changing Planck constant led Pascual Jordan in 1964 to a modification of general relativity predicting slow expansion of planets. The recent measurement of the gravitational constant imply that the upper bound for the relative change of gravitational constant is 10 time too small to produce large enough rate of expansion. Also many other theories have been proposed but they are in general conflict with modern physics.

3. The most modern version of Expanding Earth theory is by Australian geologist Samuel W. Carey. He calculated that in Cambrian period (about 500 million years ago) all continents were stuck together and covered the entire Earth. Deep seas began to evolve then.

Summary of TGD based theory of Expanding Earth

TGD based model differs from the tectonic plate model but allows subduction which cannot imply considerable back-flow of magma. Let us sum up the basic assumptions and implications.

1. The expansion is or was due to a quantum phase transition increasing the value of gravitational Planck constant and forced by the cosmic expansion in the average sense.

2. Tectonic plates do not participate to the expansion and therefore new plate must be formed and the flow of magma from the crests of mid ocean ridges is needed. The decomposition of a single plate covering the entire planet to plates to create the mid ocean ridges is necessary for the generation of new tectonic plate. The decomposition into tectonic plates is thus prediction rather than assumption.

3. The expansion forced the decomposition of Pangeia super-continent covering entire Earth for about 530 million years ago to split into tectonic plates which began to recede as new non-expanding tectonic plate was generated at the ridges creating expanding sea floor. The initiation of the phase transition generated formation of deep seas.

4. The eruption of plasma from the crests of ocean ridges generated oceanic tectonic plates which did not participate to the expansion by density reduction but by growing in size. This led to a reduction of density in the interior of the Earth roughly by a factor 1/8. From the upper bound for the age of the seafloor one can conclude that the period lasted for about 185 million years after which it transformed to convective flow in which the material returned back to the Earth interior. Subduction at continent-ocean floor boundaries and downwards double bending of tectonic plates at the boundaries between two ocean floors were the mechanisms. Thus tectonic plate theory would be more or less the correct description for the recent situation.

5. One can consider the possibility that the subducted tectonic plate does not transform to magma but is fused to the tectonic layer below continent so that it grows to an iceberg like structure. This need not lead to a loss of the successful predictions of plate tectonics explaining the generation of mountains, earthquake zones, zones of volcanic activity, etc...

6. From the video of Adams it becomes clear that the tectonic flow is East-West asymmetric in the sense that the western side is more irregular at large distances from the ocean ridge at the western side. If the magma rotates with slightly lower velocity than the surface of Earth (like liquid in a rotating vessel), the erupting magma would rotate slightly slower than the tectonic plate and asymmetry would be generated.

7. If the planet has not experienced a phase transition increasing the value of Planck constant, there is no need for the decomposition to tectonic plates and one can understand why there is no clear evidence for tectonic plates and subduction in other planets. The conductive flow of magma could occur below this plate and remain invisible.

The biological implications might provide a possibility to test the hypothesis.

1. Great steps of progress in biological evolution are associated with catastrophic geological events generating new evolutionary pressures forcing new solutions to cope in the new situation. Cambrian explosion indeed occurred about 530 years ago (the book "Wonderful Life" of Stephen Gould [?] explains this revolution in detail) and led to the emergence of multicellular creatures, and generated huge number of new life forms living in seas. Later most of them suffered extinction: large number of phylae and groups emerged which are not present nowadays.

   Thus Cambrian explosion is completely exceptional as compared to all other dramatic events in the evolution in the sense that it created something totally new rather than only making more complex something which already existed. Gould also emphasizes the failure to identify any great change in the environment as a fundamental puzzle of Cambrian explosion. Cambrian explosion is also regarded in many quantum theories of consciousness (including TGD) as a revolution in the evolution of consciousness: for instance, micro-tubuli emerged at this time. The periods of expansion might be necessary for the emergence of multicellular life forms on planets and the fact that they unavoidably occur sooner or later suggests that also life develops unavoidably.

2. TGD predicts a decrease of the surface gravity by a factor 1/4 during this period. The reduction of the surface gravity would have naturally led to the emergence of dinosaurs 230 million years ago as a response coming 45 million years after the accelerated expansion ceased. Other reasons led then to the decline and eventual catastrophic disappearance of the dinosaurs. The reduction of gravity might have had some gradually increasing effects on the shape of organisms also at microscopic level and manifest itself in the evolution of genome during expansion period.

3. A possibly testable prediction following from angular momentum conservation ($\omega R^2 = constant$) is that the duration of day has increased gradually and was four times shorter during the Cambrian era. For instance, genetically coded bio-clocks of simple organisms during the expansion period could have followed the increase of the length of day with certain lag or failed to follow it completely. The simplest known circadian clock is that of the prokaryotic cyanobacteria. Recent research has demonstrated that the circadian clock of Synechococcus elongatus can be reconstituted in vitro with just the three proteins of their central oscillator. This clock has been shown to sustain a 22 hour rhythm over several days upon the addition of ATP: the rhythm is indeed faster than the circadian rhythm. For humans the average innate circadian rhythm is however 24 hours 11 minutes and thus conforms with the fact that human genome has evolved much later than the expansion ceased.

4. Scientists have found a fossil of a sea scorpion with size of 2.5 meters [?], which has lived for about 10 million years for 400 million years ago in Germany. The gigantic size would conform nicely with the much smaller value of surface gravity at that time. The finding would conform nicely with the much smaller value of surface gravity at that time. Also the emergence of trees could be understood in terms of a gradual growth of the maximum plant size as the surface gravity was reduced. The fact that the oldest known tree fossil is 385 million years old [?] conforms with this picture.

Did intra-terrestrial life burst to the surface of Earth during Cambrian expansion?

The possibility of intra-terrestrial life [?] is one of the craziest TGD inspired ideas about the evolution of life and it is quite possible that in its strongest form the hypothesis is unrealistic. One can however try to find what one obtains from the combination of the IT hypothesis with the idea of pre-Cambrian granite Earth. Could the harsh pre-Cambrian conditions have allowed only intra-terrestrial multicellular life? Could the Cambrian explosion correspond to the moment of birth for this life in the very concrete sense that the magma flow brought it into the day-light?

1. Gould emphasizes the mysterious fact that very many life forms of Cambrian explosion looked like final products of a long evolutionary process. Could the eruption of magma from the Earth interior have induced a burst of intra-terrestrial life forms to the Earth's surface? This might make sense: the life forms living at the bottom of sea do not need direct solar light so that they could have had intra-terrestrial origin. It is quite possible that Earth's mantle contained low temperature water pockets, where the complex life forms might have evolved in an environment shielded from meteoric bombardment and UV radiation.

2. Sea water is salty. It is often claimed that the average salt concentration inside cell is that of the primordial sea: I do not know whether this claim can be really justified. If the claim is true, the cellular salt concentration should reflect the salt concentration of the water inside the pockets. The water inside water pockets could have been salty due to the diffusion of the salt from ground but need not have been same as that for the ocean water (higher than for cell interior and for obvious reasons). Indeed, the water in the underground reservoirs in arid regions such as Sahara is salty, which is the reason for why agriculture is absent in these regions. Note also that the cells of marine invertebrates are osmoconformers able to cope with the changing salinity of the environment so that the Cambrian revolutionaries could have survived the change in the salt concentration of environment.

3. What applies to Earth should apply also to other similar planets and Mars [?] is very similar to Earth. The radius is .533 times that for Earth so that after quantum leap doubling the radius and thus Schumann frequency scale (7.8 Hz would be the lowest Schumann frequency) would be essentially same as for Earth now. Mass is .131 times that for Earth so that surface gravity would be .532 of that for Earth now and would be reduced to .131 meaning quite big dinosaurs! have learned that Mars probably contains large water reservoirs in it's interior and that there is an un-identified source of methane gas usually assigned with the presence of life. Could it be that Mother Mars is pregnant and just waiting for the great quantum leap when it starts to expand and gives rise to a birth of multicellular life forms. Or expressing freely how Bible describes the moment of birth: in the beginning there was only darkness and water and then God said: Let the light come!

To sum up, TGD would provide only the long sought mechanism of expansion and a possible connection with the biological evolution. It would be indeed fascinating if Planck constant changing quantum phase transitions in planetary scale would have profoundly affected the biosphere.

2.6.5 Allais effect as evidence for large values of gravitational Planck constant?

Allais effect [?, ?] is a fascinating gravitational anomaly associated with solar eclipses. It was discovered originally by M. Allais, a Nobelist in the field of economy, and has been reproduced in several experiments but not as a rule. The experimental arrangement uses so called paraconical pendulum, which differs from the Foucault pendulum in that the oscillation plane of the pendulum can rotate in certain limits so that the motion occurs effectively at the surface of sphere.

Experimental findings

Consider first a brief summary of the findings of Allais and others [?].

a) In the ideal situation (that is in the absence of any other forces than gravitation of Earth) paraconical pendulum should behave like a Foucault pendulum. The oscillation plane of the paraconical pendulum however begins to rotate.

b) Allais concludes from his experimental studies that the orbital plane approach always asymptotically to a limiting plane and the effect is only particularly spectacular during the eclipse. During solar eclipse the limiting plane contains the line connecting Earth, Moon, and Sun. Allais explains this in terms of what he calls the anisotropy of space.

c) Some experiments carried out during eclipse have reproduced the findings of Allais, some experiments not. In the experiment carried out by Jeverdan and collaborators in Romania it was found that the period of oscillation of the pendulum decreases by $\Delta f/f \simeq 5 \times 10^{-4}$ [?, ?] which happens to correspond to the constant $v_0 = 2^{-11}$ appearing in the formula of the gravitational Planck constant. It must be however emphasized that the overall magnitude of $\Delta f/f$ varies by five orders of magnitude. Even the sign of $\Delta f/f$ varies from experiment to experiment.

d) There is also quite recent finding by Popescu and Olenici, which they interpret as a quantization of the plane of oscillation of paraconical oscillator during solar eclipse [?].

TGD based models for Allais effect

I have already earlier proposed an explanation of the effect in terms of classical Z^0 force [?]. If the Z^0 charge to mass ratio of pendulum varies and if Earth and Moon are Z^0 conductors, the resulting model is quite flexible and one might hope it could explain the high variation of the experimental results.

The rapid variation of the effect during the eclipse is however a problem for this approach and suggests that gravitational screening or some more general interference effect might be present. Gravitational screening alone cannot however explain Allais effect.

A model based on the idea that gravitational interaction is mediated by topological light rays (MEs) and that gravitons correspond to a gigantic value of the gravitational Planck constant however explains the Allais effect as an interference effect made possible by macroscopic quantum coherence in astrophysical length scales. Equivalence Principle fixes the model to a high degree and one ends up with an explicit formula for the anomalous gravitational acceleration and the general order of magnitude and the large variation of the frequency change as being due to the variation of the distance ratio $r_{S,P}/r_{M,P}$ (S, M,and P refer to Sun, Moon, and pendulum respectively). One can say that the pendulum acts as an interferometer.

2.6.6 Applications to elementary particle physics, nuclear physics, and condensed matter physics

The hierarchy of Planck constants could have profound implications for even elementary particle physics since the strong constraints on the existence of new light particles coming from the decay widths of intermediate gauge bosons can be circumvented because direct decays to dark matter are not possible. On the other hand, if light scaled versions of elementary particles exist they must be dark since otherwise their existence would be visible in these decay widths. The constraints on the existence of dark nuclei and dark condensed matter are much milder. Cold fusion and some other anomalies of nuclear and condensed matter physics - in particular the anomalies of water- might have elegant explanation in terms of dark nuclei.

Leptohadron hypothesis

TGD suggests strongly the existence of lepto-hadron physics [?]. Lepto-hadrons are bound states of color excited leptons and the anomalous production of e^+e^- pairs in heavy ion collisions finds a nice explanation as resulting from the decays of lepto-hadrons with basic condensate level $k = 127$ and having typical mass scale of one MeV. The recent indications on the existence of a new fermion with quantum numbers of muon neutrino and the anomaly observed in the decay of orto-positronium give further support for the lepto-hadron hypothesis. There is also evidence for anomalous production of low energy photons and e^+e^- pairs in hadronic collisions.

The identification of lepto-hadrons as a particular instance in the predicted hierarchy of dark matters interacting directly only via graviton exchange allows to circumvent the lethal counter arguments against the lepto-hadron hypothesis (Z^0 decay width and production of colored lepton jets in e^+e^- annihilation) even without assumption about the loss of asymptotic freedom.

PCAC hypothesis and its sigma model realization lead to a model containing only the coupling of the lepto-pion to the axial vector current as a free parameter. The prediction for e^+e^- production cross section is of correct order of magnitude only provided one assumes that lepto-pions (or electro-pions) decay to lepto-nucleon pair $e^+_{ex}e^-_{ex}$ first and that lepto-nucleons, having quantum numbers of electron and having mass only slightly larger than electron mass, decay to lepton and photon. The peculiar production characteristics are correctly predicted. There is some evidence that the resonances decay to a final state containing $n > 2$ particle and the experimental demonstration that lepto-nucleon pairs are indeed in question, would be a breakthrough for TGD.

During 18 years after the first published version of the model also evidence for colored μ has emerged [?]. Towards the end of 2008 CDF anomaly [?] gave a strong support for the colored excitation of τ. The lifetime of the light long lived state identified as a charged τ-pion comes out correctly and the identification of the reported 3 new particles as p-adically scaled up variants of neutral τ-pion predicts their masses correctly. The observed muon jets can be understood in terms of the special reaction kinematics for the decays of neutral τ-pion to 3 τ-pions with mass scale smaller by a factor 1/2 and therefore almost at rest. A spectrum of new particles is predicted. The discussion of CDF anomaly [?] led to a modification and generalization of the original model for lepto-pion production and the predicted production cross section is consistent with the experimental estimate.

Cold fusion, plasma electrolysis, and burning salt water

The article of Kanarev and Mizuno [?] reports findings supporting the occurrence of cold fusion in NaOH and KOH hydrolysis. The situation is different from standard cold fusion where heavy water D_2O is used instead of H_2O.

In nuclear string model nucleon are connected by color bonds representing the color magnetic body of nucleus and having length considerably longer than nuclear size. One can consider also dark nuclei for which the scale of nucleus is of atomic size [?]. In this framework can understand the cold fusion reactions reported by Mizuno as nuclear reactions in which part of what I call dark proton string having negatively charged color bonds (essentially a zoomed up variant of ordinary nucleus with large Planck constant) suffers a phase transition to ordinary matter and experiences ordinary strong interactions with the nuclei at the cathode. In the simplest model the final state would contain only ordinary nuclear matter. The generation of plasma in plasma electrolysis can be seen as a process analogous to the positive feedback loop in ordinary nuclear reactions.

Rather encouragingly, the model allows to understand also deuterium cold fusion and leads to a solution of several other anomalies.

1. The so called lithium problem of cosmology (the observed abundance of lithium is by a factor 2.5 lower than predicted by standard cosmology [?]) can be resolved if lithium nuclei transform partially to dark lithium nuclei.

2. The so called $H_{1.5}O$ anomaly of water [?, ?, ?, ?] can be understood if 1/4 of protons of water forms dark lithium nuclei or heavier dark nuclei formed as sequences of these just as ordinary nuclei are constructed as sequences of ^4He and lighter nuclei in nuclear string model. The results force to consider the possibility that nuclear isotopes unstable as ordinary matter can be stable dark matter.

3. The mysterious behavior burning salt water [?] can be also understood in the same framework.

4. The model explains the nuclear transmutations observed in Kanarev's plasma electrolysis. This kind of transmutations have been reported also in living matter long time ago [?, ?]. Intriguingly, several biologically important ions belong to the reaction products in the case of NaOH electrolysis. This raises the question whether cold nuclear reactions occur in living matter and are responsible for generation of biologically most important ions.

2.6.7 Applications to biology and neuroscience

The notion of field or magnetic body regarded as carrier of dark matter with large Planck constant and quantum controller of ordinary matter is the basic idea in the TGD inspired model of living matter.

Do molecular symmetries in living matter relate to non-standard values of Planck constant?

Water is exceptional element and the possibility that G_a as symmetry of singular factor space of CD in water and living matter is intriguing.

1. There is evidence for an icosahedral clustering in water [?]. Synaptic contacts contain clathrin molecules which are truncated icosahedrons and form lattice structures and are speculated to be involved with quantum computation like activities possibly performed by microtubules. Many viruses have the shape of icosahedron. One can ask whether these structures could be formed around templates formed by dark matter corresponding to 120-fold covering of CP_2 points by CD points and having $\hbar(CP_2) = 5\hbar_0$ perhaps corresponding color confined light dark quarks. Of course, a similar covering of CD points by CP_2 could be involved.

2. It should be noticed that single nucleotide in DNA double strands corresponds to a twist of $2\pi/10$ per single DNA triplet so that 10 DNA strands corresponding to length $L(151) = 10$ nm (cell membrane thickness) correspond to $3 \times 2\pi$ twist. This could be perhaps interpreted as evidence for group C_{10} perhaps making possible quantum computation at the level of DNA.

3. What makes realization of G_a as a symmetry of singular factor space of CD is that the biomolecules most relevant for the functioning of brain (DNA nucleotides, aminoacids acting as neurotransmitters, molecules having hallucinogenic effects) contain aromatic 5- and 6-cycles.

These observations led to an identification of the formula for Planck constant (two alternatives were allowed by the condition that Planck constant is algebraic homomorphism) which was not consistent with the model for dark gravitons. If one accepts the proposed formula of Planck constant, the dark space-time sheets with large Planck constant correspond to factor spaces of both $\hat{CD}\backslash M^2$ and of $CP_2\backslash S^2_I$. This identification is of course possible and it remains to be seen whether it leads to any problems. For gravitational space-time sheets only coverings of both CD and CP_2 make sense and the covering group G_a has very large order and does not correspond to geometric symmetries analogous to those of molecules.

High T_c super-conductivity in living matter

The model for high T_c super-conductivity realized as quantum critical phenomenon predicts the basic scales of cell membrane [?] from energy minimization and p-adic length scale hypothesis. This leads to the vision that cell membrane and possibly also its scaled up dark fractal variants define Josephson junctions generating Josephson radiation communicating information about the nearby environment to the magnetic body.

Any model of high T_c superconductivity should explain various strange features of high T_c superconductors. One should understand the high value of T_c, the ambivalent character of high T_c super conductors suggesting both BCS type Cooper pairs and exotic Cooper pairs with non-vanishing spin, the existence of pseudogap temperature $T_{c_1} > T_c$ and scaling law for resistance for $T_c \leq T < T_{c_1}$, the role of fluctuating charged stripes which are anti-ferromagnetic defects of a Mott insulator, the existence of a critical doping, etc...[?, ?].

There are reasons to believe that high T_c super-conductors correspond to quantum criticality in which at least two (cusp catastrophe as in van der Waals model), or possibly three or even more phases, are competing. A possible analogy is provided by the triple critical point for water vapor, liquid phase and ice coexist. Instead of long range thermal fluctuations long range quantum fluctuations manifesting themselves as fluctuating stripes are present [?].

The TGD based model for high T_c super-conductivity [?] relies on the notions of quantum criticality, general ideas of catastrophe theory, dynamical Planck constant, and many-sheeted space-time. The 4-dimensional spin glass character of space-time dynamics deriving from the vacuum degeneracy of the Kähler action defining the basic variational principle would realize space-time correlates for quantum fluctuations.

1. Two kinds of super-conductivities and ordinary non-super-conducting phase would be competing at quantum criticality at T_c and above it only one super-conducting phase and ordinary conducting phase located at stripes representing ferromagnetic defects making possible formation of $S = 1$ Cooper pairs.

2. The first super-conductivity would be based on exotic Cooper pairs of large $\hbar$ dark electrons with $\hbar = 2^{11}\hbar_0$ and able to have spin $S = 1$, angular momentum $L = 2$, and total angular momentum $J = 2$. Second type of super-conductivity would be based on BCS type Cooper pairs having vanishing spin and bound by phonon interaction. Also they have large $\hbar$ so that gap energy and critical temperature are scaled up in the same proportion. The exotic Cooper pairs are possible below the pseudo gap temperature $T_{c_1} > T_c$ but are unstable against decay to BCS type Cooper pairs which above T_c are unstable against a further decay to conduction electrons flowing along stripes. This would reduce the exotic super-conductivity to finite conductivity obeying the observed scaling law for resistance.

3. The mere assumption that electrons of exotic Cooper pairs feed their electric flux to larger space-time sheet via *two* elementary particle sized wormhole contacts rather than only *one* wormhole contacts implies that the throats of wormhole contacts defining analogs of Higgs field must carry quantum numbers of quark and anti-quark. This inspires the idea that cylindrical space-time sheets, the radius of which turns out to be about about 5 nm, representing zoomed up dark electrons of Cooper pair with Planck constant $\hbar = 2^{11}\hbar_0$ are colored and bound by a scaled up variant of color force to form a color confined state. Formation of Cooper pairs would have nothing to do with direct interactions between electrons. Thus high T_c super-conductivity could be seen as a first indication for the presence of scaled up variant of QCD in mesoscopic length scales.

This picture leads to a concrete model for high T_c superconductors as quantum critical superconductors [?]. p-Adic length scale hypothesis stating that preferred p-adic primes $p \simeq 2^k$, k integer, with primes (in particular Mersenne primes) preferred, makes the model quantitative.

1. An unexpected prediction is that coherence length ξ is actually $\hbar_{eff}/\hbar_0 = 2^{11}$ times longer than the coherence length 5-10 Angstroms deduced theoretically from gap energy using conventional theory and varies in the range $1 - 5$ μm, the cell nucleus length scale. Hence type I super-conductor would be in question with stripes as defects of anti-ferromagnetic Mott insulator serving as duals for the magnetic defects of type I super-conductor in nearly critical magnetic field.

2. At quantitative level the model reproduces correctly the four poorly understood photon absorption lines and allows to understand the critical doping ratio from basic principles.

3. The current carrying structures have structure locally similar to that of axon including the double layered structure of cell membrane and also the size scales are predicted to be same. One of the characteristic absorption lines has energy of .05 eV which corresponds to the Josephson energy for neuronal membrane for activation potential $V = 50$ mV. Hence the idea that axons are high T_c superconductors is highly suggestive. Dark matter hierarchy coming in powers $\hbar/\hbar_0 = 2^{k11}$ suggests hierarchy of Josephson junctions needed in TGD based model of EEG [?].

Magnetic body as a sensory perceiver and intentional agent

The hypothesis that dark magnetic body serves as an intentional agent using biological body as a motor instrument and sensory receptor is consistent with Libet's findings about strange time delays of consciousness. Magnetic body would carry cyclotron Bose-Einstein condensates of various ions. Magnetic body must be able to perform motor control and receive sensory input from biological body.

Cell membrane would be a natural sensor providing information about cell interior and exterior to the magnetic body and dark photons at appropriate frequency range would naturally communicate this information. The strange quantitative co-incidences with the physics of cell membrane and high T_c super-conductivity support the idea that Josephson radiation generated by Josephson currents of dark electrons through cell membrane is responsible for this communication [?].

Also fractally scaled up versions of cell membrane at higher levels of dark matter hierarchy (in particular those corresponding to powers $n = 2^{k11}$) are possible and the model for EEG indeed relies on this hypothesis. The thickness for the fractal counterpart of cell membrane thickness would be 2^{44} fold and of order of depth of ionosphere! Although this looks weird it is completely consistent with the notion of magnetic body as an intentional agent.

Motor control would be most naturally performed via genome: this is achieved if flux sheets traverse through DNA strands. Flux quantization for large values of Planck constant requires rather large widths for the flux sheets. If flux sheet contains sequences of genomes like the page of book contains lines of text, a coherent gene expression becomes possible at level of organs and even populations and one can speak about super- and hyper-genomes. Introns might relate to the collective gene expression possibly realized electromagnetically rather than only chemically [**?**, **?**].

Dark cyclotron radiation with photon energy above thermal energy could be used for coordination purposes at least. The predicted hierarchy of copies of standard model physics leads to ask whether also dark copies of electro-weak gauge bosons and gluons could be important in living matter. As already mentioned, dark W bosons could make possible charge entanglement and non-local quantum bio-control by inducing voltage differences and thus ionic currents in living matter.

The identification of plasmoids as rotating magnetic flux structures carrying dark ions and electrons as primitive life forms is natural in this framework. There exists experimental support for this identification [**?**] but the main objection is the high temperature involved: this objection could be circumvented if large $\hbar$ phase is involved. A model for the pre-biotic evolution relying also on this idea is discussed in [**?**].

At the level of biology there are now several concrete applications leading to a rich spectrum of predictions. Magnetic flux quanta would carry charged particles with large Planck constant.

1. The shortening of the flux tubes connecting biomolecules in a phase transition reducing Planck constant could be a basic mechanism of bio-catalysis and explain the mysterious ability of biomolecules to find each other. Similar process in time direction could explain basic aspects of symbolic memories as scaled down representations of actual events.

2. The strange behavior of cell membrane suggests that a dominating portion of important biological ions are actually dark ions at magnetic flux tubes so that ionic pumps and channels are needed only for visible ions. This leads to a model of nerve pulse explaining its unexpected thermodynamical properties with basic properties of Josephson currents making it un-necessary to use pumps to bring ions back after the pulse. The model predicts automatically EEG as Josephson radiation and explains the synchrony of both kHz radiation and of EEG.

3. The DC currents of Becker could be accompanied by Josephson currents running along flux tubes making possible dissipation free energy transfer and quantum control over long distances and meridians of chinese medicine could correspond to these flux tubes.

4. The model of DNA as topological quantum computer assumes that nucleotides and lipids are connected by ordinary or "wormhole" magnetic flux tubes acting as strands of braid and carrying dark matter with large Planck constant. The model leads to a new vision about TGD in which the assignment of nucleotides to quarks allows to understood basic regularities of DNA not understood from biochemistry.

5. Each physical system corresponds to an onionlike hierarchy of field bodies characterized by p-adic primes and value of Planck constant. The highest value of Planck constant in this hierarchy provides kind of intelligence quotient characterizing the evolutionary level of the system since the time scale of planned action and memory correspond to the temporal distance between tips of corresponding causal diamond (CD). Also the spatial size of the system correlates with the Planck constant. This suggests that great evolutionary leaps correspond to the increase of Planck constant for the highest level of hierarchy of personal magnetic bodies. For instance, neurons would have much more evolved magnetic bodies than ordinary cells.

6. At the level of DNA this vision leads to an idea about hierarchy of genomes. Magnetic flux sheets traversing DNA strands provide a natural mechanism for magnetic body to control the behavior of biological body by controlling gene expression. The quantization of magnetic flux states that magnetic flux is proportional to $\hbar$ and thus means that the larger the value of $\hbar$ is the larger the width of the flux sheet is. For larger values of $\hbar$ single genome is not enough to satisfy this condition. This leads to the idea that the genomes of organs, organism, and even population, can organize like lines of text at the magnetic flux sheets and form in this manner a hierarchy of genomes responsible for a coherent gene expression at level of cell, organ, organism

and population and perhaps even entire biosphere. This would also provide a mechanism by which collective consciousness would use its biological body - biosphere.

DNA as topological quantum computer

I ended up with the recent model of tqc in bottom-up manner and this representation is followed also in the text. The model which looks the most plausible one relies on two specific ideas.

1. Sharing of labor means conjugate DNA would do tqc and DNA would "print" the outcome of tqc in terms of mRNA yielding amino-acids in the case of exons. RNA could result also in the case of introns but not always. The experience about computers and the general vision provided by TGD suggests that introns could express the outcome of tqc also electromagnetically in terms of standardized field patterns as Gariaev's findings suggest [?]. Also speech would be a form of gene expression. The quantum states braid (in zero energy ontology) would entangle with characteristic gene expressions. This argument turned out to be based on a slightly wrong belief about DNA: later I learned that both strand and its conjugate are transcribed but in different directions. The symmetry breaking in the case of transcription is only local which is also visible in DNA replication as symmetry breaking between leading and lagging strand. Thus the idea about *entire* leading strand devoted to printing and second strand to tqc must be weakened appropriately.

2. The manipulation of braid strands transversal to DNA must take place at 2-D surface. Here dancing metaphor for topological quantum computation [?] generalizes. The ends of the space-like braid are like dancers whose feet are connected by thin threads to a wall so that the dancing pattern entangles the threads. Dancing pattern defines both the time-like braid, the running of classical tqc program and its representation as a dynamical pattern. The space-like braid defined by the entangled threads represents memory storage so that tqc program is automatically written to memory as the braiding of the threads during the tqc. The inner membrane of the nuclear envelope and cell membrane with entire endoplasmic reticulum included are good candidates for dancing halls. The 2-surfaces containing the ends of the hydrophobic ends of lipids could be the parquets and lipids the dancers. This picture seems to make sense.

One ends up to the model also in top-down manner.

1. Darwinian selection for which standard theory of self-organization [?] provides a model, should apply also to tqc programs. Tqc programs should correspond to asymptotic self-organization patterns selected by dissipation in the presence of metabolic energy feed. The spatial and temporal pattern of the metabolic energy feed characterizes the tqc program - or equivalently - sub-program call.

2. Since braiding characterizes the tqc program, the self-organization pattern should correspond to a hydrodynamical flow or a pattern of magnetic field inducing the braiding. Braid strands must correspond to magnetic flux tubes of the magnetic body of DNA. If each nucleotide is transversal magnetic dipole it gives rise to transversal flux tubes, which can also connect to the genome of another cell.

3. The output of tqc sub-program is probability distribution for the outcomes of state function reduction so that the sub-program must be repeated very many times. It is represented as four-dimensional patterns for various rates (chemical rates, nerve pulse patterns, EEG power distributions,...) having also identification as temporal densities of zero energy states in various scales. By the fractality of TGD Universe there is a hierarchy of tqc's corresponding to p-adic and dark matter hierarchies. Programs (space-time sheets defining coherence regions) call programs in shorter scale. If the self-organizing system has a periodic behavior each tqc module defines a large number of almost copies of itself asymptotically. Generalized EEG could naturally define this periodic pattern and each period of EEG would correspond to an initiation and halting of tqc. This brings in mind the periodically occurring sol-gel phase transition inside cell near the cell membrane.

4. Fluid flow must induce the braiding which requires that the ends of braid strands must be anchored to the fluid flow. Recalling that lipid mono-layers of the cell membrane are liquid crystals and lipids of interior mono-layer have hydrophilic ends pointing towards cell interior, it is easy to guess that DNA nucleotides are connected to lipids by magnetic flux tubes and hydrophilic lipid ends are stuck to the flow.

5. The topology of the braid traversing cell membrane cannot affected by the hydrodynamical flow. Hence braid strands must be split during tqc. This also induces the desired magnetic isolation from the environment. Halting of tqc reconnects them and make possible the communication of the outcome of tqc.

6. There are several problems related to the details of the realization. How nucleotides A,T,C,G are coded to strand color and what this color corresponds to? The prediction that wormhole contacts carrying quark and anti-quark at their ends appear in all length scales in TGD Universe resolves the problem. How to split the braid strands in a controlled manner? High T_c super conductivity provides a partial understanding of the situation: braid strand can be split only if the supra current flowing through it vanishes. From the proportionality of Josephson current to the quantity $sin(\int 2eVdt)$ it follows that a suitable voltage pulse V induces DC supra-current and its negative cancels it. The conformation of the lipid controls whether it it can follow the flow or not. How magnetic flux tubes can be cut without breaking the conservation of the magnetic flux? The notion of wormhole magnetic field saves the situation now: after the splitting the flux returns back along the second space-time sheet of wormhole magnetic field.

To sum up, it seems that essentially all new physics involved with TGD based view about quantum biology enter to the model in crucial manner.

Quantum model of nerve pulse and EEG

In this article a unified model of nerve pulse and EEG is discussed.

1. In TGD Universe the function of EEG and its variants is to make possible communications from the cell membrane to the magnetic body and the control of the biological body by the magnetic body via magnetic flux sheets traversing DNA by inducing gene expression. This leads to the notions of super- and hyper-genome predicting coherent gene expression at level of organs and population.

2. The assignment the predicted ranged classical weak and color gauge fields to dark matter hierarchy was a crucial step in the evolution of the model, and led among other things to a model of high T_c superconductivity predicting the basic scales of cell, and also to a generalization of EXG to a hierarchy of ZXGs, WXGs, and GXGs corresponding to Z^0, W bosons and gluons.

3. Dark matter hierarchy and the associated hierarchy of Planck constants plays a key role in the model. For instance, in the case of EEG Planck constant must be so large that the energies of dark EEG photons are above thermal energy at physiological temperatures. The assumption that a considerable fraction of the ionic currents through the cell membrane are dark currents flowing along the magnetic flux tubes explains the strange findings about ionic currents through cell membrane. Concerning the model of nerve pulse generation, the newest input comes from the model of DNA as a topological quantum computer and experimental findings challenging Hodgkin-Huxley model as even approximate description of the situation.

4. The identification of the cell interior as gel phase containing most of water as structured water around cytoskeleton - rather than water containing bio-molecules as solutes as assumed in Hodkin-Huxley model - allows to understand many of the anomalous behaviors associated with the cell membrane and also the different densities of ions in the interior and exterior of cell at qualitative level. The proposal of Pollack that basic biological functions involve phase transitions of gel phase generalizes in TGD framework to a proposal that these phase transitions are induced by quantum phase transitions changing the value of Planck constant. In particular, gel-sol phase transition for the peripheral cytoskeleton induced by the primary wave would accompany nerve pulse propagation. This view about nerve pulse is not consistent with Hodkin-Huxley model.

The model leads to the following picture about nerve pulse and EEG.

1. The system would consist of two superconductors- microtubule space-time sheet and the space-time sheet in cell exterior- connected by Josephson junctions represented by magnetic flux tubes defining also braiding in the model of tqc. The phase difference between two super-conductors would obey Sine-Gordon equation allowing both standing and propagating solitonic solutions. A sequence of rotating gravitational penduli coupled to each other would be the mechanical analog for the system. Soliton sequences having as a mechanical analog penduli rotating with constant velocity but with a constant phase difference between them would generate moving kHz synchronous oscillation. Periodic boundary conditions at the ends of the axon rather than chemistry determine the propagation velocities of kHz waves and kHz synchrony is an automatic consequence since the times taken by the pulses to travel along the axon are multiples of same time unit. Also moving oscillations in EEG range can be considered and would require larger value of Planck constant in accordance with vision about evolution as gradual increase of Planck constant.

2. During nerve pulse one pendulum would be kicked so that it would start to oscillate instead of rotating and this oscillation pattern would move with the velocity of kHz soliton sequence. The velocity of kHz wave and nerve pulse is fixed by periodic boundary conditions at the ends of the axon implying that the time spent by the nerve pulse in traveling along axon is always a multiple of the same unit: this implies kHz synchrony. The model predicts the value of Planck constant for the magnetic flux tubes associated with Josephson junctions and the predicted force caused by the ionic Josephson currents is of correct order of magnitude for reasonable values of the densities of ions. The model predicts kHz em radiation as Josephson radiation generated by moving soliton sequences. EEG would also correspond to Josephson radiation: it could be generated either by moving or standing soliton sequences (latter are naturally assignable to neuronal cell bodies for which $\hbar$ should be correspondingly larger): synchrony is predicted also now.

2.7 Some mathematical speculations

2.7.1 The content of McKay correspondence in TGD framework

The possibility to assign Dynkin diagrams with the inclusions of II_1 algebras is highly suggestive concerning possible physical interpretations. The basic findings are following.

1. For $\beta = \mathcal{M} : \mathcal{N} < 4$ Dynkin diagrams code for the inclusions and correspond to simply laced Lie algebras. $SU(2)$, D_{2n+1}, and E_7 are excluded.

2. Extended ADE Dynkin diagrams coding for simply laced ADE Kac Moody algebras appear at $\beta = 4$. Also $SU(2)$ Kac Moody algebra appears.

Does TGD give rise to ADE hierarchy of gauge theories

The first question is whether any finite subgroup $G \subset SU(2)$ acting in CP_2 degrees of freedom could somehow give rise to multiplets of the corresponding gauge group having interactions described by a gauge theory. Orbifold picture suggests that might be the case.

1. The "sheets" for the space-time sheet forming an $N(G)$-fold cover of CD are in one-one correspondence with group G. This degeneracy gives rise to additional states and these states correspond to the group algebra having basis given by group characters $\chi(g)$. One obtains irreducible representations of G with degeneracies given by their dimensions. Altogether one obtains $N(G)$ states in this manner. In the case of $A(n)$ the number of these states is $n+1$, the number of the states of the fundamental representation of $SU(n+1)$. In the same manner, for D_{2n} the number of these states equals to the number of states in the fundamental representation of D_{2n}. It seems that the rule is quite general. Thus these representations would in the case of fermions give the states of the fundamental representation of the corresponding gauge group.

2. From fermion and antifermion states one can construct in a similar manner pairs giving $N(G)^2$ states defining in the case of $A(n)$ n^2-1-dimensional gauge boson multiplet plus singlet. Also other groups must give boson multiplet plus possible other multiplets. For instance, for $D(4)$ the number of states is 64 and boson multiplet is 8-dimensional so that many other spin 1 states result.

3. These findings give hopes that the orbifold multiplets could be modelled by a gauge theory based on corresponding gauge group. What is nice that this huge hierarchy of gauge theories is associated with dark matter so that the predictivity and falsifiability are not lost unlike in M-theory.

Does one obtain also a hierarchy of conformal theories with ADE Kac Moody symmetry?

Consider next the question Kac Moody interactions correspond to extended ADE diagrams are possible.

1. In this case the notion of orbifold seems to break down since the symmetry related points form a continuum $SU(2)$ and space-time surface would become 6-dimensional if the CD projection is 4-dimensional. If one takes space-time as something which emerges, one could take this possibility half seriously. A more natural natural possibility is that CD projection is 2-dimensional geodesic sphere in which case one would have string like objects so that conformal field theory with Kac-Moody algebra would emerge naturally.

2. The new degrees of freedom would define 2-dimensional continuum and it would not be completely surprising if conformal field theory based on ADE Kac Moody algebra could describe the situation. One possibility is that these continua for different inclusions correspond to $SU(2)$ decompose to an $N(G)$-fold covers of S^2/G orbifold so that also now groups G would be involved with the Jones inclusions, which might provide a hint about how to construct them. S^2/G would play the role of stringy world sheet for the conformal field theory in question. This effective re-arrangement of the topology S^2 might be due to the fact that conformal fields possess G symmetry which effectively groups points of S^2 to $n(G)$-multiplets. The localized representations of the Lie group corresponding to G would correspond to the multiplets obtained from the representations of group algebra of G as in previous case.

3. The formula for the scaling factor of CD metric would give infinite scaling factor if one identifies the scaling factor as maximal order of cyclic subgroup of $SU(2)$. As a matter fact there is no finite cyclic subgroup of this kind. The solution to the problem would be identification of the scaling factor as the order of the maximal cyclic subgroup of G so that the scaling factors would be same for the two situations related by McKay correspondence.

Generalization to CD degrees of freedom

One can ask whether the proposed picture generalizes formally also the case of CD.

1. In this case quantum groups would correspond to discrete subgroups $G\subset SL(2,C)$. Kac Moody group would correspond to G-Kac Moody algebra made local with respect to $SL(2,C)$ orbit in CD divided by G. These orbits are 3-dimensional hyperboloids H_a with a constant value of light cone proper time a so that the division by G gives fundamental domain H_a/G with a finite 3-volume.

2. The 4-dimensionality of space-time would require 1-dimensional CP_2 projection. Vacuum extremals of Kähler action would be in question. Robertson-Walker metric have 1-dimensional CP_2 projection and carry non-vanishing density of gravitational mass so that in this sense the theory would be non-trivial. G would label different lattice like cosmologies defined by tesselations with fundamental domain H_a/G.

3. The multiplets of G would correspond to collections of points, one from each cells of the lattice like structure. Macroscopic quantum coherence would be realized in cosmological scales. If one takes seriously the vision about the role of short distance p-adic physics as a generator of long

range correlations of the real physics reflected as p-adic fractality, this idea does not look so weird anymore.

Complexified modular group $SL(2, Z + iZ)$ and its subgroups are interesting as far as p-adicization is considered. The principal congruence subgroups $\Gamma(N)$ of $SL(2, Z + iZ)$ which are unit matrices modulo N define normal subgroups of the complex modular group and are especially interesting candidates for groups $G \subset SL(2, C)$. The group $\Gamma(N = p^k)$ labeling fundamental domains of the tesselation $H_a/\Gamma(N = p^k)$ defines a mathematically attractive candidate for a point set associated with the intersections of p-adic space-time sheets with real space-time sheets. Also analogous groups for algebraic extensions of Z are interesting.

The simplest discrete subgroup of SL(2,C) with infinite number of elements would corresponds to powers of boost to single direction and correspond at the non-relativistic limit to multiples of basic velocity. This could also give rise to quantization of cosmic recession velocities. There is evidence for the quantization of cosmic recession velocities (for a model in which single object produces quantized redshifts see [?]) and it is interesting to see whether they could be interpreted in terms of the lattice like periodicity in cosmological length scales implied by the effective reduction of physics to M^4_+/G_n. In [?] the values $z = 2.63, 3.45, 4.47$ of cosmic red shift are listed. These correspond to recession velocities $v = (z^2 - 1)/(z^2 + 1)$ are (0.75,0.85,0.90). The corresponding hyperbolic angles are given by $\eta = acosh(1/(1 - v^2))$ and the values of η are $(1.46, 1.92, 2.39)$. The differences $\eta(2) - \eta(1) = .466$ and $\eta(3) - \eta(2) = .467$ are same within experimental uncertainties. One has however $\eta(n)/(\eta(2) - \eta(1)) = (3.13, 4.13, 5.13)$ instead of $(3, 4, 5)$. A possible interpretation is in terms of the velocity of the observer with respect to the frame in which quantization of η happens.

Quantitative support for the interpretation

A more detailed analysis of the situation gives support for the proposed vision.

1. A given value of quantum group deformation parameter $q = exp(i\pi/n)$ makes sense for any Lie algebra but now a preferred Lie-algebra is assigned to a given value of quantum deformation parameter. At the limit $\beta = 4$ when quantum deformation parameter becomes trivial, the gauge symmetry is replaced by Kac Moody symmetry.

2. The prediction is that Kac-Moody central extension parameter should vanish for $\beta < 4$. There is an intriguing relationship to formula for the quantum phase q_{KM} associated with (possibly trivial) Kac-Moody central extension and the phase defined by ADE diagram

$$\begin{array}{lll} q_{KM} = exp(i\phi) \ , & \phi_1 = \frac{\pi}{k+h^v} \ , \\ q_{Jones} = exp(i\phi) \ , & \phi = \frac{\pi}{h} \end{array}$$

In the first formula sum of Kac-Moody central extension parameter k and dual Coxeter number h^v appears whereas Coxeter number h appears in the second formula. Internal consistency requires

$$k + h^v = h \ . \tag{2.7.1}$$

It is easy see that the dual Coxeter number h^v and Coxeter number h given by $h = (dim(g) - r)/r$, where r is the dimension of Cartan algebra of g, are identical for ADE algebras so that the Kac-Moody central extension parameter k must indeed vanish. For $SO(2n + 1)$, $Sp(n)$, G_2, and F_4 the condition $h = h^v$ does not hold true but one has $h(n) = 2n = h^v + 1$ for $SO(2n + 1)$, $h(n) = 2n = 2(h^v - 1)$ for $Sp(n)$, $h = 6 = h^v + 2$ for G_2, and $h = 12 = h^v + 3$ for F_4.

What is intriguing that G_2, which seems to play a fundamental role in the dual formulation of quantum TGD based on the identification of space-times as surfaces in hyper-octonionic space M^8 [?] is not allowed. As a matter fact, $G_2 \rightarrow SU(3)$ reduction occurs also in the dual formulation based on $G_2/SU(3)$ coset model and is required by the separate conservation of quark and lepton numbers predicted by TGD. ADE groups would be associated with the interaction between space-time sheets rather than entire dynamics and need not have anything to do with the Kac-Moody algebra associated with color and electro-weak interactions appearing in the construction of physical states [?].

3. There seems to be a concrete connection with conformal field theories. This connection would allow to understand the emergence of quantum groups appearing naturally in these theories. Quite generally, the conformal central extension parameter for unitary Virasoro representations resulting by Sugawara construction from Kac Moody representations satisfies either of the conditions

$$
\begin{aligned}
c &\geq \frac{kdim(g)}{k+h^v}+1 \ , \\
c &= \frac{kdim(g)}{k+h^v}+1-\frac{6}{(h-1)h} \ .
\end{aligned}
\tag{2.7.2}
$$

For $k=0$, which should be interesting for $\beta<4$, the second formula reduces to

$$
c = 1-\frac{6}{(h-1)h} \ . \tag{2.7.3}
$$

The formula gives the values of c for minimal conformal field theories with finite number of conformal fields and real conformal weights. Indeed, h in this formula seems to correspond to the same h as appearing in the expression $\beta\equiv \mathcal{M}:\mathcal{N}=4cos^2(\pi/h)$.

$\beta=3, h=6$ corresponds to three-state Potts model with $c=4/5$ which should thus have a gauge group for which Coxeter number is 6: the group should be either $SU(6)$ or $SO(8)$. Two-state Potts model, that is Ising model with $\beta=2, h=4$ would correspond to $c=1/2$ and to a gauge group $SU(4)$ or $SO(4)$. For $h=3$ ("one-state Potts model") with group $SU(3)$ one would have $c=0$ and vanishing conformal anomaly so that conformal degrees of freedom would become pure gauge degrees of freedom.

These observations give support for the following picture.

1. Quite generally, the number of states of the generalized β-state Potts model has an interpretation as the dimension $\beta=\mathcal{M}:\mathcal{N}$ of $\mathcal{M}$ as $\mathcal{N}$-module. Besides the models with integer number of states there is an infinite number of models for which the number of states is not an integer. The conditions $c\leq 1$ guaranteing real conformal weights and $\beta\leq 4$ correspond to each other for these models.

2. $\beta>4$ Potts models would be formally obtained by allowing h to be imaginary in the defining formula for $\mathcal{M}:\mathcal{N}$. In this case c would be however complex so that the theory would not be unitary.

3. For minimal models with $(\beta<4, c<1)$ Kac-Moody central extension parameter is vanishing so that Kac Moody algebra indeed acts like gauge symmetries and gauge symmetries would be in question. $(\beta=4, c=1)$ would define a "four-state Potts model" with infinite-dimensional unitary group acting as a gauge group. On the other hand, the appearance of extended ADE Dynkin diagrams suggests strongly that this limit is not realized but that $\beta=\mathcal{M}:\mathcal{N}=4$ corresponds to $k=1$ conformal field theory allowing Kac Moody symmetries for any ADE group, which as simply-laced groups allows vertex operator construction. The appearance of $kdim(g)/(k+g)$ in the more general formula would thus code the Kac Moody group whereas for $\beta<4$ ADE diagram codes for the preferred gauge group characterizing the minimal CFT.

4. The possibility that any ADE gauge group or Kac-Moody group can characterize the interaction between space-time sheets conforms with the idea about Universe as a Topological Quantum Computer able to simulate any conceivable quantum dynamics. Of course, one cannot exclude the possibility that only electro-weak and color symmetries are realized in this manner.

G_a as a symmetry group of magnetic body and McKay correspondence

The group $G_a \subset SU(2) \subset SL(2,C)$ means exact rotational symmetry realized in terms of CD coverings of CP_2. The 5 and 6-cycles in biochemistry (sugars, DNA,....) are excellent candidates for these symmetries. For very large values of Planck constant, say for the values $\hbar(CD)/\hbar(CP_2) = GMm/v_0 = (n_a/n_b)\hbar_0$, $v_0 = 2^{-11}$, required by the model for planetary orbits as Bohr orbits [?], G_a is huge and corresponds to either Z_{n_a} or in the case of even value of n_a to the group generated by Z_n and reflection acting on plane and containing $2n_a$ elements.

The notion of magnetic body seems to provide the only conceivable candidate for a geometric object possessing G_a as symmetries. In the first approximation the magnetic field associated with a dark matter system is expected to be modellable as a dipole field having rotational symmetry around the dipole axis. Topological quantization means that this field decomposes into flux tube like structures related by the rotations of Z_n or D_{2n}. Dark particles would have wave functions delocalized to this set of these flux quanta and span group algebra of G_a. Magnetic flux quanta are indeed assumed to mediate gravitational interactions in the TGD based model for the quantization of radii of planetary orbits and this explains the dependence of $\hbar_{gr}$ on the masses of planet and central object [?].

For the model of dark matter hierarchy appearing in the model of living matter one has $n_a = 2^{11k}$, $k = 1, 2, 3, .., 7$ for cyclotron time scales below life cycle for a magnetic field $B_d = .2$ Gauss at $k = 4$ level of hierarchy (the field strength is fixed by the model for the effects of ELF em fields on vertebrate brain at harmonics of cyclotron frequencies of biologically important ions [?]). Note that B_d scales as 2^{-11k} from the requirement that cyclotron energy is constant.

ADE correspondence between subgroups of $SU(2)$ and Lie groups in ADE hierarchy encourages to consider the possibility that TGD could mimic ADE hierarchy of gauge theories. In the case of G_a this would mean that many fermion states constructed from single fermion states, which are in one-one correspondence with the elements of G_a group algebra, would define multiplets of the gauge group corresponding to the Dynkin diagram characterizing G_a: for instance, $SU(n_a)$ in the case of Z_{n_a}. Fermion multiplet would contain n_a states and gauge boson multiplet $n_a^2 - 1$ states. This would provide enormous information processing capacity since for $n_a = 2^{11k}$ fermion multiplet would code exactly $11k$ bits of information. Magnetic body could represent binary information using the many-particle states belonging to the representations of say $SU(n_a)$ at its flux tubes.

2.7.2 Jones inclusions, the large N limit of $SU(N)$ gauge theories and AdS/CFT correspondence

The framework based on Jones inclusions has an obvious resemblance with larger N limit of $SU(N)$ gauge theories and also with the celebrated AdS/CFT correspondence [?] so that a more detailed comparison is in order.

Large N limit of gauge theories and series of Jones inclusions

The large N limit of $SU(N)$ gauge field theories has as definite resemblance with the series of Jones inclusions with the integer $n \geq 3$ characterizing the quantum phase $q = exp(i\pi/n)$ and the order of the maximal cyclic subgroup of the subgroup of $SU(2)$ defining the inclusion. Recall that all ADE groups except D_{2n+1} and E_7 are allowed (SU(2) is excluded since it would correspond to $n = 2$).

The limiting procedure keeps the value of g^2N fixed. Rather remarkably, this is equivalent with keeping αN constant but assuming $\hbar$ to scale as $n = N$. Thus the quantization of Planck constants would provide a physical laboratory for the testing of large N limit.

The observation suggesting a description of YM theories in terms of closed strings is that Feynman diagrams can be interpreted as being imbedded at closed 2-surfaces of minimal genus guaranteing that the internal lines meet except in vertices. The contribution of genus g diagrams is proportional to N^{g-1} at the large N limit. The interpretation in terms of closed partonic 2-surfaces is highly suggestive and the N^{g-1} should come from the multiple covering property of CP_2 by N CD-points (or vice versa) with the finite subgroup of $G \subset SU(2)$ defining the Jones inclusion and acting as symmetries of the surface.

Analogy between stacks of branes and multiple coverings of CD and CP_2

An important aspect of AdS/CFT dualities is a prediction of an infinite hierarchy of gauge groups, which as such is as interesting as the claimed dualities. The prediction relies on the notion Dp-branes. Dp-branes are $p+1$-dimensional surfaces of the target space at which the ends of open strings can end. In the simplest situation one considers N parallel p-branes at the limit when the distances between branes characterized by an expectation value of Higgs fields approach zero to obtain what is called N-stack of branes. There are N^2 different strings connecting the branes and the heuristic idea is that they correspond to gauge bosons of $U(N)$ gauge theory. Note that the requirement that AdS/CFT dualities exist forces the introduction of branes and the optimistic interpretation is that a non-perturbative effect of still unknown M-theory is in question. In the limit of an ideal stack one assumes that $U(N)$ gauge theory at the brane representing the stack is obtained. The branes must also carry a p-form defining gauge potential for a closed $p+1$-form. This Ramond charge is quantized and its value equals to N.

Consider now the group $G_a \times G_b \subset SL(2,C) \times SU(2) \subset SU(3)$ defining double Jones inclusion and implying the scalings $\hbar(M^4) \rightarrow n(G_b)\hbar(M^4)$ and $\hbar(CP_2) \rightarrow n(G_a)\hbar(CP_2)$. These space-time surfaces define $n(G_a)$-fold multiple coverings of CP_2 and $n(G_b)$-fold multiple coverings of CD. In CP_2 degrees of freedom the collection of G_b-related partonic 2-surfaces (/3-surfaces/4-surfaces) is highly analogous to the stack of branes. In CD degrees of freedom the stack of copies of surface typically correspond to along a circle (A_n, D_{2n} or at vertices of tedrahedron or isosahedron.

In TGD framework the interpretation strings are not needed to define gauge fields. The group algebra of G realized as discrete plane waves at G-orbit gives rise to representations of G. The hypothesis supported by few examples is that these additional degrees of freedom allow to construct multiplets of the gauge group assignable to the ADE diagram characterizing the inclusion.

AdS/CFT duality

AdS/CFT duality is a further aspect of the brane construction. The dual description of the situation is in terms of a string theory in a background in which N-brane acts as a macroscopic object giving rise to a black-hole like object in (say) 10-dimensional target space. This background has the form $AdS_5 \times X_5$, where AdS_5 is 5-dimensional hyperboloid of M^6 and thus allows $SO(4,2)$ as isometries. X_5 is compact constant curvature space. S_5 gives rise to $N = 4$ SUSY in M^4 with M^4 interpreted as a brane. The first support for the dualities comes from the symmetries: for instance, the $N = 4$ super-symmetrized isometries of $AdS_5 \times S^5$ are same as the symmetries of 4-dimensional $N = 4$ SUSY for $p = 3$ branes. N-branes can be used as models for black holes in target space and black-hole entropy can be calculated using either target space picture or conformal field theory at brane and the results turn out be the same.

Does the TGD equivalent of this duality exists in some sense?

1. As far as partonic 2-surfaces identified as 1-branes are considered, conformal field theory description is trivially true. In TGD framework the analog of Ramond charges are the integers n_a and n_b characterizing the multiplicities of the maximal Abelian subgroups having clear topological meaning. This conforms with the observation that large N limit of the gauge field theories can be formulated in terms of closed surfaces at which the Feynman diagrams are imbedded without self crossings. It seems that the integers n_a and n_b characterizing the Jones inclusion naturally take the role of Ramond charge: this does not of course exclude the possibility they can be expressed as fluxes at space-time level as will be indeed found.

2. Conformal field theory description can be generalized in the sense that one replaces the $n(G_a) \times n(G_b)$ partonic surfaces with single one and describes the new states as primary fields arranged into representations of the ADE group in question. This would mean that the standard model gauge group extends by additional factor which is however non-trivially related to it.

3. If one can accept the idea that the conformal field theory description for partons gives rise to M^4 gauge theory as an approximate description, it is not too difficult to imagine that also ADE hierarchy of gauge theories results as a description of the exotic states. One can say that CFT in p-brane is replaced now with CFT on partonic 2-surface (1-brane) analogous to a closed string.

4. In the minimal interpretation there is no need to add strings connecting the branches of the double covering of the partonic 2-surface whose function is essentially that of making possible gauge bosons as fermion anti-fermion pairs. One could of course imagine gauge fluxes as counterparts of strings but just the fact that G-invariance dictates the configurations completely forces to question this kind of dynamics.

5. There is no reason to expect the emergence of $N = 4$ super-symmetric field theory in M^4 as in the case of super-string models. The reasons should be already obvious: super-conformal generators G anticommute to L_0 proportional to mass squared rather than four-momentum and the spectrum extended by $G_a \times G_b$ degeneracy contains more states.

One can of course ask whether higher values of p could make sense in TGD framework.

1. It seems that the light-like orbits of the partonic 2-surfaces defining 2-branes do not bring in anything new since the generalized conformal invariance makes it possible the restriction to a 2-dimensional cross section of the light like causal determinant.

2. The idea of regarding space-time surface X^4 as a 3-brane in H in which some kind of conformal field theory is defined is in conflict with the basis ideas of TGD. The role of X^4 interior is to provide classical correlates for quantum dynamics to make possible quantum measurement theory and also introduce correlations between partonic 2-surfaces even in the case that partonic conformal dynamics reduces to a topological string theory. It is quantum classical correspondence which corresponds to this duality.

What is the counterpart of the Ramond charge in TGD?

The condition that there exist a p-form defining $p+1$-gauge field with p-charge equal to n_a or n_b is a rather stringent additional condition also in TGD framework. For $n < \infty$ this kind of charge is defined by Jones inclusion and represented topologically so that Ramond charge is not needed in $n < \infty$ case. By the earlier arguments one must however be able to assign integers n_a and n_b also to $G = SU(2)$ inclusions with Kac-Moody algebra characterized by an extended ADE diagram with the phases $q_i = exp(i\pi/n_i)$ relating to the monodromy of the theory. Since Jones inclusion does not define in this case the value of $n < \infty$ in any obvious manner, the counterpart of the Ramond charge is needed.

1. For partonic 2-surfaces ordinary gauge potential would define this form and the condition would state that magnetic flux equals to n so that the anyonic partonic two-surfaces would be homologically non-trivial in CP_2 degrees of freedom. String ends would define basic example of this situation. This would be the case also in M^4_+ degrees of freedom: the partonic 2-surface would essentially wind n_a times around the tip of δCD and the gauge field in question would be monopole magnetic field in δCD. This kind of situation need not correspond to anything cosmological since future and past light-cones appear in the basic definition of the scattering amplitudes.

2. For $p = 3$ Chern-Simons action for the induced CP_2 Kähler form associated with the partonic 2-surface indeed defines this kind of charge. Ramond charge should be simply N. CP_2 type extremals or their small deformations satisfy this constraint and are indeed very natural in elementary particle physics context but too restrictive in a more general context.

Note that the light-like orbits of non-deformed CP_2 extremals have light-like random curve as an M^4 projection and the conformal symmetries of M^4 obviously respect light-likeness property. Hence $SO(4,2)$ symmetry characterizing AdS$_5$/CFT is not excluded but would be broken by p-adic thermodynamics and by TGD based Higgs mechanism involving the identification of inertial momentum as average value of non-conserved gravitational momentum parallel to the light-like zitterbewegung orbit.

Can one speak about black hole like structures in TGD framework?

For AdS/CFT correspondence there is also a dynamical coupling to the target space metric. The coupling to H-metric is present also now since the overall scalings of the *CD resp.* CP_2 metrics by n_b *resp.* by n_a are involved. This applies to when multiple covering is used explicitly. In the description in which one replaces the multiple covering by ordinary $M^4 \times CP_2$, the metric suffers a genuine change and something analogous to the black-hole type metrics encountered in AsS/CFT correspondence might be encountered.

Consider as an example an n_a-fold covering of CP_2 points by M^4 points (ADE diagram A_{n_a-1}). The n-fold covering means only $n2\pi$ rotation for the phase angle ψ of CP_2 complex coordinate leads to the original point. The replacement $\psi \rightarrow \psi/n_a$ gives rise to what would look like ordinary $M^4 \times CP_2$ but with a modified CP_2 metric. The metric components containing ψ as index are scaled down by $1/n_a$ or $1/n_a^2$. Notice that Ψ effectively disappears from the dynamics at the large n_a limit.

If one uses an effective description in which covering is eliminated the metric is indeed affected at the level of imbedding space black hole like structures at the level of dynamic space might make emerge also in TGD framework at large N limit since the masses of the objects in question become large and CP_2 metric is scaled by N so that CP_2 has very large size at this limit. This need not lead to any inconsistencies if these phases are interpreted as dark matter. At the elementary particle level p-adic thermodynamics predicts that p-adic entropy is proportional to thermal mass squared which implies elementary particle black-hole analogy.

Other dualities

Also quantum classical correspondence defines in a loose sense a duality justifying the basic assumptions of quantum measurement theory. The light-like orbits of 2-D partons are characterized by a generalization of ordinary 2-D conformal invariance so that CFT part of the duality would be very natural. The dynamical target space would be replaced with the space-time surface X^4 with a dynamical metric providing classical correlates for the quantum dynamics at partonic 2-surfaces. The duality in this sense cannot be however exact since classical dynamics cannot fully represent quantum dynamics.

Classical description is not expected to be unique. The basic condition on space-time surfaces assignable to a given configuration of partonic 2-surfaces associated with the surface X_V^3 defining S-matrix element are posed by quantum classical correspondence. Both hyper-quaternionic and co-hyper-quaternionic space-time surfaces are acceptable and this would define a fundamental duality.

A concrete example about this HQ-coHQ duality would be the equivalence of space-time descriptions using 4-D CP_2 type extremals and 4-D string like objects connecting them. If one restricts to CP_2 type extremals and string like objects of from $X^2 \times Y^2$, the target space reduces effectively to M^4 and the dynamical degrees of freedom correspond in both cases to transversal M^4 degrees of freedom. Note that for CP_2 type extremals the conditions stating that random light-likeness of the M^4 projection of the CP_2 type extremal are equivalent to Virasoro conditions. CP_2 type extremals could be identified as co-HQ surfaces whereas stringlike objects would correspond to HQ aspect of the duality.

HQ-coHQ provides dual classical descriptions of same phenomena. Particle massivation would be a basic example. Higgs mechanism in a gauge theory description based on CP_2 type extremals would rely on zitterbewegung implying that the average value of gravitational mass identified as inertial mass is non-vanishing and is discussed already. Higgs field would be assigned to the wormhole contacts. The dual description for the massivation would be in terms of string tension and mass squared would be proportional to the distance between G-related points of CP_2.

These observations would suggest that also a super-conformal algebra containing $SL(2,R) \times SU(2)_L \times U(1)$ or its compact version exists and corresponds to a trivial inclusion. This is indeed the case [**?**]. The so called large $N = 4$ super-conformal algebra contains energy momentum current, 2+2 super generators G, $SU(2) \times SU(2) \times U(1)$ Kac-Moody algebra (both $SU(2)$ and SL(2,R) could be interpreted as acting on M^4 spin degrees of freedom, and 2 spin 1/2 fermionic currents having interpretation in terms of right handed neutrinos corresponding to two H-chiralities. Interestingly, the scalar generator is now missing.

2.7.3 Could McKay correspondence and Jones inclusions relate to each other?

The understanding of Langlands correspondence for general reductive Lie groups in TGD framework seems to require some physical mechanism allowing the emergence of these groups in TGD based physics. The physical idea would be that quantum dynamics of TGD is able to emulate the dynamics of any gauge theory or even stringy dynamics of conformal field theory having Kac-Moody type symmetry and that this emulation relies on quantum deformations induced by finite measurement resolution described in terms of Jones inclusions of sub-factors characterized by group G leaving elements of sub-factor invariant. Finite measurement resolution would would result simply from the fact that only quantum numbers defined by the Cartan algebra of G are measured.

There are good reasons to expect that infinite Clifford algebra has the capacity needed to realize representations of an arbitrary Lie group. It is indeed known that that any quantum group characterized by quantum parameter which is root of unity or positive real number can be assigned to Jones inclusion [?]. For $q = 1$ this would gives ordinary Lie groups. In fact, all amenable groups define unique sub-factor and compact Lie groups are amenable ones.

It was so called McKay correspondence [?] which originally stimulated the idea about TGD as an analog of Universal Turing machine able to mimic both ADE type gauge theories and theories with ADE type Kac-Moody symmetry algebra. This correspondence and its generalization might also provide understanding about how general reductive groups emerge. In the following I try to cheat the reader to believe that the tensor product of representations of SU(2) Lie algebras for Connes tensor powers of $\mathcal{M}$ could induce ADE type Lie algebras as quantum deformations for the direct sum of n copies of $SU(2)$ algebras This argument generalizes also to the case of other compact Lie groups.

About McKay correspondence

McKay correspondence [?] relates discrete finite subgroups of $SU(2)$ ADE groups. A simple description of the correspondences is as follows [?].

1. Consider the irreps of a discrete subgroup $G \subset SU(2)$ which correspond to irreps of G and can be obtained by restricting irreducible representations of $SU(2)$ to those of G. The irreducible representations of $SU(2)$ define the nodes of the graph.

2. Define the lines of graph by forming a tensor product of any of the representations appearing in the diagram with a doublet representation which is always present unless the subgroup is 2-element group. The tensor product regarded as that for $SU(2)$ representations gives representations $j - 1/2$, and $j + 1/2$ which one can decompose to irreducibles of G so that a branching of the graph can occur. Only branching to two branches occurs for subgroups yielding extended ADE diagrams. For the linear portions of the diagram the spins of corresponding SU(2) representations increase linearly as $.., j, j + 1/2, j + 1, ...$

One obtains extended Dynkin diagrams of ADE series representing also Kac-Moody algebras giving A_n, D_n, E_6, E_7, E_8. Also A_∞ and $A_{-\infty,\infty}$ are obtained in case that subgroups are infinite. The Dynkin diagrams of non-simply laced groups B_n ($SO(2n+1)$), C_n (symplectic group $Sp(2n)$ and quaternionic group $Sp(n)$), and exceptional groups G_2 and F_4 are not obtained.

ADE Dynkin diagrams labeling Lie groups instead of Kac-Moody algebras and having one node less, do not appear in this context but appear in the classification of Jones inclusions for $\mathcal{M} : \mathcal{N} < 4$. As a matter fact, ADE type Dynkin diagrams appear in very many contexts as one can learn from John Baez's This Week's Finds [?].

1. The classification of integral lattices in Rn having a basis of vectors whose length squared equals 2

2. The classification of simply laced semisimple Lie groups.

3. The classification of finite sub-groups of the 3-dimensional rotation group.

4. The classification of simple singularities . In TGD framework these singularities could be assigned to origin for orbifold CP_2/G, $G \subset SU(2)$.

5. The classification of tame quivers.

Principal graphs for Connes tensor powers $\mathcal{M}$

The thought provoking findings are following.

1. The so called principal graphs characterizing $\mathcal{M} : \mathcal{N} = 4$ Jones inclusions for $G = SU(2)$ are extended Dynkin diagrams characterizing ADE type affine (Kac-Moody) algebras. D_n is possible only for $n \geq 4$.

2. $\mathcal{M} : \mathcal{N} < 4$ Jones inclusions correspond to ordinary ADE type diagrams for a subset of simply laced Lie groups (all roots have same length) A_n ($SU(n)$), D_{2n} ($SO(2n)$), and E_6 and E_8. Thus D_{2n+1} ($SO(2n+2)$) and E_7 are not allowed. For instance, for $G = S_3$ the principal graph is not D_3 Dynkin diagram.

The conceptual background behind principal diagram is necessary if one wants to understand the relationship with McKay correspondence.

1. The hierarchy of higher commutations defines an invariant of Jones inclusion $\mathcal{N} \subset \mathcal{M}$. Denoting by $\mathcal{N}'$ the commutant of $\mathcal{N}$ one has sequences of horizontal inclusions defined as $C = \mathcal{N}' \cap \mathcal{N} \subset \mathcal{N}' \cap \mathcal{M} \subset \mathcal{N}' \cap \mathcal{M}^1 \subset ...$ and $C = \mathcal{M}' \cap \mathcal{M} \subset \mathcal{M}' \cap \mathcal{M}^1 \subset ....$ There is also a sequence of vertical inclusions $\mathcal{M}' \cap \mathcal{M}^k \subset \mathcal{N}' \cap \mathcal{M}^k$. This hierarchy defines a hierarchy of Temperley-Lieb algebras [**?**] assignable to a finite hierarchy of braids. The commutants in the hierarchy are direct sums of finite-dimensional matrix algebras (irreducible representations) and the inclusion hierarchy can be described in terms of decomposition of irreps of k^{th} level to irreps of $(k-1)^{th}$ level irreps. These decomposition can be described in terms of Bratteli diagrams [**?**, **?**].

2. The information provided by infinite Bratteli diagram can be coded by a much simpler bi-partite diagram having a preferred vertex. For instance, the number of $2k$-loops starting from it tells the dimension of k^{th} level algebra. This diagram is known as principal graph.

Principal graph emerges also as a concise description of the fusion rules for Connes tensor powers of $\mathcal{M}$.

1. It is natural to decompose the Connes tensor powers [**?**] $\mathcal{M}_k = \mathcal{M} \otimes_{\mathcal{N}} ... \otimes_{\mathcal{N}} \mathcal{M}$ to irreducible $\mathcal{M}-\mathcal{M}$, $\mathcal{N}-\mathcal{M}$, $\mathcal{M}-\mathcal{N}$, or $\mathcal{N}-\mathcal{N}$ bi-modules. If $\mathcal{M} : \mathcal{N}$ is finite this decomposition involves only finite number of terms. The graphical representation of these decompositions gives rise to Bratteli diagram.

2. If $\mathcal{N}$ has finite depth the information provided by Bratteli diagram can be represented in nutshell using principal graph. The edges of this bipartite graph connect $\mathcal{M}-\mathcal{N}$ vertices to vertices describing irreducible $\mathcal{N}-\mathcal{N}$ representations resulting in the decomposition of $\mathcal{M}-\mathcal{N}$ irreducibles. If this graph is finite, $\mathcal{N}$ is said to have finite depth.

A mechanism assigning to tensor powers Jones inclusions ADE type gauge groups and Kac-Moody algebras

The earliest proposals inspired by the hierarchy of Jones inclusions is that in $\mathcal{M} : \mathcal{N} < 4$ case it might be possible to construct ADE representations of gauge groups or quantum groups and in $\mathcal{M} : \mathcal{N} = 4$ using the additional degeneracy of states implied by the multiple-sheeted cover $H \rightarrow H/G_a \times G_b$ associated with space-time correlates of Jones inclusions. Either G_a or G_b would correspond to G. In the following this mechanism is articulated in a more refined manner by utilizing the general properties of generators of Lie-algebras understood now as a minimal set of elements of algebra from which the entire algebra can be obtained by repeated commutation operator (I have often used " Lie algebra generator" as an synonym for "Lie algebra element"). This set is finite also for Kac-Moody algebras.

1. Two observations

The explanation to be discussed relies on two observations.

1. McKay correspondence for subgroups of G ($\mathcal{M} : \mathcal{N} = 4$) *resp.* its variants ($\mathcal{M} : \mathcal{N} < 4$) and its counterpart for Jones inclusions means that finite-dimensional irreducible representations of allowed $G \subset SU(2)$ label both the Cartan algebra generators and the Lie (Kac-Moody) algebra generators of t_+ and t_- in the decomposition $g = h \oplus t_+ \oplus t_-$, where h is the Lie algebra of maximal compact subgroup.

2. Second observation is related to the generators of Lie-algebras and their quantum counterparts (see Appendix for the explicit formulas for the generators of various algebras considered). The observation is that each Cartan algebra generator of Lie- and quantum group algebras, corresponds to a triplet of generators defining an SU(2) sub-algebra. The Cartan algebra of affine algebra contains besides Lie group Cartan algebra also a derivation d identifiable as an infinitesimal scaling operator L_0 measuring the conformal weight of the Kac-Moody generators. d is exceptional in that it does not give rise to a triplet. It corresponds to the preferred node added to the Dynkin diagram to get the extended Dynkin diagram.

2. Is ADE algebra generated as a quantum deformation of tensor powers of $SU(2)$ Lie algebras representations?

The ADE type symmetry groups could result as an effect of finite quantum resolution described by inclusions of HFFs in TGD inspired quantum measurement theory.

1. The description of finite resolution typically leads to quantization since complex rays of state space are replaced as $\mathcal{N}$ rays. Hence operators, which would commute for an ideal resolution cease to do so. Therefore the algebra $SU(2)\otimes ...\otimes SU(2)$ characterized by n mutually commuting triplets, where n is the number of copies of $SU(2)$ algebra in the original situation and identifiable as quantum algebra appearing in $\mathcal{M}$ tensor powers with $\mathcal{M}$ interpreted as $\mathcal{N}$ module, could suffer quantum deformation to a simple Lie algebra with $3n$ Cartan algebra generators. Also a deformation to a quantum group could occur as a consequence.

2. This argument makes sense also for discrete groups $G \subset SU(2)$ since the representations of G realized in terms of configuration space spinors extend to the representations of SU(2) naturally.

3. Arbitrarily high tensor powers of $\mathcal{M}$ are possible and one can wonder why only finite-dimensional Lie algebra results. The fact that $\mathcal{N}$ has finite depth as a sub-factor means that the tensor products in tensor powers of $\mathcal{N}$ are representable by a finite Dynkin diagram. Finite depth could thus mean that there is a periodicity involved: the kn tensor powers decomposes to representations of a Lie algebra with $3n$ Cartan algebra generators. Thus the additional requirement would be that the number of tensor powers of $\mathcal{M}$ is multiple of n.

3. Space-time correlate for the tensor powers $\mathcal{M} \otimes_{\mathcal{N}} ... \otimes_{\mathcal{N}} \mathcal{M}$

By quantum classical correspondence there should exist space-time correlate for the formation of tensor powers of $\mathcal{M}$ regarded as $\mathcal{N}$ module. A concrete space-time realization for this kind of situation in TGD would be based on n-fold cyclic covering of H implied by the $H \rightarrow H/G_a \times G_b$ bundle structure in the case of say G_b. The sheets of the cyclic covering would correspond to various factors in the n-fold tensor power of $SU(2)$ and one would obtain a Lie algebra, affine algebra or its quantum counterpart with n Cartan algebra generators in the process naturally. The number n for space-time sheets would be also a space-time correlate for the finite depth of $\mathcal{N}$ as a factor.

Configuration space spinors could provide fermionic representations of $G \subset SU(2)$. The Dynkin diagram characterizing tensor products of representations of $G \subset SU(2)$ with doublet representation suggests that tensor products of doublet representations associated with n sheets of the covering could realize the Dynkin diagram.

Singlet representation in the Dynkin diagram associated with irreps of G would not give rise to an SU(2) sub-algebra in ADE Lie algebra and would correspond to the scaling generator. For ordinary Dynkin diagram representing gauge group algebra scaling operator would be absent and therefore also the exceptional node. Thus the difference between ($\mathcal{M} : \mathcal{N} = 4$) and ($\mathcal{M} : \mathcal{N} < 4$) cases would be that in the Kac-Moody group would reduce to gauge group $\mathcal{M} : \mathcal{N} < 4$ because Kac-Moody central charge k and therefore also Virasoro central charge resulting in Sugawara construction would vanish.

4. Do finite subgroups of $SU(2)$ play some role also in $\mathcal{M} : \mathcal{N} = 4$ case?

One can ask wonder the possible interpretation for the appearance of extended Dynkin diagrams in ($\mathcal{M} : \mathcal{N} = 4$) case. Do finite subgroups $G \subset SU(2)$ associated with extended Dynkin diagrams appear also in this case. The formal analog for $H \rightarrow G_a \times G_b$ bundle structure would be $H \rightarrow H/G_a \times SU(2)$. This would mean that the geodesic sphere of CP_2 would define the fiber. The notion of number theoretic braid meaning a selection of a discrete subset of algebraic points of the geodesic sphere of CP_2 suggests that $SU(2)$ actually reduces to its subgroup G also in this case.

5. Why Kac-Moody central charge can be non-vanishing only for $\mathcal{M} : \mathcal{N} = 4$?

From the physical point of view the vanishing of Kac-Moody central charge for $\mathcal{M} : \mathcal{N} < 4$ is easy to understand. If parton corresponds to a homologically non-trivial geodesic sphere, space-time surface typically represents a string like object so that the generation of Kac-Moody central extension would relate directly to the homological non-triviality of partons. For instance, cosmic strings are string like objects of form $X^2 \times Y^2$, where X^2 is minimal surface of M^2 and Y^2 is a holomorphic sub-manifold of CP_2 reducing to a homologically non-trivial geodesic sphere in the simplest situation. A conjecture that deserves to be shown wrong is that central charge k is proportional/equal to the absolute value of the homology (Kähler magnetic) charge h.

6. More general situation

McKay correspondence generalizes also to the case of subgroups of higher-dimensional Lie groups [?]. The argument above makes sense also for discrete subgroups of more general compact Lie groups H since also they define unique sub-factors. In this case, algebras having Cartan algebra with nk generators, where n is the dimension of Cartan algebra of H, would emerge in the process. Thus there are reasons to believe that TGD could emulate practically any dynamics having gauge group or Kac-Moody type symmetry. An interesting question concerns the interpretation of non-ADE type principal graphs associated with subgroups of SU(2).

7. Flavor groups of hadron physics as a support for HFF?

The deformation assigning to an n-fold tensor power of representations of Lie group G with k-dimensional Cartan algebra a representation of a Lie group with nk-dimensional Cartan algebra could be also seen as a dynamically generated symmetry. If quantum measurement is characterized by the choice of Lie group G defining measured quantum numbers and defining Jones inclusion characterizing the measurement resolution, the measurement process itself would generate these dynamical symmetries. Interestingly, the flavor symmetry groups of hadron physics cannot be justified from the structure of the standard model having only electro-weak and color group as fundamental symmetries. In TGD framework flavor group $SU(n)$ could emerge naturally as a fusion of n quark doublets to form a representation of $SU(n)$.

2.7.4 Farey sequences, Riemann hypothesis, tangles, and TGD

Farey sequences allow an alternative formulation of Riemann Hypothesis and subsequent pairs in Farey sequence characterize so called rational 2-tangles. In TGD framework Farey sequences relate very closely to dark matter hierarchy, which inspires *"Platonia as the best possible world in the sense that cognitive representations are optimal"* as the basic variational principle of mathematics. This variational principle supports RH.

Possible TGD realizations of tangles, which are considerably more general objects than braids, are considered. One can assign to a given rational tangle a rational number a/b and the tangles labeled by a/b and c/d are equivalent if $ad - bc = \pm 1$ holds true. This means that the rationals in question are neighboring members of Farey sequence. Very light-hearted guesses about possible generalization of these invariants to the case of general N-tangles are made.

Farey sequences

Some basic facts about Farey sequences [?] demonstrate that they are very interesting also from TGD point of view.

1. Farey sequence F_N is defined as the set of rationals $0 \leq q = m/n \leq 1$ satisfying the conditions $n \leq N$ ordered in an increasing sequence.

2. Two subsequent terms a/b and c/d in F_N satisfy the condition $ad - bc = 1$ and thus define and element of the modular group $SL(2, Z)$.

3. The number $|F(N)|$ of terms in Farey sequence is given by

$$|F(N)| = |F(N-1)| + \phi(N-1) \ . \tag{2.7.4}$$

Here $\phi(n)$ is Euler's totient function giving the number of divisors of n. For primes one has $\phi(p) = 1$ so that in the transition from p to $p+1$ the length of Farey sequence increases by one unit by the addition of $q = 1/(p+1)$ to the sequence.

The members of Farey sequence F_N are in one-one correspondence with the set of quantum phases $q_n = exp(i2\pi/n)$, $0 \leq n \leq N$. This suggests a close connection with the hierarchy of Jones inclusions, quantum groups, and in TGD context with quantum measurement theory with finite measurement resolution and the hierarchy of Planck constants involving the generalization of the imbedding space. Also the recent TGD inspired ideas about the hierarchy of subgroups of the rational modular group with subgroups labeled by integers N and in direct correspondence with the hierarchy of quantum critical phases [**?**] would naturally relate to the Farey sequence.

Riemann Hypothesis and Farey sequences

Farey sequences are used in two equivalent formulations of the Riemann hypothesis. Suppose the terms of F_N are $a_{n,N}$, $0 < n \leq |F_N|$. Define

$$d_{n,N} = a_{n,N} - \frac{n}{|F_N|} \ .$$

In other words, $d_{n,N}$ is the difference between the n:th term of the N:th Farey sequence, and the n:th member of a set of the same number of points, distributed evenly on the unit interval. Franel and Landau proved that both of the following statements

$$\sum_{n=1,\ldots,|F_N|} |d_{n,N}| = O(N^r) \ \text{ for any } \ r > 1/2 \ ,$$
$$\sum_{n=1,\ldots,|F_N|} d_{n,N}^2 = O(N^r) \ \text{ for any } \ r > 1 \ . \tag{2.7.5}$$

are equivalent with Riemann hypothesis.

One could say that RH would guarantee that the numbers of Farey sequence provide the best possible approximate representation for the evenly distributed rational numbers $n/|F_N|$.

Farey sequences and TGD

Farey sequences seem to relate very closely to TGD.

1. The rationals in the Farey sequence can be mapped to the roots of unity by the map $q \rightarrow exp(i2\pi q)$. The numbers $1/|F_N|$ are in turn mapped to the numbers $exp(i2\pi/|F_N|)$, which are also roots of unity. The statement would be that the algebraic phases defined by Farey sequence give the best possible approximate representation for the phases $exp(in2\pi/|F_N|)$ with evenly distributed phase angle.

2. In TGD framework the phase factors defined by F_N corresponds to the set of quantum phases corresponding to Jones inclusions labeled by $q = exp(i2\pi/n)$, $n \leq N$, and thus to the N lowest levels of dark matter hierarchy. There are actually two hierarchies corresponding to M^4 and CP_2 degrees of freedom and the Planck constant appearing in Schrödinger equation corresponds to the ratio n_a/n_b defining quantum phases in these degrees of freedom. $Z_{n_a \times n_b}$ appears as a conformal symmetry of "dark" partonic 2-surfaces and with very general assumptions this implies that there are only in TGD Universe [**?**, **?**].

3. The fusion of physics associated with various number fields to single coherent whole requires algebraic universality. In particular, the roots of unity, which are complex algebraic numbers, should define approximations to continuum of phase factors. At least the S-matrix associated with p-adic-to-real transitions and more generally $p_1 \rightarrow p_2$ transitions between states for which the partonic space-time sheets are p_1- resp. p_2-adic can involve only this kind of algebraic phases. One can also say that cognitive representations can involve only algebraic phases and algebraic numbers in general. For real-to-real transitions and real-to-padic transitions U-matrix might be non-algebraic or obtained by analytic continuation of algebraic U-matrix. S-matrix is by definition diagonal with respect to number field and similar continuation principle might apply also in this case.

4. The subgroups of the hierarchy of subgroups of the modular group with rational matrix elements are labeled by integer N and relate naturally to the hierarchy of Farey sequences. The hierarchy of quantum critical phases is labeled by integers N with quantum phase transitions occurring only between phases for which the smaller integer divides the larger one [**?**].

Interpretation of RH in TGD framework

Number theoretic universality of physics suggests an interpretation for the Riemann hypothesis in TGD framework. RH would be equivalent to the statement that the Farey numbers provide best possible approximation to the set of rationals $k/|F_N|$ or to the statement that the roots of unity contained by F_N define the best possible approximation for the roots of unity defined as $exp(ik2\pi/|F_N|)$ with evenly spaced phase angles. The roots of unity allowed by the lowest N levels of the dark matter hierarchy allows the best possible approximate representation for algebraic phases represented exactly at $|F_N|$:th level of hierarchy.

A stronger statement would be that the Platonia, where RH holds true would be the best possible world in the sense that algebraic physics behind the cognitive representations would allow the best possible approximation hierarchy for the continuum physics (both for numbers in unit interval and for phases on unit circle). Platonia with RH would be cognitive paradise.

One could see this also from different view point. "Platonia as the cognitively best possible world" could be taken as the "axiom of all axioms": a kind of fundamental variational principle of mathematics. Among other things it would allow to conclude that RH is true: RH must hold true either as a theorem following from some axiomatics or as an axiom in itself.

Could rational N-tangles exist in some sense?

The article of Kauffman and Lambropoulou [**?**] about rational 2-tangles having commutative sum and product allowing to map them to rationals is very interesting from TGD point of view. The illustrations of the article are beautiful and make it easy to get the gist of various ideas. The theorem of the article states that equivalent rational tangles giving trivial tangle in the product correspond to subsequent Farey numbers a/b and c/d satisfying $ad-bc=\pm 1$ so that the pair defines element of the modular group SL(2,Z).

1. Rational 2-tangles

1. The basic observation is that 2-tangles are 2-tangles in both "s- and t-channels". Product and sum can be defined for all tangles but only in the case of 2-tangles the sum, which in this case reduces to product in t-channel obtained by putting tangles in series, gives 2-tangle. The so called rational tangles are 2-tangles constructible by using addition of $\pm[1]$ on left or right of tangle and multiplication by $\pm[1]$ on top or bottom. Product and sum are commutative for rational 2-tangles but the outcome is not a rational 2-tangle in the general case. One can also assign to rational 2-tangle its negative and inverse. One can map 2-tangle to a number which is rational for rational tangles. The tangles $[0]$, $[\infty]$, $\pm[1]$, $\pm 1/[1]$, $\pm[2]$, $\pm[1/2]$ define so called elementary rational 2-tangles.

2. In the general case the sum of $M-$ and $N-$tangles is $M+N-2$-tangle and combines various $N-$tangles to a monoidal structure. Tensor product like operation giving $M+N$-tangle looks to me physically more natural than the sum.

3. The reason why general 2-tangles are non-commutative although 2-braids obviously commute is that 2-tangles can be regarded as sequences of $N-$tangles with 2-tangles appearing only as the initial and final state: N is actually even for intermediate states. Since $N > 2$-braid groups are non-commutative, non-commutativity results. It would be interesting to know whether braid group representations have been used to construct representations of $N-$tangles.

2. Does generalization to $N >> 2$ case exist?

One can wonder whether the notion of rational tangle and the basic result of the article about equivalence of tangles might somehow generalize to the $N > 2$ case.

1. Could the commutativity of tangle product allow to characterize the $N > 2$ generalizations of rational 2-tangles. The commutativity of product would be a space-time correlate for the commutativity of the S-matrices defining time like entanglement between the initial and final quantum states assignable to the N-tangle. For 2-tangles commutativity of the sum would have an analogous interpretation. Sum is not a very natural operation for N-tangles for $N > 2$. Commutativity means that the representation matrices defined as products of braid group actions associated with the various intermediate states and acting in the same representation space commute. Only in very special cases one can expect commutativity for tangles since commutativity is lost already for braids.

2. The representations of 2-tangles should involve the subgroups of N-braid groups of intermediate braids identifiable as Galois groups of N:th order polynomials in the realization as number theoretic tangles. Could non-commutative 2-tangles be characterized by algebraic numbers in the extensions to which the Galois groups are associated? Could the non-commutativity reflect directly the non-commutativity of Galois groups involved? Quite generally one can ask whether the invariants should be expressible using algebraic numbers in the extensions of rationals associated with the intermediate braids.

3. Rational 2-tangles can be characterized by a rational number obtained by a projective identification $[a, b]^T \rightarrow a/b$ from a rational 2-spinor $[a, b]^T$ to which SL(2(N-1),Z) acts. Equivalence means that the columns $[a, b]^T$ and $[c, d]^T$ combine to form element of SL(2,Z) and thus defining a modular transformation. Could more general 2-tangles have a similar representation but in terms of algebraic integers?

4. Could N-tangles be characterized by $N-1$ $2(N-1)$-component projective column-spinors $[a_i^1, a_i^2, .., a_i^{2(N-1)}]^T$, $i = 1, ...N-1$ so that only the ratios $a_i^k/a_i^{2(N-1)} \leq 1$ matter? Could equivalence for them mean that the $N-1$ spinors combine to form $N-1+N-1$ columns of $SL(2(N-1), Z)$ matrix. Could N-tangles quite generally correspond to collections of projective $N-1$ spinors having as components algebraic integers and could $ad-bc = \pm 1$ criterion generalize? Note that the modular group for surfaces of genus g is SL(2g,Z) so that $N-1$ would be analogous to g and $1 \leq N \geq 3$- braids would correspond to $g \leq 2$ Riemann surfaces.

5. Dark matter hierarchy leads naturally to a hierarchy of modular sub-groups of $SL(2, Q)$ labeled by N (the generator $\tau \rightarrow \tau + 2$ of modular group is replaced with $\tau \rightarrow \tau + 2/N$). What might be the role of these subgroups and corresponding subgroups of $SL(2(N-1), Q)$. Could they arise in "anyonization" when one considers quantum group representations of 2-tangles with twist operation represented by an N:th root of unity instead of phase U satisfying $U^2 = 1$?

How tangles could be realized in TGD Universe?

The article of Kauffman and Lambropoulou stimulated the question in what senses N-tangles could be be realized in TGD Universe as fundamental structures.

1. Tangles as number theoretic braids?

The strands of number theoretical $N-$braids correspond to roots of N:th order polynomial and if one allows time evolutions of partonic 2-surface leading to the disappearance or appearance of real roots $N-$tangles become possible. This however means continuous evolution of roots so that the coefficients of polynomials defining the partonic 2-surface can be rational only in initial and final state but not in all intermediate "virtual" states.

2. Tangles as tangled partonic 2-surfaces?

Tangles could appear in TGD also in second manner.

1. Partonic 2-surfaces are sub-manifolds of a 3-D section of space-time surface. If partonic 2-surfaces have genus $g > 0$ the handles can become knotted and linked and one obtains besides ordinary knots and links more general knots and links in which circle is replaced by figure eight and its generalizations obtained by adding more circles (eyeglasses for $N-$eyed creatures).

2. Since these 2-surfaces are space-like, the resulting structures are indeed tangles rather than only braids. Tangles made of strands with fixed ends would result by allowing spherical partons elongate to long strands with fixed ends. DNA tangles would the basic example, and are discussed also in the article. DNA sequences to which I have speculatively assigned invisible (dark) braid structures might be seen in this context as space-like "written language representations" of genetic programs represented as number theoretic braids.

2.7.5 Only the quantum variants of M^4 and M^8 emerge from local hyper-finite II_1 factors

Super-symmetry suggests that the representations of CH Clifford algebra $\mathcal{M}$ as $\mathcal{N}$ module $\mathcal{M}/\mathcal{N}$ should have bosonic counterpart in the sense that the coordinate for M^8 representable as a particular $M^2(Q)$ element should have quantum counterpart. Same would apply to M^4 coordinate representable as $M^2(C)$ element. Quantum matrix representation of $\mathcal{M}/\mathcal{N}$ as $SL_q(2,F)$ matrix, $F = C, H$ is the natural candidate for this representation. As a matter fact, this guess is not quite correct. It is the interpretation of $M_2(C)$ as a quaternionic quantum algebra whose generalization to the octonionic quantum algebra works.

Quantum variants of M^D exist for all dimensions but only spaces M^4 and M^8 and their linear sub-spaces emerge from hyper-finite factors of type II_1. This is due to the non-associativity of the octonionic representation of the gamma matrices making it impossible to absorb the powers of the octonionic coordinate to the Clifford algebra element so that the local algebra character would disappear. Even more: quantum coordinates for these spaces are commutative operators so that their spectra define ordinary M^4 and M^8 which are thus already quantal concepts.

The commutation relations for $M_{2,q}(C)$ matrices

$$\left(\begin{array}{cc} a & b \\ c & d \end{array} \right) , \tag{2.7.6}$$

read as

$$\begin{array}{ll} ab = qba \ , & ac = qac \ , \quad bd = qdb \ , cd = qdc \ , \\ [ad, da] = (q - q^{-1})bc \ , & bc = cb \ . \end{array} \tag{2.7.7}$$

These relations can be extended by postulating complex conjugates of these relations for complex conjugates $a^\dagger, b^\dagger, c^\dagger, d^\dagger$ plus the following non-vanishing commutators of type $[x, y^\dagger]$:

$$[a, a^\dagger] = [b, b^\dagger] = [c, c^\dagger] = [d, d^\dagger] = 1 \ . \tag{2.7.8}$$

The matrices representing M^4 point must be expressible as sums of Pauli spin matrices. This can be represented as following conditions on physical states

$$\begin{array}{rcl} O|phys\rangle & = & 0 \ , \\ O & \in & \{a - a^\dagger, d - d^\dagger, b - c^\dagger, c - b^\dagger\} \ . \end{array} \tag{2.7.9}$$

For instance, the first two conditions follow from the reality of Pauli sigma matrices $\sigma_x, \sigma_y, \sigma_z$. These conditions are compatible only if the operators O commute. This is the case and means also that the operators representing M^4 coordinates commute and it is possible to define quantum states for which M^4 coordinates have well-defined eigenvalues so that ordinary M^4 emerges purely quantally from quaternions whose real coefficients are made non-Hermitian operators to obtain operator complexificiation of quaternions. Also the quantum states in which M^4 coordinates are emerge naturally.

$M_{2,q}(C)$ matrices define the quantum analog of C^4 and one can wonder whether other linear subspaces can be defined consistently or whether M^4_q and thus Minkowski signature is unique. This seems to be the case. For instance, the replacement $a - \overline{a} \rightarrow a + \overline{a}$ making also time variable Euclidian is impossible since $[a + \overline{a}, d - \overline{d}] = 2(q - q^{-1})bc$ does not vanish. The observation that M^4 coordinates can be regarded as eigenvalues of commuting observables proves that quantum CD and its orbifold description are equivalent.

What about M^8: does it have analogous description? The representation of M^4 point as $M_2(C)$ matrix can be interpreted a combination of 4-D gamma matrices defining hyper-quaternionic units. Hyper-octonionic units indeed have anticommutation relations of gamma matrices of M^8 and would give classical representation of M^8. The counterpart of $M_{2,q}(C)$ would thus be obtained by replacing the coefficients of hyper-octonionic units with operators satisfying the generalization of $M_{2,q}(C)$ commutation relations. One should identify the reality conditions and find whether they are mutually consistent.

Introduce the coefficients of E^4 gamma matrices having interpretation as quaterionic units as

$$\begin{array}{ll} a_0 = ix(a+d) \ , & a_3 = x(a-d) \ , \\ a_1 = x(b+c) \ , & a_2 = x(ib-c) \ , \\ x = \frac{1}{\sqrt{2}} \ , & \end{array}$$

and write the commutations relations for them to see how the generalization should be performed.

The selections of commutative and quaternionic sub-algebras of octonion space are fundamental for TGD and quantum octonionic algebra should reflect these selections in its structure. In the case of quaternions the selection of commutative sub-algebra implies the breaking of 4-D Lorentz symmetry. In the case of octonions the selection of quaternion sub-algebra should induce the breaking of 8-D Lorentz symmetry. Quaternionic sub-algebra obeys the commutations of $M_q(2,C)$ whereas the coefficients in in the complement commute mutually and quantum commute with the complex sub-algebra. This nails down the commutation relations completely:

$$\begin{aligned} [a_0, a_3] &= -i(q - q^{-1})(a_1^2 + a_2^2) \ , \\ [a_i, a_j] &= 0 \ , \ i,j \neq 0,3 \ , \\ a_0 a_i &= q a_i a_0 \ , \ i \neq 0,3 \ , \\ a_3 a_i &= q a_i a_3 \ , \ i \neq 0,3 \ . \end{aligned} \tag{2.7.10}$$

Checking that M^8 indeed corresponds to commutative subspace defined by the eigenvalues of operators is straightforward.

The argument generalizes easily to other dimensions $D \geq 4$ but now quaternionic and octonionic units must be replaced by gamma matrices and an explicit matrix representation can be introduced. These gamma matrices can be included as a tensor factor to the infinite-dimensional Clifford algebra so that the local Clifford algebra reduces to a mere Clifford algebra. The units of quantum octonions which are just ordinary octonion units do not however allow matrix representation so that this reduction is not possible and imbedding space and space-time indeed emerge genuinely. The non-associativity of octonions would determine the laws of physics in TGD Universe!

Thus the special role of classical number fields and uniqueness of space-time and imbedding space dimensions becomes really manifest only when a quantal deformation of the quaternionic and octonionic matrix algebras is performed. It is possible to construct the quantal variants of the coset spaces $M^4 \times E^4 / G_a \times G_b$ by simply posing restrictions on the of eigen states of the commuting coordinate operators. Also the quantum variants of the space-time surface and quite generally, manifolds obtained from linear spaces by geometric constructions become possible.

2.8 Appendix

2.8.1 About inclusions of hyper-finite factors of type II_1

Many names have been assigned to inclusions: Jones, Wenzl, Ocneacnu, Pimsner-Popa, Wasserman [?]. It would seem to me that the notion Jones inclusion includes them all so that various names would correspond to different concrete realizations of the inclusions conjugate under outer automorphisms.

1. According to [?] for inclusions with $\mathcal{M} : \mathcal{N} \leq 4$ (with $A_1^{(1)}$ excluded) there exists a countable infinity of sub-factors with are pairwise non inner conjugate but conjugate to $\mathcal{N}$.

2. Also for any finite group G and its outer action there exists uncountably many sub-factors which are pairwise non inner conjugate but conjugate to the fixed point algebra of G [?]. For any amenable group G the the inclusion is also unique apart from outer automorphism [?].

Thus it seems that not only Jones inclusions but also more general inclusions are unique apart from outer automorphism.

Any *-endomorphism σ, which is unit preserving, faithful, and weakly continuous, defines a sub-factor of type II_1 factor [?]. The construction of Jones leads to a atandard inclusion sequence $\mathcal{N} \subset \mathcal{M} \subset \mathcal{M}^1 \subset ....$ This sequence means addition of projectors e_i, $i < 0$, having visualization as an addition of braid strand in braid picture. This hierarchy exists for all factors of type II. At the limit $\mathcal{M}^\infty = \cup_i \mathcal{M}^i$ the braid sequence extends from $-\infty$ to ∞. Inclusion hierarchy can be understood as a hierarchy of Connes tensor powers $\mathcal{M} \otimes_\mathcal{N} \mathcal{M} .... \otimes_\mathcal{N} \mathcal{M}$. Also the ordinary tensor powers of hyper-finite factors of type II_1 (HFF) as well as their tensor products with finite-dimensional matrix algebras are isomorphic to the original HFF so that these objects share the magic of fractals.

Under certain assumptions the hierarchy can be continued also in opposite direction. For a finite index an infinite inclusion hierarchy of factors results with the same value of index. σ is said to be basic if it can be extended to *-endomorphisms from $\mathcal{M}^1$ to $\mathcal{M}$. This means that the hierarchy of inclusions can be continued in the opposite direction: this means elimination of strands in the braid picture. For finite factors (as opposed to hyper-finite ones) there are no basic *-endomorphisms of $\mathcal{M}$ having fixed point algebra of non-abelian G as a sub-factor [?].

1. Jones inclusions

For hyper-finite factors of type II_1 Jones inclusions allow basic *-endomorphism. They exist for all values of $\mathcal{M} : \mathcal{N} = r$ with $r \in \{4cos^2(\pi/n)|n \geq 3\} \cap [4, \infty)$ [?]. They are defined for an algebra defined by projectors e_i, $i \geq 1$. All but nearest neighbor projectors commute. $\lambda = 1/r$ appears in the relations for the generators of the algebra given by $e_i e_j e_i = \lambda e_i$, $|i - j| = 1$. $\mathcal{N} \subset \mathcal{M}$ is identified as the double commutator of algebra generated by e_i, $i \geq 2$.

This means that principal graph and its dual are equivalent and the braid defined by projectors can be continued not only to $-\infty$ but that also the dropping of arbitrary number of strands is possible [?]. It would seem that ADE property of the principal graph meaning single root length codes for the duality in the case of $r \leq 4$ inclusions.

Irreducibility holds true for $r < 4$ in the sense that the intersection of $Q' \cap P = P' \cap P = C$. For $r \geq 4$ one has $dim(Q' \cap P) = 2$. The operators commuting with Q contain besides identify operator of Q also the identify operator of P. Q would contain a single finite-dimensional matrix factor less than P in this case. Basic *-endomorphisms with $\sigma(P) = Q$ is $\sigma(e_i) = e_{i+1}$. The difference between genuine symmetries of quantum TGD and symmetries which can be mimicked by TGD could relate to the irreducibility for $r < 4$ and raise these inclusions in a unique position. This difference could partially justify the hypothesis that only the groups $G_a \times G_b \subset SU(2) \times SU(2) \subset SL(2, C) \times SU(3)$ define orbifold coverings of $H_\pm = CD \times CP_2 \rightarrow H_\pm/G_a \times G_b$.

2. Wasserman's inclusion

Wasserman's construction of $r = 4$ factors clarifies the role of the subgroup of $G \subset SU(2)$ for these inclusions. Also now $r = 4$ inclusion is characterized by a discrete subgroup $G \subset SU(2)$ and is given by $(1 \otimes \mathcal{M})^G \subset (M_2(C) \times \mathcal{M})^G$. According to [?] Jones inclusions are irreducible also for $r = 4$. The definition of Wasserman inclusion for $r = 4$ seems however to imply that the identity matrices of both $\mathcal{M}^G$ and $(M(2, C) \otimes \mathcal{M})^G$ commute with $\mathcal{M}^G$ so that the inclusion should be reducible for $r = 4$.

Note that G leaves both the elements of $\mathcal{N}$ and $\mathcal{M}$ invariant whereas $SU(2)$ leaves the elements of $\mathcal{N}$ invariant. $M(2,C)$ is effectively replaced with the orbifold $M(2,C)/G$, with G acting as automoprhisms. The space of these orbits has complex dimension $d = 4$ for finite G.

For $r < 4$ inclusion is defined as $M^G \subset M$. The representation of G as outer automorphism must change step by step in the inclusion sequence $... \subset \mathcal{N} \subset \mathcal{M} \subset ...$ since otherwise G would act trivially as one proceeds in the inclusion sequence. This is true since each step brings in additional finite-dimensional tensor factor in which G acts as automorphisms so that although $\mathcal{M}$ can be invariant under $G_{\mathcal{M}}$ it is not invariant under $G_{\mathcal{N}}$.

These two inclusions might accompany each other in TGD based physics. One could consider $r < 4$ inclusion $\mathcal{N} = \mathcal{M}^G \subset \mathcal{M}$ with G acting non-trivially in $\mathcal{M}/\mathcal{N}$ quantum Clifford algebra. $\mathcal{N}$ would decompose by $r = 4$ inclusion to $\mathcal{N}_1 \subset \mathcal{N}$ with $SU(2)$ taking the role of G. $\mathcal{N}/\mathcal{N}_1$ quantum Clifford algebra would transform non-trivially under $SU(2)$ but would be G singlet.

In TGD framework the G-invariance for $SU(2)$ representations means a reduction of S^2 to the orbifold S^2/G. The coverings $H_\pm \rightarrow H_\pm/G_a \times G_b$ should relate to these double inclusions and $SU(2)$ inclusion could mean Kac-Moody type gauge symmetry for $\mathcal{N}$. Note that the presence of the factor containing only unit matrix should relate directly to the generator d in the generator set of affine algebra in the McKay construction. The physical interpretation of the fact that almost all ADE type extended diagrams ($D_n^{(1)}$ must have $n \geq 4$) are allowed for $r = 4$ inclusions whereas D_{2n+1} and E_6 are not allowed for $r < 4$, remains open.

2.8.2 Generalization from $SU(2)$ to arbitrary compact group

The inclusions with index $\mathcal{M} : \mathcal{N} < 4$ have one-dimensional relative commutant $\mathcal{N}' \cup \mathcal{M}$. The most obvious conjecture that $\mathcal{M} : \mathcal{N} \geq 4$ corresponds to a non-trivial relative commutant is wrong. The index for Jones inclusion is identifiable as the square of quantum dimension of the fundamental representation of $SU(2)$. This identification generalizes to an arbitrary representation of arbitrary compact Lie group.

In his thesis Wenzl [?] studied the representations of Hecke algebras $H_n(q)$ of type A_n obtained from the defining relations of symmetric group by the replacement $e_i^2 = (q-1)e_i + q$. H_n is isomorphic to complex group algebra of S_n if q is not a root of unity and for $q = 1$ the irreducible representations of $H_n(q)$ reduce trivially to Young's representations of symmetric groups. For primitive roots of unity $q = exp(i2\pi/l)$, $l = 4, 5...$, the representations of $H_n(\infty)$ give rise to inclusions for which index corresponds to a quantum dimension of any irreducible representation of $SU(k)$, $k \geq 2$. For $SU(2)$ also the value $l = 3$ is allowed for spin 1/2 representation.

The inclusions are obtained by dropping the first m generators e_k from $H_\infty(q)$ and taking double commutant of both H_∞ and the resulting algebra. The relative commutant corresponds to $H_m(q)$. By reducing by the minimal projection to relative commutant one obtains an inclusion with a trivial relative commutant. These inclusions are analogous to a discrete states superposed in continuum. Thus the results of Jones generalize from the fundamental representation of $SU(2)$ to all representations of all groups $SU(k)$, and in fact to those of general compact groups as it turns out.

The generalization of the formula for index to square of quantum dimension of an irreducible representation of $SU(k)$ reads as

$$\mathcal{M} : \mathcal{N} = \prod_{1 \leq r < s \leq k} \frac{sin^2\left((\lambda_r - \lambda_s + s - r)\pi/l\right)}{sin^2\left((s-r)n/l\right)} \quad . \tag{2.8.1}$$

Here λ_r is the number of boxes in the r^{th} row of the Yang diagram with n boxes characterizing the representations and the condition $1 \leq k \leq l - 1$ holds true. Only Young diagrams satisfying the condition $l - k = \lambda_1 - \lambda_{r_{max}}$ are allowed.

The result would allow to restrict the generalization of the imbedding space in such a manner that only cyclic group Z_n appears in the covering of $M^4 \rightarrow M^4/G_a$ or $CP_2 \rightarrow CP_2/G_b$ factor. Be as it may, it seems that quantum representations of any compact Lie group can be realized using the generalization of the imbedding space. In the case of $SU(2)$ the interpretation of higher-dimensional quantum representations in terms of Connes tensor products of 2-dimensional fundamental representations is highly suggestive.

The groups $SO(3,1) \times SU(3)$ and $SL(2,C) \times U(2)_{ew}$ have a distinguished position both in physics and quantum TGD and the vision about physics as a generalized number theory implies them. Also the general pattern for inclusions selects these groups, and one can say that the condition that all possible statistics are realized is guaranteed by the choice $M^4 \times CP_2$.

1. $n > 2$ for the quantum counterparts of the fundamental representation of $SU(2)$ means that braid statistics for Jones inclusions cannot give the usual fermionic statistics. That Fermi statistics cannot "emerge" conforms with the role of infinite-D Clifford algebra as a canonical representation of HFF of type II_1. $SO(3,1)$ as isometries of H gives Z_2 statistics via the action on spinors of M^4 and $U(2)$ holonomies for CP_2 realize Z_2 statistics in CP_2 degrees of freedom.

2. $n > 3$ for more general inclusions in turn excludes Z_3 statistics as braid statistics in the general case. $SU(3)$ as isometries induces a non-trivial Z_3 action on quark spinors but trivial action at the imbedding space level so that Z_3 statistics would be in question.

Bibliography

Online books about TGD

[1] M. Pitkänen (2006), *Topological Geometrodynamics: Overview*. http://tgd.wippiespace.com/public_html/tgdview/tgdview.html.

[2] M. Pitkänen (2006), *Quantum Physics as Infinite-Dimensional Geometry*. http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html.

[3] M. Pitkänen (2006), *Physics in Many-Sheeted Space-Time*. http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html.

[4] M. Pitkänen (2006), *Quantum TGD*. http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html.

[5] M. Pitkänen (2006), *TGD as a Generalized Number Theory*. http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html.

[6] M. Pitkänen (2006), *p-Adic length Scale Hypothesis and Dark Matter Hierarchy*. http://tgd.wippiespace.com/public_html/paddark/paddark.html.

[7] M. Pitkänen (2006), *TGD and Fringe Physics*. http://tgd.wippiespace.com/public_html/freenergy/freenergy.html.

Online books about TGD inspired theory of consciousness and quantum biology

[8] M. Pitkänen (2006), *Bio-Systems as Self-Organizing Quantum Systems*. http://tgd.wippiespace.com/public_html/bioselforg/bioselforg.html.

[9] M. Pitkänen (2006), *Quantum Hardware of Living Matter*. http://tgd.wippiespace.com/public_html/bioware/bioware.html.

[10] M. Pitkänen (2006), *TGD Inspired Theory of Consciousness*. http://tgd.wippiespace.com/public_html/tgdconsc/tgdconsc.html.

[11] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*. http://tgd.wippiespace.com/public_html/genememe/genememe.html.

[12] M. Pitkänen (2006), *TGD and EEG*. http://tgd.wippiespace.com/public_html/tgdeeg/tgdeeg/tgdeeg.html.

[13] M. Pitkänen (2006), *Bio-Systems as Conscious Holograms*. http://tgd.wippiespace.com/public_html/hologram/hologram.html.

[14] M. Pitkänen (2006), *Magnetospheric Consciousness*. http://tgd.wippiespace.com/public_html/magnconsc/magnconsc.html.

[15] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*. http://tgd.wippiespace.com/public_html/magnconsc/mathconsc.html.

References to the chapters of books

[A1] The chapter *An Overview about the Evolution of Quantum TGD* of [?].
http://tgd.wippiespace.com/public_html/tgdview/tgdview.html#tgdevoI.

[A2] The chapter *An Overview about Quantum TGD* of [?].
http://tgd.wippiespace.com/public_html/tgdview/tgdview.html#{tgdevoII}.

[A3] The chapter *TGD and M-Theory* of [?].
http://tgd.wippiespace.com/public_html/tgdview/tgdview.html#MTGD.

[B2] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part I* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#compl1.

[B3] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part II* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#compl2.

[B4] The chapter *Configuration Space Spinor Structure* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#cspin.

[C2] The chapter *Construction of Quantum Theory: Symmetries* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#quthe.

[C3] The chapter *Construction of Quantum Theory: S-matrix* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#towards.

[C4] The chapter *Category Theory and Quantum TGD* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#categorynew.

[C5] The chapter *Construction of S-matrix* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#smatrix.

[C6] The chapter *Is it Possible to Understand Coupling Constant Evolution at Space-Time Level?* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#rgflow.

[C8] The chapter *Appendix A: Quantum Groups and Related Structures* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#bialgebra.

[C10] The chapter *Was von Neumann Right After All* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#vNeumann.

[D1] The chapter *Basic Extremals of Kähler Action* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#class.

[D3] The chapter *The Relationship Between TGD and GRT* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#tgdgrt.

[D4] The chapter *Cosmic Strings* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#cstrings.

[D5] The chapter *TGD and Cosmology* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#cosmo.

[D6] The chapter *TGD and Astrophysics* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#astro.

[E1] The chapter *TGD as a Generalized Number Theory: p-Adicization Program* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visiona.

[E2] The chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visionb.

[E3] The chapter *TGD as a Generalized Number Theory: Infinite Primes* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visionc.

[E8] The chapter *Riemann Hypothesis and Physics* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#riema.

[E9] The chapter *Topological Quantum Computation in TGD Universe* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#tqc.

[F1] The chapter *Elementary Particle Vacuum Functionals* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#elvafu.

[F2] The chapter *Massless States and Particle Massivation* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#mless.

[F3] The chapter *p-Adic Particle Massivation: Hadron Masses* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#padmass2.

[F4] The chapter *p-Adic Particle Massivation: Hadron Masses* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#padmass3.

[F5] The chapter *p-Adic Particle Massivation: New Physics* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#padmass4.

[F7] The chapter *The Recent Status of Leptohadron Hypothesis* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#leptc.

[F8] The chapter *TGD and Nuclear Physics* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#padnucl.

[F9] The chapter *Nuclear String Model* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#nuclstring.

[F10] The chapter *Dark Nuclear Physics and Living Matter* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#exonuclear.

[F12] The chapter *Quantum Hall effect and Hierarchy of Planck Constants* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#anyontgd.

[G1] The chapter *Anomalies Related to the Classical Z^0 Force and Gravitation* of [?].
http://tgd.wippiespace.com/public_html/freenergy/freenergy.html#Zanom.

[G2] The chapter *The Notion of Free Energy and Many-Sheeted Space-Time Concept* of [?].
http://tgd.wippiespace.com/public_html/freenergy/freenergy.html#freenergy.

[H1] The chapter *Matter, Mind, Quantum* of [?].
http://tgd.wippiespace.com/public_html/tgdconsc/tgdconsc.html#conscic.

[H8] The chapter *p-Adic Physics as Physics of Cognition and Intention* of [?].
http://tgd.wippiespace.com/public_html/tgdconsc/tgdconsc.html#cognic.

[J1] The chapter *Bio-Systems as Super-Conductors: part I* of [?].
http://tgd.wippiespace.com/public_html/bioware/bioware.html#superc1.

[J2] The chapter *Bio-Systems as Super-Conductors: part II* of [?].
http://tgd.wippiespace.com/public_html/bioware/bioware.html#superc2.

[J3] The chapter *Bio-Systems as Super-Conductors: part III* of [?].
http://tgd.wippiespace.com/public_html/bioware/bioware.html#superc3.

[J6] The chapter *Coherent Dark Matter and Bio-Systems as Macroscopic Quantum Systems* of [?]. http://tgd.wippiespace.com/public_html/bioware/bioware.html#darkbio.

[J7] The chapter *About the New Physics Behind Qualia* of [?]. http://tgd.wippiespace.com/public_html/bioware/bioware.html#newphys.

[K1] The chapter *Time, Spacetime and Consciousness* of [?]. http://tgd.wippiespace.com/public_html/hologram/hologram.html#time.

[L6] The chapter *The Notion of Wave-Genome and DNA as Topological Quantum Computer* of [?]. http://tgd.wippiespace.com/public_html/genememe/genememe.html#gari.

[L7] The chapter *Evolution in Many-Sheeted Space-Time* of [?]. http://tgd.wippiespace.com/public_html/genememe/genememe.html#prebio.

[M3] The chapter *Dark Matter Hierarchy and Hierarchy of EEGs* of [?]. http://tgd.wippiespace.com/public_html/tgdeeg//tgdeeg.html#eegdark.

[Appendix] The chapter *Appendix* of [?]. http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#append.

Articles related to TGD

[MPa] M. Pitkänen (2008). *Topological Geometrodynamics: What Might Be The Basic Principles.* http://tgd.wippiespace.com/public_html/articles/tgd2008.pdf.

[MPb] M. Pitkänen (2008),*About the Nature of Time.* http://tgd.wippiespace.com/public_html/articles/time.pdf.

[MPc] M. Pitkänen (2008), *TGD Inspired Theory of Consciousness.* http://tgd.wippiespace.com/public_html/articles/tgdconsc.pdf.

[MPd] M. Pitkänen (2008), *TGD Inspired Quantum Model of Living Matter.* http://tgd.wippiespace.com/public_html/articles/quantumbio.pdf.

[MPe] M. Pitkänen (2007), *About Correspondence Between Infinite Primes, Space-time Surfaces, and Configuration Space Spinor Fields.* http://tgd.wippiespace.com/public_html/articles/brahma.pdf.

[MPf] M. Pitkänen (2008), *DNA as Topological Quantum Computer.* http://tgd.wippiespace.com/public_html/articles/dnatqcart.pdf.

[MPg] M. Pitkänen (2008),*Quantum Model for Nerve Pulse and EEG.* http://tgd.wippiespace.com/public_html/articles/pulseeg.pdf.

[MPh] M. Pitkänen (2008), *Evolution in Many-Sheeted Space-Time.* http://tgd.wippiespace.com/public_html/articles/prebiotic.pdf.

[MPi] M. Pitkänen (2008),*A Model for Protein Folding and Bio-catalysis.* http://tgd.wippiespace.com/public_html/articles/prebiotic.pdf.

[MPj] M. Pitkänen (2008),*The Notion of Wave-Genome and DNA as Topological Quantum Computer.* http://tgd.wippiespace.com/public_html/articles/gari.pdf.

Mathematics related references

[16] M. Freedman, H. Larsen, and Z. Wang (2002), *A modular functor which is universal for quantum computation*, Found. Comput. Math. 1, no 2, 183-204. Comm. Math. Phys. 227, no 3, 605-622. quant-ph/0001108.
M. H. Freedman (2001), *Quantum Computation and the localization of Modular Functors*, Found. Comput. Math. 1, no 2, 183-204.
M. H. Freedman (1998), *P/NP, and the quantum field computer*, Proc. Natl. Acad. Sci. USA 95, no. 1, 98-101.
A. Kitaev (1997), Annals of Physics, vol 303, p.2. See also *Fault tolerant quantum computation by anyons*, quant-ph/9707021.
L. H. Kauffmann and S, J. Lomonaco Jr. (2004), *Braiding operations are universal quantum gates*, arxiv.org/quant-ph/0401090.
Paul Parsons (2004) , *Dancing the Quantum Dream*, New Scientist 24. January. www.newscientist.com/hottopics.

[17] F. J. Murray, J. von Neumann (1936), *On Rings of Operators*, Ann. Math. 37,116-229.
F. J. Murray, J. von Neumann (1937), *On Rings of Operators II*, Trans. Am. Math. Soc. 41, 208-248.
F. J. Murray, J. von Neumann (1940), *On Rings of Operators III*, Ann. Math. 41,94-161.
F. J. Murray, J. von Neumann (1943), *On Rings of Operators IV*, Ann. Math. 44,716-808.

[18] J. von Neumann (1937), *Quantum Mechanics of Infinite Systems*, first published in (Redei and Stöltzner 2001, 249-268). [A mimeographed version of a lecture given at Pauli's seminar held at the Institute for Advanced Study in 1937, John von Neumann Archive, Library of Congress, Washington, D.C.]

[19] M. Redei and M. Stöltzner (eds) (2001), *John von Neumann and the Foundations of Quantum Physics*, Vol. 8, Dordrecht: Kluwer Academic Publishers. V. S. Sunder (1987), *An Invitation to von Neumann Algebras*, New York: Springer-Verlag.

[20] J. Dixmier (1981), *Von Neumann Algebras*, Amsterdam: North-Holland Publishing Company. [First published in French in 1957: Les Algebres d'Operateurs dans l'Espace Hilbertien, Paris: Gauthier-Villars.]

[21] R. Longo (2004), *Operators algebras and Index Theorems in Quantum Field Theory*, Andrejevski lectures in Göttingen 2004 (lecture notes).

[22] V. F. R. Jones (1983), *Braid groups, Hecke algebras and type II_1 factors*, Geometric methods in operator algebras, Proc. of the US-Japan Seminar, Kyoto, July 1983.

[23] P. A. M. Dirac (1939), *A New Notation for Quantum Mechanics*, Proceedings of the Cambridge Philosophical Society, 35: 416-418.
J. E. Roberts (1966), *The Dirac Bra and Ket Formalism*, Journal of Mathematical Physics, 7: 1097-1104. Halvorson, Hans and Clifton, Rob (2001),

[24] L. Schwartz (1945),*Generalisation de la Notion de Fonction, de Derivation, de Transformation de Fourier et Applications Mathematiques et Physiques*, Annales de l'Universite de Grenoble, 21: 57-74. 1951-1952,
L. Scwartz (1951-1952), *Theorie des Distributions*, Publications de l'Institut de Mathematique de l'Universite de Strasbourg, Vols 9-10, Paris: Hermann.

[25] I. Gelfand and M. Neumark (1943), *On the Imbedding of Normed Rings into the Ring of Operators in Hilbert Space*, Recueil Mathematique [Matematicheskii Sbornik] Nouvelle Serie, 12 [54]: 197-213. [Reprinted in C*-algebras: 1943-1993, in the series Contemporary Mathematics, 167, Providence, R.I. : American Mathematical Society, 1994.]
I. Gelfand and N. Ya. Vilenkin (1964), *Generalized Functions*, Volume 4, New York: Academic Press. [First published in Russian in 1961.]

[26] R. P. Feynman (1948), *Space-Time Approach to Non-Relativistic Quantum Mechanics*, Reviews of Modern Physics, 20: 367387. [It is reprinted in (Schwinger 1958).]

[27] R. J. Rivers (1987), *Path Integral Methods in Quantum Field Theory*, Cambridge: Cambridge University Press.

[28] C. N. Yang, M. L. Ge (1989), *Braid Group, Knot Theory, and Statistical Mechanics*, World Scientific.
V. F. R. Jones, *Hecke algebra representations of braid groups and link polynomial*, Ann. Math., 126(1987), 335-388.

[29] C. Kassel (1995), *Quantum Groups*, Springer Verlag.

[30] C. Gomez, M. Ruiz-Altaba, G. Sierra (1996), *Quantum Groups and Two-Dimensional Physics*, Cambridge University Press.

[31] E. Witten 1989), *Quantum field theory and the Jones polynomial*, Comm. Math. Phys. 121 , 351-399.

[32] S. Sawin (1995), *Links, Quantum Groups, and TQFT's*, q-alg/9506002.

[33] H. Saleur (1990), *Zeroes of chromatic polynomials: a new approach to the Beraha conjecture using quantum groups*, Comm. Math. Phys. 132, 657.

[34] M. Takesaki (1970), *Tomita's Theory of Modular Hilbert Algebras and Its Applications*,Lecture Notes in Mathematics 128, Springer, Berlin.

[35] R. Haag and D. Kastler (1964), *An Algebraic Approach to Quantum Field Theory*, Journal of Mathematical Physics, 5: 848-861.

[36] B. Schroer (2001), *Lectures on Algebraic Quantum Field Theory and Operator Algebras*, `http://xxx.lanl.gov/abs/math-ph/0102018`.

[37] H. J. Borchers (2000), *On Revolutionizing QFT with Tomita's Modular Theory* `http://www.lqp.uni-goettingen.de/papers/99/04/99042900.html`. J. Math. Phys. 41 (2000) 3604-3673.

[38] A. Connes (1973), *Une classification des facteurs de type III*, Ann. Sci. Ecole Norm. Sup. 6, 133-252.

[39] A. Connes (1994), *Non-commutative Geometry*, San Diego: Academic Press.

[40] Y. Ito and I. Nakamura(1996) *Hilbert schemes and simple singularities*, Proc. of EuroConference on Algebraic Geometry, Warwick 1996, ed. by K. Hulek et al., CUP, (1999),pp. 151–233, `http://www.math.sci.hokudai.ac.jp/~nakamura/ADEHilb.pdf`.

[41] V. F. R. Jones (1983), *Index for Subfactors*, Invent. Math. (72),1-25.

[42] F. M. Goodman, P. de la Harpe and V. F. R. Jones (1989), *Coxeter graphs and towers of algebras*, Springer-Verlag.

[43] V. F. R. Jones(2000), *The planar algebra of a bipartite graph*, in "Knots in Hellas′98", World Sci. Publishing (2000), 94-117.

[44] N. H. V. Temperley and E. H. Lieb (1971), *Relations between the percolation and colouring problem and other graph-theoretical problems associated with regular planar lattices:some exact results for the percolation problem*, Proc. Roy. Soc. London 322 (1971), 251-280.

[45] O. Bratteli and D. W. Robinson (1979-1981), *Operator Algebras and Quantum Statistical Mechanics*, Volumes 1-2, New York: Springer-Verlag.

[46] V. Jones (2003), *In and around the origin of quantum groups*, arXiv:math.OA/0309199.

[47] F. Wilzek (1990), *Fractional Statistics and Anyon Super-Conductivity*, World Scientific.
R. B. Laughlin (1990), Phys. Rev. Lett. 50, 1395.

[48] R. B. Laughlin (1983), Phys. Rev. Lett. 50, 1395.

[49] *Fractional quantum Hall Effect*, `http://en.wikipedia.org/wiki/Fractional_quantum_Hall_effect`.
Fractional Quantum Hall Effect, `http://www.warwick.ac.uk/~phsbm/fqhe.htm`.

[50] J.K. Jain(1989), Phys. Rev. Lett. 63, 199.

[51] J. B. Miller *et al*(2007), *Fractional Quantum Hall effect in a quantum point contact at filling fraction 5/2*, arXiv:cond-mat/0703161v2.

[52] A. Connes and D. Kreimer (1998), *Hofp algebras, renormalization, and non-commutative geometry*, Quantum Field Theory; Perspective and Prospective (C. DeWitt-Morette and J.-B.- Zueber, eds.), Kluwer, Dortrecth, 1999, 59-109. CMP 2000:05, Commun. Math. Phys. 199, 203.242(1998). MR 99h:81137.
Ibid (1999), *Renormalization in quantum field theory and the Riemann-Hilbert problem I: the Hopf algebra structure of graphs and the main theorem*, arXiv:hep-th/9912092.
Ibid (2000), *Renormalization in quantum field theory and the Riemann-Hilbert problem II: the β function, diffeomorphisms and the renormalization group*, arXiv:hep-th/0003188.

[53] A. Connes (2005), *Quantum Fields and Motives*, hep-th/0504085.

[54] V. Jones (2003), *In and around the origin of quantum groups*, arXiv:math.OA/0309199.

[55] *This Week's Finds in Mathematical Physics: Week 230*,
`http://math.ucr.edu/home/baez/week230.html`.

[56] R. Haag (1992), *Local Quantum Physics*, Springer, Berlin

[57] M. Rainer (2000), *Algebraic Quantum Field Theory on Manifolds: A Haag-Kastler Setting for Quantum Geometry*, gr-qc/9911076.

[58] S. Doplicher, R. Haag, J. E. Roberts (1971-1974), *Local Observables and Particle Statistics I and II*. Commun. Math. Phys. 23 and 35.
S. Doplicher, J. E. Roberts (1989), Ann. Math. 130, 75-119.
S. Doplicher, J. E. Roberts (1989), Invent. Math. 98, 157-218.

[59] H. P. Halvorson (2001), *Locality, Localization, and the Particle Concept: Topics in the Foundations of Quantum Field Theory*, Ph. D Dissertation.
`http://philsci-archive.pitt.edu/archive/00000346/00/main-new.pdf`.

[60] L.P. Horwitz (1996), *Hypercomplex quantum mechanics*, quant-ph/9602001.

[61] S. De Leo and K. Abdel-Khalek (1996), *Octonionic quantum mechanics and complex geometry*, Prog.Theor.Phys. 96 (1996) 823-832, hep-th/9609032.

[62] S. Okubo (1997), *Angular momentum, quaternion, octonion, and Lie-super algebra* $Osp(1,2)$, ph/9710038.

[63] M. Brenner (1999), *Quantum octonions*, Communications in Algebra 27 (1999), 2809-2831.
`http://math.usask.ca/~bremner/research/publications/qo.pdf`.

[64] G. Benkart, J. M. Perez-Izquierdo, *A quantum octonion algebra*, math.QA/9801141.

[65] John C. Baez (2001), *The Octonions*, Bull. Amer. Math. Soc. 39 (2002), 145-205.
`http://math.ucr.edu/home/baez/Octonions/octonions.html`.

[66] *The 15 most famous transcendental numbers*, `http://sprott.physics.wisc.edu/pickover/trans.html`.

[67] P. Dorey (1998). *Exact S-matrices*, arXiv.org:hep-th/9810026.

[68] G. Lechner (2005), *Towards the construction of quantum field theories from a factorizing S-matrix*, arXiv:hep-th/0502184.

[69] *Farey sequence*,
`http://en.wikipedia.org/wiki/Farey_sequence`.

[70] L. H. Kauffman and S. Lambropoulou(2006), *Hard Unknots and Collapsing Tangles*, `http://arxiv.org/abs/math/0601525`.

Theoretical physics

[71] S. M. Girvin (1999), *Quantum Hall Effect, Novel Excitations and Broken Symmetries*, cond-mat/9907002.

[72] C. Itzykson, H. Saleur, J.-B. Zuber (Editors) (1988),*Conformal Invariance and Applications to Statistical Mechanics*, Word Scientific.

[73] A. Ali (2003), *An N=8 superaffine Malcev algebra dn its N=8 Sugawara*, hep-th/01015313.

[74] A. Ali (2003), *Types of 2-dimensional $N = 4$ superconformal field theories*, Pramana, vol. 61, No. 6, pp. 1065-1078.

[75] I. R. Klebanov(2000), *TASI Lectures: Introduction to the AdS/CFT Correspondence* arXiv: hep-th/0009139.

[76] D. Evans and Y. Kawahigashi(1998), *Quantum symmetries on operator algebras*, Oxford University Press, New York. The review of the book can be found in Bulletin of the American Mathematical Society, Vol. 28, Number 3, Pages 369-377.
`http://www.ams.org/bull/2001-38-03/S0273-0979-01-00906-5/S0273-0979-01-00906-5.pdf`.

[77] M. Choda (1996), *Conjugate but non inner conjugate subfactors*, Proceedings of the American Mathematical Society, Vol. 124, Number 1, January.
`http://links.jstor.org/sici?sici=0002-9939(199601)124%3A1%3C147%3ACBNICS%3E2.0.CO%3B2-0`.

[78] H. Wenzl (1988), *Hecke algebra of type A_n and subfactors*, Invent. math. 92, 349-383.

[79] D. Monroe (2008), *Know Your Anyons.* New Scientist, vol 200, No 2676.

[80] M. Dolev, M. Heiblum, V. Umansky, Ady Stern, and D. Mahalu Nature (2008), *Observation of a quarter of an electron charge at the = 5/2 quantum Hall state.* Nature, vol 452, p 829.

[81] Paul Parsons (2004) , *Dancing the Quantum Dream*, New Scientist 24. January. www.newscientist.com/hottopics.

Experimental physics

[82] T. Ludham and L. McLerran (2003), *What Have We Learned From the Relativistic Heavy Ion Collider?*, Physics Today, October issue.
`http://www.physicstoday.org/vol-56/iss-10/p48.html`.
E. S. Reich (2005), *Black hole like phenomenon created by collider*, New Scientist 19, issue 2491.

[83] A. Helmi(2000), *Halo streams as relicts from the formation of the Milky Way*, arXiv.org/astro-ph/0008086.

[84] *Weinberg angle*, `http://en.wikipedia.org/wiki/Weinberg_angle`.

[85] X.-G. He, J. Tandean, G. Valencia (2007), *Has HyperCP Observed a Light Higgs Boson?*,Phys. Rev. D74.
`http://arxiv.org/abs/hep-ph/0610274`.
X.-G. He, J. Tandean, G. Valencia (2007), *Light Higgs Production in Hyperon Decay*, Phys. Rev. Lett. 98.
`http://arxiv.org/abs/hep-ph/0610362`.

[86] CDF Collaboration (2008), *Study of multi-muon events produced in p-pbar collisions at sqrt(s)=1.96 TeV*. http://arxiv.org/PS_cache/arxiv/pdf/0810/0810.0714v1.pdf.
P. Giromini, F. Happacher, M. J. Kim, M. Kruse, K. Pitts, F. Ptohos, S. Torre (2008), *Phenomenological interpretation of the multi-muon events reported by the CDF collaboration*. http://arxiv.org/abs/0810.5730.

[87] C. Charbonnel and F. Primas (2005), *The lithium content of the Galactic Halo stars*, astro-ph/0505247.
http://www.arxiv.org/PS_cache/astro-ph/pdf/0505/0505247v2.pdf.

[88] M. Chaplin (2005), *Water Structure and Behavior*,
http://www.lsbu.ac.uk/water/index.html.
For 41 anomalies see http://www.lsbu.ac.uk/water/anmlies.html.
For the icosahedral clustering see http://www.lsbu.ac.uk/water/clusters.html.

[89] J. K. Borchardt(2003), *The chemical formula H_2O - a misnomer*, The Alchemist 8 Aug (2003).

[90] R. A. Cowley (2004), *Neutron-scattering experiments and quantum entanglement*, Physica B 350 (2004) 243-245.

[91] R. Moreh, R. C. Block, Y. Danon, and M. Neumann (2005), *Search for anomalous scattering of keV neutrons from H2O-D2O mixtures*, Phys. Rev. Lett. 94, 185301.

[92] V. J. Emery, S. A. Kivelson,, and J. M. Tranquada (1999), *Stripe phases in high-temperature superconductors*, Perspective, Vol. 96, Issue 16, 8814-8817, August 3.
http://www.pnas.org/cgi/reprint/96/16/8814.pdf.

[93] J. Zaanen (2006), *Superconductivity: Quantum Stripe Search*, Nature vol 440, 27 April.
http://www.lorentz.leidenuniv.nl/~jan/nature03/qustripes06.pdf.

[94] *Dark energy*, http://physicsworld.com/cws/article/print/19419.

Geology and biology related references

[95] Neil Adams (2006), *Conspiracy of Science, Earth is in fact growing*,
http://www.youtube.com/watch?v=VjgidAICoQI.

[96] *A challenge to all geologists of Earth*, http://www.nealadams.com/challenge.html.

[97] *Plate tectonics*, http://en.wikipedia.org/wiki/Plate_tectonics.

[98] *Oceanic trench*, http://en.wikipedia.org/wiki/Oceanic_trench.

[99] *Expanding Earth Theory*, http://en.wikipedia.org/wiki/Expanding_earth_theory.

[100] *Orogenies*, http://en.wikipedia.org/wiki/Orogenies.

[101] *Earthquake zone*, http://en.wikipedia.org/wiki/Earthquake_zone.

[102] *Volcano*,http://en.wikipedia.org/wiki/Volcano.

[103] *Mars*,http://en.wikipedia.org/wiki/Mars.

[104] *Dinosaurs*, http://en.wikipedia.org/wiki/Dinosaur.

[105] *Circadian rhythm*, http://en.wikipedia.org/wiki/Circadian_rhythm.

[106] S. J. Gould (1991) *Wonderful Life*, Penguin Books.

[107] S. J. Braddy, M. Poschmann,O. E. Tetlie (2007), *Giant claw reveals the largest ever arthropod*, Biology Letters, November 13, 2007, http://www.journals.royalsoc.ac.uk/content/t15r2588mn27n0w1. *Scientists Find Fossil of Enormous Bug*, http://www.wtop.com/?nid=220&sid=1296318.

[108] J. O′Donoghue (2007), *How trees changed the world?*, New Scienstist, issue 2631, 24 November, http://www.newscientist.com/channel/life/mg19626311.500-how-trees-changed-the-world.html.

[109] P. P. Gariaev *et al*(2002), *The spectroscopy of bio-photons in non-local genetic regulation*, Journal of Non-Locality and Remote Mental Interactions, Vol 1, Nr 3. http://www.emergentmind.org/gariaevI3.htm .

[110] P. P. Gariaev, G. G. Tertishni, A. V. Tovmash (2007), *Experimental investigation in vitro of holographic mapping and holographic transposition of DNA in conjuction with the information pool encircling DNA*, New Medical Tehcnologies, #9, pp. 42-53. The article is in Russian but Peter Gariaev kindly provided a translation of the article to English.

[111] E. Lozneanu and M. Sanduloviciu (2003) *Minimal-cell system created in laboratory by self-organization*, Chaos, Solitons & Fractals, Volume 18, Issue 2, October, p. 335.
See also *Plasma blobs hint at new form of life*, New Scientist vol. 179 issue 2413 - 20 September 2003, page 16.

[112] *Self organization*, http://en.wikipedia.org/wiki/Self_organization.

References related to anomalies

[113] M. Chown (2004), *Quantum Rebel*, popular article about the experiment of Shahriar Afshar. New Scientist, Vol 183, No 2457.

[114] J. K. Webb *et al* (2001), *Further Evidence for Cosmological Evolution of the Fine Structure Constant*, arXiv:astro-ph/0012539.
T. Chiva and K. Kohri (2001), *Quintessence Cosmology and Varying α*, arXiv:hep-ph/0111086.

[115] J. K. Webb, M. T. Murphy, V. V. Flambaum, V. A. Dzuba, J. D. Barrow, C. W. Churchill, J. X. Prochaska and A. M. Wolfe, *Further Evidence for Cosmological Evolution of the Fine Structure Constant*, Phys. Rev. Lett., Vol. 87, No. 9, Paper No. 091301; August 27, 2001. arxiv.org/abs/astro-ph/0012539.

[116] J. D. Barrow and J. K. Webb (2005), *Inconstant Constants: Do the inner workings of nature change with time?*. Scientific American, June issue.
http://www.sciam.com/article.cfm?chanID=sa006&colID=1&articleID=0005BFE6-2965-128A-A96583414B7F0000.

[117] R. Srianand, H. Chand, P. Petitjean and B. Aracil (2004), *Limits on the Time Variation of the Electromagnetic Fine-Structure Constant in the Low Energy Limit from Absorption Lines in the Spectra of Distant Quasars*, Phys. Rev. Lett., Vol. 92, Paper No. 121302; March 26, 2004. astro-ph/0402177.

[118] D. Da Roacha and L. Nottale (2003), *Gravitational Structure Formation in Scale Relativity*, astro-ph/0310036.

[119] M. Chaplin (2006), *Water as a Network of Icosahedral Water Clusters*, http://www.lsbu.ac.uk/water/clusters.html.

[120] R. T. Lahey Jr., R. P. Taleyarkhan, R. I. Nigmatulin (2005), *Sonofusion-Fact or Fiction*, http://www.rpi.edu/%7elaheyr/Sonofusion%20Paper-pdf_Lahey_NURETH-11.pdf.
For material about sonofusion see http://members.nuvox.net/~on.jwclymer/snf/.

[121] E. Storms (2001),*Cold fusion, an objective assessment*, http://home.netcom.com/~storms2/review8.html.

[122] William G. Tifft (1976), *Discrete States Of Redshift And Galaxy Dynamics I*, Astrophysical Journal, Vol. 206:38-56, 15 May, 1976.
Ibid (1977), *Discrete States Of Redshift And Galaxy Dynamics II: Systems Of Galaxies*, Astrophysical Journal, Vol. 211:31-46, 1 Jan., 1977.
Ibid (1977) *Discrete States Of Redshift And Galaxy Dynamics III: Abnormal Galaxies*, Astrophysical Journal, 211:377-391, 15 January.
Ibid (1978), *The Discrete Redshift And Asymmetry In H I Profiles*, Astrophysical Journal, Vol. 221:449-455, 15 April.
Ibid (1978), *The Absolute Solar Motion And The Discrete Redshift*, Astrophysical Journal, Vol. 221:756-775, 1 May.
Ibid (1980), *Periodicity In The Redshift Intervals For Double Galaxies*, Astrophysical Journal, Vol. 236:70-74, 15 February.
Ibid (1979), *Structure Within Redshift-Magnitude Bands*, Astrophysical Journal, Vol. 233:799-808, 1 November.
Ibid (1982), *Quantum Effects In The Redshift Intervals For Double Galaxies*, Astrophysical Journal, Vol. 257:442-499, 15 June.
Ibid (1982), *Double Galaxy Investigations II*, Astrophysical Journal, Vol. 262:44-47, 1 November.

[123] *Allais effect*, http://en.wikipedia.org/wiki/Allais_effect.

[124] M. Allais (1959), *Should the Laws of Gravitation Be Reconsidered: Part I,II,III?*,
http://home.t01.itscom.net/allais/blackprior/allais/lawgravit/lawgrav-one.pdf
http://home.t01.itscom.net/allais/blackprior/allais/lawgravit/lawgrav-two.pdf.
M. Allais, *The Allais effect and my experiments with the paraconical pendulum 1954-1960.*
http://www.allais.info/alltrans/nasareport.pdf.

[125] G. T. Jeverdan, G. I. Rusu, and V. Antonescu, *Experiments using the Foucault pendulum during the solar eclipse of 15 February, 1961*, Biblical Astronomer 1 (55), 1820 (1981).

[126] V. A. Popescu, D. Olenici (2007), *A confirmation of the Allais and Jeverdan-Rusu-Antonescu effects during the solar eclipse from 22 September 2006, and the quantization behavior of pendulum.*
http://www.hessdalen.org/sse/program/Articol.pdf.

[127] Qian-Shen Wang *et al* (2000), *Precise measurement of gravity variations during a total solar eclipse*, Phys. Rev. D, vol. 62, 041101(R),
http://home.t01.itscom.net/allais/blackprior/wang/wangetal.pdf.

[128] Ph. M. Kanarev and T. Mizuno (2002),*Cold fusion by plasma electrolysis of water*,
http://www.guns.connect.fi/innoplaza/energy/story/Kanarev/coldfusion/.

[129] *Burning salt water*,
http://www.youtube.com/watch?v=aGg0ATfoBgo.

[130] C. L. Kervran (1972),*Biological transmutations, and their applications in chemistry, physics, biology, ecology, medicine, nutrition, agriculture, geology*, Swan House Publishing Co.

[131] P. Tompkins and C. Bird (1973), *The secret life of plants*, Harper & Row, New York.

Chapter 3

Quantum Hall effect and Hierarchy of Planck Constants

3.1 Introduction

Quantum Hall effect [?, ?, ?] occurs in 2-dimensional systems, typically a slab carrying a longitudinal voltage V causing longitudinal current j. A magnetic field orthogonal to the slab generates a transversal current component j_T by Lorentz force. j_T is proportional to the voltage V along the slab and the dimensionless coefficient is known as transversal conductivity. Classically the coefficients is proportional ne/B, where n is 2-dimensional electron density and should have a continuous spectrum. The finding that came as surprise was that the change of the coefficient as a function of parameters like magnetic field strength and temperature occurred as discrete steps of same size. In integer quantum Hall effect the coefficient is quantized to $2\nu\alpha$, $\alpha = e^2/4\pi$, such that ν is integer.

Later came the finding that also smaller steps corresponding to the filling fraction $\nu = 1/3$ of the basic step were present and could be understood if the charge of electron would have been replaced with $\nu = 1/3$ of its ordinary value. Later also QH effect with wide large range of filling fractions of form $\nu = k/m$ was observed.

The model explaining the QH effect is based on pseudo particles known as anyons [?, ?]. According to the general argument of [?] anyons have fractional charge νe. Also the TGD based model for fractionization to be discussed later suggests that the anyon charge should be νe quite generally. The braid statistics of anyon is believed to be fractional so that anyons are neither bosons nor fermions. Non-fractional statistics is absolutely essential for the vacuum degeneracy used to represent logical qubits.

In the case of Abelian anyons the gauge potential corresponds to the vector potential of the divergence free velocity field or equivalently of incompressible anyon current. For non-Abelian anyons the field theory defined by Chern-Simons action is free field theory and in well-defined sense trivial although it defines knot invariants. For non-Abelian anyons situation would be different. They would carry non-Abelian gauge charges possibly related to a symmetry breaking to a discrete subgroup H of gauge group [?] each of them defining an incompressible hydrodynamical flow. According to [?] the anyons associated with the filling fraction $\nu = 5/2$ are a good candidate for non-Abelian anyons and in this case the charge of electron is reduced to $Q = e/4$ rather than being $Q = \nu e$ [?]. This finding favors non-Abelian models [?].

Non-Abelian anyons [?, ?] are always created in pairs since they carry a conserved topological charge. In the model of [?] this charge should have values in 4-element group Z_4 so that it is conserved only modulo 4 so that charges +2 and -2 are equivalent as are also charges 3 and -1. The state of n anyon pairs created from vacuum can be show to possess 2^{n-1}-dimensional vacuum degeneracy [?]. When two anyons fuse the 2^{n-1}-dimensional state space decomposes to 2^{n-2}-dimensional tensor factors corresponding to anyon Cooper pairs with topological charges 2 and 0. The topological "spin" is ideal for representing logical qubits. Since free topological charges are not possible the notion of physical qubit does not make sense (note the analogy with quarks). The measurement of topological qubit reduces to a measurement of whether anyon Cooper pair has vanishing topological charge or not.

Topological quantum computation is perhaps the most promising application of anyons [?, ?, ?, ?, ?, ?, ?].

I have already earlier proposed the explanation of FQHE, anyons, and fractionization of quantum numbers in terms of hierarchy of Planck constants realized as a generalization of the imbedding space $H = M^4 \times CP_2$ to a book like structure [?]. The book like structure applies separately to CP_2 and to causal diamonds ($CD \subset M^4$) defined as intersections of future and past directed light-cones. The pages of the Big Book correspond to singular coverings and factor spaces of CD (CP_2) glued along 2-D subspace of CD (CP_2) and are labeled by the values of Planck constants assignable to CD and CP_2 and appearing in Lie algebra commutation relations. The observed Planck constant $\hbar$, whose square defines the scale of M^4 metric corresponds to the ratio of these Planck constants. The key observation is that fractional filling factor results if $\hbar$ is scaled up by a rational number.

In this chapter I try to formulate more precisely this idea. The outcome is a rather detailed view about anyons on one hand, and about the Kähler structure of the generalized imbedding space on the other hand.

1. Fundamental role is played by the assumption that the Kähler gauge potential of CP_2 contains a gauge part with no physical implications in the context of gauge theories but contributing to physics in TGD framework since $U(1)$ gauge transformations are representations of symplectic transformations of CP_2. Also in the case of CD it makes also sense to speak about Kähler gauge potential. The gauge part codes for Planck constants of CD and CP_2 and leads to the identification of anyons as states associated with partonic 2-surfaces surrounding the tip of CD and fractionization of quantum numbers. Explicit formulas relating fractionized charges to the coefficients characterizing the gauge parts of Kähler gauge potentials of CD and CP_2 are proposed based on some empirical input.

2. One important implication is that Poincare and Lorentz invariance are broken inside given CD although they remain exact symmetries at the level of the geometry of world of classical worlds (WCW). The interpretation is as a breaking of symmetries forced by the selection of quantization axis.

3. Anyons would basically correspond to matter at 2-dimensional "partonic" surfaces of macroscopic size surrounding the tip of the light-cone boundary of CD and could be regarded as gigantic elementary particle states with very large quantum numbers and by charge fractionization confined around the tip of CD. Charge fractionization and anyons would be basic characteristic of dark matter (dark only in relative sense). Hence it is not surprising that anyons would have applications going far beyond condensed matter physics. Anyonic dark matter concentrated at 2-dimensional surfaces would play key key role in the the physics of stars and black holes, and also in the formation of planetary system via the condensation of the ordinary matter around dark matter. This assumption was the basic starting point leading to the discovery of the hierarchy of Planck constants [?]. In living matter membrane like structures would represent a key example of anyonic systems as the model of DNA as topological quantum computer indeed assumes [?].

4. One of the basic questions has been whether TGD forces the hierarchy of Planck constants realized in terms of generalized imbedding space or not. The condition that the choice of quantization axes has a geometric correlate at the imbedding space level motivated by quantum classical correspondence of course forces the hierarchy: this has been clear from the beginning. It is now clear that also the first principle description of anyons requires the hierarchy in TGD Universe. The hierarchy reveals also new light to the huge vacuum degeneracy of TGD and reduces it dramatically at pages for which CD corresponds to a non-trivial covering or factor space, which suggests that mathematical existence of the theory necessitates the hierarchy of Planck constants. Also the proposed manifestation of Equivalence Principle at the level of symplectic fusion algebras as a duality between descriptions relying on the symplectic structures of CD and CP_2[?] forces the hierarchy of Planck constants.

The first sections of the chapter contain summary about theories of quantum Hall effect appearing already in [?]. Second section is a slightly modified version of the description of the generalized imbedding space, which has appeared already in [?, ?, ?] and containing brief description of how to understand QHE in this framework. The third section represents the basic new results about the Kähler structure of generalized imbedding space and represents the resulting model of QHE.

3.2 About theories of quantum Hall effect

The most elegant models of quantum Hall effect are in terms of anyons regarded as singularities due to the symmetry breaking of gauge group G down to a finite sub-group H, which can be also non-Abelian. Concerning the description of the dynamics of topological degrees of freedom topological quantum field theories based on Chern-Simons action are the most promising approach.

3.2.1 Quantum Hall effect as a spontaneous symmetry breaking down to a discrete subgroup of the gauge group

The system exhibiting quantum Hall effect is effectively 2-dimensional. Fractional statistics suggests that topological defects, anyons, allowing a description in terms of the representations of the homotopy group of $((R^2)^n - D)/S_n$. The gauge theory description would be in terms of spontaneous symmetry breaking of the gauge group G to a finite subgroup H by a Higgs mechanism [**?**, **?**]. This would make all gauge degrees of freedom massive and leave only topological degrees of freedom. What is unexpected that also non-Abelian topological degrees of freedom are in principle possible. Quantum Hall effect is Abelian or non-Abelian depending on whether the group H has this property.

In the symmetry breaking $G \rightarrow H$ the non-Abelian gauge fluxes defined as non-integrable phase factors $Pexp(i\oint A_\mu dx^\mu)$ around large circles (surrounding singularities (so that field approaches a pure gauge configuration) are elements of the first homotopy group of G/H, which is H in the case that H is discrete group and G is simple. An idealized manner to model the situation [**?**] is to assume that the connection is pure gauge and defined by an H-valued function which is many-valued such that the values for different branches are related by a gauge transformation in H. In the general case a gauge transformation of a non-trivial gauge field by a multi-valued element of the gauge group would give rise to a similar situation.

One can characterize a given topological singularity magnetically by an element in conjugacy class C of H representing the transformation of H induced by a 2π rotation around singularity. The elements of C define states in given magnetic representation. Electrically the particles are characterized by an irreducible representations of the subgroup of $H_C \subset H$ which commutes with an arbitrarily chosen element of the conjugacy class C.

The action of $h(B)$ resulting on particle A when it makes a closed turn around B reduces in magnetic degrees of freedom to translation in conjugacy class combined with the action of element of H_C in electric degrees of freedom. Closed paths correspond to elements of the braid group $B_n(X^2)$ identifiable as the mapping class group of the punctured 2-surface X^2 and this means that symmetry breaking $G \rightarrow H$ defines a representation of the braid group. The construction of these representations is discussed in [**?**] and leads naturally via the group algebra of H to the so called quantum double $D(H)$ of H, which is a quasi-triangular Hopf algebra allowing non-trivial representations of braid group.

Anyons could be singularities of gauge fields, perhaps even non-Abelian gauge fields, and the latter ones could be modelled by these representations. In particular, braid operations could be represented using anyons.

3.2.2 Witten-Chern-Simons action and topological quantum field theories

The Wess-Zumino-Witten action used to model 2-dimensional critical systems consists of a 2-dimensional conformally invariant term for the chiral field having values in group G combined with 2+1-dimensional term defined as the integral of Chern-Simons 3-form over a 3-space containing 2-D space as its boundary. This term is purely topological and identifiable as winding number for the map from 3-dimensional space to G. The coefficient of this term is integer k in suitable normalization. k gives the value of central extension of the Kac-Moody algebra defined by the theory.

One can couple the chiral field $g(x)$ to gauge potential defined for some subgroup of G_1 of G. If the G_1 coincides with G, the chiral field can be gauged away by a suitable gauge transformation and the theory becomes purely topological Witten-Chern-Simons theory. Pure gauge field configuration represented either as flat gauge fields with non-trivial holonomy over homotopically non-trivial paths or as multi-valued gauge group elements however remain and the remaining degrees of freedom correspond to the topological degrees of freedom.

Witten-Chern-Simons theories are labelled by a positive integer k giving the value of central extension of the Kac-Moody algebra defined by the theory. The connection with Wess-Zumino-Witten theory come from the fact that the highest weight states associated with the representations of the Kac-Moody algebra of WZW theory are in one-one correspondence with the representations R_i possible for Wilson loops in the topological quantum field theory.

In the Abelian case case 2+1-dimensional Chern-Simons action density is essentially the inner product $A \wedge dA$ of the vector potential and magnetic field known as helicity density and the theory in question is a free field theory. In the non-Abelian case the action is defined by the 3-form

$$\frac{k}{4\pi} Tr\left(A \wedge (dA + \frac{2}{3} A \wedge A)\right)$$

and contains also interaction term so that the field theory defined by the exponential of the interaction term is non-trivial.

In topological quantum field theory the usual n-point correlation functions defined by the functional integral are replaced by the functional averages for $Diff^3$ invariant quantities defined in terms of non-integrable phase factors defined by ordered exponentials over closed loops. One can consider arbitrary number of loops which can be knotted, linked, and braided. These quantities define both knot and 3-manifold invariants (the functional integral for zero link in particular). The perturbative calculation of the quantum averages leads directly to the Gaussian linking numbers and infinite number of perturbative link and not invariants.

The experience gained from topological quantum field theories defined by Chern-Simons action has led to a very elegant and surprisingly simple category theoretical approach to the topological quantum field theory [?, ?] allowing to assign invariants to knots, links, braids, and tangles and also to 3-manifolds for which braids as morphisms are replaced with cobordisms. The so called modular Hopf algebras, in particular quantum groups $Sl(2)_q$ with q a root of unity, are in key role in this approach. Also the connection between links and 3-manifolds can be understood since closed, oriented, 3-manifolds can be constructed from each other by surgery based on links [?].

Witten's article [?] "Quantum Field Theory and the Jones Polynomial" is full of ingenious constructions, and for a physicist it is the easiest and certainly highly enjoyable manner to learn about knots and 3-manifolds. For these reasons a little bit more detailed sum up is perhaps in order.

1. Witten discusses first the quantization of Chern-Simons action at the weak coupling limit $k \rightarrow \infty$. First it is shown how the functional integration around flat connections defines a topological invariant for 3-manifolds in the case of a trivial Wilson loop. Next a canonical quantization is performed in the case $X^3 = \Sigma^2 \times R^1$: in the Coulomb gauge $A_3 = 0$ the action reduces to a sum of $n = dim(G)$ Abelian Chern-Simons actions with a non-linear constraint expressing the vanishing of the gauge field. The configuration space consists thus of flat non-Abelian connections, which are characterized by their holonomy groups and allows Kähler manifold structure.

2. Perhaps the most elegant quantal element of the approach is the decomposition of the 3-manifold to two pieces glued together along 2-manifold implying the decomposition of the functional integral to a product of functional integrals over the pieces. This together with the basic properties of Hilbert of complex numbers (to which the partition functions defined by the functional integrals over the two pieces belong) allows almost a miracle like deduction of the basic results about the behavior of 3-manifold and link invariants under a connected sum, and leads to the crucial skein relations allowing to calculate the invariants by decomposing the link step by step to a union of unknotted, unlinked Wilson loops, which can be calculated exactly for $SU(N)$. The decomposition by skein relations gives rise to a partition function like representation of invariants and allows to understand the connection between knot theory and statistical physics [?]. A direct relationship with conformal field theories and Wess-Zumino-Witten model emerges via Wilson loops associated with the highest weight representations for Kac Moody algebras.

3. A similar decomposition procedure applies also to the calculation of 3-manifold invariants using link surgery to transform 3-manifolds to each other, with 3-manifold invariants being defined as

Wilson loops associated with the homology generators of these (solid) tori using representations R_i appearing as highest weight representations of the loop algebra of torus. Surgery operations are represented as mapping class group operations acting in the Hilbert space defined by the invariants for representations R_i for the original 3-manifold. The outcome is explicit formulas for the invariants of trivial knots and 3-manifold invariant of S^3 for $G = SU(N)$, in terms of which more complex invariants are expressible.

4. For $SU(N)$ the invariants are expressible as functions of the phase $q = exp(i2\pi/(k + N))$ associated with quantum groups [?]. Note that for $SU(2)$ and $k = 3$, the invariants are expressible in terms of Golden Ratio. The central charge $k = 3$ is in a special position since it gives rise to $k + 1 = 4$-vertex representing naturally 2-gate physically. Witten-Chern-Simons theories define universal unitary modular functors characterizing quantum computations [?].

3.2.3 Chern-Simons action for anyons

In the case of quantum Hall effect the Chern-Simons action has been deduced from a model of electrons as a 2-dimensional incompressible fluid [?]. Incompressibility requires that the electron current has a vanishing divergence, which makes it analogous to a magnetic field. The expressibility of the current as a curl of a vector potential b, and a detailed study of the interaction Lagrangian leads to the identification of an Abelian Chern-Simons for b as a low energy effective action. This action is Abelian, whereas the anyonic realization of quantum computation would suggest a non-Abelian Chern-Simons action.

Non-Abelian Chern-Simons action could result in the symmetry breaking of a non-Abelian gauge group G, most naturally electro-weak gauge group, to a non-Abelian discrete subgroup H [?] so that states would be labelled by representations of H and anyons would be characterized magnetically H-valued non-Abelian magnetic fluxes each of them defining its own incompressible hydro-dynamical flow. As will be found, TGD predicts a non-Abelian Chern-Simons term associated with electroweak long range classical fields.

3.2.4 Topological quantum computation using braids and anyons

By the general mathematical results braids are able to code all quantum logic operations [?]. In particular, braids allow to realize any quantum circuit consisting of single particle gates acting on qubits and two particle gates acting on pairs of qubits. The coding of braid requires a classical computation which can be done in polynomial time. The coding requires that each dancer is able to remember its dancing history by coding it into its own state.

The general ideas are following.

1. The ground states of anyonic system characterize the logical qubits, One assumes non-Abelian anyons with Z_4 -valued topological charge so that a system of n anyon pairs created from vacuum allows 2^{n-1}-fold anyon degeneracy [?]. The system is decomposed into blocks containing one anyonic Cooper pair with $Q_T \in \{2, 0\}$ and two anyons with such topological charges that the net topological charge vanishes. One can say that the states $(0, 1 - 1)$ and $(0, -1, +1))$ represent logical qubit 0 whereas the states $(2, -1, -1)$ and $(2, +1, +1)$ represent logical qubit 1. This would suggest 2^2-fold degeneracy but actually the degeneracy is 2-fold.

   Free physical qubits are not possible and at least four particles are indeed necessarily in order to represent logical qubit. The reason is that the conservation of Z^4 charge would not allow mixing of qubits 1 and 0, in particular the Hadamard 1-gate generating square root of qubit would break the conservation of topological charge. The square root of qubit can be generated only if 2 units of topological charge is transferred between anyon and anyon Cooper pair. Thus qubits can be represented as entangled states of anyon Cooper pair and anyon and the fourth anyon is needed to achieve vanishing total topological charge in the batch.

2. In the initial state of the system the anyonic Cooper pairs have $Q_T = 0$ and the two anyons have opposite topological charges inside each block. The initial state codes no information unlike in ordinary computation but the information is represented by the braid. Of course, also more general configurations are possible. Anyons are assumed to evolve like free particles except during swap operations and their time evolution is described by single particle Hamiltonians.

Free particle approximation fails when the anyons are too near to each other as during braid operations. The space of logical qubits is realized as k-code defined by the 2^{n-1} ground states, which are stable against local single particle perturbations for $k = 3$ Witten-Chern-Simons action. In the more general case the stability against n-particle perturbations with $n < [k/2]$ is achieved but the gates would become $[k/2]$-particle gates (for $k = 5$ this would give 6-particle vertices).

3. Anyonic system provides a unitary modular functor as the S-matrix associated with the anyon system whose time evolution is fixed by the pre-existing braid structure. What this means that the S-matrices associated with the braids can be multiplied and thus a unitary representation for the group formed by braids results. The vacuum degeneracy of anyon system makes this representation non-trivial. By the NP complexity of braids it is possible to code any quantum logic operation by a particular braid [?]. There exists a powerful approximation theorem allowing to achieve this coding classically in polynomial time [?]. From the properties of the R-matrices inducing gate operations it is indeed clear that two gates can be realized. The Hadamard 1-gate could be realized as 2-gate in the system formed by anyon Cooper pair and anyon.

4. In [?] the time evolution is regarded as a discrete sequence of modifications of single anyon Hamiltonians induced by swaps [?]. If the modifications define a closed loop in the space of Hamiltonians the resulting unitary operators define a representation of braid group in a dense discrete sub-group of $U(2^n)$. The swap operation is 2-local operation acting like a 2-gate and induces quantum logical operation modifying also single particle Hamiltonians. What is important that this modification maps the space of the ground states to a new one and only if the modifications correspond to a closed loop the final state is in the same code space as the initial state. What time evolution does is to affect the topological charges of anyon Cooper pairs representing qubits inside the 4-anyon batches defined by the braids.

   In quantum field theory the analog but not equivalent of this description would be following. Quite generally, a given particle in the final state has suffered a unitary transformation, which is an ordered product consisting of two kinds of unitary operators. Unitary single particle operators $U_n = Pexp(i\int_{t_n}^{t_{n+1}} H_0 dt)$ are analogs of operators describing single qubit gate and play the role of anyon propagators during no-swap periods. Two-particle unitary operators $U_{swap} = Pexp(i\int H_{swap}dt)$ are analogous to four-particle interactions and describe the effect of braid operations inducing entanglement of states having opposite values of topological charge but conserving the net topological charge of the anyon pair. This entanglement is completely analogous to spin entanglement. In particular, the braid operation mixes different states of the anyon. The unitary time development operator generating entangled state of anyons and defined by the braid structure represents the operation performed by the quantum circuit and the quantum measurement in the final state selects a particular final state.

5. Formally the computation halts with a measurement of the topological charge of the left-most anyon Cooper pair when the outcome is just single bit. If decay occurs with sufficiently high probability it is concluded that the value of the computed bit is 0, otherwise 1.

3.3 A generalization of the notion of imbedding space

In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is described. This view has developed much before the original version of this chapter was written.

The original idea was that the proposed modification of the imbedding space could explain naturally phenomena like quantum Hall effect involving fractionization of quantum numbers like spin and charge. This does not however seem to be the case. $G_a \times G_b$ implies just the opposite if these quantum numbers are assigned with the symmetries of the imbedding space. For instance, quantization unit for orbital angular momentum becomes n_a where Z_{n_a} is the maximal cyclic subgroup of G_a.

One can however imagine of obtaining fractionization at the level of imbedding space for space-time sheets, which are analogous to multi-sheeted Riemann surfaces (say Riemann surfaces associated with $z^{1/n}$ since the rotation by 2π understood as a homotopy of M^4 lifted to the space-time sheet is a non-closed curve. Continuity requirement indeed allows fractionization of the orbital quantum numbers and color in this kind of situation.

3.3.1 Both covering spaces and factor spaces are possible

The observation above stimulates the question whether it might be possible in some sense to replace H or its factors by their multiple coverings.

1. This is certainly not possible for M^4, CP_2, or H since their fundamental groups are trivial. On the other hand, the fixing of quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is a geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. H_4 represents a straight cosmic string. Quantum field theory phase corresponds to Jones inclusions with Jones index $\mathcal{M} : \mathcal{N} < 4$. Stringy phase would by previous arguments correspond to $\mathcal{M} : \mathcal{N} = 4$. Also these Jones inclusions are labelled by finite subgroups of $SO(3)$ and thus by Z_n identified as a maximal Abelian subgroup.

   One can argue that cosmic strings are not allowed in QFT phase. This would encourage the replacement $\hat{M}^4 \times \hat{CP}_2$ implying that surfaces in $M^4 \times S^2$ and $M^2 \times CP_2$ are not allowed. In particular, cosmic strings and CP_2 type extremals with M^4 projection in M^2 and thus light-like geodesic without zitterwebegung essential for massivation are forbidden. This brings in mind instability of Higgs=0 phase.

3. The covering spaces in question would correspond to the Cartesian products $\hat{M}^4{}_{n_a} \times \hat{CP}_{2n_b}$ of the covering spaces of $\hat{M}^4$ and $\hat{CP}_2$ by Z_{n_a} and Z_{n_b} with fundamental group is $Z_{n_a} \times Z_{n_b}$. One can also consider extension by replacing M^2 and S^2 with its orbit under G_a (say tedrahedral, octahedral, or icosahedral group). The resulting space will be denoted by $\hat{M}^4 \hat{\times} G_a$ *resp.* $\hat{CP}_2 \hat{\times} G_b$.

4. One expects the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tedrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

5. Also the orbifolds $\hat{M}^4/G_a \times \hat{CP}_2/G_b$ can be allowed as also the spaces $\hat{M}^4/G_a \times (\hat{CP}_2 \hat{\times} G_b)$ and $(\hat{M}^4 \hat{\times} G_a) \times \hat{CP}_2/G_b$. Hence the previous framework would generalize considerably by the allowance of both coset spaces and covering spaces.

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of M^4 factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of M^4 metric can make sense. This is consistent with the identical vanishing of Chern-Simons action in $M^2 \times S^2$.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) and classical light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

3.3.2 Do factor spaces and coverings correspond to the two kinds of Jones inclusions?

What could be the interpretation of these two kinds of spaces?

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M}:\mathcal{N}<4$ and $\mathcal{M}:\mathcal{N}=4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M}:\mathcal{N}<4$ and it naturally corresponds to the coset spaces. For $\mathcal{M}:\mathcal{N}=4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.

2. $\mathcal{M}:\mathcal{N}=4$ should correspond to the allowance of cosmic strings and other analogous objects. Does the introduction of the covering spaces bring in cosmic strings in some controlled manner? Formally the subgroup of $SU(2)$ defining the inclusion is $SU(2)$ would mean that states are $SU(2)$ singlets which is something non-physical. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$.

   For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{M}^2\hat{\times}G_a$ and $\hat{CP}_2\hat{\times}G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane.

3. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M}:\mathcal{N}<4$ and $\mathcal{M}:\mathcal{N}=4$, which both are labelled by a subset of discrete subgroups of SU(2).

4. The discrete subgroups of $SU(2)$ with fixed quantization axes possess a well defined multiplication with product defined as the group generated by forming all possible products of group elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tedrahedral, octahedral, and icosahedral groups are the generators of this algebra.

   By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2¡ generated by reflection, and tedrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized configuration space spinor fields in the configuration space labelled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

The question how do the Planck constants associated with factors and coverings relate is far from trivial and I have considered several options.

1. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of metric allowed by Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains $r^2 \equiv \hbar^2/\hbar_0^2\hbar^2(M^4)/\hbar^2(CP_2)$. This puts M^4 and CP_2 in a very symmetric role and allows much more flexibility in the identification of symmetries associated with large Planck constant phases.

2. Algebraist would argue that Planck constant must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa. This gives two options.

3. Option I: $r(X) = n$ for covering and $r(X) = 1/n$ for factor space gives $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$. This gives $r = n_a/n_b$ for $\hat{H}/G_a \times G_b$ option and $r = n_b/n_a$ for $\hat{Htimes}(G_a \times G_b)$ option with obvious formulas for hybrid cases.

4. Option II: $r(X) = 1/n$ for covering and $r(X) = n$ for factor space gives $r = r(CP_2)/r(M^4)$. This gives $r = n_b/n_a$ for $\hat{H}/G_a \times G_b$ option and $r = n_a/n_b$ for $\hat{Htimes}(G_a \times G_b)$ option with obvious formulas for the hybrid cases.

5. At quantum level the fractionization would come from the modification of fermionic anti-commutation (bosonic commutation) relations involving $\hbar$ at the right hand side so that particle number becomes a multiple of $1/m$ or m. Partonic 2-surface (wormhole throat) is highly analogous to black hole horizon and this led already years ago the notion of elementary particle horizon and generalization of the area law for black-holes [?]. The $1/\hbar$-proportionality of the black hole entropy measuring the number of states associated with black hole motivates the hypothesis that the number of states associated with single sheet of the covering proportional to $1/\hbar$ so that the total number states should remain invariant in the transition changing Planck constant. Since the number of states is obviously proportional to the number m of sheets in the covering, this is achieved for $\hbar(X) \propto 1/m$ giving $r(X) \rightarrow r(X)/n$ for factor space and $r(X) \rightarrow nr(X)$ for the covering space. Option II would be selected.

6. The second manner to distinguish between these two options is to apply the theory to concrete physical situations. Since G_a and G_b act as symmetries in M^4 and CP_2 degrees of freedom, one might of being able to distinguish between the two options if it is possible to distinguish between the action of G as symmetry of quantum states associated with covering and factor space. Also the quantization of the orbital spin quantum number at single particle level as multiples of n can be distinguished from that in multiples of $1/n$.

3.3.3 A simple model of fractional quantum Hall effect

The generalization of the imbedding space suggests that it could possible to understand fractional quantum Hall effect [?] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \tag{3.3.1}$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15..., 2/3, 3/5, 4/7, 5/9, 6/11, 7/13..., 5/3, 8/5, 11/7, 14/9...4/3, 7/5, 10$ $1/5, 2/9, 3/13..., 2/7, 3/11..., 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [?].

The model of Laughlin [?] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [?]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of imbedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are $2 \times 2 = 4$ combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this one can consider two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. In the following a model based on option II for which the number of states is conserved in the phase transition changing $\hbar$.

1. The easiest manner to understand the observed fractions is by assuming that both M^4 and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

2. With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

3. Both $\nu = 1/2$ and $\nu = 5/2$ state has been observed [?, ?]. The fractionized charge is $e/4$ in the latter case [?, ?]. Since $n_i > 3$ holds true if coverings and factor spaces are correlates for Jones inclusions, this requires $n_a = 4$ and $n_b = 8$ for $\nu = 1/2$ and $n_b = 4$ and $n_a = 10$ for $\nu = 5/2$. Correct fractionization of charge is predicted. For $n_b = 2$ also Z_2 would appear as the fundamental group of the covering space. Filling fraction $1/2$ corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [?]. $n_b = 2$ is inconsistent with the observed fractionization of electric charge for $\nu = 5/2$ and with the vision inspired by Jones inclusions.

4. A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

5. Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e\int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F \int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

6. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6 \times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of

the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the assumption about charge fractionization -although consistent with fractionization for $\nu = 5/2$, is rather adhoc. Therefore the model can be taken as a warm-up exercise only.

3.4 Quantum Hall effect, charge fractionization, and hierarchy of Planck constants

In this section the most recent view about the relationship between dark matter hierarchy and quantum Hall effect is discussed. This discussion leads to a more realistic view about FQHE allowing to formulate precisely the conditions under which anyons emerge, describes the fractionization of electric and magnetic charges in terms of the delicacies of the Kähler gauge potential of generalized imbedding space, and relates the TGD based model to the original model of Laughlin. The discussion allows also to sharpen the vision about the formulation of quantum TGD itself.

3.4.1 Quantum Hall effect

Recall first the basic facts. Quantum Hall effect (QHE) [?, ?, ?] is an essentially 2-dimensional phenomenon and occurs at the end of current carrying region for the current flowing transversally along the end of the wire in external magnetic field along the wire. For quantum Hall effect transversal Hall conductance characterizing the 2-dimensional current flow is dimensionless and quantized and given by

$$\sigma_{xy} = 2\nu\alpha_{em} \ ,$$

ν is so called filling factor telling the number of filled Landau levels in the magnetic field. In the case of integer quantum Hall effect (IQHE) ν is integer valued. For fractional quantum Hall effect (FQHE) ν is rational number. Laughlin introduced his many-electron wave wave function predicting fractional quantum Hall effect for filling fractions $\nu = 1/m$ [?]. The further attempts to understand FQHE led to the notion of anyon by Wilzeck [?]. Anyon has been compared to a vortex like excitation of a dense 2-D electron plasma formed by the current carriers. ν is inversely proportional to the magnetic flux and the fractional filling factor can be also understood in terms of fractional magnetic flux.

The starting point of the quantum field theoretical models is the effective 2-dimensionality of the system implying that the projective representations for the permutation group of n objects are representations of braid group allowing fractional statistics. This is due to the non-trivial first homotopy group of 2-dimensional manifold containing punctures. Quantum field theoretical models allow to assign to the anyon like states also magnetic charge, fractional spin, and fractional electric charge.

Topological quantum computation [?, ?, ?, ?] is one of the most fascinating applications of FQHE. It relies on the notion of braids with strands representing the orbits of of anyons. The unitary time evolution operator coding for topological computation is a representation of the element of the element of braid group represented by the time evolution of the braid. It is essential that the group involved is non-Abelian so that the system remembers the order of elementary braiding operations (exchange of neighboring strands). There is experimental evidence that $\nu = 5/2$ anyons possessing fractional charge $Q = e/4$ are non-Abelian [?, ?].

During last year I have been developing a model for DNA as topological quantum computer [?]. Therefore it is of considerable interest to find whether TGD could provide a first principle description of anyons and related phenomena. The introduction of a hierarchy of Planck constants realized in terms of generalized imbedding space with a book like structure is an excellent candidate in this respect [?]. As a rule the encounters between real world and quantum TGD have led to a more precise quantitative articulation of basic notions of quantum TGD and the same might happen also now.

3.4.2 TGD description of QHE

The proportionality $\sigma_{xy} \propto \alpha_{em} \propto 1/\hbar$ suggests an explanation of FQHE [?, ?, ?] in terms of the hierarchy of Planck constants. Perhaps filling factors and magnetic fluxes are actually integer valued but the value of Planck constant defining the unit of magnetic flux is changed from its standard value

- to its rational multiple in the most general case. The killer test for the hypothesis is to find whether higher order perturbative QED corrections in powers of α_{em} are reduced from those predicted by QED in QHE phase. The proposed general principle governing the transition to large $\hbar$ phase is states that Nature loves lazy theoreticians: if perturbation theory fails to converge, a phase transition increasing Planck constant occurs and guarantees the convergence. Geometrically the phase transition corresponds to the leakage of 3-surface from a given 8-D page to another one in the Big Book having singular coverings and factor spaces of $M^4 \times CP_2$ as pages.

Chern-Simons action for Kähler gauge potential (equivalently for induced classical color gauge field proportional to the Kähler form) defines TGD as almost topological QFT. This alone strongly suggests the emergence of quantum groups and fractionalization of quantum numbers [**?**]. The challenge is to figure out the details and see whether this framework is consistent with what is known about QHE. At least the following questions pop up immediately in mind.

1. What the effective 2-dimensionality of the system exhibiting QHE corresponds in TGD framework?

2. What happens in the phase transition leading to the phase exhibiting QHE effect?

3. What are the counterparts anyons? How the fractional electric and magnetic charges emerge at classical and quantum level.

4. The Chern-Simons action associated with the induced Kähler gauge potential is Abelian: is this consistent with the non-Abelian character of braiding matrix?

3.4.3 Quantum TGD almost topological QFT

The statement that TGD is almost topological QFT means following conjectures.

1. In TGD the fundamental physical object is light-like 3-surface X^3 connecting the light-cone boundaries of $CD \times CP_2 \subset M^4 \times CP_2$ (intersection of future and past directed light-cones) but by conformal invariance in the light-like direction of X^3 physics is locally 2-dimensional in the sense that one can regard this surface as an orbit of 2-D parton as long as one restricts to finite region of X^3. Physics at X^3 remains 3-D in discretized sense (quantum states are of course quantum superpositions of different light-like 3-surfaces).

2. At the fundamental level quantum TGD can be formulated in terms of the fermionic counterpart of Chern-Simons action fort the Kähler gauge potential associated with Kähler form of CP_2. The Dirac determinant associated with the modified Dirac action defines the vacuum functional of the theory. Dirac determinant is defined as a finite product of the values of generalizes eigenvalues (functions) of the modified Dirac operator at points defined by the strands of so called number theoretic braids which by number theoretic arguments are unique [**?**, **?**].

3. Vacuum functional equals to the exponent of Kähler action for a preferred extremal $X^4(X^3)$ of Kähler action, which plays the role of Bohr orbit and allows to realize 4-D general coordinate invariance. The boundary conditions of 4-D dynamics fixing $X^4(X^3)$ are fixed by the requirement that the tangent space of X^4 contains a preferred Minkowski plane $M^2 \subset M^4$ at each point. This plane can be interpreted as the plane of non-physical polarizations.

4. "Number theoretic compactification" states that space-time surfaces can be regarded as 4-surfaces of either hyper-octonionic M^8 or $M^4 \times CP_2$ (hyper-octonions corresponds to a sub-space of complexified octonions with Minkowskian signature of metric). The surfaces of M^8 are hyper-quaternionic in the sense that each tangent plane is hyper-quaternionic and contains (this is essential for number theoretic compactification) the preferred hyper-complex plane M^2 defined by hyper-octonionic real unit and preferred imaginary unit. The preferred extrema of Kähler action should correspond hyper-quaternionic 4-surfaces of M^8 having preferred M^2 as a tangent space at each point.

These 'must-be-trues' are of course highly non-trivial un-proven conjectures. If one gives up conjecture about the reduction of entire 4-D dynamics to that for almost topological fermions at 3-D light-like surfaces, one must assume separately that vacuum functional is exponent of Kähler function for a preferred extremal.

3.4.4 Constraints to the Kähler structure of generalized imbedding space from charge fractionization

In the following the notion of generalized imbedding space is discussed. The new element is more precise formulation of the Kähler structure by allowing Kähler gauge potential to have what looks formally as gauge parts in both M^4 and CP_2 and of no physical significance on gauge theory context. In TGD framework the gauge parts have deep physical significance since symplectic transformations act as symmetries of Kähler and Chern-Simons-Kähler action only in the case of vacuum extremals.

Hierarchy of Planck constants and book like structure of imbedding space

TGD leads to a description for the hierarchy of Planck constants in terms of the generalization of the Cartesian factors of the imbedding space $H = M^4 \times CP_2$ to book like structures. To be more precise, the generalization takes place for any region $CD \times CP_2 \subset H$, where CD corresponds to a causal diamond defined as an intersection of future and past directed light-cones of M^4. CDs play key role in the formulation of quantum TGD in zero energy ontology in which the light-like boundaries of CD connected by light-like 3-surfaces can be said to be carriers of positive and negative energy parts of zero energy states. They are also crucial for TGD inspired theory of consciousness, in particular for understanding the relationship between experienced and geometric time [?].

Both CD and CP_2 are replaced with a book like structure consisting in the most general case of singular coverings and factor spaces associated with them. A simple geometric argument identifying the square of Planck constant as scaling factor of the covariant metric tensor of M^4 (or actually CD) leads to the identification of Planck constant as the ratio $\hbar/\hbar_0 = q(M^4)/q(CP_2)$, where $q(X) = N$ holds true for the covering of X and $q(X) = 1/N$ holds true for the factor space. N is the order of the maximal cyclic subgroup of the covering/divisor group $G \subset SO(3)$. The order of G can be thus larger than N. As a consequence, the spectrum of Planck constants is in principle rational-valued. $\hbar_0$ is unique since it corresponds to the unit of rational numbers. The field structure has far reaching implications for the understanding of phase transitions changing the value of Planck constant.

The hierarchy of Planck constants relates closely to quantum measurement theory. The selection of quantization axis has a direct correlate at the level of imbedding space geometry. This means breaking of isometries of H for a given CD with preferred choice time axis (rest frame) and quantization axis of spin. For CP_2 the choice of the quantization axes of color hyper charge and isospin imply symmetry breaking $SU(3) \rightarrow U(2) \rightarrow U(1) \times U(1)$. The "world of classical worlds" (WCW) is union over all Poincare and color translates of given $CD \times CP_2$ so that these symmetries are not lost at the level of WCW although the loss can happen at the level of quantum states.

Non-vanishing of Poincare quantum numbers requires CP_2 Kähler gauge potential to have M^4 part

Since Kähler action gives rise to conserved Poincare quantum numbers as Noether charges, the natural expectation is that Poincare quantum numbers make sense as Noether charges for Chern-Simons action. The problem is that Poincare quantum numbers vanish for standard Kähler gauge potential of CP_2 since it has no M^4 part.

The way out of the difficulty relies on the delicacies of CP_2 Kähler structure.

1. One can give up the strict Cartesian product property and assume that CP_2 Kähler gauge potential has M^4 part which is pure gauge and without physical meaning in gauge theory context. In TGD framework the situation is different. The reason is that $U(1)$ gauge transformations are induced by the symplectic transformations of CP_2 and correspond to genuine dynamical symmetries acting as isometries of WCW. They act as symmetries of Kähler action only in the case of vacuum extremals and relate closely to the spin glass degeneracy of Kähler action with the counterpart of spin glass energy landscape defined by small deformations of vacuum extremals of Kähler action. This vacuum degeneracy has been one of the most fruitful challenges of TGD.

2. Requiring Lorentz invariance one can write the non-vanishing pure gauge M^4 component of Kähler gauge potential as

$$A_a = constant \ . \tag{3.4.1}$$

Here a denotes the light-cone proper time. It is of course possible that also other components are present as it indeed turns out. Using standard formula for Noether current one finds that four-momentum is non-vanishing because of the term $A_a\partial_\alpha a$ in Chern-Simons-Kähler action. From $\partial_\alpha a = m^k m_{kl}\partial_\alpha m^l/a$ momentum current T^{k0} at given point of X^3 is proportional to the average 4-velocity with respect to the tip of light-cone: $T^{k0} \propto m^k/a$. Therefore the motion in the average sense is analogous to cosmic expansion. This is natural since the structure of CD corresponding to particular quantization axes breaks Poincare symmetry.

3. $A_a = constant$ guarantees the conservation of mass squared in the case of CP_2 type extremals at least and implies that mass squared is non-vanishing. Four-momentum is also proportional to the Kähler magnetic flux over the partonic 2-surface X^2 and X^2 must be homologically non-trivial for the net value of four-momentum to be non-vanishing. X^2 could correspond to the end of cosmic string in 4-D picture. Homological non-triviality does not seem to be necessary in the case of super-symmetric counterpart of Dirac action since Kähler flux is multiplied by the fermionic bilinear so that the outcome is more general than Kähler magnetic flux.

The M^4 part of CP_2 Kähler gauge potential for the generalized imbedding space

The non-triviality of A_a transforms topological QFT to an almost topological one, but says nothing about the covering- and factor space sectors of generalized imbedding space- the pages of the book like structure defined by the generalized imbedding space. The interpretation in terms of quantum measurement theory suggests that Lorentz symmetry and color symmetry are broken to Cartan sub-groups defining quantization axes. If anyons correspond to large $\hbar$ phase, the Kähler gauge potential of CP_2 should contain in these sectors additional gauge parts in both M^4 and CP_2 responsible for charge fractionization, magnetic monopoles, and other anyonic effects.

The basic prerequisite for anyonic effects is that fundamental group is non-trivial and for M^4 the emergence of M^2 as the intersection of sheets of the singular covering implies this for the complement of M^2. In the case of CP_2 the homologically trivial geodesic S^2 is common to the coverings and factors spaces and implies the non-triviality of the fundamental group.

Let $(u = m^0 + r_M), v = m^0 - r_M, \theta, \phi)$ define light-like spherical coordinates for $M^4_\pm$. Here m^k are linear M^4 time coordinates and r_M is radial M^4 coordinate. Denote the light-cone proper time by $a = \sqrt{uv}$. The origin of coordinates lies at the either tip of CD. Coordinates are not global so that the patches assignable to positive and negative energy parts of the zero energy state must be used.

The fixing of the rest system, that is the direction of time axis, reduces Lorentz invariance to $SO(3)$. This allows A to have an additional part

$$A_u = \frac{k_1}{u^2} \ . \tag{3.4.2}$$

The functional form of A_u will be deduced in the sequel from the conservation of anyonic charges. The fixing of the direction of the spin quantization axis reduces the symmetry to $SO(2)$ and allows introduction of a further gauge component

$$A_\phi = \frac{k_2}{u^2} \ . \tag{3.4.3}$$

Clearly one has a hierarchical breaking of symmetry: Poincare group $\rightarrow$ Lorentz group$\rightarrow$ rotation group $SO(3) \rightarrow SO(2)$. Globally the symmetry is not broken since WCW is a union over all possible choices of quantization for each CDs with all possible positions of lower tip are allowed. p-Adic length scale hypothesis results if the temporal distance between upper and lower tips is quantized in multiples 2^n. The hierarchy of Planck constants however implies that distance are quantized as rational multiples of basic distance scale.

How fractional electric and magnetic charges emerge from M^4 gauge part of CP_2 Kähler gauge potential?

The Maxwell field defined by the induced CP_2 Kähler form plays fundamental role in the construction of quantum TGD. Kähler gauge potential of CP_2 contributes directly to the classical electromagnetic gauge potential. Its coupling to $M^4 \times CP_2$ spinors is different for quarks and leptons representing different conserved chiralities of H spinors and it explains different electromagnetic charges of quarks and leptons as well as different color trialities. Also classical color gauge field is proportional to Kähler form. Therefore one might hope that the gauge parts of Kähler gauge potential might contain a lot of interesting physics.

The following series of arguments try to demonstrate following three results.

1. The anomalous contribution to the Kähler gauge potential induces anomalous electric and magnetic Kähler charges and therefore also em, Z^0, and color gauge charges.
2. Anyons can be characterized as 2-surfaces surrounding the tip of CD.
3. In sectors corresponding to the non-standard value of $\hbar$ the vacuum degeneracy of Kähler and Chern-Simons actions is dramatically reduced.

Note that in this section the consideration is restricted to the gauge parts of CP_2 Kähler gauge potential in $CD \subset M^4$. Also the gauge parts in CP_2 are possible and the Kähler potential assignable to the contact structure of CD must be considered separately.

1. The gauge part of Kähler gauge potential vanishes outside CD so that it is discontinuous at light-like boundary in the direction of the light like vector defined as the gradient of $v = t - r$. This means that for partonic 2-surfaces surrounding the tip of light-cone both Kähler electric and magnetic fluxes are non-vanishing and determined by $K_i(u)$, $i = 1, 2$. By requiring that the anomalous Kähler charge is time independent, one obtains $K_1(u) = k_1/u^2$. This means that the Kähler electric gauge field has a delta function like singularity at the light-like boundares of CD which becomes carrier of Kähler charge from the view point of complement of CD. This suggests that if one has N elementary particles at partonic 2-surface X^2 surrounding the tip of CD (wormhole throats of elementary particles are condensed to X^2), the charges of particles are effectively fractionized:
$$q \rightarrow q + \frac{Q_A}{N} . \tag{3.4.4}$$
2. In the case of $A_\phi = constant$ anomalous magnetic charge results since the flux expressible as line integral $\int A_\phi d\phi$ is non-vanishing because the poles of S^2 act effectively as magnetic charges. The punctures at the poles are the correlate for the selection of the quantization axes of spin. $K_2(u) = k_2/u^2$ follows from the conservation of magnetic charge. In the case of ordinary magnetic monopole spin becomes half-odd integer valued and analogous result holds also now. The minimal coupling to the gauge part of A_ϕ defining the covariant derivative D_ϕ together with covariant constancy condition implies that spin receives a fractional part for $k_2 \neq 0$ and spin fractionization results.
3. One can see the situation also differently. The 2-D partons at the ends of light-like 3-surfaces at light-like boundaries of CD interact like particles with anomalous gauge charges but the interaction is now in light-like direction. The anomalous charges indeed characterize Chern-Simons action. For $k_1 = k_2 = 0$ corresponding to $\hbar/\hbar_0 = 1$ one has Lorentz invariance and only cosmic string like objects seem to remain to the spectrum of the theory (they dominate the very early TGD based cosmology [?]).
4. Quite generally, anyonic states can be assigned with partonic 2-surfaces surrounding surrounding the tip of CD since the fractional contribution to the gauge charge vanishes otherwise.
5. Kähler gauge potential appears in the expression of the em charge so that a fractionization of electric and magnetic em and Z^0 charges results but there is no fractionization of the weak charge. The components of the classical color gauge field are of form $G^A \propto H^A J$, H^A the

Hamiltonian of color isometry and J Kähler form. The assumption that the singular part of G^A is induced from that for J implies anomalous electric and magnetic color gauge charges located at boundaries of CD. These charges should make sense as fluxes since the $SU(3)$ holonomy is Abelian.

6. A_u contributes to the four-momentum density a term proportional to the four-vector $\partial u/\partial m^k$ which in vector notation looks like $(1, \overline{r}_M/r_M)$: thus the direction of 3-momentum rends to be same as for A_a. In the approximation that the M^4 coordinates for partonic 2-surface are constant (excellent approximation at elementary particle level) this contribution to the four-momentum is massless unlike for A_a. If the variation of the projection of A_u in Chern-Simons action is responsible for the four-momentum X^2 must carry non-vanishing homological charge for Chern-Simons action but not for its fermionic counterpart. If the variation of the projection of the singular part J_{uv} is responsible for the momentum the CP_2 projection can be 1-dimensional so that the vacuum degeneracy is reduced and the homological non-triviality in CP_2 is replaced with homological triviliaty in CD with the line connecting the tips of CD removed.

7. For $(k_1, k_2) = (0, 0)$ all space-time surfaces for which CP_2 projection is Lagrange manifold of CP_2 (generally 2-dimensional sub-manifold having vanishing induced Kähler form) are vacuum extremals For $(k_1, k_2) \neq (0, 0)$ and for partonic 2-surfaces surrounding the tip of the light-cone, the situation changes since also partonic 2-surfaces which have 1-D CP_2 projection can carry non-vanishing Kähler, em, and color charges, and even four-momentum. If M^4 projection is 2-D, the anomalous part of Kähler form contributing to the charges is completely in M^4 and the variation of of A_α in Chern-Simons action gives rise to color currents. Four-momentum can be non-vanishing even when CP_2 projection is zero-dimensional since the variation of A_a gives rise to it when X^2 surrounds the tip of CD. Hence the hierarchy of Planck constants removes partially the vacuum degeneracy. This correlation conforms with the general idea that both the vacuum degeneracy and the hierarchy of Planck constants relate closely to quantum criticality. Perhaps the hierarchy of Planck constants accompanied by the anyonic gauge parts of A makes possible to have mathematically well-define theory.

Coverings and factor spaces of CP_2 and anyonic gauge part of Kähler gauge potential in CP_2?

Nothing about possible coverings and factor spaces of CP_2 has been said above. In principle they could contribute to CP_2 Kähler gauge potential an anomalous part and would form a representation for the hierarchy of Planck constants in CP_2 degrees of freedom.

1. If Kähler gauge potential has also anyonic CP_2 part, it should fix the choice of quantization axes for color charges. Thus the anomalous components could be of form $A_{I_3} = k(I_3)$ and $A_Y = k(Y_3)$ where the angle variables vary along flow lines of I_3 and Y. Singularity would emerge both at the origin and at the 2-sphere $r = \infty$ analogous to the North pole of S^2, at which the second angle variable becomes redundant.

2. These terms would give to the anomalous Kähler magnetic charge a contribution completely analogous to that coming from A_ϕ. Also color charges would receive similar contribution.

How the values of the anomalous charges relate to the parameters characterizing the page of the Big Book?

One should be able to relate the anomalous parameters characterizing anomalous gauge potentials to the parameters n_a, n_b characterizing the coverings of CD and CP_2. Consider first various manners to understand charge fractionization.

1. The hypothesis at the end of previous section states that for n_b-fold covering of CP_2 the fractionized electric charge equals to e/n_b. This predicts charge fractionization correctly for $\nu = 5/2 = 10/4$ [?]. This simple argument could apply also to other charges. The interpretation would be that when elementary particle becomes anyonic, its charge is shared between n_b sheets of the covering of CP_2. In the case of factor space the singular factor space would appear as n_b copies meaning the presence n_b particles behaving like single particle. Charge fractionization would be only apparent in this picture.

2. This global representation of the fractionization of Kähler charge might be enough. One can however ask whether also a local representation could exist in the sense that the coupling of fermions to the gauge parts of Kähler gauge potential would represent charge fractionization at single particle level in terms of phase factors analogous to plane waves. If charge fractionization is only apparent, the total anomalous Kähler charge assignable to particles should be compensated by the total anomalous Kähler charge associated with A_u. This gives a constraint between k_1 and parameter $k(Y)$.

3. Similar argument for the Kähler magnetic charge gives a constraint between k_2 and $k(Y)$ implying $k_1 = k_2$ consistent with assumption that also the anyonic part of Kähler form is self dual. In the simplest situation $k_1 = k_2 = Nq_K k(Y)$, where N is the number of identical particles at the anyonic space-time sheet. In more general case one would have $k_1 = k_2 \sum_i N_i q_{K,i} k(Y)$. If the anyonic space-time sheet does not contain the tip of CD in its interior, the total anomalous Kähler charge associated with the fermions at it must vanish.

4. Both em and Z^0 fields contain a part proportional to Kähler form so that total anomalous gauge charges defined as fluxes should be equal to those defined as sums of elementary particle contributions.

5. Anomalous color isospin and hypercharge and corresponding magnetic charges would have also representations as color gauge fluxes by using $Q^A \propto H^A J$ restricted to Cartan algebra of color group. The couplings to the anomalous gauge parts of Kähler gauge potential in CP_2 would give rise to anomalous color charges at single particle level, and also now the condition that the total anomalous charges assignable to particles compensates that assignable to the singular part of color gauge potential is natural. Thus quite a number of consistency conditions emerge.

The foregoing discussion relates to the gauge part of Kähler gauge potential assigned to CP_2 degrees of freedom. Analogous discussion applies to the M^4 part.

1. Covariant constancy conditions appear also in Minkowski degrees of freedom and correlate the value of anomalous Poincare charges to anomalous Kähler charge. Anomalous Kähler charge k_1 gives via covariant constancy condition for induced spinors contribution to four-momentum analogous to Coulomb interaction energy with Kähler charge k_1: at point like limit the contribution is light-like. In the similar manner $k_2 = k_1$ gives rise to anomalous orbital spin via the covariant constancy condition $D_\phi \Psi = (\partial_\phi + A_\phi)\Psi = 0$ equating A_ϕ with the fractional contribution to spin. Thus both anomalous four-momentum and spin fractionization effect reflects the total anomalous Kähler charge.

2. The values of $k_1 = k_2$ should correlate directly with the order of the maximal cyclic subgroup Z_{n_a} associated with the covering/factor space of CD. For covering one should should have $k_2 = n/n_a$ since the rotation by $N \times 2\pi$ is identity transformation. For the factor space one should have $k_2 = nn_a$ since the states must remain invariant under rotations by multiples of $2\pi/N$ and spin unit becomes n_a. This picture is consistent with the scaling up of the spin unit with $\hbar/\hbar_0$. Since k_1 must be also an integer multiple of $1/n_b$, k_1 should be inversely proportional to a common factor of n_a and n_b.

That classical color hyper charge and isospin correspond to electro-weak charges is an old idea which I have not been able to kill. It is discussed also in [?] from the point of view of symplectic fusion algebras.

1. Quark color is not a spin like quantum number but corresponds to CP_2 partial waves in cm degrees of freedom of partonic 2-surface. Hence it should not relate to the classical color charges associated with classical color gauge field or with the modes of induced spinor fields at space-time sheet. These nodes can also carry color hyper charge and isospin in the sense that they are proportional to space-time projections of phase factors representing states with constant Y and I_3 (being completely analogous to angular momentum eigen states on circle).

2. In the construction of symmetric spaces the holonomy group of the spinor connection is identified as a subgroup of the isometry group. Therefore electro-weak gauge group $U(2)_{ew}$ would

correspond to $U(2) \subset SU(3)$ defining color quantization axis. If so, the phase factors assignable to the induced spinor fields could indeed represent the electromagnetic and weak charges of the particle and one would have $Y = Y_{ew}$ and $I_3 = I_{3,ew}$. Also electro-weak quantum numbers, which are spin-like, would have geometric representation as phase factors of spinors.

3. This kind of multiple representation emerges also via number theoretical compactication [?] meaning that space-time surfaces can be regarded either as surfaces in hyper-octonionic space $M^8 = M^4 \times E^4$ or $M^4 \times CP_2$. In M^8 electro-weak quantum numbers are represented as particle waves and color is spin like quantum number.

Again a word of caution is in order since the formula for charge fractionization is supported only by its success in $\nu = 5/2$ case. Also the proposed formulas are only heuristic guesses.

What about Kähler gauge potential for CD?

One can assign also to light-cone boundary- metrically equivalent with S^2, symplectic (or more precisely contact-) structure. This structure can be extended to a pseudo-symplectic structure in the entire CD. The structure is not global and one must introduce two patches corresponding to the two light-cone boundaries of CD.

This symplectic structure plays a key role in the construction of symplectic fusion algebra [?]. In TGD framework Equivalence Principle is realized in terms generalized coset construction for the super-canonical conformal algebra assignable to the light-cone boundary and super-Kac-Moody algebra assignable to the light-like 3-surfaces. The cautious proposal of [?] is that at the level of fusion algebra Equivalence Principle means the possibility to use either the symplectic fusion algebra of light-cone boundary for light-cone defined by S^2 Kähler form or the symplectic fusion algebra for light-cone boundry defined by CP_2 Kähler form.

The vacuum degeneracy of Kähler action requiring that CP_2 projection of the partonic 2-surface is non-trivial would at first seem to exclude this option. Anomalous gauge charges however remove this vacuum degeneracy for $k_1 \neq 0$ so that there are no obvious reasons excluding this manifestation of Equivalence Principle.

The Kähler gauge potential of the degenerate Kähler form assignable to the light-like boundary (basically to the $r_M = constant$ sphere S^2) and also to CD and identifiable as the Kähler form of S^2 defining its signed area can indeed contain gauge part with a structure similar that for CP_2 Kähler gauge potential and involving three rational valued constants corresponding to gauge parts A_a, A_u, and A_ϕ. The TGD based realization of the Equivalence Principle suggests that the constants associated with the two Kähler forms are identical or at least proportional to each other. One could perhaps even say that the hierarchy of Planck constants and dark matter are necessary to realize Equivalence Principle in TGD framework.

Concrete picture about gluing of different sectors of the generalized imbedding space

Intuitively the scaling of Planck constant scales up quantum lengths, in particular the size of CD. This looks trivial but one one must describe precisely what is involved to check internal consistency and also to understand how to model the quantum phase transitions changing Planck constant.

The first manner to understand the situation is to consider CD with a fixed range of M^4 coordinates. The scaling up of the covariant Kähler metric of CD by $r^2 = (\hbar/\hbar_0)^2$ scales up the size of CD by r. Another manner to see the situation is by scaling up the linear M^4 coordinates by r for the larger CD so that M^4 metric becomes same for both CDs. The smaller CD is glued to the larger one isometrically together along $(M^2 \cap CD) \subset CD$ anywhere in the interior of the larger CD. What happens is non-trivial for the following reasons.

1. The singular coverings and factor spaces are different and M^4 scaling is not a symmetry of the Kähler action so that the preferred extrema in the two cases do not relate by a simple scaling. The interpretation is in terms of the coding of the radiative corrections in powers of $\hbar$ to the shape of the preferred extremals. This becomes clear from the representation of Kähler action in which M^4 coordinates have the same range for two CDs but M^4 metric differs by r^2 factor.

2. In common M^4 coordinates the M^4 gauge part A_a of CP_2 Kähler potential for the larger CD differs by a factor $1/r$ from that for the smaller CD. This guarantees the invariance of four-momentum assignable to Chern-Simons action in the phase transition changing $\hbar$. The resulting discontinuity of A_a at M^2 is analogous to a static voltage difference between the two CDs and M^2 could be seen as an analog of Josephson junction. In absence of dissipation (expected in quantum criticality) the Kähler voltage could generate oscillatory fermion, em, and Z^0 Josephson currents between the two CDs. Fermion current would flow in opposite directions for fermions and antifermions and also for quarks and leptons since Kähler gauge potential couples to quarks and leptons with opposite signs. In presence of dissipation fermionic currents would be ohmic and could force quarks and leptons and matter and antimatter to different pages of the Big Book. Quarks inside hadrons could have nonstandard value of Planck constant.

3. The discontinuities of A_u and A_ϕ allow to assign electric and magnetic Kähler point charges $Q_K^{e/m}$ with $M^1 \subset M^2$ and having sign opposite to those assignable with $\delta CD \times CP_2$. It should be possible to identify physically M^2, the line E^1 representing quantization axis of angular momentum, and the position of Q_K.

3.4.5 In what kind of situations do anyons emerge?

Charge fractionization is a fundamental piece of quantum TGD and should be extremely general phenomenon and the basic characteristic of dark matter known to contribute 95 per cent to the matter of Universe.

1. In TGD framework scaling $\hbar = m\hbar_0$ implies the scaling of the unit of angular momentum for m-fold covering of CD only if the many particle state is Z_m singlet. Z_m singletness for many particle states allows of course non-singletness for single particle states. For factor spaces of CD the scaling $\hbar \to \hbar/m$ is compensated by the scaling $l \to ml$ for $L_z = l\hbar$ guaranteing invariance under rotations by multiples of $2\pi/m$. Again one can pose the invariance condition on many-particle states but not to individual particles so that genuine physical effect is in question.

2. There is analogy with Z_3-singletness holding true for many quark states and one cannot completely exclude the possibility that quarks are actually fractionally charged leptons with $m = 3$-covering of CP_2 reducing the value of Planck constant [?, ?] so that quarks would be anyonic dark matter with smaller Planck constant and the impossibility to observe quarks directly would reduce to the impossibility for them to exist at our space-time sheet. Confinement would in this picture relate to the fractionization requiring that the 2-surface associated with quark must surround the tip of CD. Whether this option really works remains an open question. In any case, TGD anyons are quite generally confined around the tip of CD.

3. Quite generally, one expects that dark matter and its anyonic forms emerge in situations where the density of plasma like state of matter is very high so that N-fold cover of CD reduces the density of matter by $1/N$ factor at given sheet of covering and thus also the repulsive Coulomb energy. Plasma state resulting in QHE is one examples of this. The interiors of neutron stars and black hole like structures are extreme examples of this, and I have proposed that black holes are dark matter with a gigantic value of gravitational Planck constant implying that black hole entropy -which is proportional to $1/\hbar$ - is of same order of magnitude as the entropy assignable to the spin of elementary particle. The confinement of matter inside black hole could have interpretation in terms of macroscopic anyonic 2-surfaces containing the topologically condensed elementary particles. This conforms with the TGD inspired model for the final state of star [?] inspiring the conjecture that even ordinary stars could posses onion like structure with thin layers with radii given by p-adic length scale hypothesis.

   The idea about hierarchy of Planck constants was inspired by the finding that planetary orbits can be regarded as Bohr orbits [?, ?]: the explanation was that visible matter has condensed around dark matter at spherical cells or tubular structures around planetary orbits. This led to the proposal that planetary system has formed through this kind of condensation process around spherical shells or flux tubes surrounding planetary orbits and containing dark matter.

   The question why dark matter would concentrate around flux tubes surrounding planetary orbits was not answered. The answer could be that dark matter is anyonic matter at partonic 2-surfaces whose light-like orbits define the basic geometric objects of quantum TGD. These partonic 2-surfaces could contain a central spherical anyonic 2-surface connected by radial flux tubes to flux tubes surrounding the orbits of planets and other massive objects of solar system to form connected anyonic surfaces analogous to elementary particles.

If factor spaces appear in M^4 degrees of freedom, they give rise to $Z_n \subset G_a$ symmetries. In astrophysical systems the large value of $\hbar$ necessarily requires a large value of n_a for CD coverings as the considerations of [?] - in particular the model for graviton dark graviton emission and detection - forces to conclude. The same conclusion follows also from the absence of evidence for exact orbifold type symmetries in M^4 degrees of freedom for dark matter in astrophysical scales.

4. The model of DNA as topological quantum computer [?] assumes that DNA nucleotides are connected by magnetic flux tubes to the lipids of the cell membrane. In this case, p-adically scaled down u and d quarks and their antiquarks are assumed to be associated with the ends of the flux tubes and provide a representation of DNA nucleotides. Quantum Hall states would be associated with partonic 2-surfaces assignable to the lipid layers of the cell and nuclear membranes and also endoplasmic reticulum filling the cell interior and making it macroscopic quantum system and explaining also its stability. The entire system formed in this manner would be single extremely complex anyonic surface and the coherent behavior of living system would result from the fusion of anyonic 2-surfaces associated with cells to larger anyonic surfaces giving rise to organs and organisms and maybe even larger macroscopically quantum coherent connected systems.

   In living matter one must consider the possibility that small values of n_a correspond to factor spaces of CD (consider as example aromatic cycles with Z_n symmetry with $n = 5$ or $n = 6$ appearing in the key molecules of life). Large $\hbar$ would require CP_2 factor spaces with a large value of n_b so that the integers characterizing the charges of anyonic particles would be shifted by a large integer. This is not in accordance with naive ideas about stability. One can also argue that various anomalous effects such as IQHE with ν equal to an integer multiple of n_b would have been observed in living matter.

   A more attractive option is that both CD and CP_2 are replaced with singular coverings. Spin and charge fractionization takes place but the effects are small if both n_a, n_b, and n_a/n_b are large. An interesting possibility is that the ends of the flux tubes assumed to connect DNA nucleotides to lipids of various membranes carry instead of u, d and their anti-quarks fractionally charged electrons and neutrinos and their anti-particles having $n_b = 3$ and large value of n_a. Systems such as snowflakes could correspond to large $\hbar$ zoom ups of molecular systems having subgroup of rotation group as a symmetry group in the standard sense of the word.

   The model of graviton de-coherence constructed in [?] allows to conclude that the fractionization of Planck constant has interpretation as a transition to chaos in the sense that fundamental frequencies are replaced with its sub-harmonics corresponding to the divisor of $\hbar/\hbar_0 = r/s$. The more digits are needed to represent r/s, the higher the complexity of the system. Period doubling bifurcations leading to chaos represent a special case of this. Living matter is indeed a system at the boundary of chaos (or rather, complexity) and order and larger values of n_b would give rise to the complexity having as a signature weak charge and spin fractionization effects.

5. Coverings alone are enough to produce rational number valued spectrum for $\hbar$, and one must keep in mind that the applications of theory do not allow to decide whether only singular factor spaces are really needed.

3.4.6 What happens in QHE?

This picture suggest following description for what would happens in QHE in TGD Universe.

1. Light-like 3-surfaces - locally random light-like orbits of partonic 2-surfaces- are identifiable as very tiny wormhole throats in the case of elementary particles. This is the case for electrons in particular. Partonic surfaces can be also large, even macroscopic, and the size scales up in the scaling of Planck constant. To avoid confusion, it must be emphasized that light-likeness is with respect to the induced metric and does not imply expansion with light velocity in Minkowski

space since the contribution to the induced metric implying light-likeness typically comes from CP_2 degrees of freedom. Strong classical gravitational fields are present near the wormhole throat. Second important point is that regions of space-time surface with Euclidian signature of the induced metric are implied: CP_2 type extremals representing elementary particles and having light-like random curve as CP_2 projection represents basic example of this. Hence rather exotic gravitational physics is predicted to manifest itself in everyday length scales.

2. The simplest identification for what happens in the phase transition to quantum Hall phase is that the end of wire carrying the Hall current corresponds to a partonic 2-surface having a macroscopic size. The electrons in the current correspond to similar 2-surfaces but with size of elementary particle for the ordinary value of Planck constant. As the electrons meet the end of the wire, the tiny wormhole throats of electrons suffer topological condensation to the boundary. One can say that one very large elementary particle having very high electron number is formed.

3. The end of the wire forms part of a spherical surface surrounding the tip of the CD involved so that electrons can become carriers of anomalous electric and magnetic charges.

4. Chern- Simons action for Kähler gauge potential is Abelian. This raises the question whether the representations of the number theoretical braid group are also Abelian. Since there is evidence for non-Abelian anyons, one might argue that this means a failure of the proposed approach. There are however may reasons to expect that braid group representations are non-Abelian. The action is for induced Kähler form rather than primary Maxwell field, $U(1)$ gauge symmetry is transformed to a dynamical symmetry (symplectic transformations of CP_2 representing isometries of WCW and definitely non-Abelian), and the particles of the theory belong to the representations of electro-weak and color gauge groups naturally defining the representations of braid group.

5. The finite subgroups of $SU(2)$ defining covering and factor groups are in the general case non-commutative subgroups of $SU(2)$ since the hierarchies of coverings and factors spaces are assumed to correspond to the two hierarchy of Jones inclusions to which one can assign ADE Lie algebras by McKay correspondence. The ADE Lie algebras define effective gauge symmetries having interpretation in terms of finite measurement resolution described in terms of Jones inclusion so that extremely rich structures are expected.

6. The proposed model allows charge and spin fractionization also for IQHE since $\hbar/\hbar_0 = 1$ holds true for $n_a = n_b$. There is also infinite number of anyonic states predicting a given value of ν ($(n_a, n_b) \rightarrow k(n_a, n_b)$ symmetry).

An interesting challenge is to relate concrete models of QHE to the proposed description. Here only some comments about Laughlin's wave function are made.

1. In the description provided by Lauglin wave function FQHE results from a minimization of Coulomb energy. In TGD framework the tunneling to the page of H with m sheets of covering has the same effect since the density of electrons is reduced by $1/m$ factor.

2. The formula $\nu \propto e^2 N_e / e \int BdS$ with scaling up of magnetic flux by $\hbar/\hbar_0 = m$ implies effective fractional filling factor. The scaling up of magnetic flux results from the presence of m sheets carrying magnetic field with same strength. Since the N_e electrons are shared between m sheets, the filling factor is fractional when one restricts the consideration to single sheet as one indeed does.

3. Laughlin wave function makes sense for $\nu = 1/m$, m odd, and is m:th power of the many electron wave function for IQHE and expressible as the product $\prod_{i<j}(z_i - z_j)^m$, where z represents complex coordinate for the anyonic plane. The relative orbital angular momenta of electrons satisfy $L_z \geq m$ if the value of Planck constant is standard. If Laughlin wave function makes sense also in TGD framework, then m:th power implies that many-electron wave function is singlet with respect to Z_m acting in covering and the value of relative angular momentum indeed satisfies $L_z \geq m\hbar_0$ just as in Laughlin's theory.

Bibliography

Online books about TGD

[1] M. Pitkänen (2006), *Topological Geometrodynamics: Overview*. http://tgd.wippiespace.com/public_html/tgdview/tgdview.html.

[2] M. Pitkänen (2006), *Quantum Physics as Infinite-Dimensional Geometry*. http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html.

[3] M. Pitkänen (2006), *Physics in Many-Sheeted Space-Time*. http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html.

[4] M. Pitkänen (2006), *Quantum TGD*. http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html.

[5] M. Pitkänen (2006), *TGD as a Generalized Number Theory*. http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html.

[6] M. Pitkänen (2006), *p-Adic length Scale Hypothesis and Dark Matter Hierarchy*. http://tgd.wippiespace.com/public_html/paddark/paddark.html.

[7] M. Pitkänen (2006), *TGD and Fringe Physics*. http://tgd.wippiespace.com/public_html/freenergy/freenergy.html.

Online books about TGD inspired theory of consciousness and quantum biology

[8] M. Pitkänen (2006), *Bio-Systems as Self-Organizing Quantum Systems*. http://tgd.wippiespace.com/public_html/bioselforg/bioselforg.html.

[9] M. Pitkänen (2006), *Quantum Hardware of Living Matter*. http://tgd.wippiespace.com/public_html/bioware/bioware.html.

[10] M. Pitkänen (2006), *TGD Inspired Theory of Consciousness*. http://tgd.wippiespace.com/public_html/tgdconsc/tgdconsc.html.

[11] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*. http://tgd.wippiespace.com/public_html/genememe/genememe.html.

[12] M. Pitkänen (2006), *TGD and EEG*. http://tgd.wippiespace.com/public_html/tgdeeg/tgdeeg/tgdeeg.html.

[13] M. Pitkänen (2006), *Bio-Systems as Conscious Holograms*. http://tgd.wippiespace.com/public_html/hologram/hologram.html.

[14] M. Pitkänen (2006), *Magnetospheric Consciousness*. http://tgd.wippiespace.com/public_html/magnconsc/magnconsc.html.

[15] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*. http://tgd.wippiespace.com/public_html/magnconsc/mathconsc.html.

References to the chapters of books

[A8] The chapter *Was von Neumann Right After All* of [?].
http://tgd.wippiespace.com/public_html/tgdview/tgdview.html#vNeumann.

[A9] The chapter *Does TGD Predict the Spectrum of Planck Constants?* of [?].
http://tgd.wippiespace.com/public_html/tgdview/tgdview.html#Planck.

[B1] The chapter *Identification of the Configuration Space Kähler Function* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#kahler.

[B2] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part I* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#compl1.

[B3] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part II* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#compl2.

[B4] The chapter *Configuration Space Spinor Structure* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#cspin.

[C2] The chapter *Construction of Quantum Theory: Symmetries* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#quthe.

[C3] The chapter *Construction of Quantum Theory: S-matrix* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#towards.

[C4] The chapter *Category Theory and Quantum tGD* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#categorynew.

[D4] The chapter *The Relationship Between TGD and GRT* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#tgdgrt.

[D5] The chapter *Cosmic Strings* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#cstrings.

[D6] The chapter *TGD and Cosmology* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#cosmo.

[D7] The chapter *TGD and Astrophysics* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#astro.

[D8] The chapter *Quantum Astrophysics* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#qastro.

[E1] The chapter *TGD as a Generalized Number Theory: p-Adicization Program* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visiona.

[E2] The chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visionb.

[E3] The chapter *TGD as a Generalized Number Theory: Infinite Primes* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visionc.

[E5] The chapter *p-Adic Physics: Physical Ideas* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#phblocks.

[E9] The chapter *Topological Quantum Computation in TGD Universe* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#tqc.

[L5] The chapter *DNA as Topological Quantum Computer* of [?].
http://tgd.wippiespace.com/public_html/genememe/genememe.html#dnatqc.

[16] M. Pitkänen (2008), *About the Nature of Time.* http://tgd.wippiespace.com/public_html/articles/time.pdf.

Topological quantum computation

[17] Paul Parsons (2004) , *Dancing the Quantum Dream*, New Scientist 24. January. www.newscientist.com/hottopics.

[18] M. Freedman, A. Kitaev, M. Larson, Z. Wang (2001), www.arxiv.of/quant-ph/0101025.

[19] M. Freedman, H. Larsen, and Z. Wang (2002), *A modular functor which is universal for quantum computation*, Found. Comput. Math. 1, no 2, 183-204. Comm. Math. Phys. 227, no 3, 605-622. quant-ph/0001108.

[20] M. H. Freedman (2001), *Quantum Computation and the localization of Modular Functors*, Found. Comput. Math. 1, no 2, 183-204.

[21] M. H. Freedman (1998), *P/NP, and the quantum field computer*, Proc. Natl. Acad. Sci. USA 95, no. 1, 98-101.

[22] A. Kitaev (1997), Annals of Physics, vol 303, p.2. See also *Fault tolerant quantum computation by anyons*, quant-ph/9707021.

[23] A. Kitaev (1997), *Quantum computations: algorithms and error correction*, Russian Math. Survey, 52:61, 1191-1249.

[24] C. N. Yang, M. L. Ge (1989), *Braid Group, Knot Theory, and Statistical Mechanics*, World Scientific.

[25] C. Kassel (1995), *Quantum Groups*, Springer Verlag.

[26] E. Witten 1989), *Quantum field theory and the Jones polynomial*, Comm. Math. Phys. 121 , 351-399.

[27] S. Sawin (1995), *Links, Quantum Groups, and TQFT's*, q-alg/9506002.

Quantum Hall effect

[28] S. M. Girvin (1999), *Quantum Hall Effect, Novel Excitations and Broken Symmetries*, cond-mat/9907002.

[29] R. B. Laughlin (1990), Phys. Rev. Lett. 50, 1395.

[30] F. Wilzek (1990), *Fractional Statistics and Anyon Super-Conductivity*, World Scientific.

[31] *Fractional quantum Hall Effect*, http://en.wikipedia.org/wiki/Fractional_quantum_Hall_effect.
Fractional Quantum Hall Effect, http://www.warwick.ac.uk/~phsbm/fqhe.htm.

[32] J.K. Jain(1989), Phys. Rev. Lett. 63, 199.

[33] J. B. Miller *et al*(2007), *Fractional Quantum Hall effect in a quantum point contact at filling fraction 5/2*, arXiv:cond-mat/0703161v2.

[34] A. Wojs, K.-S. Yi and J. J. Quinn (2003), *Fractional Quantum Hall States of Composite Fermions*, cond-mat/0312290.

[35] D. Monroe (2008), *Know Your Anyons*. New Scientist, vol 200, No 2676.

[36] M. Dolev, M. Heiblum, V. Umansky, Ady Stern, and D. Mahalu Nature (2008), *Observation of a quarter of an electron charge at the = 5/2 quantum Hall state*. Nature, vol 452, p 829.

[37] C. Nayak and F. Wilczek (1996), *2n-quasihole states realize 2^{n-1}-dimensional spinor braiding statistics in paired quantum Hall states*, Nucl. Phys. B479, 529-533.

[38] G. Moore and N. Read (1991), *Non-Abelians in the fractional quantum Hall effect*, Nucl. Phys. B360, 362-396.

[39] M. de Wild Propitius and F. A. Bais (1996), *Discrete Gauge Theories*, hep-th/9511201.

Anomalies

[40] D. Da Roacha and L. Nottale (2003), *Gravitational Structure Formation in Scale Relativity*, astro-ph/0310036.

Part II

QUANTUM COMPUTATION IN TGD UNIVERSE

Chapter 4

Topological Quantum Computation in TGD Universe

4.1 Introduction

Quantum computation is perhaps one of the most rapidly evolving branches of theoretical physics. TGD inspired theory of consciousness has led to new insights about quantum computation and in this chapter I want to discuss these ideas in a more organized manner.

There are three mathematically equivalent approaches to quantum computation [?]: quantum Turing machines, quantum circuits, and topological quantum computation (TQC). In fact, the realization that TGD Universe seems to be ideal place to perform TQC [?, ?] served as the stimulus for writing this chapter.

Quite generally, quantum computation allows to solve problems which are NP hard, that is the time required to solve the problem increases exponentially with the number of variables using classical computer but only polynomially using quantum computer. The topological realization of the computer program using so called braids resulting when threads are weaved to 2-dimensional patterns is very robust so that de-coherence, which is the basic nuisance of quantum computation, ceases to be a problem. More precisely, the error probability is proportional to $exp(-\alpha l)$, where l is the length scale characterizing the distance between strands of the braid [?] .

4.1.1 Evolution of basic ideas of quantum computation

The notion of quantum computation goes back to Feynman [?] who demonstrated that some computational tasks boil down to problems of solving quantum evolution of some physical system, say electrons scattering from each other. Many of these computations are NP hard, which means that the number of computational steps required grows exponentially with the number of variables involved so that they become quickly unsolvable using ordinary computers. A quicker manner to do the computation is to make a physical experiment. A further bonus is that if you can solve one NP hard problem, you can solve many equivalent NP hard problems. What is new that quantum computation is not deterministic so that computation must be carried out several times and probability distribution for the outcomes allows to deduce the answer. Often however the situation is such that it is easy to check whether the outcome provides the sought for solution.

Years later David Deutch [?] transformed Feynman's ideas into a detailed theory of quantum computation demonstrating how to encode quantum computation in a quantum system and researchers started to develop applications. One of the key factors in the computer security is cryptography which relies on the fact that the factorization of large integers to primes is a NP hard problem. Peter Shor [?] discovered an algorithm, which allows to carry out the factorization in time, which is exponentially shorter than by using ordinary computers. A second example is problem of searching a particular from a set of N items, which requires time proportional to N classically but quantally only a time proportional to $\sqrt{N}$.

The key notion is quantum entanglement which allows to store information in the relationship between systems, qubits in the simplest situation. This means that information storage capacity increases exponentially as a function of number of qubits rather than only linearly. This explains why NP hard problems which require time increasing exponentially with the number of variables can be solved using quantum computers. It also means exponentially larger information storage capacity than possible classically.

Recall that there are three equivalent approaches to quantum computation: quantum Turing machine, quantum circuits, and topology based unitary modular functor approach. In quantum circuit approach the unitary time evolution defining the quantum computation is assumed to be decomposable to a product of more elementary operations defined by unitary operators associated with quantum gates. The number of different gates needed is surprisingly small: only 1-gates generating unitary transformations of single qubit, and a 2-gate representing a transformation which together with 1-gates is able to generate entanglement are needed to generate a dense subgroup of unitary group $U(2^n)$ in the case of n-qubit system. 2-gate could be conditional NOT (CNOT). The first 1-gate can induce a phase factor to the qubit 0 and do nothing for qubit 1. Second 1-gate could form orthogonal square roots of bits 1 and 0 as superposition of 1 and 0 with identical probabilities.

The formal definition of the quantum computation using quantum circuit is as a computation of the value of a Boolean function of n Boolean arguments, for instance the k:th bit of the largest prime factor of a given integer. The unitary operator U is constructed as a product of operators associated with the basic gates. It is said that the function coding the problem belongs to the class BQP (function is computable with a bounded error in polynomial time) if there exists a classical polynomial-time (in string length) algorithm for specifying the quantum circuit. The first qubit of the outgoing n-qubit is measured and the probability that the the value is 0 determines the value of the bit to be calculated. For instance, for $p(0) \geq 2/3$ the bit is 0 and for $p(0) \geq 1/3$ the bit is 1. The evaluation of the outcome is probabilistic and requires a repeat the computation sufficiently many times.

The basic problem of quantum computation is the extremely fragility of the physical qubit (say spin). The fragility can be avoided by mapping q-bits to logical qubits realized as highly entangled states of many qubits and quantum error-correcting codes and fault tolerant methods [?, ?, ?] rely on this.

The space W of the logical qubits is known as a code space. The sub-space W of physical states of space $Y = V \otimes V..... \otimes V$ is called k-code if the effect of any k-local operator (affecting only k tensor factors of Y linearly but leaving the remaining factors invariant) followed by an orthogonal projection to W is multiplication by scalar. This means that k-local operator modify the states only in directions orthogonal to W.

These spaces indeed exist and it can be shown that the quantum information coded in W is not affected by the errors operating in fewer than $k/2$ of the n particles. Note that $k = 3$ is enough to guarantee stability with respect to 1-local errors. In this manner it is possible to correct the errors by repeated quantum measurements and by a suitable choice of the sub-space eliminate the errors due to the local changes of qubits by just performing a projection of the state back to the subspace (quantum measurement).

If the the error magnitude is below so called accuracy threshold, arbitrary long quantum computations are reliable. The estimates for this constant vary between 10^{-5} and 10^{-3}. This is beyond current technologies. Error correction is based on the representation of qubit as a logical qubit defined as a state in a linear sub-space of the tensor product of several qubits.

Topological quantum computation [?] provides an alternative approach to minimize the errors caused by de-coherence. Conceptually the modular functor approach [?, ?] is considerably more abstract than quantum circuit approach. Unitary modular functor is the S-matrix of a topological quantum field theory. It defines a unitary evolution realizing the quantum computation in macroscopic topological ground states degrees of freedom. The nice feature of this approach is that the notion of physical qubit becomes redundant and the code space defined by the logical qubits can be represented in terms topological and thus non-local degrees of freedom which are stable against local perturbations as required.

4.1.2 Quantum computation and TGD

Concerning quantum computation [?] in general, TGD TGD inspired theory of consciousness provides several new insights.

Quantum jump as elementary particle of consciousness and cognition

Quantum jump is interpreted as a fundamental cognitive process leading from creative confusion via analysis to an experience of understanding, and involves TGD counterpart of the unitary process followed by state function reduction and state preparation. One can say that quantum jump is the elementary particle of consciousness and that selves consists of sequences of quantum jump just like hadrons, nuclei, atoms, molecules,... consist basically of elementary particles. Self loses its consciousness when it generates bound state entanglement with environment. The conscious experience of self is in a well-defined sense a statistical average over the quantum jump during which self exists. During macro-temporal quantum coherence during macro-temporal quantum coherence a sequence of quantum jumps integrates effectively to a single moment of consciousness and effectively defines single unitary time evolution followed by state function reduction and preparation. This means a fractal hierarchy of consciousness very closely related to the corresponding hierarchy for bound states of elementary particles and structure formed from them.

Negentropy Maximization Principle guarantees maximal entanglement

Negentropy Maximization Principle is the basic dynamical principle constraining what happens in state reduction and self measurement steps of state preparation. Each self measurement involves a decomposition of system into two parts. The decomposition is dictated by the requirement that the reduction of entanglement entropy in self measurement is maximal. Self measurement can lead to either unentangled state or to entangled state with density matrix which is proportional to unit matrix (density matrix is the observable measured). In the latter case maximally entangled state typically involved with quantum computers results as an outcome. Hence Nature itself would favor maximally entangling 2-gates. Note however that self measurement occurs only if it increases the entanglement negentropy.

Number theoretical information measures and extended rational entanglement as bound state entanglement

The emerging number theoretical notion of information allows to interpret the entanglement for which entanglement probabilities are rational (or belong to an extension of rational numbers defining a finite extension of p-adic numbers) as bound state entanglement with positive information content. Macro-temporal quantum coherence corresponds to a formation of bound entanglement stable against state function reduction and preparation processes.

Spin glass degeneracy, which is the basic characteristic of the variational principle defining space-time dynamics, implies a huge number of vacuum degrees of freedom, and is the key mechanism behind macro-temporal quantum coherence. Spin glass degrees of freedom are also ideal candidates qubit degrees of freedom. As a matter fact, p-adic length scale hierarchy suggests that qubit represents only the lowest level in the hierarchy of qupits defining p-dimensional state spaces, p prime.

Time mirror mechanism and negative energies

The new view about time, in particular the possibility of communications with and control of geometric past, suggests the possibility of circumventing the restrictions posed by time for quantum computation. Iteration based on initiation of quantum computation again and again in geometric past would make possible practically instantaneous information processing.

Space-time sheets with negative time orientation carry negative energies. Also the possibility of phase conjugation of fermions is strongly suggestive. It is also possible that anti-fermions possess negative energies in phases corresponding to macroscopic length scales. This would explain matter-antimatter asymmetry in elegant manner. Zero energy states would be ideal for quantum computation purposes and could be even created intentionally by first generating a p-adic surface representing the state and then transforming it to a real surface.

The most predictive and elegant cosmology assumes that the net quantum numbers of the Universe vanish so that quantum jumps would occur between different kinds of vacua. Crossing symmetry makes this option almost consistent with the idea about objective reality with definite conserved total quantum numbers but requires that quantum states of 3-dimensional quantum theory represent S-matrices of 2-dimensional quantum field theory. These quantum states are thus about something. The boundaries of space-time surface are most naturally light-like 3-surfaces space-time surface and are limiting cases of space-like 3-surface and time evolution of 2-surface. Hence they would act naturally as space-time correlates for the reflective level of consciousness.

4.1.3 TGD and the new physics associated with TQC

TGD predicts the new physics making possible to realized braids as entangled flux tubes and also provides a detailed model explaing basic facts about anyons.

Topologically quantized magnetic flux tube structures as braids

Quantum classical correspondence suggests that the absolute minimization of Kähler action corresponds to a space-time representation of second law and that the 4-surfaces approach asymptotically space-time representations of systems which do not dissipate anymore. The correlate for the absence of dissipation is the vanishing of Lorentz 4-force associated with the induced Kähler field. This condition can be regarded as a generalization of Beltrami condition for magnetic fields and leads to very explicit general solutions of field equations [?].

The outcome is a general classification of solutions based on the dimension of CP_2 projection. The most unstable phase corresponds to $D = 2$-dimensional projection and is analogous to a ferromagnetic phase. $D = 4$ projection corresponds to chaotic demagnetized phase and $D = 3$ is the extremely complex but ordered phase at the boundary between chaos and order. This phase was identified as the phase responsible for the main characteristics of living systems [?, ?]. It is also ideal for quantum computations since magnetic field lines form extremely complex linked and knotted structures.

The flux tube structures representing topologically quantized fields, which have $D = 3$ -dimensional CP_2 projection, are knotted, linked and braided, and carry an infinite number of conserved topological charges labelled by representations of color group. They seem to be tailor-made for defining the braid structure needed by TQC. The boundaries of the magnetic flux tubes correspond to light-like 3-surfaces with respect to the induced metric (being thus metrically 2-dimensional and allowing conformal invariance) and can be interpreted either as 3-surfaces or time-evolutions of 2-dimensional systems so that S-matrix of 2-D system can be coded into the quantum state of conformally invariant 3-D system.

Anyons in TGD

TGD suggests a many-sheeted model for anyons used in the modelling of quantum Hall effect [?, ?, ?]. Quantum-classical correspondence requires that dissipation has space-time correlates. Hence a periodic motion should create a permanent track in space-time. This kind of track would be naturally magnetic flux tube like structure surrounding the Bohr orbit of the charged particle in the magnetic field. Anyon would be electron plus its track.

The magnetic field inside magnetic flux tubes impels the anyons to the surface of the magnetic flux tube and a highly conductive state results. The partial fusion of the flux tubes along their boundaries makes possible delocalization of valence anyons localized at the boundaries of flux tubes and implies a dramatic increase of longitudinal conductivity. When magnetic field is gradually increased the radii of flux tubes and the increase of the net flux brings in new flux tubes. The competition of these effects leads to the emergence of quantum Hall plateaus and sudden increase of the longitudinal conductivity σ_{xx}.

The simplest model explains only the filling fractions $\nu = 1/m$, m odd. The filling fractions $\nu = N/m$, m odd, require a more complex model. The transition to chaos means that periodic orbits become gradually more and more non-periodic: closed orbits fail to close after the first turn and do so only after N 2π rotations. Tracks would become N-branched surfaces. In N-branched space-time the single-valued analytic two particle wave functions $(\xi_k - \xi_l)^m$ of Laughlin [?] correspond to multiple valued wave functions $(z_k - z_l)^{m/N}$ at its M^4_+ projection and give rise to a filling fraction $\nu = N/m$. The filling fraction $\nu = N/m$, m even, requires composite fermions [?]. Anyon tracks can indeed contain up to $2N$ electrons if both directions of spin are allowed so that a rich spectroscopy is predicted: in particular anyonic super-conductivity becomes possible by 2-fermion composites. The branching gives rise to Z_N-valued topological charge.

One might think that fractional charges could be only apparent and result from the multi-branched character as charges associated with a single branch. This does not seem to be the case. Rather, the fractional charges result from the additional contribution of the vacuum Kähler charge of the anyonic flux tube to the charge of anyon. For $D = 3$ Kähler charge is topologized in the sense that the charge density is proportional to the Chern-Simons. Also anyon spin could become genuinely fractional due to the vacuum contribution of the Kähler field to the spin. Besides electronic anyons also anyons associated with various ions are predicted and certain strange experimental findings about fractional Larmor frequencies of proton in water environment [?, ?] have an elegant explanation in terms of protonic anyons with $\nu = 3/5$. In this case however the magnetic field was weaker than the Earth's magnetic field so that the belief that anyons are possible only in systems carrying very strong magnetic fields would be wrong.

In TGD framework anyons as punctures of plane would be replaced by wormhole like tubes connecting different points of the boundary of the magnetic flux tube and are predicted to always appear as pairs as they indeed do. Detailed arguments demonstrate that TGD anyons are for $N = 4$ ($\nu = 4/m$) ideal for realizing the scenario of [?] for TQC.

The TGD inspired model of non-Abelian anyons is consistent with the model of anyons based on spontaneous symmetry breaking of a gauge symmetry G to a discrete sub-group H dynamically [?]. The breaking of electro-weak gauge symmetry for classical electro-weak gauge fields occurs at the space-time sheets associated with the magnetic flux tubes defining the strands of braid. Symmetry breaking implies that elements of holonomy group span H. This group is also a discrete subgroup of color group acting as isotropy group of the many-branched surface describing anyon track inside the magnetic flux tube. Thus the elements of the holonomy group are mapped to a elements of discrete subgroup of the isometry group leading from branch to another one but leaving many-branched surface invariant.

Witten-Chern-Simons action and light-like 3-surfaces

The magnetic field inside magnetic flux tube expels anyons at the boundary of the flux tube. In quantum TGD framework light-like 3-surfaces of space-time surface and future light cone are in key role since they define causal determinants for Kähler action. They also provide a universal manner to satisfy boundary conditions. Hence also the boundaries of magnetic flux tube structures could be light like surfaces with respect to the induced metric of space-time sheet and would be somewhat like black hole horizons. By their metric 2-dimensionality they allow conformal invariance and due the vanishing of the metric determinant the only coordinate invariant action is Chern-Simons action associated Kähler gauge potential or with the induced electro-weak gauge potentials.

The quantum states associated with the light-like boundaries would be naturally "self-reflective" states in the sense that they correspond to S-matrix elements of the Witten-Chern-Simons topological field theory. Modular functors could results as restriction of the S-matrix to ground state degrees of freedom and Chern-Simons topological quantum field theory is a promising candidate for defining the modular functors [?, ?].

Braid group B_n is isomorphic to the first homotopy group of the configuration space $C_n(R^2)$ of n particles. $C_n(R^2)$ is $((R^2)_n - D)/S_n$, where D is the singularity represented by the configurations in which the positions of 2 or more particles. and be regarded also as the configuration associated with plane with $n + 1$ punctures with $n + 1$:th particle regarded as inert. The infinite order of the braid group is solely due to the 2-dimensionality. Hence the dimension $D = 4$ for space-time is unique also in the sense that it makes possible TQC.

4.1.4 TGD and TQC

Many-sheeted space-time concept, the possibility of negative energies, and Negentropy Maximization Principle inspire rather concrete ideas about TQC. NMP gives good hopes that the laws of Nature could take care of building fine-tuned entanglement generating 2-gates whereas 1-gates could be reduced to 2-gates for logical qubits realized using physical qubits realized as Z^4 charges and not existing as free qubits.

Only 2-gates are needed

The entanglement of qubits is algebraic which corresponds in TGD Universe to bound state entanglement. Negentropy Maximization Principle implies that maximal entanglement results automatically in quantum jump. This might saves from the fine-tuning of the 2-gates. In particular, the maximally entangling Yang-Baxter R-matrix is consistent with NMP.

TGD suggests a rather detailed physical realization of the model of [?] for anyonic quantum computation. The findings about strong correlation between quantum entanglement and topological entanglement are apparently contradicted by the Temperley-Lie representations for braid groups using only single qubit. The resolution of the paradox is based on the observation that in TGD framework batches containing anyon Cooper pair (AA) and single anyon (instead of two anyons as in the model of [?]) allow to represent single qubit as a logical qubit, and that mixing gate and phase gate can be represented as swap operations s_1 and s_2. Hence also 1-gates are induced by the purely topological 2-gate action, and since NMP maximizes quantum entanglement, Nature itself would take care of the fine-tuning also in this case. The quantum group representation based on $q = exp(i2\pi/5)$ is the simplest representation satisfying various constraints and is also physically very attractive. [?, ?].

TGD makes possible zero energy TQC

TGD allows also negative energies: besides phase conjugate photons also phase conjugate fermions and anti-fermions are possible, and matter-antimatter asymmetry might be only apparent and due to the ground state for which fermion energies are positive and anti-fermion energies negative.

This would make in principle possible zero energy topological quantum computations. The least one could hope wold be the performance of TQC in doubles of positive and negative energy computations making possible error detection by comparison. The TGD based model for anyon computation however leads to expect that negative energies play much more important role.

The idea is that the quantum states of light-like 3-surfaces represent 2-dimensional time evolutions (in particular modular functors) and that braid operations correspond to zero energy states with initial state represented by positive energy anyons and final state represented by negative energy anyons. The simplest manner to realize braid operations is by putting positive *resp.* negative energy anyons near the boundary of tube T_1 *resp.* T_2. Opposite topological charges are at the ends of the magnetic threads connecting the positive energy anyons at T_1 with the negative energy anyons at T_2. The braiding for the threads would code the quantum gates physically.

Before continuing a humble confession is in order: I am not a professional in the area of quantum information science. Despite this, my hope is that the speculations below might serve as an inspiration for real professionals in the field and help them to realize that TGD Universe provides an ideal arena for quantum information processing, and that the new view about time, space-time, and information suggests a generalization of the existing paradigm to a much more powerful one.

4.2 Existing view about topological quantum computation

In the sequel the evolution of ideas related to topological quantum computation, dance metaphor, and the idea about realizing the computation using a system exhibiting so called non-Abelian Quantum Hall effect, are discussed.

4.2.1 Evolution of ideas about TQC

The history of the TQC paradigm is as old as that of QC and involves the contribution of several Fields Medalists. At 1987 to-be Fields Medalist Vaughan Jones [?] demonstrated that the von Neumann algebras encountered in quantum theory are related to the theory of knots and allow to distinguish between very complex knots. Vaughan also demonstrated that a given knot can be characterized in terms an array of bits. The knot is oriented by assigning an arrow to each of its points and projected to a plane. The bit sequence is determined by a sequence of bits defined by the self-intersections of the knot's projection to plane. The value of the bit in a given intersection changes when the orientation of either line changes or when the line on top of another is moved under it. Since the logic operations performed by the gates of computer can be coded to matrices consisting of 0s and 1s, this means that tying a know can encode the logic operations necessary for computation.

String theorist Edward Witten [?, ?], also a Fields Medalist, connected the work of Jones to quantum physics by showing that performing measurements to a system described by a 3-dimensional topological quantum field theory defined by non-Abelian Chern-Simons action is equivalent with performing the computation that a particular braid encodes. The braids are determined by linked word lines of the particles of the topological quantum field theory. What makes braids and quantum computation so special is that the coding of the braiding pattern to a bit sequence gives rise to a code, which corresponds to a code solving NP hard problem using classical computer.

1989 computer scientist Alexei Kitaev [?] demonstrated that Witten's topological quantum field theory could form a basis for a computer. Then Fields Medalist Michael Freedman entered the scene and in collaboration with Kitaev, Michael Larson and Zhenghan Wang developed a vision of how to build a topological quantum computer [?, ?] using system exhibiting so called non-Abelian quantum Hall effect [?].

The key notion is Z_4 valued topological charge which has values 1 and 3 for anyons and 0 and 2 for their Cooper pairs. For a system of $2n$ non-Abelian anyon pairs created from vacuum there are n-1 anyon qubits analogous to spin . The notion of physical qubit is not needed at all and logical qubit is coded to the topological charge of the anyon Cooper pair. The basic idea is to utilize entanglement between Z_4 valued topological charges to achieve quantum information storage stable against decoherence. The swap of neighboring strands of the braid is the topological correlate of a 2-gate which as such does not generate entanglement but can give rise to a transformation such as CNOT. When combined with 1-gates taking square root of qubit and relative phase, this 2-gate is able to generate $U(2^n)$.

The swap can be represented as the so called braid Yang-Baxter R-matrix characterizing also the deviation of quantum groups from ordinary groups [?]. Quite generally, all unitary Yang-Baxter R-matrices are entangling when combined with square root gate except for special values of parameters characterizing them and thus there is a rich repertoire of topologically realized quantum gates. Temperley-Lieb representation provides a 1-qubit representation for swaps in 3-braid system [?, ?]. The measurement of qubit reduces to the measurement of the topological charge of the anyon Cooper pair: in the case that it vanishes (qubit 0) the anyon Cooper pair can annihilate and this serves as the physical signature.

4.2.2 Topological quantum computation as quantum dance

Although topological quantum computation involves very abstract and technical mathematical thinking, it is possible to illustrate how it occurs by a very elegant metaphor. With tongue in cheek one could say that topological quantum computation occurs like a dance. Dancers form couples and in this dancing floor the partners can be also of same sex. Dancers can change their partners. If the partners are of the same sex, they define bit 1 and if they are of opposite sex they define bit 0.

To simplify things one can arrange dancers into a row or several rows such that neighboring partners along the row form a couple. The simplest situation corresponds to a single row of dancers able to make twists of 180 degrees permuting the dancers and able to change the partner to a new one any time. Dance corresponds to a pattern of tracks of dancers at the floor. This pattern can be lifted to a three-dimensional pattern introducing time as a third dimension. When one looks the tracks of a row of dancers in this 2+1-dimensional space-time, one finds that the tracks of the dancers form a complex weaved pattern known as braiding. The braid codes for the computation. The braiding consists of primitive swap operations in which two neighboring word lines twist around each other.

The values of the bits giving the result of the final state of the calculation can be detected since there is something very special which partners with opposite sex can do and do it sooner or later. Just by looking which pairs do it allows to deduce the values of the bits. The alert reader has of course guessed already now that the physical characterization for the sex is as a Z^4 valued topological charge, which is of opposite sign for the different sexes forming Cooper pairs, and that the thing that partners of opposite sex can do is to annihilate! All that is needed to look for those pairs which annihilate after the dance evening to detect the 0s in the row of bits. The coding of the sex to the sign of the topological charge implies also robustness.

It is however essential that the value of topological charge for a given particle in the final state is not completely definite (this is completely general feature of all quantum computations). One can tell only with certain probability that given couple in the final state is male-female or male-male or female-female and the probabilities in question code for the braid pattern in turn coding for quantum logic circuit. Hence one must consider an ensemble of braid calculations to deduce these probabilities.

The basic computational operation permuting the neighboring topological charges is topological so that the program represented by the braiding pattern is very stable against perturbations. The values of the topological charges are also stable. Hence the topological quantum computation is a very robust process and immune to quantum de-coherence even in the standard physics context.

4.2.3 Braids and gates

In order to understand better how braids define gates one must introduce some mathematical notions related to the braids.

Braid groups

Artin introduced the braid groups bearing his name as groups generated by the elements, which correspond to the cross section between neighboring strands of the braid. The definition of these groups is discussed in detail in [**?**]. For a braid having $n+1$ strands the Artin group B_{n1} has n generators s_i. The generators satisfy certain relations. Depending on whether the line coming from left is above the the line coming from right one has s_i or s_i^{-1}. The elements s_i and s_j commute for $i<j$ and $i>j+1$: $s_is_j=s_js_i$, which only says that two swaps which do not have common lines commute. For $i=j$ and $i=j+1$ commutativity is not assumed and this correspond to the situation in which the swaps act on common lines.

As already mentioned, Artin's braid group B_n is isomorphic with the homotopy group $\pi_1((R^2)^n/S_{n+1})$ of plane with $n+1$ punctures. B_n is infinite-dimensional because the conditions $s_i^2=1$ added to the defining relations in the case of permutation group S_n are not included. The infinite-dimensionality of homotopy groups reflects the very special topological role of 2-dimensional spaces.

One must consider also variants of braid groups encountered when all particles in question are not identical particles. The reason is that braid operation must be replaced by a 2π rotation of particle A around B when the particles are not identical.

1. Consider first the situation in which all particles are non-identical. The first homotopy group of $R^2)^n-D$, where D represents points configurations for which two or more points are identical is identical with the colored braid group B_n^c defined by $n+1$ punctures in plane such that $n+1$:th is passive (punctures are usually imagined to be located on line). Since particles are not identical the braid operation must be replaced by monodromy in which i:th particle makes 2π rotation around j:th particle. This group has generators

$$\gamma_{ij}=s_i...s_{j-2}s_{j-1}^2s_{j-2}...s_i^{-1} \quad ,i<j \quad , \tag{4.2.1}$$

   and can be regarded as a subgroup of the braid group.

2. When several representatives of a given particle species are present the so called partially colored braid group B_n^{pc} is believed to describe the situation. For pairs of identical particles the generators are braid generators and for non-identical particles monodromies appear as generators. It will be found later that in case of anyon bound states, the ordinary braid group with the assumption that braid operation can lead to a temporary decay and recombination of anyons to a bound state, might be a more appropriate model for what happens in braiding.

3. When all particles are identical, one has the braid group B_n, which corresponds to the fundamental group of $C_n(R^2)=((R^2)^n-D)/S_n$. Division by S_n expresses the identicality of particles.

Extended Artin's group

Artin's group can be extended by introducing any group G and forming its tensor power $G^{\otimes^n} = G \otimes ... \otimes G$ by assigning to every strand of the braid group G. The extended group is formed from elements of $g_1 \otimes g_2 .... \otimes g_n$ and s_i by posing additional relations $g_i s_j = s_j g_i$ for $i < j$ and $i > j+1$. The interpretation of these relations is completely analogous to the corresponding one for the Artin's group.

If G allows representation in some space V one can look for the representations of the extended Artin's group in the space $V^{\otimes^n}$. In particular, unitary representations are possible. The space in question can also represent physical states of for instance anyonic system and the element g_i associated with the lines of the braid can represent the unitary operators characterizing the time development of the strand between up to the moment when it experiences a swap operation represented by s_i after this operation g_i becomes $s_i g_i s_i^{-1}$.

Braids, Yang-Baxter relations, and quantum groups

Artin's braid groups can be related directly to the so called quantum groups and Yang-Baxter relations. Yang-Baxter relations follow from the relation $s_1 s_2 s_1 = s_2 s_1 s_2$ by noticing that these operations permute the lines 123 of the braid to the order 321. By assigning to a swap operation permuting i:th and j:th line group element R_{ij} when i:th line goes over the j:th line, and noticing that $R_{ij}i$ acts in the tensor product $V_i \otimes V_j$, one can write the relation for braids in a form

$$R_{32}R_{13}R_{12} = R_{12}R_{13}R_{23} \quad .$$

Braid Yang-Baxter relations are equivalent with the so called algebraic Yang-Baxter relations encountered in quantum group theory. Algebraic R can be written as $R_a = RS$, where S is the matrix representing swap operation as a mere permutation. For a suitable choice R_a provides the fundamental representations for the elements of the quantum group $SL(n)_q$ when V is n-dimensional.

The equations represent n^6 equations for n^4 unknowns and are highly over-determined so that solving the equations is a difficult challenge. Equations have symmetries which are obvious on basis of the topological interpretation. Scaling and automorphism induced by linear transformations of V act as symmetries, and the exchange of tensor factors in $V \otimes V$ and transposition are symmetries as also shift of all indices by a constant amount (using modulo N arithmetics).

Unitary R-matrices

Quite a lot is known about the general solutions of the Yang-Baxter equations and for $n = 2$ the general unitary solutions of equations is known [**?**]. All of these solutions are entangling and define thus universal 1-gates except for certain parameter values.

The first solution is

$$R = \frac{1}{\sqrt{2}} \left(\begin{array}{cccc} 1 & \cdot & \cdot & 1 \\ \cdot & 1 & -1 & \cdot \\ \cdot & 1 & 1 & \cdot \\ -1 & \cdot & \cdot & 1 \end{array} \right) \tag{4.2.2}$$

and contains no free parameters (dots denote zeros). This R-matrix is strongly entangling. Note that the condition $R^8 = 1$ is satisfied. The defining relations for Artin's braid group allow also more general solutions obtained by multiplying R with an arbitrary phase factor. This would mean that $R^8 = 1$ constraint is not satisfied anymore. One can argue that over-all phase does not matter: on the other hand, the over all phase is visible in knot invariants defined by the trace of R.

The second and third solution come as families labelled four phases a, b, c and d:

$$R'(a,b,c,d) = \frac{1}{\sqrt{2}} \left(\begin{array}{cccc} a & \cdot & \cdot & \cdot \\ \cdot & & b & \cdot \\ \cdot & c & \cdot & \cdot \\ \cdot & \cdot & \cdot & d \end{array} \right)$$

$$R''(a,b,c,d) = \frac{1}{\sqrt{2}} \left(\begin{array}{cccc} \cdot & \cdot & \cdot & a \\ \cdot & b & \cdot & \cdot \\ \cdot & \cdot & c & \cdot \\ d & \cdot & \cdot & \cdot \end{array} \right) \tag{4.2.3}$$

These matrices are not as such entangling. The products $U_1 \otimes U_2 R V_1 \otimes V_2$, where U_i and V_i are 2×2 unitary matrices, are however entangling matrices and thus act as universal gates for $ad - bc \neq 0$ guaranteeing that the state $a|11\rangle + b|10\rangle + |01\rangle + |00\rangle$ is entangled.

It deserves to be noticed that the swap matrix

$$S = R'(1,1,1,1) = \frac{1}{\sqrt{2}} \left(\begin{array}{cccc} 1 & \cdot & \cdot & \cdot \\ \cdot & & 1 & \cdot \\ \cdot & 1 & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{array} \right) \tag{4.2.4}$$

permuting the qubits does not define universal gate. This is understandable since in this representation of braid group reduces it to permutation group and situation becomes completely classical.

One can write all solutions R of braid Yang-Baxter equation in the form $R = R_a$, where R_a is the solution of so called algebraic Yang-Baxter equation. The interpretation is that the swap matrix S represents the completely classical part of the swap operation since it acts as a mere permutation whereas R_a represents genuine quantum effects related to the swap operation.

In the article of Kauffman [?] its is demonstrated explicitly how to construct CNOT gate as a product MRN, where M and N are products of single particle gates. This article contains also a beautiful discussion about how the traces of the unitary matrices defined by the braids define knot invariants. For instance, the matrix R satisfies $R^8 = 1$ so that the invariants constructed using R as 2-gate cannot distinguish between knots containing n and $n + 8k$ sub-sequent swaps. Note however that the multiplication of R with a phase factor allows to get rid of the 8-periodicity.

Knots, links, braids, and quantum 2-gates

In [?] basic facts about knots, links, and their relation to braids are discussed. Knot diagrams are introduced, the so called Reidermeister moves and homeomorphisms of plane as isotopies of knots and links are discussed. Also the notion of braid closure producing knots or links is introduced together with the theorem of Markov stating that any knot and link corresponds to some (not unique) braid. Markov moves as braid deformations leaving corresponding knots and links invariant are discussed and it the immediate implication is that traces of the braid matrices define knot invariants. In particular, the traces of the unitary matrices defined by R-matrix define invariants having same value for the knots and links resulting in the braid closure.

In [?] the state preparation and quantum measurement allowing to deduce the absolute value of the trace of the unitary matrix associated with the braid defining the quantum computer is discussed as an example how quantum computations could occur in practice. The braid in question is product of the braid defining the invariant and trivial braid with same number n of strands. The incoming state is maximally entangled state formed $\sum_n |n\rangle \otimes |n\rangle$, where n runs over all possible bit sequences defined by the tensor product of n qubits. Quantum measurement performs a projection to this state and from the measurements it is possible to deduce the absolute value of the trace defining the knot invariant.

4.2.4 About quantum Hall effect and theories of quantum Hall effect

Using the dance metaphor for TQC, the system must be such that it is possible to distinguish between the different sexes of dancers. The proposal of [?] is that the system exhibiting so called non-Abelian Quantum Hall effect [?, ?] could make possible realization of the topological computation.

The most elegant models of quantum Hall effect are in terms of anyons regarded as singularities due to the symmetry breaking of gauge group G down to a finite sub-group H, which can be also non-Abelian. Concerning the description of the dynamics of topological degrees of freedom topological quantum field theories based on Chern-Simons action are the most promising approach.

Quantum Hall effect

Quantum Hall effect [?, ?] occurs in 2-dimensional systems, typically a slab carrying a longitudinal voltage V causing longitudinal current j. A magnetic field orthogonal to the slab generates a transversal current component j_T by Lorentz force. j_T is proportional to the voltage V along the slab and the dimensionless coefficient is known as transversal conductivity. Classically the coefficients is proportional ne/B, where n is 2-dimensional electron density and should have a continuous spectrum. The finding that came as surprise was that the change of the coefficient as a function of parameters like magnetic field strength and temperature occurred as discrete steps of same size. In integer quantum Hall effect the coefficient is quantized to $2\nu\alpha$, $\alpha = e^2/4\pi$, such that ν is integer.

Later came the finding that also smaller steps corresponding to the filling fraction $\nu = 1/3$ of the basic step were present and could be understood if the charge of electron would have been replaced with $\nu = 1/3$ of its ordinary value. Later also QH effect with wide large range of filling fractions of form $\nu = k/m$ was observed.

The model explaining the QH effect is based on pseudo particles known as anyons [?, ?]. According to the general argument of [?] anyons have fractional charge νe. Also the TGD based model for fractionization to be discussed later suggests that the anyon charge should be νe quite generally. The braid statistics of anyon is believed to be fractional so that anyons are neither bosons nor fermions. Non-fractional statistics is absolutely essential for the vacuum degeneracy used to represent logical qubits.

In the case of Abelian anyons the gauge potential corresponds to the vector potential of the divergence free velocity field or equivalently of incompressible anyon current. For non-Abelian anyons the field theory defined by Chern-Simons action is free field theory and in well-defined sense trivial although it defines knot invariants. For non-Abelian anyons situation would be different. They would carry non-Abelian gauge charges possibly related to a symmetry breaking to a discrete subgroup H of gauge group [?] each of them defining an incompressible hydrodynamical flow. Non-Abelian QH effect has not yet been convincingly demonstrated experimentally. According to [?] the anyons associated with the filling fraction $\nu = 5/2$ are a good candidate for non-Abelian anyons and in this case the charge of electron is reduced to $Q = 1/4$ rather than being $Q = \nu e$.

Non-Abelian anyons [?, ?] are always created in pairs since they carry a conserved topological charge. In the model of [?] this charge should have values in 4-element group Z_4 so that it is conserved only modulo 4 so that charges +2 and -2 are equivalent as are also charges 3 and -1. The state of n anyon pairs created from vacuum can be show to possess 2^{n-1}-dimensional vacuum degeneracy [?]: later a TGD based argument for why this is the case is constructed. When two anyons fuse the 2^{n-1}-dimensional state space decomposes to 2^{n-2}-dimensional tensor factors corresponding to anyon Cooper pairs with topological charges 2 and 0. The topological "spin" is ideal for representing logical qubits. Since free topological charges are not possible the notion of physical qubit does not make sense (note the analogy with quarks). The measurement of topological qubit reduces to a measurement of whether anyon Cooper pair has vanishing topological charge or not.

Quantum Hall effect as a spontaneous symmetry breaking down to a discrete subgroup of the gauge group

The system exhibiting quantum Hall effect is effectively 2-dimensional. Fractional statistics suggests that topological defects, anyons, allowing a description in terms of the representations of the homotopy group of $((R^2)^n - D)/S_n$. The gauge theory description would be in terms of spontaneous symmetry breaking of the gauge group G to a finite subgroup H by a Higgs mechanism [?, ?]. This would make all gauge degrees of freedom massive and leave only topological degrees of freedom. What is unexpected that also non-Abelian topological degrees of freedom are in principle possible. Quantum Hall effect is Abelian or non-Abelian depending on whether the group H has this property.

In the symmetry breaking $G \rightarrow H$ the non-Abelian gauge fluxes defined as non-integrable phase factors $Pexp(i \oint A_\mu dx^\mu)$ around large circles (surrounding singularities (so that field approaches a pure gauge configuration) are elements of the first homotopy group of G/H, which is H in the case that H is discrete group and G is simple. An idealized manner to model the situation [?] is to assume that the connection is pure gauge and defined by an H-valued function which is many-valued such that the values for different branches are related by a gauge transformation in H. In the general case a gauge transformation of a non-trivial gauge field by a multi-valued element of the gauge group would give rise to a similar situation.

One can characterize a given topological singularity magnetically by an element in conjugacy class C of H representing the transformation of H induced by a 2π rotation around singularity. The elements of C define states in given magnetic representation. Electrically the particles are characterized by an irreducible representations of the subgroup of $H_C \subset H$ which commutes with an arbitrarily chosen element of the conjugacy class C.

The action of $h(B)$ resulting on particle A when it makes a closed turn around B reduces in magnetic degrees of freedom to translation in conjugacy class combined with the action of element of H_C in electric degrees of freedom. Closed paths correspond to elements of the braid group $B_n(X^2)$ identifiable as the mapping class group of the punctured 2-surface X^2 and this means that symmetry breaking $G \rightarrow H$ defines a representation of the braid group. The construction of these representations is discussed in [?] and leads naturally via the group algebra of H to the so called quantum double $D(H)$ of H, which is a quasi-triangular Hopf algebra allowing non-trivial representations of braid group.

Anyons could be singularities of gauge fields, perhaps even non-Abelian gauge fields, and the latter ones could be modelled by these representations. In particular, braid operations could be represented using anyons.

Witten-Chern-Simons action and topological quantum field theories

The Wess-Zumino-Witten action used to model 2-dimensional critical systems consists of a 2-dimensional conformally invariant term for the chiral field having values in group G combined with 2+1-dimensional term defined as the integral of Chern-Simons 3-form over a 3-space containing 2-D space as its boundary. This term is purely topological and identifiable as winding number for the map from 3-dimensional space to G. The coefficient of this term is integer k in suitable normalization. k gives the value of central extension of the Kac-Moody algebra defined by the theory.

One can couple the chiral field $g(x)$ to gauge potential defined for some subgroup of G_1 of G. If the G_1 coincides with G, the chiral field can be gauged away by a suitable gauge transformation and the theory becomes purely topological Witten-Chern-Simons theory. Pure gauge field configuration represented either as flat gauge fields with non-trivial holonomy over homotopically non-trivial paths or as multi-valued gauge group elements however remain and the remaining degrees of freedom correspond to the topological degrees of freedom.

Witten-Chern-Simons theories are labelled by a positive integer k giving the value of central extension of the Kac-Moody algebra defined by the theory. The connection with Wess-Zumino-Witten theory come from the fact that the highest weight states associated with the representations of the Kac-Moody algebra of WZW theory are in one-one correspondence with the representations R_i possible for Wilson loops in the topological quantum field theory.

In the Abelian case case 2+1-dimensional Chern-Simons action density is essentially the inner product $A \wedge dA$ of the vector potential and magnetic field known as helicity density and the theory in question is a free field theory. In the non-Abelian case the action is defined by the 3-form

$$\frac{k}{4\pi} Tr \left(A \wedge (dA + \frac{2}{3} A \wedge A) \right)$$

and contains also interaction term so that the field theory defined by the exponential of the interaction term is non-trivial.

In topological quantum field theory the usual n-point correlation functions defined by the functional integral are replaced by the functional averages for $Diff^3$ invariant quantities defined in terms of non-integrable phase factors defined by ordered exponentials over closed loops. One can consider arbitrary number of loops which can be knotted, linked, and braided. These quantities define both knot and 3-manifold invariants (the functional integral for zero link in particular). The perturbative calculation of the quantum averages leads directly to the Gaussian linking numbers and infinite number of perturbative link and not invariants.

The experience gained from topological quantum field theories defined by Chern-Simons action has led to a very elegant and surprisingly simple category theoretical approach to the topological quantum field theory [?, ?] allowing to assign invariants to knots, links, braids, and tangles and also to 3-manifolds for which braids as morphisms are replaced with cobordisms. The so called modular Hopf algebras, in particular quantum groups $Sl(2)_q$ with q a root of unity, are in key role in this approach. Also the connection between links and 3-manifolds can be understood since closed, oriented, 3-manifolds can be constructed from each other by surgery based on links.

Witten's article [?] "Quantum Field Theory and the Jones Polynomial" is full of ingenious constructions, and for a physicist it is the easiest and certainly highly enjoyable manner to learn about knots and 3-manifolds. For these reasons a little bit more detailed sum up is perhaps in order.

1. Witten discusses first the quantization of Chern-Simons action at the weak coupling limit $k \rightarrow \infty$. First it is shown how the functional integration around flat connections defines a topological invariant for 3-manifolds in the case of a trivial Wilson loop. Next a canonical quantization is performed in the case $X^3 = \Sigma^2 \times R^1$: in the Coulomb gauge $A_3 = 0$ the action reduces to a sum of $n = dim(G)$ Abelian Chern-Simons actions with a non-linear constraint expressing the vanishing of the gauge field. The configuration space consists thus of flat non-Abelian connections, which are characterized by their holonomy groups and allows Kähler manifold structure.

2. Perhaps the most elegant quantal element of the approach is the decomposition of the 3-manifold to two pieces glued together along 2-manifold implying the decomposition of the functional integral to a product of functional integrals over the pieces. This together with the basic properties of Hilbert of complex numbers (to which the partition functions defined by the functional integrals over the two pieces belong) allows almost a miracle like deduction of the basic results about the behavior of 3-manifold and link invariants under a connected sum, and leads to the crucial skein relations allowing to calculate the invariants by decomposing the link step by step to a union of unknotted, unlinked Wilson loops, which can be calculated exactly for $SU(N)$. The decomposition by skein relations gives rise to a partition function like representation of invariants and allows to understand the connection between knot theory and statistical physics [?]. A direct relationship with conformal field theories and Wess-Zumino-Witten model emerges via Wilson loops associated with the highest weight representations for Kac Moody algebras.

3. A similar decomposition procedure applies also to the calculation of 3-manifold invariants using link surgery to transform 3-manifolds to each other, with 3-manifold invariants being defined as Wilson loops associated with the homology generators of these (solid) tori using representations R_i appearing as highest weight representations of the loop algebra of torus. Surgery operations are represented as mapping class group operations acting in the Hilbert space defined by the invariants for representations R_i for the original 3-manifold. The outcome is explicit formulas for the invariants of trivial knots and 3-manifold invariant of S^3 for $G = SU(N)$, in terms of which more complex invariants are expressible.

4. For $SU(N)$ the invariants are expressible as functions of the phase $q = exp(i2\pi/(k + N))$ associated with quantum groups. Note that for $SU(2)$ and $k = 3$, the invariants are expressible in terms of Golden Ratio. The central charge $k = 3$ is in a special position since it gives rise to $k + 1 = 4$-vertex representing naturally 2-gate physically. Witten-Chern-Simons theories define universal unitary modular functors characterizing quantum computations [?].

Chern-Simons action for anyons

In the case of quantum Hall effect the Chern-Simons action has been deduced from a model of electrons as a 2-dimensional incompressible fluid [?]. Incompressibility requires that the electron current has a vanishing divergence, which makes it analogous to a magnetic field. The expressibility of the current as a curl of a vector potential b, and a detailed study of the interaction Lagrangian leads to the identification of an Abelian Chern-Simons for b as a low energy effective action. This action is Abelian, whereas the anyonic realization of quantum computation would suggest a non-Abelian Chern-Simons action.

Non-Abelian Chern-Simons action could result in the symmetry breaking of a non-Abelian gauge group G, most naturally electro-weak gauge group, to a non-Abelian discrete subgroup H [?] so that states would be labelled by representations of H and anyons would be characterized magnetically H-valued non-Abelian magnetic fluxes each of them defining its own incompressible hydro-dynamical flow. As will be found, TGD predicts a non-Abelian Chern-Simons term associated with electroweak long range classical fields.

4.2.5 Topological quantum computation using braids and anyons

By the general mathematical results braids are able to code all quantum logic operations [?]. In particular, braids allow to realize any quantum circuit consisting of single particle gates acting on qubits and two particle gates acting on pairs of qubits. The coding of braid requires a classical computation which can be done in polynomial time. The coding requires that each dancer is able to remember its dancing history by coding it into its own state.

The general ideas are following.

1. The ground states of anyonic system characterize the logical qubits, One assumes non-Abelian anyons with Z_4 -valued topological charge so that a system of n anyon pairs created from vacuum allows 2^{n-1}-fold anyon degeneracy [?]. The system is decomposed into blocks containing one anyonic Cooper pair with $Q_T \in \{2, 0\}$ and two anyons with such topological charges that the net topological charge vanishes. One can say that the states $(0, 1-1)$ and $(0, -1, +1))$ represent logical qubit 0 whereas the states $(2, -1, -1)$ and $(2, +1, +1)$ represent logical qubit 1. This would suggest 2^2-fold degeneracy but actually the degeneracy is 2-fold.

   Free physical qubits are not possible and at least four particles are indeed necessarily in order to represent logical qubit. The reason is that the conservation of Z^4 charge would not allow mixing of qubits 1 and 0, in particular the Hadamard 1-gate generating square root of qubit would break the conservation of topological charge. The square root of qubit can be generated only if 2 units of topological charge is transferred between anyon and anyon Cooper pair. Thus qubits can be represented as entangled states of anyon Cooper pair and anyon and the fourth anyon is needed to achieve vanishing total topological charge in the batch.

2. In the initial state of the system the anyonic Cooper pairs have $Q_T = 0$ and the two anyons have opposite topological charges inside each block. The initial state codes no information unlike in ordinary computation but the information is represented by the braid. Of course, also more general configurations are possible. Anyons are assumed to evolve like free particles except during swap operations and their time evolution is described by single particle Hamiltonians.

   Free particle approximation fails when the anyons are too near to each other as during braid operations. The space of logical qubits is realized as k-code defined by the 2^{n-1} ground states, which are stable against local single particle perturbations for $k = 3$ Witten-Chern-Simons action. In the more general case the stability against n-particle perturbations with $n < [k/2]$ is achieved but the gates would become $[k/2]$-particle gates (for $k = 5$ this would give 6-particle vertices).

3. Anyonic system provides a unitary modular functor as the S-matrix associated with the anyon system whose time evolution is fixed by the pre-existing braid structure. What this means that the S-matrices associated with the braids can be multiplied and thus a unitary representation for the group formed by braids results. The vacuum degeneracy of anyon system makes this representation non-trivial. By the NP complexity of braids it is possible to code any quantum logic operation by a particular braid [?]. There exists a powerful approximation theorem allowing to achieve this coding classically in polynomial time [?]. From the properties of the R-matrices inducing gate operations it is indeed clear that two gates can be realized. The Hadamard 1-gate could be realized as 2-gate in the system formed by anyon Cooper pair and anyon.

4. In [?] the time evolution is regarded as a discrete sequence of modifications of single anyon Hamiltonians induced by swaps [?]. If the modifications define a closed loop in the space of Hamiltonians the resulting unitary operators define a representation of braid group in a dense

discrete sub-group of $U(2^n)$. The swap operation is 2-local operation acting like a 2-gate and induces quantum logical operation modifying also single particle Hamiltonians. What is important that this modification maps the space of the ground states to a new one and only if the modifications correspond to a closed loop the final state is in the same code space as the initial state. What time evolution does is to affect the topological charges of anyon Cooper pairs representing qubits inside the 4-anyon batches defined by the braids.

In quantum field theory the analog but not equivalent of this description would be following. Quite generally, a given particle in the final state has suffered a unitary transformation, which is an ordered product consisting of two kinds of unitary operators. Unitary single particle operators $U_n = Pexp(i\int_{t_n}^{t_{n+1}} H_0 dt)$ are analogs of operators describing single qubit gate and play the role of anyon propagators during no-swap periods. Two-particle unitary operators $U_{swap} = Pexp(i\int H_{swap} dt)$ are analogous to four-particle interactions and describe the effect of braid operations inducing entanglement of states having opposite values of topological charge but conserving the net topological charge of the anyon pair. This entanglement is completely analogous to spin entanglement. In particular, the braid operation mixes different states of the anyon. The unitary time development operator generating entangled state of anyons and defined by the braid structure represents the operation performed by the quantum circuit and the quantum measurement in the final state selects a particular final state.

5. Formally the computation halts with a measurement of the topological charge of the left-most anyon Cooper pair when the outcome is just single bit. If decay occurs with sufficiently high probability it is concluded that the value of the computed bit is 0, otherwise 1.

4.3 General implications of TGD for quantum computation

TGD based view about time and space-time could have rather dramatic implications for quantum computation in general and these implications deserve to be discussed briefly.

4.3.1 Time need not be a problem for quantum computations in TGD Universe

Communication with and control of the geometric past is the basic mechanism of intentional action, sensory perception, and long term memory in TGD inspired theory of consciousness. The possibility to send negative energy signals to the geometric past allows also instantaneous computations with respect to subjective time defined by a sequence of quantum jumps. The physicist of year 2100 can induce the quantum jump to turn on the quantum computer at 2050 to perform a simulation of field equations defined by the absolute minimization of Kähler action and lasting 50 geometric years, and if this is not enough iterate the process by sending the outcome of computation back to the past where it defines initial values of the next round of iteration. Time would cease to be a limiting factor to computation.

4.3.2 New view about information

The notion of information is very problematic even in the classical physics and in quantum realm this concept becomes even more enigmatic. TGD inspired theory consciousness has inspired number theoretic ideas about quantum information which are still developing. The standard definition of entanglement entropy relies on the Shannon's formula: $S = -\sum_k p_k log(p_k)$. This entropy is always non-negative and tells that the best one can achieve is entanglement with zero entropy.

The generalization of the notion of entanglement entropy to the p-adic context however led to realization that entanglement for which entanglement probabilities are rational or in an extension of rational numbers defining a finite extension of p-adics allows a hierarchy of entanglement entropies S_p labelled by primes. These entropies are defined as $S_p = -\sum_k p_k log(|p_k|_p)$, where $|p_k|_p$ denotes the p-adic norm of probability. S_p can be negative and in this case defines a genuine information measure. For given entanglement probabilities S_p has a minimum for some value p_0 of prime p, and S_{p_0} could be taken as a measure for the information carried by the entanglement in question whereas entanglement in real and p-adic continua would be entropic. The entanglement with negative entanglement entropy is identified as bound state entanglement.

Since quantum computers by definition apply states for which entanglement coefficients belong to a finite algebraic extension of rational numbers, the resulting states, if ideal, should be bound states. Also finite-dimensional extensions of p-adic numbers by transcendentals are possible. For instance, the extension by the $p-1$ first powers of e (e^p is ordinary p-adic number in R_p). As an extension of rationals this extension would be discrete but infinite-dimensional. Macro-temporal quantum coherence can be identified as being due to bound state formation in appropriate degrees of freedom and implying that state preparation and state function reduction effectively ceases to occur in these degrees of freedom.

Macro-temporal quantum coherence effectively binds a sequence of quantum jumps to single quantum jump so that the effective duration of unitary evolution is stretched from about 10^4 Planck times to arbitrary long time span. Also quantum computations can be regarded as this kind of extended moments of consciousness.

4.3.3 Number theoretic vision about quantum jump as a building block of conscious experience

The generalization of number concept resulting when reals and various p-adic number fields are fused to a book like structure obtaining by gluing them along rational numbers common to all these number fields leads to an extremely general view about what happens in quantum jump identified as basic building block of conscious experience. First of all, the unitary process U generates a formal superposition of states belonging to different number fields including their extensions. Negentropy Maximization Principle [**?**] constrains the dynamics of state preparation and state function reduction following U so that the final state contains only rational or extended rational entanglement with positive information content. At the level of conscious experience this process can be interpreted as a cognitive process or analysis leading to a state containing only bound state entanglement serving as a correlate for the experience of understanding. Thus quantum information science and quantum theory of consciousness seem to meet each other.

In the standard approach to quantum computing entanglement is not bound state entanglement. If bound state entanglement is really the entanglement which is possible for quantum computer, the entanglement of qubits might not serve as a universal entanglement currency. That is, the reduction of the general two-particle entanglement to entanglement between N qubits might not be possible in TGD framework.

The conclusion that only bound state entanglement is possible in quantum computation in human time scales is however based on the somewhat questionable heuristic assumption that subjective time has the same universal rate, that is the average increment Δt of the geometric time in single quantum jump does not depend on the space-time sheet, and is of order CP_2 time about 10^4 Planck times. The conclusion could be circumvented if one assumes that Δt depends on the space-time sheet involved: for instance, instead of CP_2 time Δt could be of order p-adic time scale T_p for a space-time sheet labelled by p-adic prime p and increase like $\sqrt{p}$. In this case the unitary operator defining quantum computation would be simply that defining the unitary process U.

4.3.4 Dissipative quantum parallelism?

The new view about quantum jump implies that state function reduction and preparation process decomposes into a hierarchy of these processes occurring in various scales: dissipation would occur in quantum parallel manner with each p-adic scale defining one level in the hierarchy. At space-time level this would correspond to almost independent quantum dynamics at parallel space-time sheets labelled by p-adic primes. In particular, dissipative processes can occur in short scales while the dynamics in longer scales is non-dissipative. This would explain why the description of hadrons as dissipative systems consisting of quarks and gluons in short scales is consistent with the description of hadrons as genuine quantum systems in long scales. Dissipative quantum parallelism would also mean that thermodynamics at shorter length scales would stabilize the dynamics at longer length scales and in this manner favor scaled up quantum coherence.

NMR systems [**?**] might represent an example about dissipative quantum parallelism. Room temperature NMR (nuclear magnetic resonance) systems use highly redundant replicas of qubits which have very long coherence times. Quantum gates using radio frequency pulses to modify the spin evolution have been implemented, and even effective Hamiltonians have been synthesized. Quantum computations and dynamics of other quantum systems have been simulated and quantum error protocols have been realized. These successes are unexpected since the energy scale of cyclotron states is much below the thermal energy. This has raised fundamental questions about the power of quantum information processing in highly mixed states, and it might be that dissipative quantum parallelism is needed to explain the successes.

Magnetized systems could realize quite concretely the renormalization group philosophy in the sense that the magnetic fields due to the magnetization at the atomic space-time sheets could define a return flux along larger space-time sheets as magnetic flux quanta (by topological flux quantization) defining effective block spins serving as thermally stabilized qubits for a long length scale quantum parallel dynamics. For an external magnetic field $B \sim 10$ Tesla the magnetic length is $L \sim 10$ nm and corresponds to the p-adic length scale $L(k = 151)$. The induced magnetization is $M \sim n\mu^2 B/T$, where n is the density of nuclei and $\mu = ge/2m_p$ is the magnetic moment of nucleus. For solid matter density the magnetization is by a factor ~ 10 weaker than the Earth's magnetic field and corresponds to a magnetic length $L \sim 15$ μm: the p-adic length scale is around $L(171)$. For 10^{22} spins per block spin used for NMR simulations the size of block spin should be $\sim$ 1mm solid matter density so that single block spin would contain roughly 10^6 magnetization flux quanta containing 10^{16} spins each. The magnetization flux quanta serving as logical qubits could allow to circumvent the standard physics upper bound for scaling up of about 10 logical qubits [**?**].

4.3.5 Negative energies and quantum computation

In TGD universe space-times are 4-surfaces so that negative energies are possible due to the fact that the sign of energy depends on time orientation (energy momentum tensor is replaced by a collection of conserved momentum currents). This has several implications. Negative energy photons having phase conjugate photons as physical correlates of photons play a key role in TGD inspired theory of consciousness and living matter and there are also indications that magnetic flux tubes structures with negative energies are important.

Negative energies makes possible communications to the geometric past, and time mirror mechanism involving generation of negative energy photons is the key mechanism of intentional action and plays central role in the model for the functioning of bio-systems. In principle this could allow to circumvent the problems due to the time required by computation by initiating computation in the geometric past and iterating this process. The most elegant and predictive cosmology is that for which the net conserved quantities of the universe vanish due the natural boundary condition that nothing flows into the future light cone through its boundaries representing the moment of big bang.

Also topological quantum field theories describe systems for which conserved quantities associated with the isometries of space-time, such as energy and momentum, vanish. Hence the natural question is whether negative energies making possible zero energy states might also make possible also zero energy quantum computations.

Crossing symmetry and Eastern and Western views about what happens in scattering

The hypothesis that all physical states have vanishing net quantum numbers (Eastern view) forces to interpret the scattering events of particle physics as quantum jumps between different vacua. This interpretation is in a satisfactory consistency with the assumption about existence of objective reality characterized by a positive energy (Western view) if crossing symmetry holds so that configuration space spinor fields can be regarded as S-matrix elements between initial state defined by positive energy particles and negative energy state defined by negative energy particles. As a matter fact, the proposal for the S-matrix of TGD at elementary particle level relies on this idea: the amplitudes for the transition from vacuum to states having vanishing net quantum numbers with positive and negative energy states interpreted as incoming and outgoing states are assumed to be interpretable as S-matrix elements.

More generally, one could require that scattering between any pair of states with zero net energies and representing S-matrix allows interpretation as a scattering between positive energy states. This requirement is satisfied if their exists an entire self-reflective hierarchy of S-matrices in the sense that the S-matrix between states representing S-matrices S_1 and S_2 would be the tensor product $S_1 \otimes S_2$. At the observational level the experience the usual sequence of observations $|m_1\rangle \rightarrow |m_2\rangle ..... \rightarrow |m_n\rangle ...$ based on belief about objective reality with non-vanishing conserved net quantum numbers would correspond to a sequence $(|m_1 \rightarrow m_2\rangle \rightarrow |m_2 \rightarrow m_3\rangle ...$ between "self-reflective" zero energy states. These sequences are expected to be of special importance since the contribution of the unit matrix to S-matrix $S = 1+iT$ gives dominating contribution unless interactions are strong. This sequence would result in the approximation that $S_2 = 1 + iT_2$ in $S = S_1 \otimes S_2$ is diagonal. The fact that the scattering for macroscopic systems tends to be in forward direction would help to create the materialistic illusion about unique objective reality.

It should be possible to test whether the Eastern or Western view is correct by looking what happens strong interacting systems where the western view should fail. The Eastern view is consistent with the basic vision about quantum jumps between quantum histories having as a counterpart the change of the geometric past at space-time level.

Negative energy anti-fermions and matter-antimatter asymmetry

The assumption that space-time is 4-surface means that the sign of energy depends on time orientation so that negative energies are possible. Phase conjugate photons [**?**] are excellent candidates for negative energy photons propagating into geometric past.

Also the phase conjugate fermions make in principle sense and one can indeed perform Dirac quantization in four manners such that a) both fermions and anti-fermions have positive/negative energies, b) fermions (anti-fermions) have positive energies and anti-fermions (fermions) have negative energies. The corresponding ground state correspond to Dirac seas obtained by applying the product of a) all fermionic and anti-fermionic annihilation (creation) operators to vacuum, b) all fermionic creation (annihilation) operators and anti-fermionic annihilation (creation) operators to vacuum. The ground states of a) have infinite vacuum energy which is either negative or positive whereas the ground states of b) have vanishing vacuum energy. The case b) with positive fermionic and negative anti-fermionic energies could correspond to long length scales in which are matter-antisymmetric due to the effective absence of anti-fermions ("effective" meaning that no-one has tried to detect the negative energy anti-fermions). The case a) with positive energies could naturally correspond to the phase studied in elementary particle physics.

If gravitational and inertial masses have same magnitude and same sign, consistency with empirical facts requires that positive and negative energy matter must have been separated in cosmological length scales. Gravitational repulsion might be the mechanism causing this. Applying naively Newton's equations to a system of two bodies with energies $E_1 > 0$ and $-E_2 < 0$ and assuming only gravitational force, one finds that the sign of force for the motion in relative coordinates is determined by the sign of the reduced mass $-E_1E_2/(E_1 - E_2)$, which is negative for $E_1 > |E_2|$: positive masses would act repulsively on smaller negative masses. For $E_1 = -E_2$ the motion in the relative coordinate becomes free motion and both systems experience same acceleration which for E_1 corresponds to a repulsive force. The reader has probably already asked whether the observed acceleration of the cosmological expansion interpreted in terms of cosmological constant due to vacuum energy could actually correspond to a repulsive force between positive and negative energy matter.

It is possible to create pairs of positive energy fermions and negative energy fermions from vacuum. For instance, annihilation of photons and phase conjugate photons could create electron and negative energy positron pairs with a vanishing net energy. Magnetic flux tubes having positive and negative energies carrying fermions and negative energy positrons pairs of photons and their phase conjugates via fermion anti-fermion annihilation. The obvious idea is to perform zero energy topological quantum computations by using anyons of positive energy and anti-anyons of negative energy plus their Cooper pairs. This idea will be discussed later in more detail.

4.4 TGD based new physics related to topological quantum computation

The absolute minimization of Kähler action is the basic dynamical principle of space-time dynamics. For a long time it remained an open question whether the known solutions of field equations are building blocks of the absolute minima of Kähler action or represent only the simplest extremals one can imagine and perhaps devoid of any real significance. Quantum-classical correspondence meant a great progress in the understanding the solution spectrum of field equations. Among other things, this principle requires that the dissipative quantum dynamics leading to non-dissipating asymptotic self-organization patterns should have the vanishing of the Lorentz 4-force as space-time correlate. The absence of dissipation in the sense of vanishing of Lorentz 4-force is a natural correlate for the absence of dissipation in quantum computations. Furthermore, absolute minimization, if it is really a fundamental principle, should represent the second law of thermodynamics at space-time level. Of course, one cannot exclude the possibility that second law of thermodynamics at space-time level could replace absolute minimization as the basic principle.

The vanishing of Lorentz 4-force generalizes the so called Beltrami conditions [**?**, **?**] stating the vanishing of Lorentz force for purely magnetic field configurations and these conditions reduce in many cases to topological conditions. The study of classical field equations predicts three phases corresponding to non-vacuum solutions of field equations possessing vanishing Lorentz force. The dimension D of CP_2 projection of the space-time sheet serves as classifier of the phases.

1. $D = 2$ phase is analogous to ferro-magnetic phase possible in low temperatures and relatively simple, $D = 4$ phase is in turn analogous to a chaotic de-magnetized high temperature phase.

2. $D = 3$ phase represents spin glass phase, kind of boundary region between order and chaos possible in a finite temperature range and is an ideal candidate for the field body serving as a template for living systems. $D = 3$ phase allows infinite number of conserved topological charges having interpretation as invariants describing the linking of the magnetic field lines. This phase is also the phase in which topological quantum computations are possible.

4.4.1 Topologically quantized generalized Beltrami fields and braiding

From the construction of the solutions of field equations in terms topologically quantized fields it is obvious that TGD Universe is tailor made for TQC.

$D = 3$ phase allows infinite number of topological charges characterizing the linking of magnetic field lines

When space-time sheet possesses a $D = 3$-dimensional CP_2 projection, one can assign to it a non-vanishing and conserved topological charge characterizing the linking of the magnetic field lines defined by Chern-Simons action density $A \wedge dA/4\pi$ for induced Kähler form. This charge can be seen as classical topological invariant of the linked structure formed by magnetic field lines. For $D = 2$ the topological charge densities vanish identically, for $D = 3$ they are in general non-vanishing and conserved, whereas for $D = 4$ they are not conserved. The transition to $D = 4$ phase can thus be used to erase quantum computer programs realized as braids. The 3-dimensional CP_2 projection provides an economical manner to represent the braided world line pattern of dancers and would be the space where the 3-dimensional quantum field theory would be defined.

The topological charge can also vanish for $D = 3$ space-time sheets. In Darboux coordinates for which Kähler gauge potential reads as $A = P_k dQ^k$, the surfaces of this kind result if one has $Q^2 = f(Q^1)$ implying $A = f dQ^1$, $f = P_1 + P_2 \partial_{Q_1} Q^2$, which implies the condition $A \wedge dA = 0$. For these space-time sheets one can introduce Q^1 as a global coordinate along field lines of A and define the phase factor $exp(i \int A_\mu dx^\mu)$ as a wave function defined for the entire space-time sheet. This function could be interpreted as a phase of an order order parameter of super-conductor like state and there is a high temptation to assume that quantum coherence in this sense is lost for more general $D = 3$ solutions. Note however that in boundaries can still remain super-conducting and it seems that this occurs in the case of anyons.

Chern-Simons action is known as helicity in electrodynamics [**?**]. Helicity indeed describes the linking of magnetic flux lines as is easy to see by interpreting magnetic field as incompressible fluid flow having A as vector potential: $B = \nabla \times A$. One can write A using the inverse of $\nabla\times$ as $A = (1/\nabla\times)B$. The inverse is non-local operator expressible as

$$\frac{1}{\nabla\times} B(r) = \int dV' \frac{(r - r')}{|r - r'|^3} \times B(r') \ ,$$

as a little calculation shows. This allows to write $\int A \cdot B$ as

$$\int dV A \cdot B = \int dV dV' B(r) \cdot \left(\frac{(r-r')}{|r-r'|^3} \times B(r') \right) \quad ,$$

which is completely analogous to the Gauss formula for linking number when linked curves are replaced by a distribution of linked curves and an average is taken.

For $D = 3$ field equations imply that Kähler current is proportional to the helicity current by a factor which depends on CP_2 coordinates, which implies that the current is automatically divergence free and defines a conserved charge for $D = 3$-dimensional CP_2 projection for which the instanton density vanishes identically. Kähler charge is not equal to the helicity defined by the inner product of magnetic field and vector potential but to a more general topological charge.

The number of conserved topological charges is infinite since the product of any function of CP_2 coordinates with the helicity current has vanishing divergence and defines a topological charge. A very natural function basis is provided by the scalar spherical harmonics of $SU(3)$ defining Hamiltonians of CP_2 canonical transformations and possessing well defined color quantum numbers. These functions define and infinite number of conserved charges which are also classical knot invariants in the sense that they are not affected at all when the 3-surface interpreted as a map from CP_2 projection to M^4_+ is deformed in M^4_+ degrees of freedom. Also canonical transformations induced by Hamiltonians in irreducible representations of color group affect these invariants via Poisson bracket action when the $U(1)$ gauge transformation induced by the canonical transformation corresponds to a single valued scalar function. These link invariants are additive in union whereas the quantum invariants defined by topological quantum field theories are multiplicative.

Also non-Abelian topological charges are well-defined. One can generalize the topological current associated with the Kähler form to a corresponding current associated with the induced electro-weak gauge fields whereas for classical color gauge fields the Chern-Simons form vanishes identically. Also in this case one can multiply the current by CP_2 color harmonics to obtain an infinite number of invariants in $D = 3$ case. The only difference is that $A \wedge dA$ is replaced by $Tr(A \wedge (dA + 2A \wedge A/3))$.

There is a strong temptation to assume that these conserved charges characterize colored quantum states of the conformally invariant quantum theory as a functional of the light-like 3-surface defining boundary of space-time sheet or elementary particle horizon surrounding wormhole contacts. They would be TGD analogs of the states of the topological quantum field theory defined by Chern-Simons action as highest weight states associated with corresponding Wess-Zumino-Witten theory. These charges could be interpreted as topological counterparts of the isometry charges of configuration space of 3-surfaces defined by the algebra of canonical transformations of CP_2.

The interpretation of these charges as contributions of light-like boundaries to configuration space Hamiltonians would be natural. The dynamics of the induced second quantized spinor fields relates to that of Kähler action by a super-symmetry, so that it should define super-symmetric counterparts of these knot invariants. The anti-commutators of these super charges would contribute to configuration space metric a part which would define a Kähler magnetic knot invariant. These Hamiltonians and their super-charge counterparts would be responsible for the topological sector of quantum TGD.

The color partial wave degeneracy of topological charges inspires the idea that also anyons could move in color partial waves identifiable in terms of "rigid body rotation" of the magnetic flux tube of anyon in CP_2 degrees of freedom. Their presence could explain non-Abelianity of Chern-Simons action and bring in new kind bits increasing the computational capacity of the topological quantum computer. The idea about the importance of macroscopic color is not new in TGD context. The fact that non-vanishing Kähler field is always accompanied by a classical color field (proportional to it) has motivated the proposal that colored excitations in macroscopic length scales are important in living matter and that colors as visual qualia correspond to increments of color quantum numbers in quantum phase transitions giving rise to visual sensations.

Knot theory, 3-manifold topology, and $D = 3$ solutions of field equations

Topological quantum field theory (TQFT) [?] demonstrates a deep connection between links and 3-topology, and one might hope that this connection could be re-interpreted in terms of imbeddings of 3-manifolds to $H = M^4_+ \times CP_2$ as surfaces having 3-dimensional CP_2 projection, call it X^3 in the sequel. $D = 3$ suggests itself because in this case Chern-Simons action density for the induced Kähler field is generically non-vanishing and defines an infinite number of classical charges identifiable as Kähler magnetic canonical covariants invariant under $Diff(M^4_+)$. The field topology of Kähler magnetic field should be in a key role in the understanding of these invariants.

1. Could 3-D CP_2 projection of 3-surface provide a representation of 3-topology?

Witten-Chern-Simons theory for a given 3-manifold defines invariants which characterize both the topology of 3-manifold and the link. Why this is the case can be understood from the construction of 3-manifolds by drilling a tubular neighborhood of a link in S^3 and by gluing the tori back to get a new 3-manifolds. The links with some moves defining link equivalences are known to be in one-one correspondence with closed 3-manifolds and the axiomatic formulation of TQFT [**?**] as a modular functor clarifies this correspondence. The question is whether the CP_2 projection of the 3-surface could under some assumptions be represented by a link so that one could understand the connection between the links and topology of 3-manifolds.

In order to get some idea about what might happen consider the CP_2 projection X^3 of 3-surface. Assume that X^3 is obtained from S^3 represented as a 3-surface in CP_2 by removing from S^3 a tubular link consisting of linked and knotted solid tori $D^2\times S^1$. Since the 3-surface is closed, it must have folds at the boundaries being thus representable as a two-valued map $S^3 \rightarrow M^4_+$ near the folds. Assume that this is the case everywhere. The two halves of the 3-surface corresponding to the two branches of the map would be glued together along the boundary of the tubular link by identification maps which are in the general case characterized by the mapping class group of 2-torus. The gluing maps are defined inside the overlapping coordinate batches containing the boundary $S^1 \times S^1$ and are maps between the pairs (Ψ_i, Φ_i), $i = 1, 2$ of the angular coordinates parameterizing the tori.

Define longitude as a representative for the $a + nb$ of the homology group of the 2-torus. The integer n defines so called framing and means that the longitude twists n times around torus. As a matter fact, TQFT requires bi-framing: at the level of Chern-Simons perturbation theory bi-framing is necessary in order to define self linking numbers. Define meridian as the generator of the homology group of the complement of solid torus in S^3. It is enough to glue the carved torus back in such a manner that meridian is mapped to longitude and longitude to minus meridian. This map corresponds to the $SL(2,C)$ element

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} .$$

Also other identification maps defined by $SL(2,Z)$ matrices are possible but one can do using only this. Note that the two component $SL(2,Z)$ spinors defined as superpositions of the generators (a, b) of the homology group of torus are candidates for the topological correlates of spinors. In the gluing process the tori become knotted and linked when seen in the coordinates of the complement of the solid tori.

This construction would represent the link surgery of 3-manifolds in terms of CP_2 projections of 3-surfaces of H. Unfortunately this representation does not seem to be the only one. One can construct closed three-manifolds also by the so called Heegaard splitting. Remove from S^3 D_g, a solid sphere with g handles having boundary S_g, and glue the resulting surface with its oppositely oriented copy along boundaries. The gluing maps are classified by the mapping class group of S_g. Any closed orientable 3-manifold can be obtained by this kind of procedure for some value of g. Also this construction could be interpreted in terms of a fold at the boundary of the CP_2 projection for a 2-valued graph $S^3 \rightarrow M^4_+$. Whether link surgery representation and Heegaard splitting could be transformed to each other by say pinching D_g to separate tori is not clear to me.

When the graph $CP_2 \rightarrow M^4_+$ is at most 2-valued, the intricacies due to the imbedding of the 3-manifold are at minimum, and the link associated with the projection should give information about 3-topology and perhaps even characterize it. Also the classical topological charges associated with Kähler Chern-Simons action could give this kind of information.

2. Knotting and linking for 3-surfaces

The intricacies related to imbedding become important in small co-dimensions and it is of considerable interest to find what can happen in the case of 3-surfaces. For 1-dimensional links and knots the projection to a plane, the shadow of the knot, characterizes the link/knot and allows to deduce link and knot invariants purely combinatorially by gradually removing the intersection points and writing a contribution to the link invariant determined by the orientations of intersecting strands and by which of them is above the other. Thus also the generalization of knot and link diagrams is of interest.

Linking of m- and n-dimensional sub-manifolds of D-dimensional manifold H_D occurs when the condition $m+n=D-1$ holds true. The n-dimensional sub-manifold intersects $m+1$-dimensional surfaces having m-dimensional manifold as its boundary at discrete points, and it is usually not possible to remove these points by deforming the surfaces without intersections in some intermediate stage. The generalization of the link diagram results as a projection $D-1$-dimensional disk D^{D-1} of H_D.

3-surfaces link in dimension $D=7$ so that the linking of 3-surfaces occurs quite generally in time=constant section of the imbedding space. A link diagram would result as a projection to $E^2 \times CP_2$, E^2 a 2-dimensional plane: putting CP_2 coordinates constant gives ordinary link diagram in E^2. For magnetic flux tubes the reduction to 2-dimensional linking by idealizing flux tubes with 1-dimensional strings makes sense.

Knotting occurs in codimension 2 that is for an n-manifold imbedded in $D=n+2$-dimensional manifold. Knotting can be understood as follows. Knotted surface spans locally $n+1$-dimensional 2-sided n+1-disk D^{n+1} (disk for ordinary knot). The portion of surface going through D^{n+1} can be idealized with a 1-dimensional thread going through it and by $n+2=D$ knotting is locally linking of this 1-dimensional thread with n-dimensional manifold. N-dimensional knots define n+1-dimensional knots by so called spinning. Take an n-knot with the topology of sphere S^n such that the knotted part is above $n+1$-plane of $n+2$-dimensional space R^{n+2} ($z \geq 0$), cut off the part below plane ($z<0$), introduce an additional dimension (t) and make a 2π rotation for the resulting knot in $z-t$ plane. The resulting manifold is a knotted S^{n+1}. The counterpart of the knot diagram would be a projection to $n+1$-dimensional sub-manifold, most naturally disk D^{n+1}, of the imbedding space.

3-surfaces could become knotted under some conditions. Vacuum extremals correspond to 4-surfaces $X^4 \subset M^4_+ \times Y^2$ whereas the four-surfaces $X^4 \subset M^4_+ \times S^2$, S^2 homologically non-trivial geodesic sphere, define their own "sub-theory". In both cases 3-surfaces in time=constant section of imbedding space can get knotted in the sense that un-knotting requires giving up the defining condition temporarily. The counterpart of the knot diagram is the projection to $E^2 \times X^2$, $X^2 = Y^2$ or S^2, where E^2 is plane of M^4_+. For constant values of CP_2 coordinates ordinary knot diagram would result. Reduction to ordinary knot diagrams would naturally occur for $D=2$ magnetic flux tubes. The knotting occurs also for 4-surfaces themselves in $M^4_+ \times X^2$: knot diagram is now defined as projection to $E^3 \times X^2$.

3. Could the magnetic field topology of 3-manifold be able to mimic other 3-topologies?

In $D=3$ case the topological charges associated with Kähler Chern-Simons term characterize the linking of the field lines of the Kähler gauge potential A. What $dA \wedge A \neq 0$ means that field lines are linked and it is not possible to define a coordinate varying along the field lines of A. This is impossible even locally since the $dA \wedge A \neq 0$ condition is equivalent with non existence of a scalar functions k and Φ such that $\nabla\Phi = kA$ guaranteing that Φ would be the sought for global coordinate.

One can idealize the situation a little bit and think of a field configuration for which magnetic flux is concentrated at one-dimensional closed lines. The vector potential would in this case be simply $A = \nabla(k\Psi + l\Phi)$, where Ψ is an angle coordinate around the singular line and Φ a coordinate along the singular circle. In this idealized situation the failure to have a global coordinate would be due to the singularities of otherwise global coordinates along one-dimensional linked and knotted circles. The reason is that the field lines of A and B rotate helically around the singular circle and the points (x, y, z) with constant values of x, y are on a helix which becomes singular at z-axis. Since the replacement of a field configuration with a non-singular field configuration but having same field line topology does not affect the global field line topology, one might hope of characterizing the field topology by its singularities along linked and knotted circles also in the general case.

Just similar linked and knotted circles are used to construct 3-manifolds in the link surgery which would suggest that the singularities of the field line topology of X^3 code the non-trivial 3-topology resulting when the singularities are removed by link surgery. Physically the longitude defining the framing $a+nb$ would correspond to the field line of A making an $n2\pi$ twist along the singular circle. Meridian would correspond to a circle in the plane of B. The bi-framing necessitated by TQFT would have a physical interpretation in terms of the helical field lines of A and B rotating around the singular circle. At the level of fields the gluing operation would mean a gauge transformation such that the meridians would become the field lines of the gauge transformed A and being non-helical could be continued to the the interior of the glued torus without singularities. Simple non-helical magnetic torus would be in question.

This means that the magnetic field patterns of a given 3-manifold could mimic the topologies of other 3-manifolds. The topological mimicry of this kind would be a very robust manner to represent information and might be directly relevant to TQC. For instance, the computation of topological invariants of 3-manifold Y^3 could be coded by the field pattern of X^3 representing the link surgery producing the 3-manifold from S^3, and the physical realization of TQC program could directly utilize the singularities of this field pattern. Topological magnetized flux tubes glued to the back-ground 3-surface along the singular field lines of A could provide the braiding.

This mimicry could also induce transitions to the new topology and relate directly to 3-manifold surgery performed by a physical system. This transition would quite concretely mean gluing of simple $D = 2$ magnetic flux tubes along their boundaries to the larger $D = 3$ space-time sheet from which similar flux tube has been cut away.

4. A connection with anyons?

There is also a possible connection with anyons. Anyons are thought to correspond to singularities of gauge fields resulting in a symmetry breaking of gauge group to a finite subgroup H and are associated with homotopically non-trivial loops of $C_n = ((R^2)^n - D)/S_n$ represented as elements of H. Could the singularities of gauge fields relate to the singularities of the link surgery so that the singularities would be more or less identifiable as anyons? Could N-branched anyons be identified in terms of framings $a + Nb$ associated with the gluing map? $D = 3$ solutions allow the so called contact structure [**?**], which means a decomposition of the coordinates of CP_2 projection to a longitudinal coordinate s and a complex coordinate w. Could this decomposition generalize the notion of effective 2-dimensionality crucial for the notion of anyon?

5. What about Witten's quantal link invariants?

Witten's quantal link invariants define natural multiplicative factors of configuration space spinor fields identifiable as representations of two 2-dimensional topological evolution. In Witten's approach these invariants are defined as functional averages of non-integrable phase factors associated with a given link in a given 3-manifold. TGD does not allow any natural functional integral over gauge field configurations for a fixed 3-surface unless one is willing to introduce fictive non-Abelian gauge fields. Although this is not a problem as such, the representation of the invariants in terms of inherent properties of the 3-surface or corresponding 4-surfaces would be highly desirable.

Functional integral representation is not the only possibility. Quantum classical correspondence combined with topological field quantization implied by the absolute minimization of Kähler action generalizing Bohr rules to the field context gives hopes that the 3-surfaces themselves might be able to represent 3-manifold invariants classically. In $D = 3$ case the quantized exponents of Kähler-Chern-Simons action and $SU(2)_L$ Chern-Simons action could define 3-manifold invariants. These invariants would satisfy the obvious multiplicativity conditions and could correspond to the phase factors due to the framing dependence of Witten's invariants identifying the loops of surgery link as Wilson loops. These phase factors are powers of $U = exp(i2\pi c/24)$, where c is the central charge of the Virasoro representation defined by Kac Moody representation. One has $c = k \times dim(g)/(k + c_g/2)$, which gives $U = exp(i2\pi k/8(k + 2))$ for $SU(2)$. The dependence on k differs from what one might naively expect. For this reason, and also because the classical Wilson loops do not depend explicitly on k, the value of k appearing in Chern-Simons action should be fixed by the internal consistency and be a constant of Nature according to TGD. The guess is that k possesses the minimal value $k = 3$ allowing a universal modular functor for $SU(2)$ with $q = exp(i2\pi/5)$.

The loops associated with the topological singularities of the Kähler gauge potential (typically the center lines of helical field configurations) would in turn define natural Wilson loops, and since the holonomies around these loops are also topologically quantized, they could define invariants of 3-manifolds obtained by performing surgery around these lines. The behavior of the induced gauge fields should be universal near the singularities in the sense that the holonomies associated with the CP_2 projections of the singularities to CP_2 would be universal. This expectation is encouraged by the notion of quantum criticality in general and in particular, by the interpretation of $D = 3$ phase as a critical system analogous to spin glass.

The exponent of Chern-Simons action can explain only the phase factors due to the framing, which are usually regarded as an unavoidable nuisance. This might be however all that is needed. For the manifolds of type $X^2 \times S^1$ all link invariants are either equal to unity or vanish. Surgery would allow to build 3-manifold invariants from those of $S^2 \times S^1$. For instance, surgery gives the invariant $Z(S^3)$ in terms of $Z(S^2 \times S^1, R_i)$ and mapping class group action coded into the linking of the field lines.

Holonomies can be also seen as multi-valued $SU(2)_L$ gauge transformations and can be mapped to a multi-valued transformations in the $SU(2)$ subgroup of $SU(3)$ acting on 3-surface as a geometric transformations and making it multi-branched. This makes sense if the holonomies define a finite group so that the gauge transformation is finitely many-valued. This description might apply to the 3-manifold resulting in a surgery defined by the Wilson loops identifiable as branched covering of the initial manifold.

The construction makes also sense for the holonomies defined by the classical $SU(3)$ gauge fields defined by the projections of the isometry currents. Furthermore, the fact that any CP_2 Hamiltonian defines a conserved topological charge in $D = 3$ phase should have a deep significance. At the level of the configuration space geometry the finite-dimensional group defining Kac Moody algebra is replaced with the group of canonical transformations of CP_2. Perhaps one could extend the notion of Wilson loop for the algebra of canonical transformations of CP_2 so that the representations R_i of the gauge group would be replaced by matrix representations of the canonical algebra. That the trace of the identity matrix is infinite in this case need not be a problem since one can simply redefine the trace to have value one.

Braids as topologically quantized magnetic fields

$D = 3$ space-time sheets would define complex braiding structures with flux tubes possessing infinite number of topological charges characterizing the linking of field lines. The world lines of the quantum computing dancers could thus correspond to the flux tubes that can get knotted, linked, and braided. This idea conforms with the earlier idea that the various knotted and linked structures formed by linear bio-molecules define some kind of computer programs.

1. Boundaries of magnetic flux tubes as light-like 3-surfaces

Field equations for Kähler action are satisfied identically at boundaries if the boundaries of magnetic flux tubes (and space-time sheet in general) are light-like in the induced metric. In M^4_+ metric the flux tubes could look static structures. Light-likeness allows an interpretation of the boundary state either as a 3-dimensional quantum state or as a time-evolution of a 2-dimensional quantum state. This conforms with the idea that quantum computation is cognitive, self reflective process so that quantum state is about something rather than something. There would be no need to force particles to flow through the braid structure to build up time-like braid whereas for time-like boundaries of magnetic flux tubes a time-like braid results only if the topologically charged particles flow through the flux tubes with the same average velocity so that the length along flux tubes is mapped to time.

Using the terminology of consciousness theory, one could say that during quantum dance the dancers are in trance being entangled to a single macro-temporally coherent state which represents single collective consciousness, and wake up to individual dancers when the dance ends. Quantum classical correspondence suggests that the generation of bound state entanglement between dancers requires tangled join along boundaries bonds connecting the space-time sheets of anyons (braid of flux tubes again!): dancers share mental images whereas direct contact between magnetic flux tubes defining the braid is not necessary. The bound state entanglement between sub-systems of unentangled systems is made possible by the many-sheeted space-time. This kind of entanglement could be interpreted as entanglement not visible in scales of larger flux tubes so that the notion is natural in the philosophy based on the idea of length scale resolution.

2. How braids are generated?

The encoding of the program to a braid could be a mechanical process: a bundle of magnetic flux tubes with one end fixed would be gradually weaved to a braid by stretching and performing the needed elementary twists. The time to perform the braiding mechanically requires classical computer program and the time needed to carry out the braiding depends polynomially on the number of strands.

The process could also occur by a quantum jump generating the braided flux tubes in single flash and perhaps even intentionally in living systems (flux tubes with negative topological charge could have negative energy so that it would require no energy to generate the structure from vacuum). The interaction with environment could be used to select the desired braids. Also ensembles of braids might be imagined. Living matter might have discovered this mechanism and used int intentionally.

3. Topological quantization, many-sheetedness, and localization

Localization of modular functors is one of the key problems in topological quantum computation (see the article of Freedman [**?**]. For anyonic computation this would mean in the ideal case a decomposition of the system into batches containing 4 anyons each so that these anyon groups interact only during swap operations.

The role of topological quantization would be to select of a portion of the magnetic field defining the braid as a macroscopic structure. Topological field quantization realizes elegantly the requirement that single particle time evolutions between swaps involve no interaction with other anyons.

Also many-sheetedness is important. The (AA) pair and two anyons would correspond braids inside braids and as it turns out this gives more flexibility in construction of quantum computation since the 1-gates associated with logical qubits of 4-batch can belong to different representation of braid group than that associated with braiding of the batches.

4.4.2 Quantum Hall effect and fractional charges in TGD

In fractional QH effect anyons possess fractional electromagnetic charges. Also fractional spin is possible. TGD explains fractional charges as being due to multi-branched character of space-time sheets. Also the Z_n-valued topological charge associated with anyons has natural explanation.

Basic TGD inspired ideas about quantum Hall effect

Quantum Hall effect is observed in low temperature systems when the intensity of a strong magnetic field perpendicular to the current carrying slab is varied adiabatically. Classically quantum Hall effect can be understood as a generation of a transversal electric field, which exactly cancels the magnetic Lorentz force. This gives $E = -j \times B/ne$. The resulting current can be also understood as due to a drift velocity proportional to $E \times B$ generated in electric and magnetic fields orthogonal to each other and allowing to cancel Lorentz force. This picture leads to the classical expression for transversal Hall conductivity as $\sigma_{xy} = ne/B$. σ_{xy} should vary continuously as a function of the magnetic field and 2-dimensional electron density n.

In quantum Hall effect σ_{xy} is piece-wise constant and quantized with relative precision of about 10^{-10}. The second remarkable feature is that the longitudinal conductivity σ_{xx} is very high at plateaus: variations by 13 orders of magnitude are observed. The system is also very sensitive to small perturbations.

Consider now what these qualitative observations might mean in TGD context.

1. Sensitivity to small perturbations means criticality. TGD Universe is quantum critical and quantum criticality reduces to the spin glass degeneracy due to the enormous vacuum degeneracy of the theory. The $D = 2$ and $D = 3$ non-vacuum phases predicted by the generalized Beltrami ansatz are this in-stability might play important role in the effect.

2. The magnetic fields are genuinely classical fields in TGD framework, and for $D = 2$ proportional to induced Kähler magnetic field. The canonical symmetries of CP_2 act like $U(1)$ gauge transformations on the induced gauge field but are not gauge symmetries since canonical transformations change the shape of 3-surface and affect both classical gravitational fields and electro-weak and color gauge fields. Hence different gauges for classical Kähler field represent magnetic fields for which topological field quanta can have widely differing and physically non-equivalent shapes. For instance, tube like quanta act effectively as insulators whereas magnetic walls parallel to the slab act as conducting wires.

   Wall like flux tubes parallel to the slab perhaps formed by a partial fusion of magnetic flux tubes along their boundaries would give rise to high longitudinal conductivity. For disjoint flux tubes the motion would be around the flux tubes and the electrons would get stuck inside these tubes. By quantum criticality and by $D < 4$ property the magnetic flux tube structures are unstable against perturbations, in particular the variation of the magnetic field strength itself. The transitions from a plateau to a new one would correspond to the decay of the magnetic walls back to disjoint flux tubes followed by a generation of walls again so that conductivity is very high outside transition regions. The variation of any parameter, such as temperature, is expected to be able to cause similar effects implying dramatic changes in Hall conductivity.

The percolation model for the quantum Hall effect represents slab as a landscape with mountains and valleys and the varied external parameter, say B or free electron density, as the sea level. For the critical values of sea level narrow regions carrying so called edge states allow liquid to fill large regions appear and implies increase of conductivity. Obviously percolation model differs from the model based on criticality for which the landscape itself is highly fragile and a small perturbation can develop new valleys and mountains.

3. The effective 2-dimensionality implies that the solutions of Schrödinger equation of electron in external magnetic field are products of any analytic function with a Gaussian representing the ground state of a harmonic oscillator. Analyticity means that the kinetic energy is completely degenerate for these solutions. The Lauhglin ansatz for the state functions of electron in the external magnetic field is many-electron generalization of these solutions: the wave functions consists of products of terms of form $(z_i - z_j)^m$, m odd integer from Fermi statistics.

   The N-particle variant of Laughlin's ansatz allows to deduce that the system is incompressible. The key observation is that the probability density for the many-particle state has an interpretation as a Boltzmann factor for a fictive two-dimensional plasma in electric field created by constant charge density [**?**, **?**]. The probability density is extremely sensitive to the changes of the positions of electrons giving rise to the constant electron density. The screening of charge in this fictive plasma implies the filling fraction $\nu = 1/m$, m odd integer and requires charge fractionization $e \rightarrow e/m$. The explanation of the filling fractions $\nu = N/m$ would require multi-valued wave functions $(z_i - z_j)^{N/m}$. In single-sheeted space-time this leads to problems. TGD suggests that these wave functions are single valued but defined on N-branched surface.

   The degeneracy with respect to kinetic energy brings in mind the spin glass degeneracy induced by the vacuum degeneracy of the Kähler action. The Dirac equation for the induced spinors is not ordinary Dirac equation but super-symmetrically related to the field equations associated with Kähler action. Also it allows vacuum degeneracy. One cannot exclude the possibility that also this aspect is involved at deeper level.

4. The fractionization of charge in quantum Hall effect challenges the idea that charged particles of the incompressible liquid are electrons and this leads to the notion of anyon. Quantum-classical correspondence inspires the idea that although dissipation is absent, it has left its signature as a track associated with electron. This track is magnetic flux tube surrounding the classical orbit of electron and electron is confined inside it. This reduces the dissipative effects and explains the increase of conductivity. The rule that there is single electron state per magnetic flux quantum follows if Bohr quantization is applied to the radii of the orbits. The fractional charge of anyon would result from a contribution of classical Kähler charge of anyon flux tube to the charge of the anyon. This charge is topologized in $D = 3$ phase.

Anyons as multi-branched flux tubes representing charged particle plus its track

Electrons (in fact, any charged particles) moving inside magnetic flux tubes move along circular paths classically. The solutions of the field equations with vanishing Lorentz 4-force correspond to asymptotic patterns for which dissipation has already done its job and is absent. Dissipation has however definite effects on the final state of the system, and one can argue that the periodic motion of the charged particle has created what might called its "track". The track would be realized as a circular or helical flux tube rotating around field lines of the magnetic field. The corresponding cyclotron states would be localized inside tracks. Simplest tracks are circular ones and correspond to absence of motion in the direction of the magnetic field. Anyons could be identified as systems formed as particles plus the tracks containing them.

1. Many-branched tracks and approach to chaos

When the system approaches chaos one expects the the periodic circular tracks become non-periodic. One however expects that this process occurs in steps so that the tracks are periodic in the sense that they close after N 2π rotations with the value of N increasing gradually. The requirement that Kähler energy stays finite suggests also this. A basic example of this kind of track is obtained when the phase angles Ψ and Φ of complex CP_2 coordinates (ξ^1, ξ^2) have finitely multi-valued dependence on the coordinate ϕ of cylindrical coordinates: $(\Psi, \Phi) = (m_1/N, m_2/N)\phi)$. The space-sheet would be many-branched and it would take N turns of 2π to get back to the point were one started. The phase factors behave as a phase of a spinning particle having effective fractional spin $1/N$. I have proposed this kind of mechanism as an explanation of so called hydrino atoms claimed to have the spectrum of hydrogen atom but with energies scaled up by N^2 [**?**, **?**]. The first guess that N corresponds to m in $\nu = 1/m$ is wrong. Rather, N corresponds to N in $\nu = N/m$ which means many-valued Laughlin wave functions in single branched space-time.

Similar argument applies also in CP_2 degrees of freedom. Only the N-multiples of 2π rotations by CP_2 isometries corresponding to color hyper charge and color iso-spin would affect trivially the point of multi-branched surface. Since the contribution of Kähler charge to electromagnetic charge corresponds also to anomalous hyper-charge of spinor field in question, an additional geometric contribution to the anomalous hypercharge would mean anomalous electromagnetic charge.

It must be emphasized the fractionization of the isometry charges is only effective and results from the interpretation of isometries as space-time transformations rather than transformation rotating entire space-time sheet in imbedding space. Also classical charges are effectively fractionized in the sense that single branch gives in a symmetric situation a fraction of $1/n$ of the entire charge. Later it will be found that also a genuine fractionization occurs and is due to the classical topologized Kähler charge of the anyon track.

2. Modelling anyons in terms of gauge group and isometry group

Anyons can be modelled in terms of the gauge symmetry breaking $SU(2)_L \rightarrow H$, where H is discrete sub-group. The breaking of gauge symmetry results by the action of multi-valued gauge transformation $g(x)$ such that different branches of the multi-valued map are related by the action of H.

1. The standard description of anyons is based on spontaneous symmetry breaking of a gauge symmetry G to a discrete sub-group H dynamically [**?**]. The gauge field has suffered multi-valued gauge transformation such that the elements of H permute the different branches of $g(x)$. The puncture is characterized by the element of the H associated with the loop surrounding puncture. In the idealized situation that gauge field vanishes, the parallel translation of a particle around puncture affects the particle state, itself a representation of G, by the element of the homotopy $\pi_1(G/H) = H$ identifiable as non-Abelian magnetic charge. Thus holonomy group corresponds to homotopy group of G/H which in turn equals to H. This in turn implies that the infinite-dimensional braid group whose elements define holonomies in turn is represented in H.

2. In TGD framework the multi-valuedness of $g(x)$ corresponds to a many-branched character of 4-surface. This in turn induces a branching of both magnetic flux tube and anyon tracks describable in terms of $H \subset SU(2)_L$ acting as an isotropy group for the boundaries of the magnetic flux tubes. H can correspond only to a non-Abelian subgroup $SU(2)_L$ of the electro-weak gauge group for the induced (classical) electro-weak gauge fields since the Chern-Simons action associated with the classical color gauge fields vanishes identically. The electro-weak holonomy group would reduce to a discrete group H around loops defined by anyonic flux tubes surrounding magnetic field lines inside the magnetic flux tubes containing anyons. The reduction to H need to occur only at the boundaries of the space-time sheet where conducting anyons would reside: boundaries indeed correspond to asymptotia in well-defined sense. Electro-weak symmetry group can be regarded as a sub-group of color group of isometries in a well-defined sense so that H can be regarded also as a subgroup of color group acting as isotropies of the multi-branched surface at least in the in regions where gauge field vanishes.

3. For branched surfaces the points obtained by moving around the puncture correspond in a good approximation to some elements of $h \in H$ leading to a new branch but the 2-surface as a whole however remains invariant. The braid group of the punctured 2-surface would be also now represented as transformations of H. The simplest situation is obtained when H is a cyclic group Z_N of the $U(1)$ group of CP_2 geodesic in such a manner that 2π rotation around symmetry axis corresponds to the generating element $exp(i2\pi/N)$ of Z_N.

Dihedral group D_n having order $2n$ and acting as symmetries of n-polygon of the plane is especially interesting candidate for H. For $n = 2$ the group is Abelian group $Z_2 \times Z_2$ whereas for $n > 2$ D_n is a non-Abelian sub-group of the permutation group S_n. The cyclic group Z_4 crucial for TQC is a sub-group of D_4 acting as symmetries of square. D_4 has a 2-dimensional faithful representation. The numbers of elements for the conjugacy classes are 1,1,2,2,2. The sub-group commuting with a fixed element of a conjugacy class is D_4 for the 1-element conjugacy classes and cyclic group Z_4 for 2-element conjugacy classes. Hence 2-valued magnetic flux would be accompanied by Z_4 valued "electric charge" identifiable as a cyclic group permuting the branches.

3. Can one understand the increase in conductivity and filling fractions at plateaus?

Quantum Hall effect involves the increase of longitudinal conductivity by a factor of order 10^{13}[**?**]. The reduction of dissipation could be understood as being caused by the fact that anyonic electrons are closed inside the magnetic flux tubes representing their tracks so that their interactions with matter and thus also dissipation are reduced.

Laughlin's theory [**?**, **?**] gives almost universal description of many aspects of quantum Hall effect and the question arises whether Laughlin's wave functions are defined on possibly multi-branched space-time sheet X^4 or at projection of X^4 to M^4_+. Since most theoreticians that I know still live in single sheeted space-time, one can start with the most conservative assumption that they are defined at the projection to M^4_+. The wave functions of one-electron state giving rise filling fraction $\nu = 1/m$ are constructed of $(z_i - z_j)^m$, where m is odd by Fermi statistics.

Also rational filling fractions of form $\nu = 1/m = N/n$ have been observed. These could relate to the presence of states whose projections to M^4 are multi-valued and which thus do not have any "classical" counterpart. For N-branched surface the single-valued wave functions $(\xi_i - \xi_j)^n$, n odd by Fermi statistics, correspond to apparently multi-valued wave functions $(z_i - z_j)^{n/N}$ at M^4 projection with fractional relative angular momenta $m = n/N$. The filling fraction would be $\nu = N/n$, n odd. All filling fractions reported in [**?**] have n odd with n varying in the range $1 - 7$. N has the values $1, 2, 3, 4, 5, 7, 9$. Also values $N = 12, 13$ for which $n = 5$ are reported [**?**].

The filling fractions $\nu = N/n = 5/2, 3/8, 3/10$ reported in [**?**] would require even values of n conflicting with Fermi statistics. Obviously Lauhglin's model fails in this case and the question is whether one these fractions could correspond to bosonic anyons, perhaps Cooper pairs of electrons inside track flux tubes. The Z_N valued charge associated with N-branched surfaces indeed allows the maximum $2N$ electrons per anyon. Bosonic anyons are indeed the building block of the TQC model of [**?**]. The anyon Cooper pairs could be this kind of states and their BE condensation would make possible genuine super-conductivity rather than only exceptionally high value of conductivity.

One can imagine also more complex multi-electron wave functions than those of Laughlin. The so called conformal blocks representing correlation functions of conformal quantum field theories are natural candidates for the wave functions [**?**] and they appear naturally as state functions of in topological quantum field theories. For instance, wave functions which are products of factors $(z^k - z^l)^2$ with the Pfaffian $Pf(A_{kl})$ of the matrix $A_{kl} = 1/(z_k - z_l)$ guaranteing anti-symmetrization have been used to explain even values of m [**?**].

4. N-branched space-time surfaces make possible Z_N valued topological charge

According to [**?**] that $2n$ non-Abelian anyon pairs with charge $1/4$ created from vacuum gives rise to a 2^{n-1}-fold degenerate ground state. It is also argued that filling fraction $5/2$ could correspond to this charge [**?**]. TGD suggests somewhat different interpretation. 4-fold branching implies automatically the Z_4-valued topological charge crucial for anyonic quantum computation. For 4-branched space-time surface the contribution of a single branch to electron's charge is indeed $1/4$ units but this has nothing to do with the actual charge fractionization. The value of ν is of form $\nu = /m$ and electromagnetic charge equals to $\nu = 4e/m$ in this kind of situation.

If anyons (electron plus flux tube representing its track) have Z_4 charges 1 and 3, their Cooper pairs have charges 0 and 2. The double-fold degeneracy for anyon's topological charge means that it possesses topological spin conserved modulo 4. In presence of $2n$ anyon pairs one would expect 2^n-fold degeneracy. The requirement that the net topological charge vanishes modulo 4 however fixes the topological charge of n:th pair so that 2^{n-1} fold degeneracy results.

A possible interpretation for Z_N-valued topological charge is as fractional angular momenta k/N

associated with the phases $exp(ik2\pi/N)$, $k = 0, 1, ..., n-1$ of particles in multi-branched surfaces. The projections of these wave functions to single-branched space-time would be many-valued. If electro-weak gauge group breaks down to a discrete subgroup H for magnetic flux tubes carrying anyonic "tracks", this symmetry breakdown could induce their multi-branched property in the sense rotation by 2π would correspond to H isometry leading to a different branch.

Topologization of Kähler charge as an explanation for charge fractionization

The argument based on what happens when one adds one anyon to the anyon system by utilizing Faraday's law [?] leads to the conclusion that anyon charge is fractional and given by νe. The anyonic flux tube along boundary of the flux tube corresponds to the left hand side in the Faraday's equation

$$\oint E \cdot dl = -\frac{d\Phi}{dt} \quad .$$

By expressing E in turns of current using transversal conductivity and integrating with respect to time, one obtains

$$Q = \nu e$$

for the charge associated with a single anyon. Hence the addition of the anyon means an addition of a fractional charge νe to the system. This argument should survive as such the 1-branched situation so that at least in this case the fractional charges should be real.

In N-branched case the closed loop $\oint E \cdot dl$ around magnetic flux tube corresponds to N-branched anyon and surrounds the magnetic flux tube N times. This would suggest so that net magnetic flux should be N times the one associated with single but unclosed 2π rotation. Hence the formula would seem to hold true as such also now for the total charge of the anyon and the conclusion is that charge fractionization is real and cannot be an effective effect due to fractionization of charge at single branch of anyon flux tube.

One of the basic differences between TGD and Maxwell's theory is the possibility of vacuum charges and this provides an explanation of the effect is in terms of vacuum Kähler charge. Kähler charge contributes $e/2$ to the charge of electron. Anyon flux tube can generate vacuum Kähler charge changing the net charge of the anyon. If the anyon charge equals to νe the conclusions are following.

1. The vacuum Kähler charge of the anyon track is $q = (\nu - 1)e$.

2. The dimension of the CP_2 projection of the anyon flux tube must be $D = 3$ since only in this case the topologization of anyon charge becomes possible so that the charge density is proportional to the Chern-Simons term $A \wedge dA/4\pi$. Anyon flux tubes cannot be super-conducting in the sense that non-integrable phase factor $exp(\int A \cdot dl)$ would define global order parameter. The boundaries of anyonic flux tubes could however remain potentially super-conducting and anyon Cooper pairs would be expelled there by Meissner effect. This gives super-conductivity in length scale of single flux tube. Conductivity and super-conductivity in long length scales requires that magnetic flux tubes are glued together along their boundaries partially.

3. By Bohr quantization anyon tracks can have $r_n = \sqrt{n} \times r_B$, $n \leq m$, where r_m corresponds to the radius of the magnetic flux tube carrying m flux quanta. Only the tracks with radius r_m contribute to boundary conductivity and super-conductivity giving $\nu = 1/m$ for singly branched surfaces.

   The states with $\nu = N/m$ cannot correspond to non-super-conducting anyonic tracks with radii r_n, $n < m$, n odd, since these cannot contribute to boundary conductivity. The many-branched character however allows an N-fold degeneracy corresponding to the fractional angular momentum states $exp(ik\phi/N)$, $k = 0, ..., N-1$ of electron inside anyon flux tubes of radius r_m. k is obviously a an excellent candidate for the Z_N-valued topological charge crucial for anyonic quantum computation. Z_4 is uniquely selected by the braid matrix R.

   Only part of the anyonic Fermi sea need to be filled so that filling fractions $\nu = k/m$, $k = 1, ..., N$ are possible. Charges νe are possible if each electron inside anyon track contributes $1/m$ units to the fractional vacuum Kähler charge. This is achieved if the radius of the anyonic flux tube grows as $\sqrt{k/m}$ when electrons are added. The anyon tracks containing several electrons give rise to composite fermions with fermion number up to $2N$ if both directions of electron spin are allowed.

4. Charge fractionization requires vacuum Kähler charge has rational values $Q_K = (\nu - 1)e$. The quantization indeed occurs for the helicity defined by Chern-Simons term $A \wedge dA/4\pi$. For compact 3-spaces without boundary the helicity can be interpreted as an integer valued invariant characterizing the linking of two disjoint closed curves defined by the magnetic field lines. This topological charge can be also related to the asymptotic Hopf invariant proposed by Arnold [**?**], which in non-compact case has a continuum of values. Vacuum Kähler current is obtained from the topological current $A \wedge dA/4\pi$ by multiplying it with a function of CP_2 coordinates completely fixed by the field equations. There are thus reasons to expect that vacuum Kähler charge and also the topological charges obtained by multiplying Chern Simons current by $SU(3)$ Hamiltonians are quantized for compact 3-surfaces but that the presence of boundaries replaces integers by rationals.

What happens in quantum Hall system when the strength of the external magnetic field is increased?

The proposed mechanism of anyonic conducitivity allows to understand what occurs in quantum Hall system when the intensity of the magnetic field is gradually increased.

1. Percolation picture encourages to think that magnetic flux tubes fuse partially along their boundaries in a transition to anyon conductivity so that the anyonic states localized at the boundaries of flux tubes become delocalized much like electrons in metals. Laughlin's states provide an idealized description for these states. Also anyons, whose tracks have Bohr radii r_m smaller than the radius r_B of the magnetic flux tube could be present but they would not participate in this localization. Clearly, the anyons at the boundaries of magnetic flux tubes are highly analogous to valence electrons in atomic physics.

2. As the intensity of the magnetic field B increases, the areas a of the flux tubes decreases as $a \propto 1/B$: this means that the existing contacts between neighboring flux tubes tend to be destroyed so that anyon conductivity is reduced. On the other hand, new magnetic flux tubes must emerge by the constancy of the average magnetic flux implying $dn/da \propto B$ for the average density of flux tubes. This increases the probability that the newly generated flux tubes can partially fuse with the existing flux tubes.

3. If the flux tubes are not completely free to move and change their shape by area preserving transformations, one can imagine that for certain value ranges of B the generation of new magnetic flux tubes is not favored since there is simply no room for the newcomers. The Fermi statistics of the anyonic electrons at the boundaries of flux tubes might relate to this non-hospitable behavior. At certain critical values of the magnetic field the sizes of flux tubes become however so small that the situation changes and the new flux tubes penetrate the system and via the partial fusion with the existing flux tubes increase dramatically the conductivity.

Also protonic anyons are possible

According to the TGD based model, any charged particle can form anyons and the strength of the magnetic field does not seem to be crucial for the occurrence of the effect and it could occur even in the Earth's magnetic field. The change of the cyclotron and Larmor frequencies of the charged particle in an external magnetic field to a value corresponding to the fractional charge provides a clear experimental signature for both the presence of anyons and for their the fractional charge.

Interestingly, water displays a strange scaling of proton's cyclotron frequency in an external magnetic field [**?**, **?**]. In an alternating magnetic field of .1551 Gauss (Eearth's field has a nominal value of .58 Gauss) a strong absorption at frequency $f = 156$ Hz was observed. The frequency was halved when D_2O was used and varied linearly with the field strength. The resonance frequency however deviated from proton's Larmor frequency, which suggests that a protonic anyon is in question. The Larmor frequency would be in this case $f_L = r \times \nu eB/2m_p$, where $r = \mu_p/\mu_B = 2.2792743$ is the ratio of proton's actual magnetic moment to its value for a point like proton. The experimental data gives

$\nu = .6003 = 3/5$ with the accuracy of 5×10^{-4} so that 3-branched protonic anyons with $m = 5$ would be responsible for the effect.

If this interpretation is correct, entire p-adic hierarchy of anyonic NMR spectroscopies associated with various atomic nuclei would become possible. Bosonic anyon atoms and Cooper pairs of fermionic anyon atom could also form macroscopic quantum phases making possible super-conductivity very sensitive to the value of the average magnetic field and bio-systems and brain could utilize this feature.

4.4.3 Does the quantization of Planck constant transform integer quantum Hall effect to fractional quantum Hall effect?

The model for topological quantum computation inspired the idea that Planck constant might be dynamical and quantized. The work of Nottale [?] gave a strong boost to concrete development of the idea and it took year and half to end up with a proposal about how basic quantum TGD could allow quantization Planck constant associated with M^4 and CP_2 degrees of freedom such that the scaling factor of the metric in M^4 degrees of freedom corresponds to the scaling of $\hbar$ in CP_2 degrees of freedom and vice versa [?]. The dynamical character of the scaling factors of M^4 and CP_2 metrics makes sense if space-time and imbedding space, and in fact the entire quantum TGD, emerge from a local version of an infinite-dimensional Clifford algebra existing only in dimension $D = 8$ [?].

The predicted scaling factors of Planck constant correspond to the integers n defining the quantum phases $q = exp(i\pi/n)$ characterizing Jones inclusions. A more precise characterization of Jones inclusion is in terms of group $G_b \subset SU(2) \subset SU(3)$ in CP_2 degrees of freedom and $G_a \subset SL(2,C)$ in M^4 degrees of freedom. In quantum group phase space-time surfaces have exact symmetry such that to a given point of M^4 corresponds an entire G_b orbit of CP_2 points and vice versa. Thus space-time sheet becomes $N(G_a)$ fold covering of CP_2 and $N(G_b)$-fold covering of M^4. This allows an elegant topological interpretation for the fractionization of quantum numbers. The integer n corresponds to the order of maximal cyclic subgroup of G.

In the scaling $\hbar_0 \rightarrow n\hbar_0$ of M^4 Planck constant fine structure constant would scale as

$$\alpha = \frac{e^2}{4\pi\hbar c} \rightarrow \frac{\alpha}{n} \quad ,$$

and the formula for Hall conductance would transform to

$$\sigma_H \rightarrow \frac{\nu}{n}\alpha \quad .$$

Fractional quantum Hall effect would be integer quantum Hall effect but with scaled down α. The apparent fractional filling fraction $\nu = m/n$ would directly code the quantum phase $q = exp(i\pi/n)$ in the case that m obtains all possible values. A complete classification for possible phase transitions yielding fractional quantum Hall effect in terms of finite subgroups $G \subset SU(2) \subset SU(3)$ given by ADE diagrams would emerge (A_n, D_{2n}, E_6 and E_8 are possible). What would be also nice that CP_2 would make itself directly manifest at the level of condensed matter physics.

4.4.4 Why 2+1-dimensional conformally invariant Witten-Chern-Simons theory should work for anyons?

Wess-Zumino-Witten theories are 2-dimensional conformally invariant quantum field theories with dynamical variables in some group G. The action contains the usual 2-dimensional kinetic term for group variables allowing conformal group action as a dynamical symmetry plus winding number defined associated with the mapping of 3-surface to G which is $Diff^4$ invariant. The coefficient of this term is quantized to integer.

If one couples this theory to a gauge potential, the original chiral field can be transformed away and only a Chern-Simons term defined for the 3-manifold having the 2-dimensional space as boundary remains. Also the coefficient k of Chern-Simons term is quantized to integer. Chern-Simons-Witten action has close connection with Wess-Zumino-Witten theory. In particular, the states of the topological quantum field theory are in one-one correspondence with highest weights of the WZW action.

The appearance of 2+1-dimensional $Diff^3$ invariant action can be understood from the fundamentals of TGD.

1. Light-like 3-surfaces of both future light-cone M^4_+ and of space-time surface X^4 itself are in a key role in the construction of quantum TGD since they define causal determinants for Kähler action.

2. At the space-time level both the boundaries of X^4 and elementary particle horizons surrounding the orbits of wormhole contacts define light-like 3-surfaces. The field equations are satisfied identically at light-like boundaries. Of course, the projections of the the light-like surfaces of X^4 to Minkowski space need not look light-like at all, and even boundaries of magnetic flux tubes could be light-like.

   Light-like 3-surfaces are metrically 2-dimensional and allow a generalized conformal invariance crucial for the construction of quantum TGD. At the level of imbedding space conformal super-canonical invariance results. At the space-time level the outcome is conformal invariance highly analogous to the Kac Moody symmetry of super string models [?, ?, ?]. In fact, there are good reasons to believe that the three-dimensional Chern-Simons action appears even in the construction of configuration space metric and give an additional contribution to the configuration space metric when the light-like boundaries of 3-surface have 3-dimensional CP_2 projection.

3. By the effective two-dimensionality the Wess-Zumino-Witten action containing Chern-Simons term is an excellent candidate for the quantum description of S-matrix associated with the light-like 3-surfaces since by the vanishing of the metric determinant one cannot define any general coordinate invariant 3-dimensional action other than Chern-Simons action. The boundaries of the braid formed by the magnetic flux tubes having light-like boundaries, perhaps having join along boundaries bonds between swapped flux tubes would define the 2+1-dimensional space-time associated with a braid, would define the arena of Witten-Chern-Simons theory describing anyons. This S-matrix can be interpreted also as characterizing either a 3-dimensional quantum state since light-like boundaries are limiting cases of space-like 3-surfaces.

4. Kähler action defines an Abelian Chern-Simons term and the induced electroweak gauge fields define a non-Abelian variant of this term. The Chern-Simons action associated with the classical color degrees of freedom vanishes as is easy to find. The classical color fields are identified as projections of Killing vector fields of color group: $A^c_\alpha = j^A_k \partial_\alpha s^k \tau_A = J_k{}^r \partial_r H^A \partial_\alpha s^k$. The classical color gauge field is proportional to the induced Kähler form: $F^c_{\alpha\beta} = H^A J_{\alpha\beta} \tau_A$. A little calculation shows that the instanton density vanishes by the identity $H_A H^A = 1$ (this identity is forced by the necessary color-singletness of the YM action density and is easy to check in the simpler case of S^2.

5. Since qubit realizes the fundamental representation of the quantum group $SU(2)_q$, $SU(2)$ is in a unique role concerning the construction of modular functors and quantum computation using Chern-Simons action. The quantum group corresponding to $q = exp(i2\pi/r)$, $r = 5$ is realized for the level $k = 3$ Chern-Simons action and satisfies the constraint $r = k + c_g$, where $c_g = 2$ is the so called dual Coxeter number of $SU(2)$ [?, ?, ?].

   The exponent non-Abelian $SU(2)_L \times U(1)$ Chern-Simons action combined with the corresponding action for Kähler form so that effective reduction to $SU(2)_L$ occurs, could appear as a multiplicative factor of the configuration space spinor fields defined in the configuration space of 3-surfaces. Since 3-dimensional quantum state would represent a 2-dimensional time evolution the role of these phase factor would be very analogous to the role of ordinary Chern-Simons action.

4.5 Topological quantum computation in TGD Universe

The general philosophy behind TQC inspires the dream that the existence of basic gates, in particular the maximally entangling 2-gate R, is guaranteed by the laws of Nature so that no fine tuning would be needed to build the gates. Negentropy Maximization Principle, originally developed in context of TGD inspired theory of consciousness, is a natural candidate for this kind of Law of Nature.

4.5.1 Concrete realization of quantum gates

The bold dream is that besides 2-gates also 1-gates are realized by the basic laws of Nature. The topological realization of the 3-braid representation in terms of Temperley-Lie algebra allows the reduction of 1-gates to 2-gates.

NMP and TQC

Quantum jump involves a cascade of self measurements in which the system under consideration can be though of as decomposing to two parts which are either un-entangled or possess rational or extended rational entanglement in the final state. The sub-system is selected by the requirement that entanglement negentropy gain is maximal in the measurement of the density matrix characterizing the entanglement of the sub-system with its complement.

In the case case that the density matrix before the self measurement decomposes into a direct sum of matrices of dimensions N_i , such that $N_i > 1$ holds true for some values of i, say i_0, the final state is a rationally entangled and thus a bound state. i_0 is fixed by the requirement that the number theoretic entropy for the final state maximally negative and equals to $klog(p)$, where p^k is the largest power of prime dividing N_{i_0}. This means that maximally entangled state results and the density matrix is proportional to a unit matrix as it is also for the entanglement produced by R. In case of R the density matrix is $1/2$ times 2-dimensional unit matrix so that bound state entanglement negentropy is 1 bit.

The question is what occurs if the density matrix contains a part for which entanglement probabilities are extended rational but not identical. In this case the entanglement negentropy is positive and one could argue that no self-measurement occurs for this state and it remains entangled. If so then the measurement of the density matrix would occur only when it increases entanglement negentropy. This looks the only sensible option since otherwise only bound state entanglement with identical entanglement probabilities would be possible. This question is relevant also because Temperley-Lieb representation using $(AA) - A - A$ system involves entanglement with entanglement probabilities which are not identical.

In the case that the 2-gate itself is not directly entangling as in case of R' and R'', NMP should select just the quantum history, that single particle gates at it guarantee maximum entanglement negentropy. Thus NMP would come in rescue and give hopes that various gates are realized by Nature.

Non-Abelian anyon systems are modelled in terms of punctures of plane and Chern-Simons action for the incompressible vector potential of hydrodynamical flow. It is interesting to find how these ideas relate to the TGD description.

Non-Abelian anyons reside at boundaries of magnetic flux tubes in TGD

In [?] anyons are modelled in terms of punctures of plane defined by the slab carrying Hall current. In TGD the punctures correspond naturally to magnetic flux tubes defining the braid. It is now however obvious under what conditions the braid containing the TGD counterpart of (AA)-A-A system can be described as a punctured disk if the flux tubes describing the tracks of valence anyons are very near to the boundaries of the magnetic flux tubes. Rather, the punctured disk is replaced with the closed boundary of the magnetic flux tube or of the structure formed by the partial fusion of several magnetic flux tubes. This microscopic description and is consistent with Laughlin's model only if it is understood as a long length scale description.

Non-Abelian charges require singularities and punctures but a two-surface which is boundary does not allow punctures. The punctures assigned with an anyon pair would become narrow wormhole threads traversing through the interior of the magnetic flux tube and connecting the punctures like wormholes connect two points of an apple. It is also possible that the threads connect the surfaces of two nearby magnetic flux tubes. The wormhole like character conforms with the fact that non-Abelian anyons appear always in pairs.

The case in which which the ends of the wormhole thread belong to different neighboring magnetic flux tubes, call them T_1 and T_2, is especially interesting as far as the model for TQC is considered. The state of $(AA) - A - A$ system before (after) the 3-braid operation would be identifiable as anyons near the surface of T_1 (T_2). If only sufficiently local operations are allowed, the braid group would be same as for anyons inside disk. This means consistency with the anyon model of [?] for TQC requiring that the dimension for the space of ground states is 2^{n-1} in a system consisting of n anyon pairs.

The possibility of negative energies allows inspires the idea that the anyons at T_2 have negative energies so that the anyon system would have a vanishing net energy. This would conform with the idea that the scattering from initial to final state is equivalent with the creation of zero energy state for which initial (final) state particles have positive (negative) energies, and with the fact that the boundaries of magnetic flux tubes are light-like systems for which 3-D quantum state is representation for a 2-D time evolution.

Since the correlation between anyons at the ends of the wormhole thread is purely topological, the most plausible option is that they behave as free anyons dynamically. Assuming 4-branched anyon surfaces, the charges of anyons would be of form $Q = \nu_A e$, $\nu_A = 4/m$, m odd.

Consider now the representation of 3-braid group. That the mapping class group for the 3-braid system should have a 2-dimensional representation is obvious from the fact that the group has same generators as the mapping class group for torus which is represented by as $SL(2,Z)$ matrices acting on the homology of torus having two generators a, b corresponding to the two non-contractible circles around torus. 3-braid group would be necessarily represented in Temperley-Lieb representation.

The character of the anyon bound state is important for braid representations.

1. If anyons form loosely bound states (AA), the electrons are at different tracks and the charge is additive in the process so that one has $Q_{AA} = 2Q_A = 8/m$, m odd, which is at odds with statistics. It might be that the naive rule of assigning fractional charge to the state does not hold true for loosely bound bosonic anyons. In this case $(AA) - A$ system with charge states $((1,-1),1)$ and $((1,1),-1)$ would be enough for realizing 1-gates in TQC. The braid operation s_2 of Temperley-Lieb representation represented $(A_1A_2) - A_3 \rightarrow (A_1A_3) - A_2$ would correspond to an exchange of the dance partner by a temporary decay of (A_1A_2) followed by a recombination to a quantum superposition of (A_1A_2) and (A_1A_3) and could be regarded as an ordinary braid operation rather than monodromy. The relative phase 1-gate would correspond to s_1 represented as braid operation for A_1 and A_2 inside (A_1A_2).

2. If anyons form tightly bound states (AA) in the sense that single anyonic flux tube carries two electrons, charge need not be additive so that bound states could have charges $Q = 4/2m_1$ so that the vacuum Kähler charge $Q_K = 4(1/m_1 - 2/m)$ would be created in the process. This would stabilize (AA) state and would mean that the braid operation $(A_1A_2) - A_3 \rightarrow (A_1A_3) - A_2$ cannot occur via a temporary decay to free anyons and it might be necessary to replace 3-braid group by a partially colored 3-braid group for $(AA) - A - A$ system which is sub-group of 3-braid group and has generators s_1^2 (two swaps for $(AA) - A$) and s_2 (swap for $A - A$) instead of s_1 and s_2. Also in this case a microscopic mechanism changing the value of (AA) Z^4 charge is needed and the situation might reduce to the case a) after all.

   The Temperley Lieb representation for this group is obtained by simply taking square of the generator inducing entanglement (s_2 rather than s_1 in the notation used!). The topological charge assignments for $(AA) - A - A$ system are $((1,-1),1,-1)$ and $((1,1),-1,-1)$. s_1^2 would correspond to the group element generating $(AA) - A$ entanglement and s_2 acting on $A - A$ pair would correspond to phase generating group element.

Braid representations and 4-branched anyon surfaces

Some comments about braid representations in relation to Z_N- valued topological charges are in order.

1. Yang-Baxter braid representation using the maximally entangling braid matrix R is especially attractive option. For anyonic computation with Z_4-valued topological charge R is the unique 2-gate conserving the net topological charge (note that the mixing of the $|1,1\rangle$ and $|-1,-1\rangle$ is allowed). On the other, R allows only the conservation of Z_4 value topological charge. This suggests that the the entanglement between logical qubits represented by $(AA) - A - A$ batches is is generated by R. The physical implication is that only $\nu = 4/n$ 4-branched anyons could be used for TQC.

2. In TGD framework the entangling braid representation inside batches responsible for 1-gates need not be the same since batches correspond to magnetic flux tubes. In standard physics con-

text it would be harder to defend this kind of assumption. As will be found 3-braid Temperley-Lieb representation is very natural for 1-gates. The implication is that the n-braid system with braids represented as 4-batches would have 2^n-dimensional space of logical qubits in fact identical with the space of realizable qubits.

3. Also n-braid Temperley-Lieb representations are possible and the explicit expressions of the braiding matrices for 6-braid case suggest that Z_4 topological charge is conserved also now [?]. In this case the dimension of the space of logical qubits is for highly favored value of quantum group parameter $q = exp(i\pi/5)$ given by the Fibonacci number $F(n)$ for n-braid case and behaves as Φ^{4n} asymptotically so that this option would be more effective. From $\Phi^4 = 1 + 3\Phi \simeq 8.03$ one can say that single 4-batch carries 3 bits of information instead of one. This is as it must be since topological charge is not conserved inside batches separately for this option.

4. $(AA) - A$ representation based on Z_4-valued topological charge is unique in that the space of logical qubits would be the space of topologically realizable qubits. Quantum superposition of logical qubits could could be represented $(AA) - A$ entangled state of form $a|2, -1\rangle + b|0, 1\rangle$ generated by braid action. Relative phase could be generated by braid operation acting on the entangled state of anyons of (AA) Cooper pair. Since the superposition of logical cubits corresponds to an entangled state $a|2, -1\rangle + b|0, 1\rangle$ for which coefficients are extended rational numbers, the number theoretic realization of the bound state property could pose severe conditions on possible relative phases.

4.5.2 Temperley-Lieb representations

The articles of Kaufmann [?] and Freedman [?, ?] provide enjoyable introduction to braid groups and to Tempeley-Lie representations. In the sequel Temperley-Lieb representations are discussed from TGD view point.

Temperley-Lieb representation for 3-braid group

In [?] it is explained how the so called Temperley-Lie algebra defined by 2×2-matrices I, U_1, U_2 satisfying the relations $U_1^2 = dU_1$, $U_2^2 = dU_2$, $U_1U_2U_1 = U_2$, $U_2U_1U_2 = U_1$ allows a unitary representation of Artin's braid group by unitary 2×2 matrices. The explicit representations of the matrices U_1 and U_2 (note that U_i/d acts as a projector) given by

$$\begin{aligned} U_1 &= \begin{pmatrix} d & 0 \\ 0 & 0 \end{pmatrix} \ , \\ U_2 &= \begin{pmatrix} \frac{1}{d} & \sqrt{1-\frac{1}{d^2}} \\ \sqrt{1-\frac{1}{d^2}} & d-\frac{1}{d} \end{pmatrix} \ . \end{aligned} \tag{4.5.1}$$

Note that the eigenvalues of U_i are d and 0. The representation of the elements s_1 and s_2 of the 3-braid group is given by

$$\begin{aligned} \Phi(s_1) &= AI + A^{-1}U_1 = \begin{pmatrix} -U^{-3} & 0 \\ 0 & U \end{pmatrix} \ , \\ \Phi(s_2) &= AI + A^{-1}U_2 = \begin{pmatrix} -\frac{U^3}{d} & \frac{U^{-1}}{\sqrt{1-(1/d)^2}} \\ \frac{U^{-1}}{\sqrt{1-(1/d)^2}} & \frac{U^{-5}}{d} \end{pmatrix} \ , \\ U &= exp(i\phi) \ . \end{aligned} \tag{4.5.2}$$

Here the condition $d = -A^2 - A^{-2}$ is satisfied. For $A = exp(i\phi)$, with $|\phi| \leq \pi/6$ or $|\pi - \phi| \leq \pi/6$, the representation is unitary. The constraint comes from the requirement $d > 1$. From the basic representation it follows that the eigenvalues of $\Phi(s_i)$ are $-exp(-3i\phi)$ and $exp(i\phi)$.

Tihs 3-braid representation is a special case of a more general Temperley-Lieb-Jones representation discussed in [?] using notations $A = \sqrt{-1}exp(-i2\pi/4r)$, $s = A^2$, and $q = A^4$. In this case all eigenvalues of all representation matrices are -1 and $q = exp(-i2\pi/r)$. This representation results by multiplying Temperley-Lieb representation above with an over-all phase factor $exp(4i\phi)$ and by the replacement $A = exp(i\phi) \rightarrow \sqrt{-1}A$.

Constraints on the parameters of Temperley-Lieb representation

The basic mathematical requirement is that besides entangling 2-gate there is minimum set of 1-gates generating infinite sub-group of $U(2)$. Further conditions come from the requirement that a braid representation is in question. In the proposal of [?, ?] the 1-gates are realized using Temperley-Lieb 3-braid representation. It is found that there are strong constraints to the representation and that relative phase gate generating the phase $exp(i\phi) = exp(i2\pi/5)$ is the simplest solution to the constraints.

The motivation comes from the findings made already by Witten in his pioneering work related to the topological quantum field theories and one can find a good representation about what is involve din [?].

Topological quantum field theories can produce unitary modular functors when the $A = q^{1/4} = exp(i\phi)$ characterizing the quantum group multiplication is a root of unity so that the quantum enveloping algebra $U(Sl(2))_q$ defined as the quantum version of the enveloping algebra $U(Sl(2))$ is not homomorphic with $U(Sl(2))$ and theory does not trivialize. Besides this, q must satisfy some consistency conditions. First of all, $A^{4n} = 1$ must be satisfied for some value of n so that A is either a primitive l:th, $2l$:th of unity for l odd, or $4l$:th primitive root of unity.

This condition relates directly to the fact that the quantum integers $[n]_q = (A^{2n} - A^{-2n})/(A^2 - A^{-2})$ vanish for $n \geq l$ so that the representations for a highest weight n larger than l are not irreducible. This implies that the theory simplifies dramatically since these representations can be truncated away but can cause also additional difficulties in the definition of link invariants. Indeed, as Witten found in his original construction, the topological field theories are unitary for $U(Sl(2))_q$ only for $A = exp(ik\pi/2l)$, k not dividing $2l$, and $A = exp(i\pi/l)$, l odd (no multiples are allowed) [?]. $n = 2l = 10$, which is the physically favored choice, corresponds to the relative phase $4\phi = 2\pi/5$.

Golden Mean and quantum computation

Temperley-Lieb representation based on $q = exp(i2\pi/5)$ is highly preferred physically.

1. One might hope that the Yang-Baxter representation based on maximally entangling braid matrix R might work. $R^8 = 1$ constraint is however not consistent with Temperley-Lieb representations. The reason is that $\Phi^8(s_1) = 1$ gives $\phi = \pi/4 > \pi/6$ so that unitarity constraint is not satisfied. $\phi = exp(i2\pi/16)$ corresponding $r = 4$ and to the matrix $\Phi(s_2) = \hat{R} = exp(i2\pi/16) \times R$ allows to satisfy the unitarity constraint. This would look like a very natural looking selection since $\Phi(s_2)$ would act as a Hadamard gate and NMP would imply identical entanglement probabilities if a bound state results in a quantum jump. Unfortunately, s_1 and s_2 do not generate a dense subgroup of $U(2)$ in this case as shown in [?].

2. $\phi = \pi/10$ corresponding to $r = 5$ and Golden Mean satisfies all constraints coming from quantum computation and knot theory. That is it spans a dense subgroup of $U(2)$, and allows the realization of modular functor defined by Witten-Chern-Simons $SU(2)$ action for $k = 3$, which is physically highly attractive since the condition

$$r = k + c_g(SU(2))$$

connecting r, k and the dual Coxeter number $c_g(SU(N)) = n$ in WCS theories is satisfied for $SU(2)$ in this case for $r = 5$ and $k = 3$.

$SU(2)$ would have interpretation as the left-handed electro-weak gauge group $SU(2)_L$ associated with classical electro-weak gauge fields. The symmetry breaking of $SU(2)_L$ down to a discrete subgroup of $SU(2)_L$ yielding anyons would relate naturally to this. The conservation of the topologized Kähler charge would correlate with the fact that there is no symmetry breaking in the classical $U(1)$ sector. $k = 3$ Chern-Simons theory is also known to share the same universality class as simple 4-body Hamiltonian [?] (larger values of k would correspond to $k + 1$-body Hamiltonians).

3. Number theoretical vision about intentional systems suggests that the preferred relative phases are algebraic numbers or more generally numbers which belong to a finite-dimensional extension of p-adic numbers. The idea about p-adic cognitive evolution as a gradual generation of increasingly complex algebraic extensions of rationals allows to see the extension containing Golden Mean $\Phi = (1+\sqrt{5})/2$ as one of the simplest extensions. The relative phase $exp(i4\phi) = exp(i2\pi/5)$ is expressible in an extension containing $\sqrt{\Phi}$ and Φ: one has $cos(4\phi) = (\Phi - 1)/2$ and $sin(4\phi) = \sqrt{5\Phi}/2$.

The general number theoretical ideas about cognition support the view that Golden Mean is in a very special role in the number theoretical world order. This would be due to the fact that $log(\Phi)/\pi$ is a rational number. This hypothesis would explain scaling hierarchies based on powers of Golden Mean. One could argue that the geometry of the braid should reflect directly the value of the $A = exp(i2\phi)$. The angle increment per single DNA nucleotide is $\phi/2 = 2\pi/10$ for DNA double strand (note that q would be $exp(i\pi/10)$, which raises the question whether DNA might be a topological quantum computer.

Bratteli diagram for $n = 5$ case, Fibonacci numbers, and microtubuli

Finite-dimensional von Neumann algebras can be conveniently characterized in terms of Bratteli diagrams [?]. For instance, the diagram a) of the figure **??** at the end of the chapter represents the inclusion $N \subset M$, where $N = M_2(C)\otimes C$, $M = M_6(C)\otimes M_3(C)\otimes C$. The diagram expresses the imbeddings of elements $A\otimes x$ of $M_2(C)\otimes C$ to $M_6(C)$ as a tensor product $A_1\otimes A_2\otimes x$

$$\begin{aligned} A_1 &= \begin{pmatrix} A & . & . \\ . & A & . \\ . & . & A \end{pmatrix} , \\ A_2 &= \begin{pmatrix} A & . \\ . & x \end{pmatrix} . \end{aligned} \tag{4.5.3}$$

Bratteli diagrams of infinite-dimensional von Neumann algebras are obtained as limiting cases of finite-dimensional ones.

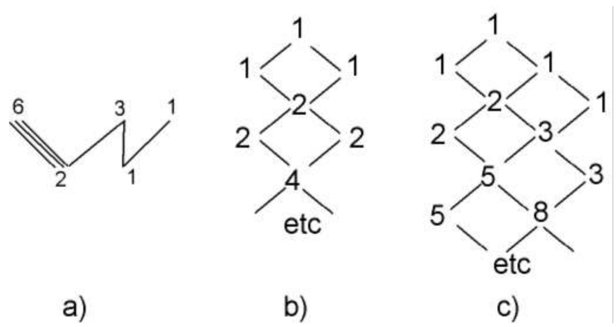

Figure 4.1: a) Illustration of Bratteli diagram. b) and c) give Bratteli diagrams for $n = 4$ and $n = 5$ Temperley Lieb algebras

2. Temperley Lieb algebras approximate II_1 factors

The hierarchy of inclusions of with $|M_{i+1} : M_i| = r$ defines a hierarchy of Temperley-Lieb algebras characterizable using Bratteli diagrams. The diagrams b) and c) of the figure **??** at the end of the chapter characterize the Bratteli diagrams for $n = 4$ and $n = 5$. For $n = 4$ the dimensions of algebras come in powers of 2 in accordance with the fact $r = 2$ is the dimension of the effective tensor factor of II_1.

For $n=5$ and $B_m=\{1,e_1,...,e_m\}$ the dimensions of the two tensor factors of the Temperley Lieb-representation are two subsequent Fibonacci numbers F_{m-1}, F_m ($F_{m+1}=F_m+F_{m-1}$, $F_1=1, F_2=1$) so that the dimension of the tensor product is $dim(B_m)=F_mF_{m-1}$. One has $dim(B_{m+1})/dim(B_m)=F_m/F_{m-2}\rightarrow \Phi^2=1+\Phi$, the dimension of the effective tensor factor for the corresponding hierarchy of II_1 factors. Hence the two dimensional hierarchies "approximate" each other. In fact, this result holds completely generally.

The fact that r is approximated by an integer in braid representations is highly interesting from the point of view of TQC. For 3-braid representation the dimension of Temperley-Lieb representation is 2 for all values of n so that 3-braid representation defines single (topo)logical qubit as $(AA)-A-A$ realization indeed assumes. One could optimistically say that TGD based physics automatically realizes topological qubit in terms of 3-braid representation and the challenge is to understand the details of this realization.

2. Why Golden Mean should be favored?

The following argument suggests a physical reason for why just Golden Mean should be favored in the magnetic flux tube systems.

1. Arnold [?] has shown that if Lorentz 3-force satisfies the condition $F_B=q(\nabla\times B)\times B=q\nabla\Phi$, then the field lines of the magnetic field lie on $\Phi=constant$ tori. On the other hand, the vanishing of the Lorentz 4-forces for solutions of field equations representing asymptotic self-organized states, which are the "survivors" selected by dissipation, equates magnetic force with the negative of the electric force expressible as qE, $E=-\nabla\Phi+\partial_tA$, which is gradient if the vector potential does not depend on time. Since the vector potential depends on three CP_2 coordinates only for $D=3$, this seems to be the case.

2. The celebrated Kolmogorov-Arnold-Moser (KAM) theorem is about the stability of systems, whose orbits are on invariant tori characterized by the frequencies associated with the n independent harmonic oscillator like degrees of freedom. The theorem states that the tori for which the frequency ratios are rational are highly unstable against perturbations: this is due to resonance effects. The more "irrational" the frequencies are, the higher the stability of the orbits is, and the most stable situation corresponds to frequencies whose ratio is Golden Mean. In quantum context the frequencies for wave motion on torus would correspond to multiples $\omega_i=n2\pi/L_i$, L_i the circumference of torus. This poor man's argument would suggest that the ratio of the circumferences of the most stable magnetic tori should be given by Golden Mean in the most stable situation: perhaps one might talk about Golden Tori!

3. Golden Mean and microtubuli

What makes this observation so interesting is that Fibonacci numbers appear repeatedly in the geometry of living matter. For instance, micro-tubuli, which are speculated to be systems performing quantum computation, represent in their structure the hierarchy Fibonacci numbers $5,8,13$, which brings in mind the tensor product representation $5\otimes 8$ of B_5 (5 braid strands!) and leads to ask whether this Temperley-Lieb representation could be somehow realized using microtubular geometry.

According to the arguments of [?] the state of n anyons corresponds to 2^{n-1} topological degrees of freedom and code space corresponds to F_n-dimensional sub-space of this space. The two conformations of tubulin dimer define the standard candidate for qubit, and one could assume that the conformation correlates strongly with the underlying topological qubit. A sequence of 5 *resp.* 8 tubulin dimers would give 2^4 *resp.* 2^7-dimensional space with $F_5=5$- *resp.* $F_7=13$-dimensional code sub-space so that numbers come out nicely. The changes of tubulin dimer conformations would be induced by the braid groups B_4 and B_7. B_4 would be most naturally realized in terms of a unit of 5-dimers by regarding the 4 first tubulins as braided punctures and 5th tubulin as the passive puncture. B_7 would be realized in a similar manner using a unit of 8 tubulin dimers.

Flux tubes would connect the subsequent dimers along the helical 5-strand *resp.* 8-strand defined by the microtubule. Nearest neighbor swap for the flux tubes would induce the change of the tubulin conformation and induce also entanglement between neighboring conformations. A full 2π helical twist along microtubule would correspond to 13 basic steps and would define a natural TQC program module. In accordance with the interpretation of II_1 factor hierarchy, (magnetic or electric) flux tubes could be assumed to correspond to $r=2$ II_1 factor and thus carry 2-dimensional representations of

$n = 5$ or $n = 4$ 3-braid group. These qubits could be realized as topological qubits using $(AA) - A$ system.

Topological entanglement as space-time correlate of quantum entanglement

Quantum-classical correspondence encourages to think that bound state formation is represented at the space-time level as a formation of join along boundaries bonds connecting the boundaries of 3-space sheets. In particular, the formation of entangled bound states would correspond to a topological entanglement for the join along boundaries bonds forming braids. The light-likeness of the boundaries of the bonds gives a further support for this identification. During macro-temporal quantum coherence a sequence of quantum jumps binds effectively to single quantum jump and subjective time effectively ceases to run. The light-likeness for the boundaries of bonds means that geometric time stops and is thus natural space-time correlate for the subjective experience during macro-temporal quantum coherence.

Also the work with TQC lends support for a a deep connection between quantum entanglement and topological entanglement in the sense that the knot invariants constructed using entangling 2-gate R can detect linking. Temperley-Lieb representation for 3-braids however suggests that topological entanglement allows also single qubit representations for with quantum entanglement plays no role. One can however wonder whether the entanglement might enter into the picture in some natural manner in the quantum computation of Temperley-Lieb representation. The idea is simple: perhaps the physics of $(AA) - A - A$ system forces single qubit representation through the simple fact that the state space reduces in 4-batch to single qubit by topological constraints.

For TQC the logical qubits correspond to entangled states of anyon Cooper pair (AA) and second anyon A so that the quantum superposition of qubits corresponds to an entangled state in general. Several arguments suggest that logical qubits would provide Temperley-Lieb representation in a natural manner.

1. The number of braids inside 4-anyon batch (or 3-anyon batch in case that (AA) can decay temporarily during braid operation) 3 so that by the universality this system allows to compute the unitary Temperley-Lieb braid representation. The space of logical qubits equals to the entire state space since the number of qubits represented by topological ground state degeneracy is 1 instead of the expected three since $2n$ anyon system gives rise to 2^{n-1}-fold vacuum degeneracy. The degeneracy is same even when two of the anyons fuse to anyon Cooper pair. Thus it would seem that the 3-braid system in question automatically produces 1-qubit representation of 3-braid group.

2. The braiding matrices $\Phi(s_1)$ and $\Phi(s_2)$ are different and only $\Phi(s_2)$ mixes qubit values. This can be interpreted as the presence of two inherently different braid operations such that only the second braiding operation can generate entanglement of states serving as building blocks of logical qubits. The description of anyons as 2-dimensional wormholes led to precisely this picture. The braid group reduces to braid group for one half of anyons since anyon and its partner at the end of wormhole are head and feet of single dancer, and the anyon pair (AA) forming bound state can change partner during swap operation with anyon A and this generates quantum entanglement. The swap for anyons inside (AA) can generate only relative phase.

3. The vanishing of the topological charge in a pairwise manner is the symmetry which reduces the dimension of the representation space to 2^{n-1} as already found. For $n = 4$ only single topological qubit results. The conservation and vanishing of the net topological charge inside each batch gives a constraint, which is satisfied by the maximally entangling R-matrix R so that it could take care of braiding between different 4-batches and one would have different braid representation for 4-batches and braids consisting of them. Topological quantization justifies this picture physically. Only phase generating *physical* 1-gates are allowed since Hadamard gate would break the conservation of topological charge whereas for *logical* 1-gates entanglement generating 2-gates can generate mixing without the breaking of the conservation of topological charges.

Summary

It deserves to summarize the key elements of the proposed model for which the localization (in the precise sense defined in [**?**]) made possible by topological field quantization and Z_4 valued topological charge are absolutely essential prerequisites.

1. $2n$-anyon system has 2^{n-1}-fold ground state degeneracy, which for $n = 2$ leaves only single logical qubit. In standard physics framework $(AA) - A - A$ is minimal option because the total homology charge of the system must vanish. In TGD $(AA) - A$ system is enough to represent 3-braid system if the braid operation between AA and A can be realized as an exchange of the dancing partner. This option makes sense because the anyons with opposite topological charges at the ends of wormhole threads can be negative energy anyons representing the final state of the braid operation. A pair of magnetic flux tubes is needed to realize single anyon-system containing braid.

2. Maximally entangling R-matrix realizes braid interactions between $(AA) - A$ systems realized as 3-braids inside larger braids and the space of logical qubits is equivalent with the space of realizable qubits. The topological charges are conserved separately for each $(AA) - A$ system. Also the more general realization based on n-braid representations of Temperley-Lieb algebra is formally possible but the different topological realization of braiding operations does not support this possibility.

3. Temperley-Lieb 3-braid representation for $(AA) - A - A$ system allows to realize also 1-gates as braid operations so that topology would allow to avoid the fine-tuning associated with 1-gates. Temperley-Lieb representation for $\phi = exp(i\pi/10)$ satisfies all basic constraints and provides representation of the modular functor expressible using $k = 3$ Witten-Chern-Simons action. Physically 1-gates are realizable using Φ_1 acting as phase gate for anyon pair inside (AA) and $\Phi(s_2)$ entangling (AA) and A by partner exchange. The existence of single qubit braid representations apparently conflicting with the identification of topological entanglement as a correlate of quantum entanglement has an explanation in terms of quantum computation under topological symmetries.

4.5.3 Zero energy topological quantum computations

As already described, TGD suggests a radical re-interpretation for matter antimatter asymmetry in long length scales. The asymmetry would be due to the fact that ground state for fermion system corresponds to infinite sea of negative energy fermions and positive energy anti-fermions so that fermions would have positive energies and anti-fermions negative energies.

The obvious implication is the possibility to interpret scattering between positive energy states as a creation of a zero energy state with outgoing particles represented as negative energy particles. The fact that the quantum states of 3-dimensional light-like boundaries of 3-surfaces represent evolutions of 2-dimensional quantum systems suggests a realization of topological quantum computations using physical boundary states consisting of positive energy anyons representing the initial state of anyon system and negative energy anyons representing the outcome of the braid operation.

The simplest scenario simply introduces negative energy charge conjugate of the $(AA) - A$ system so that no deviations from the proposed scenario are needed. Both calculation and its conjugate are performed. This picture is the only possible one if one assumes that given space-time sheet contains either positive or negative energy particles but not both and very natural if one assumes ordinary fermionic vacuum. The quantum computing system would could be generated without any energy costs and even intentionally by first generating the p-adic space-time sheets responsible for the magnetic flux tubes and anyons and then transformed to their real counterparts in quantum jump. This double degeneracy is analogous to that associated with DNA double strand and could be used for error correction purposes: if the calculation has been run correctly both anyon Cooper pairs and their charge conjugates should decay with the same probability.

Negative energies could have much deeper role in TQC. This option emerges naturally in the wormhole handle realization of TQC. The TGD realization of 1-gates in 3-braid Temperley-Lieb representation uses anyons of opposite topological charges at the opposite ends of threads connecting magnetic flux tube boundaries. Single 3-braid unit would correspond to positive energy electronic anyons at the first flux tube boundary and negative energy positronic anyons at the second flux tube boundary. The sequences of 1-gates represented as 3-braid operations would be coded by a sequence of 3-braids representing generators of 3-braid group along a pair of magnetic flux tubes. Of course, also n-braid operations could be coded in the similar manner in series. Hence TQC could be realized using only two magnetic flux tubes with n-braids connecting their boundaries in series.

Condensed matter physicist would probably argue that all this could be achieved by using electrons in strand and holes in the conjugate strand instead of negative energy positrons: this would require only established physics. One can however ask whether negative energy positrons could appear routinely in condensed matter physics. For instance, holes might in some circumstances be generated by a creation of an almost zero energy pair such that positron annihilates with a fermion below the Fermi surface. The signature for this would be a photon pair consisting of ordinary and phase conjugate photons.

The proposed interpretation of the S-matrix in the Universe having vanishing net quantum numbers encourages to think that the S-matrices of 2+1-dimensional field theories based on Witten-Chern-Simons action defined in the space of zero (net) energy states could define physical states for quantum TGD. Thus the 2+1-dimensional S-matrix could define quantum states of 4-dimensional theory having interpretation as states representing "self-reflective" level representing in itself the S-matrix of a lower-dimensional theory. The identification of the quantum state as S-matrix indeed makes sense for light-like surfaces which can be regarded as limiting cases of space-like 3-surfaces defining physical state and time-like surfaces defining a time evolution of the state of 2-dimensional system.

Time evolution would define also an evolution in topological degrees of freedom characterizing ground states. Quantum states associated with light-like (with respect to the induced metric of space-time sheet) 3-dimensional boundaries of say magnetic flux tubes would define quantum computations as modular functors. This conforms with quantum-classical correspondence since braids, the classical states, indeed define quantum computations.

The important implication would be that a configuration which looks static would code for the dynamic braiding. One could understand the quantum computation in this framework as signals propagating through the strands and being affected by the gate. Even at the limit when the signal propagates with light velocity along boundary of braid the situation looks static from outside. Time evolution as a state could be characterized as sequence of many-anyon states such that basic braid operations are realized as zero energy states with initial state realized using positive energy anyons and final state realized using negative energy energy anyons differing by the appropriate gate operation from the positive energy state.

In the case of n-braid system the state representing the S-matrix $S = S^1S^2....S^n$ associated with a concatenation of n elementary braid operations would look like

$$\begin{aligned} |S\rangle &= P_{k_1}S^1_{k_1k_2}P_{k_2}S^2_{k_2k_3}P_{k_3}S^3_{k_3k_4}... \ , \\ P_k &= |k,<\rangle|k,>\rangle \ . \end{aligned} \quad (4.5.4)$$

Here S^k are S-matrices associated with gates representing simple braiding operations s_k for $n+1$ threads connecting the magnetic flux tubes. P_k represents a trivial transition $|k\rangle \rightarrow |k \rightarrow k\rangle$ as zero energy state $|k,>0\rangle|k,<\rangle$. The states P_k represent matrix elements of the identification map from positive energy Hilbert space to its negative energy dual.

What would happen can be visualized in two alternative manners.

1. For this option the braid maps occur always from flux tube 1 to flux tube 2. A braiding transition from 1 to 2 is represented by S^{k_1}; a trivial transition from 2 to 1 is represented by P_k; a braiding transition from 1 to 2 is represented by S^{k_2}, etc... In this case flux tube 1 contains positive energy anyons and flux tube 2 the negative energy anyons.

2. An alternative representation is the one in which P_k represents transition along the strand so that S^k *resp.* S^{k+1} corresponds to braiding transition from strand 1 to 2 *resp.* 2 to 1. In this case both flux tubes contain both positive and negative energy anyons.

4.6 Appendix: A generalization of the notion of imbedding space

In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is described. This view has developed much before the original version of this chapter was written.

The original idea was that the proposed modification of the imbedding space could explain naturally phenomena like quantum Hall effect involving fractionization of quantum numbers like spin and charge. This does not however seem to be the case. $G_a \times G_b$ implies just the opposite if these quantum numbers are assigned with the symmetries of the imbedding space. For instance, quantization unit for orbital angular momentum becomes n_a where Z_{n_a} is the maximal cyclic subgroup of G_a.

One can however imagine of obtaining fractionization at the level of imbedding space for space-time sheets, which are analogous to multi-sheeted Riemann surfaces (say Riemann surfaces associated with $z^{1/n}$ since the rotation by 2π understood as a homotopy of M^4 lifted to the space-time sheet is a non-closed curve. Continuity requirement indeed allows fractionization of the orbital quantum numbers and color in this kind of situation.

4.6.1 Both covering spaces and factor spaces are possible

The observation above stimulates the question whether it might be possible in some sense to replace H or its factors by their multiple coverings.

1. This is certainly not possible for M^4, CP_2, or H since their fundamental groups are trivial. On the other hand, the fixing of quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is a geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. Zero energy ontology forces to modify this picture somewhat. In zero energy ontology causal diamonds (CDs) defined as the intersections of future and past directed light-cones are loci for zero energy states containing positive and negative energy parts of state at the two light-cone boundaries. The location of CD in M^4 is arbitrary but p-adic length scale hypothesis suggests that the temporal distances between tips of CD come as powers of 2 using CP_2 size as unit. Thus M^4 is replaces by CD and $\hat{M}^4$ is replaced with $\hat{CD}$ defined in obvious manner.

3. H_4 represents a straight cosmic string inside CD. Quantum field theory phase corresponds to Jones inclusions with Jones index $\mathcal{M} : \mathcal{N} < 4$. Stringy phase would by previous arguments correspond to $\mathcal{M} : \mathcal{N} = 4$. Also these Jones inclusions are labeled by finite subgroups of $SO(3)$ and thus by Z_n identified as a maximal Abelian subgroup.

   One can argue that cosmic strings are not allowed in QFT phase. This would encourage the replacement $\hat{CD} \times \hat{CP}_2$ implying that surfaces in $CD \times S^2$ and $(M^2 \cap CD) \times CP_2$ are not allowed. In particular, cosmic strings and CP_2 type extremals with M^4 projection in M^2 and thus light-like geodesic without zitterwebegung essential for massivation are forbidden. This brings in mind instability of Higgs=0 phase.

4. The covering spaces in question would correspond to the Cartesian products $\hat{CD}_{n_a} \times \hat{CP}_{2n_b}$ of the covering spaces of $\hat{CD}$ and $\hat{CP}_2$ by Z_{n_a} and Z_{n_b} with fundamental group is $Z_{n_a} \times Z_{n_b}$. One can also consider extension by replacing $M^2 \cap CD$ and S^2 with its orbit under G_a (say tedrahedral, octahedral, or icosahedral group). The resulting space will be denoted by $\hat{CD} \hat{\times} G_a$ *resp.* $\hat{CP}_2 \hat{\times} G_b$.

5. One expects the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds $M^2 \cap CD$ or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of $M^2 \cap CD$ the quantization axes for angular momentum

would be replaced by the set of quantization axes going through the vertices of tedrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

6. Also the orbifolds $\hat{CD}/G_a \times \hat{CP_2}/G_b$ can be allowed as also the spaces $\hat{CD}/G_a \times (\hat{CP_2}\hat{\times}G_b)$ and $(\hat{CD}\hat{\times}G_a) \times \hat{CP_2}/G_b$. Hence the previous framework would generalize considerably by the allowance of both coset spaces and covering spaces.

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $(M^2 \cap CD) \times CP_2$ takes place? It would seem that the covariant metric of M^4 factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of M^4 metric can make sense. This is consistent with the identical vanishing of Chern-Simons action in $M^2 \times S^2$.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in CD degrees of freedom. This is not the case. Light-likeness in $(M^2 \cap CD) \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset (M^2 \cap CD) \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $(M^2 \cap CD) \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of $(M^2 \cap CD)$ as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunneling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) and classical light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

4.6.2 Do factor spaces and coverings correspond to the two kinds of Jones inclusions?

What could be the interpretation of these two kinds of spaces?

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M} : \mathcal{N} < 4$ and it naturally corresponds to the coset spaces. For $\mathcal{M} : \mathcal{N} = 4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.

2. $\mathcal{M} : \mathcal{N} = 4$ should correspond to the allowance of cosmic strings and other analogous objects. Does the introduction of the covering spaces bring in cosmic strings in some controlled manner? Formally the subgroup of $SU(2)$ defining the inclusion is $SU(2)$ would mean that states are $SU(2)$ singlets which is something non-physical. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$.

   For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{CD}\hat{\times}G_a$ and $\hat{CP}_2\hat{\times}G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane.

3. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M}:\mathcal{N}<4$ and $\mathcal{M}:\mathcal{N}=4$, which both are labeled by a subset of discrete subgroups of SU(2).

4. The discrete subgroups of $SU(2)$ with fixed quantization axes possess a well defined multiplication with product defined as the group generated by forming all possible products of group elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tedrahedral, octahedral, and icosahedral groups are the generators of this algebra.

By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2¡ generated by reflection, and tedrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized configuration space spinor fields in the configuration space labelled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

The question how do the Planck constants associated with factors and coverings relate is far from trivial and I have considered several options.

1. If one assumes that $\hbar^2(X)$, $X=M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of metric allowed by Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains $r^2\equiv\hbar^2/\hbar_0^2\hbar^2(M^4)/\hbar^2(CP_2)$. This puts M^4 and CP_2 in a very symmetric role and allows much more flexibility in the identification of symmetries associated with large Planck constant phases.

2. Algebraist would argue that Planck constant must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X)=\hbar(X)\hbar_0=n$ for covering and $r(X)=1/n$ for factor space or vice versa. This gives two options.

3. Option I: $r(X)=n$ for covering and $r(X)=1/n$ for factor space gives $r\equiv\hbar/\hbar_0=r(M^4)/r(CP_2)$. This gives $r=n_a/n_b$ for $\hat{H}/G_a\times G_b$ option and $r=n_b/n_a$ for $\hat{H}\hat{times}(G_a\times G_b)$ option with obvious formulas for hybrid cases.

4. Option II: $r(X)=1/n$ for covering and $r(X)=n$ for factor space gives $r=r(CP_2)/r(M^4)$. This gives $r=n_b/n_a$ for $\hat{H}/G_a\times G_b$ option and $r=n_a/n_b$ for $\hat{H}\hat{times}(G_a\times G_b)$ option with obvious formulas for the hybrid cases.

5. At quantum level the fractionization would come from the modification of fermionic anti-commutation (bosonic commutation) relations involving $\hbar$ at the right hand side so that particle number becomes a multiple of $1/n$ or n. If one postulates that the total number states is invariant in the transition, the increase in the number of sheets is compensated by the increase of the fundamental phase space volume proportional to $\hbar$. This would give $r(X)\rightarrow r(X)/n$ for factor space and $r(X)\rightarrow nr(X)$ for the covering space to compensate the n-fold reduction/increase of states. This would favor Option II.

6. The second manner to distinguish between these two options is to apply the theory to concrete physical situations. Since G_a and G_b act as symmetries in CD and CP_2 degrees of freedom, one might of being able to distinguish between the two options if it is possible to distinguish between the action of G as symmetry of quantum states associated with covering and factor space. Also the quantization of the orbital spin quantum number at single particle level as multiples of n can be distinguished from that in multiples of $1/n$.

4.6.3 A simple model of fractional quantum Hall effect

The generalization of the imbedding space suggests that it could possible to understand fractional quantum Hall effect [?] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \tag{4.6.1}$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15..., 2/3, 3/5, 4/7, 5/9, 6/11, 7/13..., 5/3, 8/5, 11/7, 14/9...4$ $1/5, 2/9, 3/13..., 2/7, 3/11..., 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [?].

The model of Laughlin [?] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [?]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of imbedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are four combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this there are two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. On the following just for fun consideration option I is considered although the conservation of number of states in the phase transition changing $\hbar$ favors option II.

1. The easiest manner to understand the observed fractions is by assuming that both M^4 and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

2. With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

3. The appearance of $\nu = 5/2$ has been observed [?]. The fractionized charge is $e/4$ in this case. Since $n_i > 3$ holds true if coverings are correlates for Jones inclusions, this requires to $n_b = 4$ and $n_a = 10$. n_b predicting a correct fractionization of charge. The alternative option would be $n_b = 2$ that also Z_2 would appear as the fundamental group of the covering space. Filling fraction $1/2$ corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [?]. $n_b = 2$ is however inconsistent with the observed fractionization of electric charge and with the vision inspired by Jones inclusions.

4. A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is

that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

5. Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e\int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F \int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

6. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6\times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the identification of charge unit is rather ad hoc. Therefore this model can be taken only as a warm-up exercise. In [**?**] Quantum Hall effect and charge fractionization are discussed in detail and one ends up with a rather detailed view about the delicacies of the Kähler structure of generalized imbedding space.

Bibliography

Online books about TGD

[1] M. Pitkänen (2006), *Topological Geometrodynamics: Overview*. http://tgd.wippiespace.com/public_html/tgdview/tgdview.html.

[2] M. Pitkänen (2006), *Quantum Physics as Infinite-Dimensional Geometry*. http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html.

[3] M. Pitkänen (2006), *Physics in Many-Sheeted Space-Time*. http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html.

[4] M. Pitkänen (2006), *Quantum TGD*. http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html.

[5] M. Pitkänen (2006), *TGD as a Generalized Number Theory*. http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html.

[6] M. Pitkänen (2006), *p-Adic length Scale Hypothesis and Dark Matter Hierarchy*. http://tgd.wippiespace.com/public_html/paddark/paddark.html.

[7] M. Pitkänen (2006), *TGD and Fringe Physics*. http://tgd.wippiespace.com/public_html/freenergy/freenergy.html.

Online books about TGD inspired theory of consciousness and quantum biology

[8] M. Pitkänen (2006), *Bio-Systems as Self-Organizing Quantum Systems*. http://tgd.wippiespace.com/public_html/bioselforg/bioselforg.html.

[9] M. Pitkänen (2006), *Quantum Hardware of Living Matter*. http://tgd.wippiespace.com/public_html/bioware/bioware.html.

[10] M. Pitkänen (2006), *TGD Inspired Theory of Consciousness*. http://tgd.wippiespace.com/public_html/tgdconsc/tgdconsc.html.

[11] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*. http://tgd.wippiespace.com/public_html/genememe/genememe.html.

[12] M. Pitkänen (2006), *TGD and EEG*. http://tgd.wippiespace.com/public_html/tgdeeg/tgdeeg/tgdeeg.html.

[13] M. Pitkänen (2006), *Bio-Systems as Conscious Holograms*. http://tgd.wippiespace.com/public_html/hologram/hologram.html.

[14] M. Pitkänen (2006), *Magnetospheric Consciousness*. http://tgd.wippiespace.com/public_html/magnconsc/magnconsc.html.

[15] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*. http://tgd.wippiespace.com/public_html/magnconsc/mathconsc.html.

References to the chapters of books

[A8] The chapter *Was von Neumann Right After All* of [?].
http://tgd.wippiespace.com/public_html/tgdview/tgdview.html#vNeumann.

[A9] The chapter *Does TGD Predict the Spectrum of Planck Constants?* of [?].
http://tgd.wippiespace.com/public_html/tgdview/tgdview.html#Planck.

[B1] The chapter *Identification of the Configuration Space Kähler Function* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#kahler.

[B2] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part I* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#compl1.

[B3] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part II* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#compl2.

[B4] The chapter *Configuration Space Spinor Structure* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#cspin.

[C2] The chapter *Construction of Quantum Theory: Symmetries* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#quthe.

[C3] The chapter *Construction of Quantum Theory: S-matrix* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#towards.

[D1] The chapter *Basic Extremals of Kähler Action* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#class.

[D6] The chapter *TGD and Astrophysics* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#astro.

[E2] The chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visionb.

[E6] The chapter *Fusion of p-Adic and Real Variants of Quantum TGD to a More General Theory* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#mblocks.

[E8] The chapter *Riemann Hypothesis and Physics* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#riema.

[E11] The chapter *Langlands Program and TGD* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdeeg/tgdnumber.html#Langlandia.

[F9] The chapter *Nuclear String Physics* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#nuclstring.

[F12] The chapter *Quantum Hall effect and Hierarchy of Planck Constants* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#anyontgd.

[G2] The chapter *The Notion of Free Energy and Many-Sheeted Space-Time Concept* of [?].
http://tgd.wippiespace.com/public_html/freenergy/freenergy.html#freenergy.

[H2] The chapter *Negentropy Maximization Principle* of [?].
http://tgd.wippiespace.com/public_html/tgdconsc/tgdconsc.html#nmpc.

[I4] The chapter *Quantum Control and Coordination in Bio-systems: Part I* of [?].
http://tgd.wippiespace.com/public_html/bioselforg/bioselforg.html#qcococI.

[I5] The chapter *Quantum Control and Coordination in Bio-Systems: Part II* of [?].
http://tgd.wippiespace.com/public_html/bioselforg/bioselforg.html#qcococII.

[K4] The chapter *Bio-Systems as Conscious Holograms* of [?].
http://tgd.wippiespace.com/public_html/hologram/hologram.html#hologram.

[L1] The chapter *Genes and Memes* of [?].
http://tgd.wippiespace.com/public_html/genememe/genememe.html#genememec.

[L2] The chapter *Many-Sheeted DNA* of [?].
http://tgd.wippiespace.com/public_html/genememe/genememe.html#genecodec.

[M3] The chapter *Dark Matter Hierarchy and Hierarchy of EEGs* of [?].
http://tgd.wippiespace.com/public_html/tgdeeg/tgdeeg/tgdeeg.html#eegdark.

[N4] The chapter *Pre-Biotic Evolution in Many-Sheeted Space-Time* of [?].
http://tgd.wippiespace.com/public_html/magnconsc/magnconsc.html#prebio.

Mathematics related references

[16] C. N. Yang, M. L. Ge (1989), *Braid Group, Knot Theory, and Statistical Mechanics*, World Scientific.

[17] A. Lakthakia (1994), *Beltrami Fields in Chiral Media*, Series in Contemporary Chemical Physics - Vol. 2, World Scientific, Singapore.

[18] V. I. Arnold (1986), Sel. Math. Sov. 5, 327.

[19] D. Reed (1995), in *Advanced Electromagnetism: Theories, Foundations, Applications*, edited by T. Barrett (Chap. 7), World Scientific, Singapore.

[20] G. E. Marsh (1995), *Helicity and Electromagnetic Field Topology* in *Advanced Electromagnetism*, Eds. T. W. Barrett and D. M. Grimes, Word Scientific.

Quantum computation

[21] *Quantum Information Science*, Report of NSF Workshop, October 28-29, 1999. Arlington Virginia. National Science Foundation.

[22] Paul Parsons (2004) , *Dancing the Quantum Dream*, New Scientist 24. January. www.newscientist.com/hottopics.

[23] M. Freedman, A. Kitaev, M. Larson, Z. Wang (200?), www.arxiv.of/quant-ph/0101025.

[24] R. Feynman (1982), *Simulating physics with computers*, Int. J. Theor. Phys. 21, 467-488.
Ibid (1986), *Quantum mechanical computers*, Found. Phys., 16, 507-603.

[25] D. Deutch (1985), *Quantum theory, the Church-Turing principle, and the universal quantum computer*, Proc. Roy. Soc. London, A400, 97-117.

[26] P. Shor (1994), *Algorithms for quantum computation, discrete logarithms, and factoring*, Proc. 35th Annual Symposium on Foundations of Computer Science, IEEE Computer Society Press, Los Alamitos, CA, 124-134.

[27] J. Preskill (1997), *Fault tolerant quantum computation*, quant-ph/9712048.

[28] P. Shor (1995), *Scheme for reducing de-coherence in quantum computer memory*, Phys. Rev. A 52, 2493.

[29] D. Gottesman (1998), *Theory of fault tolerant quantum computation*, Phys. Lett. A57, 127-137.

[30] S. Sawin (1995), *Links, Quantum Groups, and TQFT's*, q-alg/9506002.

[31] C. Kassel (1995), *Quantum Groups*, Springer Verlag.

[32] E. Witten 1989), *Quantum field theory and the Jones polynomial*, Comm. Math. Phys. 121 , 351-399.

[33] K. Walker (1991), *On Witten's 3-manifold invariants*, preprint, `http://xmission.com/~kwalker/math/`.

[34] V. F. R. Jones (1987), *Hecke algebra representations of braid groups and link polynomial*, Ann. Math., 126, 335-388.

[35] L. H. Kauffmann and S. J. Lomonaco Jr. (2004), *Braiding operations are universal quantum gates*, arxiv.org/quant-ph/0401090.

[36] H. Dye (2003), *Unitary solutions to the Yang-Baxter equations in dimension four*, Quantum Information Processing, Vol 2 Ns 1,2, April, 117,150 (quant-ph/0211050).

[37] M. Freedman, H. Larsen, and Z. Wang (2002), *A modular functor which is universal for quantum computation*, Found. Comput. Math. 1, no 2, 183-204. Comm. Math. Phys. 227, no 3, 605-622. quant-ph/0001108.

[38] M. H. Freedman (2001), *Quantum Computation and the localization of Modular Functors*, Found. Comput. Math. 1, no 2, 183-204.

[39] M. H. Freedman (1998), *P/NP, and the quantum field computer*, Proc. Natl. Acad. Sci. USA 95, no. 1, 98-101.

[40] A. Kitaev (1997), Annals of Physics, vol 303, p.2. See also *Fault tolerant quantum computation by anyons*, quant-ph/9707021.

[41] A. Kitaev (1997), *Quantum computations: algorithms and error correction*, Russian Math. Survey, 52:61, 1191-1249.

Quantum Hall effect

[42] S. M. Girvin (1999), *Quantum Hall Effect, Novel Excitations and Broken Symmetries*, cond-mat/9907002.

[43] R. B. Laughlin (1990), Phys. Rev. Lett. 50, 1395.

[44] F. Wilzek (1990), *Fractional Statistics and Anyon Super-Conductivity*, World Scientific.

[45] *Fractional quantum Hall Effect*, `http://en.wikipedia.org/wiki/Fractional_quantum_Hall_effect`.
Fractional Quantum Hall Effect, `http://www.warwick.ac.uk/~phsbm/fqhe.htm`.

[46] J.K. Jain(1989), Phys. Rev. Lett. 63, 199.

[47] J. B. Miller *et al*(2007), *Fractional Quantum Hall effect in a quantum point contact at filling fraction 5/2*, arXiv:cond-mat/0703161v2.

[48] A. Wojs, K.-S. Yi and J. J. Quinn (2003), *Fractional Quantum Hall States of Composite Fermions*, cond-mat/0312290.

[49] M. de Wild Propitius and F. A. Bais (1996), *Discrete Gauge Theories*, hep-th/9511201.

[50] C. Nayak and F. Wilczek (1996), *2n-quasihole states realize 2^{n-1}-dimensional spinor braiding statistics in paired quantum Hall states*, Nucl. Phys. B479, 529-533.

[51] G. Moore and N. Read (1991), *Non-Abelians in the fractional quantum Hall effect*, Nucl. Phys. B360, 362-396.

References related to physics anomalies

[52] V. V. Shkunov and B. Ya. Zeldowich (1985), *Optical Phase Conjugation*, Scientific American, December issue.

[53] D. Da Roacha and L. Nottale (2003), *Gravitational Structure Formation in Scale Relativity*, astro-ph/0310036.

[54] M. Chaplin (2005), *Water Structure and Behavior*,
http://www.lsbu.ac.uk/water/index.html.
For 41 anomalies see http://www.lsbu.ac.uk/water/anmlies.html.
For the icosahedral clustering see http://www.lsbu.ac.uk/water/clusters.html.
J. K. Borchardt(2003), *The chemical formula H2O - a misnomer*, The Alchemist 8 Aug (2003).
R. A. Cowley (2004), *Neutron-scattering experiments and quantum entanglement*, Physica B 350 (2004) 243-245.
R. Moreh, R. C. Block, Y. Danon, and M. Neumann (2005), *Search for anomalous scattering of keV neutrons from H2O-D2O mixtures*, Phys. Rev. Lett. 94, 185301.

[55] L. P. Semikhana and Yu. A. Lyubinov (1998), *Effects of Weak Magnetic Fields on Dielectric Loss in Ordinary Water and Heavy Water*, Moscow University Physics Bulletin, 43, 60-64.

[56] R. Mills *et al* (2003), *Spectroscopic and NMR identification of novel hybrid ions in fractional quantum energy states formed by an exothermic reaction of atomic hydrogen with certain catalysts.* http://www.blacklightpower.com/techpapers.html .

Life science related references

[57] P. Gariaev *et al* (2000), *The DNA-wave-biocomputer*, CASYS'2000, Fourth International Conference on Computing Anticipatory Systems, Liege, 2000. Abstract Book, Ed. M. Dubois.

[58] A. Coghlan (2004), *Our genome 'reads' junk as well as genes*, New Scientist, 21 February.

[59] Articles about TGD inspired theory of consciousness in previous issues of Journal of Non-Locality and Remote Mental Interactions.
http://www.emergentmind.org.

[60] W. A. Tiller, W. E. Dibble Jr., and M. J. Kohane (2001), *Conscious Acts of Creation: the Emergence of a New Physics*, p. 63.
Pavior Publishing. http://www.pavior.com/.

[61] E. Lozneanu and M. Sanduloviciu (2003) *Minimal-cell system created in laboratory by self-organization*, Chaos, Solitons & Fractals, Volume 18, Issue 2, October, p. 335.
See also *Plasma blobs hint at new form of life*, New Scientist vol. 179 issue 2413 - 20 September 2003, page 16.

Chapter 5

DNA as Topological Quantum Computer

5.1 Introduction

Large values of Planck constant makes possible all kinds of quantum computations [?, ?, ?, ?]. What makes topological quantum computation (tqc) [?, ?, ?, ?, ?] so attractive is that the computational operations are very robust and there are hopes that external perturbations do not spoil the quantum coherence in this case. The basic problem is how to create, detect, and control the dark matter with large $\hbar$. The natural looking strategy would be to assume that living matter, say a system consisting of DNA and cell membranes, performs tqc and to look for consequences.

There are many questions. How the tqc could be performed? Does tqc hypothesis might allow to understand the structure of living cell at a deeper level? What does this hypothesis predict about DNA itself? One of the challenges is to fuse the vision about living system as a conscious hologram with the DNA as tqc vision. The experimental findings of Peter Gariaev [?, ?] might provide a breakthrough in this respect. In particular, the very simple experiment in which one irradiates DNA sample using ordinary light in UV-IR range and photographs the scattered light seems to allow an interpretation as providing a photograph of magnetic flux tubes containing dark matter. If this is really the case, then the bottle neck problem of how to make dark matter visible and how to manipulate it would have been resolved in principle. The experiment of Gariaev and collaborators [?] also show that the photographs are obtained only in the presence of DNA sample. This leaves open the question whether the magnetic flux tubes associated with instruments are there in absence of DNA and only made visible by DNA or generated by the presence of DNA.

5.1.1 Basic ideas of tqc

The basic idea of topological quantum computation (tqc) is to code tqc programs to braiding patterns (analogous to linking and knotting). A nice metaphor for tqc is as dance. Dancing pattern in time direction defines the tqc program. This kind of patterns are defined by any objects moving around so that the Universe might be performing topological quantum computation like activities in all scales.

One assigns to the strands of the braid elementary particles. The S-matrix coding for tqc is determined by purely topological consideration as a representation for braiding operation. It is essential that the particles are in anyonic phase: this means in TGD framework that the value of Planck constant differs from its standard value. Tqc as any quantum computation halts in state function reduction which corresponds to the measurement of say spins of the particles involved.

As in the case of ordinary computers one can reduce the hardware to basic gates. The basic 2-gate is represented by a purely topological operation in which two neighboring braid strands are twisted by π. 1-particle gate corresponds to a phase multiplication of the quantum state associated with braid strand. This operation is not purely topological and requires large Planck constant to overcome the effects of thermal noise.

In TGD framework tqc differs somewhat from the ordinary one.

1. Zero energy ontology means that physical states decompose into pairs of positive and negative energy states at boundaries of causal diamond formed by future and past directed lightcones containing the particles at their light-like boundaries. The interpretation is as an event, say particle scattering, in positive energy ontology. The time like entanglement coefficients define S-matrix, or rather M-matrix, and this matrix can be interpreted as coding for physical laws in the structure of physical state as quantum superposition of statements "A implies B" with A and B represented as positive and negative energy parts of quantum state. The halting of topological quantum computation would select this kind of statement.

2. The new view about quantum state as essentially 4-D notion implies that the outcome of tqc is expressed as a four-dimensional pattern at space-time sheet rather than as time=constant final state. All kinds of patterns would provide a representation of this kind. In particular, holograms formed by large $\hbar$ photons emitted by Josephson currents, including EEG as a special case, would define particular kind of representation of outcome.

5.1.2 Identification of hardware of tqc and tqc programs

One challenge is to identify the hardware of tqc and realization of tqc programs.

1. Living cell is an excellent candidate in this respect. The lipid layers of the cell membrane is 2-D liquid crystal and the 2-D motion of lipids would define naturally the braiding if the lipids are connected to DNA nucleotides. This motion might be induced by the self organization patterns of metabolically driven liquid flow in the vicinity of lipid layer both in interior and exterior of cell membrane and thus self-organization patters of the water flow would define the tqc programs.

2. This identification of braiding implies that tqc as dancing pattern is coded automatically to memory in the sense that lipids connected to nucleotides are like dancers whose feet are connected to the wall of the dancing hall define automatically space-like braiding as the threads connected to their feet get braided. This braiding would define universal memory realized not only as tissue memory but related also to water memory [**?**].

3. It is natural to require that the genetic code is somehow represented as property of braids strands. This is achieved if strands are "colored" so that A,T,C,G correspond to four different "colors". This leads to the hypothesis that flux tubes assignable to nucleotides are wormhole magnetic flux tubes such that the ends of the two sheets carry quark and antiquark (*resp.* antiquark and quark) quantum numbers. This gives mapping A,T,C,G to u, u_c, d, d_c. These quarks are not ordinary quarks but their scaled variants predicted by the fractal hierarchy of color and electro-weak physics. Chiral selection in living matter could be explained by the hierarchy of weak physics. The findings of topologist Barbara Shipman about mathematical structure of honeybee dance led her to proposed that the color symmetries of quarks are in some mysterious manner involved with honeybee cognition and this model would justify her intuition [**?**].

4. One should identify the representation of qubit. Ordinary spin is not optimal since the representation of 1-gates would require a modification of direction of magnetic field in turn requiring modification of direction of flux tubes. A more elegant representation is based on quark color which means effectively 3-valued logic: true, false, and undefined, also used in ordinary computers and is natural in a situation in which information is only partial. In this case 1-gates would correspond to color rotations for space-time sheets requiring no rotation of the magnetic field.

In this framework genes define the hardware of tqc rather than genetic programs. This means that the evolution takes place also at the level of tqc programs meaning that strict genetic determinism fails. There are also good reasons to believe that these tqc programs can be inherited to some degree. This could explain the huge differences between us and our cousins in spite of almost the identical genetic codes and explains also cultural evolution and the observation that our children seem to learn more easily those things that we have already learned [**?**]. It must be added that DNA as tqc paradigm seems to generalized: DNA, lipids, proteins, water molecules,... can have flux tubes connecting them together and this is enough to generate braidings and tqc programs. Even water could be performing simple tqc or at least building memory representations based on braiding of flux tubes connecting water molecules.

5.1.3 How much tqc resembles ordinary computation?

If God made us to his own image one can ask whether we made computers images of ourselves in some respects. Taking this seriously one ends up asking whether facts familiar to us from ordinary computers and world wide web might have counterparts in DNA as tqc paradigm.

1. Can one identify program files as space-like braiding patterns. Can one differentiate between program files and data files?

2. In ordinary computers electromagnetic signalling is in key role. The vision about living matter as conscious holograms suggests that this is the case also now. In particular, the idea that entire biosphere forms a tqc web communicating electromagnetically information and control signals looks natural. Topological light rays (MEs) make possible precisely targeted communications with light velocity without any change in pulse shape. Gariaev's findings [?] that the irradiation of DNA by laser light induces emission of radio wave photons having biological effects on living matter at distances of tens of kilometers supports this kind of picture. Also the model of EEG in which the magnetic body controls the biological body also from astrophysical distances conforms with this picture.

3. The calling of computer programs by simply clicking the icon or typing the name of program followed by return is an extremely economic manner to initiate complex computer programs. This also means that one can construct arbitrarily complex combinations from given basic modules and call this complex by a single name if the modules are able to call each other. This kind of program call mechanism could be realized at the level of tqc by DNA. Since the intronic portion of genome increases with the evolutionary level and is about 98 per cent for humans, one can ask whether introns would contain representations for names of program modules. If so, introns would express themselves electromagnetically by transcribing the nucleotide to a temporal pattern of electromagnetic radiation activating desired subprogram call, presumably the conjugate of intronic portion as DNA sequence. A hierarchical sequence of subprogram calls proceeding downwards at intronic level and eventually activating the tqc program leading to gene expression is suggestive.

   Gariaev [?] has found that laser radiation scattering from given DNA activates only genomes which contain an address coded as temporal pattern for the direction of polarization plane. If flux tubes are super-conducting and there is strong parity breaking (chiral selection) then Faraday rotation for photons traveling through the wormhole flux tube code nucleotide to an angle characterizing the rotation of polarization plane. User id and password would be kind of immune system against externally induced gene expression.

4. Could nerve pulses establish only the connection between receiver and sender neurons as long magnetic flux tubes? Real communication would take place by electromagnetic signals along the flux tube, using topological light ray (ME) attached to flux tube, and by entanglement. Could neural transmitters specify which parts of genomes are in contact and thus serve as a kind of directory address inside the receiving genome?

5.1.4 Basic predictions of DNA as tqc hypothesis

DNA as tqc hypothesis leads to several testable predictions about DNA itself.

Anomalous em charge

The model for DNA as tqc assigns to flux tubes starting from DNA an anomalous em charge. This means that the total charge of DNA nucleotide using e as unit is $Q=-2+Q(q)$, where -2 is the charge of phosphate group and $Q(q)=-/+2/3,+/-1/3$ is the electromagnetic charge of quark associated with "upper" sheet of wormhole magnetic flux tube. If the phosphate group is not present one has $Q=Q(q)$. In the presence of phosphate bonds the anomalous charge makes possible the coding of nucleotides to the rotation of angle of polarization plane resulting as photon travels along magnetic flux tube. The anomalous em charge should be visible as an anomalous voltage created by DNA. It would be relatively easy to test this prediction by using various kinds of DNA:s.

Does breaking of matter antimatter and isospin symmetries happen at the level of DNA and mRNA?

The nice feature of the model is that it allows to interpret the slightly broken A-G and T-C symmetries of genetic code with respect to the third nucleotide Z of codon XYZ in terms of the analog of strong isospin symmetry at quark level at wormhole magnetic flux tubes. Also matter-antimatter dichotomy has a chemical analog in the sense that if the letter Y of codon corresponds to quark u, d (antiquark u_c, d_c), the codon codes for hydrophobic (hydrophilic) aminoacid. It is also known that the first letter X of the codon codes for the reaction path leading from a precursor to an aminoacid. These facts play a key role in the model for code of protein folding and catalysis. The basic assumption generalizing base pairing for DNA nucleotides is that wormhole flux tubes can connect an aminoacid inside protein only to molecules (aminoacids, DNA, mRNA, or tRNA) for which Y letter is conjugate to that associated with the aminoacid. This means that the reduction of Planck constant leading to the shortening of the flux tube can bring only these aminoacids together so that only these molecules can find each other in biocatalysis: this would mean kind of code of bio-catalysis.

The fact that matter-antimatter and isospin symmetries are broken in Nature suggests that the same occurs at the level of DNA for quarks and anti-quarks coding for nucleotides. One would expect that genes and other parts of genome differ in the sense that the anomalous em charge, isospin, and net quark number (vanishes for matter antimatter symmetric situation) differ for them. From Wikipedia [?] one learns that there are rules about distribution of nucleotides which cannot be understood on basis of chemistry. The rules could be understood in terms of new physics. Chargaff's rules state that these symmetries hold true in one per cent approximation at the level of entire chromosomes. Szybalski's rules [?] state that they fail for genes. There is also a rule stating that in good approximation both strands contain the same portion of DNA transcribed to mRNA. This implies that at mRNA level the sign of matter antimatter asymmetry is always the same: this is analogous to the breaking of matter antimatter asymmetry in cosmology (only matter is observed).

It would be interesting to study systematically the breaking of these symmetries for a sufficiently large sample of genes and also other in parts of genome where a compensating symmetry breaking must occur. that the irradiation of DNA by laser light induces emission of radio wave photons having biological effects on living matter at distances of tens of kilometers supports this kind of picture. Also the model of EEG in which magnetic body controls biological body from astrophysical distances conforms with this picture.

5.2 How quantum computation in TGD Universe differs from standard quantum computation?

Many problems of quantum computation in standard sense might relate to a wrong view about quantum theory. If TGD Universe is the physical universe, the situation would improve in many respects. There is the new fractal view about quantum jump and observer as "self"; there is p-adic length scale hierarchy and hierarchy of Planck constants as well as self hierarchy; there is a new view about entanglement and the possibility of irreducible entanglement carrying genuine information and making possible quantum superposition of fractal quantum computations and quantum parallel dissipation; there is zero energy ontology, the notion of M-matrix allowing to understand quantum theory as a square root of thermodynamics, the notion of measurement resolution allowing to identify M-matrix in terms of Connes tensor product; there is also the notion of magnetic body providing one promising realization for braids in tqc, etc... This section gives a short summary of these aspects of TGD.

There is also a second motivation for this section. Quantum TGD and TGD inspired theory of consciousness involve quite a bundle of new ideas and the continual checking of internal consistency by writing it through again and again is of utmost importance. This section can be also seen as this kind of checking. I can only represent apologies to the benevolent reader: this is a work in rapid progress.

5.2.1 General ideas related to topological quantum computation

Topological computation relies heavily on the representation of tqc program as a braiding. There are many kinds of braidings. Number theoretic braids are defined by the orbits of minima of vacuum expectation of Higgs at lightlike partonic 3-surfaces (and also at space-like 3-surfaces). There are braidings defined by Kähler gauge potential (possibly equivalent with number theoretic ones) and by Kähler magnetic field. Magnetic flux tubes and partonic 2-surfaces interpreted as strands of define braidings whose strands are not infinitely thin. A very concrete and very complex time-like braiding is defined by the motions of people at the surface of globe: perhaps this sometimes purposeless-looking fuss has a deeper purpose: maybe those at the higher levels of dark matter hierarchy are using us to carry out complex topological quantum computations)!

General vision about quantum computation

In TGD Universe the hierarchy of Planck constants gives excellent prerequisites for all kinds of quantum computations. The general vision about quantum computation (tqc) would result as a special case and would look like follows.

1. Time-like entanglement between positive and negative energy parts of zero energy states would define the analogs of qc-programs. Space-like quantum entanglement between ends of strands whose motion defines time-like braids would provide a representation of q-information.

2. Both time- and space-like quantum entanglement would correspond to Connes tensor product expressing the finiteness of the measurement resolution between the states defined at ends of space-like braids whose orbits define time like braiding. The characterization of the measurement resolution would thus define both possible q-data and tq-programs as representations for "laws of physics".

3. The braiding between DNA strands with each nucleotide defining one strand transversal to DNA realized in terms of magnetic flux tubes was my first bet for the representation of space-like braiding in living matter. It turned out that the braiding is more naturally defined by flux tubes connecting nucleotides to the lipids of nuclear-, cell-, and endoplasma membranes. Also braidings between other microtubules and axonal membrane can be considered. The conjectured hierarchy of genomes giving rise to quantum coherent gene expressions in various scales would correspond to computational hierarchy.

About the relation between space-like and time-like number theoretic braidings

The relationship between space- and time-like braidings is interesting and there might be some connections also to 4-D topological gauge theories suggested by geometric Langlands program discussed in the previous posting and also in [?].

1. The braidings along light-like surfaces modify space-like braiding if the moving ends of the space-like braids at partonic 3-surfaces define time-like braids. From tqc point of view the interpretation would be that tqc program is written to memory represented as the modification of space-like braiding in 1-1 correspondence with the time-like braiding.

2. The orbits of space-like braids define codimension two sub-manifolds of 4-D space-time surface and can become knotted. Presumably time-like braiding gives rise to a non-trivial "2-braid". Could also the "2-braiding" based on this knotting be of importance? Do 2-connections of n-category theorists emerge somehow as auxiliary tools? Could 2-knotting bring additional structure into the topological QFT defined by 1-braidings and Chern-Simons action?

3. The strands of dynamically evolving braids could in principle go through each other so that time evolution can transform braid to a new one also in this manner. This is especially clear from standard representation of knots by their planar projections. The points where intersection occurs correspond to self-intersection points of 2-surface as a sub-manifold of space-time surface. Topological QFT:s are also used to classify intersection numbers of 2-dimensional surfaces understood as homological equivalence classes. Now these intersection points would be associated with "braid cobordism".

Quantum computation as quantum superposition of classical computations?

It is often said that quantum computation is quantum super-position of classical computations. In standard path integral picture this does not make sense since between initial and final states represented by classical fields one has quantum superposition over *all* classical field configurations representing classical computations in very abstract sense. The metaphor is as good as the perturbation theory around the minimum of the classical action is as an approximation.

In TGD framework the classical space-time surface is a preferred extremal of Kähler action so that apart from effects caused by the failure of complete determinism, the metaphor makes sense precisely. Besides this there is of course the computation associated with the spin like degrees of freedom in which one has entanglement and which one cannot describe in this manner.

For tqc a particular classical computation would reduce to the time evolution of braids and would be coded by 2-knot. Classical computation would be coded to the manipulation of the braid. Note that the branching of strands of generalized number theoretical braids has interpretation as classical communication.

The identification of topological quantum states

Quantum states of tqc should correspond to topologically robust degrees of freedom separating neatly from non-topological ones.

1. The generalization of the imbedding space inspired by the hierarchy of Planck constants suggests an identification of this kind of states as elements of the group algebra of discrete subgroup of $SO(3)$ associated with the group defining covering of M^4 or CP_2 or both in large $\hbar$ sector. One would have wave functions in the discrete space defined by the homotopy group of the covering transforming according to the representations of the group. This is by definition something robust and separated from non-topological degrees of freedom (standard model quantum numbers). There would be also a direct connection with anyons.

2. An especially interesting group is dodecahedral group corresponding to the minimal quantum phase $q = exp(2\pi/5)$ (Golden Mean) allowing a universal topological quantum computation: this group corresponds to Dynkin diagram for E_8 by the ALE correspondence. Interestingly, neuronal synapses involve clathrin molecules [**?**] associated with microtubule ends possessing dodecahedral symmetry.

Some questions

A conjecture inspired by the inclusions of HFFs is that these states can be also regarded as representations of various gauge groups which TGD dynamics is conjectured to be able to mimic so that one might have connection with non-Abelian Chern-Simons theories where topological S-matrix is constructed in terms of path integral over connections: these connections would be only an auxiliary tool in TGD framework.

1. Do these additional degrees of freedom give only rise to topological variants of gauge- and conformal field theories? Note that if the earlier conjecture that entire dynamics of these theories could be mimicked, it would be best to perform tqc at quantum criticality where either M^4 or CP_2 dynamical degrees of freedom or both disappear.

2. Could it be advantageous to perform tqc near quantum criticality? For instance, could one construct magnetic braidings in the visible sector near q-criticality using existing technology and then induce phase transition changing Planck constant by varying some parameter, say temperature.

5.2.2 Fractal hierarchies

Fractal hierarchies are the essence of TGD. There is hierarchy of space-time sheets labelled by preferred p-adic primes. There is hierarchy of Planck constants reflecting a book like structure of the generalized imbedding space and identified in terms of a hierarchy of dark matters. These hierarchies correspond at the level of conscious experience to a hierarchy of conscious entities - selves: self experiences its sub-selves as mental images.

Fractal hierarchies mean completely new element in the model for quantum computation. The decomposition of quantum computation to a fractal hierarchy of quantum computations is one implication of this hierarchy and means that each quantum computation proceeds from longer to shorter time scales $T_n = T_0 2^{-n}$ as a cascade like process such that at each level there is a large number of quantum computations performed with various values of input parameters defined by the output at previous level. Under some additional assumptions to be discussed later this hierarchy involves at a given level a large number of replicas of a given sub-module of tqc so that the output of single fractal sub-module gives automatically probabilities for various outcomes as required.

5.2.3 Irreducible entanglement and possibility of quantum parallel quantum computation

The basic distinction from standard measurement theory is irreducible entanglement not reduced in quantum jump.

NMP and the possibility of irreducible entanglement

Negentropy Maximimization Principle (NMP) states that entanglement entropy is minimized in quantum jump. For standard Shannon entropy this would lead to a final state which corresponds to a ray of state space. If entanglement probabilities are rational - or even algebraic - one can replace Shannon entropy with its number theoretic counterpart in which p-adic norm of probability replaces the probability in the argument of logarithm: $log(p_n) \rightarrow log(|p_n|_p)$. This entropy can have negative values. It is not quite clear whether prime p should be chosen to maximize the number theoretic negentropy or whether p is the p-adic prime characterizing the light-like partonic 3-surface in question.

Obviously NMP favors generation of irreducible entanglement which however can be reduced in U process. Irreducible entanglement is something completely new and the proposed interpretation is in terms of experience of various kinds of conscious experiences with positive content such as understanding.

Quantum superposition of unitarily evolving quantum states generalizes to a quantum superposition of quantum jump sequences defining dissipative time evolutions. Dissipating quarks inside quantum coherent hadrons would provide a basic example of this kind of situation.

Quantum parallel quantum computations and conscious experience

The combination of quantum parallel quantum jump sequences with the fractal hierarchies of scales implies the possibility of quantum parallel quantum computations. In ordinary quantum computation halting selects single computation but in the recent case arbitrarily large number of computations can be carried out simultaneously at various branches of entangled state. The probability distribution for the outcomes is obtained using only single computation.

One would have quantum superposition of space-time sheets (assignable to the maxima of Kähler function) each representing classically the outcome of a particular computation. Each branch would correspond to its own conscious experience but the entire system would correspond to a self experiencing consciously the outcome of computation as intuitive and holistic understanding, and abstraction. Emotions and emotional intellect could correspond to this kind of non-symbolic representation for the outcome of computation as analogs for collective parameters like temperature and pressure.

Delicacies

There are several delicacies involved.

1. The above argument works for factors of type I. For HFFs of type II_1 the finite measurement resolution characterized in terms of the inclusion $\mathcal{N} \subset \mathcal{M}$ mean is that state function reduction takes place to $\mathcal{N}$-ray. There are good reasons to expect that the notion of number theoretic entanglement negentropy generalizes also to this case. Note that the entanglement associated with $\mathcal{N}$ is below measurement resolution.

2. In TGD inspired theory of consciousness irreducible entanglement makes possible sharing and fusion of mental images. At space-time level the space-time sheets corresponding to selves are disjoint but the space-time sheets topologically condensed at them are joined typically by what I call join along boundaries bonds identifiable as braid strands (magnetic flux quanta). In topological computation with finite measurement resolution this kind of entanglement with environment would be below the natural resolution and would not be a problem.

3. State function reduction means quantum jump to an eigen state of density matrix. Suppose that density matrix has rational elements. Number theoretic vision forces to ask whether the quantum jump to eigen state is possible if the eigenvalues of ρ do not belong to the algebraic extension of rationals and p-adic numbers used. If not, then one would have number theoretically irreducible entanglement depending on the algebraic extension used. If the eigenvalues actually define the extension there would be no restrictions: this option is definitely simpler.

4. Fuzzy quantum logic [**?**] brings also complications. What happens in the case of quantum spinors that spin ceases to be observable and one cannot reduce the state to spin up or spin down. Rather, one can measure only the eigenvalues for the probability operator for spin up (and thus for spin down) so that one has fuzzy quantum logic characterized by quantum phase. Inclusions of HFFs are characterized by quantum phases and a possible interpretation is that the quantum parallelism related to the finite measurement resolution could give rise to fuzzy qubits. Also the number theoretic quantum parallelism implied by number theoretic NMP could effectively make probabilities as operators. The probabilities for various outcomes would correspond to outcomes of quantum parallel state function reductions.

5.2.4 Connes tensor product defines universal entanglement

Both time-like entanglement between quantum states with opposite quantum numbers represented by M-matrix and space-like entanglement reduce to Connes tensor dictated highly uniquely by measurement resolution characterized by inclusion of HFFs of type II_1

Time-like and space-like entanglement in zero energy ontology

If hyper-finite factors of II_1 are all that is needed then Connes tensor product defines universal S-matrix and the most general situation corresponds to a direct sum of them. M-matrix for each summand is product of Hermitian square root of density matrix and unitary S-matrix multiplied by a square root of probability having interpretation as analog for Boltzmann weight or probability defined by density matrix (note that it is essential to have $Tr(Id) = 1$ for factors of type II_1. If factor of type I_∞ are present situation is more complex. This means that quantum computations are highly universal and M-matrices are characterized by the inclusion $\mathcal{N} \subset \mathcal{M}$ in each summand defining measurement resolution. Hermitian elements of $\mathcal{N}$ act as symmetries of M-matrix. The identification of the reducible entanglement characterized by Boltzmann weight like parameters in terms of thermal equilibrium would allow to interpret quantum theory as square root of thermodynamics.

If the entanglement probabilities defined by S-matrix and assignable to $\mathcal{N}$ rays do not belong to the algebraic extension used then a full state function reduction is prevented by NMP. Ff the generalized Boltzmann weights are also algebraic then also thermal entanglement is irreducible. In p-adic thermodynamics for Virasoro generator L_0 and using some cutoff for conformal weights the Boltzmann weights are rational numbers expressible using powers of p-adic prime p.

Effects of finite temperature

Usually finite temperature is seen as a problem for quantum computation. In TGD framework the effect of finite temperature is to replace zero energy states formed as pairs of positive and negative energy states with a superposition in which energy varies.

One has an ensemble of space-time sheets which should represent nearly replicas of the quantum computation. There are two cases to be considered.

1) If the thermal entanglement is reducible then each space-time sheet gives outcome corresponding to a well defined energy and one must form an average over these outcomes.

2) If thermal entanglement is irreducible each space-time sheet corresponds to a quantum superposition of space-time sheets, and if the outcome is represented classically as rates and temporal field patterns, it should reflect thermal average of the outcomes as such.

If the degrees of freedom assignable to topological quantum computation do not depend on the energy of the state, thermal width does not affect at all the relevant probabilities. The probabilities are actually affected even in the case of tqc since 1-gates are not purely topological and the effects of temperature in spin degrees of freedom are unavoidable. If T grows the probability distribution for the outcomes flattens and it becomes difficult to select the desired outcome as that appearing with the maximal probability.

5.2.5 Possible problems related to quantum computation

At least following problems are encountered in quantum computation.

1. How to preserve quantum coherence for a long enough time so that unitary evolution can be achieved?

2. The outcome of calculation is always probability distribution: for instance, the output with maximum probability can correspond to the result of computation. The problem is how to replicate the computation to achieve the desired accuracy. Or more precisely, how to produce replicas of the hardware of quantum computer defined in terms of classical physics?

3. How to isolate the quantum computer from the external world during computation and despite this feed in the inputs and extract the outputs?

The notion of coherence region in TGD framework

In standard framework one can speak about coherence in two senses. At the level of Schrödinger amplitudes one speaks about coherence region inside which it makes sense to speak about Schrödinger time evolution. This notion is rather defined.

In TGD framework coherence region is identifiable as a region inside which the modified Dirac equation holds true. Strictly speaking, this region corresponds to a light-like partonic 3-surface whereas 4-D space-time sheet corresponds to coherence region for classical fields. p-Adic length scale hierarchy and hierarchy of Planck constants means that arbitrarily large coherence regions are possible.

The precise definition for the notion of coherence region and the presence of scale hierarchies imply that the coherence in the case of single quantum computation is not a problem in TGD framework. De-coherence time or coherence time correspond to the temporal span of space-time sheet and a hierarchy coming in powers of two for a given value of Planck constant is predicted by basic quantum TGD. p-Adic length scale hypothesis and favored values of Planck constant would naturally reflect this fundamental fractal hierarchy.

De-coherence of density matrix and replicas of tqc

Second phenomenological description boils down to the assumption that non-diagonal elements of the density matrix in some preferred basis (involving spatial localization of particles) approach to zero. The existence of more or less faithful replicas of space-time sheet in given scale allows to identify the counterpart of this notion in TGD context. De-coherence would mean a loss of information in the averaging of M-matrix and density matrix associated with these space-time sheets.

Topological computations are probabilistic. This means that one has a collection of space-time sheets such that each space-time sheet corresponds to more or less the same tqc and therefore the same M-matrix. If M is too random (in the limits allowed by Connes tensor product), the analog of generalized phase information represented by its "phase" - S-matrix - is useless.

In order to avoid de-coherence in this sense, the space-time sheets must be approximate copies of each other. Almost copies are expected to result by dissipation leading to asymptotic self-organization patterns depending only weakly on initial conditions and having also space-time correlates. Obviously, the role of dissipation in eliminating effects of de-coherence in tqc would be something new. The enormous symmetries of M-matrix, the uniqueness of S-matrix for given resolution and parameters characterizing braiding, fractality, and generalized Bohr orbit property of space-time sheets, plus dissipation give good hopes that almost replicas can be obtained.

Isolation and representations of the outcome of tqc

The interaction with environment makes quantum computation difficult. In the case of topological quantum computation this interaction corresponds to the formation of braid strands connecting the computing space-time sheet with space-time sheets in environment. The environment is four-dimensional in TGD framework and an isolation in time direction might be required. The space-time sheets responsible for replicas of tqc should not be connected by light-like braids strands having time-like projections in M^4.

Length scale hierarchy coming in powers of two and finite measurement resolution might help considerably. Finite measurement resolution means that those strands which connect space-time sheets topologically condensed to the space-time sheets in question do not induce entanglement visible at this level and should not affect tqc in the resolution used.

Hence only the elimination of strands responsible for tqc at given level and connecting computing space-time sheet to space-time sheets at same level in environment is necessary and would require magnetic isolation. Note that super-conductivity might provide this kind of isolation. This kind of elimination could involve the same mechanism as the initiation of tqc which cuts the braid strands so the initiation and isolation might be more or less the same thing.

Strands reconnect after the halting of tqc and would make possible the communication of the outcome of computation along strands by using say em currents in turn generating generalized EEG, nerve pulse patterns, gene expression, etc... halting and initiation could be more or less synonymous with isolation and communication of the outcome of tqc.

How to express the outcome of quantum computation?

The outcome of quantum computation is basically a representation of probabilities for the outcome of tqc. There are two representations for the outcome of tqc. Symbolic representation which quite generally is in terms of probability distributions represented in terms "classical space-time" physics. The rates for various processes having basically interpretation as geometro-temporal densities would represent the probabilities just as in the case of particle physics experiment. For tqc in living matter this would correspond to gene expression, neural firing, EEG patterns,...

A representation as a conscious experience is another (and actually the ultimate) representation of the outcome. It need not have any symbolic counterpart since it is felt. Intuition, emotions and emotional intelligence would naturally relate to this kind of representation made possible by irreducible entanglement. This representation would be based on fuzzy qubits and would mean that the outcome would be true or false only with certain probability. This unreliability would be felt consciously.

The proposed model of tqc combined with basic facts about theta waves [**?**, **?**] to be discussed in the subsection about the role of supra currents in tqc suggests that EEG rhythm (say theta rhythm) and correlated firing patterns correspond to the isolation at the first half period of tqc and random firing at second half period to the sub-sequent tqc:s at shorter time scales coming as negative powers of 2. The fractal hierarchy of time scales would correspond to a hierarchy of frequency scales for generalized EEG and power spectra at these scales would give information about the outcome of tqc. Synchronization would be obviously an essential element in this picture and could be understood in terms of classical dynamics which defines space-time surface as a generalized Bohr orbit.

Tqc would be analogous to the generation of a dynamical hologram or "conscious hologram" [**?**]. EEG rhythm would correspond to reference wave generated by magnetic body as control and coordination signal and the contributions of spikes to EEG generated by neurons would correspond to the incoming wave interfering with the reference wave.

How data is feeded into submodules of tqc?

Scale hierarchy obviously gives tqc a fractal modular structure and the question is how data is feeded to submodules at shorter length scales. There are certainly interactions between different levels of scale hierarchy. The general ideas about master-slave hierarchy assigned with self-organization support the hypothesis that these interactions are directed from longer to shorter scales and have interpretation as a specialization of input data to tqc sub-modules represented by smaller space-time sheets of hierarchy. The call of submodule would occur when the tqc of the calling module halts and the result of computation is expressed as a 4-D pattern. The lower level module would start only after the halting of tqc (with respect to subjective time at least) and the durations of resulting tqc's would come as $T_n = 2^{-n}T_0$ that geometric series of tqc's would become possible. There would be entire family of tqc's at lower level corresponding to different values of input parameters from calling module.

One of the ideas assigned to hyper-computation [?] is that one can have infinite series of computations with durations comings as negative powers of 2 (Zeno paradox obviously inspires this idea). In TGD framework there can be however only a finite series of these tqc's since CP_2 time scale poses a lower bound for the duration of tqc. One might of course ask whether the spectrum of Planck constant could help in this respect.

The role of dissipation and energy feed

Dissipation plays key role in the theory of self-organizing systems [?]. Its role is to serve as a Darwinian selector. Without an external energy feed the outcome is a situation in which all organized motions disappear. In presence of energy feed highly unique self-organization patterns depending only very weakly on the initial conditions emerge.

In the case of tqc one function of dissipation would be to drive the braidings to static standard configurations, and perhaps even effectively eliminate fluctuations in non-topological degrees of freedom. Note that magnetic fields are important for 1-gates. Magnetic flux conservation however saves magnetic fields from dissipation.

External energy feed is needed in order to generate new braidings. For the proposed model of cellular tqc the flow of intracellular water induces the braiding and requires energy feed. Also now dissipation would drive this flow to standard patterns coding for tqc programs. Metabolic energy would be also needed in order to control whether lipids can flow or not by generating cis type unsaturated bonds. Obviously, energy flows defining self organization patterns would define tqc programs.

Is it possible to realize arbitrary tqc?

The 4-D spin glass degeneracy of TGD Universe due to the enormous vacuum degeneracy of Kähler action gives good hopes that the classical dynamics for braidings allows to realize every possible tqc program. As a consequence, space-time sheets decompose to maximal non-deterministic regions representing basic modules of tqc. Similar decomposition takes place at the level of light-like partonic 3-surfaces and means decomposition to 3-D regions inside which conformal invariance eliminates light-like direction as dynamical degree of freedom so that the dynamics is effectively that of 2-dimensional object. Since these 3-D regions behave as independent units as far as longitudinal conformal invariance is considered, one can say that light-like 3-surfaces are 3-dimensional in discretized sense. In fact, for 2-D regions standard conformal invariance implies similar effective reduction to 1-dimensional dynamics realized in terms of a net of strings and means that 2-dimensionality is realized only in discretized sense.

5.3 DNA as topological quantum computer

Braids [?] code for topological quantum computation. One can imagine many possible identifications of braids but this is not essential for what follows. What is highly non-trivial is that the motion of the ends of strands defines both time-like and space-like braidings with latter defining in a well-defined sense a written version of the tqc program, kind of log file. The manipulation of braids is a central element of tqc and if DNA really performs tqc, the biological unit modifying braidings should be easy to identify. An obvious signature is the 2-dimensional character of this unit.

5.3.1 Conjugate DNA as performer of tqc and lipids as quantum dancers

In this section the considerations are restricted to DNA as tqc. It is however quite possible that also RNA and other biomolecules could be involved with tqc like process.

Sharing of labor

The braid strands must begin from DNA double strands. Precisely which part of DNA does perform tqc? Genes? Introns[?]? Or could it be conjugate DNA which performs tqc? The function of conjugate

DNA has indeed remained a mystery and sharing of labor suggests itself.

Conjugate DNA would do tqc and DNA would "print" the outcome of tqc in terms of RNA yielding amino-acids in the case of exons. RNA could the outcome in the case of introns. The experience about computers and the general vision provided by TGD suggests that introns could express the outcome of tqc also electromagnetically in terms of standardized field patterns. Also speech would be a form of gene expression. The quantum states braid would entangle with characteristic gene expressions. This hypothesis will be taken as starting point in the following considerations.

Cell membranes as modifiers of braidings defining tqc programs?

The manipulation of braid strands transversal to DNA must take place at 2-D surface. The ends of the space-like braid are dancers whose dancing pattern defines the time-like braid, the running of classical tqc program. Space-like braid represents memory storage and tqc program is automatically written to memory during the tqc. The inner membrane of the nuclear envelope and cell membrane with entire endoplasmic reticulum included are good candidates for dancing hall. The 2-surfaces containing the ends of the hydrophobic ends of lipids could be the parquets and lipids the dancers. This picture seems to make sense.

1. Consider first the anatomy of membranes. Cell membrane [?] and membranes of nuclear envelope [?] consist of 2 lipid [?] layers whose hydrophobic ends point towards interior. There is no water here nor any direct perturbations from the environment or interior milieu of cell. Nuclear envelope consists of two membranes having between them an empty volume of thickness 20-40 nm. The inner membrane consists of two lipid layers like ordinary cell membrane and outer membrane is connected continuously to endoplasmic reticulum [?], which forms a highly folded cell membrane. Many biologists believe that cell nucleus is a prokaryote, which began to live in symbiosis with a prokaryote defining the cell membrane.

2. What makes dancing possible is that the phospholipid layers of the cell membrane are liquid crystals [?]: the lipids can move freely in the horizontal direction but not vertically. "Phospho" could relate closely to the metabolic energy needs of dancers. If these lipids are self-organized around braid strands, their dancing patterns along the membrane surface would be an ideal manner to modify braidings since the lipids would have standard positions in a lattice. This would be like dancing on a chessboard. Note that the internal structure of lipid does not matter in this picture since it is braid color dicated by DNA nucleotide which matters. As a matter fact, living matter is full of self-organizing liquid crystals and one can wonder whether the deeper purpose of their life be running and simultaneous documentation of tqc programs?

3. Ordinary computers have an operating system [?]: a collection of standard programs - the system - and similar situation should prevail now. The "printing" of outputs of tqc would represent example of this kind of standard program. This tqc program should not receive any input from the environment of the nucleus and should therefore correspond to braid strands connecting conjugate strand with strand. Braid strands would go only through the inner nuclear membrane and return back and would not be affected much since the volume between inner and outer nuclear membranes is empty. This assumption looks ad hoc but it will be found that the requirement that these programs are inherited as such in the cell replication necessitates this kind of structure (see the section "Cell replication and tqc").

4. The braid strands starting from the conjugate DNA could traverse several time through the highly folded endoplasmic reticulum but without leaving cell interior and return back to nucleus and modify tqc by intracellular input. Braid strands could also traverse the cell membrane and thus receive information about the exterior of cell. Both of these tqc programs could be present also in prokaryotes [?] but the braid strands would always return back to the DNA, which can be also in another cell. In multicellulars (eukaryotes [?]) braid strands could continue to another cell and give rise to "social" tqc programs performed by the multicellular organisms. Note that the topological character of braiding does not require isolation of braiding from environment. It might be however advantageous to have some kind of sensory receptors amplifying sensory input to standardized re-braiding patterns. Various receptors in cell membrane would serve this purpose.

5. Braid strands can end up at the parquet defined by ends of the inner phospholipid layer: their distance of inner and outer parquet is few nanometers. They could also extend further.

   i) If one is interested in connecting cell nucleus to the membrane of another cell, the simpler option is the formation of hole defined by a protein attached to cell membrane. In this case only the environment of the second cell affects the braiding assignable to the first cell nucleus.

   ii) The bi-layered structure of the cell membrane could be essential for the build-up of more complex tqc programs since the strands arriving at two nearby hydrophobic 2-surfaces could combine to form longer strands. The formation of longer strands could mean the fusion of the two nearby hydrophobic two-surfaces in the region considered. In fact, tqc would begin with the cutting of the strands so that non-trivial braiding could be generated via lipid dance and tqc would halt when strands would recombine and define a modified braiding. This would allow to connect cell nucleus and cell membrane to a larger tqc unit and cells to multicellular tqc units so that the modification of tqc programs by feeding the information from the exteriors of cells - essential for the survival of multicellulars - would become possible.

Gene expression and other basic genetic functions from tqc point of view

It is useful to try to imagine how gene expression might relate to the halting of tqc. There are of course myriads of alternatives for detailed realizations, and one can only play with thoughts to build a reasonable guess about what might happen.

1. Qubits for transcription factors and other regulators

Genetics is consistent with the hypothesis that genes correspond to those tqc moduli whose outputs determine whether genes are expressed or not. The naive first guess would be that the value of single qubit determines whether the gene *is* expressed or not. Next guess replaces "*is* " with "*can be*".

Indeed, gene expression involves promoters, enhancers and silencers [?]. Promoters are portions of the genome near genes and recognized by proteins known as transcription factors [?]. Transcription factors bind to the promoter and recruit RNA polymerase, an enzyme that synthesizes RNA. In prokaryotes RNA polymerase itself acts as the transcription factor. For eukaryotes situation is more complex: at least seven transcription factors are involved with the recruitment of the RNA polymerase II catalyzing the transcription of the messenger RNA. There are also transcription factors for transcription factors and transcription factor for the transcription factor itself.

The implication is that several qubits must have value "Yes" for the actual expression to occur since several transcription factors are involved with the expression of the gene in general. In the simplest situation this would mean that the computation halts to a measurement of single qubit for subset of genes including at least those coding for transcription factors and other regulators of gene expression.

2. Intron-exon qubit

Genes would have very many final states since each nucleotide is expected to correspond to at least single qubit. Without further measurements that state of nucleotides would remain highly entangled for each gene. Also these other qubits are expected to become increasingly important during evolution.

For instance, eukaryotic gene expression involves a transcription of RNA and splicing out of pieces of RNA which are not translated to amino-acids (introns). Also the notion of gene is known to become increasingly dynamical during the evolution of eukaryotes so that the expressive power of genome increases. A single qubit associated with each codon telling whether it is spliced out or not would allow maximal flexibility. Tqc would define what genes are and the expressive power of genes would be due to the evolution of tqc programs: very much like in the case of ordinary computers. Stopping sign codon and starting codon would automatically tell where the gene begins and ends if the corresponding qubit is "Yes". In this picture the old fashioned static genes of prokaryotes without splicings would correspond to tqc programs for which the portions of genome with a given value of splicing qubit are connected.

3. What about braids between DNA, RNA, tRNA and amino-acids

This simplified picture might have created the impression that amino-acids are quantum outsiders obeying classical bio-chemistry. For instance, transcription factors would in this picture end up to the promoter by a random process and "Print" would only increase the density of the transcription factor. If DNA is able to perform tqc, it would however seem very strange if it would be happy with this rather dull realization of other central functions of the genetic apparatus.

One can indeed consider besides the braids connecting DNA and its conjugate - crucial for the success of replication - also braids connecting DNA to mRNA and other forms of RNA, mRNA to tRNA, and tRNA to amino-acids. These braids would provide the topological realization of the genetic code and would increase dramatically the precision and effectiveness of the transcription and translation if these processes correspond to quantum transitions at the level of dark matter leading more or less deterministically to the desired outcome at the level of visible matter be it formation of DNA doublet strand, of DNA-mRNA association, of mRNA-tRNA association or tRNA-amino-acid association.

For instance, a temporary reduction of the value of Planck constant for these braids would contract these to such a small size that these associations would result with a high probability. The increase of Planck constant for braids could in turn induce the transfer of mRNA from the nucleus, the opening of DNA double strand during transcription and mitosis.

Also DNA-amino-acid braids might be possible in some special cases. The braiding between regions of DNA at which proteins bind could be a completely general phenomenon. In particular, the promoter region of gene could be connected by braids to the transcription factors of the gene and the halting of tqc computation to printing command could induce the reduction of Planck constant for these braids inducing the binding of the transcription factor binds to the promoter region. In a similar manner, the region of DNA at which RNA polymerase binds could be connected by braid strands to the RNA polymerase.

How braid color is represented?

If braid strands carry 4-color (A,T,C,G) then also lipid strands should carry this kind of 4-color. The lipids whose hydrophobic ends can be joined to form longer strand should have same color. This color need not be chemical in TGD Universe.

Only braid strands of the same color can be connected as tqc halts. This poses strong restrictions on the model.

1. Do braid strands appear as patches possessing same color?

Color conservation is achieved if the two lipid layers decompose in a similar manner into regions of fixed color and the 2-D flow is restricted inside this kind of region at both layers. A four-colored map of cell membrane would be in question! Liquid crystal structure [?] applies only up to length scale of $L(151) = 10$ nm and this suggests that lipid layer decomposes into structural units of size $L(151)$ defining also cell membrane thickness. These regions might correspond to minimal regions of fixed color containing $N \sim 10^2$ lipids.

The controversial notion of lipid raft [?] was inspired by the immiscibility of ordered and disordered liquid phases in a liquid model of membrane. The organization to connected regions of particular phase could be a phenomenon analogous to a separation of phases in percolation. Many cell functions implicate the existence of lipid rafts. The size of lipid rafts has remained open and could be anywhere between 1 and 1000 nm. Also the time scale for the existence of a lipid raft is unknown. A line tension between different regions is predicted in hydrodynamical model but not observed. If the decomposition into ordered and disordered phases is time independent, ordered phases could correspond to those involved with tqc and possess a fixed color. If disordered phases contain no braid strands the mixing of different colors is avoided. The problem with this option is that it restricts dramatically the possible braidings.

If one takes this option seriously, the challenge is to make patches and patch color (A,T,C,G) visible. Perhaps one could try to mark regions of portions of lipid layer by some marker to find whether the lipid layer decomposes to non-mixing regions.

Quantum criticality suggests that that the patches of lipid layer have a fractal structure corresponding to a hierarchy of tqc program modules. The hydrodynamics would be thus fractal: patches containing patches.... moving with respect to each other would correspond to braids containing braids containing ... such that sub-braids behave as braid strands. In principle this is also a testable prediction.

2. Does braid color corresponds to some chemical property?

The conserved braid color is not necessary for the model but would imply genetic coding of the tqc hardware so that sexual reproduction would induce an evolution of tqc hardware. Braid color would also make the coupling of foreign DNA to the tqc performed by the organism difficult and realize an immune system at the level of quantum information processing.

The conservation of braid color poses however considerable problems. The concentration of braid strands of the same color to patches would guarantee the conservation but would restrict the possible braiding dramatically. A more attractive option is that the strands of same color find each other automatically by energy minimization after the halting of tqc. Electromagnetic Coulomb interaction would be the most natural candidate for the interaction in question. Braid color would define a faithful genetic code at the level of nucleotides. It would induce long range correlation between properties of DNA strand and the dynamics of cell immediately after the halting of tqc.

The idea that color could be a chemical property of phospholipids does not seem plausible. The lipid asymmetry of the inner and outer monolayers excludes the assignment of color to the hydrophilic groups PS, PI, PE, PCh. Fatty acids have $N = 14, ..., 24$ carbon atoms and $N = 16$ and 18 are the most common cases so that one could consider the possibility that the 4 most common feet pairs could correspond to the resulting combinations. It is however extremely difficult to understand how long range correlation between DNA nucleotide and fatty acid pair could be created.

3. Does braid color correspond to neutral quark pairs?

It seems that the color should be a property of the braid strand. In TGD inspired model of high T_c super-conductivity [**?**] wormhole contacts having u and $\overline{d}$ and d and $\overline{u}$ quarks at the two wormhole throats feed electron's gauge flux to larger space-time sheet. The long range correlation between electrons of Cooper pairs is created by color confinement for an appropriate scaled up variant of chromo-dynamics which are allowed by TGD. Hence the neutral pairs of colored quarks whose members are located the ends of braid strand acting like color flux tube connecting the nucleotide to the lipid could code DNA color to QCD color.

For the pairs $u\overline{d}$ with net em charge the quark and anti-quark have the same sign of em charge and tend to repel each other. Hence the minimization of electro-magnetic Coulomb energy favors the neutral configurations $u\overline{u}$, $d\overline{d}$ and $u\overline{u}$, and $d\overline{d}$ coding for A,T,C,G in some order.

After the halting of tqc only these pairs would form with a high probability. The reconnection of the strands would mean a formation of a short color flux tube between the strands and the annihilation of quark pair to gluon. Note that single braid strand would connect DNA color and its conjugate rather than identical colors so that braid strands connecting two DNA strands (conjugate strands) should always traverse through an even (odd) number of cell membranes. The only plausible looking option is that nucleotides A,T,G,C are mapped to pairs of quark and anti-quarks at the ends of braid strand. Symmetries pose constraints on this coding.

1. By the basic assumptions charge conjugation must correspond to DNA conjugation so that one A and T would be coded to quark pair, say $q\overline{q}$ and its conjugate $\overline{q}q$. Same for C and G.

2. An additional aesthetically appealing working hypothesis is that *both* A and G with the same number of aromatic cycles (three) correspond to $q\overline{q}$ (or its conjugate).

   This would leave four options:

$$\begin{array}{lll} (A,G) \rightarrow (u\overline{u}, d\overline{d}) & , & (T,C) \rightarrow (\overline{u}u, \overline{d}d) \ , \\ (A,G) \rightarrow (d\overline{d}, u\overline{u}) & , & (T,C) \rightarrow (\overline{d}d), \overline{u}u) \ , \\ (T,C) \rightarrow (u\overline{u}, d\overline{d}) & , & (A,G) \rightarrow (\overline{u}u, \overline{d}d) \ , \\ (T,C) \rightarrow (d\overline{d}, u\overline{u}) & , & (A,G) \rightarrow (\overline{d}d), \overline{u}u) \ . \end{array} \tag{5.3.1}$$

   It is an experimental problem to deduce which of these correspondences - if any - is realized.

Some general predictions

During tqc the lipids of the two lipid layers should define independent units of lipid hydrodynamics whereas after halting of tqc they should behave as single dynamical unit. Later it will be found that these two phases should correspond to high T_c superconductivity for electrons (Cooper pairs would bind the lipid pair to form single unit) and its absence. This prediction is testable.

The differentiation of cells should directly correspond to the formation of a mapping of a particular part of genome to cell membrane. For neurons the gene expression is maximal which conforms with the fact that neurons can have very large size. Axon might be also part of the map. Stem cells represent the opposite extreme and in this case minimum amount of genome should be mapped to cell membrane. The prediction is that the evolution of cell should be reflected in the evolution of the genome-membrane map.

Quantitative test for the proposal

There is a simple quantitative test for the proposal. A hierarchy of tqc programs is predicted, which means that the number of lipids in the nuclear inner membrane should be larger or at least of the same order of magnitude that the number of nucleotides. For definiteness take the radius of the lipid molecule to be about 5 Angstroms (probably somewhat too large) and the radius of the nuclear membrane about 2.5 μm.

For our own species the total length of DNA strand is about one meter and there are 30 nucleotides per 10 nm. This gives 6.3×10^7 nucleotides: the number of intronic nucleotides is only by few per cent smaller. The total number of lipids in the nuclear inner membrane is roughly 10^8. The number of lipids is roughly twice the number nucleotides. The number of lipids in the membrane of a large neuron of radius of order 10^{-4} meters is about 10^{11}. The fact that the cell membrane is highly convoluted increases the number of lipids available. Folding would make possible to combine several modules in sequence by the proposed connections between hydrophobic surfaces.

5.3.2 How quantum states are realized?

Quantum states should be assigned to the ends of the braid strands and therefore to the nucleotides of DNA and conjugate DNA. The states should correspond to many-particle states of anyons and fractional electrons and quarks and anti-quarks are the basic candidates.

Anyons represent quantum states

The multi-sheeted character of space-time surface as a 4-surface in a book like structure having as pages covering spaces of the imbedding space (very roughly, see the appendix) would imply additional degrees of freedom corresponding to the group algebra of the group $G \supset Z_n$ defining the covering. Especially interesting groups are tedra-hedral, octahedral, and icosahedral groups whose action does not map any plane to itself. Group algebra would give rise to $n(G)$ quantum states. If electrons are labeled by elements of group algebra this gives $2^{n(G)}$-fold additional degeneracy corresponding to many-electron states at sheets of covering. The vacuum state would be excluded so that $2^{n(G)} - 1$ states would result. If only Cooper pairs are allowed one would have $m_n = 2^{n(G)-1} - 1$ states.

This picture suggests the fractionization of some fermionic charges such as em charge, spin, and fermion number. This aspect is discussed in detail in the Appendix. Single fermion state would be replaced by a set of states with fractional quantum numbers and one would have an analogy with the full electronic shell of atom in the sense that a state containing maximum number of anyonic fermions with the same spin direction would have the quantum numbers of the ordinary fermion.

One can consider two alternative options.

1. The fractionization of charges inspired the idea that catalytic hot spots correspond to "half" hydrogen bonds containing dark fractionally charged electron meaning that the Fermi sea for electronic anyons is not completely filled [**?**]. The formation of hydrogen bond would mean a fusion of "half hydrogen bond" and its conjugate having by definition a compensating fractional charges guaranteing that the net em charge and electron number of the resulting state are those of the ordinary electron pair and the state is stable as an analog of the full electron shell. Half hydrogen bonds would assign to bio-molecules "names" as sequences of half hydrogen bonds and only molecules whose "names" are conjugates of each other would form stable hydrogen bonded pairs. Therefore symbolic dynamics would enter the biology via bio-catalysis. Concerning quantum computation the problem is that the full shell assigned to hydrogen bond corresponds to only single state and cannot carry information.

2. The assignment of braids and fractionally charged anyonic quarks and anti-quarks would realize very similar symbolic dynamics. One cannot exclude the possibility that leptonic charges fractionize to same values as quark charges.

This suggest the following picture.

1. One could assign the fractional quantum numbers to the quarks and anti-quarks at the ends of the flux tubes defining the braid strands. This hypothesis is consistent with the correspondence between nucleotides and quarks and assigns anyonic quantum states to the ends of the braid. Wormhole magnetic fields would distinguish between matter in vivo and in vitro. This option is certainly favored by Occam's razor in TGD Universe.

2. Hydrogen bonds connect the DNA strands which suggests that fractionally charged quantum states at the ends of braids might be assignable to the ends of hydrogen bonds. The model for plasma electrolysis of Kanarev [?] leads to a proposal that new physics is involved with hydrogen bonds. The presence of fractionally charged particles at the ends of bond might provide alternative explanation for the electrostatic properties of hydrogen bonds usually explained in terms of a modification electronic charge distribution by donor-acceptor mechanism. There would exists entire hierarchy of hydrogen bonds corresponding to the increasing values of Planck constant. DNA and even hydrogen bonds associated with water might correspond to a larger value of Planck constant for mammals than for bacteria.

3. The model for protein folding code [?] leads to a cautious conclusion that flux tubes are prerequisites for the formation of hydrogen bonds although not identifiable with them. The model predicts also the existence of long flux tubes between acceptors of hydrogen bonds (such as $O =$, and aromatic rings assignable to DNA nucleotides, amino-acid backbone, phosphates, XYP, $X = A, T, G, C$, $Y = M, D, T$). This hypothesis would allow detailed identification of places to which quantum states are assigned.

Hierarchy of genetic codes defined by Mersenne primes

The model for the hierarchy of genetic codes inspires the question whether the favored values of $n(G) - 1$ correspond to Mersenne primes [?]. The table below lists the lowest hierarchies. Most of them are short.

$$\begin{array}{lll}
\{M_n\} & \{n(G)\} & \\
 & \{n_b\} & \\
\{2,7,127,2^{127}-1,?\} & \{4,8,128,2^{127},?\} & \{2,6,126,2^{126},?\} \\
\{5,31,2^{31}-1\} & \{6,32,2^{31}\} & \{4,30,2^{30}\} \\
\{13,2^{13}-1\} & \{14,2^{13}\} & \{12,2^{12}\} \\
\{17,2^{17}-1\} & \{18,2^{17}\} & \{16,2^{16}\} \\
\{19,2^{19}-1\} & \{20,2^{19}\} & \{18,2^{18}\} \\
\{61,2^{61}-1\} & \{62,2^{61}\} & \{60,2^{60}\} \\
\{89,2^{89}-1\} & \{90,2^{89}\} & \{88,2^{88}\} \\
\{107,2^{107}-1\} & \{108,2^{107}\} & \{106,2^{106}\}
\end{array} \qquad (5.3.2)$$

The number of states assignable to M_n is $M_n = 2^n - 1$ which does not correspond to full n bits: the reason is that one of the states is not physically realizable. 2^{n-1} states have interpretation as maximal number of mutually consistent statements and to $n_b = n - 1$ bits. The table above lists the values of n_b for Mersenne primes.

Notice that micro-tubules decompose into 13 parallel helices consisting of 13 tubulin dimers. Could these helices with the conformation of the last tubulin dimer serving as a kind of parity bit realize M_{13} code?

There would be a nice connection with the basic phenomenology of ordinary computers. The value of the integer $n - 1$ associated with Mersenne primes would be analogous to the number of bits of the basic information unit of processor. During the evolution of PCs it has evolved from 8 to 32 and is also power of 2.

5.3.3 The role of high T_c superconductivity in tqc

A simple model for braid strands leads to the understanding of how high T_c super conductivity assigned with cell membrane [?] could relate to tqc. The most plausible identification of braid strands is as magnetic or wormhole magnetic flux tubes consisting of pairs of flux tubes connected by wormhole contacts whose throats carry fermion and anti-fermion such that their rotational motion at least partially generates the antiparallel magnetic fluxes at the two sheets of flux tube. The latter option is favored by the model of tqc but one must of course keep mind open for variants of the model involving only ordinary flux tubes. Both kinds of flux tubes can carry charged particles such as protons, electrons, and biologically important ions as dark matter with large Planck constant and the model for nerve pulse and EEG indeed relies on this assumption [?].

Currents at space-like braid strands

If space-like braid strands are identified as idealized structures obtained from 3-D tube like structures by replacing them with 1-D strands, one can regard the braiding as a purely geometrical knotting of braid strands.

The simplest realization of the braid strand as magnetic flux tube would be as a hollow cylindrical surface connecting conjugate DNA nucleotide to cell membrane and going through 5- and/or 6- cycles associated with the sugar backbone of conjugate DNA nucleotides. The free electron pairs associated with the aromatic cycles would carry the current creating the magnetic field needed.

For wormhole magnetic flux one would have pair of this kind of hollow cylinders connected by wormhole contacts and carrying opposite magnetic fluxes. In this case the currents created by wormhole contacts would give rise to the antiparallel magnetic fluxes at the space-time sheets of wormhole contact and could serve as controllers of tqc. I have indeed proposed long time ago that so called wormhole Bose-Einstein condensates might be fundamental for the quantum control in living matter [?]. In this case the presence of supra currents at either sheet would generate asymmetry between the magnetic fluxes.

There are two extreme options for both kinds of magnetic fields. For B-option magnetic field is parallel to the strand and vector potential rotates around it. For A-option vector potential is parallel to the strand and magnetic field rotates around it. The general case corresponds to the hybrid of these options and involves helical magnetic field, vector potential, and current.

1. For B-option current flowing around the cylindrical tube in the transversal direction would generate the magnetic field. The splitting of the flux tube would require that magnetic flux vanishes requiring that the current should go to zero in the process. This would make possible selection of a part of DNA strand participating to tqc.

2. For A-option the magnetic field lines of the braid would rotate around the cylinder. This kind of field is created by a current in the direction of cylinder. In the beginning of tqc the strand would split and the current of electron pairs would stop flowing and the magnetic field would disappear. Also now the initiation of computation would require stopping of the current and should be made selectively at DNA.

   The control of the tqc should rely on currents of electron pairs (perhaps Cooper pairs) associated with the braid strands. Supra currents would have quantized values and they are therefore very attractive candidates. The (supra) currents could also bind lipids to pairs so that they would define single dynamical unit in 2-D hydrodynamical flow. One can also think that Cooper pairs with electrons assignable to different members of lipid pair bind it to a single dynamical unit.

Do supra currents generate magnetic fields?

Energetic considerations favor the possibility that supra currents create the magnetic fields associated with the braid strands defined by magnetic flux tubes. In the case of wormhole magnetic flux tubes supra currents could generate additional magnetic fields present only at the second sheet of the flux tube.

Supra current would be created by a voltage pulse ΔV, which gives rise to a constant supra current after it has ceased. Supra current would be destroyed by a voltage pulse of opposite sign. Therefore voltage pulses could define an elegant fundamental control mechanism allowing to select the parts of genome participating to tqc. This kind of voltage pulse could be collectively initiated at cell membrane or at DNA. Note that constant voltage gives rise to an oscillating supra current.

Josephson current through the cell membrane would be also responsible for dark Josephson radiation determining that part of EEG which corresponds to the correlate of neuronal activity [?]. Note that TGD predicts a fractal hierarchy of EEGs and that ordinary EEG is only one level in this hierarchy. The pulse initiating or stopping tqc would correspond in EEG to a phase shift by a constant amount

$$\Delta\Phi = Ze\Delta VT/\hbar \ ,$$

where T is the duration of pulse and ΔV its magnitude.

The contribution of Josephson current to EEG responsible for beta and theta bands interpreted as satellites of alpha band should be absent during tqc and only EEG rhythm would be present. The periods dominated by EEG rhythm should be observed as EEG correlates for problem solving situations (say mouse in a maze) presumably involving tqc. The dominance of slow EEG rhythms during sleep and meditation would have interpretation in terms of tqc.

Topological considerations

The existence of supra current requires that the flow allows for a complex phase $exp(i\Psi)$ such that supra current is proportional to $\nabla\Psi$. This requires integrability in the sense that one can assign to the flow lines of A or B (combination of them in the case of A-B braid) a coordinate variable Ψ varying along the flow lines. In the case of a general vector field X this requires $\nabla\Psi = \Phi X$ giving $\nabla \times X = -\nabla\Phi/\Phi$ as an integrability condition. This condition defines what is known as Beltrami flow [?].

The perturbation of the flux tube, which spoils integrability in a region covering the entire cross section of flux tube means either the loss of super-conductivity or the disappearance of the net supra current. In the case of the A-braid, the topological mechanism causing this is the increase in the dimension of the CP_2 projection of the flux tube so that it becomes 3-D [?], where I have also considered the possibility that 3-D character of CP_2 projection is what transforms the living matter to a spin glass type phase in which very complex self-organization patterns emerge. This would conform with the idea that in tqc takes place in this phase.

Fractal memory storage and tqc

If Josephson current through cell membrane ceases during tqc, tqc manifests itself as the presence of only EEG rhythm characterized by an appropriate cyclotron frequency. Synchronous neuron firing might therefore relate to tqc. The original idea that a phase shift of EEG is induced by the voltage initiating tqc - although wrong - was however useful in that it inspired the question whether the initiation of tqc could have something to do with what is known as a place coding by phase shifts performed by hippocampal pyramidal cells [?, ?]. The playing with this idea provides important insights about the construction of quantum memories and demonstrates the amazing explanatory power of the paradigm once again.

The model also makes explicit important conceptual differences between tqc a la TGD and in the ordinary sense of word: in particular those related to different view about the relation between subjective and geometric time.

1. In TGD tqc corresponds to the unitary process U taking place following by a state function reduction and preparation. It replaces configuration space ("world of classical worlds") spinor field with a new one. Configuration space spinor field represent generalization of time evolution of Schrödinger equation so that a quantum jump occurs between entire time evolutions. Ordinary tqc corresponds to Hamiltonian time development starting at time $t = 0$ and halting at $t = T$ to a state function reduction.

2. In TGD the expression of the result of tqc is essentially 4-D pattern of gene expression (spiking pattern in the recent case). In usual tqc it would be 3-D pattern emerging as the computation halts at time t. Each moment of consciousness can be seen as a process in which a kind of 4-D statue is carved by starting from a rough sketch and proceeding to shorter details and building

fractally scaled down variants of the basic pattern. Our life cycle would be a particular example of this process and would be repeated again and again but of course not as an exact copy of the previous one.

1. Empirical findings

The place coding by phase shifts was discovered by O′Reefe and Recce [?]. In [?] Y. Yamaguchi describes the vision in which memory formation by so called theta phase coding is essential for the emergence of intelligence. It is known that hippocampal pyramidal cells have "place property" being activated at specific "place field" position defined by an environment consisting of recognizable objects serving as landmarks. The temporal change of the percept is accompanied by a sequence of place unit activities. The theta cells exhibit change in firing phase distributions relative to the theta rhythm and the relative phase with respect to theta phase gradually increases as the rat traverses the place field. In a cell population the temporal sequence is transformed into a phase shift sequence of firing spikes of individual cells within each theta cycle.

Thus a temporal sequence of percepts is transformed into a phase shift sequence of individual spikes of neurons within each theta cycle along linear array of neurons effectively representing time axis. Essentially a time compressed representation of the original events is created bringing in mind temporal hologram. Each event (object or activity in perceptive field) is represented by a firing of one particular neuron at time τ_n measured from the beginning of the theta cycle. τ_n is obtained by scaling down the real time value t_n of the event. Note that there is some upper bound for the total duration of memory if scaling factor is constant.

This scaling down - story telling - seems to be a fundamental aspect of memory. Our memories can even abstract the entire life history to a handful of important events represented as a story lasting only few seconds. This scaling down is thought to be important not only for the representation of the contextual information but also for the memory storage in the hippocampus. Yamaguchi and collaborators have also found that the gradual phase shift occurs at half theta cycle whereas firings at the other half cycle show no correlation [?]. One should also find an interpretation for this.

2. TGD based interpretation of findings

How this picture relates to TGD based 4-D view about memory in which primary memories are stored in the brain of the geometric past?

1. The simplest option is the initiation of tqc like process in the beginning of each theta cycle of period T and having geometric duration $T/2$. The transition $T \rightarrow T/2$ conforms nicely with the fundamental hierarchy of time scales comings as powers defining the hierarchy of measurement resolutions and associated with inclusions of hyperfinite factors of type II_1 [?]. That firing is random at second half of cycle could simply mean that no tqc is performed and that the second half is used to code the actual events of "geometric now".

2. In accordance with the vision about the hierarchy of Planck constants defining a hierarchy of time scales of long term memories and of planned action, the scaled down variants of memories would be obtained by down-wards scaling of Planck constant for the dark space-time sheet representing the original memory. In principle a scaling by any factor $1/n$ (actually by any rational) is possible and would imply the scaling down of the geometric time span of tqc and of light-like braids. One would have tqc's inside tqc's and braids within braids (flux quanta within flux quanta). The coding of the memories to braidings would be an automatic process as almost so also the formation of their zoomed down variants.

3. A mapping of the time evolution defining memory to a linear array of neurons would take place. This can be understood if the scaled down variant (scaled down value of $\hbar$) of the space-time sheet representing original memory is parallel to the linear neuron array and contains at scaled down time value t_n a stimulus forcing n^{th} neuron to fire. The 4-D character of the expression of the outcome of tqc allows to achieve this automatically without complex program structure.

To sum up, it seems that the scaling of Planck constant of time like braids provides a further fundamental mechanism not present in standard tqc allowing to build fractally scaled down variants of not only memories but tqc's in general. The ability to simulate in shorter time scale is a certainly very important prerequisite of intelligent and planned behavior. This ability has also a space-like counterpart: it will be found that the scaling of Planck constant associated with space-like braids connecting bio-molecules might play a fundamental role in DNA replication, control of transcription by proteins, and translation of mRNA to proteins. A further suggestive conclusion is that the period T associated with a given EEG rhythm defines a sequence of tqc's having geometric span $T/2$ each: the rest of the period would be used to perceive the environment of the geometric now. The fractal hierarchy of EEGs would mean that there are tqc's within tqc's in a very wide range of time scales.

5.3.4 Codes and tqc

TGD suggests the existence of several (genetic) codes besides 3-codon code [**?**, **?**]. The experience from ordinary computers and the fact that genes in general do not correspond to $3n$ nucleotides encourages to take this idea more seriously. The use of different codes would allow to tell what kind of information a given piece of DNA strand represents. DNA strand would be like a drawing of building containing figures (3-code) and various kinds of text (other codes). A simple drawing for the building would become a complex manual containing mostly text as the evolution proceeds: for humans 96 per cent of code would corresponds to introns perhaps obeying some other code.

The hierarchy of genetic codes is obtained by starting from n basic statements and going to the meta level by forming all possible statements about them (higher order logics) and throwing away one which is not physically realizable (it would correspond to empty set in the set theoretic realization). This allows $2^n - 1$ statements and one can select 2^{n-1} mutually consistent statements (half of the full set of statements) and say that these are true and give kind of axiomatics about world. The remaining statements are false. DNA would realize only the true statements.

The hierarchy of Mersenne primes $M_n = 2^n - 1$ with $M_{n(next)} = M_{M_n}$ starting from $n = 2$ with $M_2 = 3$ gives rise to 1-code with 4 codons, 3-code with 64 codons, and $3 \times 21 = 63$-code with 2^{126} codons [**?**] realized as sequences of 63 nucleotides (the length of 63-codon is about $2L(151)$, roughly twice the cell membrane thickness. It is not known whether this Combinatorial Hierarchy continues ad infinitum. Hilbert conjectured that this is the case.

In the model of pre-biotic evolution also 2-codons appear and 3-code is formed as the fusion of 1- and 2-codes. The problem is that 2-code is not predicted by the basic Combinatorial Hierarchy associated with $n = 2$.

There are however also other Mersenne hierarchies and the next hierarchy allows the realization of the 2-code. This Combinatorial Hierarchy begins from Fermat prime $n = 2^k + 1 = 5$ with $M_5 = 2^5 - 1 = 31$ gives rise to a code with 16 codons realized as 2-codons (2 nucleotides). Second level corresponds to Mersenne prime $M_{31} = 2^{31} - 1$ and a code with $2^{30=15\times 2}$ codons realized by sequences of 15 3-codons containing 45 nucleotides. This corresponds to DNA length of 15 nm, or length scale $3L(149)$, where $L(149) = 5$ nm defines the thickness of the lipid layer of cell membrane. $L(151) = 10$ nm corresponds to 3 full 2π twists for DNA double strand. The model for 3-code as fusion of 1- and 2-codes suggests that also this hierarchy - which probably does not continue further - is realized.

There are also further short Combinatorial hierarchies corresponding to Mersenne primes [**?**].

1. $n = 13$ defines Mersenne prime M_{13}. The code would have $2^{12=6\times 2}$ codons representable as sequences of 6 nucleotides or 2 3-codons. This code might be associated with microtubuli.

2. The Fermat prime $17 = 2^4 + 1$ defines Mersenne prime M_{17} and the code would have $2^{16=8\times 2}$ codons representable as sequences of 8 nucleotides.

3. $n = 19$ defines Mersenne prime M_{19} and code would have $2^{18=9\times 2}$ codons representable as sequences of 9 nucleotides or three DNA codons.

4. The next Mersennes are M_{31} belonging to $n = 5$ hierarchy, M_{61} with $2^{60=30\times 2}$ codons represented by 30-codons. This corresponds to DNA length $L(151) = 10$ nm (cell membrane thickness). M_{89} (44-codons), M_{107} (53-codons) and M_{127} (belonging to the basic hierarchy) are the next Mersennes. Next Mersenne corresponds to M_{521} (260-codon) and to completely super-astrophysical p-adic length scale and might not be present in the hierarchy.

   This hierarchy is realized at the level of elementary particle physics and might appear also at the level of DNA. The 1-, 2-, 3-, 6-, 8-, and 9-codons would define lowest Combinatorial Hierarchies.

5.4 How to realize the basic gates?

In order to have a more concrete view about realization of tqc, one must understand how quantum computation can be reduced to a construction of braidings from fundamental unitary operations. The article "Braiding Operators are Universal Quantum Gates" by Kaufmann and Lomonaco [?] contains a very lucid summary of how braids can be used in topological quantum computation.

1. The identification of the braiding operator R - a unitary solution of Yang-Baxter equation - as a universal 2-gate is discussed. In the following I sum up only those points which are most relevant for the recent discussion.

2. One can assign to braids both knots and links and the assignment is not unique without additional conditions. The so called braid closure assigns a unique knot to a given braid by connecting n^{th} incoming strand to n^{th} outgoing strand without generating additional knotting. All braids related by so called Markov moves yield the same knot. The Markov trace (q-trace actually) of the unitary braiding S-matrix U is a knot invariant characterizing the braid closure.

3. Braid closure can be mimicked by a topological quantum computation for the original n-braid plus trivial n-braid and this leads to a quantum computation of the modulus of the Markov trace of U. The probability for the diagonal transition for one particular element of Bell basis (whose states are maximally entangled) gives the modulus squared of the trace. The closure can be mimicked quantum computationally.

5.4.1 Universality of tqc

Quantum computer is universal if all unitary transformations of n^{th} tensor power of a finite-dimensional state space V can be realized. Universality is achieved by using only two kinds of gates. The gates of first type are single particle gates realizing arbitrary unitary transformation of $U(2)$ in the case of qubits. Only single 2-particle gate is necessary and universality is guaranteed if the corresponding unitary transformation is entangling for some state pair. The standard choice for the 2-gate is CNOT acting on bit pair (t, c). The value of the control bit c remains of course unchanged and the value of the target bit changes for $c = 1$ and remains unchanged for $c = 0$.

5.4.2 The fundamental braiding operation as a universal 2-gate

The realization of CNOT or gate equivalent to it is the key problem in topological quantum computation. For instance, the slow de-coherence of photons makes quantum optics a promising approach but the realization of CNOT requires strongly nonlinear optics. The interaction of control and target photon should be such that for second polarization of the control photon target photon changes its direction but keeps it for the second polarization direction.

For braids CNOT can be be expressed in terms of the fundamental braiding operation e_n representing the exchange of the strands n and $n+1$ of the braid represented as a unitary matrix R acting on $V_n \otimes V_{n+1}$.

The basic condition on R is Yang-Baxter equation expressing the defining condition $e_n e_{n+1} e_n = e_{n+1} e_n e_{n+1}$ for braid group generators. The solutions of Yang-Baxter equation for spinors are well-known and CNOT can be expressed in the general case as a transformation of form $A_1 \otimes A_2 R A_3 \otimes A_4$ in which single particle operators A_i act on incoming and outgoing lines. 3-braid is the simplest possible braid able to perform interesting tqc, which suggests that genetic codons are associated with 3-braids.

The dance of lipids on chessboard defined by the lipid layer would reduce R to an exchange of neighboring lipids. For instance, the matrix $R = DS$, $D = diag(1,1,1,-1)$ and $S = e_{11} + e_{23} + e_{32} + e_{44}$ the swap matrix permuting the neighboring spins satisfies Yang-Baxter equation and is entangling.

5.4.3 What the replacement of linear braid with planar braid could mean?

Standard braids are essentially linear objects in plane. The possibility to perform the basic braiding operation for the nearest neighbors in two different directions must affect the situation somehow.

1. Classically it would seem that the tensor product defined by a linear array must be replaced by a tensor product defined by the lattice defined by lipids. Braid strands would be labelled by two indices and the relations for braid group would be affected in an obvious manner.

2. The fact that DNA is a linear structure would suggests that the situation is actually effectively one-dimensional, and that the points of the lipid layer inherit the linear ordering of nucleotides of DNA strand. One can however ask whether the genuine 2-dimensionality could provide a mathematical realization for possible long range correlations between distant nucleotides n and $n+N$ for some N. p-Adic effective topology for DNA might become manifest via this kind of correlations and would predict that N is power of some prime p which might depend on organism's evolutionary level.

3. Quantum conformal invariance would suggest effective one-dimensionality in the sense that only the observables associated with a suitably chosen linear braid commute. One might also speak about topological quantum computation in a direction transversal to the braid strands giving a slicing of the cell membrane to parallel braid strands. This might mean an additional computational power.

4. Partonic picture would suggest a generalization of the linear braid to a structure consisting of curves defining the decomposition of membrane surface regions such that conformal invariance applies separately in each region: this would mean breaking of conformal invariance and 2-dimensionality in discrete sense. Each region would define a one parameter set of topological quantum computations. These regions might corresponds to genes. If each lipid defines its own conformal patch one would have a planar braid.

5.4.4 Single particle gates

The realization of single particle gates as $U(2)$ transformations leads naturally to the extension of the braid group by assigning to the strands sequences of group elements satisfying the group multiplication rules. The group elements associated with a n^{th} strand commute with the generators of braid group which do not act on n^{th} strand. G would be naturally subgroup of the covering group of rotation group acting in spin degrees of spin 1/2 object. Since $U(1)$ transformations generate only an overall phase to the state, the presence of this factor might not be necessary. A possible candidate for $U(1)$ factor is as a rotation induced by a time-like parallel translation defined by the electromagnetic scalar potential $\Phi = A_t$.

One of the challenges is the realization of single particle gates representing $U(2)$ rotation of the qubit. The first thing to come mind was that $U(2)$ corresponds to $U(2)$ rotation induced by magnetic field and electric fields. A more elegant realization is in terms of $SU(3)$ rotation, where $SU(3)$ is color group associated with strong interactions. This looks rather weird but there is direct evidence for the prediction that color $SU(3)$ is associated with tqc and thus cognition: something that does not come first in mind! I have myself written text about the strange finding of topologist Barbara Shipman suggesting that quarks are in some mysterious manner involved with honeybee dance and proposed an interpretation.

The realization of qubit as ordinary spin

A possible realization for single particle gate $s \subset SU(2)$ would be as $SU(2)$ rotation induced by a magnetic pulse. This transformation is fixed by the rotation axis and rotation angle around this axes. This kind of transformation would result by applying to the strand a magnetic pulse with magnetic field in the direction of rotation axes. The duration of the pulse determines the rotation angle. Pulse could be created by bringing a magnetic flux tube to the system, letting it act for the required time, and moving it away. $U(1)$ phase factor could result from the electromagnetic gauge potential as a non-integrable phase factor $exp(ie \int A_t dt/\hbar)$ coming from the presence of scale potential $\Phi = A_t$ in the Hamiltonian.

Conrete model for realization of 1-gates in terms of ordinary rotations

What could be the simplest realization of the $U(2)$ transformation in the case of cell membrane assuming that it corresponds to ordinary rotation?

1. There should be a dark spin 1/2 particle associated with each lipid, electron or proton most plausibly. TGD based model for high T_c superconductivity [?] predicts that Cooper pairs correspond to pairs of cylindrical space-time sheets with electrons at the two space-time sheets. The size scale of the entire Cooper corresponds to p-adic length scale $L(151)$ defining the thickness of the cell membrane and cylindrical structure to $L(149)$, the thickness of lipid layer so that electrons are the natural candidates for tqc. The Cooper pair BE condensate would fuse the lipid pairs to form particles of lipid liquid.

2. Starting of tqc requires the splitting of electron Cooper pairs and its halting the formation of Cooper pairs again. The initiation of tqc could involve increase of temperature or an introduction of magnetic field destroying the Cooper pairs. Tqc could be also controlled by supra currents flowing along cylindrical flux tubes connecting 5- and/or aromatic cycles of conjugate DNA nucleotides to the cell membrane. The cutting of the current flow would make it possible for braid strand to split and tqc to begin.

3. By shifting a magnetic flux tube or sheet parallel to the cell membrane to the position of the portion of membrane participating to tqc is the simplest manner to achieve this. Halting could be achieved by removing the flux tube. The unitary rotation induced by the constant background magnetic field would not represent gate and it should be possible to eliminate its effect from tqc proper.

4. The gate would mean the application of a magnetic pulse much stronger than background magnetic field on the braid strands ending at the lipid layer. The model for the communication of sensory data to the magnetic body requires that magnetic flux tubes go through the cell membrane. This would suggest that the direction of the magnetic flux tube is temporarily altered and that the flux tube then covers part of the lipid for the required period of time.

   The realization of the single particle gates requires electromagnetic interactions. That single particle gates are not purely topological transformations could bring in the problems caused by a de-coherence due to electromagnetic perturbations. The large values of Planck constant playing a key role in the TGD based model of living matter could save the situation. The large value of $\hbar$ would be also required by the anyonic character of the system necessary to obtain R-matrix defining a universal 2-gate.

   The minimum time needed to inducing full 2π rotation around the magnetic axes would be essentially the inverse of cyclotron frequency for the particle in question in the magnetic field considered: $T = 1/f_c = 2\pi m/ZeB$. For electrons in the dark magnetic field of $B = .2$ Gauss assigned to living matter in the quantum model of EEG this frequency would be about $f_c = .6$ MHz. For protons one would have $f_c = 300$ Hz. For a magnetic field of Tesla the time scales would be reduced by a factor 2×10^{-5}.

The realization of 1-gate in terms of color rotations

One can criticize the model of 1-gates based on ordinary spin. The introduction of magnetic pulses does not look an attractive idea and seems to require additional structures besides magnetic flux tubes (MEs?). It would be much nicer to assign the magnetic field with the flux tubes defining the braid strands. The rotation of magnetic field would however require changing the direction of braid strands. This does not look natural. Could one do without this rotation by identifying spin like degree of freedom in some other manner? This is indeed possible.

TGD predicts a hierarchy of copies of scaled up variants of both weak and color interactions and these play a key role in TGD inspired model of living matter. Both weak isospin and color isospin could be considered as alternatives for the ordinary spin as a realization of qubit in TGD framework. Below color isospin is discussed but one could consider also a realization in terms of nuclei and their exotic counterparts [?] differing only by the replacement of neutral color bond between nuclei of nuclear string with a charged one. Charge entanglement between nuclei would guarantee overall charge conservation.

1. Each space-time sheet of braid strands contains quark and antiquark at its ends. Color isospin and hypercharge label their states. Two of the quarks of the color triplet form doublet with respect to color isospin and the third is singlet and has different hyper charge Y. Hence qubit

could be realized in terms of color isospin I_3 instead of ordinary spin but third quark would be inert in the Boolean sense. Qubit could be also replaced with qutrit and isospin singlet could be identified as a statement with ill-defined truth value. Trits are used also in ordinary computers. In TGD framework finite measurement resolution implies fuzzy qubits and the third state might relate to this fuzziness. Also Gödelian interpretation can be considered: the quark state with vanishing isospin would be associated with counterparts of undecidable propositions to which one cannot assign truth value (consider sensory input which is so ambiguous that one cannot tell what is there or a situation in which one cannot decide whether to do something or not). Note that hyper-charge would induce naturally the U(1) factor affecting the over all phase of qubit but affecting differently to the third quark.

2. Magnetic flux tubes are also color magnetic flux tubes carrying non-vanishing classical color gauge field in the case that they are non-vacuum extremals. The holonomy group of classical color field is an Abelian subgroup of the $U(1) \times U(1)$ Cartan subgroup of color group. Classical color magnetic field defines the choice of quantization axes for color quantum numbers. For instance, magnetic moment is replaced with color magnetic moment and this replacement is in key role in simple model for color magnetic spin spin splittings between spin 0 and 1 mesons as well as spin 1/2 and 3/2 baryons.

3. There is a symmetry breaking of color symmetry to subgroup $U(1)_{I_3} \times U(1)_Y$ and color singletness is in TGD framework replaced by a weaker condition stating that physical states have vanishing net color quantum numbers. This makes possible the measurement of color quantum numbers in the manner similar to that for spin. For instance, color singlet formed by quark and antiquark with opposite color quantum numbers can in the measurement of color quantum numbers of quark reduce to a state in which quark has definite color quantum numbers. This state is a superposition of states with vanishing Y and I_3 in color singlet and color octet representations. Strong form of color confinement would not allow this kind of measurement.

4. Color rotation in general changes the directions of quantization axis of I_3 and Y and generates a new state basis. Since $U(1) \times U(1)$ leaves the state basis invariant, the space defined by the choices of quantization axes is 6-dimensional flag manifold $F = SU(3)/U(1) \times U(1)$. In contrast to standard model, color rotations in general do not leave classical electromagnetic field invariant since classical em field is a superposition of color invariant induced Kähler from and color non-invariant part proportional classical Z^0 field. Hence, although the magnetic flux tube retains its direction and shape in M^4 degrees of freedom, its electromagnetic properties are affected and this is visible at the level of classical electromagnetic interactions.

5. If color isospin defines the qubit or qutrit in topological quantum computation, color quantum numbers and the flag manifold F should have direct relevance for cognition. Amazingly, there is a direct experimental support for this! Years ago topologist Barbara Shipman made the intriguing observation that honeybee dance can be understood in terms of a model involving the flag manifold F [**?**]. This led her to propose that quarks are in some mysterious manner involved with the honeybee dance. My proposal [**?**] was that color rotations of the space-time sheets associated with neurons represent geometric information: sensory input would be coded to color rotations defining the directions of quantization axes for I_3 and Y. Subsequent state function reduction would provide conscious representations in terms of trits characterizing for instance sensory input symbolically.

   In [**?**] I introduced the notions of geometric and sensory qualia corresponding to two choices involved with the quantum measurement: the choice of quantization axes performed by the measurer and the "choice" of final state quantum numbers in state function reduction. In the case of honeybee dance geometric qualia could code information about the position of the food source. The changes of color quantum numbers in quantum jump were identified as visual colors. In state function reduction one cannot speak about change of quantum numbers but about their emergence. Therefore one must distinguish between color qualia and the conscious experience defined by the emergence of color quantum numbers: the latter would have interpretation as qutrit.

Summarizing, this picture suggests that 1-gates of DNA tqc (understood as "dance of lipids") are defined by color rotations of the ends of space-like braid strands and at lipids. The color rotations would be induced by sensory and other inputs to the system. Topological quantum computation would be directly related to conscious experience and sensory and other inputs would fix the directions of the color magnetic fields.

5.5 About realization of braiding

The most plausible identification of braid strands is as magnetic or wormhole magnetic flux tubes. Flux tubes can contain charged particles such as protons, electrons, and biologically important ions as dark matter with large Planck constant and the model for nerve pulse and EEG indeed relies on this assumption [?].

5.5.1 Could braid strands be split and reconnect all the time?

As far as braiding alone is considered, braid strands could be split all the time. In other words, there would be no continuation of strands through the cell membrane. Computation would halt when lipids lose their unsaturated cis bonds so that they cannot follow the liquid flow. The conservation of strand color would be trivially true but would not have any implications. Supra currents would not be needed to control tqc and there would be no connection with generalized EEG. It is not obvious how the gene expression for the outcome of tqc could take place since the strands would not connect genome to genome. For these reasons this option does not look attractive.

The models for prebiotic evolution [?] and protein folding [?] lead to a conclusion that braids can connect all kind of bio-molecules to each other and also water molecules and bio-molecules Thus DNA tqc would represent only one example of tqc like activities performed by the living matter. The conclusion is that braidings are dynamical with reconnection of flux tubes representing a fundamental transformation changing the braiding and thus also tqc programs.

5.5.2 What do braid strands look like?

In the following the anatomy of braid strands is discussed at general level and then identification in terms of flux tubes of magnetic body is proposed.

Braid strands as nearly vacuum extremals

The braid strands should be nearly quantum critical sub-manifolds of $M^4 \times CP_2$ so that phase transitions changing Planck constant and thus their length can take place easily (DNA replication, binding of mRNA molecules to DNA during transcription, binding of transcription factors to promoters, binding of tRNA-amino-acid complexes to mRNA...).

Depending on whether phase transition takes place in M^4 or CP_2 degrees of freedom, either their M^4 projection belongs to $M^2 \subset M^4$ or their CP_2 projection to the homological trivial geodesic sphere $S^2 \subset CP_2$. In the latter case a vacuum extremal is in question. Maximal quantum criticality means $X^4 \subset M^2 \times S^2$ so that one has straight string with a vanishing string tension. The almost vacuum extremal property guarantees the braid strands can be easily generated from vacuum.

An additional requirement is that the gravitational mass is small. For objects of type $M^2 \times X^2_g$, $X^2_g \subset E^2 \times CP_2$, the gravitational mass vanishes for $g = 1$ (genus) and is of order CP_2 mass otherwise and negative for $g > 1$. Torus topology is the unique choice. A simple model for the braid strand is as a small non-vacuum deformation of $X^4 = M^2 \times X^2_g \subset M^2 \subset E^2 \times S^2$, $g = 1$. As a special case one has $X^4 = M^2 \times S^1 \times S^1 \subset M^2 \subset E^2 \times S^1$, for which M^4 projection is a hollow cylinder, which could connect the aromatic 5- or 6-cycle of sugar backbone to another DNA strand, lipid, or amino-acid.

Braid strands as flux tubes of color magnetic body

One can make this model more detailed by feeding in simple physical inputs. The flux tubes carry magnetic field when the supra current is on. In TGD Universe all classical fields are expressible in terms of the four CP_2 coordinates and their gradients so that em, weak, color and gravitational fields are not independent as in standard model framework. In particular, the ordinary classical em field is necessarily accompanied by a classical color field in the case of non-vacuum extremals. This predicts color and ew fields in arbitrary long scales and quantum classical correspondence forces to conclude that there exists fractal hierarchy of electro-weak and color interactions.

Since the classical color gauge field is proportional to Kähler form, its holonomy group is Abelian so that effectively $U(1)\times U(1) \subset SU(3)$ gauge field is in question. The generation of color flux requires colored particles at the ends of color flux tube so that the presence of pairs of quark and antiquark assignable to the pairs of wormhole throats at the ends of the tube is unavoidable if one accepts quantum classical correspondence.

In the case of cell, a highly idealized model for color magnetic flux tubes is as flux tubes of a dipole field. The preferred axis could be determined by the position of the centrosomes forming a T shaped structure. DNA strands would define the idealized dipole creating this field: DNA is indeed negatively charged and electronic currents along DNA could create the magnetic field. The flux tubes of this field would go through nuclear and cell membrane and return back unless they end up to another cell. This is indeed required by the proposed model of tqc.

It has been assumed that the initiation of tqc means that the supra current ceases and induces the splitting of braid strands. The magnetic flux need not however disappear completely. As a matter fact, its presence forced by the conservation of magnetic flux seems to be crucial for the conservation of braiding. Indeed, during tqc magnetic and color magnetic flux could return from lipid to DNA along another space-time sheet at a distance of order CP_2 radius from it. For long time ago I proposed that this kind of structures -which I christened "wormhole magnetic fields" - might play key role in living matter [?]. The wormhole contacts having quark and antiquark at their opposite throats and coding for A,T,C,G would define the places where the current flows to the "lower" space-time sheet to return back to DNA. Quarks would also generate the remaining magnetic field and supra current could indeed cease.

The fact that classical em fields and thus classical color fields are always present for non-vacuum extremals means that also the motion of any kind of particles (space-time sheets), say water flow, induces a braiding of magnetic flux tubes associated with molecules in water if the temporary splitting of flux tubes is possible. Hence the prerequisites for tqc are met in extremely general situation and tqc involving DNA could have developed from a much simpler form of tqc performed by water giving perhaps rise to what is known as water memory [?, ?, ?, ?]. This would also suggest that the braiding operation is induced by the a controlled flow of cellular water.

5.5.3 How to induce the basic braiding operation?

The basic braiding operation requires the exchange of two neighboring lipids. After some basic facts about phospholipids the simplest model found hitherto is discussed.

Some facts about phospholipids

Phospholipids [?] - which form about 30 per cent of the lipid content of the monolayer - contain phosphate group. The dance of lipids requires metabolic energy and the hydrophilic ends of the phospholipid could provide it. They could also couple the lipids to the flow of water in the vicinity of the lipid monolayer possibly inducing the braiding. Of course, the causal arrow could be also opposite.

The hydrophilic part of the phospholipid is a nitrogen containing alcohol such as serine, inositol or ethanolamine, or an organic compound such as choline. Phospholipids are classified into 3 kinds of phosphoglycerides [?] and sphingomyelin.

1. *Phosphoglycerides*

In cell membranes, phosphoglycerides are the more common of the two phospholipids, which suggest that they are involved with tqc. One speaks of phosphotidyl X, where X= serine, inositol, ethanolamine is the nitrogen containing alcohol and X=Ch the organic compound. The shorthand notion OS, PI, PE, PCh is used.

The structure of the phospholipid is most easily explained using the dancer metaphor. The two fatty chains define the hydrophobic feet of the dancer, glycerol and phosphate group define the body providing the energy to the dance, and serine, inositol, ethanolamine or choline define the hydrophilic head of the dancer (perhaps "deciding" the dancing pattern).

There is a lipid asymmetry in the cell membrane. PS, PE, PI in cytoplasmic monolayer (alcohols). PC (organic) and sphingomyelin in outer monolayer. Also glycolipids are found only in the outer monolayer. The asymmetry is due to the manner that the phospholipids are manufactured.

PS [?] in the inner monolayer is negatively charged and its presence is necessary for the normal functioning of the cell membrane. It activates protein kinase C which is associated with memory function. PS slows down cognitive decline in animals models. This encourages to think that the hydrophilic polar end of at least PS is involved with tqc, perhaps to the generation of braiding via the coupling to the hydrodynamic flow of cytoplasm in the vicinity of the inner monolayer.

2. Fatty acids

The fatty acid chains in phospholipids and glycolipids usually contain an even number of carbon atoms, typically between 14 and 24 making 5 possibilities altogether. The 16- and 18-carbon fatty acids are the most common. Fatty acids [?] may be saturated or unsaturated, with the configuration of the double bonds nearly always cis.

The length and the degree of unsaturation of fatty acids chains have a profound effect on membranes fluidity as unsaturated lipids create a kink, preventing the fatty acids from packing together as tightly, thus decreasing the melting point (increasing the fluidity) of the membrane. The number of unsaturaded cis bonds and their positions besides the number of Carbon atoms characterizes the lipid. Quite generally, there are $3n$ Carbons after each bond. The creation of unsatured bond by removing H atom from the fatty acid could be an initiating step in the basic braiding operation creating room for the dancers. The bond should be created on both neighboring lipids simultaneously.

Could hydrodynamic flow induce braiding operations?

One can imagine several models for what might happen during the braiding operation in the lipid bilayer [?]. One such view is following.

1. The creation of unsaturated bond and involving elimination of H atom from fatty acid would lead to cis configuration and create the room needed by dancers. This operation should be performed for both lipids participating in the braiding operation. After the braiding it might be necessary to add H atom back to stabilize the situation. The energy needed to perform either or both of these operations could be provided by the phosphate group.

2. The hydrophilic ends of lipids couple the lipids to the surrounding hydrodynamic flow in the case that the lipids are able to move. This coupling could induce the braiding. The primary control of tqc would thus be by using the hydrodynamic flow by generating localized vortices. There is considerable evidence for water memory [?] but its mechanism remains to be poorly understood. If also water memory is realized in terms of the braid strands connecting fluid particles, DNA tqc could have evolved from water memory.

3. Sol-gel phase transition is conjectured to be important for the quantum information processing of cell [?]. In the transition which can occur cyclically actin filaments (also at EEG frequencies) are assembled and lead to a gel phase resembling solid. Sol phase could correspond to tqc and gel to the phase following the halting of tqc. Actin filaments might be assignable with braid strands or bundles of them and shield the braiding. Also microtubules might shield bundles of braid strands.

4. Only inner braid strands are directly connected to DNA which also supports the view that only the inner monolayer suffers a braiding operation during tqc and that the outer monolayer should be in a "freezed" state during it. There is a net negative charge associated with the inner monolayer possibly relating to its participation to the braiding. The vigorous hydrodynamical flows known to take place below the cell membrane could induce the braiding.

5.5.4 Some qualitative tests

In life sciences the standard manner to test a model is to look whether the function of the system is affected in the predicted manner if one somehow interferes the system. Now interfering with tqc should affect the gene expression resulting otherwise.

1. Lipid layer hydrodynamics is predicted to allow two fundamental phases. The pairs of lipids should behave like single dynamical unit in super-conducting phase and as independent units

in non-super-conducting phase. The application of magnetic field or increase of temperature should induce a transition between these two phases. These phase transitions applied selectively to the regions of cell membrane should affect gene expression. One could prevent halting of tqc by applying an external magnetic field and thus prevent gene expression. One could dream of deducing gene-membrane mapping with endoplasmic reticulum included.

2. The temperature range in which quantum critical high T_c super-conductivity is possible is probably rather narrow and should correspond to the temperature range in which cell membrane is functional. Brain is functional in a very narrow range of temperatures. Selective freezing of cell membrane might provide information about gene map provided by cell membrane.

3. One could do various things to the cell membrane. One could effectively remove part of it, freeze, or heat some part of the lipid liquid and look whether this has effects on gene expression. The known effects of ELF em fields on the behavior and physiology of vertebrates [?] might relate to the fact that these fields interfere with tqc.

4. Artificially induced braiding by inducing a motion of lipids by some kind of stirring during tqc could induce/affect gene expression.

5. The application of external dark magnetic fields could affect gene expression. Tqc could be initiated artificially in some part of cell membrane by the application of dark magnetic field. Running tqc could be halted by an application of dark magnetic field interfering to zero with the background field. The application of magnetic pulses would affect tqc and thus gene expression. The problem is how to create dark magnetic fields in given length scale (range of magnetic field strength). Perhaps one could generate first ordinary magnetic field and then transform it to dark magnetic field by $\hbar$ changing phase transition. This could be achieved by a variation of some macroscopic parameters such as temperature, magnetic field strength, and analog of doping fraction appearing in standard high T_c super-conductivity.

6. Artificially induced scalings of $\hbar$ by varying temperature and parameters such as pH should induce or stop DNA replication, DNA-mRNA transcription and the translation of mRNA to proteins.

5.6 A model for flux tubes

Biochemistry represents extremely complex and refined choreography. It is hard to believe that this reduces to a mere unconscious and actually apparent fight for chemical survival. In TGD Universe consciousness would be involved even at the molecular level and magnetic body would be the choreographer whose dance would induce the molecular activities. This picture combined with the idea of standard plugs and terminals at which flux tubes end, leads to a to a picture allowing to get rather concrete picture about DNA as topological quantum computer. It becomes also possible to formulate a model for protein folding in which DNA codons can be said to code for both amino-acid sequences and their folding. Hence the information loss thought to occur because of the many-to-one character of the code does not really happen. The model is discussed in [?].

5.6.1 Flux tubes as a correlate for directed attention

Molecular survival is the standard candidate for the fundamental variational principle motivating the molecular intentional actions. There is entire hierarchy of selves and the survival at the higher level of hierarchy would force co-operation and altruistic behavior at the lower levels. One might hope that this hypothesis reduces to Negentropy Maximization Principle [?], which states that the information contents of conscious experience is maximized. If this picture is accepted, the evolution of molecular system becomes analogous to the evolution of a society.

Directed attention is the basic aspect of consciousness and the natural guess would be that directed attention corresponds to the formation of magnetic flux tubes between subject and target. The directedness property requires some manner to order the subject and target.

1. The ordering by the values of Planck constant is what first comes in mind. The larger space-time sheet characterized by a larger value of Planck constant and thus at a higher level of evolutionary hierarchy would direct its attention to the smaller one.

2. Also the ordering by the value of p-adic prime characterizing the size scale of the space-time sheet could be considered but in this case directedness could be questioned.

3. Attention can be directed also to thoughts. Could this mean that attention is directed from real space-time sheets to p-adic space-time sheets for various values of primes but not vice versa? Or could the direction be just the opposite at least in the intentional action transforming p-adic space-time sheet to real space-time sheet? Perhaps directions are opposite for cognition and intention.

The generation of wormhole magnetic flux tubes could be the correlate for the directed attention, not only at molecular level, but quite generally. Metaphorically, the strands of braid would be the light rays from the eyes of the perceiver to the target and their braiding would code the motions of the target to a topological quantum computation like activity and form a memory representation at least. The additional aspect of directed attention would be the coloring of the braid strands, kind of coloring for the virtual light rays emerging from the eyes of the molecular observer. In the case of DNA this can induce a coloring of braid strands emerging from amino-acids and other molecules so that it would indeed become possible to assign to amino-acid the conjugate of the middle nucleotide of the codon XYZ coding for it.

Attention can be also redirected. For this process there is a very nice topological description as a reconnection of flux tubes. What happens is that flux tubes $A \rightarrow B$ and $C \rightarrow D$ fuse for a moment and become flux tubes $A \rightarrow D$ and $C \rightarrow B$. This process is possible only if the strands have the same color so that the values of the quark charges associated with A and B are the same.

This kind of process can modify tqc programs. For instance, in the case of the flux tubes coming from nucleotides X and X_c and ending to the lipid layer this process means that X and X_c and corresponding lipids become connected and genome builds memory representation about this process via similar link. If proteins are connected with mRNA connected to DNA in this manner, this process would allow the formation of flux tubes between amino-acids of two proteins in such a manner that protein would inherit from DNA codon the color of the middle nucleotide and its interactions effectively reduce to base pairing.

DNA would have memory representation about molecular processes via these changing braiding topologies, and one could say that these molecular processes reflect the bodily motions of the magnetic body. Entire molecular dynamics of the organism could represent an enormous tqc induced by the motor activities of the magnetic body. At the level of sensory experience similar idea has been discussed earlier [**?**]: out of body experiences (OBEs) and illusions such as train illusion could be understood in terms of motor action of magnetic body inducing virtual sensory percepts.

Attention can be also switched on and off. Here the structure of the lipid ends containing two nearby situated $= O$:s suggests the mechanism: the short flux tube connecting $= O$:s disappears. The minimization of Coulomb interaction energy at each end implies that re-appearance of the flux tubes creates a short flux tube with the original strand color. Note that the conservation of magnetic flux allows this option only for wormhole magnetic flux tubes.

5.6.2 Does directed attention generate memory representations and tqc like processes?

Directed attention induces braiding if the target is moving and changing its shape. This gives rise to a memory representation of the behavior of the object of attention and also to a tqc like process. A considerable generalization of tqc paradigm suggests itself.

Tqc could be induced by the braiding between DNA and lipids, DNA and proteins via folding processes, DNA RNA braiding and braiding between DNA and its conjugate, DNA and protein braiding. The outcome of tqc would be represented as the temporal patterns of biochemical concentrations and rates and there would be hierarchy of p-adic time scales and those associated with the dark matter hierarchy.

For instance, the protein content of lipid membranes is about 50 per cent and varies between 25-75 per cent so that protein folding and lipid flow could define tqc programs as self-organization patterns.

The folding of protein is dynamical process: alpha helices are created and disappear in time scale of 10^{-7} seconds and the side chains of protein can rotate.

The details of the tqc like process depend on what one assumes. The minimal scenario is deduced from the transcription and translation processes and from the condition that magnetic body keeps control or at least keeps book about what happens using genome as a tool. The picture would be essentially what one might obtain by applying a rough model for web in terms of nodes and links. The reader is encouraged to use paper and pencil to make the following description more illustrative.

1. Assume that mRNA and DNA remain connected by flux tubes after transcription and that only reconnection process can cut this connection so that mRNA inherits the conjugate colors of DNA. Assume same for mRNA and tRNA. Assume that amino-acid associated with tRNA has similar flux tube connections with the nucleotides of tRNA. Under these assumptions amino-acid inherits the conjugate colors of DNA nucleotides via the connection line DNA-mRNA-tRNA-amino-acid faith-fully if all links are correspond to quark pairs rather than their superpositions. Wobble pairing for Z nucleotide could actually correspond to this kind of superposition.

2. One can consider several options for the amino-acid-acid DNA correspondence but trial-and-error work showed that a realistic folding code is obtained only if X, Y, and Z correspond to $O-H$, $O=$, and NH_2 in the constant part of free amino-acid. During translation the formation of the peptide bond between amino-acids dehydration leads to a loss of $O-H$ and one H from NH_2. The flux tube from tRNA to $O-H$ becomes a flux tube to water molecule inheriting the color of X so that $O=-NH_2$ of the amino-acid inside protein represents the conjugate of YZ.

3. Hydrogen bonding between NH and $O=$ in alpha helices and beta sheets reduces effectively to base pairing taking place only if the condition $Y_1 = Z_2$ (briefly $Y = Z$ in the sequel) is satisfied. This is extremely restrictive condition on the gene coding the amino-acid unless one assumes quantum counterpart of wobble base pairing for mRNA or tRNA-amino-acid pairing in the case of Z nucleotide (as one indeed must do). Note that the $O=$ atom of the amino-acid is in a special role in that it can have hydrogen bond flux tubes to donors and flux tube connections with $O=$:s of other amino-acids, the residues of amino-acids containing acceptors (say $O=$ or aromatic ring), and with the aromatic rings of say ATP.

4. The recombination process for two conjugate DNA-mRNA-tRNA-amino-acid links can transform the flux tubes in such manner that one obtains link between the $=O$:s of amino-acids A_1 and A_2 characterized by Y and Y_c. Besides hydrogen bonding this mechanism could be central in the enzyme substrate interaction. The process would pair tRNAs corresponding to Y and Y_c together to give DNA-mRNA-tRNA-tRNA-mRNA-DNA link providing a memory representation about amino-acid pairing $A_1 - A_2$. One could say that magnetic body creates with the mediation of the genome dynamical tqc programs to which much of the bio-molecular activity reduces. Not all however, since two amino-acid pairs $A_1 - A_2$ and $A_3 - A_4$ can recombine to $A_1 - A_4$ and $A_3 - A_2$ without DNA knowing anything about it. Magnetic body would however know.

5. The constant part of non-hydrogen bonded amino-acid inside protein would behave like Y_cZ_c if amino-acid is coded by XYZ. The $COOH$ end of protein would behave like $X_cY_cZ_c$. Also flux tubes connecting the residue groups become possible and protein does not behave like single nucleotide anymore. By color inheritance everything resulting in the reconnection process between $O=$ and NH_2 and residues reduces in a well-defined sense to the genetic code.

5.6.3 Realization of flux tubes

The basic questions about flux are following. Where do they begin, where do they end, and do they have intermediate plugs which allow temporary cutting of the flux tube.

Where do flux tubes begin from?

The view about magnetic body as a controller of biological body using genome as a control tool suggests that DNA is to a high degree responsible for directed attention and other molecules as targets so that flux tubes emanate from DNA nucleotides. The reason would be that the aromatic cycles of DNA correspond to larger value of Planck constant. Some chemical or geometric property of DNA nucleotides or of DNA nucleotides of DNA strand could raise them to the role of subject. Aromatic cycle property correlates with the symmetries associated with large value of Planck constant and is the best candidate for this property.

If this picture is accepted then also some amino-acid residues might act as subjects/objects depending on the option. Phe, His, Trp, Tyr contain aromatic cycle. The derivatives of Trp and Tyr act as neurotransmitters and His is extremely effective nucleophilic catalyst. This would make possible more specific catalytic mechanisms through the pairing of Phe, His, Trp, and Tyr with residues having flux tube terminals.

This raises the question about the physical interaction determining the color of the strand emerging from the aromatic cycle. The interaction energy of quark at the end of flux tube with the classical electromagnetic fields of nuclei and electrons of the ring should determine this. The wormhole contact containing quark/antiquark at the throat at space-time sheet containing nuclei and electrons could also delocalize inside the ring. One of the earliest hypothesis of TGD inspired model for living matter was that wormhole Bose-Einstein condensates could be crucial for understanding of the behavior of biomolecules [?]. Wormhole throats with quark and antiquark at their throats appear also in the model of high T_c superconductivity [?]. As far as couplings are considered, these wormhole contacts are in many respects analogous to the so called axions predicted by some theories of elementary particle physics. The wormhole contact like property is by no means exceptional: all gauge bosons correspond to wormhole contacts in TGD Universe.

The only manner for the electronic space-time sheet to feed its electromagnetic gauge flux to larger space-time sheets using exactly two wormhole contacts is to use wormhole contacts with $\overline{u}$ and d at their "upper" throat (T, G). For proton one would have $\overline{d}$ and u at their "upper" throat (A, C). The presence of electron or proton at nucleotide space-time sheet near the end of flux tube might allow to understand the correlation. The transfer of electrons and protons between space-time sheets with different p-adic length scale is basic element of TGD based model of metabolism so that there might be some relation.

Acceptors as plugs and donors as terminals of flux tubes?

Standardization constraint suggests that flux tubes are attached to standard plugs and terminals. The explicit study of various biological molecules and the role of water in biology gives some hints.

1. An attractive idea is that $=O$ serves as a plug to which flux arrives and from which it can also continue. For the minimal option suggested by hydrogen bonding $O=$ could be connected to two donors and $O=$ could not be connected to $O=$. The assumption that the flux tube can connect also two $O=$:s represents a hypothesis going outside the framework of standard physics. A stronger assumption is that all acceptors can act as plugs. For instance, the aromatic rings of DNA nucleotides could act as acceptors and be connected to a sequence of $O=$ plugs eventually terminating to a hydrogen bond.

2. Donors such as $O-H$ would in turn correspond to a terminal at which flux tube can end. One might be very naive and say that conscious bio-molecules have learned the fundamental role of oxygen and water in the metabolism and become very attentive to the presence of $=O$ and $O-H$. $=O$ appears in $COOH$ part of each amino-acid so that this part defines the standard plug. $=O$ appears also in the residues of Asp, Glu, Asn, Gln. $O-H$ groups appear inside the residues of Asp,Glu and Ser, Thr.

3. Hydrogen bonds $X-H--Y$ have the basic defining property associated with directed attention, namely the asymmetry between donor X and acceptor Y. Hence there is a great temptation consider the possibility that hydrogen bonds correspond to short flux tubes, that flux tubes could be seen as generalized hydrogen bonds. Quite generally, Y could be seen as the object of directed attention of X characterized by larger value of Planck constant. The assumption that two $O=$:s, or even two acceptors of a hydrogen bond, can be connected by a flux tube means more than a generalization of hydrogen bond the connection with a donor would correspond only to the final step in the sequence of flux tubes and plugs giving rise to a directed attention.

4. This hypothesis makes the model rather predictive. For instance, $N-H$, NH_2, $O-H$ and much less often $C-H$ and $S-H$ are the basic donors in the case of proteins whereas $O=$, $-O-$,

$-N = S - S$, $-S^-$ and aromatic rings are the basic acceptors. Reconnection process should be involved with the dynamics of ordinary hydrogen bonding. Reconnection process implies inheritance of the flux tube color and means a realization of the symbol based dynamics. It turns out that this hypothesis leads to a model explaining basic qualitative facts about protein folding.

What about ions like Ca^{++} and Mg_{++}: can flux tubes attach also to these? Could n flux tubes terminate to an n-valent ion? This assumption leads to a model of the gel phase in which Ca^{++} ions serve as cross links between proteins in the sense that Ca^{++} ion is connected by flux tubes to two proteins whereas monovalent ions such as Na^+ cannot perform such a function. Gel-sol phase transition would be induced by a flow of Na^+ ions to the interior of cell inducing a reconnection process so that in the sol phase proteins would be connected to Na^+ ions.

5.6.4 Flux tubes and DNA

The model of DNA as topological quantum computer gives useful guide lines in the attempt to form a vision about flux tubes. It was assumed that braid strands defined by "wormhole magnetic" flux tubes join nucleotides to lipids and can continue through the nuclear or cell membrane but are split during tqc. The hydrophilic ends of lipids attach to water molecules and self-organization patterns for the water flow in gel phase induce a 2-D flow in the lipid layer which is liquid crystal defining tqc programs at the classical level as braidings. The flow indeed induces braiding if one assumes that during topological computation the connection through the cell membrane is split and reconnected after the halting of tqc.

The challenge is to understand microscopically how the flux tube joins DNA nucleotide to the phospholipid [**?**]. Certainly the points at which the flux tubes attach should be completely standard plugs and the formation of polypeptide bonds is an excellent guide line here. Recall that phospholipid, the tqc dancer, has two hydrophobic legs and head. Each leg has at the hydrophilic end O=C-O-C part joining it to glyceride connected to monophosphate group in turn connected to a hydrophilic residue R. The most often appearing residues are serine, inositol, ethanolamine, and choline. Only three of these appear in large quantities and there is asymmetry between cell exterior and interior.

Let us denote by $= O_1$ and $= O_2$ the two oxygens (maybe analogs of right and left hemispheres!) in question. The proposal is that DNA nucleotide and $= O_1$ are connected by a flux tube: the asymmetry between right and left lipid legs should determine which of the legs is "left leg" and which $O =$ is the "left brain hemisphere". $= O_2$, the "holistic right brain hemisphere", connects in turn to the flux tube coming from the other symmetrically situated $= O_2$ at the outer surface of the second lipid layer. Besides this $= O_1$ and $= O_2$ are connected by a flux tube serving as switch on both sides of the membrane.

During tqc the short $O = -O =$ flux tube would experience reconnection with a flux tube acting as hydrogen bond between water molecules so that the connection is split and $O =$:s form hydrogen bonds. The reversal of this reconnection creates the connection again and halts the computation. The lipid residue R couples with the flow of the liquid in gel phase. Since $= O$ is in question the quark or antiquark at the end can correspond to the DNA nucleotide in question. The necessary complete correlation between quark and antiquark charges at the ends of flux tubes associated with $= O_1$ and $= O_2$ can be understood as being due to the minimization of Coulomb interaction energy.

If one is ready to accept magnetic flux tubes between all acceptors then the aromatic rings of nucleotides known to be acceptors could be connected by a flux tube to the $O =$ atom of the lipid or to some intermediate $O =$ atom. The phosphate groups associated with nucleotides of DNA strand contain also $= O$, which could act as a plug to which the flux tube from the nucleotide is attached. The detailed charge structure of the aromatic ring(s) should determine the quark-nucleotide correspondence. The connection line to the lipid could involve several intermediate $O =$ plugs and the first plug in the series would be the $O =$ atom of the monophosphate of the nucleotide.

There is a strong temptation to assume that subset of XYP molecules, $X = A, G, T, C$, $Y = M, D, T$ act as standard plugs with X and phosphates connected by flux tubes to a string. This would make it possible to engineer braid strands from standard pieces connected by standard plugs. DNA nucleotide XMP would have flux tube connection to the aromatic ring of X and the $O =$ of last P would be connected to next plug of the communication line. If so, a close connection with metabolism and topological quantum computation would emerge.

1. Phosphorylation [?] would be an absolutely essential for both metabolism and buildup of connection lines acting as braid strands. Phosphorylation is indeed known to be the basic step activating enzymes. In eukaryotes the phosphorylation takes plane amino-acids most often for ser but also thr, and trp with aromatic rings are phosphorylated. Mitochondrions have specialized to produce ATP in oxidative phosphorylation from ADP and photosynthesis produces ATP. All these activities could be seen as a production of standard plugs for braid strands making possible directed attention and quantum information processing at molecular level.

2. As already noticed, $O = -O =$ flux tubes could also act as switches inducing a shortcut of the flux tube connection by reconnecting with a hydrogen bond connecting two water molecules. This is an essential step in the model for how DNA acts as topological quantum computer. De-phosphorylation might be standard manner to realized this process.

3. This picture would fit with the fact that XYP molecules, in particular AMP, ADP, and ATP, appear in bio-molecules involved with varying functions such as signalling, control, and metabolism. $= O$ might act as a universal plug to which flux tubes from electronegative atoms of information molecules can attach their flux tubes. This would also provide a concrete realization of the idea that information molecules (neurotransmitters, hormones) are analogous to links in Internet [?]: they would not represent the information but establish a communication channel. The magnetic flux tube associated with the information molecule would connect it to another cell and by the join to $= O$ plug having flux tube to another cell, say to its nucleus, would create a communication or control channel.

5.7 Some predictions related to the representation of braid color

Even in the rudimentary form discussed above the model makes predictions. In particular, the hypothesis that neutral quark pairs represent braid color is easily testable.

5.7.1 Anomalous em charge of DNA as a basic prediction

The basic prediction is anomalous charge of DNA. Also integer valued anomalous charge for the structural units of genome is highly suggestive.

The selection of the working option - if any such exists - is indeed experimentally possible. The anomalous charge coupling to the *difference* of the gauge potentials at the two space-time sheets defines the signature of the wormhole contact at the DNA end of braid strand. The effective (or anomalous) em charge is given as sum of quark charges associated with DNA space-time sheet:

$$Q_a = [n(A)-n(T)]Q(q_A)+[n(G)-n(C)]Q(q_G) \tag{5.7.1}$$

is predicted. The four possible options for charge are given explicitly in the table below

$$\begin{array}{l} Q_a = [n(A)-n(T)]\frac{2}{3} - [n(G)-n(C)]\frac{1}{3} \ , \\ Q_a = -[n(A)-n(T)]\frac{1}{3} + [n(G)-n(C)]\frac{2}{3} \ , \\ Q_a = -[n(A)-n(T)]\frac{2}{3} + [n(G)-n(C)]\frac{1}{3} \ , \\ Q_a = [n(A)-n(T)]\frac{1}{3} - [n(G)-n(C)]\frac{2}{3} \ . \end{array} \tag{5.7.2}$$

Second option is obtained from the first option $(A,T,G,C) \rightarrow (u,\overline{u},d,\overline{d})$ by permuting u and d quark in the correspondence and the last two options by performing charge conjugation for quarks in the first two options.

The anomalous charge is experimentally visible only if the external electromagnetic fields at the two sheets are different. The negative charge of DNA due to the presence of phosphate groups implies that the first sheet carries different em field so that this is indeed the case.

The presence effective em charge depending on the details of DNA sequence means that electromagnetism differentiates between different DNA:s strands and some strands might be more favored dynamically than others. It is interesting to look basic features of DNA from this view point. Vertebral mitochondrial code has full $A \leftrightarrow G$ and $C \leftrightarrow T$ symmetries with respect to the third nucleotide of the codon and for the nuclear code the symmetry is almost exact. In the above scenario A and C *resp.* G and T would have different signs and magnitudes of em charge but they would correspond to different weak isospin states for the third quark so that this symmetry would be mathematically equivalent to the isospin symmetry of strong interactions.

The average gauge potential due to the anomalous charge per length at space-time sheet containing ordinary em field of a straight portion of DNA strand is predicted to be proportional to

$$\frac{dQ_a}{dl} = [p(A) - p(T)]Q(q_A) + [p(G) - p(C)]Q(q_G)\frac{1}{\Delta L} \ ,$$

where ΔL corresponds to the length increment corresponding to single nucleotide and $p(X)$ represents the frequency for nucleotide X to appear in the sequence. Hence the strength of the anomalous scalar potential would depend on DNA and vanish for DNA for which A and T *resp.* G and C appear with the same frequency.

5.7.2 Chargaff's second parity rule and the vanishing of net anomalous charge

Chargaff's second parity rule states that the frequencies of nucleotides for single DNA strand satisfy the conditions $p(A) \simeq p(T)$ and $p(C) \simeq p(G)$ (I am grateful for Faramarz Faghihi for mentioning this rule and the related article [?] to me). This rule holds true in a good approximation. In the recent context the interpretation would be as the vanishing of the net anomalous charge of the DNA strand and thus charge conjugation invariance. Stability of DNA might explain the rule and the poly-A tail in the untranslated mRNA could relate stabilization of DNA and mRNA strands.

Together with $p(A) + p(T) + p(G) + p(C) = 1$ Chargaff's rule implies the conditions

$$\begin{array}{ll} p(A) + p(C) \simeq 1/2 \ , & p(A) + p(G) \simeq 1/2 \ , \\ p(T) + p(C) \simeq 1/2 \ , & p(T) + p(G) \simeq 1/2 \ . \end{array} \tag{5.7.3}$$

An interesting empirical finding [?] is that only some points at the line $p(A)+p(C) \simeq 1/2$ are realized in the case of human genome and that these points are in a good accuracy expressible in terms of Fibonacci numbers resulting as a prediction of optimization problem in which Fibonacci numbers are however put in by hand. $p(A) = p(G) = p(C) = p(T) = 1/4$ results as a limiting case. The poly-A tail of mRNA (not coded by DNA) could reflect to the compensation of this asymmetry for translated mRNA.

The physical interpretation would be as a breaking of isospin symmetry in the sense that isospin up and down states for quarks (A and G *resp.* T and C) do not appear with identical probabilities. This need not have any effect on protein distributions if the asymmetry corresponds to asymmetry for the third nucleotide of the codon having $A \leftrightarrow G$ and $T \leftrightarrow C$ symmetries as almost exact symmetries. This of course if protein distribution is invariant under this symmetry for the first two codons.

The challenge would be to understand the probabilities $p_3(X)$ for the third codon from a physical model for the breaking of isospin symmetry for the third codon in the sense that u and $\overline{u}$ at DNA space-time sheet are more favored than d and $\overline{d}$ or vice versa. There is an obvious analogy with spontaneous breaking of vacuum symmetry.

5.7.3 Are genes and other genetic sub-structures singlets with respect to QCD color?

Genes are defined usually as transcribed portions of DNA. Genes are however accompanied by promoter regions and other regions affecting the transcription so that the definition of what one really means with gene is far from clear. In the recent case gene would be naturally tqc program module and gene in standard sense would only correspond to its sub-module responsible for the translated mRNA output of tqc.

Whatever the definition of gene is, genes as tqc program modules could be dynamical units with respect to color interaction and thus QCD color singlets (QCD color should not be confused with braid color) or equivalently - possess integer valued anomalous em charge.

One can consider two alternative working hypothesis - in a well-defined sense diametrical opposites of each other.

1. The division of the gene into structural sub-units correlates with the separation into color singlets. Thus various structural sub-units of gene (say transcribed part, translated part, intronic portions, etc...) would be color singlets.

2. Also different genetic codes that I have discussed in [?] could distinguish between different structural sub-units. For this option only gene - understood as tqc unit with un-transcribed regions included - would be color singlet.

Color singletness condition is unavoidable for mRNA and leads to a testable prediction about the length of poly-A tail added to the transcribed mRNA after translation.

The condition of integer valued anomalous charge for coding regions

In the case of coding region of gene the condition for integer charge is replaced by the conditions

$$n(A)+n(G) \ \ mod \ \ 3=0 \ \ , \qquad n(C)+n(T) \ \ mod \ \ 3=0 \ \ . \tag{5.7.4}$$

These conditions are not independent and it suffices to check whether either of them is satisfied. The conditions are consistent with $A \leftrightarrow G$ and $T \leftrightarrow C$ symmetries of the third nucleotide. Note that the contribution of the stop codon (TAA, TGA or TAG) and initiating codon ATG to the A+G count is one unit.

General condition for integer valued anomalous charge

The anomalous charge of gene or even that of an appropriate sub-unit of gene is integer valued implies in the general case

$$n(A)-n(T)+n(G)-n(C) \ \ mod \ \ 3=0 \ \ . \tag{5.7.5}$$

Note that this condition does not assume that gene corresponds to $3n$ nucleotides (as I had accustomed to think). The surprising (to me) finding was that gene and also mRNA coding region of the gene in general fails to satisfy $3n$ rule. This rule is of course by no means required: only the regions coding for proteins can be thought of as consisting of DNA triplets.

A possible interpretation is in terms of TGD based model for pre-biotic evolution [?] according to which genetic code (or 3-code) was formed as a fusion of 2-code and 1-code. 2-code and 1-code could still be present in genome and be associated with non-translated regions of mRNA preceding and following the translated region. The genes of 2-code and coding for RNA would have $2n$ nucleotides and the genes of 1-code could also consist of odd number of nucleotides.

There might be analogy with drawings for a building. These contain both figures providing information about building and text giving meta-level information about how to interpret figures. Figures could correspond to 3-code coding for proteins and text could be written with other codes and give instructions for the transcription and translation processes. Prokaryotic code would contain mostly figures (CDS). In eukaryotic code intronic portions could carry rich amounts of this kind of meta-level information. In the case of mRNA untranslated region preceding $5'$ end could provide similar information.

1. Repeating sequences consisting of n copies of same repeating unit could obey 1-code or 2-code. The simplest building blocks of repeating sequences are AT and CG having vanishing anomalous em charge. TATATA.... and CGCGCG... indeed appear often. Also combinations of CG and AT could repeat: so called mini-satellites are CG rich repeating sequences. Interpretation in terms of 2-code suggests itself.

2. Triplet of the unit ATTCG with integer charge repeats also often: in this case 3-code suggests itself. Telomeres of vertebrates consist of a repeating unit TTAGGG which does not have integer charge: this unit appears also as 8-nucleotide variant which suggests 2-code. Color singletness would require that this unit appears $3n$ times.

3. I have also proposed that intronic regions could obey memetic code [?] predicting that intronic codon can be represented as a sequence of 21 3-codons (implying 2^{63} 63-codons!). Individual intronic segments need not satisfy this rule, only their union if even that. Direct experimentation with gene bank data show that neither introns nor their union correspond to integer multiples of 63 nor 3 or 2 in general.

Color singletness conditions for gene

Gene is usually defined as the sequence of DNA coding for mRNA. mRNA involves also two untranslated regions (UTRs) [?].

1. The $5'$ end of mRNA contains $5'$ cap (methylated G) and $5'$ untranslated region (UTR). The latter can be several kb long for eukaryotes. Methylated G is not coded by DNA but added so that it does not contribute to A+G-T-C count at DNA level.

2. mRNA continues after the stop codon as $3'$ UTR. Translation assigns to UTR also a poly-A tail (up to several hundreds A:s) not coded by DNA and not contributing to A+G-T-C count in the case of DNA. This region contains also AAUAAA which does not contribute to A+G-T-C count of mRNA.

   One could argue that any amino-acid sequence must allow coding and that one function of UTRs is to guarantee integer valued charge for the part of gene beginning from the initiating codon. Of course, also the non-transcribed regions of DNA not included in the standard definition of gene could take care of this.

Color singletness conditions for mRNA

Both poly-A tail and G gap are known to relate to the stabilization of mRNA. The mechanism could be addition of an anomalous charge compensating for the anomalous charge of mRNA to guarantee that second Chargaff's rule is satisfied in a good approximation: this hypothesis is testable.

Second function would be to guarantee color-singletness property. Color singletness would mean that transcribed mRNA + cap G + poly-A tail as a separate unit must be QCD color singlet at DNA space-time sheet. mRNA stability requires the condition

$$n(A)-n(T)+n(G)-n(C)+n_{tail}(A)+1 \ \ mod \ \ 3=0 \tag{5.7.6}$$

to be satisfied. The knowledge of gene would thus predict $n_{tail}(A) \ \ mod \ \ 3$. This hypothesis is testable.

Chargaff's rule for mRNA

If Chargaff's rule applies also to mRNA strands one obtains one of the following predictions

$$\begin{aligned}
&2\left[n(A)+n_{tail}(A)-n(T)\right]-\left[n(G)+1-n(C)\right]\simeq 0 \ \ ,\\
&-\left[n(A)+n_{tail}(A)-n(T)\right]+2\left[n(G)+1-n(C)\right]\simeq 0 \ \ ,\\
&-2\left[n(A)+n_{tail}(A)-n(T)\right]+\left[n(G)+1-n(C)\right]\simeq 0 \ \ ,\\
&\left[n(A)+n_{tail}(A)-n(T)\right]-2\left[n(G)+1-n(C)\right]\simeq 0 \ \ .
\end{aligned} \tag{5.7.7}$$

Here $n_{tail}(A)$ includes also AAUAA contributing 3 units to it plus possible other structures appearing in the tail added to the translated mRNA. The presence of poly-A tail which could also compensate for the ordinary negative charge of translated part of mRNA would suggest that A corresponds to u or $\overline{d}$ corresponding to options 1 and 4.

Moving genes and repeating elements

Transposons [?, ?] are moving or self-copying genes. Moving genes cut from initial position and past to another position of double strand. Copying genes copy themselves first to RNA and them to a full DNA sequence which is then glued to the double strand by cut and paste procedure. They were earlier regarded as mere parasites but now it is known that their transcription is activated under stress situations so that they help DNA to evolve. In tqc picture their function would be to modify tqc hardware. For copying transposons the cutting of DNA strand occurs usually at different points for DNA and cDNA so that "sticky ends" result ("overhang" and its complement) [?]. Often the overhang has four nucleotides. The copied transposon have ends which are reversed conjugates of each other so that transposons are palindromes as are also DNA hairpins. This is suggestive of the origin of transposons.

In order to avoid boring repetitions let us denote by "satisfy P" for having having integer valued (or even vanishing) Q_a. The predictions are following:

1) The double strand parts associated with the segments of DNA produced by cutting should satisfy P.

2) The cutting of DNA should take place only at positions separated by segments satisfying P.

3) The overhangs should satisfy P.

4) Transposons should satisfy P: their reverse ends certainly satisfy P.

In the example mentioned in [?] the overhang is $CTAG$ and has vanishing Q_a. The cut site $CCTAGG$ has also vanishing Q_a. It is known [?] that transposons - repeating regions themselves - tend to attach to the repeating regions of DNA [?].

1. There are several kinds of repeating regions. 6-10 base pair long sequences can be repeated in untranslated regions up to 10^5 times and whole genes can repeat themselves $50 - 10^4$ times.

2. Repeats are classified into tandems (say TTAGGG associated with telomeres), interspersed repetitive DNA (nuclear elements), and transposable repeat elements. Interspersed nuclear elements (INEs) are classified LINEs (long), SINEs (short), TLTRs (Transposable elements with Long Terminal Repeats), and DNA transposons themselves.

3. LINEs contain AT rich regions. SINEs known as alus (about 280 bps) contain GC rich regions whereas mariner elements (about 80 bps) are flanked by TA pairs. LTRs have length 300-1000 bps. DNA transposons are flanked with two short inverted repeat sequences flanking the reading frame: "inverted" refers to the palindrome property already mentioned.

AT and CG have vanishing Q_a so that their presence in LINEs and SINEs would make the cutting and pasting easy allowing to understand why transposons favor these regions. Viruses are known to contain long repeating terminal sequences (LTR). One could also check whether DNA decomposes to regions satisfying P and surrounded by repeating sequences which satisfy P separately or as whole as in the case DNA transposons.

Tests

Some checks of the color singletness hypothesis were made for human genome [?].

1. For the coding sequences (CDSs) the strong prediction in general fails as expected (condition would pose restrictions on possible amino-acid contents).

2. Color singletness condition fails for genes defined in terms of translated part of mRNA (with gap and poly-A tail excluded). The un-transcribed regions of DNA involved with the gene expression (promoter region, etc...) could guarantee the color singletness. They could also stabilize DNA by bringing in compensating anomalous charge to guarantee second Chargaff's rule. Different genetic codes could distinguish between the subunits of gene.

3. To test color singletness conditions for mRNA one should know the length of poly-A tail. Unfortunately, I do not have access to this information.

4. The computation of total anomalous charges for a handful of genes, introns, and repeat units for some gene bank examples in the case of human genome indicates that both of them tend to carry net em charge which is largest for $(a, g) \leftrightarrow (\overline{d}, \overline{u})$ correspondence. The charge is in the range 5-10 per cent from the charge associated with the phosphates (-2 units per nucleotide). For second option giving negative charge (permute u and d) the anomalous charge is few per cent smaller.

   By Chargaff's law the regions outside genes responsible for the control of gene expression must contain a compensating charge of opposite sign. Kind of spontaneous symmetry breaking of charge conjugation symmetry $A \leftrightarrow T, G \leftrightarrow C$ and analogous to matter antimatter symmetry seems to take place. That control regions and translated regions have opposite densities of anomalous charge might also help in the control gene expression.

5. The poly-A tail of mRNA would carry compensating positive anomalous charge: the RNA-quark assignment could be conjugate to the DNA-quark assignment as suggested by what takes place in transcription. For instance, for the option $A \rightarrow \overline{d}$, the prediction for the length of polytail for $A \rightarrow \overline{d}$ option would be about $n_{tail}/n_{mRNA} \simeq 3p_a(mRNA)$ where $N(mRNa)$ is the number of nucleotides in transcribed mRNA and $p_a(mRNA)$ is the per cent of anomalous charge which is typically 5-10 per cent. For $p_a(mRNA) = 10$ per cent this gives as much as 30 per cent. For $A \rightarrow \overline{u}$ option one has $n_{tail}/n_{mRNA} \simeq 3p_a(mRNA)/2$. In this case also p_a is considerably smaller, typically by a factor of of order 2-3 per cent and even below per cent in some cases. Hence the relative length of tail would around 3-5 per cent. This option is perhaps more since it minimizes anomalous charge and maximizes the effectiveness of charge compensation by poly-A tail.

6. The predictions for transposons and their cut and past process should be easily testable.

5.7.4 Summary of possible symmetries of DNA

The following gives a list of possible symmetries of DNA inspired by the identification of braid color.

Color confinement in strong form

The states of quarks and anti-quarks associated with DNA both wormhole wormhole throats of braided (living) DNA strand can be color singlets and have thus integer valued anomalous em charge. The resulting prediction depends on the assignment of quarks and antiquarks to A,T,C,G which in principle should be determined by the minimization of em interaction energy between quark and nucleotide. For instance $2(A-T)-(G-C) \ \ mod \ \ 3 = 0$ for a piece of living DNA which could make possible color singletness. As a matter fact, color singletness conditions are equivalent for all possible for braid color assignments. This hypothesis might be weakened. For instance, it could hold true only for braided parts of DNA and this braiding are dynamical. It could also hold for entire braid with both ends included only: in this case it does not pose any conditions on DNA.

Questions: Do all living DNA strands satisfy this rule? Are only the double stranded parts of DNA braided and satisfy the rule. What about loops of hairpins?

Matter antimatter asymmetry at quark level

$A \leftrightarrow T$ and $G \leftrightarrow C$ corresponds to charge conjugation at the level of quarks (quark $\leftrightarrow$ antiquark). Chargaff's rules states $A \simeq T$ and $C \simeq G$ for long DNA strands and mean matter-antimatter symmetry in the scale of DNA strand. Double strand as a whole is matter anti-matter symmetric.

Matter-antimatter asymmetry is realized functionally at the level of DNA double strand in the sense that only DNA strand is transcribed. The study of some examples shows that genes defined as transcribed parts of DNA do not satisfy Chargaff's rule. This inspires the hypothesis about the breaking of matter antimatter symmetry. Genes have non-vanishing net $A-T$ and $C-G$ and therefore also net Q_a with sign opposite to that in control regions. Just as the Universe is matter-antimatter asymmetric, also genes would be matter-antimatter asymmetric.

Isospin symmetry at quark level

$A \leftrightarrow G$ and $T \leftrightarrow A$ correspond change of anomalous em charge by 1 unit and these operations respect color confinement condition. Local modifications of DNA inducing these changes should be preferred. The identification for the symmetries $A \leftrightarrow G$ and $T \leftrightarrow A$ for the third nucleotide of code is as isospin symmetries. For the vertebrate mitochondrial code the symmetry exact and for nuclear code slightly broken.

Matter antimatter asymmetry and isospin symmetries for the first two nucleotides

The first two nucleotides of the codon dictate to a high degree which amino-acid is coded. This inspires the idea that 3-code has emerged as fusion of 1- and 2-codes in some sense. There are two kinds of 2-codons. The codons of type A have fractional em charge and net quark number (consisting of either matter or antimatter at quark level) and are not able to form color singlets. The codons of type B have integer em charge and vanishing quark number (consisting of matter and antimatter) and are able to form color singlets. The 2-codons of type A (resp. B) are related by isospin rotations and there should be some property distinguishing between types A and B. There indeed is: if 2-codon is matter-antimatter symmetric, 1-codon is not and vice versa.

1. For almost all type A codons the amino-acid coded by the codon does not depend on the last nucleotide. There are two exceptions in the case of the nuclear code: (leu,leu,phe,phe) and (ile,ile,ile,met). For human mitochondrial code one has (ile,ile,ile,ile) and thus only one exception to the rule. The breaking of matter-antimatter symmetry for the third nucleotide is thus very small.

2. For codons of type B the 4-columns code always for two doublets in the case of vertebrate mitochondrial code so that for codons with vanishing net quark number the breaking of matter-antimatter symmetry for the third nucleotide is always present.

Em stability

Anomalous em charge Q_a vanishes for DNA and perhaps also mRNA strand containing also the G cap and poly-A tail which could compensate for the Q_a of the transcribed region so that

$$2(A-T)-(G-C) \simeq 0$$

or some variant of it holds true. Chargaff's rules for long DNA strands imply the smallness of Q_a.

Summary of testable working hypothesis

Following gives a summary of testable working hypothesis related to the isospin symmetry and color singletness. The property of having integer valued/vanishing Q_a is referred to as property P.

1. Gene plus control region and also DNA repeats should have property P. Transcribed and control regions of gene have Q_a with opposite signs.

2. Transposons, repeating regions, the overhangs associated with the cut and paste of transposon, and the DNA strands resulting in cutting should have property P. This could explain why transposons can paste themselves to AT and GC ($Q_a = 0$) rich repeating regions of DNA. The points at which DNA can be cut should differ by a DNA section having property P. This gives precise predictions for the points at which transposons and pieces of viral DNA can join and could have implications for genetic engineering.

3. If also mRNA is braided, it has property P. This can be only true if the poly-A tail compensates for the non-vanishing Q_a associated with the translated region.

4. Living hairpins should have property P. If only double helix parts of hairpins are braided, the prediction is trivially true by the palindrome property. tRNA or at least parts of it could be braided. Braids could end to the nuclear membrane or mRNA or to the amino-acid attachable

to tRNA. For stem regions Q_a is integer valued. The fact that the nucleotide of the anticodon corresponding to the third nucleotide of codon can base pair with several nucleotides of mRNA suggests that $I(nositol)$ can have Q_a opposite to that of A,T,C and U opposite to that of A,G. For 2-anticodon the pairing would be unique. This would give a lot of freedom to achieve property P in weak sense for tRNA. Braid structure for tRNA + amino-acid could be different that for tRNA alone and also in the translation the braid structure could change.

5. Telomeres [?] are of special interests as far as anomalous em charge is considered. Chromosomes are not copied completely in cell replication, and one function of telomeres is to guarantee that the translated part of genome replicates completely for sufficiently many cell divisions. Telomeres consists of 3-20 kilobases long repetitions of TTAGGG, and there is a 100-300 kilobases long repeating sequence between telomere and the rest of the chromosome. Telomeres can form can also 4-stranded structures. Telomere end contains a hair-pin loop as a single stranded part, which prevents the action of DNA repair enzymes on the chromosome end. Telomerase is a reverse transcriptase enzyme involved with the synthesis of telomeres using RNA strand as a template but since its expression is repressed in many types of human cells, telomere length shortens in each cell replication. In the case of germ cells, stem cells and white blood cells telomerase is expressed and telomere length preserved. Telomere shortening is known to relate to ageing related diseases. On the other hand, overactive telomere expression seems to correlate with cancer.

   If telomeres possess braid strands, the compensation of Q_a might provide an additional reason for their presence. If this the case and if telomeres are strict multiples of TTAGGG, the shortening of telomeres generates a non-vanishing Q_a unless something happens for the active part of DNA too. Color singletness condition should however remain true: the disappearance of $3n$ multiples of TTAGGG in each replication is the simplest guess for what might happen. In any case, DNA strands would become unstable in cell replication. Q_a could be reduced by a partial death of DNA in the sense that some portions of braiding disappear. Also this would induce ill functioning of tqc harware perhaps related to ageing related diseases. Perhaps evolution has purposefully developed this ageing mechanism since eternal life would stop evolution.

6. Also aminoacids could be braided. Q_a could vary and correspond to Q_a for one of the codons coding for it. The aminoacid sequences of catalysts attaching to DNA strand should have opposite Q_a for each codon-aminoacid pair so that aminoacid would attach only to the codons coding for it. The TGD based model for nerve pulse [?] inspires the proposal that magnetic flux tubes connecting microtubules to the axonal membrane allow tqc during nerve pulse propagation when axonal membrane makes transition from gel like phase to liquid crystal phase. Aminoacids of tubulin dimers would be connected by 3-braids, smallest interesting braid, to groups of 3-lipids in axonal membrane and tubulin dimers would define fundamental tqc modules.

5.7.5 Empirical rules about DNA and mRNA supporting the symmetry breaking picture

Somewhat surprisingly, basic facts which can be found from Wikipedia, support the proposed vision about symmetry breaking although, the mechanism of matter antimatter symmetry breaking is more complex than the first guess. I am grateful for Dale Trenary for references which made possible to realize this. Before continuing some comments about the physical picture are in order.

1. The vanishing of the induced Kähler field means that the space-time sheet of DNA is a highly unstable vacuum extremal. The non-vanishing of the induced Kähler electric field is thus a natural correlate for both the stability and the non-vanishing quark number density (matter antimatter asymmetry). The generation of matter antimatter asymmetry induces a net density of anomalous em charge, isospin, and quark number in the portion of DNA considered. This in turn generates not only longitudinal electric field but also a longitudinal Kähler electric field along DNA.

2. Weak electric fields play a key role in living matter. There are electric fields associated with embryos, central nervous system, individual neurons, and microtubules and their direction de-

termines the direction of a process involved (head-to-tail direction, direction of propagation of nerve pulse, ...).

3. Same mechanism is expected to be at work also in the case of DNA and RNA. In the case of gene the direction of transcription could be determined by the direction of the electric field created by gene and telomeres at the ends of chromosomes carrying a net anomalous quark number could be partially responsible for the generation of this field. In the case of mRNA the direction of translation would be determined in the similar manner. The net anomalous em charges of poly-A tail and the transcribed part of mRNA would have opposite signs so that a longitudinal electric field would result.

It will be found that this picture is consistent with empirical findings about properties of DNA.

Breaking of matter antimatter symmetry and isospin symmetry for entire genome

Chargaff's rules are not exact and the breaking gives important information about small breakings of isospin and matter-antimatter symmetries at the level of entire genome. The basic parameters are em charge per nucleotide, isospin per nucleotide, the amount of quark number per nucleotide, and the ratio of u and d type matters coded by $(G+C)/(A+T)$ ratio. Recall that there are four options for the map of A,T,C,G to quarks and antiquarks and for option 3) *resp.* 4) the anomalous em charge is opposite to that for 1) *resp.* 2).

The following table gives A,T,C,G contents (these data are from Wikipedia [?]), the amount of quark charge per nucleotide for the options 1) *resp.* 2) given by $dq_1/dn = p[2(A-T)-G-C)]/3$ *resp.* $dq_2/dn = p[A-T-2(G-C)]/3$, the amount $dI_3/dn = p(A-G+C-T]/2$ of isospin per nucleotide, the amount $d(q-\overline{q})/dn = p(A-T+G-C)$ of quark number per nucleotide, and $(A+T)/(C+G)$ ratio for *entire genomes* in some cases. It will be found that so called Szybalski's rules state that for coding regions there is breaking of the approximate matter antimatter asymmetry.

Note that matter antimatter asymmetry in the scale of entire genome has largest positive value for human genome and negative value only for yeast genome: this case the magnitude of the asymmetry is largest.

	$Human$	$Chicken$	$Grass-hopper$	$Sea\ Urchin$	$Wheat$	$Yeast$	$E.Coli$
$p(A)$	0.3090	0.2880	0.2930	0.3280	0.2730	0.3130	0.2470
$p(T)$	0.2940	0.2920	0.2930	0.3210	0.2710	0.3290	0.2360
$p(C)$	0.1990	0.2050	0.2050	0.1770	0.2270	0.1870	0.2600
$p(G)$	0.1980	0.2170	0.2070	0.1730	0.2280	0.1710	0.2570
$\frac{dq_1}{dn}$	0.0103	-0.0067	-0.0007	0.0060	0.0010	-0.0053	0.0083
$\frac{dq_2}{dn}$	0.0057	-0.0093	-0.0013	0.0050	-0.0000	0.0053	0.0057
$\frac{dI_3}{dn}$	0.0080	-0.0080	-0.0010	0.0055	0.0005	0.0000	0.0070
$\frac{d(q-\overline{q})}{dn}$	0.0140	0.0080	0.0020	0.0030	0.0030	-0.0320	0.0080
$\frac{p(A+T)}{p(G+C)}$	1.5189	1.3744	1.4223	1.8543	1.1956	1.7933	0.9342

(5.7.8)

For option 2) the amount of anomalous charge is about .0057e per nucleotide and thus about $3\times 10^7 e$ for entire human DNA having length of about 1.8 meters. The inspection of tables of [?] shows that the anomalous em charge for the repeating sequence defining the telomere is always non-vanishing and has always the same sign. Telomeres for human chromosomes consist of TTAGGG repetitions with anomalous em charge with magnitude $5e/3$ for all options and have a length measured in few kbases. Human genome as has 24 chromosomes so that the total anomalous em charge of telomeres is roughly $24\times(5/18)\times x10^3 e \sim .8\times 10^3 xe$, $1<x<10$. The anomalous em charge of telomeres is three orders of magnitude smaller than that of entire DNA but if DNA is quantum critical system the change the total anomalous em charge and quark number due to the shortening of telomeres could induce instabilities of DNA (due to the approach to vacuum extremal) contributing to ageing. Note that the small net value of quark number in all the cases considered might be necessary for overall stability of DNA. Telomeres are also known to prevent the ends of chromosomes to stick to each other. This could be partially due to the Coulomb repulsion due to the anomalous em charge.

According to [?] Chargaff's rules do not apply to viral organellar genomes (mitochondria [?], plastids) or single stranded viral DNA and RNA genomes. Thus approximate matter antimatter symmetry fails for DNA:s of organelles involved with metabolism. This might relate to the fact that the coding portion of DNA is very high and repeats are absent. Chargaff's rule applies not only to nucleotides but also for oligonucleotides which corresponds to DNA or RNA sequences with not more than 20 bases. This means that for single strand oligonucleotides and their conjugates appear in pairs. Matter antimatter asymmetry would be realized as presence of matter blobs and their conjugates. This might relate to the mechanism how the sequences of oligonucleotides are generated from DNA and its conjugate.

Breaking of matter antimatter symmetry for coding regions

As noticed, one can consider three type of symmetry breaking parameters for DNA in DNA as tqc model. There are indeed three empirical parameters of this kind. Chargaff' rules have been already discussed and correspond to approximate matter antimatter symmetry. The second asymmetry parameter would measure the asymmetry between $u\overline{u}$ and $d\overline{d}$ type matter. $p(G+C)$ corresponds to the fraction of $d\overline{d}$ type quark matter for option 1) and $u\overline{u}$ matter for option 2). It is known that G+C fraction $p(G+C)$ characterizes genes [?] and the value of $p(G+C)$ is proportional to the length of the coding sequence [?, ?].

Besides Chargaff' rules holding true for entire genome also Szybalski's rules [?] hold true but only for coding coding regions. The biological basis of neither rules is not understood. The interpretation of Chargaff's rules would be in terms of approximate matter antimatter symmetry and the vanishing of net isospin at the level of quarks whereas Szybalski's rule would state the breaking of these symmetries non-coding regions. Hence all the three basic empirical rules would have a nice interpretation in DNA as tqc picture.

Consider now Szybalski's rules in more detail.

1. In most bacterial genomes (which are generally 80-90 % coding) genes are arranged in such a fashion that approximately 50 % of the coding sequence lies on either strand. Note that either strand can act as a template (this came as a surprise for me). Szybalski, in the 1960s, showed that in bacteriophage coding sequences purines (A and G) exceed pyrimidines (C and T). This rule has since been confirmed in other organisms and known as Szybalski's rule [?, ?]. While Szybalski's rule generally holds, exceptions are known to exist.

   Interpretation. A breaking of matter antimatter symmetry occurs in coding regions such that the net breakings are opposite for regions using different templates and thus different directions of transcription (promoter to the right/left of coding region).

2. One can actually characterize Szybalski's rules more precisely. By Chargaff's rules one has $p(A+T) \simeq 1 - p(G+C))$. In coding regions with low value of $p(G+C)$ $p(A)$ is known to be higher than on the average whereas for high value of $p(G+C)$ $p(G)$ tends to higher than on the average.

   Interpretation. These data do not fix completely the pattern of breaking of the approximate matter antimatter symmetry.

   i) It could take place for both kinds of quark matter ($u\overline{u}$ and $d\overline{d}$): both $p(A)$ and $p(G)$ would increase from its value for entire genome but the dominance of A over G or vice versa would explain the observation.

   ii) The breaking could also occur only for the dominating type of quark matter ($u\overline{u}$ or $d\overline{d}$) in which case only $p(A)$ or $p(G)$ would increase from the value for entire genome.

   Also a net isospin is generated which is of opposite sign for short and long coding sequences so that there must be some critical length of the coding sequences for which isospin per nucleotide vanishes. This length should have biological meaning.

3. For mRNA $A+G$ content is always high. This is possible only because the template part of the DNA which need not be always the same strand varies so that if it is strand it has higher $A+G$ content and if it is conjugate strand it has higher $T+C$ content.

Interpretation. mRNA breaks always matter antimatter symmetry and the sign of matter antimatter asymmetry is always the same. Thus mRNA is analogous to matter in observed universe. The poly-A tail added to the end of mRNA after transcription to stabilize it would reduce the too large values of isospin and anomalous em charge per nucleon due to the fact that mRNA does not contain regions satisfying Chargaff's rules. It would also generate the needed longitudinal electric field determining the direction of translation. In the case of DNA the breaking of matter antimatter symmetry is realized at the functional level by a varying direction of transcription and variation of template strand so that matter antimatter symmetry for the entire DNA is only slightly broken. Direction of transcription would be determined by the direction of the electric field. The stability of long DNA sequences might require approximate matter antimatter symmetry for single DNA strand if it is long. In the case of simple genomes (mitochondrial, plastid, and viral) the small size of the genome, the high fraction of coding regions, and the absence of repeating sequences might make approximate matter antimatter symmetry un-necessary. An interesting working hypothesis is that the direction of transcription is always the same for these genomes.

One can try to use this information to fix the most probable option for nucleotide quark correspondence.

1. In nuclear physics the neutron to proton ratio of nucleus increases as nucleus becomes heavier so that the nuclear isospin becomes negative: $I_3 < 0$. The increase of the nuclear mass corresponds to the increase for the length of the coding region. Since G/A fraction increases with the length of coding region, G should correspond to either d quark $((Q_a < 0, I_3 = -1/2))$ or its charge conjugate d_c $(Q_a < 0)$. Hence option 1) or its charge conjugate would be favored.

2. If one takes very seriously the analogy with cosmic matter antimatter asymmetry then matter should dominate and only $(A, G, T, C) \rightarrow (u, d, \overline{u}, \overline{d})$ option would remain.

Szybalski's findings leave open the question whether non-coding regions obey the Chargaff' rules in good approximation or whether also they appear as pairs with opposite matter antimatter asymmetry. Introns are belong to coding regions in the sense that they are transcribed to mRNA. Splicing however cuts them off from mRNA. It is not clear whether introns break the approximate matter antimatter symmetry or not. If breaking takes place it might mean that introns code for something but not chemically. On the other hand, the absence of asymmetry might serve at least partially as a signal telling that introns must be cut off before translation. Many interesting questions represent itself. For instance, how the symmetry breaking parameters, in particular matter antimatter asymmetry parameter, depend on genes. The correlation with gene length is the most plausible guess.

5.7.6 Genetic codes and tqc

TGD suggests the existence of several genetic codes besides 3-codon code [?, ?]. The experience from ordinary computers and the fact that genes in general do not correspond to $3n$ nucleotides encourages to take this idea more seriously. The use of different codes would allow to tell what kind of information a given piece of DNA strand represents. DNA strand would be like a drawing of building containing figures (3-code) and various kinds of text (other codes). A simple drawing for the building would become a complex manual containing mostly text as the evolution proceeds: for humans 96 per cent of code would corresponds to introns perhaps obeying some other code.

The hierarchy of genetic codes is obtained by starting from n basic statements and going to the meta level by forming all possible statements about them (higher order logics) and throwing away one which is not physically realizable (it would correspond to empty set in the set theoretic realization). This allows $2^n - 1$ statements and one can select 2^{n-1} mutually consistent statements (half of the full set of statements) and say that these are true and give kind of axiomatics about world. The remaining statements are false. DNA would realize only the true statements.

The hierarchy of Mersenne primes $M_n = 2^n - 1$ with $M_{n(next)} = M_{M_n}$ starting from $n = 2$ with $M_2 = 3$ gives rise to 1-code with 4 codons, 3-code with 64 codons, and $3 \times 21 = 63$-code with 2^{126} codons [?] realized as sequences of 63 nucleotides (the length of 63-codon is about $2L(151)$, roughly twice the cell membrane thickness. It is not known whether this Combinatorial Hierarchy continues ad infinitum. Hilbert conjectured that this is the case.

In the model of pre-biotic evolution also 2-codons appear and 3-code is formed as the fusion of 1- and 2-codes. The problem is that 2-code is not predicted by the basic Combinatorial Hierarchy associated with $n = 2$.

There are however also other Mersenne hierarchies and the next hierarchy allows the realization of the 2-code. This Combinatorial Hierarchy begins from Fermat prime $n = 2^k + 1 = 5$ with $M_5 = 2^5 - 1 = 31$ gives rise to a code with 16 codons realized as 2-codons (2 nucleotides). Second level corresponds to Mersenne prime $M_{31} = 2^{31} - 1$ and a code with $2^{30=15\times2}$ codons realized by sequences of 15 3-codons containing 45 nucleotides. This corresponds to DNA length of 15 nm, or length scale $3L(149)$, where $L(149) = 5$ nm defines the thickness of the lipid layer of cell membrane. $L(151) = 10$ nm corresponds to 3 full 2π twists for DNA double strand. The model for 3-code as fusion of 1- and 2-codes suggests that also this hierarchy - which probably does not continue further - is realized.

There are also further short Combinatorial hierarchies corresponding to Mersenne primes [?].

1. $n = 13$ defines Mersenne prime M_{13}. The code would have $2^{12=6\times2}$ codons representable as sequences of 6 nucleotides or 2 3-codons. This code might be associated with microtubuli.

2. The Fermat prime $17 = 2^4 + 1$ defines Mersenne prime M_{17} and the code would have $2^{16=8\times2}$ codons representable as sequences of 8 nucleotides.

3. $n = 19$ defines Mersenne prime M_{19} and code would have $2^{18=9\times2}$ codons representable as sequences of 9 nucleotides or three DNA codons.

4. The next Mersennes are M_{31} belonging to $n = 5$ hierarchy, M_{61} with $2^{60=30\times2}$ codons represented by 30-codons. This corresponds to DNA length $L(151) = 10$ nm (cell membrane thickness). M_{89} (44-codons), M_{107} (53-codons) and M_{127} (belonging to the basic hierarchy) are the next Mersennes. Next Mersenne corresponds to M_{521} (260-codon) and to completely super-astrophysical p-adic length scale and might not be present in the hierarchy.

   This hierarchy is realized at the level of elementary particle physics and might appear also at the level of DNA. The 1-, 2-, 3-, 6-, 8-, and 9-codons would define lowest Combinatorial Hierarchies.

5.8 Cell replication and tqc

DNA as tqc model leads to quite detailed ideas about the evolution of the genetic code and the mechanisms of bio-catalysis and of protein folding [?]. These applications in turn leads to a considerable generalization of DNA as tqc concept [?]. The presence of braiding leads also to a revision of the model of nerve pulse and EEG [?, ?]. Here the discussion is restricted to one particular example. One can look what happens in the cell replication in the hope of developing more concrete ideas about tqc in multicellular system. This process must mean a replication of the braid's strand system and a model for this process gives concrete ideas about how multicellular system performs tqc.

5.8.1 Mitosis and tqc

Mitosis is the form of cell replication yielding soma cells and it is interesting what constraints this process gives on tqc and whether the special features of this process could be understood from computational point of view.

1. During mitosis chromosomes [?] are replicated. During this process the strands connecting chromosomes become visible: the pattern brings in mind flux tubes of magnetic field. For prokaryotes the replication of chromosomes is followed by the fission of the cell membrane. Also plant nuclei separated by cellulose walls suffer fission after the replication of chromosomes. For animals nuclear membranes break down before the replication suggesting that nuclear tqc programs are reset and newly formed nuclei start tqc from a clean table. For eukaryotes cell division is controlled by centrosomes [?]. The presence of centrosomes is not necessary for the survival of the cell or its replication but is necessary for the survival of multicellular. This conforms with the proposed picture.

2. If the conjugate strands are specialized in tqc, the formation of new double strands does not involve braids in an essential manner. The formation of conjugate strand should lead to also

to a generation of braid strands unless they already exist as strands connecting DNA and its conjugate and are responsible for "printing". These strands need not be short. The braiding associated with printing would be hardware program which could be genetically determined or at least inherited as such so that the strands should be restricted inside the inner cell membrane or at most traverse the inner nuclear membrane and turn back in the volume between inner membrane and endoplasmic reticulum.

The return would be most naturally from the opposite side of nuclear membrane which suggest a breaking of rotational symmetry to axial symmetry. The presence of centriole implies this kind of symmetry breaking: in neurons this breaking becomes especially obvious. The outgoing braid strands would be analogous to axon and returning braid strands to dendrites. Inner nuclear membrane would decompose the braiding to three parts: one for strand, second for conjugate strand, and a part in the empty space inside nuclear envelope.

3. The formation of new DNA strands requires recognition relying on "strand color" telling which nucleotide can condense at it. The process would conserve the braidings connecting DNA to the external world. The braidings associated with the daughter nuclei would be generated from the braiding between DNA and its conjugate. As printing software they should be identical so that the braiding connecting DNA double strands should be a product of a braiding and its inverse. This would however mean that the braiding is trivial. The division of the braid to three parts hinders the transformation to a trivial braid if the braids combine to form longer braids only during the "printing" activity.

4. The new conjugate strands are formed from the old strands associated with printing. In the case of plants the nuclear envelope does not disintegrate and splits only after the replication of chromosomes. This would suggest that plant cells separated by cell walls perform only intracellular tqc. Hermits do not need social skills. In the case of animals nuclear envelope disintegrates. This is as it must be since the process splits the braids connecting strand and conjugate strands so that they can connect to the cell membrane. The printing braids are inherited as such which conforms with the interpretation as a fixed software.

5. The braids connecting mother and daughter cells to extranuclear world would be different and tqc braidings would give to the cell a memory about its life-cycle. The age ordering of cells would have the architecture of a tree defined by the sequence of cell replications and the life history of the organism. The 4-D body would contain kind of log file about tqc performed during life time: kind of fundamental body memory.

6. Quite generally, the evolution of tqc programs means giving up the dogma of genetic determinism. The evolution of tqc programs during life cycle and the fact that half of them is inherited means kind of quantum Lamarckism [?]. This inherited wisdom at DNA level might partly explain why we differ so dramatically from our cousins.

5.8.2 Sexual reproduction and tqc

Meiosis [?] produces gametes in which the pair of chromosomes from parents is replaced with single chromosome obtained as chimera of the chromosomes of parents. Meiosis is the basic step of sexual reproduction and it is interesting to study it from tqc point of view.

1. Sexual reproduction of eukaryotes relies on haploid cells differing from diploid cells in that chromatids do not possess sister chromatids. Whereas mitosis produces from single diploid [?] cell two diploid cells, meiosis gives rise to 4 haploid [?] cells. The first stage is very much like mitosis. DNA and chromosomes duplicate but cell remains a diploid in the sense that there is only single centrosome: in mitosis also centrosome duplicates. After this the cell membrane divides into two. At the next step the chromosomes in daughter cells split into two sister chromosomes each going into its own cell. The outcome is four haploid cells.

2. The presence of only single chromatid [?] in haploids means that germ cells would perform only one half of the "social" tqc performed by soma cells [?] who must spend their life cycle as a member of cell community. In some cells the tqc would be performed by chromatids of both

father and mother making perhaps possible kind of stereo view about world and a model for couple - the simplest possible social structure.

3. This brings in mind the sensory rivalry between left and right brain: could it be that the two tqc's give competing computational views about world and how to act in it? We would have inside us our parents and their experiences as a pair of chromatids representing chemical chimeras of chromatid pairs possessed by the parents: as a hardware - one might say. Our parents would have the same mixture in software via sharing and fusion of chromatid mental images or via quantum computational rivalry. What is in software becomes hardware in the next generation.¡/p¿¡p¿

4. The ability of sexual reproduction to generate something new relates to meiosis. During meiosis genetic recombination [?] occurs via chromosomal crossover which in string model picture would mean splitting of chromatids and the recombination of pieces in a new manner $(A_1+B_1)+(A_2+B_2) \rightarrow (A_1+B_2)+(A_2+B_1)$ takes place in crossover and $(A_1+B_1+C_1)+(A_2+B_2+C_2) \rightarrow (A_1+B_2+C_1)+(A_2+B_1+C_2)$ in double crossover. New hardware for tqc would result by combining pieces of existing hardware. What this means in terms of braids should be clarified.

5. Fertilization is in well-define sense the inverse of meiosis. In fertilization the chromatids of spermatozoa and ova combine to form the chromatids of diploid cell. The recombination of genetic programs during meiosis becomes visible in the resulting tqc programs.

5.8.3 What is the role of centrosomes and basal bodies?

Centrosomes [?] and basal bodies [?] form the main part of Microtubule Organizing Center [?]. They are somewhat mysterious objects and at first do not seem to fit to the proposed picture in an obvious manner.

1. Centrosomes consist two centrioles [?] forming a T shaped antenna like structure in the center of cell. Also basal bodies consist of two centrioles but are associated with the cell membrane. Centrioles and basal bodies have cylindrical geometry consisting of nine triplets of microtubules along the wall of cylinder. Centrosome is associated with nuclear membrane during mitosis.

2. The function of basal bodies which have evolved from centrosomes seems to be the motor control (both cilia [?] and flagella [?]) and sensory perception (cilia). Cell uses flagella and cilia to move and perceive. Flagella and cilia are cylindrical structures associated with the basal bodies. The core of both structures is axoneme having $9\times 2+2$ microtubular structure. So called primary cilia do not posses the central doublet and the possible interpretation is that the inner doublet is involved with the motor control of cilia. Microtubules [?] of the pairs are partially fused together.

3. Centrosomes are involved with the control of mitosis [?]. Mitosis can take place also without them but the organism consisting of this kind of cells does not survive. Hence the presence of centrosomes might control the proper formation of tqc programs. The polymerization of microtubules [?] is nucleated at microtubule self-organizing center which can be centriole or basal body. One can say that microtubules which are highly dynamical structures whose length is changing all the time have their second end anchored to the self-organizing center. Since this function is essential during mitosis it is natural that centrosome controls it.

4. The key to the understanding of the role of centrosomes and basal bodies comes from a paradox. DNA and corresponding tqc programs cannot be active during mitosis. What does then control mitosis?

   i) Perhaps centrosome and corresponding tqc program represents the analog of the minimum seed program in computer allowing to generate an operating system [?] like Windows 2000 (the files from CD containing operating system must be read!). The braid strands going through the microtubules of centrosome might define the corresponding tqc program. The isolation from environment by the microtubular surface might be essential for keeping the braidings defining these programs strictly unchanged.

ii) The RNA defining the genome of centrosome (yes: centrosome has its own genome defined by RNA rather than DNA [?]!) would define the hardware for this tqc. The basal bodies could be interpreted as a minimal sensory-motor system needed during mitosis.

iii) As a matter fact, centrosome and basal bodies could be seen as very important remnants of RNA era believed by many biologists to have preceded DNA era. This assumption is also made in TGD inspired model of prebiotic evolution [?].

iv) Also other cellular organelles possessing own DNA and own tqc could remain partly functional during mitosis. In particular, mitochondria are necessary for satisfying energy needs during the period when DNA is unable to control the situation so that they must have some minimum amount of own genome.

5. Neurons [?] do not possess centrosome which explains why they cannot replicate. The centrioles are replaced with long microtubules associated with axons and dendrites. The system consisting of microtubules corresponds to a sensory-motor system controlled by the tqc programs having as a hardware the RNA of centrosomes and basal bodies. Also this system would have a multicellular part.

6. Intermediate filaments [?], actin filaments [?], and microtubules [?] are the basic building elements of the eukaryotic cytoskeleton [?]. Microtubules, which are hollow cylinders with outer radius of 24 nm, are especially attractive candidates for structures carrying bundles of braid strands inside them. The microtubular outer-surfaces could be involved with signalling besides other well-established functions. It would seem that microtubules cannot be assigned with tqc associated with nuclear DNA but with RNA of centrosomes and could contain corresponding braid strand bundles. It is easy to make a rough estimate for the number of strands and this would give an estimate for the amount of RNA associated with centrosomes. Also intermediate filaments and actin filaments might relate to cellular organelles having their own DNA.

5.9 Appendix: A generalization of the notion of imbedding space

In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is described. This view has developed much before the original version of this chapter was written.

The original idea was that the proposed modification of the imbedding space could explain naturally phenomena like quantum Hall effect involving fractionization of quantum numbers like spin and charge. This does not however seem to be the case. $G_a \times G_b$ implies just the opposite if these quantum numbers are assigned with the symmetries of the imbedding space. For instance, quantization unit for orbital angular momentum becomes n_a where Z_{n_a} is the maximal cyclic subgroup of G_a.

One can however imagine of obtaining fractionization at the level of imbedding space for space-time sheets, which are analogous to multi-sheeted Riemann surfaces (say Riemann surfaces associated with $z^{1/n}$ since the rotation by 2π understood as a homotopy of M^4 lifted to the space-time sheet is a non-closed curve. Continuity requirement indeed allows fractionization of the orbital quantum numbers and color in this kind of situation.

5.9.1 Both covering spaces and factor spaces are possible

The observation above stimulates the question whether it might be possible in some sense to replace H or its factors by their multiple coverings.

1. This is certainly not possible for M^4, CP_2, or H since their fundamental groups are trivial. On the other hand, the fixing of quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is a geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. Zero energy ontology forces to modify this picture somewhat. In zero energy ontology causal diamonds (CDs) defined as the intersections of future and past directed light-cones are loci for zero energy states containing positive and negative energy parts of state at the two light-cone boundaries. The location of CD in M^4 is arbitrary but p-adic length scale hypothesis suggests that the temporal distances between tips of CD come as powers of 2 using CP_2 size as unit. Thus M^4 is replaces by CD and $\hat{M}^4$ is replaced with $\hat{CD}$ defined in obvious manner.

3. H_4 represents a straight cosmic string inside CD. Quantum field theory phase corresponds to Jones inclusions with Jones index $\mathcal{M} : \mathcal{N} < 4$. Stringy phase would by previous arguments correspond to $\mathcal{M} : \mathcal{N} = 4$. Also these Jones inclusions are labeled by finite subgroups of $SO(3)$ and thus by Z_n identified as a maximal Abelian subgroup.

   One can argue that cosmic strings are not allowed in QFT phase. This would encourage the replacement $\hat{CD} \times \hat{CP}_2$ implying that surfaces in $CD \times S^2$ and $(M^2 \cap CD) \times CP_2$ are not allowed. In particular, cosmic strings and CP_2 type extremals with M^4 projection in M^2 and thus light-like geodesic without zitterwebegung essential for massivation are forbidden. This brings in mind instability of Higgs=0 phase.

4. The covering spaces in question would correspond to the Cartesian products $\hat{CD}_{n_a} \times \hat{CP}_{2n_b}$ of the covering spaces of $\hat{CD}$ and $\hat{CP}_2$ by Z_{n_a} and Z_{n_b} with fundamental group is $Z_{n_a} \times Z_{n_b}$. One can also consider extension by replacing $M^2 \cap CD$ and S^2 with its orbit under G_a (say tedrahedral, octahedral, or icosahedral group). The resulting space will be denoted by $\hat{CD} \hat{\times} G_a$ *resp.* $\hat{CP}_2 \hat{\times} G_b$.

5. One expects the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds $M^2 \cap CD$ or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of $M^2 \cap CD$ the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tedrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

6. Also the orbifolds $\hat{CD}/G_a \times \hat{CP}_2/G_b$ can be allowed as also the spaces $\hat{CD}/G_a \times (\hat{CP}_2 \hat{\times} G_b)$ and $(\hat{CD} \hat{\times} G_a) \times \hat{CP}_2/G_b$. Hence the previous framework would generalize considerably by the allowance of both coset spaces and covering spaces.

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $(M^2 \cap CD) \times CP_2$ takes place? It would seem that the covariant metric of M^4 factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of M^4 metric can make sense. This is consistent with the identical vanishing of Chern-Simons action in $M^2 \times S^2$.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in CD degrees of freedom. This is not the case. Light-likeness in $(M^2 \cap CD) \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset (M^2 \cap CD) \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $(M^2 \cap CD) \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of $(M^2 \cap CD)$ as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunneling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single

homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) and classical light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

5.9.2 Do factor spaces and coverings correspond to the two kinds of Jones inclusions?

What could be the interpretation of these two kinds of spaces?

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M} : \mathcal{N} < 4$ and it naturally corresponds to the coset spaces. For $\mathcal{M} : \mathcal{N} = 4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.

2. $\mathcal{M} : \mathcal{N} = 4$ should correspond to the allowance of cosmic strings and other analogous objects. Does the introduction of the covering spaces bring in cosmic strings in some controlled manner? Formally the subgroup of $SU(2)$ defining the inclusion is $SU(2)$ would mean that states are $SU(2)$ singlets which is something non-physical. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$.

   For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{CD}\hat{\times}G_a$ and $\hat{CP_2}\hat{\times}G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane.

3. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$, which both are labeled by a subset of discrete subgroups of SU(2).

4. The discrete subgroups of $SU(2)$ with fixed quantization axes possess a well defined multiplication with product defined as the group generated by forming all possible products of group elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tedrahedral, octahedral, and icosahedral groups are the generators of this algebra.

By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2¡ generated by reflection, and tedrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized configuration space spinor fields in the configuration space labelled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

The question how do the Planck constants associated with factors and coverings relate is far from trivial and I have considered several options.

1. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of metric allowed by Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains $r^2 \equiv \hbar^2/\hbar_0^2\hbar^2(M^4)/\hbar^2(CP_2)$. This puts M^4 and CP_2 in a very symmetric role and allows much more flexibility in the identification of symmetries associated with large Planck constant phases.

2. Algebraist would argue that Planck constant must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa. This gives two options.

3. Option I: $r(X) = n$ for covering and $r(X) = 1/n$ for factor space gives $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$. This gives $r = n_a/n_b$ for $\hat{H}/G_a \times G_b$ option and $r = n_b/n_a$ for $\hat{H}\hat{times}(G_a \times G_b)$ option with obvious formulas for hybrid cases.

4. Option II: $r(X) = 1/n$ for covering and $r(X) = n$ for factor space gives $r = r(CP_2)/r(M^4)$. This gives $r = n_b/n_a$ for $\hat{H}/G_a \times G_b$ option and $r = n_a/n_b$ for $\hat{H}\hat{times}(G_a \times G_b)$ option with obvious formulas for the hybrid cases.

5. At quantum level the fractionization would come from the modification of fermionic anti-commutation (bosonic commutation) relations involving $\hbar$ at the right hand side so that particle number becomes a multiple of $1/n$ or n. If one postulates that the total number states is invariant in the transition, the increase in the number of sheets is compensated by the increase of the fundamental phase space volume proportional to $\hbar$. This would give $r(X) \rightarrow r(X)/n$ for factor space and $r(X) \rightarrow nr(X)$ for the covering space to compensate the n-fold reduction/increase of states. This would favor Option II.

6. The second manner to distinguish between these two options is to apply the theory to concrete physical situations. Since G_a and G_b act as symmetries in CD and CP_2 degrees of freedom, one might of being able to distinguish between the two options if it is possible to distinguish between the action of G as symmetry of quantum states associated with covering and factor space. Also the quantization of the orbital spin quantum number at single particle level as multiples of n can be distinguished from that in multiples of $1/n$.

5.9.3 A simple model of fractional quantum Hall effect

The generalization of the imbedding space suggests that it could possible to understand fractional quantum Hall effect [?] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \tag{5.9.1}$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15..., 2/3, 3/5, 4/7, 5/9, 6/11, 7/13..., 5/3, 8/5, 11/7, 14/9...$ 1/5, 2/9, 3/13..., 2/7, 3/11..., 1/7.... with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [?].

The model of Laughlin [?] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [?]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of imbedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are four combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this there are two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. On the following just for fun consideration option I is considered although the conservation of number of states in the phase transition changing $\hbar$ favors option II.

1. The easiest manner to understand the observed fractions is by assuming that both M^4 and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

2. With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

3. The appearance of $\nu = 5/2$ has been observed [?]. The fractionized charge is $e/4$ in this case. Since $n_i > 3$ holds true if coverings are correlates for Jones inclusions, this requires to $n_b = 4$ and $n_a = 10$. n_b predicting a correct fractionization of charge. The alternative option would be $n_b = 2$ that also Z_2 would appear as the fundamental group of the covering space. Filling fraction 1/2 corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [?]. $n_b = 2$ is however inconsistent with the observed fractionization of electric charge and with the vision inspired by Jones inclusions.

4. A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

5. Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e \int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F \int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

6. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6 \times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the identification of charge unit is rather ad hoc. Therefore this model can be taken only as a warm-up exercise.

Bibliography

Online books about TGD

[1] M. Pitkänen (2006), *Topological Geometrodynamics: Overview*.
http://tgd.wippiespace.com/public_html/tgdview/tgdview.html.

[2] M. Pitkänen (2006), *Quantum Physics as Infinite-Dimensional Geometry*.
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html.

[3] M. Pitkänen (2006), *Physics in Many-Sheeted Space-Time*.
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html.

[4] M. Pitkänen (2006), *Quantum TGD*.
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html.

[5] M. Pitkänen (2006), *TGD as a Generalized Number Theory*.
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html.

[6] M. Pitkänen (2006), *p-Adic length Scale Hypothesis and Dark Matter Hierarchy*.
http://tgd.wippiespace.com/public_html/paddark/paddark.html.

[7] M. Pitkänen (2006), *TGD and Fringe Physics*.
http://tgd.wippiespace.com/public_html/freenergy/freenergy.html.

Online books about TGD inspired theory of consciousness and quantum biology

[8] M. Pitkänen (2006), *Bio-Systems as Self-Organizing Quantum Systems*.
http://tgd.wippiespace.com/public_html/bioselforg/bioselforg.html.

[9] M. Pitkänen (2006), *Quantum Hardware of Living Matter*.
http://tgd.wippiespace.com/public_html/bioware/bioware.html.

[10] M. Pitkänen (2006), *TGD Inspired Theory of Consciousness*.
http://tgd.wippiespace.com/public_html/tgdconsc/tgdconsc.html.

[11] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*.
http://tgd.wippiespace.com/public_html/genememe/genememe.html.

[12] M. Pitkänen (2006), *TGD and EEG*.
http://tgd.wippiespace.com/public_html/tgdeeg/tgdeeg.html.

[13] M. Pitkänen (2006), *Bio-Systems as Conscious Holograms*.
http://tgd.wippiespace.com/public_html/hologram/hologram.html.

[14] M. Pitkänen (2006), *Magnetospheric Consciousness*.
http://tgd.wippiespace.com/public_html/magnconsc/magnconsc.html.

[15] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*.
http://tgd.wippiespace.com/public_html/magnconsc/mathconsc.html.

References to the chapters of books

[A8] The chapter *Was von Neumann Right After All* of [?].
http://tgd.wippiespace.com/public_html/tgdview/tgdview.html#vNeumann.

[A9] The chapter *Does TGD Predict the Spectrum of Planck Constants?* of [?].
http://tgd.wippiespace.com/public_html/tgdview/tgdview.html#Planck.

[B1] The chapter *Identification of the Configuration Space Kähler Function* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#kahler.

[B2] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part I* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#compl1.

[B3] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part II* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#compl2.

[B4] The chapter *Configuration Space Spinor Structure* of [?].
http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#cspin.

[C2] The chapter *Construction of Quantum Theory: Symmetries* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#quthe.

[C3] The chapter *Construction of Quantum Theory: S-matrix* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#towards.

[D1] The chapter *Basic Extremals of Kähler Action* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#class.

[D6] The chapter *TGD and Astrophysics* of [?].
http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#astro.

[E2] The chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visionb.

[E6] The chapter *Fusion of p-Adic and Real Variants of Quantum TGD to a More General Theory* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#mblocks.

[E8] The chapter *Riemann Hypothesis and Physics* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#riema.

[E9] The chapter *Topological Quantum Computation in TGD Universe* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#tqc.

[E11] The chapter *Langlands Program and TGD* of [?].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#Langlandia.

[F9] The chapter *Nuclear String Physics* of [?].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#nuclstring.

[G2] The chapter *The Notion of Free Energy and Many-Sheeted Space-Time Concept* of [?].
http://tgd.wippiespace.com/public_html/freenergy/freenergy.html#freenergy.

[H2] The chapter *Negentropy Maximization Principle* of [?].
http://tgd.wippiespace.com/public_html/tgdconsc/tgdconsc.html#nmpc.

[H11] The chapter *TGD Based Model for OBEs* of [?].
http://tgd.wippiespace.com/public_html/tgdconsc/tgdconsc.html#OBE.

[I4] The chapter *Quantum Control and Coordination in Bio-systems: Part I* of [?]. http://tgd.wippiespace.com/public_html/bioselforg/bioselforg.html#qcococI.

[I5] The chapter *Quantum Control and Coordination in Bio-Systems: Part II* of [?]. http://tgd.wippiespace.com/public_html/bioselforg/bioselforg.html#qcococII.

[J1] The chapter *Bio-Systems as Super-Conductors: part I* of [?]. http://tgd.wippiespace.com/public_html/bioware/bioware.html#superc1.

[J4] The chapter *Quantum Antenna Hypothesis* of [?]. http://tgd.wippiespace.com/public_html/bioware/bioware.html#tubuc.

[J5] The chapter *Wormhole Magnetic Fields* of [?]. http://tgd.wippiespace.com/public_html/bioware/bioware.html#wormc.

[J7] The chapter *About the New Physics Behind Qualia* of [?]. http://tgd.wippiespace.com/public_html/bioware/bioware.html#newphys.

[K3] The chapter *General Theory of Qualia* of [?]. http://tgd.wippiespace.com/public_html/hologram/hologram.html#qualia.

[K4] The chapter *Bio-Systems as Conscious Holograms* of [?]. http://tgd.wippiespace.com/public_html/hologram/hologram.html#hologram.

[L1] The chapter *Genes and Memes* of [?]. http://tgd.wippiespace.com/public_html/genememe/genememe.html#genememec.

[L2] The chapter *Many-Sheeted DNA* of [?]. http://tgd.wippiespace.com/public_html/genememe/genememe.html#genecodec.

[L7] The chapter *A Model for Protein Folding and Bio-catalysis* of [?]. http://tgd.wippiespace.com/public_html/genememe/genememe.html#foldcat.

[M2] The chapter *Quantum Model for Nerve Pulse* of [?]. http://tgd.wippiespace.com/public_html/tgdeeg/tgdeeg.html#pulse.

[M3] The chapter *Dark Matter Hierarchy and Hierarchy of EEGs* of [?]. http://tgd.wippiespace.com/public_html/tgdeeg/tgdeeg.html#eegdark.

[N4] The chapter *Pre-Biotic Evolution in Many-Sheeted Space-Time* of [?]. http://tgd.wippiespace.com/public_html/magnconsc/magnconsc.html#prebio.

[tqc] The chapter *Topological Quantum Computation in TGD Universe* of [?]. http://tgd.wippiespace.com/public_html/tgd/tgd.html#tqc.

Mathematical references

[16] *Mersenne prime*, http://en.wikipedia.org/wiki/Mersenne_prime.

[17] *Braid theory*, http://en.wikipedia.org/wiki/Braid_theory.

[18] B. Shipman (1998) *The geometry of momentum mappings on generalized flag manifolds, connections with a d mechanics and the dance of honeybee.* http://math.cornell.edu/~oliver/Shipman.gif.
B. Shipman (1998), *On the geometry of certain isospectral sets in the full Kostant-Toda lattice.* http://nyjm.albany.edu:8000/PacJ/1997/Shipman.html.
B. Shipman (1998), *A symmetry of order two in the full Kostant-Toda lattice.* http://www.math.rochester.edu:8080/u/shipman/symmetrypaper/.

Quantum Computation

[19] R. Feynman (1982), *Simulating physics with computers*, Int. J. Theor. Phys. 21, 467-488. *Ibid* (1986), *Quantum mechanical computers*, Found. Phys., 16, 507-603.

[20] D. Deutch (1985), *Quantum theory, the Church-Turing principle, and the universal quantum computer*, Proc. Roy. Soc. London, A400, 97-117.

[21] P. Shor (1994), *Algorithms for quantum computation, discrete logarithms, and factoring*, Proc. 35th Annual Symposium on Foundations of Computer Science, IEEE Computer Society Press, Los Alamitos, CA, 124-134.

[22] *Quantum Information Science*, Report of NSF Workshop, October 28-29, 1999. Arlington Virginia. National Science Foundation.

[23] Paul Parsons (2004) , *Dancing the Quantum Dream*, New Scientist 24. January. www.newscientist.com/hottopics.

[24] M. Freedman, A. Kitaev, M. Larson, Z. Wang (2001), www.arxiv.of/quant-ph/0101025.

[25] L. H. Kauffmann and S. J. Lomonaco Jr. (2004), *Braiding operations are universal quantum gates*, arxiv.org/quant-ph/0401090.

[26] M. Freedman, H. Larsen, and Z. Wang (2002), *A modular functor which is universal for quantum computation*, Found. Comput. Math. 1, no 2, 183-204. Comm. Math. Phys. 227, no 3, 605-622. quant-ph/0001108.
M. H. Freedman (2001), *Quantum Computation and the localization of Modular Functors*, Found. Comput. Math. 1, no 2, 183-204.
M. H. Freedman (1998), *P/NP, and the quantum field computer*, Proc. Natl. Acad. Sci. USA 95, no. 1, 98-101.

[27] A. Kitaev (1997), Annals of Physics, vol 303, p.2. See also *Fault tolerant quantum computation by anyons*, quant-ph/9707021.
A. Kitaev (1997), *Quantum computations: algorithms and error correction*, Russian Math. Survey, 52:61, 1191-1249.

[28] *Hypercomputation*, `http://en.wikipedia.org/wiki/Hypercomputing`.

[29] A. Khrennikov (2004), *Bell's inequality for conditional probabilities as a test for quantum like behaviour of mind*, arXiv:quant-ph/0402169.

[30] E. Conte, O. Todarello, A. Federici, J. P. Zbilut (2008), *Minds States Follow Quantum Mechanics During Perception and Cognition of Ambigious Figures: A Final Experimental Confirmation*, arXiv:0802.1835v1 [physics.gen-ph].

Quantum Hall effect

[31] S. M. Girvin (1999), *Quantum Hall Effect, Novel Excitations and Broken Symmetries*, cond-mat/9907002.

[32] R. B. Laughlin (1990), Phys. Rev. Lett. 50, 1395.

[33] F. Wilzek (1990), *Fractional Statistics and Anyon Super-Conductivity*, World Scientific.

[34] *Fractional quantum Hall Effect*, `http://en.wikipedia.org/wiki/Fractional_quantum_Hall_effect`.
Fractional Quantum Hall Effect, `http://www.warwick.ac.uk/~phsbm/fqhe.htm`.

[35] J.K. Jain(1989), Phys. Rev. Lett. 63, 199.

[36] J. B. Miller *et al*(2007), *Fractional Quantum Hall effect in a quantum point contact at filling fraction 5/2*, arXiv:cond-mat/0703161v2.

[37] A. Wojs, K.-S. Yi and J. J. Quinn (2003), *Fractional Quantum Hall States of Composite Fermions*, cond-mat/0312290.

[38] M. de Wild Propitius and F. A. Bais (1996), *Discrete Gauge Theories*, hep-th/9511201.

[39] C. Nayak and F. Wilczek (1996), *2n-quasihole states realize 2^{n-1}-dimensional spinor braiding statistics in paired quantum Hall states*, Nucl. Phys. B479, 529-533.

[40] G. Moore and N. Read (1991), *Non-Abelians in the fractional quantum Hall effect*, Nucl. Phys. B360, 362-396.

Life science related references

[41] A. Coghlan (2004), *Our genome 'reads' junk as well as genes*, New Scientist, 21 February.

[42] Articles about TGD inspired theory of consciousness in previous issues of Journal of Non-Locality and Remote Mental Interactions.
`http://www.emergentmind.org`.

[43] W. A. Tiller, W. E. Dibble Jr., and M. J. Kohane (2001), *Conscious Acts of Creation: the Emergence of a New Physics*, p. 63.
Pavior Publishing. `http://www.pavior.com/`.

[44] E. Lozneanu and M. Sanduloviciu (2003) *Minimal-cell system created in laboratory by self-organization*, Chaos, Solitons & Fractals, Volume 18, Issue 2, October, p. 335.
See also *Plasma blobs hint at new form of life*, New Scientist vol. 179 issue 2413 - 20 September 2003, page 16.

[45] A. Coghlan (2007), *Junk DNA makes compulsive reading*, New Scientist, issue 2608, 16 June.
`http://www.newscientist.com/contents/issue/2608.html`.

[46] The ENCODE Project Consortium, *Identification and analysis of functional elements in one % of the human genome by the ENCODE pilot project*, `http://www.nature.com/nature/journal/v447/n7146/edsumm/e070614-01.html`.

[47] *Supercoil*, `http://en.wikipedia.org/wiki/Supercoil`.

[48] *Chromatin*, `http://en.wikipedia.org/wiki/Chromatin`.

[49] *Nucleosome*, `http://en.wikipedia.org/wiki/Nucleosome`.

[50] *Geometric Langlands program*,
`http://matpitka.blogspot.com/2007/11/geometric-langlands.html`.

[51] *Some ideas about topological quantum computation in TGD Universe*,
`http://matpitka.blogspot.com/2007/12/some-ideas-about-topological-quantum.html`.

[52] *DNA as topological quantum computer*,
`http://matpitka.blogspot.com/2007/11/dna-as-topological-quantum-computer.html`.

[53] *Clathrin*, `http://en.wikipedia.org/wiki/Clathrin`.

[54] J. O'Keefe, M. L. Recce (2004), *Phase relationship between hippocampal place units and the EEG theta rhythm*, Hippocampus, Vol 3, Issue 3, p.317-330.
`http://www3.interscience.wiley.com/cgi-bin/abstract/109711451/ABSTRACT`.

[55] Y. Yamaguchi (2003), *Laboratory for dynamics of emergent intelligence*,
`http://www.brain.riken.jp/reports/annual_reports/2003/2003-doc/0243.pdf`.

[56] *Introns*, http://en.wikipedia.org/wiki/Introns.

[57] http://matpitka.blogspot.com/2007/06/introns-transcribed-to-rna-inside-cells.html.

[58] *Cell membrane*, http://en.wikipedia.org/wiki/Cell_membrane.

[59] *Nuclear envelope*, http://en.wikipedia.org/wiki/Nuclear_envelope.

[60] *Lipid*, http://en.wikipedia.org/wiki/Lipid.

[61] *Lipid raft*, http://en.wikipedia.org/wiki/Lipid_raft.

[62] *Endoplasmic reticulum*, http://en.wikipedia.org/wiki/Endoplasmic_reticulum.

[63] *Liquid crystal*, http://en.wikipedia.org/wiki/Liquid_crystal.

[64] *Operating system*, http://en.wikipedia.org/wiki/Operating_system.

[65] *Prokaryotes*, http://en.wikipedia.org/wiki/Prokaryote.

[66] *Eukaryotes*, http://en.wikipedia.org/wiki/Eukaryote.

[67] *Promoter*, http://en.wikipedia.org/wiki/Promoter.

[68] *Transcription factors*, http://en.wikipedia.org/wiki/Transcription_factors.

[69] *Lamarckism*, http://en.wikipedia.org/wiki/Lamarckism.

[70] *Ploidy*, http://en.wikipedia.org/wiki/Ploidy.

[71] *Mitosis*, http://en.wikipedia.org/wiki/Mitosis.

[72] *Meiosis*, http://en.wikipedia.org/wiki/Meiosis.

[73] ,*Chromatid*, http://en.wikipedia.org/wiki/Chromatid.

[74] *Soma cell*, http://en.wikipedia.org/wiki/Soma_cell.

[75] *Genetic recombination*, http://en.wikipedia.org/wiki/Genetic_recombination.

[76] *Human Genome Project Information*, http://www.ornl.gov/sci/techresources/Human_Genome/.

[77] $5'$ *untranslated region*, http://en.wikipedia.org/wiki/Five_prime_untranslated_region.
$3'$ *untranslated region* http://en.wikipedia.org/wiki/Three_prime_untranslated_region.

[78] M. E. B. Yamagishi and A. I. Shimabukuro (2006), *Nucleotide Frequencies in Human Genome and Fibonacci Numbers*, arXiv:q-bio/0611041.

[79] *Fertilization*, http://en.wikipedia.org/wiki/Fertilization.

[80] *Cilia*, http://en.wikipedia.org/wiki/Cilia.

[81] *Flagella*, http://en.wikipedia.org/wiki/Flagella.

[82] *Centrosome*, http://en.wikipedia.org/wiki/Centrosome.

[83] *Basal body*, http://en.wikipedia.org/wiki/Basal_body.

[84] *Microtubule organizing center*, http://en.wikipedia.org/wiki/Microtubule_organizing_center.

[85] *Centriole*, http://en.wikipedia.org/wiki/Centriole.

[86] *Microtubule*, http://en.wikipedia.org/wiki/Microtubule.

[87] *Intermediate filament*, http://en.wikipedia.org/wiki/Intermediate_filament.

[88] *Actin filament*, http://en.wikipedia.org/wiki/Actin_filament.

[89] *Cytoskeleton*, http://en.wikipedia.org/wiki/Cytoskeleton.

[90] *Neuron*, http://en.wikipedia.org/wiki/Neuron.

[91] *Phospholipid*, http://en.wikipedia.org/wiki/Phospholipid.

[92] *Phosphoglyceride*, http://en.wikipedia.org/wiki/Phosphoglyceride.

[93] *Phosphorylation*, http://en.wikipedia.org/wiki/Phosphorylation.

[94] *Fatty acids*, http://en.wikipedia.org/wiki/Fatty_acids.

[95] *Lipid bilayer*, http://en.wikipedia.org/wiki/Lipid_bilayer.

[96] *Phosphatidyl serine*, http://en.wikipedia.org/wiki/Phosphatidyl_serine.

[97] S. Hameroff , *What is Consciousness?*, http://www.quantumconsciousness.org/presentations/whatisconsciousness.html.

[98] R. Adler (2008), *DNA 'fabricator' constructs walking DNA*, New Scientist˙ http://technology.newscientist.com/channel/tech/dn13192-dna-fabricator-constructs-walking-dna.html.

[99] *Programming biomolecular self-assembly pathways*, Y. Peng Yin *et al* (2007), Nature 451, 318-322 (17 January 2008).

[100] S. J. Green, D. Lubrich, A. J Turberfield (2006), *DNA Hairpins: Fuel for Autonomous DNA Devices*, Biophysical Journal, Oct 15, http://findarticles.com/p/articles/mi_qa3938/is_200610/ai_n16779588/pg_1.

[101] *Stem loop*, http://en.wikipedia.org/wiki/Hairpin_loop.

[102] *Palindrome*, http://en.wikipedia.org/wiki/Palindrome.

[103] *Inosine*, http://en.wikipedia.org/wiki/Inosine.

[104] *tRNA*, http://en.wikipedia.org/wiki/tRNA.

[105] *Wobble base pair*, http://en.wikipedia.org/wiki/Wobble_base_pair.

[106] *Transfer RNA*, http://www.tulane.edu/~biochem/nolan/lectures/rna/trnaz.htm.

[107] http://www.staff.uni-bayreuth.de/~btc914/search/index.html.

[108] M. Nakata *et al* (2007), *End-to-End Stacking and Liquid Crystal Condensation of 6 to 20Base Pair DNA Duplexes*, Science 2007, 318, 1276. http://www.sciencemag.org/cgi/content/abstract/318/5854/1276.

[109] *Transposon*, http://en.wikipedia.org/wiki/Transposon.
Retrotransposon, http://en.wikipedia.org/wiki/Retrotransposon.

[110] D. F. Voytas (2008), *Fighting fire with fire*, Nature vol 451, January.

[111] *Sticky ends*, http://en.wikipedia.org/wiki/Sticky_ends.

[112] *Recombinant DNA and gene cloning*, http://users.rcn.com/jkimball.ma.ultranet/BiologyPages/R/RecombinantDNA.html.

[113] *DNA repeat sequences and disease*, http://www.neuro.wustl.edu/NEUROMUSCULAR/mother/dnarep.htm#dnareptype.

[114] *Telomeres*, http://en.wikipedia.org/wiki/Telomeres.
Are Telomeres the key to Ageing and Cancer?, http://learn.genetics.utah.edu/features/telomeres/.

[115] *Mitochondria*, http://en.wikipedia.org/wiki/Mitochondria.

[116] *Isochore*, http://en.wikipedia.org/wiki/Isochore_(genetics).
GC content, http://en.wikipedia.org/wiki/GC_content.

[117] G. Kudla, A. Helwak, and L. Lipinski (2004) *Gene Conversion and GC-Content Evolution in Mammalian Hsp70*, Mol. Biol. Evol. 21(7), http://mbe.oxfordjournals.org/cgi/content/full/21/7/1438.

[118] *Chargaff's rules*, http://en.wikipedia.org/wiki/Chargaff's_rules.

[119] P. J. Lao, and D. R. Forsdyke (2000), *Thermophilic Bacteria Strictly Obey Szybalski's Transcription Direction Rule and Politely Purine-Load RNAs with Both Adenine and Guanine*, Vol. 10, Issue 2, 228-236, February. http://www.genome.org/cgi/content/full/10/2/228.

[120] P. Gariaev *et al* (2000), *The DNA-wave-biocomputer*, CASYS'2000, Fourth International Conference on Computing Anticipatory Systems, Liege, 2000. Abstract Book, Ed. M. Dubois.

[121] P. Gariaev, *Brief introduction into WaveGenetics. Its scope and opporturnities.*, http://www.wavegenetics.jino-net.ru.

[122] P. P. Gariaev *et al*(2002), *The spectroscopy of bio-photons in non-local genetic regulation*, Journal of Non-Locality and Remote Mental Interactions, Vol 1, Nr 3.
http://www.emergentmind.org/gariaevI3.htm.

[123] P. P. Gariaev, V. I. Chudin, G. G. Komissarov, A. A. Berezin , A. A. Vasiliev (1991), *Holographic Associative Memory of Biological Systems*, Proceedings SPIE - The International Society for Optical Engineering. Optical Memory and Neural Networks. v.1621, p. 280- 291. USA.

[124] P. P. Gariaev, G. G. Tertishni, A. V. Tovmash (2007), *Experimental investigation in vitro of holographic mapping and holographic transposition of DNA in conjuction with the information pool encircling DNA*, New Medical Tehcnologies, #9, pp. 42-53. The article is in Russian but Peter Gariaev kindly provided a translation of the article to English.

[125] F. A. Popp, B. Ruth, W. Bahr, J. Bhm, P. Grass (1981), G. Grolig, M. Rattemeyer, H. G. Schmidt and P. Wulle: *Emission of Visible and Ultraviolet Radiation by Active Biological Systems.* Collective Phenomena(Gordon and Breach), 3, 187-214.

[126] *Faraday effect*, http://en.wikipedia.org/wiki/Faraday_effect.

[127] *Water Memory*, http://en.wikipedia.org/wiki/Water_memory.

[128] L. Milgrom (2001), *Thanks for the memory.* An article in Guardian about the work of professor M. Ennis of Queen's University Belfast supporting the observations of Dr. J. Benveniste about water memory.
http://www.guardian.co.uk/Archive/Article/0,4273,4152521,00.html.

[129] J. Benveniste *et al* (1988). *Human basophil degranulation triggered by very dilute antiserum against IgE.* Nature 333:816-818.

[130] J. Benveniste *et al* (198?). *Transatlantic transfer of digitized antigen signal by telephone link.* Journal of Allergy and Clinical Immunology. 99:S175 (abs.). For recent work about digital biology and further references about the work of Benveniste and collaborators see http://www.digibio-.com/.

[131] E. Strand (editor) (2007), *Proceedings of the 7^{th} European SSE Meeting August 17-19, 2007, Röros, Norway.* Society of Scientific Exploration: http://www.scientificexploration.org/.

[132] *Self organization*, http://en.wikipedia.org/wiki/Self_organization.

[133] R. Sheldrake (1995), *A New Science of Life: The Hypothesis of Morphic Resonance*, Inner Traditions Intl Ltd.

Part III

CATEGORIES, NUMBER THEORY AND CONSCIOUSNESS

Chapter 6

Category Theory, Quantum TGD, and TGD Inspired Theory of Consciousness

6.1 Introduction

Goro Kato has proposed an ontology of consciousness relying on category theory [?, ?]. Physicist friendly summary of the basic concepts of category theory can be found in [?]) whereas the books [?, ?] provide more mathematically oriented representations. Category theory has been proposed as a new approach to the deep problems of modern physics, in particular quantization of General Relativity. To mention only one example, C. J. Isham [?] has proposed that topos theory could provide a new approach to quantum gravity in which space-time points would be replaced by regions of space-time and that category theory could geometrize and dynamicize even logic by replacing the standard Boolean logic with a dynamical logic dictated by the structure of the fundamental category purely geometrically [?].

Although I am an innocent novice in this field and know nothing about the horrible technicalities of the field, I have a strong gut feeling that category theory might provide the desired systematic approach to quantum TGD proper, the general theory of consciousness, and the theory of cognitive representations [?].

6.1.1 Category theory as a purely technical tool

Category theory could help to disentangle the enormous technical complexities of the quantum TGD and to organize the existing bundle of ideas into a coherent conceptual framework. The construction of the geometry of the configuration space ("world of classical worlds")[?, ?, ?, ?, ?, ?], of classical configuration space spinor fields [?], and of S-matrix [?] using a generalization of the quantum holography principle are especially natural applications. Category theory might also help in formulating the new TGD inspired view about number system as a structure obtained by "gluing together" real and p-adic number fields and TGD as a quantum theory based on this generalized notion of number [?, ?, ?, ?, ?].

6.1.2 Category theory based formulation of the ontology of TGD Universe

It is interesting to find whether also the ontology of quantum TGD and TGD inspired theory of consciousness based on the trinity of geometric, objective and subjective existences [?] could be expressed elegantly using the language of the category theory.

There are indeed natural and non-trivial categories involved with many-sheeted space-time and the geometry of the configuration space ("the world of classical worlds"); with configuration space spinor fields; and with the notions of quantum jump, self and self hierarchy. Functors between these categories could express more precisely the quantum classical correspondences and self-referentiality of quantum states allowing them to express information about quantum jump sequence.

i) Self hierarchy has a structure of category and corresponds functorially to the hierarchical structure of the many-sheeted space-time.
ii) Quantum jump sequence has a structure of category and corresponds functorially to the category formed by a sequence of maximally deterministic regions of space-time sheet.
iii) Even the quantum jump could have space-time correlates made possible by the generalization of the Boolean logic to what might be space-time correlate of quantum logic and allowing to identify space-time correlate for the notion of quantum superposition.
iv) The category of light cones with inclusion as an arrow defining time ordering appears naturally in the construction of the configuration space geometry and realizes the cosmologies within cosmologies scenario. In particular, the notion of the arrow of psychological time finds a nice formulation unifying earlier two different explanations.

6.1.3 Other applications

One can imagine also other applications.

1. Categories posses inherent logic [?] based on the notion of sieves relying on the notion of presheaf which generalizes Boolean logic based on inclusion. In TGD framework inclusion is naturally replaced by topological condensation and this leads to a two-valued logic realizing space-time correlate of quantum logic based on the notions of quantum sieve and quantum topos.

   This suggests the possibility to geometrize the logic of both geometric, objective and subjective existences and perhaps understand why ordinary consciousness experiences the world through Boolean logic and Zen consciousness experiences universe through logic in which the law of excluded middle is not true. Interestingly, the p-adic logic of cognition is naturally 2-valued whereas the real number based logic of sensory experience allows excluded middle (is the person at the door in or out, in and out, or neither in nor out?). The quantum logic naturally associated with spinors (in the "world of classical worlds") is consistent with the logic based on quantum sieves.

2. Simple Boolean logic of right and wrong does not seem to be ideal for understanding moral rules. Same applies to the beauty-ugly logic of aesthetic experience. The logic based on quantum sieves would perhaps provide a more flexible framework.

3. Cognition is categorizing and category theory suggests itself as a tool for understanding cognition and self hierarchies and the abstraction processes involved with conscious experience. Here the new elements associated with the ontology of space-time due to the generalization of number concept would be central. Category theory could be also helpful in the modelling of conscious communications, in particular the telepathic communications based on sharing of mental images involving the same mechanism which makes possible space-time correlates of quantum logic and quantum superposition.

6.2 What categories are?

In the following the basic notions of category theory are introduced and the notion of presheaf and category induced logic are discussed.

6.2.1 Basic concepts

Categories [?, ?, ?] are roughly collections of objects A, B, C... and morphisms $f(A \to B)$ between objects A and B such that decomposition of two morphisms is always defined. Identity morphisms map objects to objects. Topological/linear spaces form a category with continuous/linear maps acting as morphisms. Also algebraic structures of a given type form a category: morphisms are now homomorphisms. Practically any collection of mathematical structures can be regarded as a category. Morphisms can can be very general: for instance, partial ordering $a \leq b$ can define morphism $f(A \to B)$.

Functors between categories map objects to objects and morphisms to morphisms so that a product of morphisms is mapped to the product of the images and identity morphism is mapped to identity morphism. Group representation is example of this kind of a functor: now group action in group is mapped to a linear action at the level of the representations. Commuting square is an easy visual manner to understand the basic properties of a functor, see Fig. ??.

The product $C = AB$ for objects of categories is defined by the requirement that there are projection morphisms π_A and π_B from C to A and B and that for any object D and pair of morphisms $f(D \rightarrow A)$ and $g(D \rightarrow B)$ there exist morphism $h(D \rightarrow C)$ such that one has $f = \pi_A h$ and $g = \pi_B h$. Graphically (see Fig. ??) this corresponds to a square diagram in which pairs A,B and C,D correspond to the pairs formed by opposite vertices of the square and arrows DA and DB correspond to morphisms f and g, arrows CA and CB to the morphisms π_A and π_B and the arrow h to the diagonal DC.

Examples of product categories are Cartesian products of topological and linear spaces, of differentiable manifolds, groups, etc. Also tensor products of linear spaces satisfies these axioms. One can define also more advanced concepts such as limits and inverse limits. Also the notions of sheafs, presheafs, and topos are important.

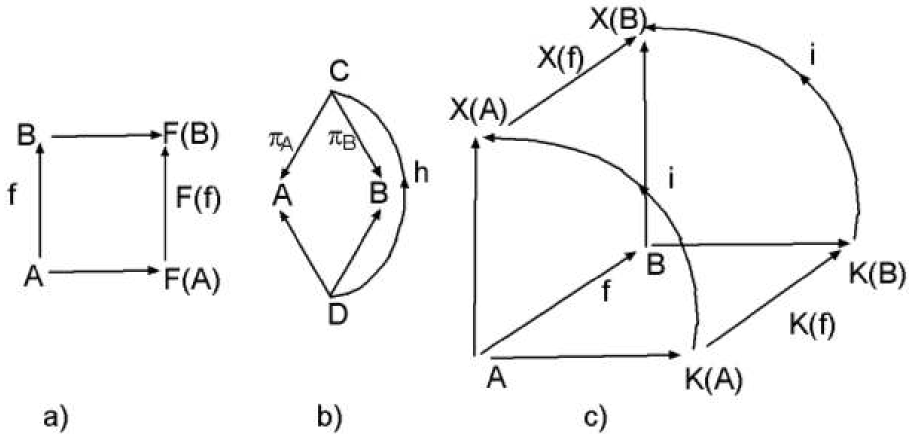

Figure 6.1: Commuting diagram associated with the definition of a) functor, b) product of objects of category, c) presheaf K as sub-object of presheaf X ("two pages of book".)

6.2.2 Presheaf as a generalization for the notion of set

Presheafs can be regarded as a generalization for the notion of set. Presheaf is a functor X that assigns to any object of a category **C** an object in the category **Set** (category of sets) and maps morphisms to morphisms (maps between sets for **C**). In order to have a category of presheafs, also morphisms between presheafs are needed. These morphisms are called natural transformations $N : X(A) \rightarrow Y(A)$ between the images $X(A)$ and $Y(A)$ of object A of **C**. They are assumed to obey the commutativity property $N(B)X(f) = Y(f)N(A)$ which is best visualized as a commutative square diagram. Set theoretic inclusion $i : X(A) \subset Y(A)$ is obviously a natural transformation.

An easy manner to understand and remember this definition is commuting diagram consisting of two pages of book with arrows of natural transformation connecting the corners of the pages: see Fig. ??.

As noticed, presheafs are generalizations of sets and a generalization for the notion of subset to a sub-object of presheaf is needed and this leads to the notion of topos [?, ?]. In the classical set theory a subset of given sets X can be characterized by a mapping from set X to the set $\Omega = \{true, false\}$ of Boolean statements. Ω itself belongs to the category **C**. This idea generalizes to sub-objects whose objects are collections of sets: Ω is only replaced with its Cartesian power. It can be shown that in the case of presheafs associated with category **C** the sub-object classifier Ω can be replaced with a more general algebra, so called Heyting algebra [?, ?] possessing the same basic operations as Boolean algebra (and, or, implication arrow, and negation) but is not in general equivalent with any Boolean algebra. What is important is that this generalized logic is inherent to the category **C** so that many-valued logic ceases to be an ad hoc construct in category theory.

In the theory of presheafs sub-object classifier Ω, which belongs to **Set**, is defined as a particular presheaf. Ω is defined by the structure of category **C** itself so that one has a geometrization of the notion of logic implied by the properties of category. The notion of sieve is essential here. A sieve for an object A of category **C** is defined as a collection of arrows $f(A \to ...)$ with the property that if $f(A \to B)$ is an arrow in sieve and if $g(B \to C)$ is any arrow then $gf(A \to C)$ belongs to sieve.

In the case that morphism corresponds to a set theoretic inclusion the sieve is just either empty set or the set of all sets of category containing set A so that there are only two sieves corresponding to Boolean logic. In the case of a poset (partially ordered set) sieves are sets for which all elements are larger than some element.

6.2.3 Generalized logic defined by category

The presheaf $\Omega : \mathbf{C} \to \mathbf{Set}$ defining sub-object classifier and a generalization of Boolean logic is defined as the map assigning to a given object A the set of all sieves on A. The generalization of maps $X \to \Omega$ defining subsets is based on the the notion of sub-object K. K is sub-object of presheaf X in the category of presheaves if there exist natural transformation $i : K \to X$ such that for each A one has $K(A) \subset X(A)$ (so that sub-object property is reduced to subset property).

The generalization of the map $X \to \Omega$ defining subset is achieved as follows. Let K be a sub-object of X. Then there is an associated characteristic arrow $\chi^K : X \to \Omega$ generalizing the characteristic Boolean valued map defining subset, whose components $\chi^K_A : X(A) \to \Omega(A)$ in **C** is defined as

$$\chi^K_A(x) = \{f(A \to B)|X(f)(x) \in K(B)\} \ .$$

By using the diagrammatic representation of Fig. **??** for the natural transformation i defining sub-object, it is not difficult to see that by the basic properties of the presheaf K $\chi^K_A(x)$ is a sieve. When morphisms f are inclusions in category Set, only two sheaves corresponding to all sets containing X and empty sheaf result. Thus binary valued maps are replaced with sieve-valued maps and sieves take the role of possible truth values. What is also new that truths and logic are in principle context dependent since each object A of **C** serves as a context and defines its own collection of sieves.

The generalization for the notion of point of set X exists also and corresponds to a selection of single element γ_A in the set $X(A)$ for each A object of **C**. This selection must be consistent with the action of morphisms $f(A \to B)$ in the sense that the matching condition $X(f)(\gamma_A) = \gamma_B$ is satisfied. It can happen that category of presheaves has no points at all since the matching condition need not be satisfied globally.

It turns out that TGD based notion of subsystem leads naturally to what might be called quantal versions of topos, presheaves, sieves and logic.

6.3 Category theory and consciousness

Category theory is basically about relations between objects, rather than objects themselves. Category theory is not about Platonic ideas, only about relations between them. This suggests a possible connection with TGD and TGD inspired theory of consciousness where the sequences quantum jumps between quantum histories defining selves have a role similar to morphisms and quantum states themselves are like Platonic ideas not conscious as such. Also the fact that it is not possible to write any formula for the contents of conscious experience although one can say a lot about its general structure bears a striking similarity to the situation in category theory.

6.3.1 The ontology of TGD is tripartistic

The ontology of TGD involves a trinity of existences.

1. Geometric existence or existence in the sense of classical physics. Objects are 3-surfaces in 8-D imbedding space, matter as res extensa. Quantum gravitational holography assigns to a 3-surface X^3 serving as a causal determinant space-time sheet $X^4(X^3)$ defining the classical physics associated with X^3 as a generalization of Bohr orbit. X^3 can be seen as a 3-D hologram representing the information about this 4-D space-time sheet

The geometry of configuration space of 3-surfaces, "the world of classical worlds" corresponds to a higher level geometric existence serving as the fixed arena for the quantum dynamics. The basic vision is that the existence requirement for Kähler geometry in the infinite-dimensional context fixes the infinite-dimensional geometric existence uniquely.

2. Quantum states defined as classical spinor fields in the world of classical worlds, and provide the quantum descriptions of possible physical realities that the probably never-reachable ultimate theory gives as solutions of field equations. The solutions *are* the objective realities in the sense of quantum theory: theory and theory about world are one and the same thing: there is no separate 'reality' behind the solutions of the field equations.

3. Subjective existence corresponds to quantum jumps between the quantum states identified as moment of consciousness. Just as quantum numbers characterize physical states, the increments of quantum numbers in quantum jump are natural candidates for qualia, and this leads to a concrete quantum model for sensory qualia and sensory perception [**?**].

Quantum jump has a complex anatomy: counterpart for the unitary U process of Penrose followed by a counterpart of the state function reduction followed by a counterpart of the state preparation process yielding a classical state in Boolean and geometrical sense. State function preparation and reduction are nondeterministic processes and preparation is analogous to analysis since it decomposes at each step the already existing unentangled subsystems to unentangled subsystems if possible.

Quantum jump is the elementary particle of consciousness and selves are like atoms, molecules,... built from these. Self is by definition a system able to not develop bound state quantum entanglement with environment and loses consciousness when this occurs. Selves form a hierarchy very much analogous to the hierarchy of states formed from elementary particles. Self experiences its sub-selves as mental images. Selves form objects of a category in which arrows connect sub-selves to selves.

Macro-temporal and macroscopic quantum coherence corresponds to the formation of bound states [**?**]: in this process state function reduction and preparation effective cease in appropriate degrees of freedom. In TGD framework one can assign to bound state entanglement negative entropy identifiable as a genuine measure for information [**?**]. The bound state entanglement stable against state function preparation would thus serve as a correlate for the experience of understanding, and one could compare quantum jump to a brainstorm followed by an analysis leading to an experience of understanding.

Quantum classical correspondence relates the three levels of existence to each other. It states that both quantum states and quantum jump sequences have space-time correlates. This is made possible by p-adic and classical non-determinism, which are characteristic features of TGD space-time. p-Adic non-determinism makes it possible to map quantum jump sequences to p-adic space-time sheets: this gives rise to cognitive representations. The non-determinism of Kähler action makes possible symbolic sensory representations of quantum jump sequences of which language is the basic example.

The natural identification of the correlates of quantum states is as maximal deterministic regions of space-time sheet. The final states of quantum jump define a sequence of quantum states so that quantum jump sequence (contents of consciousness) has the decomposition of space-time sheet to maximal deterministic regions as a space-time correlate. Thus space-time surface can be said to define a symbolic (and unfaithful) representation for the contents of consciousness. Since configuration space spinor field is defined in the world of classical worlds, this means that quantum states carry information about quantum jump sequence and self reference becomes possible. System can become conscious about what it *was* (not "is") conscious of.

The possibility to represent quantum jump sequences at space-time level is what makes possible practical mathematics, cognition, and symbolic representations. The generation of these representations in turn means generation of reflective levels of consciousness and thus explains self-referential nature of consciousness. This feedback makes also possible the evolution of mathematical consciousness: mathematician without paper and pencil (or computer keyboard!) cannot do very much.

Category theory might help to formulate more precisely the quantum classical correspondence and self referentiality as structure respecting functors from the categories associated with subjective existence to the categories of quantum and classical existence and from the category of quantum existence to that of classical existence.

6.3.2 The new ontology of space-time

Classical worlds are space-time surfaces and have much richer ontology than the space-time of general relativity. Space-time is many-sheeted possessing a hierarchy of parallel space-time sheets topologically condensed at larger space-time sheets and identifiable as geometric correlates for physical objects in various length scales (see Fig. ??). Topological field quantization allows to assign to any material system "field body": this has important implications for quantum biology in TGD Universe [?].

TGD leads to a generalization of the notion of real numbers obtained by gluing real number field and p-adic number fields R_p, labelled by primes $p = 2, 3, 5, ...$ and their extensions together along common rationals (very roughly) to form a "book like" structure [?, ?, ?, ?, ?]. p-Adic space-time sheets are interpreted as space-time correlates of cognition and intentionality. The transformation of intention to action corresponds to a quantum jump replacing p-adic space-time sheet with a real one.

The p-adic notion of distance differs dramatically from its real counterpart. Two rationals infinitesimally near p-adically are infinitely distance in real sense. This means that p-adic space-time sheets have literally infinite size in the real sense and cognition and intentionality cannot be localized in brain. Biological body serves only as a sensory receptor and motor instrument utilizing symbolic representations built by brain.

The notion of infinite numbers (primes, rationals, reals, complex numbers and also quaternions and octonions)[?] inspired by TGD inspired theory of consciousness leads to a further generalization. One can form ratios of infinite rationals to get ordinary rational numbers in the real sense and division by its inverse gives numbers which are units in the real sense but not in various p-adic senses ($p = 2, 3, 5, ...$).

This means that each space-time point is infinitely structured (note also that configuration space points are 3-surfaces and infinitely structure too!) but this structure is not seen at the level of real physics. The infinite hierarchy of infinite primes implies that single space-time point is in principle able to represent the physical quantum state of the entire universe in its structure cognitively. There are several interpretations: space-time points are algebraic holograms realizing Brahman=Atman identity; the Platonia of mathematical ideas resides at every space-time point, space-time points are the monads of Leibniz or the nodes of Indra's web...

One might hope that category theory could be of help in formulating more precisely this intuitive view about space-time which generalizes also to the other two levels of ontology.

6.3.3 The new notion of sub-system and notions of quantum presheaf and "quantum logic" of sub-systems

TGD based notion subsystem differs from the standard one already at the classical level [?]. The relationship of having wormhole contacts to a larger space-time sheet would correspond to the basic morphism and would correspond to inclusion in category Set. Note that same space-time sheet can have wormhole contacts to several larger space-time sheets (see Fig. ??). The wormhole contacts are surrounded by light like 3-surfaces somewhat analogous to black hole horizons. They act as causal determinants and define 3-dimensional quantum gravitational holograms. Also other causal determinants are possible but light-likeness seems to a common feature of them.

Subsystem does not correspond to a mere subset geometrically as in standard physics and the functors mapping quantum level to space-time level are not maps to the category of sets but to that of space-time sheets, and thus pre-sheafs are replaced with what might be called quantum pre-sheafs. Boolean algebra and also Heyting algebra are replaced with their quantum variants.

1. The set theoretic inclusion $\subset$ in the definition of Heyting algebra is replaced by the arrow $A \to B$ representing a sequence of topological condensations connecting the space-time sheet A to B. The arrow from A to B is possible only if A is smaller than B, more precisely: if the p-adic prime $p(A)$ characterizing A is larger (or equal) than $p(B)$. The relation $\in$ of being a point of the space-time sheet A is not utilized at all.

2. Sieves at A are defined, not in terms of arrow sequences $f(A \to B)$, but as arrow sequences $f(B \to A)$: the wormhole contact roads leading from sheet B down to A. If there is a road from B to A then all roads to $C \to B$ combine with roads $B \to A$ to give roads $C \to A$ and thus define elements of the sieve.

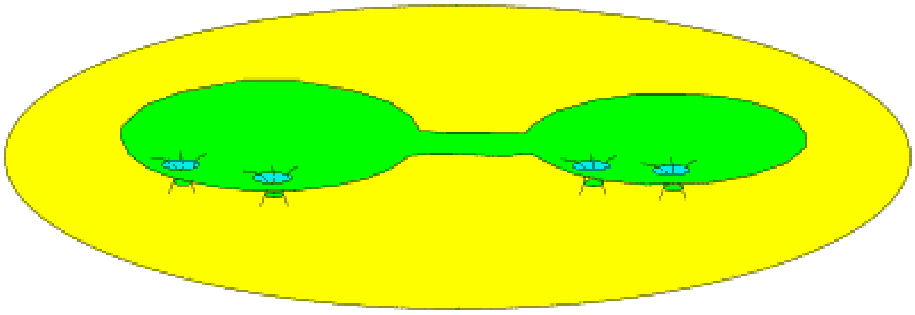

Figure 6.2: a) Wormhole contacts connect interiors space-time parallel space-time sheets (at a distance of about 10^4 Planck lengths) and join along boundaries bonds of possibly macroscopic size connect boundaries of space-time sheets. b) Wormhole contacts connecting space-time sheet to several space-time sheets could represent space-time correlate of quantum superposition. c) Space-time correlate for bound state entanglement making possible sharing of mental images.

3. X is quantum presheaf if it is a functor from the a category C to the category of space-time sheets. A sub-object of X is presheaf K such that for every A there is a road from $K(A)$ to $X(A)$.

4. Let K be a sub-object of the pre-sheaf X. The elements of the corresponding quantum Heyting algebra at A are defined as the collections of roads $f(B, A)$ leading via $K(A)$ to $K(X)$. This collection is either empty or contains all the roads via $K(A)$ to $K(X)$. A two-valued logic results trivially.

5. The difference with respect to Boolean logic comes from the fact space-time sheet can condense simultaneously to several disjoint space-time sheets whereas a given set cannot be a subset of two disjoint sets (see Fig. **??**).

One can ask whether this property of "quantum logic" allows a space-time correlate even for the superposition of orthogonal quantum states as simultaneous topological condensation at several space-time sheets. This interpretation would be consistent with the hypothesis that bound state entanglement has the formation of join along boundaries bonds (JABs) as a space-time correlate. Topologically condensed JAB-connected space-time sheets could indeed condense simultaneously on several space-time sheets. It however seems that this interpretation is not consistent with quantum superpositions.

The new notion of sub-system at space-time level forces to modify the notion of sub-system at quantum level. The subsystem defined by a smaller space-time sheet is not describable as a simple tensor factor but the relation is given by the morphism representing the property of being sub-system. In the chapter "Was von Neumann Right After All" [**?**] a mathematical formulation for this relationship is proposed in terms of so called Jones inclusions of von Neumann algebras of type II_1, which seem to provide the proper mathematical framework for quantum TGD. Wormhole contacts would represent space-time correlate for inclusion as a generalized tensor factor rather than inclusion as a direct summand as in quantum superposition.

Space-time correlate for ordinary quantum logic

The proposed "quantum logic" for subsystems based on topological condensation by the formation of wormhole contacts does not seem to correspond to the formation of quantum superpositions and the usual quantum logic. The most non-intuitive aspect of quantum logic is represented by the quantum superposition of mutually exclusive options represented by orthogonal quantum states.

In the double-slit experiment this corresponds to the possibility of single photon to travel along the paths going through the two slits simultaneously and to interfere on the screen. In TGD framework this would correspond quite literally to the decay of the 3-surface describing photon to two pieces which travel through the slits and fuse together before the screen. More generally, the space-time correlate for this aspect of quantum logic would be splitting of 3-surface to several pieces. In string models where the splitting of string means creation of 2-particle state (2-photon state in the case of double slit experiment), which at state space-level corresponds to a tensor product product state. Therefore the ontologies of string models and TGD differ in a profound manner.

In quantum measurement the projection to an eigen state of observables means that a quantum jump in which all branches except one become vacuum extremal occurs. What is also new that by the classical non-determinism space-time surface can also represent a quantum jump sequence. For instance, the states before and after the reduction correspond to space-time regions. This picture allows to understand the recent findings of Afshar [?, ?], which challenge Copenhagen interpretation.

6.3.4 Does quantum jump allow space-time description?

Quantum jump consists of a unitary process, state function reduction and state preparation. The geometrical realization of "quantum logic" suggests that simultaneous topological condensation to several space-time sheets could be a space-time correlate for the maximally entangled superposition of quantum states created in the U-process. Quantal multi-verse states would functorially correspond to classical multi-verse states: something which obviously came in my mind for long time ago but seemed stupid. State function reduction would lead to the splitting of the wormhole contacts and as a result maximally reduced state would result: one cannot however exclude bound state entanglement due to interactions mediated by wormhole contacts.

State function preparation would correspond to a sequence of splittings for join along boundaries bonds serving as prerequisites for entanglement in the degrees of freedom associated with second quantized induced spinor fields at space-time sheets. An equivalent process is the decay of 3-sheet to two pieces interpretable as de-coherence. For instance, the splitting of photon beam in the modified double slit experiment by Afshar [?, ?], which challenges the existing interpretations of quantum theory and provides support for TGD based theory of quantum measurement relying on classical non-determinism, would correspond to this process.

State preparation yields states in which no dissipation occurs. The space-time correlates are asymptotic solutions of field equations for which classical counterpart of dissipation identified as Lorentz 4-force vanishes: this hypothesis indeed leads to very general solutions of field equations [?]. The non-determinism at quantum level would correspond to the non-determinism for the evolution of induced spinor fields at space-time level.

6.3.5 Brief summary of the basic categories relating to the self hierarchy

Category theory suggests the identification of space-time sheets as basic objects of the space-time category. Space-time sheets are natural correlates for selves and the arrow describing sub-self property is mapped to the arrow of being topologically condensed space-time sheet. Category theoretically this would mean the existence of a functor from the the category defined by self hierarchy to the hierarchy of space-time sheets.

The highly non-trivial implication of the new notion of sub-system is that same sub-self can be sub-self of several selves: mental images can be shared so that consciousness would not be so private as usually believed. Sharing involves also fusion of mental images. Sub-selves of different selves form a bound state and fuse to single sub-self giving rise to stereo consciousness (fusion of right and left visual fields is the basic example).

The formation of join along boundaries bonds connecting the boundaries of a sub-self space-time sheets is the space-time correlate for this process. The ability of subsystems to entangle when systems remain un-entangled is completely new and due to the new notion of subsystem (subsystem is separated by elementary particle horizon from system). Sharing of mental images and the possibility of time-like entanglement also possible telepathic quantum communications: for instance, TGD based model of episodal memories relies on this mechanism [?].

The hierarchy of space-time sheets functorially replicates itself at the level of quantum states and of subjective existence. Quantum states have a hierarchical structure corresponding to the decomposition of space-time to space-time sheets. The sequence of quantum jumps decomposes into parallel sequences of quantum jumps occurring at different parallel space-time sheets characterized by p-adic length scales. The possibility of quantum parallel dissipation (quarks inside hadrons) is one important implication: although dissipation and de-coherence occur in short length and time scales, quantum coherence is preserved in longer length and time scales. This is of utmost importance for understanding how wet and hot brain can be macroscopic quantum system [?].

The self hierarchy has also counterpart at the level of Platonia made possible by infinitely structured points of space-time. The construction of infinite primes is analogous to a repeated second quantization of an arithmetic quantum field theory such that the many particle states of previous level representing infinite primes at that level become elementary particles at the next level of construction. This hierarchy reflect itself as the hierarchy of units and as a hierarchy of levels of mathematical consciousness.

The steps in quantum jump, or equivalently the sequence of final states of individual steps would define the objects of the category associated with the quantum jump. The first step would be the formation of a larger number of wormhole contacts during U process followed by their splitting to minimum in the state function reduction. Formation and splitting of contacts would define arrows now. During the state preparation each decay to separate 3-sheets would define arrow from connecting initial state to both final states.

6.3.6 The category of light cones, the construction of the configuration space geometry, and the problem of psychological time

Light-like 7-surfaces of imbedding space are central in the construction of the geometry of the world of classical worlds. The original hypothesis was that space-times are 4-surfaces of $H = M^4_+ \times CP_2$, where M^4_+ is the future light cone of Minkowski space with the moment of big bang identified as its boundary $\delta H = \delta M^4_+ \times CP_2$: "the boundary of light-cone". The naive quantum holography would suggest that by classical determinism everything reduces to the light cone boundary. The classical non-determinism of Kähler action forces to give up this naive picture which also spoils the full Poincare invariance.

The new view about energy and time forces to conclude that space-time surfaces approach vacua at the boundary of the future light cone. The world of classical worlds, call it CH, would consist of classical universes having a vanishing inertial 4-momentum and other conserved quantities and being created from vacuum: big bang would be replaced with a "silent whisper amplified to a big bang". The net gravitational mass density can be non-vanishing since gravitational momentum is difference of inertial momenta of positive and negative energy matter: Einstein's Equivalence Principle is exact truth only at the limit when the interaction between positive and negative energy matter can be neglected [?].

Poincare invariant theory results if one replaces CH with the union of its copies $CH(a)$ associated with the light cones $M^4_+(a)$ with a specifying the position of the dip of $M^4_+(a)$ in M^4. Also past directed light-cones $M^4_-(a)$ are allowed. The unions and intersections of the light cones with inclusion as a basic arrow would form category analogous to the category Set with inclusion defining the arrow of time. This category formalizes the ideas that cosmology has a fractal Russian doll like structure, that the cosmologies inside cosmologies are singularity free, and that cosmology is analogous to an organic evolution and organic evolution to a mini cosmology [?].

The view also unifies the proposed two explanations for the arrow of psychological time [?].

1. The mind like space-time sheets representing conscious self drift quantum jump by quantum jump towards geometric future whereas the matter like space-time sheets remain stationary. The self of the organism presumably consisting mostly of topological field quanta, would be like a passenger in a moving train seeing the changing landscape. The organism would be a mini cosmology drifting quantum jump to the geometric future. Also selves living in the reverse direction of time are possible.

2. Psychological time corresponds to a phase transition front in which intentions represented by p-adic space-time sheets transform to actions represented by real space-time sheets moving to the direction of geometric future. The motion would be due to the drift of $M^4_+(a)$. The very fact that the mini cosmology is created from vacuum, implies that space-time sheets of both negative and positive field energy are abundantly generated as realizations of intentions. The intentional resources are richest near the boundary of $M^4_+(a)$ and depleted during the ageing with respect to subjective time as asymptotic self-organization patterns are reached. Interestingly,

mini cosmology can be seen as a fractally scaled up variant of quantum jump. The realization of intentions as negative energy signals (phase conjugate light) sent to the geometric past and inducing a positive energy response (say neural activity) is consistent with the TGD based models for motor action and long term memory [?].

6.4 More precise characterization of the basic categories and possible applications

In the following the categories associated with self and quantum jump are discussed in more precise manner and applications to communications and cognition are considered.

6.4.1 Intuitive picture about the category formed by the geometric correlates of selves

Space-time surface $X^4(X^3)$ decomposes into regions obeying either real or p-adic topology and each region of this kind corresponds to an unentangled subsystem or self lasting at least one quantum jump. By the localization in the zero modes these decompositions are equivalent for all 3-surfaces X^3 in the quantum superposition defined by the prepared configuration space spinor fields resulting in quantum jumps. There is a hierarchy of selves since selves can contain sub-selves. The entire space-time surface $X^4(X^3)$ represents the highest level of the self hierarchy.

This structure defines in a natural manner a category. Objects are all possible sub-selves contained in the self hierarchy: sub-self is set consisting of lower level sub-selves, which in turn have a further decomposition to sub-selves, etc... The naive expectation is that geometrically sub-self belongs to a self as a subset and this defines an inclusion map acting as a natural morphism in this category. This expectation is not quite correct. More natural morphisms are the arrows telling that self as a set of sub-selves contains sub-self as an element. These arrows define a structure analogous to a composite of hierarchy trees.

To be more precise, for a single space-time surface $X^4(X^3)$ this hierarchy corresponds to a subjective time slice of the self hierarchy defined by a single quantum jump. The sequence of hierarchies associated with a sequence of quantum jumps is a natural geometric correlate for the self hierarchy. This means that the objects are now sequences of submoments of consciousness. Sequences are not arbitrary. Self must survive its lifetime although sub-selves at various levels can disappear and reappear (generation and disappearance of mental images). Geometrically this means typically a phase transition transforming real or p_1-adic to p_2-adic space-time region with same topology as the environment. Also sub-selves can fuse to single sub-self. The constraints on self sequences must be such that it takes these processes into account. Note that these constraints emerge naturally from the fact that quantum jumps sequences define the sequences of surfaces $X^4(X^3)$.

By the rich anatomy of the quantum jump there is large number of quantum jumps leading from a given initial quantum history to a given final quantum history. One could envisage quantum jump also as a discrete path in the space of configuration space spinor fields leading from the initial state to the final state. In particular, for given self there is an infinite number of closed elementary paths leading from the initial quantum history back to the initial quantum history and these paths in principle give all possible conscious information about a given quantum history/idea: kind of self morphisms are in question (analogous to, say, group automorhisms). Information about point of space is obtained only by moving around and coming back to the point, that is by studying the surroundings of the point. Self in turn can be seen as a composite of elementary paths defined by the quantum jumps. Selves can define arbitrarily complex composite closed paths giving information about a given quantum history.

6.4.2 Categories related to self and quantum jump

The categories defined by moments of consciousness and the notion of self

Since quantum jump involves state reduction and the sequence of self measurement reducing all entanglement except bound state entanglement, it defines a hierarchy of unentangled subsystems allowing interpretation as objects of a category. Arrows correspond to subsystem-system relationship and the two subsystems resulting in self measurement to the system. What subsystem corresponds mathematically is however not at all trivial and the naive description as a tensor factor does not work. Rather, a definition relying on the notion of p-adic length scale cutoff identified as a fundamental aspect of nature and consciousness is needed.

It is not clear what the statement that self corresponds to a subsystem which remains unentangled in subsequent quantum jump means concretely since subsystem can certainly change in some limits. What is clear that bound state entanglement between selves means a loss of consciousness. Category theory suggests that there should exist a functor between categories defined by two subsequent moments of consciousness. This functor maps submoments of consciousness to submoments of consciousness and arrows to arrows. Two subsequent submoments of consciousness belong to same sub-self is the functor maps the first one to the latter one. Thus category theory would play essential role in the precise definition of the notion of self.

The sequences of moments of consciousness form a larger category containing sub-selves as sequences of unentangled subsystems mapped to each other by functor arrows functoring subsequent quantum jumps to each other.

What might then be the ultimate characterizer of the self-identity? The theory of infinite primes suggests that space-time surface decomposes into regions labelled by finite p-adic primes. These primes must label also real regions rather than only p-adic ones, and one could understand this as resulting from a resonant transformation of intention to action. A p-adic space-time region characterized by prime p can transform to a real one or vice versa in quantum jump if the sizes of real and p-adic regions are characterized by the p-adic length scale L_p (or n-ary p-adic length scale $L_p(n)$. One can also consider the possibility that real region is accompanied by a p-adic region characterized by a definite prime p and providing a cognitive self-representation of the real region.

If this view is correct, the p-adic prime characterizing a given real or p-adic space-time sheet is the ultimate characterizer of the self-identity. Self identity is lost in bound state entanglement with another space-time sheet (at least when a space-time sheet with smaller value of the p-adic prime joins by join along boundaries bond to a one with a higher value of the p-adic prime). Self identity is also lost if a space-time sheet characterized by a given p-adic prime disappears in quantum jump.

The category associated with quantum jump sequences

There are several similarities between the ontologies and epistemologies of TGD and of category theory. Conscious experience is always determined by the discrete paths in the space of configuration space spinor fields defined by a quantum jump connecting two quantum histories (states) and is never determined by single quantum history as such (quantum states are unconscious). Also category theory is about relations between objects, not about objects directly: self-morphisms give information about the object of category (in case of group composite paths would correspond to products of group automorphisms). Analogously closed paths determined by quantum jump sequences give information about single quantum history. The point is however that it is impossible to have direct knowledge about the quantum histories: they are not conscious.

One can indeed define a natural category, call it **QSelf**, applying to this situation. The objects of the category **QSelf** are initial quantum histories of quantum jumps and correspond to prepared quantum states. The discrete path defining quantum jump can be regarded as an elementary morphism. Selves are composites of elementary morphisms of the initial quantum history defined by quantum jumps: one can characterize the morphisms by the number of the elementary morphisms in the product. Trivial self contains no quantum jumps and corresponds to the identity morphism, null path. Thus the collection of all possible sequences of quantum jumps, that is collections of selves allows a description in terms of category theory although the category in question is not a subcategory of the category **Set**.

Category **QSelf** does not possess terminal and initial elements (for terminal (initial) element T there is exactly one arrow $A \rightarrow T$ ($T \rightarrow A$) for every A: now there are always many paths between quantum histories involved).

6.4.3 Communications in TGD framework

Goro Kato identifies communications between conscious entities as natural maps between them whereas in TGD natural maps bind submoments of consciousness to selves. In TGD framework quantum measurement and the sharing of mental images are the basic candidates for communications. The problem is that the identification of communications as sharing of mental images is not consistent with the naive view about subsystem as a tensor factor. Many-sheeted space-time however forces length scale dependent notion of subsystem at space-time level and this saves the situation.

What communications are?

Communication is essentially generation of desired mental images/sub-selves in receiver. Communication between selves need not be directly conscious: in this case communication would generate mental images at some lower level of self hierarchy of receiver: for instance generate large number of sub-sub-selves of similar type. This is like communications between organizations. Communication can be also vertical: self can generate somehow sub-self in some sub-sub....sub-self or sub-sub...sub-self can generate sub-self of self somehow. This is communication from boss to the lower levels organization or vice versa.

These communications should have direct topological counterparts. For instance, the communication between selves could correspond to an exchange of mental image represented as a space-time region of different topology inside sender self space-time sheet. The sender self would simply throw this space-time region to a receiver self like a ball. This mechanism applies also to vertical communications since the ball could be also thrown from a boss to sub...sub-self at some lower level of hierarchy and vice versa.

The sequence of space-time surfaces provides a direct topological counterpart for communication as throwing balls representing sub-selves. Quantum jump sequence contains space-time surfaces in which the regions corresponding to receiver and sender selves are connected by a join along boundaries bond (perhaps massless extremal) representing classically the communication: during the communication the receiver and sender would form single self. The cartoon vision about rays connecting the eyes of communication persons would make sense quite concretely.

More refined means of communication would generate sub-selves of desired type directly at the end of receiver. In this case it is not so obvious how the sequence $X^(X^3)$ of space-time surfaces could represent communication. Of course, one can question whether communication is really what happens in this kind of situation. For instance, sender can affect the environment of receiver to be such that receiver gets irritated (computer virus is good manner to achieve this!) but one can wonder whether this is real communication.

Communication as quantum measurement?

Quantum measurement generates one-one map between the states of the entangled systems resulting in quantum measurement. Both state function reduction and self measurement give rise to this kind of map. This map could perhaps be interpreted as quantum communication between unentangled subsystems resulting in quantum measurement. For the state reduction process the space-time correlates are the values of zero modes. For state preparation the space-time correlates should correspond to classical spinor field modes correlating for the two subsystems generated in self measurement.

Communication as sharing of mental images

It has become clear that the sharing of mental images induced by quantum entanglement of sub-selves of two separate selves represents genuine conscious communication which is analogous telepathy and provides general mechanism of remote mental interactions making possible even molecular recognition mechanisms.

1. The sharing of mental images is not possible unless one assumes that self hierarchy is defined by using the notion of length scale resolution defined by p-adic length scale. The notion of scale of resolution is indeed fundamental for all quantum field theories (renormalization group invariance) for all quantum field theories and without it the practical modelling of physics would not be possible. The notion reflects directly the length scale resolution of conscious experience. For a given sub-self of self the resolution is given by the p-adic length scale associated with the sub-self space-time sheet.

2. Length scale resolution emerges naturally from the fact that sub-self space-time sheets having Minkowskian signature of metric are separated from the one representing self by wormhole

contacts with Euclidian signature of metric. The signature of the induced metric changes from Minkowskian signature to Euclidian signature at 'elementary particle horizons' surrounding the throats of the wormhole contacts and having degenerate induced metric. Elementary particle horizons are thus metrically two-dimensional light like surfaces analogous to the boundary of the light cone and allow conformal invariance. Elementary particle horizons act as causal horizons. Topologically condensed space-time sheets are analogous to black hole interiors and due to the lack of the causal connectedness the standard description of sub-selves as tensor factors of the state space corresponding to self is not appropriate.

Hence systems correspond, not to the space-time sheets plus entire hierarchy of space-time sheets condensed to it, but rather, to space-time sheets with holes resulting when the space-time sheets representing subsystems are spliced off along the elementary particle horizons around wormhole contacts. This does not mean that all information about subsystem is lost: subsystem space-time sheet is only replaced by the elementary particle horizon. In analogy with the description of the black hole, some parameters (mass, charges,...) characterizing the classical fields created by the sub-self space-time sheet characterize sub-self.

One can say that the state space of the system contains 'holes'. There is a hierarchy of state spaces labelled by p-adic primes defining length scale resolutions. This picture resolves a long-standing puzzle relating to the interpretation of the fact that particle is characterize by both classical and quantum charges. Particle cannot couple simultaneously to both and this is achieved if quantum charge is associated with the lowest level description of the particle as CP_2 extremal and classical charges to its description at higher levels of hierarchy.

3. The immediate implication indeed is that it is possible to have a situation in which two selves are unentangled although their sub-selves (mental images) are entangled. This corresponds to the fusion and sharing of mental images. The sharing of the mental images means that union of disjoint hierarchy trees with levels labelled by p-adic primes p is replaced by a union of hierarchy threes with horizontal lines connecting subsystems at the same level of hierarchy. Thus the classical correspondence defines a category of presheaves with both vertical arrows replaced by sub-self-self relationship, horizontal arrows representing sharing of mental images, and natural maps representing binding of submoments of consciousness to selves.

Comparison with Goro Kato's approach

It is of interest to compare Goro Kato's approach with TGD approach. The following correspondence suggests itself.

1. In TGD each quantum jumps defines a category analogous to the Goro Kato's category of open sets of some topological space but set theoretic inclusion replaced by topological condensation. The category defined by a moment of consciousness is dynamical whereas the category of open sets is non-dynamical.

2. The assignment of a 3-surface acting as a causal determinant to each unentangled subsystem defined by a moment of consciousness defines a unique "quantum presheaf" which is the counterpart of the presheaf in Goro Kato's theory. The conscious entity of Kato's theory corresponds to the classical correlate for a moment of consciousness.

3. Natural maps between the causal determinants correspond to the space-time correlates for the functor arrows defining the threads connecting submoments of consciousness to selves. In Goro Kato's theory natural maps are interpreted as communications between conscious entities. The sharing of mental images by quantum entanglement between subsystems of unentangled systems defines horizontal bi-directional arrows between subsystems associated with same moment of consciousness and is counterpart of communication in TGD framework. It replaces the union of disjoint hierarchy trees associated with various unentangled subsystems with hierarchy trees having horizontal connections defining the bi-directional arrows. The sharing of mental images is not possible if subsystem is identified as a tensor factor and thus without taking into account length scale resolution.

6.4.4 Cognizing about cognition

There are close connections with basic facts about cognition.

1. Categorization means classification and abstraction of common features in the class formed by the objects of a category. Already quantum jump defines category with hierarchical structure and can be regarded as consciously experienced analysis in which totally entangled entire universe $U\Psi_i$ decomposes to a product of maximally unentangled subsystems. The sub-selves of self are like elements of set and are experienced as separate objects whereas sub-sub-selves of sub-self self experiences as an average: they belong to a class or category formed by the sub-self. This kind of averaging occurs also for the contributions of quantum jumps to conscious experience of self.

2. The notions of category theory might be useful in an attempt to construct a theory of cognitive structures since cognition is indeed to high degree classification and abstraction process. The sub-selves of a real self indeed have p-adic space-time sheets as geometric correlates and thus correspond to cognitive sub-selves, thoughts. A meditative experience of empty mind means in case of real self the total absence of thoughts.

3. Predicate logic provides a formalization of the natural language and relies heavily on the notion of n-ary relation. Binary relations $R(a, b)$ corresponds formally to the subset of the product set $A \times B$. For instance, statements like 'A does something to B' can be expressed as a binary relation, particular kind of arrow and morphism ($A \leq B$ is a standard example). For sub-selves this relation would correspond to a dynamical evolution at space-time level modelling the interaction between A and B. The dynamical path defined by a sequence of quantum jumps is able to describe this kind of relationships too at level of conscious experience. For instance, 'A touches B' would involve the temporary fusion of sub-selves A and B to sub-self C.

6.5 Logic and category theory

Category theory allows naturally more general than Boolean logics inherent to the notion of topos associated with any category. Basic question is whether the ordinary notion of topos algebra based on set theoretic inclusion or the notion of quantum topos based on topological condensation is physically appropriate. Starting from the quasi-Boolean algebra of open sets one ends up to the conclusion that quantum logic is more natural. Also configuration space spinor fields lead naturally to the notion of quantum logic.

6.5.1 Is the logic of conscious experience based on set theoretic inclusion or topological condensation?

The algebra of open sets with intersections and unions and complement defined as the interior of the complement defines a modification of Boolean algebra having the peculiar feature that the points at the boundary of the closure of open set cannot be said to belong to neither interior of open set or of its complement. There are two options concerning the interpretation.

1. 3-valued logic could be in question. It is however not possible to understand this three-valuedness if one defines the quasi-Boolean algebra of open sets as Heyting algebra. The resulting logic is two-valued and the points at boundaries of the closure do not correspond neither to the statement or its negation. In p-adic context the situation changes since p-adic open sets are also closed so that the logic is strictly Boolean. That our ordinary cognitive mind is Boolean provides a further good reason for why cognition is p-adic.

2. These points at the boundary of the closure belong to both interior and exterior in which case a two-valued "quantum logic" allowing superposition of opposite truth values is in question. The situation is indeed exactly the same as in the case of space-time sheet having wormhole contacts to several space-time sheets.

The quantum logic brings in mind Zen consciousness [?] (which I became fascinated of while reading Hofstadter's book "Gödel, Escher,Bach" [?]) and one can wonder whether selves having real space-time sheets as geometric correlates and able to live simultaneously in many parallel worlds correspond to Zen consciousness and Zen logic. Zen logic would be also logic of sensory experience whereas cognition would obey strictly Boolean logic.

The causal determinants associated with space-time sheets correspond to light like 3-surfaces which could elementary particle horizons or space-time boundaries and possibly also 3-surfaces separating two maximal deterministic regions of a space-time sheet. These surfaces act as 3-dimensional quantum holograms and have the strange Zen property that they are neither space-like nor time-like so that they represent both the state and the process. In the TGD based model for topological quantum computation (TQC) light-like boundaries code for the computation so that TQC program code would be equivalent with the running program [?].

6.5.2 Do configuration space spinor fields define quantum logic and quantum topos

I have proposed already earlier that configuration space spinor fields define what might be called quantum logic. One can wonder whether configuration space spinors could also naturally define what might be called quantum topos since the category underlying topos defines the logic appropriate to the topos. This question remains unanswered in the following: I just describe the line of though generalizing ordinary Boolean logic.

Finite-dimensional spinors define quantum logic

Spinors at a point of an $2N$-dimensional space span 2^N-dimensional space and spinor basis is in one-one correspondence with Boolean algebra with N different truth values (N bits). 2N=2-dimensional case is simple: Spin up spinor= true and spin-dow spinor=false. The spinors for $2N$-dimensional space are obtained as an N-fold tensor product of 2-dimensional spinors (spin up,spin down): just like in the case of Cartesian power of Ω.

Boolean spinors in a given basis are eigen states for a set N mutually commuting sigma matrices providing a representation for the tangent space group acting as rotations. Boolean spinors define N Boolean statements in the set Ω^N so that one can in a natural manner assign a set with a Boolean spinor. In the real case this group is $SO(2N)$ and reduces to $SU(N)$ for Kähler manifolds. For pseudo-euclidian metric some non-compact variant of the tangent space group is involved. The selections of N mutually commuting generators are labelled by the flag-manifold $SO(2N)/SO(2)^N$ in real context and by the flag-manifold $U(N)/U(1)^N$ in the complex case. The selection of these generators defines a collection of N 2-dimensional linear subspaces of the tangent space.

Spinors are in general complex superpositions of spinor basis which can be taken as the product spinors. The quantum measurement of N spins representing the Cartan algebra of $SO(2N)$ ($SU(N)$) leads to a state representing a definite Boolean statement. This suggests that quantum jumps as moments of consciousness quite generally make universe classical, not only in geometric but also in logical sense. This is indeed what the state preparation process for the configuration space spinor field seems to do.

Quantum logic for finite-dimensional spinor fields

One can generalize the idea of the spinor logic also to the case of spinor fields. For a given choice of the local spinor basis (which is unique only modular local gauge rotation) spinor field assigns to each point of finite-dimensional space a quantum superposition of Boolean statements decomposing into product of N statements.

Also now one can ask whether it is possible to find a gauge in which each point corresponds to definite Boolean statement and is thus an eigen state of a maximal number of mutually commuting rotation generators Σ_{ij}. This is not trivial if one requires that Dirac equation is satisfied. In the case of flat space this is certainly true and constant spinors multiplied by functions which solve d'Alembert equation provide a global basis.

The solutions of Dirac equation in a curved finite-dimensional space do not usually possess a definite spin direction globally since spinor curvature means the presence of magnetic spin-flipping interaction and since there need not exist a global gauge transformation leading to an eigen state of the local Cartan algebra everywhere. What might happen is that the local gauge transformation becomes singular at some point: for instance, the direction of spin would be radial around given point and become ill defined at the point. This is much like the singularities for vector fields on sphere. The spinor field having this kind of singularity should vanish at singularity but the local gauge rotation rotating spin in same direction everywhere is necessarily ill-defined at the singularity.

In fact, this can be expressed using the language of category theory. The category in question corresponds to a presheaf which assigns to the points of the base space the fiber space of the spinor bundle. The presence of singularity means that there are no global section for this presheaf, that is a continuous choice of a non-vanishing spinor at each point of the base space. The so called Kochen-Specker theorem discussed in [?] is closely related to a completely analogous phenomenon involving non-existence of global sections and thus non-existence of a global truth value.

Thus in case of curved spaces is not necessarily possible to have spinor field basis representing globally Boolean statements and only the notion of locally Boolean logic makes sense. Indeed, one can select the basis to be eigen state of maximal set of mutually commuting rotation generators in single point of the compact space. Any such choice does.

Quantum logic and quantum topos defined by the prepared configuration space spinor fields

The prepared configuration space spinor fields occurring as initial and final states of quantum jumps are the natural candidates for defining quantum logic. The outcomes of the quantum jumps resulting in the state preparation process are maximally unentangled states and are as close to Boolean states as possible.

Configuration space spinors correspond to fermionic Fock states created by infinite number of fermionic (leptonic and quarklike) creation and annihilation operators. The spin degeneracy is replaced by the double-fold degeneracy associated with a given fermion mode: given state either contains fermion or not and these two states represent true and false now. If configuration space were flat, the Fock state basis with definite fermion and anti-fermion numbers in each mode would be in one-one correspondence with Boolean algebra.

Situation is however not so simple. Finite-dimensional curved space is replaced with the fiber degrees of freedom of the configuration space in which the metric is non-vanishing. The precise analogy with the finite-dimensional case suggests that if the curvature form of the configuration space spinor connection is nontrivial, it is impossible to diagonalize even the prepared maximally unentangled configuration space spinor fields Ψ_i in the entire fiber of the configuration space (quantum fluctuating degrees of freedom) for given values of the zero modes. Local singularities at which the spin quantum numbers of the diagonalized but vanishing configuration space spinor field become ill-defined are possible also now.

In the infinite-dimensional context the presence of the fermion-anti-fermion pairs in the state means that it does not represent a definite Boolean statement unless one defines a more general basis of configuration space spinors for which pairs are present in the states of the state basis: this generalization is indeed possible. The sigma matrices of the configuration space appearing in the spinor connection term of the Dirac operator of the configuration space indeed create fermion-fermion pairs. What is decisive, is not the absence of fermion-anti-fermion pairs, but the possibility that the spinor field basis cannot be reduced to eigen states of the local Cartan algebra in fiber degrees of freedom globally.

Also for bound states of fermions (say leptons and quarks) it is impossible to reduce the state to a definite Boolean statement even locally. This would suggest that fermionic logic does not reduce to a completely Boolean logic even in the case of the prepared states.

Thus configuration space spinor fields could have interpretation in terms of non-Boolean quantum logic possessing Boolean logics only as sub-logics and define what might be called quantum topos. Instead of Ω^N-valued maps the values for the maps are complex valued quantum superpositions of truth values in Ω^N.

An objection against the notion of quantum logic is that Boolean algebra operations AND and OR do not preserve fermion number so that quantum jump sequences leading from the product state defined by operands to the state representing the result of operation are therefore not possible. One manner to circumvent the objection is to consider the sub-algebra spanned by fermion and anti-fermion pairs for given mode so that fermion number conservation is not a problem. The objection can be also circumvented for pairs of space-time sheets with opposite time orientations and thus opposite signs of energies for particles. One can construct the algebra in question as pairs of many fermion states consisting of positive energy fermion and negative energy anti-fermion so that all states have vanishing fermion number and logical operations become possible. Pairs of MEs with opposite time orientations are excellent candidates for carries of these fermion-anti-fermion pairs.

Quantum classical correspondence and quantum logic

The intuitive idea is that the global Boolean statements correspond to sections of Z^2 bundle. Möbius band is a prototype example here. The failure of a global statement would reduce to the non-existence of global section so that true would transforms to false as one goes around full 2π rotation.

One can ask whether fermionic quantum realization of Boolean logic could have space-time counterpart in terms of Z_2 fiber bundle structure. This would give some hopes of having some connection between category theoretical and fermionic realizations of logic. The following argument stimulated by email discussion with Diego Lucio Rapoport suggests that this might be the case.

1. The hierarchy of Planck constants realized using the notion of generalized imbedding space involves only groups $Z_{n_a} \times Z_{n_b}$, $n_a, n_b \neq 2$ if one takes Jones inclusions as starting point. There is however no obvious reason for excluding the values $n_a = 2$ and $n_b = 2$ and the question concerns physical interpretation. Even if one allows only $n_i \geq 3$ one can ask for the physical interpretation for the factorization $Z_{2n} = Z_2 \times Z_n$. Could it perhaps relate to a space-time correlates for Boolean two-valuedness?

2. An important implication of fiber bundle structure is that the partonic 2-surfaces have $Z_{n_a} \times Z_{n_b} = Z_{n_a n_b}$ as a group of conformal symmetries. I have proposed that n_a or n_b is even for fermions so that Z_2 acts as a conformal symmetry of the partonic 2-surface. Both n_a and n_b would be odd for truly elementary bosons. Note that this hypothesis makes sense also for $n_i \geq 3$.

3. Z_2 conformal symmetry for fermions would imply that all partonic 2-surfaces associated with fermions are hyper-elliptic. As a consequence elementary particle vacuum functionals defined in modular degrees of freedom would vanish for fermions for genus $g > 2$ so that only three fermion families would be possible in accordance with experimental facts. Since gauge bosons and Higgs correspond to pairs of partonic 2-surfaces (the throats of the wormhole contact) one has 9 gauge boson states labelled by the pairs (g_1, g_2) which can be grouped to SU(3) singlet and octet. Singlet corresponds to ordinary gauge bosons.

   Super-canonical bosons are truly elementary bosons in the sense that they do not consist of fermion-antifermion pairs. For them both n_a and n_b should be odd if the correspondence is taken seriously and all genera would be possible. The super-conformal partners of these bosons have the quantum numbers of right handed neutrino. Since both spin directions are possible, one can ask whether Boolean Z_2 must be present also now. This need not be the case, ν_R generates only super-symmetries and does not define a family of fermionic oscillator operators. The electro-weak spin of ν_R is frozen and it does not couple at all to electro-weak intersections. Perhaps (only) odd values of n_i are possible in this case.

4. If fermionic Boolean logic has a space-time correlate, one can wonder whether the fermionic Z_2 conformal symmetry might correspond to a space-time correlate for the Boolean true-false dichotomy. If the partonic 2-surface contains points which are fixed points of Z_2 symmetry, there exists no everywhere non-vanishing sections. Furthermore, induced spinor fields should vanish at the fixed points of Z_2 symmetry since they correspond to singular orbifold points so that one could not actually have a situation in which true and false are true simultaneously. Global sections could however fail to exist since CP_2 spinor bundle is non-trivial.

6.5.3 Category theory and the modelling of aesthetic and ethical judgements

Consciousness theory should allow to model model the logics of ethics and aesthetics. Evolution (representable as p-adic evolution in TGD framework) is regarded as something positive and is a good candidate for defining universal ethics in TGD framework. Good deeds are such that they support this evolution occurring in statistical sense in any case. Moral provides a practical model for what good deeds are and moral right-wrong statements are analogous to logical statements. Often however the two-valued right-wrong logic seems to be too simplistic in case of moral statements. Same applies to aesthetic judgements. A possible application of the generalized logics defined by the inherent structure of categories relates to the understanding of the dilemmas associated with the moral and aesthetic rules.

As already found, quantum versions of sieves provide a formal generalization of Boolean truth values as a characteristic of a given category. Generalized moral rules could perhaps be seen as sieve valued statements about deeds. Deeds are either right or wrong in what might be called Boolean moral code. One can also consider Zen moral in which some deeds can be said to be right and wrong simultaneously. Some deeds could also be such that there simply exists no globally consistent moral rule: this would correspond to the nonexistence of what is called global section assigning to each object of the category consisting of the pairs formed by a moral agents and given deed) a sieve simultaneously.

6.6 Platonism, Constructivism, and Quantum Platonism

During years I have been trying to understand how Category Theory and Set Theory relate to quantum TGD inspired view about fundamentals of mathematics and the outcome section is added to this chapter several years after its first writing. I hope that reader does not experience too unpleasant discontinuity. I managed to clarify my thoughts about what these theories are by reading the article Structuralism, Category Theory and Philosophy of Mathematics by Richard Stefanik [**?**]. Blog discussions and email correspondence with Sampo Vesterinen have been very stimulating and inspired the attempt to represent TGD based vision about the unification of mathematics, physics, and consciousness theory in a more systematic manner.

Before continuing I want to summarize the basic ideas behind TGD vision. One cannot understand mathematics without understanding mathematical consciousness. Mathematical consciousness and its evolution must have direct quantum physical correlates and by quantum classical correspondence these correlates must appear also at space-time level. Quantum physics must allow to realize number as a conscious experience analogous to a sensory quale. In TGD based ontology there is no need to postulate physical world behind the quantum states as mathematical entities (theory is the reality). Hence number cannot be any physical object, but can be identified as a quantum state or its label and its number theoretical anatomy is revealed by the conscious experiences induced by the number theoretic variants of particle reactions. Mathematical systems and their axiomatics are dynamical evolving systems and physics is number theoretically universal selecting rationals and their extensions in a special role as numbers, which can can be regarded elements of several number fields simultaneously.

6.6.1 Platonism and structuralism

There are basically two philosophies of mathematics.

1. Platonism assumes that mathematical objects and structures have independent existence. Natural numbers would be the most fundamental objects of this kind. For instance, each natural number has its own number-theoretical anatomy decomposing into a product of prime numbers defining the elementary particles of Platonia. For quantum physicist this vision is attractive, and even more so if one accepts that elementary particles are labelled by primes (as I do)! The problematic aspects of this vision relate to the physical realization of the Platonia. Neither Minkowski space-time nor its curved variants understood in the sense of set theory have no room for Platonia and physical laws (as we know them) do not seem to allow the realization of all imaginable internally consistent mathematical structures.

2. Structuralist believes that the properties of natural numbers result from their relations to other natural numbers so that it is not possible to speak about number theoretical anatomy in the Platonic sense. Numbers as such are structureless and their relationships to other numbers provide them with their apparent structure. According to [**?**] structuralism is however not enough for the purposes of number theory: in combinatorics it is much more natural to use intensional definition for integers by providing them with inherent properties such as decomposition into

primes. I am not competent to take any strong attitudes on this statement but my physicist's intuition tells that numbers have number theoretic anatomy and that this anatomy can be only revealed by the morphisms or something more general which must have physical counterparts. I would like to regard numbers are analogous to bound states of elementary particles. Just as the decays of bound states reveal their inner structure, the generalizations of morphisms would reveal to the mathematician the inherent number theoretic anatomy of integers.

6.6.2 Structuralism

Set theory and category theory represent two basic variants of structuralism and before continuing I want to clarify to myself the basic ideas of structuralism: the reader can skip this section if it looks too boring.

Set theory

Structuralism has many variants. In set theory [?] the elements of set are treated as structureless points and sets with the same cardinality are equivalent. In number theory additional structure must be introduced. In the case of natural numbers one introduces the notion of successor and induction axiom and defines the basic arithmetic operations using these. Set theoretic realization is not unique. For instance, one can start from empty set Φ identified as 0, identify 1 as $\{\Phi\}$, 2 as $\{0,1\}$ and so on. One can also identify 0 as Φ, 1 as $\{0\}$, 2 as $\{\{0\}\}$,.... For both physicist and consciousness theorist these formal definitions look rather weird.

The non-uniqueness of the identification of natural numbers as a set could be seen as a problem. The structuralist's approach is based on an extensional definition meaning that two objects are regarded as identical if one cannot find any property distinguishing them: object is a representative for the equivalence class of similar objects. This brings in mind gauge fixing to the mind of physicists.

Category theory

Category theory [?] represents a second form of structuralism. Category theorist does not worry about the ontological problems and dreams that all properties of objects could be reduced to the arrows and formally one could identify even objects as identity morphisms (looks like a trick to me). The great idea is that functors between categories respecting the structure defined by morphisms provide information about categories. Second basic concept is natural transformation which maps functors to functors in a structure preserving manner. Also functors define a category so that one can construct endless hierarchy of categories. This approach has enormous unifying power since functors and natural maps systemize the process of generalization. There is no doubt that category theory forms a huge piece of mathematics but I find difficult to believe that arrows can catch all of it.

The notion of category can be extended to that of n-category: in [?] I described a geometric realization of this hierarchy in which one defines 1-morphisms by parallel translations, 2-morphisms by parallel translations of parallel translations, and so on. In infinite-dimensional space this hierarchy would be infinite. Abstractions about abstractions about.., thoughts about thoughts about, statements about statements about..., is the basic idea behind this interpretation. Also the hierarchy of logics of various orders corresponds to this hierarchy. This encourages to see category theoretic thinking as being analogous to higher level self reflection which must be distinguished from the direct sensory experience.

In the case of natural numbers category theoretician would identify successor function as the arrow binding natural numbers to an infinitely long string with 0 as its end. If this approach would work, the properties of numbers would reflect the properties of the successor function.

6.6.3 The view about mathematics inspired by TGD and TGD inspired theory of consciousness

TGD based view might be called quantum Platonism. It is inspired by the requirement that both quantum states and quantum jumps between them are able to represent number theory and that all quantum notions have also space-time correlates so that Platonia should in some sense exist also at the level of space-time. Here I provide a brief summary of this view as it is now. The articles "TGD"

[?] and "TGD inspired theory of consciousness" [?] provide an overview about TGD and TGD inspired theory of consciousness.

Physics is fixed from the uniqueness of infinite-D existence and number theoretic universality

1. The basic philosophy of quantum TGD relies on the geometrization of physics in terms of infinite-dimensional Kähler geometry of the "world of classical worlds" (configuration space), whose uniqueness is forced by the mere mathematical existence. Space-time dimension and imbedding space $H = M^4 \times CP_2$ are fixed among other things by this condition and allow interpretation in terms of classical number fields. Physical states correspond to configuration space spinor fields with configuration space spinors having interpretation as Fock states. Rather remarkably, configuration space Clifford algebra defines standard representation of so called hyper finite factor of II_1, perhaps the most fascinating von Neumann algebra.

2. Number theoretic universality states that all number fields are in a democratic position. This vision can be realized by requiring generalization of notions of imbedding space by gluing together real and p-adic variants of imbedding space along common algebraic numbers. All algebraic extensions of p-adic numbers are allowed. Real and p-adic space-time sheets intersect along common algebraics. The identification of the p-adic space-time sheets as correlates of cognition and intentionality explains why cognitive representations at space-time level are always discrete. Only space-time points belonging to an algebraic extension of rationals associated contribute to the data defining S-matrix. These points define what I call number theoretic braids. The interpretation in of algebraic discreteness terms of a physical realization of axiom of choice is highly suggestive. The axiom of choice would be dynamical and evolving quantum jump by quantum jump as the algebraic complexity of quantum states increases.

Holy trinity of existence

In TGD framework one would have 3-levelled ontology numbers should have representations at all these levels [?].

1. Subjective existence as a sequence of quantum jumps giving conscious sensory representations for numbers and various geometric structures would be the first level.

2. Quantum states would correspond to Platonia of mathematical ideas and mathematician- or if one is unwilling to use this practical illusion- conscious experiences about mathematic ideas, would be in quantum jumps. The quantum jumps between quantum states respecting the symmetries characterizing the mathematical structure would provide conscious information about the mathematical ideas not directly accessible to conscious experience. Mathematician would live in Plato's cave. There is no need to assume any independent physical reality behind quantum states as mathematical entities since quantum jumps between these states give rise to conscious experience. Theory-reality dualism disappears since the theory is reality or more poetically: painting is the landscape.

3. The third level of ontology would be represented by classical physics at the space-time level essential for quantum measurement theory. By quantum classical correspondence space-time physics would be like a written language providing symbolic representations for both quantum states and changes of them (by the failure of complete classical determinism of the fundamental variational principle). This would involve both real and p-adic space-time sheets corresponding to sensory and cognitive representations of mathematical concepts. This representation makes possible the feedback analogous to formulas written by mathematician crucial for the ability of becoming conscious about what one was conscious of and the dynamical character of this process allows to explain the self-referentiality of consciousness without paradox.

This ontology releases a deep Platonistic sigh of relief. Since there are no physical objects, there is no need to reduce mathematical notions to objects of the physical world. There are only quantum states identified as mathematical entities labelled naturally by integer valued quantum numbers; conscious experiences, which must represent sensations giving information about the number theoretical anatomy of a given quantum number; and space-time surfaces providing space-time correlates for quantum physics and therefore also for number theory and mathematical structures in general.

Factorization of integers as a direct sensory perception?

Both physicist and consciousness theorist would argue that the set theoretic construction of natural numbers could not be farther away from how we experience integers. Personally I feel that neither structuralist's approach nor Platonism as it is understood usually are enough. Mathematics is a conscious activity and this suggests that quantum theory of consciousness must be included if one wants to build more satisfactory view about fundamentals of mathematics.

Oliver Sack's book *The man who mistook his wife for a hat* [?] (see also [?]) contains fascinating stories about those aspects of brain and consciousness which are more or less mysterious from the view point of neuroscience. Sacks tells in his book also a story about twins who were classified as idiots but had amazing number theoretical abilities. I feel that this story reveals something very important about the real character of mathematical consciousness.

The twins had absolutely no idea about mathematical concepts such as the notion of primeness but they could factorize huge numbers and tell whether they are primes. Their eyes rolled wildly during the process and suddenly their face started to glow of happiness and they reported a discovery of a factor. One could not avoid the feeling that they quite concretely saw the factorization process. The failure to detect the factorization served for them as the definition of primeness. For them the factorization was not a process based on some rules but a direct sensory perception.

The simplest explanation for the abilities of twins would in terms of a model of integers represented as string like structures consisting of identical basic units. This string can decay to strings. If string containing n units decaying into $m > 1$ identical pieces is not perceived, the conclusion is that a prime is in question. It could also be that decay to units smaller than 2 was forbidden in this dynamics. The necessary connection between written representations of numbers and representative strings is easy to build as associations.

This kind theory might help to understand marvellous feats of mathematicians like Ramanujan who represents a diametrical opposite of Groethendienck as a mathematician (when Groethendienck was asked to give an example about prime, he mentioned 57 which became known as Groethendienck prime!).

The lesson would be that one very fundamental representation of integers would be, not as objects, but conscious experiences. Primeness would be like the quale of redness. This of course does not exclude also other representations.

Experience of integers in TGD inspired quantum theory of consciousness

In quantum physics integers appear very naturally as quantum numbers. In quantal axiomatization or interpretation of mathematics same should hold true.

1. In TGD inspired theory of consciousness [?] quantum jump is identified as a moment of consciousness. There is actually an entire fractal hierarchy of quantum jumps consisting of quantum jumps and this correlates directly with the corresponding hierarchy of physical states and dark matter hierarchy. This means that the experience of integer should be reducible to a certain kind of quantum jump. The possible changes of state in the quantum jump would characterize the sensory representation of integer.

2. The quantum state as such does not give conscious information about the number theoretic anatomy of the integer labelling it: the change of the quantum state is required. The above geometric model translated to quantum case would suggest that integer represents a multiplicatively conserved quantum number. Decays of this this state into states labelled by integers n_i such that one has $n = \prod_i n_i$ would provide the fundamental conscious representation for the number theoretic anatomy of the integer. At the level of sensory perception based the space-time correlates a string-like bound state of basic particles representing n=1.

3. This picture is consistent with the Platonist view about integers represented as structured objects, now labels of quantum states. It would also conform with the view of category theorist in the sense that the arrows of category theorist replaced with quantum jumps are necessary to gain conscious information about the structure of the integer.

Infinite primes and arithmetic consciousness

Infinite primes [?] were the first mathematical fruit of TGD inspired theory of consciousness and the inspiration for writing this posting came from the observation that the infinite primes at the lowest level of hierarchy provide a representation of algebraic numbers as Fock states of a super-symmetric arithmetic QFT so that it becomes possible to realize quantum jumps revealing the number theoretic anatomy of integers, rationals, and perhaps even that of algebraic numbers.

1. Infinite primes have a representation as Fock states of super-symmetric arithmetic QFT and at the lowest level of hierarchy they provide representations for primes, integers, rationals and algebraic numbers in the sense that at the lowest level of hierarchy of second quantizations the simplest infinite primes are naturally mapped to rationals whereas more complex infinite primes having interpretation as bound states can be mapped to algebraic numbers. Conscious experience of number can be assigned to the quantum jumps between these quantum states revealing information about the number theoretic anatomy of the number represented. It would be wrong to say that rationals only label these states: rather, these states represent rationals and since primes label the particles of these states.

2. More concretely, the conservation of number theoretic energy defined by the logarithm of the rational assignable with the Fock state implies that the allowed decays of the state to a product of infinite integers are such that the rational can decompose only into a product of rationals. These decays could provide for the above discussed fundamental realization of multiplicative aspects of arithmetic consciousness. Also additive aspects are represented since the exponents k in the powers p^k appearing in the decomposition are conserved so that only the partitions $k = \sum_i k_i$ are representable. Thus both product decompositions and partitions, the basic operations of number theorist, are represented.

3. The higher levels of the hierarchy represent a hierarchy of abstractions about abstractions bringing strongly in mind the hierarchy of n-categories and various similar constructions including n:th order logic. It also seems that the n+1:th level of hierarchy provides a quantum representation for the n:th level. Ordinary primes, integers, rationals, and algebraic numbers would be the lowest level, -the initial object- of the hierarchy representing nothing at low level. Higher levels could be reduced to them by the analog of category theoretic reductionism in the sense that there is arrow between n:th and n+1:th level representing the second quantization at this level. On can also say that these levels represent higher reflective level of mathematical consciousness and the fundamental sensory perception corresponds the lowest level.

4. Infinite primes have also space-time correlates. The decomposition of particle into partons can be interpreted as a infinite prime and this gives geometric representations of infinite primes and also rationals. The finite primes appearing in the decomposition of infinite prime correspond to bosonic or fermionic partonic 2-surfaces. Many-sheeted space-time provides a representation for the hierarchy of second quantizations: one physical prediction is that many particle bound state associated with space-time sheet behaves exactly like a boson or fermion. Nuclear string model is one concrete application of this idea: it replaces nucleon reductionism with reductionism occurs first to strings consisting of $A \leq 4$ nuclei and which in turn are strings consisting of nucleons. A further more speculative representation of infinite rationals as space-time surfaces is based on their mapping to rational functions.

Number theoretic Brahman=Atman identity

The notion of infinite primes leads to the notion of algebraic holography in which space-time points possess infinitely rich number-theoretic anatomy. This anatomy would be due to the existence of infinite number of real units defined as ratios of infinite integers which reduce to unit in the real sense and various p-adic senses. This anatomy is not visible in real physics but can contribute directly to mathematical consciousness [?].

The anatomies of single space-time point could represent the entire world of classical worlds and quantum states of universe: the number theoretic anatomy is of course not visible in the structure of these these states. Therefore the basic building brick of mathematics - point- would become the Platonia able to represent all of the mathematics consistent with the laws of quantum physics. Space-time points would evolve, becoming more and more complex quantum jump by quantum jump. Configuration space and quantum states would be represented by the anatomies of space-time points. Some space-time points are more "civilized" than others so that space-time decomposes into "civilizations" at different levels of mathematical evolution.

Paths between space-time points represent processes analogous to parallel translations affecting the structure of the point and one can also define n-parallel translations up to $n = 4$ at level of space-time and $n = 8$ at level of imbedding space. At level of world of classical worlds whose points are representable as number theoretical anatomies arbitrary high values of n can be realized.

It is fair to say that the number theoretical anatomy of the space-time point makes it possible self-reference loop to close so that structured points are able to represent the physics of associated with with the structures constructed from structureless points. Hence one can speak about algebraic holography or number theoretic Brahman=Atman identity.

Finite measurement resolution, Jones inclusions, and number theoretic braids

In the history of physics and mathematics the realization of various limitations have been the royal road to a deeper understanding (Uncertainty Principle, Gödel's theorem). The precision of quantum measurement, sensory perception, and cognition are always finite. In standard quantum measurement theory this limitation is not taken into account but forms a corner stone of TGD based vision about quantum physics and of mathematics too as I want to argue in the following.

The finite resolutions has representation both at classical and quantum level.

1. At the level of quantum states finite resolution is represented in terms of Jones inclusions N subset M of hyper-finite factors of type II_1 (HFFs)[**?**]. N represents measurement resolution in the sense that the states related by the action of N cannot be distinguished in the measurement considered. Complex rays are replaced by N rays. This brings in noncommutativity via quantum groups [**?**]. Non-commutativity in TGD Universe would be therefore due to a finite measurement resolution rather than something exotic emerging in the Planck length scale. Same applies to p-adic physics: p-adic space-time sheets have literally infinite size in real topology!

2. At the space-time level discretization implied by the number theoretic universality could be seen as being due to the finite resolution with common algebraic points of real and p-adic variant of the partonic 3-surface chosen as representatives for regions of the surface. The solutions of modified Dirac equation are characterized by the prime in question so that the preferred prime makes itself visible at the level of quantum dynamics and characterizes the p-adic length scale fixing the values of coupling constants. Discretization could be also understood as effective non-commutativity of imbedding space points due to the finite resolution implying that second quantized spinor fields anticommute only at a discrete set of points rather than along stringy curve.

In this framework it is easy to imagine physical representations of number theoretical and other mathematical structures.

1. Every compact group corresponds to a hierarchy of Jones inclusions corresponding to various representations for the quantum variants of the group labelled by roots of unity. I would be surprised if non-compact groups would not allow similar representation since HFF can be regarded as infinite tensor power of n-dimensional complex matrix algebra for any value of n. Somewhat paradoxically, the finite measurement resolution would make possible to represent Lie group theory physically [**?**].

2. There is a strong temptation to identify the Galois groups of algebraic numbers as the infinite permutation group S_∞ consisting of permutations of finite number of objects, whose projective representations give rise to an infinite braid group B_∞. The group algebras of these groups are HFFs besides the representation provided by the spinors of the world of classical worlds having physical identification as fermionic Fock states. Therefore physical states would provide a direct representation also for the more abstract features of number theory [**?**].

3. Number theoretical braids crucial for the construction of S-matrix provide naturally representations for the Galois groups G associated with the algebraic extensions of rationals as diagonal

imbeddings $G\times G\times ....$ to the completion of S_∞ representable also as the action on the completion of spinors in the world of classical worlds so that the core of number theory would be represented physically [?]. At the space-time level number theoretic braid having G as symmetries would represent the G. These representations are analogous to global gauge transformations. The elements of S_∞ are analogous to local gauge transformations having a natural identification as a universal number theoretical gauge symmetry group leaving physical states invariant.

Hierarchy of Planck constants and the generalization of imbedding space

Jones inclusions inspire a further generalization of the notion of imbedding space obtained by gluing together copies of the imbedding space H regarded as coverings $H \rightarrow H/G_a \times G_b$. In the simplest scenario $G_a \times G_b$ leaves invariant the choice of quantization axis and thus this hierarchy provides imbedding space correlate for the choice of quantization axes inducing these correlates also at space-time level and at the level of world of classical worlds [?].

Dark matter hierarchy is identified in terms of different sectors of H glued together along common points of base spaces and thus forming a book like structure. For the simplest option elementary particles proper correspond to maximally quantum critical systems in the intersection of all pages. The field bodies of elementary particles are in the interiors of the pages of this "book".

One can assign to Jones inclusions quantum phase $q = exp(i2\pi/n)$ and the groups Z_n acts as exact symmetries both at level of M^4 and CP_2. In the case of M^4 this means that space-time sheets have exact Z_n rotational symmetry. This suggests that the algebraic numbers q^m could have geometric representation at the level of sensory perception as Z_n symmetric objects. We need not be conscious of this representation in the ordinary wake-up consciousness dominated by sensory perception of ordinary matter with $q = 1$. This would make possible the idea about transcendentals like π, which do not appear in any finite-dimensional extension of even p-adic numbers (p-adic numbers allow finite-dimensional extension by since e^p is ordinary p-adic number). Quantum jumps in which state suffers an action of the generating element of Z_n could also provide a sensory realization of these groups and numbers $exp(i2\pi/n)$.

Planck constant is identified as the ratio n_a/n_b of integers associated with M^4 and CP_2 degrees of freedom so that a representation of rationals emerge again. The so called ruler and compass rationals whose definition involves only a repeated square root operation applied on rationals are cognitively the simplest ones and should appear first in the evolution of mathematical consciousness. The successful [?] quantum model for EEG is only one of the applications providing support for their preferred role. Other applications are to Bohr quantization of planetary orbits interpreted as being induced by the presence of macroscopically quantum coherent dark matter [?].

6.6.4 Farey sequences, Riemann hypothesis, tangles, and TGD

Farey sequences allow an alternative formulation of Riemann Hypothesis and subsequent pairs in Farey sequence characterize so called rational 2-tangles. In TGD framework Farey sequences relate very closely to dark matter hierarchy, which inspires *"Platonia as the best possible world in the sense that cognitive representations are optimal"* as the basic variational principle of mathematics. This variational principle supports RH.

Possible TGD realizations of tangles, which are considerably more general objects than braids, are considered. One can assign to a given rational tangle a rational number a/b and the tangles labelled by a/b and c/d are equivalent if $ad - bc = \pm 1$ holds true. This means that the rationals in question are neighboring members of Farey sequence. Very light-hearted guesses about possible generalization of these invariants to the case of general N-tangles are made.

Farey sequences

Some basic facts about Farey sequences [?] demonstrate that they are very interesting also from TGD point of view.

1. Farey sequence F_N is defined as the set of rationals $0 \leq q = m/n \leq 1$ satisfying the conditions $n \leq N$ ordered in an increasing sequence.

2. Two subsequent terms a/b and c/d in F_N satisfy the condition $ad - bc = 1$ and thus define and element of the modular group $SL(2, Z)$.

3. The number $|F(N)|$ of terms in Farey sequence is given by

$$|F(N)| \quad = \quad |F(N-1)| + \phi(N-1) \ . \tag{6.6.1}$$

Here $\phi(n)$ is Euler's totient function giving the number of divisors of n. For primes one has $\phi(p) = 1$ so that in the transition from p to $p+1$ the length of Farey sequence increases by one unit by the addition of $q = 1/(p+1)$ to the sequence.

The members of Farey sequence F_N are in one-one correspondence with the set of quantum phases $q_n = exp(i2\pi/n)$, $0 \leq n \leq N$. This suggests a close connection with the hierarchy of Jones inclusions, quantum groups, and in TGD context with quantum measurement theory with finite measurement resolution and the hierarchy of Planck constants involving the generalization of the imbedding space. Also the recent TGD inspired ideas about the hierarchy of subgroups of the rational modular group with subgroups labelled by integers N and in direct correspondence with the hierarchy of quantum critical phases [**?**] would naturally relate to the Farey sequence.

Riemann Hypothesis and Farey sequences

Farey sequences are used in two equivalent formulations of the Riemann hypothesis. Suppose the terms of F_N are $a_{n,N}$, $0 < n \leq |F_N|$. Define

$$d_{n,N} = a_{n,N} - \frac{n}{|F_N|} \ .$$

In other words, $d_{n,N}$ is the difference between the n:th term of the N:th Farey sequence, and the n:th member of a set of the same number of points, distributed evenly on the unit interval. Franel and Landau proved that both of the following statements

$$\begin{aligned} \sum_{n=1,\ldots,|F_N|} |d_{n,N}| = O(N^r) \ \text{ for any } \ r > 1/2 \ , \\ \sum_{n=1,\ldots,|F_N|} d_{n,N}^2 = O(N^r) \ \text{ for any } \ r > 1 \ . \end{aligned} \tag{6.6.2}$$

are equivalent with Riemann hypothesis.

One could say that RH would guarantee that the numbers of Farey sequence provide the best possible approximate representation for the evenly distributed rational numbers $n/|F_N|$.

Farey sequences and TGD

Farey sequences seem to relate very closely to TGD.

1. The rationals in the Farey sequence can be mapped to the roots of unity by the map $q \rightarrow exp(i2\pi q)$. The numbers $1/|F_N|$ are in turn mapped to the numbers $exp(i2\pi/|F_N|)$, which are also roots of unity. The statement would be that the algebraic phases defined by Farey sequence give the best possible approximate representation for the phases $exp(in2\pi/|F_N|)$ with evenly distributed phase angle.

2. In TGD framework the phase factors defined by F_N corresponds to the set of quantum phases corresponding to Jones inclusions labelled by $q = exp(i2\pi/n)$, $n \leq N$, and thus to the N lowest levels of dark matter hierarchy. There are actually two hierarchies corresponding to M^4 and CP_2 degrees of freedom and the Planck constant appearing in Schrödinger equation corresponds to the ratio n_a/n_b defining quantum phases in these degrees of freedom. $Z_{n_a \times n_b}$ appears as a conformal symmetry of "dark" partonic 2-surfaces and with very general assumptions this implies that there are only in TGD Universe [**?**, **?**].

3. The fusion of physics associated with various number fields to single coherent whole requires algebraic universality. In particular, the roots of unity, which are complex algebraic numbers, should define approximations to continuum of phase factors. At least the S-matrix associated with p-adic-to-real transitions and more generally $p_1 \rightarrow p_2$ transitions between states for which the partonic space-time sheets are p_1- resp. p_2-adic can involve only this kind of algebraic phases. One can also say that cognitive representations can involve only algebraic phases and algebraic numbers in general. For real-to-real transitions and real-to-padic transitions U-matrix might be non-algebraic or obtained by analytic continuation of algebraic U-matrix. S-matrix is by definition diagonal with respect to number field and similar continuation principle might apply also in this case.

4. The subgroups of the hierarchy of subgroups of the modular group with rational matrix elements are labelled by integer N and relate naturally to the hierarchy of Farey sequences. The hierarchy of quantum critical phases is labelled by integers N with quantum phase transitions occurring only between phases for which the smaller integer divides the larger one [?].

Interpretation of RH in TGD framework

Number theoretic universality of physics suggests an interpretation for the Riemann hypothesis in TGD framework. RH would be equivalent to the statement that the Farey numbers provide best possible approximation to the set of rationals $k/|F_N|$ or to the statement that the roots of unity contained by F_N define the best possible approximation for the roots of unity defined as $exp(ik2\pi/|F_N|)$ with evenly spaced phase angles. The roots of unity allowed by the lowest N levels of the dark matter hierarchy allows the best possible approximate representation for algebraic phases represented exactly at $|F_N|$:th level of hierarchy.

A stronger statement would be that the Platonia, where RH holds true would be the best possible world in the sense that algebraic physics behind the cognitive representations would allow the best possible approximation hierarchy for the continuum physics (both for numbers in unit interval and for phases on unit circle). Platonia with RH would be cognitive paradise.

One could see this also from different view point. "Platonia as the cognitively best possible world" could be taken as the "axiom of all axioms": a kind of fundamental variational principle of mathematics. Among other things it would allow to conclude that RH is true: RH must hold true either as a theorem following from some axiomatics or as an axiom in itself.

Could rational N-tangles exist in some sense?

The article of Kauffman and Lambropoulou [?] about rational 2-tangles having commutative sum and product allowing to map them to rationals is very interesting from TGD point of view. The illustrations of the article are beautiful and make it easy to get the gist of various ideas. The theorem of the article states that equivalent rational tangles giving trivial tangle in the product correspond to subsequent Farey numbers a/b and c/d satisfying $ad - bc = \pm 1$ so that the pair defines element of the modular group SL(2,Z).

1. Rational 2-tangles

1. The basic observation is that 2-tangles are 2-tangles in both "s- and t-channels". Product and sum can be defined for all tangles but only in the case of 2-tangles the sum, which in this case reduces to product in t-channel obtained by putting tangles in series, gives 2-tangle. The so called rational tangles are 2-tangles constructible by using addition of $\pm[1]$ on left or right of tangle and multiplication by $\pm[1]$ on top or bottom. Product and sum are commutative for rational 2-tangles but the outcome is not a rational 2-tangle in the general case. One can also assign to rational 2-tangle its negative and inverse. One can map 2-tangle to a number which is rational for rational tangles. The tangles $[0]$, $[\infty]$, $\pm[1]$, $\pm 1/[1]$, $\pm[2]$, $\pm[1/2]$ define so called elementary rational 2-tangles.

2. In the general case the sum of $M-$ and $N-$tangles is $M + N - 2$-tangle and combines various $N-$tangles to a monoidal structure. Tensor product like operation giving $M + N$-tangle looks to me physically more natural than the sum.

3. The reason why general 2-tangles are non-commutative although 2-braids obviously commute is that 2-tangles can be regarded as sequences of $N-$tangles with 2-tangles appearing only as the initial and final state: N is actually even for intermediate states. Since $N > 2$-braid groups are non-commutative, non-commutativity results. It would be interesting to know whether braid group representations have been used to construct representations of $N-$tangles.

2. Does generalization to $N >> 2$ case exist?

One can wonder whether the notion of rational tangle and the basic result of the article about equivalence of tangles might somehow generalize to the $N > 2$ case.

1. Could the commutativity of tangle product allow to characterize the $N > 2$ generalizations of rational 2-tangles. The commutativity of product would be a space-time correlate for the commutativity of the S-matrices defining time like entanglement between the initial and final quantum states assignable to the N-tangle. For 2-tangles commutativity of the sum would have an analogous interpretation. Sum is not a very natural operation for N-tangles for $N >$ 2. Commutativity means that the representation matrices defined as products of braid group actions associated with the various intermediate states and acting in the same representation space commute. Only in very special cases one can expect commutativity for tangles since commutativity is lost already for braids.

2. The representations of 2-tangles should involve the subgroups of N-braid groups of intermediate braids identifiable as Galois groups of N:th order polynomials in the realization as number theoretic tangles. Could non-commutative 2-tangles be characterized by algebraic numbers in the extensions to which the Galois groups are associated? Could the non-commutativity reflect directly the non-commutativity of Galois groups involved? Quite generally one can ask whether the invariants should be expressible using algebraic numbers in the extensions of rationals associated with the intermediate braids.

3. Rational 2-tangles can be characterized by a rational number obtained by a projective identification $[a, b]^T \rightarrow a/b$ from a rational 2-spinor $[a, b]^T$ to which SL(2(N-1),Z) acts. Equivalence means that the columns $[a, b]^T$ and $[c, d]^T$ combine to form element of SL(2,Z) and thus defining a modular transformation. Could more general 2-tangles have a similar representation but in terms of algebraic integers?

4. Could N-tangles be characterized by $N-1$ $2(N-1)$-component projective column-spinors $[a_i^1, a_i^2, .., a_i^{2(N-1)}]^T$, $i = 1, ...N-1$ so that only the ratios $a_i^k/a_i^{2(N-1)} \leq 1$ matter? Could equivalence for them mean that the $N-1$ spinors combine to form $N-1+N-1$ columns of $SL(2(N-1), Z)$ matrix. Could N-tangles quite generally correspond to collections of projective $N-1$ spinors having as components algebraic integers and could $ad-bc = \pm 1$ criterion generalize? Note that the modular group for surfaces of genus g is SL(2g,Z) so that $N-1$ would be analogous to g and $1 \leq N \geq 3$- braids would correspond to $g \leq 2$ Riemann surfaces.

5. Dark matter hierarchy leads naturally to a hierarchy of modular sub-groups of $SL(2, Q)$ labelled by N (the generator $\tau \rightarrow \tau + 2$ of modular group is replaced with $\tau \rightarrow \tau + 2/N$). What might be the role of these subgroups and corresponding subgroups of $SL(2(N-1), Q)$. Could they arise in "anyonization" when one considers quantum group representations of 2-tangles with twist operation represented by an N:th root of unity instead of phase U satisfying $U^2 = 1$?

How tangles could be realized in TGD Universe?

The article of Kauffman and Lambropoulou stimulated the question in what senses N-tangles could be be realized in TGD Universe as fundamental structures.

1. Tangles as number theoretic braids?

The strands of number theoretical $N-$braids correspond to roots of N:th order polynomial and if one allows time evolutions of partonic 2-surface leading to the disappearance or appearance of real roots $N-$tangles become possible. This however means continuous evolution of roots so that the coefficients of polynomials defining the partonic 2-surface can be rational only in initial and final state but not in all intermediate "virtual" states.

2. Tangles as tangled partonic 2-surfaces?

Tangles could appear in TGD also in second manner.

1. Partonic 2-surfaces are sub-manifolds of a 3-D section of space-time surface. If partonic 2-surfaces have genus $g > 0$ the handles can become knotted and linked and one obtains besides ordinary knots and links more general knots and links in which circle is replaced by figure eight and its generalizations obtained by adding more circles (eyeglasses for $N-$eyed creatures).

2. Since these 2-surfaces are space-like, the resulting structures are indeed tangles rather than only braids. Tangles made of strands with fixed ends would result by allowing spherical partons elongate to long strands with fixed ends. DNA tangles would the basic example, and are discussed also in the article. DNA sequences to which I have speculatively assigned invisible (dark) braid structures might be seen in this context as space-like "written language representations" of genetic programs represented as number theoretic braids.

6.7 Quantum Quandaries

John Baez's [?] discusses in a physicist friendly manner the possible application of category theory to physics. The lessons obtained from the construction of topological quantum field theories (TQFTs) suggest that category theoretical thinking might be very useful in attempts to construct theories of quantum gravitation.

The point is that the Hilbert spaces associated with the initial and final state n-1-manifold of n-cobordism indeed form in a natural manner category. Morphisms of Hilb in turn are unitary or possibly more general maps between Hilbert spaces. TQFT itself is a functor assigning to a cobordism the counterpart of S-matrix between the Hilbert spaces associated with the initial and final n-1-manifold. The surprising result is that for $n \leq 4$ the S-matrix can be unitary S-matrix only if the cobordism is trivial. This should lead even string theorist to raise some worried questions.

In the hope of feeding some category theoretic thinking into my spine, I briefly summarize some of the category theoretical ideas discussed in the article and relate it to the TGD vision, and after that discuss the worried questions from TGD perspective. That space-time makes sense only relative to imbedding space would conform with category theoretic thinking.

6.7.1 The *-category of Hilbert spaces

Baez considers first the category of Hilbert spaces. Intuitively the definition of this category looks obvious: take linear spaces as objects in category Set, introduce inner product as additional structure and identify morphisms as maps preserving this inner product. In finite-D case the category with inner product is however identical to the linear category so that the inner product does not seem to be absolutely essential. Baez argues that in infinite-D case the morphisms need not be restricted to unitary transformations: one can consider also bounded linear operators as morphisms since they play key role in quantum theory (consider only observables as Hermitian operators). For hyper-finite factors of type II_1 inclusions define very important morphisms which are not unitary transformations but very similar to them. This challenges the belief about the fundamental role of unitarity and raises the question about how to weaken the unitarity condition without losing everything.

The existence of the inner product is essential only for the metric topology of the Hilbert space. Can one do without inner product as an inherent property of state space and reduce it to a morphism? One can indeed express inner product in terms of morphisms from complex numbers to Hilbert space and their conjugates. For any state Ψ of Hilbert space there is a unique morphisms T_Ψ from C to Hilbert space satisfying $T_\Psi(1) = \Psi$. If one assumes that these morphisms have conjugates T^*_Ψ mapping Hilbert space to C, inner products can be defined as morphisms $T^*_\Phi T_\Psi$. The Hermitian conjugates of operators can be defined with respect to this inner product so that one obtains *-category. Reader has probably realized that T_Ψ and its conjugate correspond to ket and bra in Dirac's formalism.

Note that in TGD framework based on hyper-finite factors of type II_1 (HFFs) the inclusions of complex rays might be replaced with inclusions of HFFs with included factor representing the finite measurement resolution. Note also the analogy of inner product with the representation of space-times as 4-surfaces of the imbedding space in TGD.

6.7.2 The monoidal *-category of Hilbert spaces and its counterpart at the level of nCob

One can give the category of Hilbert spaces a structure of monoid by introducing explicitly the tensor products of Hilbert spaces. The interpretation is obvious for physicist. Baez describes the details of this identification, which are far from trivial and in the theory of quantum groups very interesting things happen. A non-commutative quantum version of the tensor product implying braiding is possible and associativity condition leads to the celebrated Yang-Baxter equations: inclusions of HFFs lead to quantum groups [?] too.

At the level of nCob the counterpart of the tensor product is disjoint union of n-1-manifolds. This unavoidably creates the feeling of cosmic loneliness. Am I really a disjoint 3-surface in emptiness which is not vacuum even in the geometric sense? Cannot be true!

This horrifying sensation disappears if n-1-manifolds are n-1-surfaces in some higher-dimensional imbedding space so that there would be at least something between them. I can emit a little baby manifold moving somewhere perhaps being received by some-one somewhere and I can receive radiation from some-one at some distance and in some direction as small baby manifolds making gentle tosses on my face!

This consoling feeling could be seen as one of the deep justifications for identifying fundamental objects as light-like partonic 3-surfaces in TGD framework. Their ends correspond to 2-D partonic surfaces at the boundaries of future or past directed light-cones (states of positive and negative energy respectively) and are indeed disjoint but not in the desperately existential sense as 3-geometries of General Relativity.

This disjointness has also positive aspect in TGD framework. One can identify the color degrees of freedom of partons as those associated with CP_2 degrees of freedom. For instance, SU(3) analogs for rotational states of rigid body become possible. 4-D space-time surfaces as preferred extremals of Kähler action connect the partonic 3-surfaces and bring in classical representation of correlations and thus of interactions. The representation as sub-manifolds makes it also possible to speak about positions of these sub-Universes and about distances between them. The habitants of TGD Universe are maximally free but not completely alone.

6.7.3 TQFT as a functor

The category theoretic formulation of TQFT relies on a very elegant and general idea. Quantum transition has as a space-time correlate an n-dimensional surface having initial final states as its n-1-dimensional ends. One assigns Hilbert spaces of states to the ends and S-matrix would be a unitary morphism between the ends. This is expressed in terms of the category theoretic language by introducing the category nCob with objects identified as n-1-manifolds and morphisms as cobordisms and *-category Hilb consisting of Hilbert spaces with inner product and morphisms which are bounded linear operators which do not however preserve the unitarity. Note that the morphisms of nCob cannot anymore be identified as maps between n-1-manifolds interpreted as sets with additional structure so that in this case category theory is more powerful than set theory.

TQFT is identified as a functor nCob $\rightarrow$ Hilb assigning to n-1-manifolds Hilbert spaces, and to cobordisms unitary S-matrices in the category Hilb. This looks nice but the surprise is that for $n \leq 4$ unitary S-matrix exists only if the cobordism is trivial so that topology changing transitions are not possible unless one gives up unitarity.

This raises several worried questions.

1. Does this result mean that in TQFT sense unitary S-matrix for topology changing transitions from a state containing n_i closed strings to a state containing $n_f \neq n_i$ strings does not exist? Could the situation be same also for more general non-topological stringy S-matrices? Could the non-converging perturbation series for S-matrix with finite individual terms matrix fail to no non-perturbative counterpart? Could it be that M-theory is doomed to remain a dream with no hope of being fulfilled?

2. Should one give up the unitarity condition and require that the theory predicts only the relative probabilities of transitions rather than absolute rates? What the proper generalization of the S-matrix could be?

3. What is the relevance of this result for quantum TGD?

6.7.4 The situation is in TGD framework

The result about the non-existence of unitary S-matrix for topology changing cobordisms allows new insights about the meaning of the departures of TGD from string models.

Cobordism cannot give interesting selection rules

When I started to work with TGD for more than 28 years ago, one of the first ideas was that one could identify the selection rules of quantum transitions as topological selection rules for cobordisms. Within week or two came the great disappointment: there were practically no selection rules. Could one revive this naive idea? Could the existence of unitary S-matrix force the topological selection rules after all? I am skeptic. If I have understood correctly the discussion of what happens in 4-D case [?] only the exotic diffeo-structures modify the situation in 4-D case.

Light-like 3-surfaces allow cobordism

In the physically interesting GRT like situation one would expect the cobordism to be mediated by a space-time surface possessing Lorentz signature. This brings in metric and temporal distance. This means complications since one must leave the pure TQFT context. Also the classical dynamics of quantum gravitation brings in strong selection rules related to the dynamics in metric degrees of freedom so that TQFT approach is not expected to be useful from the point of view of quantum gravity and certainly not the limit of a realistic theory of quantum gravitation.

In TGD framework situation is different. 4-D space-time sheets can have Euclidian signature of the induced metric so that Lorentz signature does not pose conditions. The counterparts of cobordisms correspond at fundamental level to light-like 3-surfaces, which are arbitrarily except for the light-likeness condition (the effective 2-dimensionality implies generalized conformal invariance and analogy with 3-D black-holes since 3-D vacuum Einstein equations are satisfied). Field equations defined by the Chern-Simons action imply that CP_2 projection is at most 2-D but this condition holds true only for the extremals and one has functional integral over all light-like 3-surfaces. The temporal distance between points along light-like 3-surface vanishes. The constraints from light-likeness bring in metric degrees of freedom but in a very gentle manner and just to make the theory physically interesting.

Feynmann cobordism as opposed to ordinary cobordism

In string model context the discouraging results from TQFT hold true in the category of nCob, which corresponds to trouser diagrams for closed strings or for their open string counterparts. In TGD framework these diagrams are replaced with a direct generalization of Feynman diagrams for which 3-D light-like partonic 3-surfaces meet along their 2-D ends at the vertices. In honor of Feynman one could perhaps speak of Feynman cobordisms. These surfaces are singular as 3-manifolds but vertices are nice 2-manifolds. I contrast to this, in string models diagrams are nice 2-manifolds but vertices are singular as 1-manifolds (say eye-glass type configurations for closed strings).

This picture gains a strong support for the interpretation of fermions as light-like throats associated with connected sums of CP_2 type extremals with space-time sheets with Minkowski signature and of bosons as pairs of light-like wormhole throats associated with CP_2 type extremal connecting two space-time sheets with Minkowski signature of induced metric. The space-time sheets have opposite time orientations so that also zero energy ontology emerges unavoidably. There is also consistency TGD based explanation of the family replication phenomenon in terms of genus of light-like partonic 2-surfaces.

One can wonder what the 4-D space-time sheets associated with the generalized Feynman diagrams could look like? One can try to gain some idea about this by trying to assign 2-D surfaces to ordinary Feynman diagrams having a subset of lines as boundaries. In the case of 2→2 reaction open string is pinched to a point at vertex. 1→2 vertex, and quite generally, vertices with odd number of lines, are impossible. The reason is that 1-D manifolds of finite size can have either 0 or 2 ends whereas in higher-D the number of boundary components is arbitrary. What one expects to happen in TGD context is that wormhole throats which are at distance characterized by CP_2 fuse together in the vertex so that some kind of pinches appear also now.

Zero energy ontology

Zero energy ontology gives rise to a second profound distinction between TGD and standard QFT. Physical states are identified as states with vanishing net quantum numbers, in particular energy. Everything is creatable from vacuum - and one could add- by intentional action so that zero energy ontology is profoundly Eastern. Positive *resp.* negative energy parts of states can be identified as states associated with 2-D partonic surfaces at the boundaries of future *resp.* past directed light-cones, whose tips correspond to the arguments of n-point functions. Each incoming/outgoing particle would define a mini-cosmology corresponding to not so big bang/crunch. If the time scale of perception is much shorter than time interval between positive and zero energy states, the ontology looks like the Western positive energy ontology. Bras and kets correspond naturally to the positive and negative energy states and phase conjugation for laser photons making them indeed something which seems to travel in opposite time direction is counterpart for bra-ket duality.

Finite temperature S-matrix defines genuine quantum state in zero energy ontology

In TGD framework one encounters two S-matrix like operators.

1. There is U-matrix between zero energy states. This is expected to be rather trivial but very important from the point of view of description of intentional actions as transitions transforming p-adic partonic 3-surfaces to their real counterparts.

2. The S-matrix like operator describing what happens in laboratory corresponds to the time-like entanglement coefficients between positive and negative energy parts of the state. Measurement of reaction rates would be a measurement of observables reducing time like entanglement and very much analogous to an ordinary quantum measurement reducing space-like entanglement. There is a finite measurement resolution described by inclusion of HFFs and this means that situation reduces effectively to a finite-dimensional one.

p-Adic thermodynamics strengthened with p-adic length scale hypothesis predicts particle masses with an amazing success. At first the thermodynamical approach seems to be in contradiction with the idea that elementary particles are quantal objects. Unitarity is however *not* necessary if one accepts that only relative probabilities for reductions to pairs of initial and final states interpreted as particle reactions can be measured.

The beneficial implications of unitarity are not lost if one replaces QFT with thermal QFT. Category theoretically this would mean that the time-like entanglement matrix associated with the product of cobordisms is a product of these matrices for the factors. The time parameter in S-matrix would be replaced with a complex time parameter with the imaginary part identified as inverse temperature. Hence the interpretation in terms of time evolution is not lost.

In the theory of hyper-finite factors of type III_1 the partition function for thermal equilibrium states and S-matrix can be neatly fused to a thermal S-matrix for zero energy states and one could introduce p-adic thermodynamics at the level of quantum states. It seems that this picture applies to HFFs by restriction. Therefore the loss of unitarity S-matrix might after all turn to a victory by more or less forcing both zero energy ontology and p-adic thermodynamics.

6.8 How to represent algebraic numbers as geometric objects?

I have found the blogs of mathematicians very interesting, in particular "Kea's blog" [?] has provided many stimuli in my attempts to gain some intuition about categories and their possible application to quantum TGD. Kea has generously explained what are the deep problems of category theoretic approach to mathematics and given references to articles: thanks to these references also this section saw the day light.

These blogs are also interesting because they allow to get some grasp about very different styles of thinking of a mathematician and physicist. For mathematician it is very important that the result is obtained by a strict use of axioms and deduction rules. Physicist is a cognitive opportunist: it does not matter how the result is obtained by moving along axiomatically allowed paths or not, and the new result is often more like a discovery of a new axiom and physicist is ever-grateful for Gödel for giving justification for what sometimes admittedly degenerates to a creative hand-waving. For physicist ideas form a kind of bio-sphere and the fate of the individual idea depends on its ability to survive, which is determined by its ability to become generalized, its consistency with other ideas, and ability to interact with other ideas to produce new ideas.

6.8.1 Can one define complex numbers as cardinalities of sets?

During few days before writing this we have had in Kea's blog a little bit of discussion inspired by the problem related to the categorification of basic number theoretical structures. I have learned that sum and product are natural operations for the objects of category. For instance, one can define sum as in terms of union of sets or direct sum of vector spaces and product as Cartesian product of sets and tensor product of vector spaces: rigs [**?**] are example of categories for which natural numbers define sum and product.

Subtraction and division are however problematic operations. Negative numbers and inverses of integers do not have a realization as a number of elements for any set or as dimension of vector space. The naive physicist inside me asks immediately: why not go from statics to dynamics and take operations (arrows with direction) as objects: couldn't this allow to define subtraction and division? Is the problem that the axiomatization of group theory requires something which purest categorification does not give? Or aren't the numbers representable in terms of operations of finite groups not enough? In any case cyclic groups would allow to realize roots of unity as operations (Z_2 would give -1).

One could also wonder why the algebraic numbers might not somehow result via the representations of permutation group of infinite number of elements containing all finite groups and thus Galois groups of algebraic extensions as subgroups? Why not take the elements of this group as objects of the basic category and continue by building group algebra and hyper-finite factors of type II_1 isomorphic to spinors of world of classical worlds, and so on.

After having written the first half of the section, I learned that something similar to the transition from statics to dynamics is actually carried out but by manner which is by many orders of magnitudes more refined than the proposal above and that I had never been able to imagine. The article *Objects of categories as complex numbers* of Marcelo Fiore and Tom Leinster [**?**] describes a fascinating idea summarized also by John Baez [**?**] about how one can assign to the objects of a category complex numbers as roots of a polynomial $Z = P(Z)$ defining an isomorphism of object. Z is the element of a category called rig, which differs from ring in that integers are replaced with natural numbers. One can replace Z with a complex number $|Z|$ defined as a root of polynomial. $|Z|$ is interpreted formally as the cardinality of the object. It is essential to have natural numbers and thus only product and sum are defined. This means a restriction: for instance, only complex algebraic numbers associated with polynomials having natural numbers as coefficients are obtained. Something is still missing.

Note that this correspondence assumes the existence of complex numbers and one cannot say that complex numbers are categorified. Maybe basic number fields must be left outside categorification. One can however require that all of them have a concrete set theoretic representation rather than only formal interpretation as cardinality so that one still encounters the problem how to represent algebraic complex number as a concrete cardinality of a set.

6.8.2 In what sense a set can have cardinality -1?

The discussion in Kea's blog led me to ask what the situation is in the case of p-adic numbers. Could it be possible to represent the negative and inverse of p-adic integer, and in fact any p-adic number, as a geometric object? In other words, does a set with -1 or $1/n$ or even $\sqrt{-1}$ elements exist? If this were in some sense true for all p-adic number fields, then all this wisdom combined together might provide something analogous to the adelic representation for the norm of a rational number as product of its p-adic norms. As will be found, alternative interpretations of complex algebraic numbers as p-adic numbers representing cardinalities of p-adic fractals emerge. The fractal defines the manner how one must do an infinite sum to get an infinite real number but finite p-adic number.

Of course, this representation might not help to define p-adics or reals categorically but might help to understand how p-adic cognitive representations defined as subsets for rational intersections of real and p-adic space-time sheets could represent p-adic number as the number of points of p-adic fractal having infinite number of points in real sense but finite in the p-adic sense. This would also give a fundamental cognitive role for p-adic fractals as cognitive representations of numbers.

How to construct a set with -1 elements?

The basic observation is that p-adic -1 has the representation

$$-1 = (p-1)/(1-p) = (p-1)(1+p+p^2+p^3....)$$

As a real number this number is infinite or -1 but as a p-adic number the series converges and has p-adic norm equal to 1. One can also map this number to a real number by canonical identification taking the powers of p to their inverses: one obtains p in this particular case. As a matter fact, any rational with p-adic norm equal to 1 has similar power series representation.

The idea would be to represent a given p-adic number as the infinite number of points (in real sense) of a p-adic fractal such that p-adic topology is natural for this fractal. This kind of fractals can be constructed in a simple manner: from this more below. This construction allows to represent any p-adic number as a fractal and code the arithmetic operations to geometric operations for these fractals.

These representations - interpreted as cognitive representations defined by intersections of real and p-adic space-time sheets - are in practice approximate if real space-time sheets are assumed to have a finite size: this is due to the finite p-adic cutoff implied by this assumption and the meaning a finite resolution. One can however say that the p-adic space-time itself could by its necessarily infinite size represent the *idea* of given p-adic number faithfully.

This representation applies also to the p-adic counterparts of algebraic numbers in case that they exist. For instance, roughly one half of p-adic numbers have square root as ordinary p-adic number and quite generally algebraic operations on p-adic numbers can give rise to p-adic numbers so that also these could have set theoretic representation. For $p \ mod \ 4 = 1$ also $\sqrt{(-1)}$ exists: for instance, for $p = 5$: $2^2 = 4 = -1 \ mod \ 5$ guarantees this so that also imaginary unit and complex numbers would have a fractal representation. Also many transcendentals possess this kind of representation. For instance $exp(xp)$ exists as a p-adic number if x has p-adic norm not larger than 1: also $log(1+xp)$ does so.

Hence a quite impressive repertoire of p-adic counterparts of real numbers would have representation as a p-adic fractal for some values of p. Adelic vision would suggest that combining these representations one might be able to represent quite a many real numbers. In the case of π I do not find any obvious p-adic representation (for instance $sin(\pi/6) = 1/2$ does not help since the p-adic variant of the Taylor expansion of $\pi/6 = arcsin(1/2)$ does not converge p-adically for any value of p). It might be that there are very many transcendentals not allowing fractal representation for any value of p.

Conditions on the fractal representations of p-adic numbers

Consider now the construction of the fractal representations in terms of rational intersections of real real and p-adic space-time sheets. The question is what conditions are natural for this representation if it corresponds to a cognitive representation is realized in the rational intersection of real and p-adic space-time sheets obeying same algebraic equations.

1. Pinary cutoff is the analog of the decimal cutoff but is obtained by dropping away high positive rather than negative powers of p to get a finite real number: example of pinary cutoff is $-1 = (p-1)(1+p+p^2+...) \rightarrow (p-1)(1+p+p^2)$. This cutoff must reduce to a fractal cutoff meaning a finite resolution due to a finite size for the real space-time sheet. In the real sense the p-adic fractal cutoff means not forgetting details below some scale but cutting out all above some length scale. Physical analog would be forgetting all frequencies below some cutoff frequency in Fourier expansion.

   The motivation comes from the fact that TGD inspired consciousness assigns to a given biological body there is associated a field body or magnetic body containing dark matter with large $\hbar$ and quantum controlling the behavior of biological body and so strongly identifying with it so as to belief that this all ends up to a biological death. This field body has an onion like fractal structure and a size of at least order of light-life. Of course, also larger onion layers could be present and would represent those levels of cognitive consciousness not depending on the sensory input on biological body: some altered states of consciousness could relate to these levels. In any case, the larger the magnetic body, the better the numerical skills of the p-adic mathematician.

2. Lowest pinary digits of $x = x_0+x_1p+x_2p^2+...$, $x_n \leq p$ must have the most reliable representation since they are the most significant ones. The representation must be also highly redundant to guarantee reliability. This requires repetitions and periodicity. This is guaranteed if the representation is hologram like with segments of length p^n with digit x_n represented again and again in all segments of length p^m, $m > n$.

3. The TGD based physical constraint is that the representation must be realizable in terms of induced classical fields assignable to the field body hierarchy of an intelligent system interested in artistic expression of p-adic numbers using its own field body as instrument. As a matter, sensory and cognitive representations are realized at field body in TGD Universe and EEG is in a fundamental role in building this representation. By p-adic fractality fractal wavelets are the most natural candidate. The fundamental wavelet should represent the p different pinary digits and its scaled up variants would correspond to various powers of p so that the representation would reduce to a Fourier expansion of a classical field.

Concrete representation

Consider now a concrete candidate for a representation satisfying these constraints.

1. Consider a p-adic number

$$y = p^{n_0}x, \ \ x = \sum x_n p^n \ \ , \ \ n \geq n_0 = 0 \ \ .$$

If one has a representation for a p-adic unit x the representation of is by a purely geometric fractal scaling of the representation by p^n. Hence one can restrict the consideration to p-adic units.

2. To construct the representation take a real line starting from origin and divide it into segments with lengths $1, p, p^2, ...$. In TGD framework this scalings come actually as powers of $p^{1/2}$ but this is just a technical detail.

3. It is natural to realize the representation in terms of periodic field patterns. One can use wavelets with fractal spectrum $p^n\lambda_0$ of "wavelet lengths", where λ_0 is the fundamental wavelength. Fundamental wavelet should have p different patterns correspond to the p values of pinary digit as its structures. Periodicity guarantees the hologram like character enabling to pick n:th digit by studying the field pattern in scale p^n anywhere inside the field body.

4. Periodicity guarantees also that the intersections of p-adic and real space-time sheets can represent the values of pinary digits. For instance, wavelets could be such that in a given p-adic scale the number of rational points in the intersection of the real and p-adic space-time sheet equals to x_n. This would give in the limit of an infinite pinary expansion a set theoretic realization of any p-adic number in which each pinary digit x_n corresponds to infinite copies of a set with x_n elements and fractal cutoff due to the finite size of real space-time sheet would bring in a finite precision. Note however that p-adic space-time sheet necessarily has an infinite size and it is only real world realization of the representation which has finite accuracy.

5. A concrete realization for this object would be as an infinite tree with $x_n + 1 \leq p$ branches in each node at level n ($x_n + 1$ is needed in order to avoid the splitting tree at $x_n = 0$). In 2-adic case -1 would be represented by an infinite pinary tree. Negative powers of p correspond to the of the tree extending to a finite depth in ground.

6.8.3 Generalization of the notion of rig by replacing naturals with p-adic integers

Previous considerations do not relate directly to category theoretical problem of assigning complex numbers to objects. It however turns out that p-adic approach allows to generalize the proposal of [?] by replacing natural numbers with p-adic integers in the definition of rig so that any algebraic complex number can define cardinality of an object of category allowing multiplication and sum and that these complex numbers can be replaced with p-adic numbers if they make sense as such so that previous arguments provide a concrete geometric representation of the cardinality. The road to the realization this simple generalization required a visit to the John Baez's Weekly Finds (Week 102) [?].

The outcome was the realization that the notion of rig used to categorify the subset of algebraic numbers obtained as roots of polynomials with *natural number* valued coefficients generalizes trivially by replacing natural numbers by *p-adic integers*. As a consequence one obtains beautiful p-adicization of the generating function F(x) of structure as a function which converges p-adically for any rational $x = q$ for which it has prime p as a positive power divisor.

Effectively this generalization means the replacement of natural numbers as coefficients of the polynomial defining the rig with all rationals, also negative, and *all* complex algebraic numbers find a category theoretical representation as "cardinalities". These cardinalities have a dual interpretation as p-adic integers which in general correspond to infinite real numbers but are mappable to real numbers by canonical identification and have a geometric representation as fractals.

Mapping of objects to complex numbers and the notion of rig

The idea of rig approach is to categorify the notion of cardinality in such a manner that one obtains a *subset* of algebraic complex numbers as cardinalities in the category-theoretical sense. One can assign to an object a polynomial with coefficients, which are *natural numbers* and the condition $Z = P(Z)$ says that $P(Z)$ acts as an isomorphism of the object. One can interpret the equation also in terms of complex numbers. Hence the object is mapped to a complex number Z defining a root of the polynomial interpreted as an ordinary polynomial: it does not matter which root is chosen. The complex number Z is interpreted as the "cardinality" of the object but I do not really understand the motivation for this. The deep further result is that also more general polynomial equations $R(|Z|) = Q(|Z|)$ satisfied by the generalized cardinality Z imply $R(Z) = Q(Z)$ as isomorphism.

I try to reproduce what looks the most essential in the explanation of John Baez and relate it to my own ideas but take this as my talk to myself and visit This Week's Finds [?], one of the many classics of Baez, to learn of this fascinating idea.

1. Baez considers first the ways of putting a given structure to n-element set. The set of these structures is denoted by F_n and the number of them by $|F_n|$. The generating function $|F|(x) = \sum_n |F_n| x^n$ packs all this information to a single function.

   For instance, if the structure is binary tree, this function is given by $T(x) = \sum_n C_{n-1} x^n$, where C_{n-1} are Catalan numbers and n¿0 holds true. One can show that T satisfies the formula

   $$T = X + T^2 \quad ,$$

   since any binary tree is either trivial or decomposes to a product of binary trees, where two trees emanate from the root. One can solve this second order polynomial equation and the power expansion gives the generating function.

2. The great insight is that one can also work directly with structures. For instance, by starting from the isomorphism $T = 1 + T^2$ applying to an object with cardinality 1 and substituting T^2 with $(1 + T^2)^2$ repeatedly, one can deduce the amazing formula $T^7(1) = T(1)$ mentioned by Kea, and this identity can be interpreted as an isomorphism of binary trees.

3. This result can be generalized using the notion of rig category [?]. In rig category one can add and multiply but negatives are not defined as in the case of ring. The lack of subtraction and division is still the problem and as I suggested in previous posting p-adic integers might resolve the problem.

Whenever Z is object of a rig category, one can equip it with an isomorphism $Z = P(Z)$ where $P(Z)$ is polynomial with *natural numbers* as coefficients and one can assign to object "cardinality" as any root of the equation $Z = P(Z)$. Note that set with n elements corresponds to $P(|Z|) = n$. Thus subset of algebraic complex numbers receive formal identification as cardinalities of sets. Furthermore, if the cardinality satisfies another equation $Q(|Z|) = R(|Z|)$ such that neither polynomial is constant, then one can construct an isomorphism $Q(Z) = R(Z)$. Isomorphisms correspond to equations!

4. This is indeed nice that there is something which is not so beautiful as it could be: why should we restrict ourselves to *natural numbers* as coefficients of $P(Z)$? Could it be possible to replace them with integers to obtain *all complex algebraic numbers* as cardinalities? Could it be possible to replace natural numbers by p-adic integers?

p-Adic rigs and Golden Object as p-adic fractal

The notions of generating function and rig generalize to the p-adic context.

1. The generating function $F(x)$ defining isomorphism Z in the rig formulation converges p-adically for any p-adic number containing p as a factor so that the idea that all structures have p-adic counterparts is natural. In the real context the generating function typically diverges and must be defined by analytic continuation. Hence one might even argue that p-adic numbers are more natural in the description of structures assignable to finite sets than reals.

2. For rig one considers only polynomials $P(Z)$ (Z corresponds to the generating function F) with coefficients which are natural numbers. Any p-adic integer can be however interpreted as a non-negative integer: natural number if it is finite and "super-natural" number if it is infinite. Hence can generalize the notion of rig by replacing natural numbers by p-adic integers. The rig formalism would thus generalize to arbitrary polynomials with integer valued coefficients so that all complex algebraic numbers could appear as cardinalities of category theoretical objects. Even rational coefficients are allowed. This is highly natural number theoretically.

3. For instance, in the case of binary trees the solutions to the isomorphism condition $T = p + T^2$ giving $T = [1 \pm (1-4p)^{1/2}]/2$ and T would be complex number $[p \pm (1-4p)^{1/2}]/2$. $T(p)$ can be interpreted also as a p-adic number by performing power expansion of square root in case that the p-adic square root exists: this super-natural number can be mapped to a real number by the canonical identification and one obtains also the set theoretic representations of the category theoretical object $T(p)$ as a p-adic fractal. This interpretation of cardinality is much more natural than the purely formal interpretation as a complex number. This argument applies completely generally. The case $x = 1$ discussed by Baez gives $T = [1 \pm (-3)^{1/2}]/2$ allows p-adic representation if $-3 == p-3$ is square mod p. This is the case for $p = 7$ for instance.

4. John Baez [?] poses also the question about the category theoretic realization of "Golden Object", his big dream. In this case one would have $Z = G = -1 + G^2 = P(Z)$. The polynomial on the right hand side does not conform with the notion of rig since -1 is not a natural number. If one allows p-adic rigs, $x = -1$ can be interpreted as a p-adic integer $(p-1)(1+p+...)$, positive and infinite and "super-natural", actually largest possible p-adic integer in a well defined sense.

   A further condition is that Golden Mean converges as a p-adic number: this requires that $\sqrt{5}$ must exist as a p-adic number: $(5 = 1+4)^{1/2}$ certainly converges as power series for $p = 2$ so that Golden Object exists 2-adically. By using [?] of Euler, one finds that 5 is square mod p only if p is square mod 5. To decide whether given p is Golden it is enough to look whether p mod 5 is 1 or 4. For instance, $p = 11, 19, 29, 31$ ($=M_5$) are Golden. Mersennes M_k, $k = 3, 7, 127$ and Fermat primes are not Golden. One representation of Golden Object as p-adic fractal is the p-adic series expansion of $[1/2 \pm 5^{1/2}]/2$ representable geometrically as a binary tree such that there are $0 \leq x_n + 1 \leq p$ branches at each node at height n if n:th p-adic coefficient is x_n. The "cognitive" p-adic representation in terms of wavelet spectrum of classical fields is discussed in the previous posting.

5. It would be interesting to know how quantum dimensions of quantum groups assignable to Jones inclusions [?, ?, ?] relate to the generalized cardinalities. The root of unity property of

quantum phase ($q^{n+1} = q$) suggests $Q = Q^{n+1} = P(Q)$ as the relevant isomorphism. For Jones inclusions the cardinality $q = exp(i2\pi/n)$ would not be however equal to quantum dimension $D(n) = 4cos^2(\pi/n)$.

Is there a connection with infinite integers?

Infinite primes [?] correspond to Fock states of a super-symmetric arithmetic quantum field theory and there is entire infinite hierarchy of them corresponding to repeated second quantization. Also infinite primes and rationals make sense. Besides free Fock states spectrum contains at each level also what might be identified as bound states. All these states can be mapped to polynomials. Since the roots of polynomials represent complex algebraic numbers and as they seem to characterize objects of categories, there are reasons to expect that infinite rationals might allow also interpretation in terms of say rig categories or their generalization. Also the possibility to identify space-time coordinate as isomorphism of a category might be highly interesting concerning the interpretation of quantum classical correspondence.

6.9 Gerbes and TGD

The notion of gerbes has gained much attention during last years in theoretical physics and there is an abundant gerbe-related literature in hep-th archives. Personally I learned about gerbes from the excellent article of Jouko Mickelson [?] (Jouko was my opponent in PhD dissertation for more than two decades ago: so the time flows!).

I have already applied the notion of bundle gerbe in TGD framework in the construction of the Dirac determinant which I have proposed to define the Kähler function for the configuration space of 3-surfaces (see the chapter "Configuration Space Spinor Structure"). The insights provided by the general results about bundle gerbes discussed in [?] led, not only to a justification for the hypothesis that Dirac determinant exists for the modified Dirac action, but also to an elegant solution of the conceptual problems related to the construction of Dirac determinant in the presence of chiral symmetry. Furthermore, on basis of the special properties of the modified Dirac operator there are good reasons to hope that the determinant exists even without zeta function regularization. The construction also leads to the conclusion that the space-time sheets serving as causal determinants must be geodesic sub-manifolds (presumably light like boundary components or "elementary particle horizons"). Quantum gravitational holography is realized since the exponent of Kähler function is expressible as a Dirac determinant determined by the local data at causal determinants and there would be no need to find absolute minima of Kähler action explicitly.

In the sequel the emergence of 2-gerbes at the space-time level in TGD framework is discussed and shown to lead to a geometric interpretation of the somewhat mysterious cocycle conditions for a wide class of gerbes generated via the $\wedge d$ products of connections associated with 0-gerbes. The resulting conjecture is that gerbes form a graded-commutative Grassmman algebra like structure generated by -1- and 0-gerbes. 2-gerbes provide also a beautiful topological characterization of space-time sheets as structures carrying Chern-Simons charges at boundary components and the 2-gerbe variant of Bohm-Aharonov effect occurs for perhaps the most interesting asymptotic solutions of field equations especially relevant for anyonics systems, quantum Hall effect, and living matter [?].

6.9.1 What gerbes roughly are?

Very roughly and differential geometrically, gerbes can be regarded as a generalization of connection. Instead of connection 1-form (0-gerbe) one considers a connection $n+1$-form defining n-gerbe. The curvature of n-gerbe is closed $n+2$-form and its integral defines an analog of magnetic charge. The notion of holonomy generalizes: instead of integrating n-gerbe connection over curve one integrates its connection form over n+1-dimensional closed surface and can transform it to the analog of magnetic flux.

There are some puzzling features associated with gerbes. Ordinary $U(1)$-bundles are defined in terms of open sets U_α with gauge transformations $g_{\alpha\beta} = g_{\beta\alpha}^{-1}$ defined in $U_\alpha \cap U_\beta$ relating the connection forms in the patch U_β to that in patch U_α. The 3-cocycle condition

$$g_{\alpha\beta}g_{\beta\gamma}g_{\gamma\alpha} = 1 \quad (6.9.1)$$

makes it possible to glue the patches to a bundle structure.

In the case of 1-gerbes the transition functions are replaced with the transition functions $g_{\alpha\beta\gamma} = g^{-1}_{\gamma\beta\alpha}$ defined in triple intersections $U_\alpha \cap U_\beta \cap U_\gamma$ and 3-cocycle must be replaced with 4-cocycle:

$$g_{\alpha\beta\gamma}g_{\beta\gamma\delta}g_{\gamma\delta\alpha}g_{\delta\alpha\beta} = 1 \ . \quad (6.9.2)$$

The generalizations of these conditions to n-gerbes is obvious.

In the case of 2-intersections one can build a bundle structure naturally but in the case of 3-intersections this is not possible. Hence the geometric interpretation of the higher gerbes is far from obvious. One possible interpretation of non-trivial 1-gerbe is as an obstruction for lifting projective bundles with fiber space CP_n to vector bundles with fiber space C^{n+1} [?]. This involves the lifting of the holomorphic transition functions g_α defined in the projective linear group $PGL(n+1,C)$ to $GL(n+1,C)$. When the 3-cocycle condition for the lifted transition functions $\overline{g}_{\alpha\beta}$ fails it can be replaced with 4-cocycle and one obtains 1-gerbe.

6.9.2 How do 2-gerbes emerge in TGD?

Gerbes seem to be interesting also from the point of view of TGD, and TGD approach allows a geometric interpretation of the cocycle conditions for a rather wide class of gerbes.

Recall that the Kähler form J of CP_2 defines a non-trivial magnetically charged and self-dual $U(1)$-connection A. The Chern-Simons form $\omega = A \wedge J = A \wedge dA$ having CP_2 Abelian instanton density $J \wedge J$ as its curvature form and can thus be regarded as a 3-connection form of a 2-gerbe. This 2-gerbe is induced by 0-gerbe.

The coordinate patches U_α are same as for $U(1)$ connection. In the transition between patches A and ω transform as

$$\begin{aligned} A &\rightarrow A + d\phi \ , \\ \omega &\rightarrow \omega + dA_2 \ , \\ A_2 &= \phi \wedge J \ . \end{aligned} \quad (6.9.3)$$

The transformation formula is induced by the transformation formula for $U(1)$ bundle. Somewhat mysteriously, there is no need to define anything in the intersections of U_α in the recent case.

The connection form of the 2-gerbe can be regarded as a second $\wedge d$ power of Kähler connection:

$$A_3 \equiv A \wedge dA \ . \quad (6.9.4)$$

The generalization of this observation allows to develop a different view about n-gerbes generated as $\wedge d$ products of 0-gerbes.

The hierarchy of gerbes generated by 0-gerbes

Consider a collection of $U(1)$ connections $A^{i)}$. They generate entire hierarchy of gerbe-connections via the $\wedge d$ product

$$A_3 = A^{1)} \wedge dA^{2)} \quad (6.9.5)$$

defining 2-gerbe having a closed curvature 4-form

$$F_4 = dA^{1)} \wedge dA^{2)} \ . \quad (6.9.6)$$

$\wedge d$ product is commutative apart from a gauge transformation and the curvature forms of $A^{1)} \wedge dA^{2)}$ and $A^{2)} \wedge dA^{1)}$ are the same.

Quite generally, the connections A_m of $m-1$ gerbe and A_n of $n-1$-gerbe define $m+n+1$ connection form and the closed curvature form of $m+n$-gerbe as

$$\begin{aligned} A_{m+n+1} &= A_m^{1)} \wedge dA_n^{2)} \ , \\ F_{m+n+2} &= dA_m^{1)} \wedge dA_n^{2)} \ . \end{aligned} \tag{6.9.7}$$

The sequence of gerbes extends up to $n = D-2$, where D is the dimension of the underlying manifold. These gerbes are not the most general ones since one starts from 0-gerbes. One can of course start from $n > 0$-gerbes too.

The generalization of the $\wedge d$ product to the non-Abelian situation is not obvious. The problems stem from the that the Lie-algebra valued connection forms $A^{1)}$ and $A^{2)}$ appearing in the covariant version $D = d + A$ do not commute.

6.9.3 How to understand the replacement of 3-cycles with n-cycles?

If n-gerbes are generated from 0-gerbes it is possible to understand how the intersections of the open sets emerge. Consider the product of 0-gerbes as the simplest possible case. The crucial observation is that the coverings U_α for $A^{1)}$ and V_β for $A^{2)}$ need not be same (for CP_2 this was the case). One can form a new covering consisting of sets $U_\alpha \cap V_{\alpha_1}$. Just by increasing the index range one can replace V with U and one has covering by $U_\alpha \cap U_{\alpha_1} \equiv U_{\alpha\alpha_1}$.

The transition functions are defined in the intersections $U_{\alpha\alpha_1} \cap U_{\beta\beta_1} \equiv U_{\alpha\alpha_1\beta\beta_1}$ and cocycle conditions must be formulated using instead of intersections $U_{\alpha\beta\gamma}$ the intersections $U_{\alpha\alpha_1\beta\beta_1\gamma\gamma_1}$. Hence the transition functions can be written as $g_{\alpha\alpha_1\beta\beta_1}$ and the 3-cocycle are replaced with 5-cocycle conditions since the minimal co-cycle corresponds to a sequence of 6 steps instead of 4:

$$U_{\alpha\alpha_1\beta\beta_1} \to U_{\alpha_1\beta\beta_1\gamma} \to U_{\beta\beta_1\gamma\gamma_1} \to U_{\beta_1\gamma\gamma_1\alpha} \to U_{\gamma\gamma_1\alpha\alpha_1} \ .$$

The emergence of higher co-cycles is thus forced by the modification of the bundle covering necessary when gerbe is formed as a product of lower gerbes. The conjecture is that any even gerbe is expressible as a product of 0-gerbes.

An interesting application of the product structure is at the level of configuration space of 3-surfaces ("world of classical worlds"). The Kähler form of the configuration space defines a connection 1-form and this generates infinite hierarchy of connection $2n+1$-forms associated with $2n$-gerbes.

6.9.4 Gerbes as graded-commutative algebra: can one express all gerbes as products of -1 and 0-gerbes?

If one starts from, say 1-gerbes, the previous argument providing a geometric understanding of gerbes is not applicable as such. One might however hope that it is possible to represent the connection 2-form of any 1-gerbe as a $\wedge d$ product of a connection 0-form ϕ of "-1"-gerbe and connection 1-form A of 0-gerbe:

$$A_2 = \phi dA \equiv A \wedge d\phi \ ,$$

with different coverings for ϕ and A. The interpretation as an obstruction for the modification of the underlying bundle structure is consistent with this interpretation.

The notion of -1-gerbe is not well-defined unless one can define the notion of -1 form precisely. The simplest possibility that 0-form transforms trivially in the change of patch is not consistent. One could identify contravariant n-tensors as $-n$-forms and d for them as divergence and d^2 as the antisymmetrized double divergence giving zero. ϕ would change in a gauge transformation by a divergence of a vector field. The integral of a divergence over closed M vanishes identically so that if the integral of ϕ over M is non-vanishing it corresponds to a non-trivial 0-connection. This interpretation of course requires the introduction of metric.

The requirement that the minimal intersections of the patches for 1-gerbes are of form $U_{\alpha\beta\gamma}$ would be achieved if the intersections patches can be restricted to the intersections $U_{\alpha\beta\gamma}$ defined by $U_\alpha \cap V_\gamma$

and $U_\beta \cap V_\gamma$ (instead of $U_\beta \cap V_\delta$), where the patches V_γ would be most naturally associated with -1-gerbe. It is not clear why one could make this restriction. The general conjecture is that any gerbe decomposes into a multiple $\wedge d$ product of -1 and 0-gerbes just like integers decompose into primes. The $\wedge d$ product of two odd gerbes is anti-commutative so that there is also an analogy with the decomposition of the physical state into fermions and bosons, and gerbes for a graded-commutative super-algebra generalizing the Grassmann algebra of manifold to a Grassmann algebra of gerbe structures for manifold.

6.9.5 The physical interpretation of 2-gerbes in TGD framework

2-gerbes could provide some insight to how to characterize the topological structure of the many-sheeted space-time.

1. The cohomology group H^4 is obviously crucial in characterizing 2-gerbe. In TGD framework many-sheetedness means that different space-time sheets with induced metric having Minkowski signature are separated by elementary particle horizons which are light like 3-surfaces at which the induced metric becomes degenerate. Also the time orientation of the space-time sheet can change at these surfaces since the determinant of the induced metric vanishes.

   This justifies the term elementary particle horizon and also the idea that one should treat different space-time sheets as generating independent direct summands in the homology group of the space-time surface: as if the space-time sheets not connected by join along boundaries bonds were disjoint. Thus the homology group H^4 and 2-gerbes defining instanton numbers would become important topological characteristics of the many-sheeted space-time.

2. The asymptotic behavior of the general solutions of field equations can be classified by the dimension D of the CP_2 projection of the space-time sheet. For $D = 4$ the instanton density defining the curvature form of 2-gerbe is non-vanishing and instanton number defines a topological charge. Also the values of the Chern-Simons invariants associated with the boundary components of the space-time sheet define topological quantum numbers characterizing the space-time sheet and their sum equals to the instanton charge. CP_2 type extremals represent a basic example of this kind of situation. From the physical view point $D = 4$ asymptotic solutions correspond to what might be regarded chaotic phase for the flow lines of the Kähler magnetic field. Kähler current vanishes so that empty space Maxwell's equations are satisfied.

3. For $D = 3$ situation is more subtle when boundaries are present so that the higher-dimensional analog of Aharonov-Bohm effect becomes possible. In this case instanton density vanishes but the Chern-Simons invariants associated with the boundary components can be non-vanishing. Their sum obviously vanishes. The space-time sheet can be said to be a neutral C-S multipole. Separate space-time sheets can become connected by join along boundaries bonds in a quantum jump replacing a space-time surface with a new one. This means that the cohomology group H^4 as well as instanton charges and C-S charges of the system change.

Concerning the asymptotic dynamics of the Kähler magnetic field, $D = 3$ phase corresponds to an extremely complex but highly organized phase serving as an excellent candidate for the modelling of living matter. Both the TGD based description of anyons and quantum Hall effect and the model for topological quantum computation based on the braiding of magnetic flux tubes rely heavily on the properties $D = 3$ phase [**?**].

The non-vanishing of the C-S form implies that the flow lines of the Kähler magnetic are highly entangled and have as an analog mixing hydrodynamical flow. In particular, one cannot define non-trivial order parameters, say phase factors, which would be constant along the lines. The interpretation in terms of broken super-conductivity suggests itself. Kähler current can be non-vanishing so that there is no counterpart for this phase at the level of Maxwell's equations.

6.10 Appendix: Category theory and construction of S-matrix

The construction of configuration space geometry, spinor structure and of S-matrix involve difficult technical and conceptual problems and category theory might be of help here. As already found, the application of category theory to the construction of configuration space geometry allows to understand how the arrow of psychological time emerges.

The construction of the S-matrix involves several difficult conceptual and technical problems in which category theory might help. The incoming states of the theory are what might be called free states and are constructed as products of the configuration space spinor fields. One can effectively regard them as being defined in the Cartesian power of the configuration space divided by an appropriate permutation group. Interacting states in turn are defined in the configuration space.

Cartesian power of the configuration space of 3-surfaces is however in geometrical sense more or less identical with the configuration space since the disjoint union of N 3-surfaces is itself a 3-surface in configuration space. Actually it differs from configuration space itself only in that the 3-surfaces of many particle state can intersect each other and if one allows this, one has paradoxical self-referential identification $CH = \overline{CH^2}/S_2 = ... = \overline{CH^N}/S_N...$, where over-line signifies that intersecting 3-surfaces have been dropped from the product.

Note that arbitrarily small deformation can remove the intersections between 3-surfaces and four-dimensional general coordinate invariance allows always to use non-intersecting representatives. In case of the spinor structure of the Cartesian power this identification means that the tensor powers SCH^N of the configuration space spinor structure are in some sense identical with the spinor structure SCH of the configuration space. Certainly the oscillator operators of the tensor factors must be assumed to be mutually anti-commuting.

The identities $CH = \overline{CH^2}/S_2 = ..$ and corresponding identities $SCH = SCH^2 = ...$ for the space SCH of configuration space spinor fields might imply very deep constraints on S-matrix. What comes into mind are counterparts for the Schwinger-Dyson equations of perturbative quantum field theory providing defining equations for the n-point functions of the theory [?]. The isomorphism between SCH^2 and SCH is actually what is needed to calculate the S-matrix elements. Category theory might help to understand at a general level what these self-referential and somewhat paradoxical looking identities really imply and perhaps even develop TGD counterparts of Schwinger-Dyson equations.

There is also the issue of bound states. The interacting states contain also bound states not belonging to the space of free states and category theory might help also here. It would seem that the state space must be constructed by taking into account also the bound states as additional 'free' states in the decomposition of states to product states.

A category naturally involved with the construction of the S-matrix (or U-matrix) is the space of the absolute minima $X^4(X^3)$ of the Kähler action which might be called interacting category. The canonical transformations acting as isometries of the configuration space geometry act naturally as the morphisms of this category. The group $Diff^4$ of general coordinate transformations in turn acts as gauge symmetries.

S-matrix relates free and interacting states and is induced by the classical interactions induced by the absolute minimization of Kähler action. S-matrix elements are essentially Glebch-Gordan coefficients relating the states in the tensor power of the interacting supercanonical representation with the interacting supercanonical representation itself. More concretely, N-particle free states can be seen as configuration space spinor fields in CH^N obtained as tensor products of ordinary CH spinor fields. Free states correspond classically to the unions of space-time surfaces associated with the 3-surfaces representing incoming particles whereas interacting states correspond classically to the space-time surfaces associated with the unions of the 3-surfaces defining incoming states. These two states define what might be called free and interacting categories with canonical transformations acting as morphisms.

The classical interaction is represented by a functor $S : \overline{CH^N}/S_N \rightarrow CH$ mapping the classical free many particle states, that is objects of the product category defined by $\overline{CH^N}/S_N$ to the interacting category CH. This functor assigns to the union $\cup_i X^4(X_i^3)$ of the absolute minima $X^4(X_i^3)$ of Kähler action associated with the incoming, free states X_i^3 the absolute minimum $X^4(\cup X_i^3)$ associated with the union of three-surfaces representing the outgoing interacting state. At quantum level this functor maps the state space SCH^N associated with $\cup_i X^4(X_i^3)$ to SCH in a unitary manner. An important constraint on S-matrix is that it acts effectively as a flow in zero modes correlating the quantum numbers in fiber degrees of freedom in one-to-one manner with the values of zero modes so that quantum jump $U\Psi_i \rightarrow \Psi_0...$ gives rise to a quantum measurement.

Bibliography

Online books about TGD

[1] M. Pitkänen (2006), *Topological Geometrodynamics: Overview*. http://www.helsinki.fi/~matpitka/tgdview/tgdview.html.

[2] M. Pitkänen (2006), *Quantum Physics as Infinite-Dimensional Geometry*. http://www.helsinki.fi/~matpitka/tgdgeom/tgdgeom.html.

[3] M. Pitkänen (2006), *Physics in Many-Sheeted Space-Time*. http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html.

[4] M. Pitkänen (2006), *Quantum TGD*. http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html.

[5] M. Pitkänen (2006), *TGD as a Generalized Number Theory*. http://www.helsinki.fi/~matpitka/tgdnumber/tgdnumber.html.

[6] M. Pitkänen (2006), *p-Adic length Scale Hypothesis and Dark Matter Hierarchy*. http://www.helsinki.fi/~matpitka/paddark/paddark.html.

[7] M. Pitkänen (2006), *TGD and Fringe Physics*. http://www.helsinki.fi/~matpitka/freenergy/freenergy.html.

Online books about TGD inspired theory of consciousness and quantum biology

[8] M. Pitkänen (2006), *Bio-Systems as Self-Organizing Quantum Systems*. http://www.helsinki.fi/~matpitka/bioselforg/bioselforg.html.

[9] M. Pitkänen (2006), *Quantum Hardware of Living Matter*. http://www.helsinki.fi/~matpitka/bioware/bioware.html.

[10] M. Pitkänen (2006), *TGD Inspired Theory of Consciousness*. http://www.helsinki.fi/~matpitka/tgdconsc/tgdconsc.html.

[11] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*. http://www.helsinki.fi/~matpitka/genememe/genememe.html.

[12] M. Pitkänen (2006), *TGD and EEG*. http://www.helsinki.fi/~matpitka/tgdeeg/tgdeeg/tgdeeg.html.

[13] M. Pitkänen (2006), *Bio-Systems as Conscious Holograms*. http://www.helsinki.fi/~matpitka/hologram/hologram.html.

[14] M. Pitkänen (2006), *Magnetospheric Consciousness*. http://www.helsinki.fi/~matpitka/magnconsc/magnconsc.html.

[15] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*. http://www.helsinki.fi/~matpitka/magnconsc/mathconsc.html.

References to the chapters of books

[A1] The chapter *An Overview about the Evolution of Quantum TGD* of [?].
http://www.helsinki.fi/~matpitka/tgdview/tgdview.html#tgdevoI.

[A2] The chapter *An Overview about Quantum TGD* of [?].
http://www.helsinki.fi/~matpitka/tgdview/tgdview.html#tgdevoII.

[A9] The chapter *Does TGD Predict the Spectrum of Planck Constants?* of [?].
http://www.helsinki.fi/~matpitka/tgdview/tgdview.html#Planck.

[B1] The chapter *Identification of the Configuration Space Kähler Function* of [?].
http://www.helsinki.fi/~matpitka/tgdgeom/tgdgeom.html#kahler.

[B2] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part I* of [?].
http://www.helsinki.fi/~matpitka/tgdgeom/tgdgeom.html#compl1.

[B3] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part II* of [?].
http://www.helsinki.fi/~matpitka/tgdgeom/tgdgeom.html#compl2.

[B4] The chapter *Configuration Space Spinor Structure* of [?].
http://www.helsinki.fi/~matpitka/tgdgeom/tgdgeom.html#cspin.

[C1] The chapter *Construction of Quantum Theory: Symmetries* of [?].
http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html#quthe.

[C4] The chapter *Construction of S-matrix* of [?].
http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html#smatrix.

[C6] The chapter *Construction of Quantum Theory: S-matrix* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#towards.

[C8] The chapter *Was von Neumann Right After All* of [?].
http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html#vNeumann.

[C12] The chapter *Appendix A: Quantum Groups and Related Structures* of [?].
http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#bialgebra.

[D1] The chapter *Basic Extremals of Kähler Action* of [?].
http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html#class.

[D5] The chapter *TGD and Cosmology* of [?].
http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html#cosmo.

[D6] The chapter *TGD and Astrophysics* of [?].
http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html#astro.

[E1] The chapter *TGD as a Generalized Number Theory: p-Adicization Program* of [?].
http://www.helsinki.fi/~matpitka/tgdnumber/tgdnumber.html#visiona.

[E2] The chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [?].
http://www.helsinki.fi/~matpitka/tgdnumber/tgdnumber.html#visionb.

[E3] The chapter *TGD as a Generalized Number Theory: Infinite Primes* of [?].
http://www.helsinki.fi/~matpitka/tgdnumber/tgdnumber.html#visionc.

[E6] The chapter *Fusion of p-Adic and Real Variants of Quantum TGD to a More General Theory* of [?].
http://www.helsinki.fi/~matpitka/tgdnumber/tgdnumber.html#mblocks.

[E9] The chapter *Topological Quantum Computation in TGD Universe* of [**?**].
http://www.helsinki.fi/~matpitka/tgdnumber/tgdnumber.html#tqc.

[E11] The chapter *Langlands Program and TGD* of [**?**].
http://www.helsinki.fi/~matpitka/tgdnumber/tgdeeg/tgdnumber.html#Langland.

[F1] The chapter *Elementary Particle Vacuum Functionals* of [**?**].
http://www.helsinki.fi/~matpitka/paddark/paddark.html#elvafu.

[H2] The chapter *Negentropy Maximization Principle* of [**?**].
http://www.helsinki.fi/~matpitka/tgdconsc/tgdconsc.html#nmpc.

[H3] The chapter *Self and Binding* of [**?**].
http://www.helsinki.fi/~matpitka/tgdconsc/tgdconsc.html#selfbindc.

[H8] The chapter *p-Adic Physics as Physics of Cognition and Intention* of [**?**].
http://www.helsinki.fi/~matpitka/tgdconsc/tgdconsc.html#cognic.

[K1] The chapter *Time, Spacetime and Consciousness* of [**?**].
http://www.helsinki.fi/~matpitka/hologram/hologram.html#time.

[K2] The chapter *Macro-Temporal Quantum Coherence and Spin Glass Degeneracy* of [**?**].
http://www.helsinki.fi/~matpitka/hologram/hologram.html#macro.

[K3] The chapter *General Theory of Qualia* of [**?**].
http://www.helsinki.fi/~matpitka/hologram/hologram.html#qualia.

[M3] The chapter *Dark Matter Hierarchy and Hierarchy of EEGs* of [**?**].
http://www.helsinki.fi/~matpitka/tgdeeg/tgdeeg/tgdeeg.html#eegdark.

Articles related to TGD

[16] M. Pitkänen (2007), *TGD*,
http://www.helsinki.fi/~matpitka/TGDbrief.pdf.

[17] M. Pitkänen (2007), *TGD inspired theory of consciousness*,
http://www.helsinki.fi/~matpitka/tgdconsc.pdf.

[18] M. Pitkänen (2007), *First edge of the cube*,
http://matpitka.blogspot.com/2007/05/first-edge-of-cube.html.

Mathematics related references

[19] R. Goldblatt (1984), *The Categorial Analysis of Logic*. North-Holland, Amsterdam.

[20] C. J. Isham and J. Butterfield (1999), *Some Possible Roles for Topos Theory in Quantum Theory and Quantum Gravity*. arXiv:gr-gc/9910005 .

[21] D. R. Hofstadter (1980), *Gödel, Escher, Bach: an Eternal Braid*, Penguin Books.

[22] G. Kato (2002), *Sheaf Cohomology of Conscious Entity*. To appear in Int. J. of Comp. Antic. Systems.

[23] G. Kato and D. C. Struppa (2001), *Category Theory and Consciousness*. Proceedings of the International Conference Tokyo '99 held at United Nations University, in press.

[24] B. Mitchell (1965), *Theory of Categories*. Academic Press.

[25] H. Schubert (1972), *Categories*. Springer-Verlag, New York, Heidelberg.

[26] Iztykson, Zuber (1980),"Quantum Field Theory", 549, New York: Mc Graw-Hill Inc.

[27] Richard Stefanik (1994), *Structuralism, Category Theory and Philosophy of Mathematics* Washington: MSG Press.

[28] *Set theory*,
http://en.wikipedia.org/wiki/Set_theory.

[29] *Category theory*,
http://en.wikipedia.org/wiki/Category_theory.

[30] John Baez (2007), *Quantum Quandaries*,
http://math.ucr.edu/home/baez/quantum/node1.html.

[31] http://math.ucr.edu/home/baez/quantum/ruberman.html.

[32] *Farey sequence*,
http://en.wikipedia.org/wiki/Farey_sequence.

[33] L. H. Kauffman and S. Lambropoulou(2006), *Hard Unknots and Collapsing Tangles*,
http://arxiv.org/abs/math/0601525.

[34] http://kea-monad.blogspot.com/2007/08/blogrolling-on.html.

[35] M. Fiore and T. Leinster (2007), *Objects of categories as complex numbers*,
http://www.arxiv.org/abs/math.CT/0212377.

[36] *Weekly Finds (Week 102)*,
http://math.ucr.edu/home/baez/week202.html.

[37] *Quadratic resiprocity theorem*,
http://en.wikipedia.org/wiki/Quadratic_reciprocity.

[38] J. Mickelson (2002), *Gerbes, (Twisted) K-Theory, and the Supersymmetric WZW Model*, hep-th/0206139.

Experimental physics

[39] M. Chown (2004), *Quantum Rebel*, popular article about the experiment of Shahriar Afshar. New Scientist, Vol 183, No 2457.

Consciousness

[40] O. Sacks (1998), *The man who mistook his wife for a hat*, Touchstone books. (First edition 1985).

[41] S. Suzuki (1988), *Zen Mind, Beginner's Mind*. Waterhill, New York & Tokyo.

Chapter 7

Infinite Primes and Consciousness

7.1 Introduction

This chapter is devoted to the possible role of infinite primes in TGD and TGD inspired theory of consciousness.

The notion of prime seems to capture something very essential about what it is to be elementary building block of matter and has become a fundamental conceptual element of TGD. The notion of prime gains it generality from its reducibility to the notion of prime ideal of an algebra. Thus the notion of prime is well-defined, not only in case of quaternions and octonions, but also in the case of hyper-quaternions and -octonions, which are especially natural physically and for which numbers having zero norm correspond physically to light-like 8-vectors. Many interpretations for infinite primes have been competing for survival but it seems that the recent state of TGD allows to exclude some of them from consideration.

7.1.1 The notion of infinite prime

p-Adic unitarity implies that each quantum jump involves unitarity evolution U followed by a quantum jump to some sector D_p of the configuration space labeled by a p-adic prime. Simple arguments show that the p-adic prime characterizing the 3-surface representing the entire universe increases in a statistical sense. This leads to a peculiar paradox: if the number of quantum jumps already occurred is infinite, this prime is most naturally infinite. On the other hand, if one assumes that only finite number of quantum jumps have occurred, one encounters the problem of understanding why the initial quantum history was what it was. Furthermore, since the size of the 3-surface representing the entire Universe is infinite, p-adic length scale hypothesis suggest also that the p-adic prime associated with the entire universe is infinite.

These arguments motivate the attempt to construct a theory of infinite primes and to extend quantum TGD so that also infinite primes are possible. Rather surprisingly, one can construct infinite primes by repeating a procedure analogous to a quantization of a super symmetric quantum field theory. At given level of hierarchy one can identify the decomposition of space-time surface to p-adic regions representing selves with the corresponding decomposition of the infinite prime to primes at lower level of infinity: at the basic level are finite primes for which one cannot find any formula.

This and other observations suggest that the Universe of quantum TGD might basically provide a physical representation of number theory allowing also infinite primes. The proposal suggests also a possible generalization of real numbers to a number system akin to hyper-reals introduced by Robinson in his non-standard calculus [**?**] providing rigorous mathematical basis for calculus. In fact, some rather natural requirements lead to a unique generalization for the concepts of integer, rational and real. Somewhat surprisingly, infinite integers and reals can be regarded as infinite-dimensional vector spaces with integer and real valued coefficients respectively and this raises the question whether the tangent space for the configuration space of 3-surfaces could be regarded as the space of generalized 8-D hyper-octonionic numbers.

7.1.2 Generalization of ordinary number fields

The introduction of infinite primes, integers, and rationals leads also to a generalization of real numbers since an infinite algebra of real units defined by finite ratios of infinite rationals multiplied by ordinary rationals which are their inverses becomes possible. These units are not units in the p-adic sense and have a finite p-adic norm which can be differ from one. This construction generalizes also to the case of hyper- quaternions and -octonions although non-commutativity and in case of hyper-octonions also non-associativity pose technical problems. Obviously this approach differs from the standard introduction of infinitesimals in the sense that sum is replaced by multiplication meaning that the set of real units becomes infinitely degenerate.

7.1.3 Infinite primes and physics in TGD Universe

Several different views about how infinite primes, integers, and rationals might be relevant in TGD Universe have emerged.

Infinite primes, cognition, and intentionality

The correlation of infinite primes with cognition is the first fascinating possibility and this possibility has stimulated several ideas.

1. The hierarchy of infinite primes associated with algebraic extensions of rationals leading gradually towards algebraic closure of rationals would in turn define cognitive hierarchy corresponding to algebraic extensions of p-adic numbers.

2. Infinite primes form an infinite hierarchy so that the points of space-time and imbedding space can be seen as infinitely structured and able to represent all imaginable algebraic structures. Certainly counter-intuitively, single space-time point might be even capable of representing the quantum state of the entire physical Universe in its structure. For instance, in the real sense surfaces in the space of units correspond to the same real number 1, and single point, which is structure-less in the real sense could represent arbitrarily high-dimensional spaces as unions of real units. For real physics this structure is completely invisible and is relevant only for the physics of cognition. One can say that Universe is an algebraic hologram, and there is an obvious connection both with Brahman=Atman identity of Eastern philosophies and Leibniz's notion of monad.

3. One can assign to infinite primes at n^{th} level of hierarchy rational functions of n rational arguments which form a natural hierarchical structure in that highest level corresponds to a polynomial with coefficients which are rational functions of the arguments at the lower level. One can solve one of the arguments in terms of lower ones to get a hierarchy of algebraic extensions. At the lowest level algebraic extensions of rationals emerge, at the next level algebraic extensions of space of rational functions of single variable, etc... This would suggest that infinite primes code for the correlation between quantum states and the algebraic extensions appearing in their their physical description and characterizing their cognitive correlates. The hierarchy of infinite primes would also correlate with a hierarchy of logics of various orders (hierarchy of statements about statements about...).

Infinite primes and super-symmetric quantum field theory

Consider next the physical interpretation.

1. The discovery of infinite primes suggested strongly the possibility to reduce physics to number theory. The construction of infinite primes can be regarded as a repeated second quantization of a super-symmetric arithmetic quantum field theory. This suggests that configuration space spinor fields or at least the ground states of associated super-conformal representations could be mapped to infinite primes in both bosonic and fermionic degrees of freedom. The process might generalize so that it applies in the case of quaternionic and octonionic primes and their hyper counterparts. This hierarchy of second quantizations means enormous generalization of physics to what might be regarded a physical counterpart for a hierarchy of abstractions about

abstractions about.... The ordinary second quantized quantum physics corresponds only to the lowest level infinite primes.

2. The ordinary primes appearing as building blocks of infinite primes at the first level of the hierarchy could be identified as coding for p-adic primes assignable to fermionic and bosonic partons identified as 2-surfaces of a given space-time sheet. The hierarchy of infinite primes would correspond to hierarchy of space-time sheets defined by the topological condensate. This leads also to a precise identification of p-adic and real variants of bosonic partonic 2-surfaces as correlates of intention and action and pairs of p-adic and real fermionic partons as correlates for cognitive representations.

3. The idea that infinite primes characterize quantum states of the entire Universe, perhaps ground states of super-conformal representations, if not all states, could be taken further. Could 8-D hyper-octonions correspond to 8-momenta in the description of TGD in terms of 8-D hyper-octonion space M^8? Could 4-D hyper-quaternions have an interpretation as four-momenta? The problems caused by non-associativity and non-commutativity however suggests that it is perhaps wiser to restrict the consideration to infinite primes associated with rationals and their algebraic extensions.

Here however emerges the idea about the number theoretic analog of color confinement. Rational (infinite) primes allow not only a decomposition to (infinite) primes of algebraic extensions of rationals but also to algebraic extensions of quaternionic and octonionic (infinite) primes. The physical analog is the decomposition of a particle to its more elementary constituents. This fits nicely with the idea about number theoretic resolution represented as a hierarchy of Galois groups defined by the extensions of rationals and realized at the level of physics in terms of Jones inclusions [**?**] defined by these groups having a natural action on space-time surfaces, induced spinor fields, and on configuration space spinor fields representing physical states [**?**].

Infinite primes and physics as number theory

The hierarchy of algebraic extensions of rationals implying corresponding extensions of p-adic numbers suggests that Galois groups, which are the basic symmetry groups of number theory, should have concrete physical representations using induced spinor fields and configuration space spinor fields and also infinite primes and real units formed as infinite rationals. These groups permute zeros of polynomials and thus have a concrete physical interpretation both at the level of partonic 2-surfaces dictated by algebraic equations and at the level of braid hierarchy. The vision about the role of hyperfinite factors of II_1 and of Jones inclusions as descriptions of quantum measurements with finite measurement resolution leads to concrete ideas about how these groups are realized.

Space-time correlates of infinite primes

One can assign to infinite primes at the n^{th} level of hierarchy rational functions of n arguments with arguments ordered in a hierarchical manner. It would be nice to assign some concrete interpretation to the polynomials of n arguments in the extension of field of rationals.

1. Do infinite primes code for space-time surfaces?

Infinite primes code naturally for Fock states in a hierarchy of super-symmetric arithmetic quantum field theories. Quantum classical correspondence leads to ask whether infinite primes could also code for the space-time surfaces serving as symbolic representations of quantum states. This would a generalization of algebraic geometry would emerge and could reduce the dynamics of Kähler action to algebraic geometry and organize 4-surfaces to a physical hierarchy according to their algebraic complexity. Note that this conjecture which should be consistent with several conjectures about the dynamics of space-time surfaces (space-time surfaces as preferred extrema of Kähler action, as Kähler calibrations, as quaternionic or co-quaternionic (as associative or co-associative) 4-surfaces of hyper-octonion space M^8.

The most promising variant of this idea is based on the conjecture that hyper-octonion real-analytic maps define foliations of $HO = M^8$ by hyper-quaternionic space-time surfaces providing in turn preferred extremals of Kähler action. This would mean that lowest level infinite primes would define hyper-analytic maps $HO \rightarrow HO$ as polynomials. The intuitive expectation is that higher levels should give rise to more complex HO analytic maps.

The basic objections against the idea is the failure of associativity. The only manner to guarantee associativity is to assume that the arguments oh_n in the polynomial are not independent but that one has $h_i = f_i(h_{i-1}$, $i = 2, ..., n$ where f_i is hyper-octonion real-analytic function. This assumption means that one indeed obtains foliation of M^8 by hyper-quaternionic surfaces also now and that these foliations become increasingly complex as n increases. One could of course consider also the possibility that the hierarchy of infinite primes is directly mapped to functions of single hyper-octonionic argument $h_n = ... = h_1 = h$.

2. What about the interpretation of zeros and poles of rational functions associated with infinite primes

If one accepts this interpretation of infinite primes, one must reconsider the interpretation of the zeros and also poles of the functions $f(o)$ defined by the infinite primes. The set of zeros and poles consists of discrete points and this suggests an interpretation in terms of preferred points of M^8, which appear naturally in the quantization of quantum TGD [?] if one accepts the ideas about hyper-finite factors of type II_1 [?] and the generalization of the notion of imbedding space and quantization of Planck constant [?].

The M^4 projection of the preferred point would code for the position tip of future or past light-cone $\delta M^4_\pm$ whereas E^4 projection would choose preferred origin for coordinates transforming linearly under $SO(4)$. At the level of CP_2 the preferred point would correspond to the origin of coordinates transforming linearly under $U(2) \subset SU(3)$. These preferred points would have interpretation as arguments of n-point function in the construction of S-matrix and theory would assign to each argument of n-point function (not necessarily so) "big bang" or "big crunch".

Also configuration space CH (the world of classical worlds) would decompose to a union CH_h of the classical world consisting of 3-surfaces inside $\delta M^4_\pm \times CP_2$ with CP_2 possessing also a preferred point. The necessity of this decomposition in M^4 degrees of freedom became clear long time ago.

3. Why effective 1-dimensionality in algebraic sense?

The identification of arguments (via hyper-octonion real-analytic map in the most general case) means that one consider essentially functions of single variable in the algebraic sense of the word. Rational functions of single variable defined on curve define the simplest function fields having many resemblances with ordinary number fields, and it is known that the dimension $D = 1$ is completely exceptional in algebraic sense [?].

1. Langlands program [?] is based on the idea that the representations of Galois groups can be constructed in terms of so called automorphic functions to which zeta functions relate via Mellin transform. The zeta functions associated with 1-dimensional algebraic curve on finite field F_q, $q = p^n$, code the numbers of solutions to the equations defining algebraic curve in extensions of F_q which form a hierarchy of finite fields F_{q^m} with $m = kn$ [?]: these conjectures have been proven. Algebraic 1-dimensionality is also responsible for the deep results related to the number theoretic Langlands program as far as 1-dimensional function fields on finite fields are considered [?, ?]. In fact, Langlands program is formulated only for algebraic extensions of 1-dimensional function fields.

2. The exceptional character of algebraically 1-dimensional surfaces is responsible the successes of conformal field theory inspired approach to the realization of the geometric Langlands program [?]. It is also responsible for the successes of string models.

3. Effective 1-dimensionality in the sense that the induced spinor fields anti-commute only along 1-D curve of partonic 2-surface is also crucial for the stringy aspects of quantum TGD [?].

4. Associativity is a key axiom of conformal field theories and would dictate both classical and quantum dynamics of TGD in the approach based on hyper-finite factors of type II_1[?]. Hence it is rather satisfactory outcome that the mere associativity for octonionic polynomials forces algebraic 1-dimensionality.

7.1.4 About literature

The reader not familiar with the basic algebra of quaternions and octonions is encouraged to study some background material: the home page of Tony Smith provides among other things an excellent introduction to quaternions and octonions [?]. String model builders are beginning to grasp the potential importance of octonions and quaternions and the articles about possible applications of octonions [?, ?, ?] provide an introduction to octonions using the language of physicist.

Personally I found quite frustrating to realize that I had neglected totally learning of the basic ideas of algebraic geometry, despite its obvious potential importance for TGD and its applications in string models. This kind of losses are the price one must pay for working outside the scientific community. It is not easy for a physicist to find readable texts about algebraic geometry and algebraic number theory from the bookshelves of mathematical libraries. The book "Algebraic Geometry for Scientists and Engineers" by Abhyankar [?], which is not so elementary as the name would suggest, introduces in enjoyable manner the basic concepts of algebraic geometry and binds the basic ideas with the more recent developments in the field. "Problems in Algebraic Number Theory" by Esmonde and Murty [?] in turn teaches algebraic number theory through exercises which concretize the abstract ideas. The book "Invitation to Algebraic Geometry" by K. E. Smith. L. Kahanpää, P. Kekäläinen and W. Traves is perhaps the easiest and most enjoyable introduction to the topic for a novice. It also contains references to the latest physics inspired work in the field.

7.2 Infinite primes, integers, and rationals

By the arguments of introduction p-adic evolution leads to a gradual increase of the p-adic prime p and at the limit $p \to \infty$ Omega Point is reached in the sense that the negentropy gain associated with quantum jump can become arbitrarily large. There several interesting questions to be answered. Does the topology R_P at the limit of infinite P indeed approximate real topology? Is it possible to generalize the concept of prime number and p-adic number field to include infinite primes? This is is possible is suggested by the fact that sheets of 3-surface are expected to have infinite size and thus to correspond to infinite p-adic length scale. Do p-adic numbers R_P for sufficiently large P give rise to reals by canonical identification? Do the number fields R_P provide an alternative formulation/generalization of the non-standard analysis based on the hyper-real numbers of Robinson [?]? Is it possible to generalize the adelic formula [?] so that one could generalize quantum TGD so that it allows effective p-adic topology for infinite values of p-adic prime? It must be emphasized that the consideration of infinite primes need not be a purely academic exercise: for infinite values of p p-adic perturbation series contains only two terms and this limit, when properly formulated, could give excellent approximation of the finite p theory for large p.

It turns out that there is not any unique infinite prime nor even smallest infinite prime and that there is an entire hierarchy of infinite primes. Somewhat surprisingly, R_P is not mapped to entire set of reals nor even rationals in canonical identification: the image however forms a dense subset of reals. Furthermore, by introducing the corresponding p-adic number fields R_P, one necessarily obtains something more than reals: one might hope that for sufficiently large infinite values of P this something might be regarded as a generalization of real numbers to a number field containing both infinite numbers and infinitesimals.

The pleasant surprise is that one can find a general construction recipe for infinite primes and that this recipe can be characterized as a repeated second quantization procedure in which the many boson states of the previous level become single boson states of the next level of the hierarchy: this realizes Cantor's definition 'Set as Many allowing to regard itself as One' in terms of the basic concepts of quantum physics. Infinite prime allows decomposition to primes at lower level of infinity and these primes can be identified as primes labeling various space-time sheets which are in turn geometric correlates of selves in TGD inspired theory of consciousness. Furthermore, each infinite prime defines decomposition of a fictive many particle state to a purely bosonic part and to part for which fermion number is one in each mode. This decomposition corresponds to the decomposition of the space-time surface to cognitive and material space-time sheets. Thus the concept of infinite prime suggests completely unexpected connection between quantum field theory, TGD based theory of consciousness and number theory by providing in its structure nothing but a symbolic representation of mathematician and external world!

The definition of the infinite integers and rationals is a straightforward procedure. Infinite primes also allow generalization of the notion of ordinary number by allowing infinite-dimensional space of real units which are however non-equivalent in p-adic sense. This means that space-time points are infinitely structured. The fact that this structure completely invisible at the level of real physics suggests that it represents the space-time correlate for mathematical cognition.

7.2.1 The first level of hierarchy

In the following the concept of infinite prime is developed gradually by stepwise procedure rather than giving directly the basic definitions. The hope is that the development of the concept in the same manner as it actually occurred would make it easier to understand it.

Step 1

One could try to define infinite primes P by starting from the basic idea in the proof of Euclid for the existence of infinite number of primes. Take the product of all finite primes and add 1 to get a new prime:

$$\begin{aligned} P &= 1+X \ , \\ X &= \prod_p p \ . \end{aligned} \tag{7.2.1}$$

If P were divisible by finite prime then $P-X=1$ would be divisible by finite prime and one would encounter contradiction. One could of course worry about the possible existence of infinite primes smaller than P and possibly dividing P. The numbers $N=P-k$, $k>1$, are certainly not primes since k can be taken as a factor. The number $P'=P-2=-1+X$ could however be prime. P is certainly not divisible by $P-2$. It seems that one cannot express P and $P-2$ as product of infinite integer and finite integer. Neither it seems possible to express these numbers as products of more general numbers of form $\prod_{p\in U} p+q$, where U is infinite subset of finite primes and q is finite integer.

Step 2

P and $P-2$ are not the only possible candidates for infinite primes. Numbers of form

$$\begin{aligned} P(\pm,n) &= \pm 1+nX \ , \\ k(p) &= 0,1,..... \ , \\ n &= \prod_p p^{k(p)} \ , \\ X &= \prod_p p \ , \end{aligned} \tag{7.2.2}$$

where $k(p)\neq 0$ holds true only in finite set of primes, are characterized by a integer n, and are also good prime candidates. The ratio of these primes to the prime candidate P is given by integer n. In general, the ratio of two prime candidates $P(m)$ and $P(n)$ is rational number m/n telling which of the prime candidates is larger. This number provides ordering of the prime candidates $P(n)$. The reason why these numbers are good canditates for infinite primes is the same as above. No finite prime p with $k(p)\neq 0$ appearing in the product can divide these numbers since, by the same arguments as appearing in Euclid's theorem, it would divide also 1. On the other hand it seems difficult to invent any decomposition of these numbers containing infinite numbers. Already at this stage one can notice the structural analogy with the construction of multiboson states in quantum field theory: the numbers $k(p)$ correspond to the occupation numbers of bosonic states of quantum field theory in one-dimensional box, which suggests that the basic structure of QFT might have number theoretic interpretation in some very general sense. It turns out that this analogy generalizes.

Step 3

All $P(n)$ satisfy $P(n)\geq P(1)$. One can however also the possibility that $P(1)$ is not the smallest infinite prime and consider even more general candidates for infinite primes, which are smaller than $P(1)$. The trick is to drop from the infinite product of primes $X=\prod_p p$ some primes away by dividing it by integer $s=\prod_{p_i} p_i$, multiply this number by an integer n not divisible by any prime dividing s and to add to/subtract from the resulting number nX/s natural number ms such that m expressible as a product of powers of only those primes which appear in s to get

$$\begin{aligned} &P(\pm,m,n,s) = n\frac{X}{s} \pm ms \ , \\ &m = \prod_{p|s} p^{k(p)} \ , \\ &n = \prod_{p|\frac{X}{s}} p^{k(p)}, \ \ k(p) \geq 0 \ . \end{aligned} \tag{7.2.3}$$

Here $x|y$ means 'x divides y'. To see that no prime p can divide this prime candidate it is enough to calculate $P(\pm,m,n,s)$ modulo p: depending on whether p divides s or not, the prime divides only the second term in the sum and the result is nonzero and finite (although its precise value is not known). The ratio of these prime candidates to $P(+,1,1,1)$ is given by the rational number n/s: the ratio does not depend on the value of the integer m. One can however order the prime candidates with given values of n and s using the difference of two prime candidates as ordering criterion. Therefore these primes can be ordered.

One could ask whether also more general numbers of the form $n\frac{X}{s} \pm m$ are primes. In this case one cannot prove the indivisibility of the prime candidate by p not appearing in m. Furthermore, for $s \ mod \ 2 = 0$ and $m \ mod \ 2 \neq 0$, the resulting prime candidate would be even integer so that it looks improbable that one could obtain primes in more general case either.

Step 4

An even more general series of candidates for infinite primes is obtained by using the following ansatz which in principle is contained in the original ansatz allowing infinite values of n

$$\begin{aligned} &P(\pm,m,n,s|r) = nY^r \pm ms \ , \\ &Y = \frac{X}{s} \ , \\ &m = \prod_{p|s} p^{k(p)} \ , \\ &n = \prod_{p|\frac{X}{s}} p^{k(p)}, \ \ k(p) \geq 0 \ . \end{aligned} \tag{7.2.4}$$

The proof that this number is not divisible by any finite prime is identical to that used in the previous case. It is not however clear whether the ansatz for given r is not divisible by infinite primes belonging to the lower level. A good example in $r = 2$ case is provided by the following unsuccessful ansatz

$$\begin{aligned} &N = (n_1Y + m_1s)(n_2Y + m_2s) = \frac{n_1n_2X^2}{s^2} - m_1m_2s^2 \ , \\ &Y = \frac{X}{s} \ , \\ &n_1m_2 - n_2m_1 = 0 \ . \end{aligned}$$

Note that the condition states that n_1/m_1 and $-n_2/m_2$ correspond to the same rational number or equivalently that (n_1, m_1) and (n_2, m_2) are linearly dependent as vectors. This encourages the guess that all other $r = 2$ prime candidates with finite values of n and m at least, are primes. For higher values of r one can deduce analogous conditions guaranteing that the ansatz does not reduce to a product of infinite primes having smaller value of r. In fact, the conditions for primality state that the polynomial $P(n,m,r)(Y) = nY^r + m$ with integer valued coefficients ($n > 0$) defined by the prime candidate is irreducible in the field of integers, which means that it does not reduce to a product of lower order polynomials of same type.

Step 5

A further generalization of this ansatz is obtained by allowing infinite values for m, which leads to the following ansatz:

$$\begin{aligned} &P(\pm,m,n,s|r_1,r_2) = nY^{r_1} \pm ms \ , \\ &m = P_{r_2}(Y)Y + m_0 \ , \\ &Y = \frac{X}{s} \ , \\ &m_0 = \prod_{p|s} p^{k(p)} \ , \\ &n = \prod_{p|Y} p^{k(p)}, \ \ k(p) \geq 0 \ . \end{aligned} \tag{7.2.5}$$

Here the polynomial $P_{r_2}(Y)$ has order r_2 is divisible by the primes belonging to the complement of s so that only the finite part m_0 of m is relevant for the divisibility by finite primes. Note that the part proportional to s can be infinite as compared to the part proportional to Y^{r_1}: in this case one must however be careful with the signs to get the sign of the infinite prime correctly. By using same arguments as earlier one finds that these prime candidates are not divisible by finite primes. One must also require that the ansatz is not divisible by lower order infinite primes of the same type. These conditions are equivalent to the conditions guaranteing the polynomial primeness for polynomials of form $P(Y) = nY^{r_1} \pm (P_{r_2}(Y)Y + m_0)s$ having integer-valued coefficients. The construction of these polynomials can be performed recursively by starting from the first order polynomials representing first level infinite primes: Y can be regarded as formal variable and one can forget that it is actually infinite number.

By finite-dimensional analogy, the infinite value of m means infinite occupation numbers for the modes represented by integer s in some sense. For finite values of m one can always write m as a product of powers of $p_i|s$. Introducing explicitly infinite powers of p_i is not in accordance with the idea that all exponents appearing in the formulas are finite and that the only infinite variables are X and possibly S (formulas are symmetric with respect to S and X/S). The proposed representation of m circumvents this difficulty in an elegant manner and allows to say that m is expressible as a product of infinite powers of p_i despite the fact that it is not possible to derive the infinite values of the exponents of p_i.

Summarizing, an infinite series of candidates for infinite primes has been found. The prime candidates $P(\pm, m, n, s)$ labeled by rational numbers n/s and integers m plus the primes $P(\pm, m, n, s|r_1, r_2)$ constructed as r_1:th or r_2:th order polynomials of $Y = X/s$: the latter ansatz reduces to the less general ansatz of infinite values of n are allowed.

One can ask whether the $p\ mod\ 4 = 3$ condition guaranteing that the square root of -1 does not exist as a p-adic number, is satisfied for $P(\pm, m, n, s)$. $P(\pm, 1, 1, 1)\ mod\ 4$ is either 3 or 1. The value of $P(\pm, m, n, s)\ mod\ 4$ for odd s on n only and is same for all states containing even/odd number of $p\ mod\ = 3$ excitations. For even s the value of $P(\pm, m, n, s)\ mod\ 4$ depends on m only and is same for all states containing even/odd number of $p\ mod\ = 3$ excitations. This condition resembles G-parity condition of Super Virasoro algebras. Note that either $P(+, m, n, s)$ or $P(-, m, n, s)$ but not both are physically interesting infinite primes ($2m\ mod\ 4 = 2$ for odd m) in the sense of allowing complex Hilbert space. Also the additional conditions satisfied by the states involving higher powers of X/s resemble to Virasoro conditions. An open problem is whether the analogy with the construction of the many-particle states in super-symmetric theory might be a hint about more deeper relationship with the representation of Super Virasoro algebras and related algebras.

It is not clear whether even more general prime candidates exist. An attractive hypothesis is that one could write explicit formulas for all infinite primes so that generalized theory of primes would reduce to the theory of finite primes.

7.2.2 Infinite primes form a hierarchy

By generalizing using general construction recipe, one can introduce the second level prime candidates as primes not divisible by any finite prime p or infinite prime candidate of type $P(\pm, m, n, s)$ (or more general prime at the first level: in the following we assume for simplicity that these are the only infinite primes at the first level). The general form of these prime candidates is exactly the same as at the first level. Particle-analogy makes it easy to express the construction receipe. In present case 'vacuum primes' at the lowest level are of the form

$$\begin{array}{l} \frac{X_1}{S} \pm S \ , \\ X_1 = X\prod_{P(\pm,m,n,s)} P(\pm, m, n, s) \ , \\ S = s\prod_{P_i} P_i \ , \\ s = \prod_{p_i} p_i \ . \end{array} \tag{7.2.6}$$

S is product or ordinary primes p and infinite primes $P_i(\pm, m, n, s)$. Primes correspond to physical states created by multiplying X_1/S (S) by integers not divisible by primes appearing S (X_1/S). The integer valued functions $k(p)$ and $K(p)$ of prime argument give the occupation numbers associated with X/s and s type 'bosons' respectively. The non-negative integer-valued function $K(P) = K(\pm, m, n, s)$ gives the occupation numbers associated with the infinite primes associated with X_1/S and S type 'bosons'. More general primes can be constructed by mimicking the previous procedure.

One can classify these primes by the value of the integer $K_{tot} = \sum_{P|X/S} K(P)$: for a given value of K_{tot} the ratio of these prime candidates is clearly finite and given by a rational number. At given level the ratio P_1/P_2 of two primes is given by the expression

$$\frac{P_1(\pm,m_1,n_1,s_1K_1,S_1}{P_2(\pm,m_2,n_2,s_2,K,S_2)} = \frac{n_1s_2}{n_2s_1}\prod_{\pm,m,n,s}(\frac{n}{s})^{K_1^+(\pm,n,m,s)-K_2^+(\pm,n,m,s)} \quad . \tag{7.2.7}$$

Here K_i^+ denotes the restriction of $K_i(P)$ to the set of primes dividing X/S. This ratio must be smaller than 1 if it is to appear as the first order term $P_1P_2 \rightarrow P_1/P_2$ in the canonical identification and again it seems that it is not possible to get all rationals for a fixed value of P_2 unless one allows infinite values of N expressed neatly using the more general ansatz involving higher power of S.

7.2.3 Construction of infinite primes as a repeated quantization of a super-symmetric arithmetic quantum field theory

The procedure for constructing infinite primes is very much reminiscent of the second quantization of an super-symetric arithmetic quantum field theory in which single particle fermion and boson states are labeled by primes. In particular, there is nothing especially frightening in the particle representation of infinite primes: theoretical physicists actally use these kind of representations quite routinely.

1. The binary-valued function telling whether a given prime divides s can be interpreted as a fermion number associated with the fermion mode labeled by p. Therefore infinite prime is characterized by bosonic and fermionic occupation numbers as functions of the prime labeling various modes and situation is super-symmetric. X can be interpreted as the counterpart of Dirac sea in which every negative energy state state is occupied and $X/s \pm s$ corresponds to the state containing fermions understood as holes of Dirac sea associated with the modes labeled by primes dividing s.

2. The multiplication of the 'vacuum' X/s with $n = \prod_{p|X/s} p^{k(p)}$ creates $k(p)$ 'p-bosons' in mode of type X/s and multiplication of the 'vacuum' s with $m = \prod_{p|s} p^{k(p)}$ creates $k(p)$ 'p-bosons'. in mode of type s (mode occupied by fermion). The vacuum states in which bosonic creation operators act, are tensor products of two vacuums with tensor product represented as sum

$$|vac(\pm)\rangle = |vac(\frac{X}{s})\rangle \otimes |vac(\pm s)\rangle \leftrightarrow \frac{X}{s} \pm s \tag{7.2.8}$$

obtained by shifting the prime powers dividing s from the vacuum $|vac(X)\rangle = X$ to the vacuum ± 1. One can also interpret various vacuums as many fermion states. Prime property follows directly from the fact that any prime of the previous level divides either the first or second factor in the decomposition $NX/S \pm MS$.

3. This picture applies at each level of infinity. At a given level of hierarchy primes P correspond to all the Fock state basis of all possible many-particle states of second quantized super-symmetric theory. At the next level these many-particle states are regarded as single particle states and further second quantization is performed so that the primes become analogous to the momentum labels characterizing various single-particle states at the new level of hierarchy.

4. There are two nonequivalent quantizations for each value of S due to the presence of $\pm$ sign factor. Two primes differing only by sign factor are like G-parity $+1$ and -1 states in the sense that these primes satisfy $P\ mod\ 4 = 3$ and $P\ mod\ 4 = 1$ respectively. The requirement that -1 does not have p-adic square root so that Hilbert space is complex, fixes G-parity to say $+1$. This observation suggests that there exists a close analogy with the theory of Super Virasoro algebras so that quantum TGD might have interpretation as number theory in infinite context. An alternative interpretation for the $\pm$ degeneracy is as counterpart for the possibility to choose the fermionic vacuum to be a state in which either all positive or all negative energy fermion states are occupied.

5. One can also generalize the construction to include polynomials of $Y = X/S$ to get infinite hierarchy of primes labeled by the two integers r_1 and r_2 associated with the polynomials in question. An entire hierarchy of vacuums labeled by r_1 is obtained. A possible interpretation of these primes is as counterparts for the bound states of quantum field theory. The coefficient for the power $(X/s)^{r_1}$ appearing in the highest term of the general ansatz, codes the occupation numbers associated with vacuum $(X/s)^{r_1}$. All the remaining terms are proportional to s and combine to form, in general infinite, integer m characterizing various infinite occupation numbers for the subsystem characterized by s. The additional conditions guaranteing prime number property are equivalent with the primality conditions for polynomials with integer valued coefficients and resemble Super Virasoro conditions. For $r_2 > 0$ bosonic occupation numbers associated with the modes with fermion number one are infinite and one cannot write explicit formula for the boson number.

6. One could argue that the analogy with super-symmetry is not complete. The modes of Super Virasoro algebra are labeled by natural number whereas now modes are labeled by prime. This need not be a problem since one can label primes using natural number n. Also 8-valued spin index associated with fermionic and bosonic single particle states in TGD world is lacking (space-time is surface in 8-dimensional space). This index labels the spin states of 8-dimensional spinor with fixed chirality. One could perhaps get also spin index by considering infinite octonionic primes, which correspond to vectors of 8-dimensional integer lattice such that the length squared of the lattice vector is ordinary prime:

$$\sum_{k=1,\ldots,8} n_k^2 = \text{ prime } .$$

   Thus one cannot exclude the possibility that TGD based physics might provide representation for octonions extended to include infinitely large octonions. The notion of prime octonion is well defined in the set of integer octonions and it is easy to show that the Euclidian norm squared for a prime octonion is prime. If this result generalizes then the construction of generalized prime octonions would generalize the construction of finite prime octonions. It would be interesting to know whether the results of finite-dimensional case might generalize to the infinite-dimensional context. One cannot exclude the possibility that prime octonions are in one-one correspondence with physical states in quantum TGD.

These observations suggest a close relationship between quantum TGD and the theory of infinite primes in some sense: even more, entire number theory and mathematics might be reducible to quantum physics understood properly or equivalently, physics might provide the representation of basic mathematics. Of course, already the uniqueness of the basic mathematical structure of quantum TGD points to this direction. Against this background the fact that 8-dimensionality of the imbedding space allows introduction of octonion structure (also p-adic algebraic extensions) acquires new meaning. Same is also suggested by the fact that the algebraic extensions of p-adic numbers allowing square root of real p-adic number are 4- and 8-dimensional.

What is especially interesting is that the core of number theory would be concentrated in finite primes since infinite primes are obtained by straightforward procedure providing explicit formulas for them. Repeated quantization provides also a model of abstraction process understood as construction of hierarchy of natural number valued functions about functions about ...... At the first level infinite primes are characterized by the integer valued function $k(p)$ giving occupation numbers plus subsystem-complement division (division to thinker and external world!). At the next level prime is characterized in a similar manner. One should also notice that infinite prime at given level is characterized by a pair $(R = MN, S)$ of integers at previous level. Equivalently, infinite prime at given level is characterized by fermionic and bosonic occupation numbers as functions in the set of primes at previous level.

7.2.4 Construction in the case of an arbitrary commutative number field

The basic construction recipe for infinite primes is simple and generalizes even to the case of algebraic extensions of rationals. Let $K = Q(\theta)$ be an algebraic number field (see the Appendix of [?] for the basic definitions). In the general case the notion of prime must be replaced by the concept of irreducible defined as an algebraic integer with the property that all its decompositions to a product of two integers are such that second integer is always a unit (integer having unit algebraic norm, see Appendix of [?]).

Assume that the irreducibles of $K = Q(\theta)$ are known. Define two irreducibles to be equivalent if they are related by a multiplication with a unit of K. Take one representative from each equivalence class of units. Define the irreducible to be positive if its first non-vanishing component in an ordered basis for the algebraic extension provided by the real unit and powers of θ, is positive. Form the counterpart of Fock vacuum as the product X of these representative irreducibles of K.

The unique factorization domain (UFD) property (see Appendix of [?]) of infinite primes does not require the ring O_K of algebraic integers of K to be UFD although this property might be forced somehow. What is needed is to find the primes of K; to construct X as the product of all irreducibles of K but not counting units which are integers of K with unit norm; and to apply second quantization to get primes which are first order monomials. X is in general a product of powers of primes. Generating infinite primes at the first level correspond to generalized rationals for K having similar representation in terms of powers of primes as ordinary rational numbers using ordinary primes.

7.2.5 Mapping of infinite primes to polynomials and geometric objects

The mapping of the generating infinite primes to first order monomials labeled by their rational zeros is extremely simple at the first level of the hierarchy:

$$P_{\pm}(m,n,s) = \frac{mX}{s} \pm ns \rightarrow x_{\pm} \pm \frac{m}{sn} \quad . \tag{7.2.9}$$

Note that a monomial having zero as its root is not obtained. This mapping induces the mapping of all infinite primes to polynomials.

The simplest infinite primes are constructed using ordinary primes and second quantization of an arithmetic number theory corresponds in one-one manner to rationals. Indeed, the integer $s = \prod_i p_i^{k_i}$ defining the numbers k_i of bosons in modes k_i, where fermion number is one, and the integer r defining the numbers of bosons in modes where fermion number is zero, are co-prime. Moreover, the generating infinite primes can be written as $(n/s)X \pm ms$ corresponding to the two vacua $V = X \pm 1$ and the roots of corresponding monomials are positive *resp.* negative rationals.

More complex infinite primes correspond sums of powers of infinite primes with rational coefficients such that the corresponding polynomial has rational coefficients and roots which are not rational but belong to some algebraic extension of rationals. These infinite primes correspond simply to products of infinite primes associated with some algebraic extension of rationals. Obviously the construction of higher infinite primes gives rise to a hierarchy of higher algebraic extensions.

It is possible to continue the process indefinitely by constructing the Dirac vacuum at the n:th level as a product of primes of previous levels and applying the same procedure. At the second level Dirac vacuum $V = X \pm 1$ involves X which is the product of all primes at previous levels and in the polynomial correspondence X thus correspond to a new independent variable. At the n:th level one would have polynomials $P(q_1|q_2|...)$ of q_1 with coefficients which are rational functions of q_2 with coefficients which are.... The hierarchy of infinite primes would be thus mapped to the functional hierarchy in which polynomial coefficients depend on parameters depending on ....

At the second level one representation of infinite primes would be as algebraic curve resulting as a locus of $P(q_1|q_2) = 0$: this certainly makes sense if q_1 and q_2 commute. At higher levels the locus is a higher-dimensional surface.

7.2.6 How to order infinite primes?

One can order the infinite primes, integers and rationals. The ordering principle is simple: one can decompose infinite integers to two parts: the 'large' and the 'small' part such that the ratio of the small part with the large part vanishes. If the ratio of the large parts of two infinite integers is different from one or their sign is different, ordering is obvious. If the ratio of the large parts equals to one, one can perform same comparison for the small parts. This procedure can be continued indefinitely.

In case of infinite primes ordering procedure goes like follows. At given level the ratios are rational numbers. There exists infinite number of primes with ratio 1 at given level, namely the primes with same values of N and same S with MS infinitesimal as compared to NX/S. One can order these primes using either the relative sign or the ratio of $(M_1S_1)/(M_2S_2)$ of the small parts to decide which of the two is larger. If also this ratio equals to one, one can repeat the process for the small parts of M_iS_i. In principle one can repeat this process so many times that one can decide which of the two primes is larger. Same of course applies to infinite integers and also to infinite rationals build from primes with infinitesimal MS. If NS is not infinitesimal it is not obvious whether this procedure works. If $N_iX_i/M_iS_i = x_i$ is finite for both numbers (this need not be the case in general) then the ratio $\frac{M_1S_1}{M_2S_2}\frac{(1+x_2)}{(1+x_1)}$ provides the needed criterion. In case that this ratio equals one, one can consider use the ratio of the small parts multiplied by $\frac{(1+x_2)}{(1+x_1)}$ of M_iS_i as ordering criterion. Again the procedure can be repeated if needed.

7.2.7 What is the cardinality of infinite primes at given level?

The basic problem is to decide whether Nature allows also integers S , $R = MN$ represented as infinite product of primes or not. Infinite products correspond to subsystems of infinite size (S) and infinite total occupation number (R) in QFT analogy.

1. One could argue that S should be a finite product of integers since it corresponds to the requirement of finite size for a physically acceptable subsystem. One could apply similar argument to R. In this case the set of primes at given level has the cardinality of integers ($alef_0$) and the cardinality of all infinite primes is that of integers. If also infinite integers R are assumed to involve only finite products of infinite primes the set of infinite integers is same as that for natural numbers.

2. NMP is well defined in p-adic context also for infinite subsystems and this suggests that one should allow also infinite number of factors for both S and $R = MN$. Super symmetric analogy suggests the same: one can quite well consider the possibility that the total fermion number of the universe is infinite. It seems however natural to assume that the occupation numbers $K(P)$ associated with various primes P in the representations $R = \prod_P P^{K(P)}$ are finite but nonzero for infinite number of primes P. This requirement applied to the modes associated with S would require the integer m to be explicitly expressible in powers of $P_i|S$ ($P_{r_2} = 0$) whereas all values of r_1 are possible. If infinite number of prime factors is allowed in the definition of S, then the application of diagonal argument of Cantor shows that the number of infinite primes is larger than $alef_0$ already at the first level. The cardinality of the first level is $2^{alef_0}2^{alef_0} == 2^{alef_0}$. The first factor is the cardinality of reals and comes from the fact that the sets S form the set of all possible subsets of primes, or equivalently the cardinality of all possible binary valued functions in the set of primes. The second factor comes from the fact that integers $R = NM$ (possibly infinite) correspond to all natural number-valued functions in the set of primes: if only finite powers $k(p)$ are allowed then one can map the space of these functions to the space of binary valued functions bijectively and the cardinality must be 2^{alef_0}. The general formula for the cardinality at given level is obvious: for instance, at the second level the cardinality is the cardinality of all possible subsets of reals. More generally, the cardinality for a given level is the cardinality for the subset of possible subsets of primes at the previous level.

7.2.8 How to generalize the concepts of infinite integer, rational and real?

The allowance of infinite primes forces to generalize also the concepts concepts of integer, rational and real number. It is not obvious how this could be achieved. The following arguments lead to a possible generalization which seems practical (yes!) and elegant.

Infinite integers form infinite-dimensional vector space with integer coefficients

The first guess is that infinite integers N could be defined as products of the powers of finite and infinite primes.

$$N = \prod_k p_k^{n_k} = nM \ , \ n_k \geq 0 \ , \tag{7.2.10}$$

where n is finite integer and M is infinite integer containing only powers of infinite primes in its product expansion.

It is not however not clear whether the sums of infinite integers really allow similar decomposition. Even in the case that this decomposition exists, there seems to be no way of deriving it. This would suggest that one should regard sums

$$\sum_i n_i M_i$$

of infinite integers as infinite-dimensional linear space spanned by M_i so that the set of infinite integers would be analogous to an infinite-dimensional algebraic extension of say p-adic numbers such that each coordinate axes in the extension corresponds to single infinite integer of form $N = mM$. Thus the most general infinite integer N would have the form

$$N = m_0 + \sum m_i M_i \ . \tag{7.2.11}$$

This representation of infinite integers indeed looks promising from the point of view of practical calculations. The representation looks also attractive physically. One can interpret the set of integers N as a linear space with integer coefficients m_0 and m_i:

$$N = m_0|1\rangle + \sum m_i|M_i\rangle \ . \tag{7.2.12}$$

$|M_i\rangle$ can be interpreted as a state basis representing many-particle states formed from bosons labeled by infinite primes p_k and $|1\rangle$ represents Fock vacuum. Therefore this representation is analogous to a quantum superposition of bosonic Fock states with integer, rather than complex valued, superposition coefficients. If one interprets M_i as orthogonal state basis and interprets m_i as p-adic integers, one can define inner product as

$$\langle N_a, N_b\rangle = m_0(a)m_0(b) + \sum_i m_i(a)m_i(b) \ . \tag{7.2.13}$$

This expression is well defined p-adic number if the sum contains only enumerable number of terms and is always bounded by p-adic ultrametricity. It converges if the p-adic norm of of m_i approaches to zero when M_i increases.

Generalized rationals

Generalized rationals could be defined as ratios $R = M/N$ of the generalized integers. This works nicely when M and N are expressible as products of powers of finite or infinite primes but for more general integers the definition does not look attractive. This suggests that one should restrict the generalized rationals to be numbers having the expansion as a product of positive and negative primes, finite or infinite:

$$N = \prod_k p_k^{n_k} = \frac{n_1 M_1}{nM} \ . \tag{7.2.14}$$

Generalized reals form infinite-dimensional real vector space

One could consider the possibility of defining generalized reals as limiting values of the generalized rationals. A more practical definition of the generalized reals is based on the generalization of the pinary expansion of ordinary real number given by

$$\begin{aligned} x &= \sum_{n\geq n_0} x_n p^{-n} \ , \\ x_n &\in \{0,..,p-1\} \ . \end{aligned} \tag{7.2.15}$$

It is natural to try to generalize this expansion somehow. The natural requirement is that sums and products of the generalized reals and canonical identification map from the generalized reals to generalized p-adcs are readily calculable. Only in this manner the representation can have practical value.

These requirements suggest the following generalization

$$\begin{aligned} X &= x_0 + \sum_N x_N p^{-N} \ , \\ N &= \sum_i m_i M_i \ , \end{aligned} \tag{7.2.16}$$

where x_0 and x_N are ordinary reals. Note that N runs over infinite integers which has *vanishing finite part*. Note that generalized reals can be regarded as infinite-dimensional linear space such that each infinite integer N corresponds to one coordinate axis of this space. One could interpret generalized real as a superposition of bosonic Fock states formed from single single boson state labeled by prime p such that occupation number is either 0 or infinite integer N with a vanishing finite part:

$$X = x_0|0\rangle + \sum_N x_N|N> \ . \tag{7.2.17}$$

The natural inner product is

$$\langle X,Y\rangle = x_0y_0 + \sum_N x_N y_N \ . \tag{7.2.18}$$

The inner product is well defined if the number of N:s in the sum is enumerable and x_N approaches zero sufficiently rapidly when N increases. Perhaps the most natural interpretation of the inner product is as R_p valued inner product.

The sum of two generalized reals can be readily calculated by using only sum for reals:

$$X+Y = x_0 + y_0 + \sum_N (x_N + y_N)p^{-N} \ , \tag{7.2.19}$$

The product XY is expressible in the form

$$XY = x_0y_0 + x_0Y + Xy_0 + \sum_{N_1,N_2} x_{N_1}y_{N_2}p^{-N_1-N_2} \ , \tag{7.2.20}$$

If one assumes that infinite integers form infinite-dimensional vector space in the manner proposed, there are no problems and one can calculate the sums $N_1 + N_2$ by summing component wise manner the coefficients appearing in the sums defining N_1 and N_2 in terms of infinite integers M_i allowing expression as a product of infinite integers.

Canonical identification map from ordinary reals to p-adics

$$x = \sum_k x_k p^{-k} \ \rightarrow \ x_p = \sum_k x_k p^k \ ,$$

generalizes to the form

$$x = x_0 + \sum_N x_N p^{-N} \ \rightarrow \ (x_0)_p + \sum_N (x_N)_p p^N \ , \tag{7.2.21}$$

so that all the basic requirements making the concept of generalized real calculationally useful are satisfied.

There are several interesting questions related to generalized reals.

1. Are the extensions of reals defined by various values of p-adic primes mathematically equivalent or not? One can map generalized reals associated with various choices of the base p to each other in one-one manner using the mapping

$$X = x_0 + \sum_N x_N p_1^{-N} \rightarrow x_0 + \sum_N x_N p_2^{-N} . \tag{7.2.22}$$

   The ordinary real norms of *finite* (this is important!) generalized reals are identical since the representations associated with different values of base p differ from each other only infinitesimally. This would suggest that the extensions are physically equivalent. It these extensions are not mathematically equivalent then p-adic primes could have a deep role in the definition of the generalized reals.

2. One can generalize previous formulas for the generalized reals by replacing the coefficients x_0 and x_i by complex numbers, quaternions or octonions so as to get generalized complex numbers, quaternions and octonions. Also inner product generalizes in an obvious manner. The 8-dimensionality of the imbedding space provokes the question whether it might be possible to regard the infinite-dimensional configuration space of 3-surfaces, or rather, its tangent space, as a Hilbert space realization of the generalized octonions. This kind of identification could perhaps reduce TGD based physics to generalized number theory.

7.2.9 Comparison with the approach of Cantor

The main difference between the approach of Cantor and the proposed approach is that Cantor uses only the basic arithmetic concepts such as sum and multiplication and the concept of successor defining ordering of both finite and infinite ordinals. Cantor's approach is also purely set theoretic. The problems of purely set theoretic approach are related to the question what the statement 'Set is Many allowing to regard itself as One' really means and to the fact that there is no obvious connection with physics. The proposed approach is based on the introduction of the concept of prime as a basic concept whereas ordering is based on the use of ratios: using these one can recursively define ordering and get precise quantitative information based on finite reals. Together with canonical identification the concept of infinite primes becomes completely physical in the sense that all probabilities are always finite real numbers. The 'Set is Many allowing to regard itself as One' is defined as quantum physicist would define it: many particle states become single particle states in the second quantization describing the counterpart for the construction of the set of subsets of a given set. One could also say that integer as such corresponds to set as 'One' and its decomposition to a product of primes corresponds to the set as 'Many'. The concept of prime, the ultimate 'One', has as its physical counterpart the concept of elementary particle understood in very general sense. The new element is the physical interpretation: the sum of two numbers whose ratio is zero correspond to completely physical finite-subsystem-infinite complement division and the iterated construction of the set of subsets of a set at given level is basically p-adic evolution understood in the most general possible sense and realized as a repeated second quantization. What is attractive is that this repeated second quantization can be regarded also as a model of abstraction process and actually the process of abstraction itself.

The possibility to interpret the construction of infinite primes either as a repeated bosonic quantization involving subsystem-complement division or as a repeated super-symmetric quantization could have some deep meaning. A possible interpretation consistent with these two pictures is based on the hypothesis that fermions provide a reflective level of consciousness in the sense that the 2^N element Fock basis of many-fermion states formed from N single-fermion states can be regarded as a set of all possible statements about N basic statements. Statements about whether a given element of set X belongs to some subset S of X are certainly the fundamental statements from the point of view of mathematics. Hence one could argue that many-fermion states provide cognitive representation for the subsets of some set. Single fermion states represent the points of the set and many-fermion states represent possible subsets.

7.3 Generalizing the notion of infinite prime to the non-commutative context

The notion of prime and more generally, that of irreducible, makes sense also in more general number fields and even algebras. The considerations of [?] suggests that the notion of infinite prime should be generalized to the case of complex numbers, quaternions, and octonions as well as to their hyper counterparts which seem to be physically the most interesting ones [?]. Also the hierarchy of infinite primes should generalize as well as the representation of infinite primes as polynomials and as space-time surfaces. The proposed number theoretic realization of the dynamics defined by the absolute minimization of Kähler action can be realized if it is possible to assign hyper-octonion analytic functions to infinite hyper-octonionic primes [?].

7.3.1 General view about the construction of generalized infinite primes

The consideration of basic objections against quaternionic and octonionic infinite primes allows to identify the basic philosophical ideas serving as guidelines for the construction of infinite primes.

Infinite primes should be commutative and associative

The basic objections against (hyper-)quaternionic and (hyper-)octonionic infinite primes relate to the non-commutativity and non-associativity.

1. In the case of quaternionic infinite primes non-commutativity, and in the case of octonionic infinite primes also non-associativity, might be expected to cause difficulties in the definition of X. Fortunately, the fact that all conjugates of a given finite prime appear in the product defining X, implies that the contribution from each irreducible with a given norm p is real and X is real. Therefore the multiplication and division of X with quaternionic or octonionic primes is a well-defined procedure, and generating infinite primes are well-defined apart from the degeneracy due to non-commutativity and non-associativity of the finite number of lower level primes. Also the products of infinite primes are well defined, since by the reality of X it is possible to tell how the products AB and BA differ. Of course, also infinite primes representing physical states containing infinite numbers of fermions and bosons are possible and infinite primes of this kind must be analogous to generators of a free algebra for which AB and BA are not related in any manner.

2. The sums of products of monomials of generating infinite primes define higher level infinite primes and also here non-commutativity and associativity cause potential difficulties. The assignment of a monomial to a quaternionic or octonionic infinite prime is not unique since the rational obtained by dividing the finite part mr with the integer n associated with infinite part can be defined either as $(1/n) \times mr$ or $mr \times (1/n)$ and the resulting non-commuting rationals are different.

If the polynomial associated with infinite prime has real-rational coefficients these difficulties do not appear. This would imply universality in the sense that the polynomials as such would not contain information about the number field in question. This number theoretic universality is highly attractive also physically.

The reduction of the roots of polynomials to complex roots encourages the idea about the analogy with quantum measurement theory. Although it is possible to define more general infinite primes, it seems that the primes having representation as space-time surface are reducible to those represented by polynomials with real-rational coefficients. This would mean that the number field field would not be seen at all in the characterization of the polynomial. The roots of the polynomial would be in general complex and effective 2-dimensionality would prevail in this sense. Complex planes of quaternions and octonions space define maximal commutative sub-fields of them. In the case of hyper-quaternions and hyper-octonions hyper-complex planes take the role of maximal sub-algebra which is closed and at the same time commutative. Interestingly, the hyper-octonionic solution ansatz involves a local fixing of a hyper-complex algebra at each point of $HO = M^8$ physically equivalent with the fixing the space of longitudinal polarizations.

At space-time level this should correspond to effective 2-dimensionality in the sense that quantum states and space-time surfaces are coded by the data associated with 2-dimensional partonic surfaces at the intersections of 3-D and 7-D light-like causal determinants. The tangent spaces of these surfaces should be dual to the local hyper-complex longitudinal polarization planes. The induced selection of the transversal polarization plane at each space-time point could be also seen as the number theoretical analog for the selection of a rest frame and of quantization axis for spin.

Commutativity requirement for infinite primes allows real-rationals or possibly algebraic extensions of them as the coefficients of the polynomials formed from hyper-octonionic infinite primes. If only infinite primes with complex rational coefficients are allowed and only the vacuum state $V_{\pm} = X \pm 1$ involving product over all primes of the number field, would reveal the number field. One could thus construct the generating infinite primes using the notion of hyper-octonionic prime for any algebraic extension of rationals.

Do hyper-octonionic infinite primes correspond to space-time surfaces?

The general philosophy behind the construction of infinite primes involves at least the following ideas.

1. Quantum TGD should result as an algebraic continuation of rational number based physics to various number fields. Similar continuation principle should hold true also for infinite primes. This means that the formal expressions for infinite primes should be essentially same as those associated with the infinite primes associated with the field or rational numbers or complex rationals. As far as space-time representation in terms of polynomials is considered, this means that the polynomials involved should have real coefficients. An analogous situation should prevail at the higher levels of the hierarchy.

2. Hyper-octonionic primes are favored physically and if they have representation as polynomials or more general rational functions of hyper-octonion with real-rational coefficients, it is possible to assign to each prime a 4-parameter foliation of $M^4 \times CP_2$ hyper-quaternionic space-time surfaces by the construction of [**?**]. Also the dual of the foliation defines a foliation and canonically imbedded M^4 and CP_2 provide a basic example of dual 4-surfaces. The foliations are parameterized by functions $HO = M^8 \rightarrow S^6$ fixing the preferred octonionic imaginary unit. A possible identification is in terms of vacuum degeneracy. The fixing of the imaginary unit means fixing of complex plane of octonions and the physical interpretation is as a local fixing of longitudinal polarization directions having interpretation as gauge degrees of freedom.

The decomposition of rational infinite primes to hyper-octonionic could have a physical meaning

The requirement that hyper-octonionic infinite primes correspond at the highest level to polynomials with rational coefficients would mean an effective reducibility to rational infinite primes.

The reduction to rational infinite primes does not mean trivialization of the theory. One can decompose infinite rational primes to a product of hyper-octonionic primes just as one can decompose them to a product of primes in algebraic extensions of rational numbers and this decomposition might have a physical interpretation as a decomposition of a particle to its composites if one accepts the idea that the hierarchy of algebraic extensions corresponds to a hierarchy of increasing measurement resolutions. The reduction to a rational infinite prime implies that hyper-octonionic primes and their conjugates appear in a pairwise manner in the products involved. Hence the net values of the transversal parts of infinite hyper-octonic 8-momenta vanish and one could speak about the vanishing of transversal M^8 momenta in M^8 context. In H context this brings in mind the vanishing of transversal M^4 momenta for hadron and vanishing of color quantum numbers.

Commutativity and associativity for infinite primes does not imply commutativity and associativity for corresponding polynomials

The commutativity of infinite primes is not enough to eliminate completely the effects due to non-commutativity and non-associativity in case of corresponding polynomials. For the hyper-octonionic infinite primes at higher levels of hierarchy non-associativity causes delicate effects since the grouping of infinite primes affects the polynomial associated with the infinite prime and thus space-time surface associated with the infinite prime. Only for arguments $h_1, ..h_n$ restricted to a 2-dimensional subspace H_2 of M^8 the effects due to non-commutativity and non-associativity are completely absent and this conforms nicely with the notion of effective 2-dimensionality meaning that the physical on-associativity and non-commutativity are trivial and correspond to gauge degrees of freedom.

The unique solution to the problems is to assign to infinite hyper-octonionic primes polynomials for which all arguments h_i are identical $h_n = ... = h_1 = h$. A more general solution would be based on the assumption that the arguments of the polynomial are related by hyper-octonion real-analytic rational function. This option also allows to assign to hyper-octonionic infinite primes 4-D surfaces in a natural manner if hyper-octonion real-analyticity gives rise to a foliation of M^8 by quaternionic 4-surfaces. In this framework the proposed mapping of infinite primes to space-time surfaces could be seen as being natural because hyper-octonionic primes are associated with a maximal algebraic completion.

The interpretation of two vacuum primes in terms of positive and negative energy Fock states

In the rational case the positivity of primes means that $V_{\pm} = X \pm 1$ correspond to two non-equivalent Fock vacua. For hyper-octonionic primes the two vacua correspond to the to different signs of energy related by time reflection since the units with $n_0 < 0$ correspond to time reflection combined with Lorentz boost. The real part of a hyper-octonionic generating prime can be made non-vanishing by an application of a suitable boost represented by unit.

In TGD the time-orientation of the space-time sheet can be also negative and this means that energies can be either positive or negative [**?**, **?**]. The interpretation of the two vacua is as vacua associated with space-time sheets of negative and positive time orientation. The possibility that the sign of inertial energy is negative has profound implications and defines one of the most important differences between TGD and competing theories.

Physically it would be desirable that also more complex infinite primes having interpretation as representations of bound states could be interpreted as composites of states of unique positive and negative energy generating primes. If the positive and negative energy infinite primes correspond to states with fermion numbers, one must assume that the polynomials of the generating infinite primes are superpositions of products of monomials of degree n_+ and n_- with respect to the generating infinite primes $P_{\pm}(m, n, s)$ such that $n = n_+ - n_-$ is constant.

The vacua $X \pm 1$ can be interpreted as rational infinite primes, which are however not constructible from rational vacuum $X = \prod_p p$ by a finite number of steps since each rational prime p appears with some power $N(p)$ counting the number of positive primes with norm

$$N(\pi) = h_0^2 - \sum h_i^2 = p \quad .$$

Thus one has

$$X = \prod_{\pi > 0} \pi = \prod_p p^{N(p)} \quad .$$

Numbers with components in real algebraic extensions of rationals would pop-up dynamically, when one factorizes polynomials which are irreducible in the field of rationals.

If algebraic extensions of rationals are allowed as a fundamental number field, $N(\pi)$ must be replaced with

$$N(\pi) = N_K(h_0^2 - \sum_i h_i^2) = p \quad .$$

Only one representative of positive primes related by a multiplication with real Dirichlet units representable as fractal scalings can be included (note that the number of Dirichlet units is always infinite for the real extensions of rationals). This gives a finite number of primes for given p. This option is however not attractive physically since it is in conflict with the idea that algebraic extensions pop up dynamically from the representations of the polynomial as space-time surface.

7.3.2 Quaternionic and octonionic primes and their hyper counterparts

The loss of commutativity and associativity implies that the definitions of (hyper-)quaternionic and (hyper-)octonionic primes are not completely straightforward.

Basic facts about quaternions and octonions

Both quaternions and octonions allow both Euclidian norm and the Minkowskian norm defined as a trace of the linear operator defined by the multiplication with octonion. Minkowskian norm has the metric signature of $H = M^4 \times CP_2$ or $M^4_+ \times CP_2$ so that H can be regarded locally as an octonionic space. Both norms are a multiplicative and the notions of both quaternionic and octonionic prime are well defined despite non-associativity. Quaternionic and octonionic primes have length squared equal to rational prime.

In the case of quaternions different basis of imaginary units I, J, K are related by 3-dimensional rotation group and different quaternionic basis span a 3-dimensional sphere. There is 2-sphere of complex structures since imaginary unit can be any unit vector of imaginary 3-space.

A basis for octonionic imaginary units J, K, L, M, N, O, P can be chosen in many manners and fourteen-dimensional subgroup G_2 of the group $SO(7)$ of rotations of imaginary units is the group labeling the octonionic structures related by octonionic automorphisms to each other. It deserves to be mentioned that G_2 is unique among the simple Lie-groups in that the ratio of the square roots of lengths for long and short roots of G_2 Lie-algebra are in ratio $3:1$ [?]. For other Lie-groups this ratio is either 2:1 or all roots have same length. The set of equivalence classes of the octonion structures is $SO(7)/G_2 = S^7$. In the case of quaternions there is only one equivalence class.

The group of automorphisms for octonions with a fixed imaginary part is $SU(3)$. The coset space $S^6 = G_2/SU(3)$ labels possible complex structures of the octonion space specified by a selection of a preferred imaginary unit. $SU(3)/U(2) = CP_2$ could be thought of as the space of octonionic structures giving rise to a given quaternionic structure with complex structure fixed. This can be seen as follows. The units $1, I$ are $SU(3)$ singlets whereas J, J_1, J_2 and K, K_1, K_2 form $SU(3)$ triplet and antitriplet. Under $U(2)$ J and K transform like objects having vanishing $SU(3)$ isospin and suffer only a $U(1)$ phase transformation determined by multiplication with complex unit I and are mixed with each other in orthogonal mixture. Thus $1, I, J, K$ is transformed to itself under $U(2)$.

Quaternionic and octonionic primes

Quaternionic primes with $p \ mod \ 4 = 1$ can correspond to (n_1, n_2) with n_1 even and n_2 odd or vice versa. For $p \ mod \ 4 = 3$ (n_1, n_2, n_3) with n_i odd is the minimal option. In this case there is however large number of primes having only two components: in particular, Gaussian primes with $p \ mod \ 4 = 1$ define also quaternionic primes. Purely real Gaussian primes with $p \ mod \ 4 = 3$ with norm $z\overline{z}$ equal to p^2 are not quaternionic primes, and are replaced with 3-component quaternionic primes allowing norm equal to p. Similar conclusions hold true for octonionic primes.

The reality condition for polynomials associated with Gaussian infinite primes requires that the products of generating prime and its conjugate are present so that the outcome is a real polynomial of second order.

Hyper primes

The notion of prime generalizes to hyper-quaternionic and octonionic case. The factorization $n_0^2 - n_3^2 = (n_0 + n_3)(n_0 - n_3)$ implies that any hyper-quaternionic and -octonionic primes can be represented as $(n_0, n_3, 0, ...) = (n_3 + 1, n_3, 0, ...)$, $n_3 = (p-1)/2$ for $p > 2$. $p = 2$ is exceptional: a representation with minimal number of components is given by $(2, 1, 1, 0, ...)$. Notice that the interpretation of hyper-quaternionic primes (or integers) as four-momenta implies that it is not possible to find rest system for them: only a system where energy is minimum is possible.

The notion of "irreducible" (see Appendix of [?]) is defined as the equivalence class of primes related by a multiplication with a unit and is more fundamental than that of prime. All Lorentz boosts of a hyper prime combine to form an irreducible. Note that the units cannot correspond to real particles in corresponding arithmetic quantum field theory.

If the situation for $p > 2$ is effectively 2-dimensional in the sense that it is always possible to transform the hyper prime to a 2-component form by multiplying it by a suitable unit representing

Lorentz boost, the theory for time-like hyper primes effectively reduces to the 2-dimensional hyper-complex case when irreducibles are chosen to belong to H_2. The physical counterpart for the choice of H_2 would be the choice of the plane of longitudinal polarizations, or equivalently, of quantization axis for spin. This hypothesis is physically highly attractive since it would imply number theoretic universality and conform with the effective 2-dimensionality. Of course, the hyper-octonionic primes related by $SO(7,1)$ boosts need not represent physically equivalent states.

Also the rigorous notion of hyper primeness seems to require effective 2-dimensionality. If effective 2-dimensionality holds true, hyper integers have a decomposition to a product of hyper primes multiplied by a suitable unit. The representation is obtained by Lorentz boosting the hyper integer first to a 2-component form and then decomposing it to a product of hyper-complex primes.

The situation becomes certainly more complex if also space-like primes with negative norm squared $n_0^2 - n_1^2 - ... = -p$ are allowed. Gaussian primes with $p \ mod4 \ = 1$ are representable as space-like primes of form $(0, n_1, n_2, 0)$: $n_1^2 + n_2^2 = p$. Space-like primes with $p \ mod \ 4 = 3$ have at least 3 non-vanishing components which are odd integers.

7.3.3 Hyper-octonionic infinite primes

The infinite-primes associated with hyper-octonions are the most natural ones physically because of the underlying Lorentz invariance and the possibility to interpret them as 8-momenta with mass squared equal to prime. M^8 is consistent with the metric signature of the tangent space of H, and the four additional momentum components bring strongly in mind the tangent space counterpart of CP_2 contribution to the mass squared. Also the interpretation of quaternionic part of finite hyper-octonionic primes in terms of electro-weak and color quantum numbers could be considered since the total number of them is $2 + 2 = 4$.

Construction recipe at the lowest level of hierarchy assuming reduction to rational infinite primes

The condition that allowed hyper-octonionic infinite primes correspond to decompositions of rational infinite primes to products of their hyper-octonionic counterparts is the simplest manner to define them and generalizes the decomposition of rational infinite primes to products of primes in algebraic extensions of rationals.

This allows primes in algebraic extensions of rationals containing $\sqrt{-1}$ only if one interprets the commuting unit of hyper-octonionic integers as imaginary unit associated with the algebraic extensions of rationals. Composites of infinite primes in complexification of octonions would be in question. The reality of the coefficients of the polynomials assignable to infinite primes would also mean that the M^8 coordinates of M^8 stay real.

The physical interpretation for the reduction to rational infinite primes would be in terms of number theoretic analog of color confinement meaning decomposition of particles to their composites becoming visible in an improved algebraic resolution. Also the interpretation in terms of non-commutative geometry in transversal degrees of freedom meaning that only longitudinal momenta corresponding to non-vanishing of only hyper-complex part of hyper-octonionic 8-momentum. Indeed, the commutation relations $xy = qyz$, $q = exp(i\pi/n)$ for quantum plane would allow the vanishing of x and y identified now as components of transversal momentum.

More general construction recipe at the lowest level of hierarchy

The following argument represents the construction recipe for the first level hyper-octonionic primes without the assumption about the reduction to rational infinite primes.

1. Infinite prime property requires that X must be defined by taking one representative from each equivalence class representing irreducible and forming the product of their conjugates. The representative hyper-octonionic primes can be taken to be time-like positive energy primes. The conjugates of each irreducible appear in X so for a given norm p the net result is real for each rational prime p.

   The number of conjugates depends on the number of non-vanishing components of the the prime with norm p in the minimal representation having minimal energy. Several primes with a given norm p not related by a multiplication with unit or by automorphism are in principle possible. The degeneracy is determined by the number of elements of a subgroup of Galois group acting non-trivially on the prime. Galois group is generated by the permutations of 7 imaginary units and 7 conjugations of units consistent with the octonionic product. X is proportional to $p^{N(p)}$ where $N(p)$ in principle depends on p.

2. If the conjectured effective 2-dimensionality holds true, the situation reduces effectively to hyper-complex case and X is product of the squares of all primes multiplied by a power of 2. In the case of ordinary infinite primes there are two different vacuum primes $X \pm 1$. This is the case also now. Since the sign of the time-like component part corresponds to the sign of energy, the sign degeneracy $X \pm 1$ for the vacua could relate to the degeneracy corresponding to positive and negative energy space-time sheets. An alternative interpretation is in terms of fermion-antifermion degeneracy.

3. The product X of all hyper-octonionic irreducibles can be regarded as the counterpart of Dirac vacuum in a rather concrete sense. Moreover, in the hyper-quaternionic and octonionic case the norm of X is analogous to the Dirac determinant of a fermionic field theory with prime valued mass spectrum and integer valued momentum components. The inclusion of only irreducible eliminates from the infinite product defining Dirac determinant product over various Lorentz boosts of $p^k\gamma_k - m$.

4. An interesting question is what happens when the finite part of an infinite prime is multiplied by light like integer k. The obvious guess is that k describes the presence of a massless particle. If the resulting infinite integer is multiplied with conjugates $k_{c,i}$ of k an integer of form $\prod_i k_{c,i} mX/n$ having formally zero norm results. It would thus seem that there is a kind of gauge invariance in the sense that infinite primes for which both finite and infinite part are multiplied with the same light-like primes, are divisors of zero and correspond to gauge degrees of freedom.

5. More complex infinite hyper-octonionic primes can be always decomposed to products of generating infinite primes which correspond to polynomials with zeros in algebraic extensions of rationals so that the resulting polynomial has real-rational coefficients but has no rational zeros. An interpretation as bound states is suggestive and the replacement of the zero of corresponding polynomial with non-rational number is analogous to the change of particle rest mass in bound state formation. The sign of energy is well defined for each factor of this kind.

6. Hyper-octonionic infinite primes correspond to real-rational polynomials if all conjugates of given hyper-octonionic prime occur in the definition of generating infinite primes. The reality requirement satisfied in this manner would exclude the presence of light-like factors in the finite part of the infinite prime. Physically the presence of these factors would seem to be desirable (at least in the finite part of the infinite prime) since they could be interpreted physically as representations of massless particles. The reality condition can be also satisfied for a product of conjugates of infinite primes. In this case the constant part of the resulting infinite primes vanishes.

Zeta function and infinite primes

Fermionic Zeta function is expressible as a product of fermionic partition functions $Z_{F,p} = 1 + p^{-z}$ and could be seen as an image of X under algebraic homomorphism mapping prime p to $Z_{F,p}$ defining an analog of prime in the commutative function algebra of complex numbers. For hyper-octonionic infinite primes the contribution of each p to the norm of X is same finite power of p since only single representative from each Lorentz equivalence class is included, and there is one-one correspondence with ordinary primes so that an appropriate power of ordinary ζ_F might be regarded as a representation of X also in the case of hyper-octonionic primes.

Infinite primes suggest a generalization of the notion of ζ function. Real-rational infinite prime $X \pm 1$ would correspond to $\zeta_F \pm 1$. General infinite prime is mapped to a generalized zeta function by dividing ζ_F with the product of partition functions $Z_{F,p}$ corresponding to fermions kicked out from sea. The same product multiplies '1'. The powers p^n present in either factor correspond to the presence of n bosons in mode p and to a factor $Z^n_{p,B}$ in corresponding summand of the generalized Zeta. In the case of hyper-octonionic infinite primes some power of Z_F multiplied by p-dependent powers $Z_{F,p}^{n(p)}$ of fermionic partition functions with $n(p) \to 0$ for $p \to \infty$ should replace the image of X. If effective 2-dimensionality holds true $n(p) = 2$ holds true for $p > 2$.

For zeros of ζ_F which are same as those of Riemann ζ the image of infinite part of infinite prime vanishes and only the finite part is represented faithfully. Whether this might have some physical meaning is an interesting question.

7.3.4 Mapping of the hyper-octonionic infinite primes to polynomials

Infinite primes can be mapped to polynomial primes which in turn have geometric representation as algebraic surfaces. This inspires the idea that physics could be reduced to algebraic number theory and algebraic geometry [**?**, **?**, **?**] in some general sense. In the following consideration is restricted to hyper-octonionic primes which are the most interesting ones on basis of the considerations of [**?**].

Mapping of infinite primes to polynomials at the first level of the hierarchy

The mapping of the generating infinite primes to first order monomials labeled by their rational zeros is extremely simple at the first level of the hierarchy:

$$P_{\pm}(m,n,s) = \frac{mX}{s} \pm ns \to h \pm \frac{m}{sn} \quad .$$

Note that a monomial having zero as its root is not obtained. This mapping induces the mapping of all infinite primes to polynomials.

The simplest infinite primes are constructed using ordinary primes and second quantization of an arithmetic number theory corresponds in one-one manner to rationals. Indeed, the integer $s = \prod_i p_i^{k_i}$ defining the numbers k_i of bosons in modes k_i, where fermion number is one, and the integer r defining the numbers of bosons in modes where fermion number is zero, are co-prime. Moreover, the generating infinite primes can be written as $(n/s)X \pm ms$ corresponding to the two vacua $V = X \pm 1$ and the roots of corresponding monomials are positive *resp.* negative rationals.

More complex infinite primes correspond sums of powers of infinite primes with rational coefficients such that the corresponding polynomial has real coefficients and roots which are not rational but belong to some algebraic extension of rationals. These infinite primes correspond simply to products of infinite primes associated with some algebraic extension of rationals. Obviously the construction of higher infinite primes gives rise to a hierarchy of higher algebraic extensions.

The representation of higher level infinite primes as polynomials

It is possible to continue the process indefinitely by constructing the Dirac vacuum at the n:th level as a product of primes of previous levels and applying the same procedure. At the second level Dirac vacuum $V = X \pm 1$ involves X which is the product of all primes at previous levels and in the polynomial correspondence X thus correspond to a new independent variable. At the n:th level one has polynomials $P(h_1|h_2|...)$ of h_1 with coefficients which are real-rational functions of h_2 with coefficients which are.... The hierarchy of infinite primes is thus mapped to the functional hierarchy in which polynomial coefficients depend on parameters depending on ....

The so called Slaving Hierarchy appearing in Haken's theory of self-organization has similar form: the non-dynamical coupling parameters of the system depend on slowly varying external parameters which in turn depend on... The lowest level of the hierarchy corresponding to the ordinary rationals takes the role of the highest boss in the hierarchy of infinite primes.

For higher level infinite primes the effects of non-commutativity and non-associativity cannot be avoided except when the arguments are restricted to the same hyper-complex sub-space of M^8 defining the polarization plane. The non-associativity implies that the grouping of the arguments in the polynomial matters and affects the space-time surface. It is not clear whether non-associativity and non-commutative can be really allowed for infinite primes.

A very attractive manner to avoid effects of non-associativity is to assume that all infinite primes are reducible to rational infinite primes and that representations in terms of infinite primes associated with various extensions of rationals (algebraic extensions of rationals and of non-commutative and non-associative completions of rationals) emerge from the decompositions of rational primes to these primes.

7.3.5 Mapping of infinite primes to space-time surfaces

At the lowest level of hierarchy the mapping of hyper-octonionic infinite primes to 4-surfaces is a special case of assigning to a hyper-octonion analytic function a foliation of imbedding space by 4-surfaces. At higher levels of hierarchy the mapping of infinite primes to space-time surfaces requires a generalization of this procedure and the constraints from non-commutativity and non-associativity dictate the generalization completely.

Associativity as the basic constraint

On basis of the general vision about how hyper-octonion analytic maps of M^8 to itself correspond to four-surfaces in $M^4 \times CP_2$ and perhaps also absolute minima of Kähler action, it is clear that the hyper-octonionic polynomials defined by the infinite primes at the first level of hierarchy indeed define a foliations of $M^4 \times CP_2$ by four-dimensional surfaces with an additional degeneracy corresponding to the possibility to choose freely the map $f : HO \to S^6$ characterizing the choice of preferred imaginary octonionic unit, or equivalently the plane defined by time-like polarizations. There is also a degeneracy related to the choice of the origin of M^8 coordinates and due to the $SO(7,1)$ invariance acting at the level of $M^8 = HO$.

The basic objection is that the polynomials representing infinite are ill defined at the higher levels of hierarchy due to the problems caused by non-associativity even in case that one restricts the consideration to rational functions with real coefficients. The only resolution of this objection is that the arguments h_i are functionally independent so that one can express h_i, $i > 1$ as hyper-octonion real-analytic function of h_1. Rational functions look especially natural and one can consider also the identification $h_n = h_{n-1} = ... = h_1$.

This assumption reduces the representation to one-dimensional case and if hyper-octonion real-analytic functions define foliations of imbedding space by quaternionic space-time surfaces, one obtains a hierarchy of increasingly complex space-time surfaces. An open question is whether the hierarchy of infinite primes indeed corresponds to a hierarchy of space-time sheets.

The requirement that the theory allows p-adicization is not only a challenge but also a heavy constraint. If everything is rational at the basic level in the proposed sense, there are indeed good hopes for the p-adicization at space-time level. This optimistic view is also encouraged by the recent formulation of quantum TGD as almost topological conformal field theory [**?**].

The ordering of the arguments of the polynomials characterizes the thoughts about thoughts hierarchy as a hierarchy in which algebraic complexity increases and, as already noticed, also the Slaving Hierarchy. h_n corresponds to the highest level of the hierarchy and h_1 to its lowest level. Topological condensate indeed gives rise to this kind of hierarchy very naturally. This hierarchy is not lost even in the reduction of variables to single hyper-octonionic variable.

The identification allows a generalization of the basic philosophy of algebraic geometry. The rational functions associated with infinite primes have natural ordering with respect to their degree and dimension of algebraic extension of rationals associated with the roots of these polynomials. This makes sense for both functions of n complex arguments and single hyper-octonionic argument. Hence the space-time surfaces can be ordered in a natural manner with respect to their algebraic complexity. One could hope that this kind of ordering might be of decisive help in the physical interpretation of the predictions of the theory.

The most elegant theory results if all infinite primes are assumed to reduce to rational infinite primes and that the decomposition to primes in algebraic completions of rationals and to quaternionic, octonionic, hyper-octonionic infinite primes and their variants in the complexification of quaternions and octonions reflects to or is at least analogous to the possibility to decompose a particle into its more elementary constituents. One might hope that number theoretic analog of color confinement translates to a deep physical principle.

Interaction between infinite primes fixes the scaling of the polynomials associated with infinite primes

The assignment of a polynomial with an infinite prime is unique only up to an over-all scaling and the following argument suggests that the only physically acceptable scaling corresponds to the normalization of the constant term, call it c, of the polynomial to $c = 1$.

In algebraic geometry the zeros of polynomials as their representations has the property that the product of polynomials corresponds to a union of disjoint surfaces and there is no interaction between the surfaces. For infinite integers represented in terms of hyper-quaternionic surfaces this is not the case. This raises the question whether this state of affairs makes possible a realistic number theoretical description of interactions. This description could the counterpart for the description based on the absolute minima of Kähler action which are not simply disjoint unions of absolute minima associated with two 3-surfaces. It would also be analog for the description of the interaction between different space-time sheets in terms of polynomials defined by higher level infinite primes.

This interaction should be consistent with the idea that the interaction of the systems described by infinite primes is weak in some space-time regions. This is certainly the case if the polynomial approaches constant equal to one. To see what happens consider the product of polynomials associated with two infinite primes. The expectation is that in the regions where second hyper-octonion analytic polynomials P_1 approaches to a constant value, which must be real by real-analyticity, the product of infinite primes defines a 4-surface which resembles the surfaces associated with P_2.

The product of hyper-octonion analytic functions $g_1 = a_1 + b_1\overline{h}$ and $g_2 = a_2 + b_2\overline{h}$ is $a_1a_2 + b_1b_2\overline{h} \cdot \overline{h} + (a_1b_2 - a_2b_1)\overline{h}$. If b_1 approaches to zero, the product behaves as $a_1a_2 + a_1b_2\overline{h}$, so that a_1 should approach to $a_1 = 1$ in order that interaction would be negligible.

The observation would suggest that the mapping of infinite primes to polynomials must involve a scaling taking care that the constant term appearing in the polynomial equals to one. This kind of scaling is of course possible. It would however mean that infinite primes with polynomials for which constant term vanishes are not allowed. This would mean that products of conjugates of infinite primes for which finite part is proportional to a light-like integer are not allowed since in this case the constant term vanishes. This is true if one assumes that hyper-octonionic infinite primes reduce to rational infinite primes.

7.4 How to interpret the infinite hierarchy of infinite primes?

From the foregoing it should be clear that infinite primes might play key role in quantum physics. One can even consider the possibility that physics reduces to a generalized number theory, and that infinite primes are crucial for understanding mathematically consciousness and cognition. Of course, one must leave open the question whether infinite primes really provide really the mathematics of consciousness or whether they are only a beautiful but esoteric mathematical construct. In this spirit the following subsections give only different points of view to the problem with no attempt to a coherent overall view.

7.4.1 Infinite primes and hierarchy of super-symmetric arithmetic quantum field theories

Infinite primes are a generalization of the notion of prime. They turn out to provide number theoretic correlates of both free, interacting and bound states of a super-symmetric arithmetic quantum field theory. The mapping of infinite primes to polynomials in turn allows to assign to infinite prime space-time surface as a geometric correlate. Hence infinite primes serve as a bridge between classical and quantum and realize quantum classical correspondence stating that quantum states have classical counterparts, and has served as a basic heuristic guideline of TGD. More precisely, the natural hypothesis is that infinite primes code for the ground states of super-canonical representations (for instance, ordinary particles correspond to states of this kind).

Generating infinite primes as counterparts of Fock states of a super-symmetric arithmetic quantum field theory

The basic construction recipe for infinite primes is simple and generalizes to the quaternionic case.

1. Form the product of all primes and call it X:

$$X = \prod_p p \ .$$

2. Form the vacuum states

$$V_{\pm} = X \pm 1 \ \ .$$

3. From these vacua construct all *generating* infinite primes by the following process. Kick out from the Dirac sea some negative energy fermions: they correspond to a product s of first powers of primes: $V \to X/s \pm s$ (s is thus square-free integer). This state represents a state with some fermions represented as holes in Dirac sea but no bosons. Add bosons by multiplying by integer r, which decomposes into parts as $r = mn$: m corresponding to bosons in X/s is product of powers of primes dividing X/s and n corresponds to bosons in s and is product of powers of primes dividing s. This step can be described as $X/s \pm s \to mX/s \pm ns$.

Generating infinite primes are thus in one-one correspondence with the Fock states of a super-symmetric arithmetic quantum field theory and can be written as

$$P_{\pm}(m,n,s) = \frac{mX}{s} \pm ns \ \ ,$$

where X is product of all primes at previous level. s is square free integer. m and n have no common factors, and neither m and s nor n and X/s have common factors.

The physical analog of the process is the creation of Fock states of a super-symmetric arithmetic quantum field theory. The factorization of s to a product of first powers of primes corresponds to many-fermion state and the decomposition of m and n to products of powers of prime correspond to bosonic Fock states since p^k corresponds to k-particle state in arithmetic quantum field theory.

More complex infinite primes as counterparts of bound states

Generating infinite primes are not all that are possible. One can construct also polynomials of the generating primes and under certain conditions these polynomials are non-divisible by both finite primes and infinite primes already constructed.

The physical counterpart of n:th order irreducible polynomial is as a bound state of n particles whereas infinite integers constructed as products of infinite primes correspond to non-bound but interacting states. This process can be repeated at the higher levels by defining the vacuum state to be the product of all primes at previous levels and repeating the process. A repeated second quantization of a super-symmetric arithmetic quantum field theory is in question.

The fact that more general infinite primes can be constructed as polynomials of the generating infinite primes, suggest strongly that infinite primes can be mapped to ordinary polynomials by replacing the argument X in $V_{\pm} = X \pm 1$ with variable h. This indeed turns out to be the case. This correspondence allows to deduce that more general infinite primes correspond to irreducible polynomials of generating infinite primes not allowing decomposition to a product of generating infinite primes.

The infinite primes represented by irreducible polynomials correspond to quantum states obtained by mapping the superposition of the products of the generating infinite primes to a superposition of the corresponding Fock states. If complex rationals are the coefficient field for infinite integers, this gives rise to states in a complex Hilbert space and irreducibility corresponds to a superposition of states with varying particle number and the presence of entanglement. For instance, the superpositions of several products of type $\prod_{i=1,..,n} P_i$ of n generating infinite primes are possible and in general give rise to irreducible infinite primes decomposing into a product of infinite primes in algebraic extension of rationals.

7.4.2 Prime Hilbert spaces and infinite primes

There is a result of quantum information science providing an additional reason why for p-adic physics. Suppose that one has N-dimensional Hilbert space which allows $N+1$ unbiased basis. This means that the moduli squared for the inner product of any two states belonging to different basis equals to $1/N$. If one knows all transition amplitudes from a given state to all states of all $N+1$ mutually unbiased basis, one can fully reconstruct the state. For $N = p^n$ dimensional $N+1$ unbiased basis can be found and the article of Durt[**?**] gives an explicit construction of these basis by applying the properties of finite fields. Thus state spaces with p^n elements - which indeed emerge naturally in p-adic framework - would be optimal for quantum tomography. For instance, the discretization of one-dimensional line with length of p^n units would give rise to p^n-dimensional Hilbert space of wave functions.

The observation motivates the introduction of prime Hilbert space as as a Hilbert space possessing dimension which is prime and it would seem that this kind of number theoretical structure for the category of Hilbert spaces is natural from the point of view of quantum information theory. One might ask whether the tensor product of mutually unbiased bases in the general case could be constructed as a tensor product for the bases for prime power factors. This can be done but since the bases cannot have common elements the number of unbiased basis obtained in this manner is equal to $M+1$, where M is the smallest prime power factor of N. It is not known whether additional unbiased bases exists.

Hierarchy of prime Hilbert spaces characterized by infinite primes

The notion of prime Hilbert space provides also a new interpretation for infinite primes, which are in 1-1 correspondence with the states of a supersymmetric arithmetic QFT. The earlier interpretation was that the hierarchy of infinite primes corresponds to a hierarchy of quantum states. Infinite primes could also label a hierarchy of infinite-D prime Hilbert spaces with product and sum for infinite primes representing unfaithfully tensor product and direct sum.

1. At the lowest level of hierarchy one could interpret infinite primes as homomorphisms of Hilbert spaces to generalized integers (tensor product and direct sum mapped to product and sum) obtained as direct sum of infinite-D Hilbert space and finite-D Hilbert space. (In)finite-D Hilbert space is (in)finite tensor product of prime power factors. The map of N-dimensional Hilbert space to the set of N-orthogonal states resulting in state function reduction maps it to N-element set and integer N. Hence one can interpret the homomorphism as giving rise to a kind of shadow on the wall of Plato's cave projecting (shadow quite literally!) the Hilbert space to generalized integer representing the shadow. In category theoretical setting one could perhaps see generalize integers as shadows of the hierarchy of Hilbert spaces.

2. The interpretation as a decomposition of the universe to a subsystem plus environment does not seem to work since in this case one would have tensor product. Perhaps the decomposition could be to degrees of freedom to those which are above and below measurement resolution. One could of course consider decomposition to a tensor product of bosonic and fermionic state spaces.

3. The construction of the Hilbert spaces would reduce to that of infinite primes. The analog of the fermionic sea would be infinite-D Hilbert space which is tensor product of all prime Hilbert spaces H_p with given prime factor appearing only once in the tensor product. One can "add n bosons" to this state by replacing of any tensor factor H_p with its n+1:th tensor power. One can "add fermions" to this state by deleting some prime factors H_p from the tensor product and adding their tensor product as a finite-direct summand. One can also "add n bosons" to this factor.

4. At the next level of hierarchy one would form infinite tensor product of all infinite-dimensional prime Hilbert spaces obtained in this manner and repeat the construction. This can be continued ad infinitum and the construction corresponds to abstraction hierarchy or a hierarchy of statements about statements or a hierarchy of n:th order logics. Or a hierarchy of space-time sheets of many-sheeted space-time. Or a hierarchy of particles in which certain many-particle states at the previous level of hierarchy become particles at the new level (bosons and fermions). There are many interpretations.

5. Note that at the lowest level this construction can be applies also to Riemann Zeta function. ζ would represent fermionic vacuum and the addition of fermions would correspond to a removal of a product of corresponding factors ζ_p from ζ and addition of them to the resulting truncated ζ function. The addition of bosons would correspond to multiplication by a power of appropriate ζ_p. The analog of ζ function at the next level of hierarchy would be product of all these modified ζ functions and might well fail to exist since the product might typically converge to either zero or infinity.

Hilbert spaces assignable to infinite integers and rationals make also sense

1. Also infinite integers make sense since one can form tensor products and direct sums of infinite primes and of corresponding Hilbert spaces. Also infinite rationals exist and this raises the question what kind of state spaces inverses of infinite integers mean.

2. Zero energy ontology suggests that infinite integers correspond to positive energy states and their inverses to negative energy states. Zero energy states would be always infinite rationals with real norm which equals to real unit.

3. The existence of these units would give for a given real number an infinite rich number theoretic anatomy so that single space-time point might be able to represent quantum states of the entire universe in its anatomy (number theoretical Brahman=Atman). Also the world of classical worlds (light-like 3-surfaces of the imbedding space) might be imbeddable to this anatomy so that basically one would have just space-time surfaces in 8-D space and configuration space would have representation in terms of space-time based on generalized notion of number. Note that infinitesimals around a given number would be replaced with infinite number of number-theoretically non-equivalent real units multiplying it.

Should one generalize the notion of von Neumann algebra?

Especially interesting are the implications of the notion of prime Hilbert space concerning the notion of von Neumann algebra -in particular the notion of hyper-finite factors of type II_1 playing a key role in TGD framework. Does the prime decomposition bring in additional structure? Hyper-finite factors of type II_1 are canonically represented as infinite tensor power of 2×2 matrix algebra having a representation as infinite-dimensional fermionic Fock oscillator algebra and allowing a natural interpretation in terms of spinors for the world of classical worlds having a representation as infinite-dimensional fermionic Fock space.

Infinite primes would correspond to something different: a tensor product of all $p\times p$ matrix algebras from which some factors are deleted and added back as direct summands. Besides this some factors are replaced with their tensor powers. Should one refine the notion of von Neumann algebra so that one can distinguish between these algebras as physically non-equivalent? Is the full algebra tensor product of this kind of generalized hyper-finite factor and hyper-finite factor of type II_1 corresponding to the vibrational degrees of freedom of 3-surface and fermionic degrees of freedom? Could p-adic length scale hypothesis - stating that the physically favored primes are near powers of 2 - relate somehow to the naturality of the inclusions of generalized von Neumann algebras to HFF of type II_1?

7.4.3 Do infinite hyper-octonionic primes represent quantum numbers associated with Fock states?

Hyper-octonionic primes involve so much structure that one can seriously consider the possibility that they could code quantum numbers of elementary particles which in accordance with quantum-classical correspondence would be coded to the shape of space-time surfaces.

Hyper-octonionic infinite primes as representations for quantum numbers of Fock states?

Configuration space spinor fields assign infinite number of quantum states to a given 3-surface as components of configuration space spinor. This suggests that there cannot be one-to-one correspondence between Fock states and space-time surfaces except in the approximation that one replaces configuration space spinor field with single 'quantum average space-time'. This forces to consider critically the identification of the hyper-octonionic primes as quantum numbers.

Perhaps a more realistic identification of infinite primes is as coding for the quantum numbers for the ground states of the representations of super-canonical and Kac-Moody algebras. This identification would be in an agreement with the view that space-time surfaces represent only the classical aspects of physics but not quantum fluctuations. Arithmetic quantum field theory should represent only the sector of ground states of quantum TGD.

It is interesting to check whether hyper-octonionic infinite primes could allow a realistic coding for the quantum numbers of ground states of super Kac-Moody representations.

1. If it is assumed that each prime in the finite part of X corresponds to a fermion, the requirement that the Fock state possesses a well-defined fermion number poses constraints on the structure of the polynomial associated with the infinite prime. A product of generating infinite primes in algebraic extension of real-rationals interpreted as representing states for which rest mass is changed by bound state interactions, would however resolve these constraints. Also superpositions of products are allowed but in this case net fermion numbers associated with various monomials must be same.

2. Hyper-octonionic infinite prime could be interpreted as coding for the relationship between particle four-momentum represented by the hyper-quaternionic part of infinite prime and the quantum numbers associated with CP_2 degrees of freedom represented by the quaternionic part of the infinite prime. Electro-weak isospin and hyper charge and corresponding color quantum numbers indeed give rise to four quantum numbers.

   Mass squared formula for infinite primes, and more generally, infinite integers would be the basic string mass formula. For bound states the mass squared values would be primes in algebraic extension of rationals.

3. Space-like hyper-octonionic primes do not seem to be natural in the case of hyper-octonionic option. Octonionic option would allow them but in this case the interpretation in terms of momenta is lost. This not so plausible option would allow as a special case Gaussian and Eisenstein primes discussed in [?]. Eisenstein primes correspond to algebraic extension involving $\sqrt{3}$. These primes correspond to time-like primes obtained by multiplying the prime with a suitable unit. The degeneracies of these primes due to units defined by complex phases are 4 and 8. One can ask whether these degeneracies might relate to the spin states of imbedding space spinors.

4. If the proposed interpretation is taken at face value, the question about distinction between quarks and leptons at the level of infinite primes, arises. Somehow the two different chiralities for induced imbedding space spinor fields should have space-time correlates. If the primes $p \ mod \ 4 = 1$ and $p \ mod \ 4 = 3$ correspond to leptons and quarks or vice versa it would be possible to assign to each generating infinite prime lepton or quark number. Bosons could be regarded as fermion-antifermion bound states and bosonic surfaces would correspond to the composites of two infinite primes with either $p \ mod \ 4 = 1$ or $p \ mod \ 4 = 3$ or superposition of this kind of monomials.

5. Since only polynomials with real coefficients are possible, kind of number theoretic analog of color confinement occurs, and requires that at least two generating infinite primes with the hyper-octonionic zero of the corresponding monomial with components belonging to an algebraic extension of real rationals appears in the state. This confinement has counterpart at the level of super-canonical conformal weights which are complex and expressible in terms of zeros of Riemann Zeta: only states with real net conformal weight are possible.

6. One can imagine several interpretations for the two vacua $V_{\pm} = X \pm 1$.
   i) The most plausible interpretation for these vacua is in terms of matter and antimatter and thus as representations for states having opposite fermion number. In number theoretic bound states represented by higher degree polynomials both matter and antimatter particles can occur.
   ii) A less plausible interpretation is as positive and negative energy vacua associated with the space-time sheets of opposite time orientation predicted by TGD. The fact that negative energy particles do not seem to appear in elementary particle reactions inspires the hypothesis that negative energies are associated with higher level infinite primes and correspond to the infinite primes defining the denominators of the rational functions appearing in the definitions of higher level infinite primes. Phase conjugate photons would be a basic example of negative energy particles.
   iii) Also the interpretation in terms of the vacua of associated Ramond and NS type super canonical algebras can be considered.

There are also other degrees of freedom besides Super Kac Moody degrees fo freedom.

1. Zero modes are an essential part of TGD and would correspond to the degrees of freedom associated with the maps $HO \rightarrow S^6$ and their generalization to the higher levels of the hierarchy. Physical interpretation would be as a imbedding space dependent selection of longitudinal degrees of freedom in turn fixing at space-time level the spin quantization axis and the transversal degrees of freedom associated with polarizations of massless particles.

2. There is no obvious relation between super-canonical conformal weights and infinite primes. Perhaps the reason is that these quantum numbers are associated with configuration space spinor fields.

Family replication phenomenon and commutative sub-manifolds of space-time surface

The idea that complex Abelian sub-manifolds of space-time sheets are in preferred role by their commutativity in hyper-octonionic sense, is consistent with the topological explanation of family replication phenomenon [**?**] by interpreting different particle families as particles with corresponding 3-surface having boundary with genus $g = 0, 1, 2, ...$

The representations $p = f(q)$ of the algebraic surfaces with real-analytic f, when restricted to complex numbers, define 2-dimensional Riemann surfaces in 4-dimensional complex space. These surfaces are characterized by genus so that genus emerges in very natural manner from the theory.

If the boundary component has same genus as the genus defined by hyper-quaternionicity, then the notion of elementary particle vacuum functional makes sense, and p-adic mass calculations [**?**, **?**, **?**, **?**] which rely crucially on the notion of genus, remain unchanged. natural possibility is that the 2-surface where hyper-quaternions are commutative in fact corresponds to a boundary component of 3-surface. The 2-dimensional intersections of 3-D light-like causal determinants X_l^3 and 7-D light-like causal determinants defined by boundaries of future and past light-cones of M^4 are natural candidates for partonic 2-surfaces. If this picture is correct, one can also answer the troublesome question 'What is the two-dimensional sub-manifold of 3-dimensional boundary of space-time surface to which one assigns elementary particle vacuum functional?'. This question is of high relevance since the conformal equivalence class of boundary component depends on how the boundary component is identified.

7.4.4 The physical interpretation of infinite integers at the first level of the hierarchy

The idea that primes are for the number theory what elementary particles are for physics, suggests that the decomposition of an infinite integer to a product of infinite primes corresponds to the decomposition of a physical system to elementary systems allowing no further decomposition.

Higher degree polynomial primes as bound states

The sums for the products of infinite primes defining irreducible polynomials define infinite primes describing many particle states and the interpretation as composites of space-time surfaces associated with simpler 'effective' generating infinite primes belonging to the extension of quaternions is natural and leads to a dynamical generation of algebraic symmetries. A natural interpretation is as topological composites formed from space-time surfaces describing bound states. Each root of the polynomial equation defining a branch of the space-time surface would correspond to a particle present in the composite. Indeed, n:th order irreducible polynomial factors to product of monomials $x - l$, $l \notin K$. If the polynomial differs only slightly from a product of prime polynomials, it is natural to interpret the slight change of the roots as a slight change of the composite states induced by the mutual interaction.

Infinite integers as interacting many particle states

The space-time surfaces representing infinite integers could represent many-particle states. The space-time surface associated with the integer is in general not a union of the space-time surfaces associated with the primes composing the integer. This means that classical description of interactions emerges automatically. The description of classical states in terms of infinite integers is completely analogous to the description of many particle states as finite integers in arithmetic quantum field theory.

The finite primes which correspond to particles of an arithmetic quantum field theory present in Fock state, correspond to the space-time sheets of finite size serving as the building blocks of the space-time sheet characterized by infinite prime. Real topology is the space-time topology in the regions, where matter resides whereas 'mind stuff' corresponds to the regions obeying p-adic topology. This is in accordance with the fact that the physics based on real numbers is so successful. The success of p-adic physics could be understood as resulting from the fact that it describes the physics of the mind like regions mimicking the physics of the real matter-like regions.

7.4.5 What is the interpretation of the higher level infinite primes?

Interesting questions are related to the higher level infinite primes obtained by taking X to be a product of all lower level primes and repeating the construction.

Infinite hierarchy of infinite primes

Infinite hierarchy of infinite primes codes for a hierarchy of Fock states such that many-particle Fock states of a given level serve as elementary particles at next level. The unavoidable conclusion is that higher levels represent totally new physics not described by the standard quantization procedures. In particular, the assignment of fermion/boson property to arbitrarily large system would be in some sense exact. Topologically these higher level particles could correspond to space-time sheets containing many-particle states and behaving as higher level elementary particles.

This view suggests that the generating quantum numbers are present already at the lowest level and somehow coded by the hyper-octonionic primes taking the role of momentum quantum number they have in arithmetic quantum field theories. The task is to understand whether and how hyper-octonionic primes can code for quantum numbers predicted by quantum TGD.

The quantum numbers coding higher level states are collections of quantum numbers of lower level states. At geometric level the replacement of the coefficients of polynomials with rational functions is the equivalent of replacing single particle states with new single particle states consisting of many-particle states.

Rationals of the previous level appear at given level

What is remarkable is that the rationals formed from the integers of $n-1$:th level label the simplest primes of n:th level. The numerator and denominator of the rational number correspond to a pair of integers representing physical states at previous level, which suggests that the new states are higher level physical states representing information about pairs of physical states at the previous level. The most natural guess is that the states of the pair correspond to the initial and final states of a quantum jump. In this manner the infinite hierarchy give rise to physical states representing increasingly abstract information about dynamics. The fact that I am a physical system ponder physics problems could be seen as a direct evidence for the existence for these higher levels of physical existence.

At the next level physical states represent information about pairs of quantum jumps which in TGD inspired theory of consciousness correspond to memories about primary conscious experiences determined by quantum jumps. They clearly represent experiences about experiences. At n:th level quantum jump represent n-fold abstraction giving conscious information about experiences about.....about experiences.

TGD allows space-time sheets with both positive and negative time orientation and the sign of classical energy correlates with the orientation of the space-time sheet. This leads to a radical revision of the energy concept and clarifies the relationship between gravitational and inertial energy. The interpretation of the numerator and denominator of the infinite rational in terms of positive and negative energy space-time sheets looks natural. Of course, one must be ready to consider the possibility that "energy" might be replaced by some other conserved quantity. This interpretation would also explain why negative energy particles appear only at higher organization level of matter and are not detected in accelerators. Indeed, the basic TGD applications relate to quantum biology, consciousness [?], and free energy [?].

The interpretation of particle reactions as quantum jumps between zero energy states is implied by this vision, and this interpretation is consistent with crossing symmetry. Zero energy states can be seen also as representations of quantum jumps with positive and negative energy components of the state identifiable as counterparts of initial and final states. One could say that all states of the entire Universe, even at classical space-time level, represent reflective level of existence, being always about something. Only in the approximation that positive and negative energy components of the state do not interact the western view about objective reality with conserved energy makes sense.

7.4.6 Infinite primes and the structure of many-sheeted space-time

The mapping of infinite primes to space-time surfaces codes the structure of infinite prime to the structure of space-time surface in rather non-implicit manner, and the question arises about the concrete correspondence between the structure of infinite prime and topological structure of the space-time surface.

A possible interpretation for the lowest level infinite primes

The concrete prediction of the general vision is that the hierarchy of infinite primes should correspond to the hierarchy of space-time sheets. The challenge is to find space-time counterparts for infinite primes at the lowest level of hierarchy.

One could hope that the Fock space structure of infinite prime would have a more concrete correspondence with the structure of the many-sheeted space-time. One might that the space-time sheets labeled by primes p would directly correspond to the primes appearing in the definition of infinite prime. This expectation seems to be too simplistic.

1. What seems to be a safe guess is that the simplest infinite primes at the lowest level of the hierarchy should correspond to elementary particles. If inverses of infinite primes correspond to negative energy space-time sheets, this would explain why negative energy particles are not encountered in elementary particle physics.

2. More complex infinite primes at the lowest level of the hierarchy could be interpreted in terms of structures formed by connecting these structures by join along boundaries bonds to get space-time correlates of bound states. Even simplest infinite primes must correspond to bound state structures if the condition that the corresponding polynomial has real-rational coefficients is taken seriously.

Infinite primes at lowest level of hierarchy correspond to several finite primes rather than single finite prime. The number of finite primes is however finite. This conforms with the idea that this level indeed corresponds to space-time sheets associated with elementary particles.

1. A possible interpretation for multi-p property is in terms of multi-p p-adic fractality prevailing in the interior of space-time surface. The effective p-adic topology of these space-time sheets would depend on length scale. In the longest scale the topology would correspond to p_n, in some shorter length scale there would be smaller structures with $p_{n-1} < p_n$-adic topology, and so on... . A good metaphor would be a wave containing ripples, which in turn would contain still smaller ripples. The multi-p p-adic fractality would be assigned with the 4-D space-time sheets associated with elementary particles.

2. Effective 2-dimensionality would suggest that p-adic topologies could be assigned with the 2-dimensional partonic surfaces or corresponding 3-D light-like causal determinants. Thus infinite prime would characterize at the lowest level space-time sheet and corresponding partonic 2-surfaces. This interpretation is consistent with the fact that modified Dirac operator assigns to its generalized eigen modes p-adic prime p characterizing the p-adic topology of corresponding p-adic parton obeying same algebraic equations.

How to interpret higher level infinite primes?

A possible interpretation for higher level infinite primes is in terms of q-adicity assignable to the function spaces defined by the rational functions assignable to them. The role of finite prime p would be taken by the rational function defined by the infinite prime. This interpretation makes sense both when one assigns to infinite primes functions of rational arguments $q_1, ...q_n$ or when one identifies these arguments. This function space is q-adic for some rational number q. At the lowest level the infinite prime indeed defines naturally an ordinary rational number.

At higher levels of the hierarchy one can assign to infinite prime an infinite rational number of previous level. By continuing the assignments of lower level rationals to the infinite primes appearing in this infinite rational one ends up with an assignment of a unique rational number with a given infinite prime. This rational serves as a good candidate for a rational defining the q-adicity. The question is whether this q-adicity can be assign with space-time topology or some function space topology.

1. The modified Dirac operator associated with a partonic 2-surface assignable to the largest space-time sheet of topological condensation hierarchy would naturally assign q to its eigen modes. It is however not clear whether one can assign to partonic 2-surface characterized by algebraic equations unique q-adic space-time sheet. The problem is that q-adic numbers do not form number field so that the algebraic equations defining the partonic 2-surface need not make sense.

2. The q-adic function spaces might have a natural interpretation in terms of the fields assignable to the space-time sheet by replacing complex argument with quaternionic one. One possible interpretation is that primes appearing in the lowest level infinite prime correspond to partonic 2-surfaces and infinite prime itself defines q-adic topology for a functions space assignable to the space-time sheet. The q-adic topology associated with the function space associated with a space-time sheet containing topologically condensed space-time sheets would be characterized by the infinite prime and corresponding polynomial determined by the infinite primes associated with the topologically condensed space-time sheets that it contains. Note that the modified Dirac operator would assign to partonic 2-surfaces at all levels of hierarchy a p-adic prime.

3. Quantum criticality suggests strongly that configuration space of 3-surfaces effectively reduces to discrete spin glass energy landscape corresponding to the maxima of Kähler function. Spin glass property suggests strongly that this space obeys ultrametric topology. Therefore a natural conjecture is that the q-adic topology can be assigned with this space.

7.4.7 How infinite integers could correspond to p-adic effective topologies?

Besides the hierarchy of space-time sheets, TGD predicts, or at least suggests, several hierarchies such as the hierarchy of infinite primes, hierarchy of Jones inclusions [?], dark matter hierarchy characterized by increasing values of $\hbar$ [?, ?], the hierarchy of extensions of given p-adic number field, and the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. There are good reasons to expect that these hierarchies are closely related. Number theoretical considerations allow to develop more quantitative vision about the relationship between the hierarchy of infinite primes and p-adic length scale hierarchy.

How to define the notion of elementary particle?

p-Adic length scale hierarchy forces to reconsider carefully also the notion of elementary particle. p-Adic mass calculations led to the idea that particle can be characterized uniquely by single p-adic prime characterizing its mass squared [?, ?, ?]. It however turned out that the situation is probably not so simple.

The work with modelling dark matter suggests that particle could be characterized by a collection of p-adic primes to which one can assign weak, color, em, gravitational interactions, and possibly also other interactions [?, ?, ?]. It would also seem that only the space-time sheets containing common primes in this collection can interact. This leads to the notions of relative and partial darkness. An entire hierarchy of weak and color physics such that weak bosons and gluons of given physics are characterized by a given p-adic prime p and also the fermions of this physics contain space-time sheet characterized by same p-adic prime, say M_{89} as in case of weak interactions. In this picture the decay widths of weak bosons do not pose limitations on the number of light particles if weak interactions for them are characterized by p-adic prime $p \neq M_{89}$. Same applies to color interactions.

The p-adic prime characterizing the mass of the particle would perhaps correspond to the largest p-adic prime associated with the particle. Graviton which corresponds to infinitely long ranged interactions, could correspond to the same p-adic prime or collection of them common to all particles.

This might apply also to photons. Infinite range might mean that the join along boundaries bonds mediating these interactions can be arbitrarily long but their transversal sizes are characterized by the p-adic length scale in question.

The natural question is what this collection of p-adic primes characterizing particle means? The hint about the correct answer comes from the number theoretical vision, which suggests that at fundamental level the branching of boundary components to two or more components, completely analogous to the branching of line in Feynman diagram, defines vertices [**?**].

1. If space-time sheets correspond holographically to multi-p p-adic topology such that largest p determines the mass scale, the description of particle reactions in terms of branchings indeed makes sense. This picture allows also to understand the existence of different scaled up copies of QCD and weak physics. Multi-p p-adicity could number theoretically correspond to q-adic topology for $q = m/n$ a rational number consistent with p-adic topologies associated with prime factors of m and n ($1/p$-adic topology is homeomorphic with p-adic topology).

2. One could also imagine that different p-adic primes in the collection correspond to different space-time sheets condensed at a larger space-time sheet or boundary components of a given space-time sheet. If the boundary topologies for gauge bosons are completely mixed, as the model of hadrons forces to conclude, this picture is consistent with the topological explanation of the family replication phenomenon and the fact that only charged weak currents involve mixing of quark families. The problem is how to understand the existence of different copies of say QCD. The second difficult question is why the branching leads always to an emission of gauge boson characterized by a particular p-adic prime, say M_{89}, if this p-adic prime does not somehow characterize also the particle itself.

What effective p-adic topology really means?

The need to characterize elementary particle p-adically leads to the question what p-adic effective topology really means. p-Adic mass calculations leave actually a lot of room concerning the answer to this question.

1. At the fundamental level this problem seems to be well understood now. By the almost topological QFT property of quantum real and p-adic variants of light-like partonic 3-surfaces can satisfy same algebraic equations. Modified Dirac operator assigns well-defined p-adic prime p to its eigenmodes with non-vanishing eigenvalues. Zero modes are an exception.

2. The naivest option would be that each space-time sheet corresponds to single p-adic prime. This view is not favored by the view that each particle corresponds to a collection of p-adic primes each characterizing one particular interaction that the particle in question participates. A more natural possibility is that the boundary components of space-time sheet, and more generally, light-like 3-surfaces serving as causal determinants, correspond to different p-adic primes.

3. This implies that a given space-time sheet to several p-adic primes. Indeed, a power series in powers of given integer n gives rise to a well-defined power series with respect to all prime factors of n and effective multi-p-adicity could emerge at the level of field equations in this manner in the interior of space-time sheets. One could say that space-time sheet corresponds to several p-adic primes through its effective p-adic topology in a hologram like manner. This option is the most natural as far as physical interpretation is considered. It is also supported by the number theoretical considerations predicting the value of gravitational coupling constant.

An attractive hypothesis is that only space-time sheets characterized by integers n_i having common prime factors can be connected by join along boundaries bonds and can interact by particle exchanges and that each prime p in the decomposition corresponds to a particular interaction mediated by an elementary boson characterized by this prime.

Do infinite primes code for effective q-adic space-time topologies?

As found, one can assign to a given infinite prime a rational number. The most obvious question concerns the possible space-time interpretation of this rational number. Also the question arises about the possible relation with the integers characterizing space-time sheets having interpretation in terms of multi-p-adicity. On can assign to any rational number $q = m/n$ so called q-adic topology. This topology is not consistent with number field property like p-adic topologies. Hence the rational number q assignable to infinite prime could correspond to an effective q-adic topology.

If this interpretation is correct, arithmetic fermion and boson numbers could be coded into effective q-adic topology of the space-time sheets characterizing the non-determinism of Kähler action in the relevant length scale range. For instance, the power series of $q > 1$ in positive powers with integer coefficients in the range $[0, q)$ define q-adically converging series, which also converges with respect to the prime factors of m and can be regarded as a p-adic power series. The power series of q in negative powers define in similar converging series with respect to the prime factors of n.

I have proposed earlier that the integers defining infinite rationals and thus also the integers m and n characterizing finite rational could correspond at space-time level to particles with positive *resp.* negative time orientation with positive *resp.* negative energies. Phase conjugate laser beams would represent one example of negative energy states. With this interpretation super-symmetry exchanging the roles of m and n and thus the role of fermionic and bosonic lower level primes would correspond to a time reversal.

1. The first interpretation is that there is single q-adic space-time sheet and that positive and negative energy states correspond to primes associated with m and n respectively. Positive (negative) energy space-time sheets would thus correspond to p-adicity ($1/p$-adicity) for the field modes describing the states.

2. Second interpretation is that particle (in extremely general sense that entire universe can be regarded as a particle) corresponds to a pair of positive and negative energy space-time sheets labeled by m and n characterizing the p-adic topologies consistent with $m-$ and n-adicities. This looks natural since Universe has necessary vanishing net quantum numbers. Unless one allows the non-uniqueness due to $m/n = mr/nr$, positive and negative energy space-time sheets can be connected only by # contacts so that positive and negative energy space-time sheets cannot interact via the formation of $\#_B$ contacts and would be therefore dark matter with respect to each other. Antiparticles would also have different mass scales. If the rate for the creation of # contacts and their CP conjugates are slightly different, say due to the presence of electric components of gauge fields, matter antimatter asymmetry could be generated primordially.

These interpretations generalize to higher levels of the hierarchy. There is a homomorphism from infinite rationals to finite rationals. One can assign to a product of infinite primes the product of the corresponding rationals at the lower level and to a sum of products of infinite primes the sum of the corresponding rationals at the lower level and continue the process until one ends up with a finite rational. Same applies to infinite rationals. The resulting rational $q = m/n$ is finite and defines q-adic effective topology, which is consistent with all the effective p-adic topologies corresponding to the primes appearing in factorizations of m and n. This homomorphism is of course not 1-1.

q would associate with the particle q-adic topology consistent with a collection of p-adic topologies corresponding to the prime factors of m and n and characterizing the interactions that the particle can participate directly. In a very precise sense particles would represent both infinite and finite numbers.

Under what conditions boundary components can be connected by $\#_B$ contact?

Assume that particles are characterized by a p-adic prime determining it mass scale plus p-adic primes characterizing the gauge bosons to which they couple and assume that $\#_B$ contacts mediate gauge interactions. Assume that these primes label the boundary components of the space-time sheet representing the particle or more general light-like 3-surfaces. The question is what kind of space-time sheets can be connected by $\#_B$ contacts.

The first working hypothesis that comes in mind is that the p-adic primes associated with the two boundary components connected by $\#_B$ contact must be identical. If the notion of multi-p p-adicity is accepted, space-time sheets are characterized by integers and the largest prime dividing the integer might characterize the mass of the particle. This makes sense if the p-adic temperature $T = 1/n$ associated with small primes is small enough. In this case a common prime factor p for the integers characterizing the two space-time sheets could be enough for the possibility of $\#_B$ contact and this contact would be characterized by this prime. If no common prime factors exist, only # contacts could connect the space-time sheets. This option conforms with the number theoretical vision. This option would predict that the transition to large $\hbar$ phase occurs simultaneously for all interactions.

What about the integer characterizing graviton?

If one accepts the hypothesis that graviton couples to both visible and dark matter, graviton should be characterized by an integer dividing the integers characterizing all particles. This leaves two options.

Option I: gravitational constant characterizes graviton number theoretically

The argument leading to an expression for gravitational constant in terms of CP_2 length scale led to the proposal that the product of primes $p \leq 23$ are common to all particles and one interpretation was in terms of multi-fractality. If so, graviton would be characterized by a product of some or all primes $p \leq 23$ and would thus correspond to a very small p-adic length scale. This might be also the case for photon although it would seem that photon cannot couple to dark matter always. $p = 23$ might characterize the transversal size of the massless extremal associated with the space-time sheet of graviton.

Option II: gravitons are characterized by Mersenne prime M_{127}

The arguments related to the model of coupling constant evolution [?] lead to the proposal that graviton coupling strength behaves as L_p^2 as a function of the p-adic length scale and that effective renormalization group invariance of tge gravitational coupling strength is due to the fact that gravitational interactions are carried by $\#_B$ contacts which correspond to Mersenne prime M_{127}. This would mean that each elementary particle contains partonic 2-surface labeled by M_{127}. This is possible if the p-adic temperature associated with M_{127} is $T = 1/n$, $n > 1$, for all particles lighter than electron so that p-adic thermodynamics does not contribute appreciably to the mass squared of the particle.

Option III: graviton behaves as a unit with respect to multiplication

One can also argue that if the largest prime assignable to a particle characterizes the size of the particle space-time sheet it does not make sense to assign any finite prime to a massless particle like graviton. Perhaps graviton corresponds to simplest possible infinite prime $P = X \pm 1$, X the product of all primes.

As found, one can assign to any infinite prime, integer, and rational a rational number $q = m/n$ to which one can assign a q-adic topology as effective space-time topology and as a special case effective p-adic topologies corresponding to prime factors of m and n.

In the case of $P = X \pm 1$ the rational number would be equal to ± 1. Graviton could thus correspond to $p = 1$-adic effective topology. The "prime" $p = 1$ indeed appears as a factor of any integer so that graviton would couple to any particle. Formally the 1-adic norm of any number would be 1 or 0 which would suggest that a discrete topology is in question.

The following observations help in attempts to interpret this.

1. CP_2 type extremals having interpretation as gravitational instantons are non-deterministic in the sense that M^4 projection is random light-like curve. This condition implies Virasoro conditions which suggests interpretation in terms topological quantum theory limit of gravitation involving vanishing four-momenta but non-vanishing color charges. This theory would represent gravitation at the ultimate CP_2 length scale limit without the effects of topological condensation. In longer length scales a hierarchy of effective theories of gravitation corresponds to the coupling of space-time sheets by join along boundaries bonds would emerge and could give rise to "strong gravities" with strong gravitational constant proportional to L_p^2. It is quite possible that the M-theory based vision about duality between gravitation and gauge interactions applies to electro-weak interactions and in these "strong gravities".

2. p-Adic length scale hypothesis $p \simeq 2^k$, k integer, implies that $L_k \propto \sqrt{k}$ corresponds to the size scale of causal horizon associated with # contact. For $p = 1$ k would be zero and the causal horizon would contract to a point which would leave only generalized Feynman diagrams consisting of CP_2 type vacuum extremals moving along random light-like orbits and obeying Virasoro conditions so that interpretation as a kind of topological gravity suggests itself.

3. $p = 1$ effective topology could make marginally sense for vacuum extremals with vanishing Kähler form and carrying only gravitational charges. The induced Kähler form vanishes identically by

the mere assumption that X^4, be it continuous or discontinuous, belongs to $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2.

Why topological graviton, or whatever the particle represented by CP_2 type vacuum extremals should be called, should correspond to the weakest possible notion of continuity? The most plausible answer is that discrete topology is *consistent* with any other topology, in particular with any p-adic topology. This would express the fact that CP_2 type extremals can couple to any p-adic prime. The vacuum property of CP_2 type extremals implies that the splitting off of CP_2 type extremal leaves the physical state invariant and means effectively multiplying integer by $p = 1$.

It seems that Option I suggested by the deduction of the value of gravitational constant looks more plausible as far as the interpretation of gravitation is considered. This does not however mean that CP_2 type vacuum extremals carrying color quantum numbers could not describe gravitational interactions in CP_2 length scale.

7.4.8 An alternative interpretation for the hierarchy of functions defined by infinite primes

Suppose that infinite primes code for the ground states of super-conformal representations. Supersymmetry suggests that the corresponding polynomials or their zeros could code for the moduli space associated with these states. At the limit of algebraic closure of rationals the vanishing of the polynomial would code for a complex codimension one surface of C^n at n:th level of hierarchy.

The recent progress in the understanding of S-matrix [?] relies on the idea that the data needed to construct S-matrix is provided by the intersection of real and p-adic parton 2-surface obeying same algebraic equations. Quantum TGD is almost topological QFT since only the light-likeness of orbits of partonic 2-surfaces brings in the notion of metric. This leads to the idea that the braiding S-matrices of topological quantum field theories generalize to give a realistic S-matrix in TGD framework. The number theoretical braids at partonic 2-surface for which the strands of the braid project to the same point of the geodesic sphere S^2 of CP_2 play a key role in this approach. Braids are thus characterized by complex numbers labeling the points of S^2.

In this framework the natural idea would be that that the n, in general complex, algebraic numbers, code for the positions of braids and that vanishing of the polynomial gives correlation between the positions of braids so that the position of n^{th} level braid is fixed almost uniquely once the positions of lower level braids are known. One must however admit that this kind of correlation does not look too convincing and that the interpretation involves ad hoc elements such as the selection of the geodesic sphere. It must be however added that infinite primes could allow several mutually consistent interpretations and that this interpretation or some interpretation analogous to it might make sense.

7.5 Does the notion of infinite-P p-adicity make sense?

In this section speculations related to infinite-P p-adicity are represented in the form of shy questions in order to not irritate too much the possible reader. The basic open question causing tension is whether infinite primes relate only to the physics of cognition or whether they might allow to say something non-trivial about the physics of matter too.

The obvious question is whether the notion of p-adic number field makes sense makes sense for infinite primes and whether it might have some physical relevance. One can certainly introduce power series in powers of any infinite prime P and the coefficients can be taken to belong to any ordinary number field. In the representation by polynomials P-Adic power series correspond to Laurent series in powers of corresponding polynomial and are completely finite.

For straightforward generalization of the norm all powers of infinite-P prime have vanishing norm. The infinite-p p-adic norm of infinite-p p-adic integer would be given by its finite part so that in this sense positive powers of P would represent infinitesimals. For Laurent series this would mean that the lowest term would give the whole approximation in the real topology. For finite-primes one could however replace the norm as a power of p by a power of some other number. This would allow to have a finite norm also for P-adic primes. Since the simplest P-adic primes at the lowest level of hierarchy define naturally a rational one might consider the possibility of defining the norm of P as the inverse of this rational.

7.5.1 Does infinite-P p-adicity reduce to q-adicity?

Any non-vanishing p-adic number is expressible as a product of power of p multiplied by a p-adic unit which can be infinite as a normal integer and has pinary expansion in powers of p:

$$x = p^n(x_0 + \sum_{k>0} x_k p^k) \ , \ x_k \in \{0,..,p-1\} \ , \ x_0 > 0 \ . \tag{7.5.1}$$

The p-adic norm of x is given by $N_p(x) = p^{-n}$. Each unit has p-adic inverse which for finite integers is always infinite as an ordinary integer.

To define infinite-P p-adic numbers one must generalize the pinary expansion to a infinite-P p-adic expansion of an infinite rational. In particular, one must identify what the statement 'infinite integer modulo P' means when P is infinite prime, and what are the infinite integers N satisfying the condition $N < P$. Also one must be able to construct the p-adic inverse of any infinite prime. The correspondence of infinite primes with polynomials allows to construct infinite-P p-adics in a straightforward manner.

Consider first the infinite integers at the lowest level.

1. Infinite-P p-adics at the first level of hierarhcy correspond to Laurent series like expansions using an irreducible polynomial P of degree n representing infinite prime. The coefficients of the series are numbers in the coefficient fields. Modulo p operation is replaced with modulo polynomial P operation giving a unique result and one can calculate the coefficients of the expansion in powers of P by the same algorithm as in the case of the ordinary p-adic numbers. In the case of n-variables the coefficients of Taylor series are naturally rational functions of at most $n-1$ variables. For infinite primes this means rationals formed from lower level infinite-primes.

2. Infinite-P p-adic units correspond to expansions of this type having non-vanishing zeroth order term. Polynomials take the role of finite integers. The inverse of a infinite integer in P-adic number field is obtained by developing the polynomial counterpart of $1/N$ in the following manner. Express N in the form $N = N_0(1 + x_1P + ..)$, where N_0 is polynomial with degree at most equal to $n-1$. The factor $1/(1 + x_1P + ...)$ can be developed in geometric series so that only the calculation of $1/N_0$ remains. Calculate first the inverse $\hat{N}_0^{-1}$ of N_0 as an element of the 'finite field' defined by the polynomials modulo P: a polynomial having degree at most equal to $n-1$ results. Express $1/N_0$ as

$$\frac{1}{N_0} = \hat{N}_0^{-1}(1 + y_1P + ...)$$

and calculate the coefficients in the expansion iteratively using the condition $N \times (1/N) = 1$ by applying polynomial modulo arithmetics. Generalizing this, one can develop any rational function to power series with respect to polynomial prime P. The expansion with respect to a polynomial prime can in turn be translated to an expansion with respect to infinite prime and also mapped to a superposition of Fock states.

3. What about the norm of infinite-P p-adic integers? Ultra-metricity suggest a straightforward generalization of the usual p-adic norm. The direct generalization of the finite-p p-adic norm would mean the identification of infinite-P p-adic norm as P^{-n}, where n corresponds to the lowest order term in the polymomial expansion. Thus the norm would be infinite for $n < 0$, equal to one for $n = 0$ and vanish for $n > 0$. Any polynomial integer N would have vanishing norm with respect to those infinite-P p-adics for which P divides N. Essentially discrete topology would result.

This seems too trivial to be interesting. One can however replace P^{-n} with a^{-n}, where a is any finite number a without losing the multiplicativity and ultra-metricity properties of the norm. The function space associated with the polynomial defined by P serves as a guideline also now. This space is naturally q-adic for some rational number q. At the lowest level the infinite prime defines naturally an ordinary rational number as the zero of the polynomial as is clear from the definition of the polynomial. At higher levels of the hierarchy the rational number is rational function of lower level infinite primes and by continuing the assignments of lower level rational functions to the infinite primes one ends up with an assignment of a unique rational number with a given infinite prime serving as an excellent candidate for a rational defining the q-adicity.

7.5.2 q-Adic topology determined by infinite prime as a local topology of the configuration space

Since infinite primes correspond to polynomials, infinite-P p-adic topology, which by previous considerations would be actually q-adic topology, is a natural candidate for a topology in function spaces, in particular in the configuration space of 3-surfaces.

This view conforms also with the idea of algebraic holography. The sub-spaces of configuration space can be modelled in terms of function spaces of rational functions, their algebraic extensions, and their P-adic completions. The mapping of the elements of these spaces to infinite rationals would make possible the correspondence between configuration space and number theoretic anatomy of point of the imbedding space.

The q-adic norm for these function spaces is in turn consistent with the ultra-metricity for the space of maxima of Kähler functions conjectured to be all that is needed to construct S-matrix. Ultra-metricity conforms nicely with the expected four-dimensional spin glass degeneracy due to the enormous vacuum degeneracy meaning that maxima of Kähler function define the analog of spin glass free energy landscape. That only maxima of Kähler function would be needed would mean that radiative corrections to the configuration space integral would vanish as quantum criticality indeed requires. This TGD can be regarded as an analog of for an integrable quantum theory. Quantum criticality is absolutely essential for guaranteing that S-matrix and U-matric elements are algebraic numbers which in turn guarantees number theoretic universality of quantum TGD.

7.5.3 The interpretation of the discrete topology determined by infinite prime

Also $p = 1$-adic topology makes formally sense and corresponds to a discrete topology in which all rationals have unit norm. It results also results if one naively generalizes p-adic topology to infinite-p p-adic topology by defining the norm of infinite prime at the lowest level of hierarchy as $|P|_P = 1/P = 0$. In this topology the distance between two points is either 1 or 0 and this topology is the roughest possible topology one can imagine.

It must be however noticed that if one maps infinite-P p-adics to real by the formal generalization of the canonical identification then one obtains real topology naturally if coefficients of powers of P are taken to be reals. This would mean that infinite-P p-adic topology would be equivalent with real topology.

Consider now the possible interpretations.

1. At the level of function spaces infinite-p p-adic topology in the naive sense has a completely natural interpretation and states that the replacement of the Taylor series with its lowest term.

2. The formal possibility of $p = 1$-adic topology at space-time level suggests a possible interpretation for the mysterious infinite degeneracy caused by the presence of the absolute minima of the Kähler function: one can add to any absolute minimum a vacuum extremal, which behaves completely randomly except for the constraints forcing the surface to be a vacuum extremal. This non-determinism is much more general than the non-determinism involving a discrete sequence of bifurcations (I have used the term association sequence about this kind of sequences). This suggests that one must replace the concept of 3-surface with a more general one, allowing also continuous association sequences consisting of a continuous family of space-like 3-surfaces with infinitesimally small time like separations. These continuous association sequences would be analogous to vacuum bubbles of the quantum field theories.

One can even consider the possibility that vacuum extremals are non-differentiable and even discontinuous obeying only effective $p = 1$-adic topology. Also modified Dirac operator vanishes identically in this case. Since vacuum surfaces are in question, $p = 1$ regions cannot correspond to material sheets carrying energy and also the identification as cognitive space-time sheets is questionable. Since

$p = 1$, the smallest possible prime in generalized sense, it must represent the lowest possible level of evolution, primordial chaos. Quantum classical correspondence suggests that $p = 1$ level is indeed present at the space-time level and might realized by the mysterious vacuum extremals.

7.6 Infinite primes and mathematical consciousness

The mathematics of infinity relates naturally with the mystery of consciousness and religious and mystic experience. In particular, mathematical cognition might have as a space-time correlate the infinitely structured space-time points implied by the introduction of infinite-dimensional space of real units defined by infinite (hyper-)octonionic rationals having unit norm in the real sense. I hope that the reader takes this section as a noble attempt to get a glimpse about unknown rather than final conclusions.

7.6.1 Infinite primes, cognition and intentionality

Somehow it is obvious that infinite primes must have some very deep role to play in quantum TGD and TGD inspired theory of consciousness. What this role precisely is has remained an enigma although I have considered several detailed interpretations, one of them above.

In the following an interpretation allowing to unify the views about fermionic Fock states as a representation of Boolean cognition and p-adic space-time sheets as correlates of cognition is discussed. Very briefly, real and p-adic partonic 3-surfaces serve as space-time correlates for the bosonic super algebra generators, and pairs of real partonic 3-surfaces and their algebraically continued p-adic variants as space-time correlates for the fermionic super generators. Intentions/actions are represented by p-adic/real bosonic partons and cognitions by pairs of real partons and their p-adic variants and the geometric form of Fermi statistics guarantees the stability of cognitions against intentional action. It must be emphasized that this interpretation is not identical with the one discussed above since it introduces different identification of the space-time correlates of infinite primes.

Infinite primes very briefly

Infinite primes have a decomposition to infinite and finite parts allowing an interpretation as a many-particle state of a super-symmetric arithmetic quantum field theory for which fermions and bosons are labeled by primes. There is actually an infinite hierarchy for which infinite primes of a given level define the building blocks of the infinite primes of the next level. One can map infinite primes to polynomials and these polynomials in turn could define space-time surfaces or at least light-like partonic 3-surfaces appearing as solutions of Chern-Simons action so that the classical dynamics would not pose too strong constraints.

The simplest infinite primes at the lowest level are of form $m_B X/s_F + n_B s_F$, $X = \prod_i p_i$ (product of all finite primes). The simplest interpretation is that X represents Dirac sea with all states filled and $X/s_F + s_F$ represents a state obtained by creating holes in the Dirac sea. m_B, n_B, and s_F are defined as $m_B = \prod_i p_i^{m_i}$, $n_B = \prod_i q_i^{n_i}$, and $s_F = \prod_i q_i$, m_B and n_B have no common prime factors. The integers m_B and n_B characterize the occupation numbers of bosons in modes labeled by p_i and q_i and $s_F = \prod_i q_i$ characterizes the non-vanishing occupation numbers of fermions.

The simplest infinite primes at all levels of the hierarchy have this form. The notion of infinite prime generalizes to hyper-quaternionic and even hyper-octonionic context and one can consider the possibility that the quaternionic components represent some quantum numbers at least in the sense that one can map these quantum numbers to the quaternionic primes.

The obvious question is whether configuration space degrees of freedom and configuration space spinor (Fock state) of the quantum state could somehow correspond to the bosonic and fermionic parts of the hyper-quaternionic generalization of the infinite prime. That hyper-quaternionic (or possibly hyper-octonionic) primes would define as such the quantum numbers of fermionic super generators does not make sense. It is however possible to have a map from the quantum numbers labeling super-generators to the finite primes. One must also remember that the infinite primes considered are only the simplest ones at the given level of the hierarchy and that the number of levels is infinite.

Precise space-time correlates of cognition and intention

The best manner to end up with the proposal about how p-adic cognitive representations relate bosonic representations of intentions and actions and to fermionic cognitive representations is through the following arguments.

1. In TGD inspired theory of consciousness Boolean cognition is assigned with fermionic states. Cognition is also assigned with p-adic space-time sheets. Hence quantum classical correspondence suggets that the decomposition of the space-time into p-adic and real space-time sheets should relate to the decomposition of the infinite prime to bosonic and fermionic parts in turn relating to the above mention decomposition of physical states to bosonic and fermionic parts.

   If infinite prime defines an association of real and p-adic space-time sheets this association could serve as a space-time correlate for the Fock state defined by configuration space spinor for given 3-surface. Also spinor field as a map from real partonic 3-surface would have as a space-time correlate a cognitive representation mapping real partonic 3-surfaces to p-adic 3-surfaces obtained by algebraic continuation.

2. Consider first the concrete interpretation of integers m_B and n_B. The most natural guess is that the primes dividing $m_B = \prod_i p^{m_i}$ characterize the effective p-adicities possible for the real 3-surface. m_i could define the numbers of disjoint partonic 3-surfaces with effective p_i-adic topology and associated with with the same real space-time sheet. These boundary conditions would force the corresponding real 4-surface to have all these effective p-adicities implying multi-p-adic fractality so that particle and wave pictures about multi-p-adic fractality would be mutually consistent. It seems natural to assume that also the integer n_i appearing in $m_B = \prod_i q_i^{n_i}$ code for the number of real partonic 3-surfaces with effective q_i-adic topology.

3. Fermionic statistics allows only single genuinely q_i-adic 3-surface possibly forming a pair with its real counterpart from which it is obtained by algebraic continuation. Pairing would conform with the fact that n_F appears both in the finite and infinite parts of the infinite prime (something absolutely essential concerning the consistency of interpretation!).

   The interpretation could be as follows.

   i) Cognitive representations must be stable against intentional action and fermionic statistics guarantees this. At space-time level this means that fermionic generators correspond to pairs of real effectively q_i-adic 3-surface and its algebraically continued q_i-adic counterpart. The quantum jump in which q_i-adic 3-surface is transformed to a real 3-surface is impossible since one would obtain two identical real 3-surfaces lying on top of each other, something very singular and not allowed by geometric exclusion principle for surfaces. The pairs of boson and fermion surfaces would thus form cognitive representations stable against intentional action.

   ii) Physical states are created by products of super algebra generators Bosonic generators can have both real or p-adic partonic 3-surfaces as space-time correlates depending on whether they correspond to intention or action. More precisely, m_B and n_B code for collections of real and p-adic partonic 3-surfaces. What remains to be interpreted is why m_B and n_B cannot have common prime factors (this is possible if one allows also infinite integers obtained as products of finite integer and infinite primes).

   iii) Fermionic generators to the pairs of a real partonic 3-surface and its p-adic counterpart obtained by algebraic continuation and the pictorial interpretation is as fermion hole pair. Unrestricted quantum super-position of Boolean statements requires that many-fermion state is accompanied by a corresponding many-antifermion state. This is achieved very naturally if real and corresponding p-adic fermion have opposite fermion numbers so that the kicking of negative energy fermion from Dirac sea could be interpreted as creation of real-p-adic fermion pairs from vacuum.

   If p-adic space-time sheets obey same algebraic expressions as real sheets (rational functions with algebraic coefficients), the Chern-Simons Noether charges associated with real partons defined as integrals can be assigned also with the corresponding p-adic partons if they are rational or algebraic numbers. This would allow to circumvent the problems related to the p-adic integration. Therefore one can consider also the possibility that p-adic partons carry

Noether charges opposite to those of corresponding real partons sheet and that pairs of real and p-adic fermions can be created from vacuum. This makes sense also for the classical charges associated with Kähler action in space-time interior if the real space-time sheet obeying multi-p p-adic effective topology has algebraic representation allowing interpretation also as p-adic surface for all primes involved.

iv) This picture makes sense if the partonic 3-surfaces containing a state created by a product of super algebra generators are unstable against decay to this kind of 3-surfaces so that one could regard partonic 3-surfaces as a space-time representations for a configuration space spinor field.

4. Are alternative interpretations possible? For instance, could $q = m_B/m_B$ code for the effective q-adic topology assignable to the space-time sheet. That q-adic numbers form a ring but not a number field casts however doubts on this interpretation as does also the general physical picture.

Number theoretical universality of S-matrix

The discreteness of the intersection of the real space-time sheet and its p-adic variant obtained by algebraic continuation would be a completely universal phenomenon associated with all fermionic states. This suggests that also real-to-real S-matrix elements involve instead of an integral a sum with the arguments of an n-point function running over all possible combinations of the points in the intersection. S-matrix elements would have a universal form which does not depend on the number field at all and the algebraic continuation of the real S-matrix to its p-adic counterpart would trivialize. Note that also fermionic statistics favors strongly discretization unless one allows Dirac delta functions.

7.6.2 The generalization of the notion of ordinary number field

The notion of infinite rationals leads also to the generalization of the notion of a finite number. The obvious generalization would be based on the allowance of infinitesimals. Much more interesting approach is however based on the observation that one obtains infinite number of real units by taking two infinite primes with a finite rational valued ratio q and by dividing this ratio by ordinary rational number q. As a real number the resulting number differs in no manner from ordinary unit of real numbers but in p-adic sense the points are not equivalent. This construction generalizes also to quaternionic and octonionic case.

Space-time points would become structured since infinite rationals normed to unity define naturally a gigantically infinite-dimensional free algebra generated by the units serving in well-define sense as Mother of All Algebras. The units of the algebra multiplying ordinary rational numbers (and also other elements) of various number fields are invisible at the level of real physics so that the interpretation as the space-time correlate of mathematical cognition realizing the idea of monad is natural. Universe would be an algebraic hologram with single point being able to represent the state of the Universe in its structure. Infinite rationals would allow the realization of the Platonia of all imaginable mathematical constructs at the level of space-time.

The generalized units for quaternions and octonions

In the case of real and complex rationals the group of generalized units generated by primes *resp.* infinite Gaussian primes is commutative. In the case of unit quaternions and hyper-quaternions group becomes non-commutative and in case of unit hyper-octonions the group is replaced by a kind non-associative generalization of group.

For infinite primes for which only finite number of bosonic and fermionic modes are excited it is possible to tell how the products AB and BA of two infinite primes explicitly since the finite hyper-octonionic primes can be assumed to multiply the infinite integer X from say left.

Situation changes if infinite number of bosonic excitations are present since one would be forced to move finite H- or O-primes past a infinite number of primes in the product AB. Hence one must simply assume that the group G generated by infinite units with infinitely many bosonic excitations is a free group. Free group interpretation means that non-associativity is safely localized inside infinite primes and reduced to the non-associativity of ordinary hyper-octonions. Needless to say free group is the best one can hope of achieving since free group allows maximal number of factor groups.

The free group G can be extended into a free algebra A by simply allowing superpositions of units with coefficients which are real-rationals or possibly complex rationals. Again free algebra fulfils the dreams as system with a maximal representative power. The analogy with quantum states defined as functions in the group is highly intriguing and unit normalization would correspond to the ordinary normalization of Schrödinger amplitudes. Obviously this would mean that single point is able to mimic quantum physics in its structure. Could state function reduction and preparation be represented at the level of space-time surfaces so that initial and final 3-surfaces would represent pure states containing only bound state entanglement represented algebraically, and could the infinite rationals generating the group of quaternionic units (no sums over them) represent pure states?

The free algebra structure of A together with the absolutely gigantic infinite-dimensionality of the endless hierarchy of infinite rational units suggests that the resulting free algebra structure is universal in the sense that any algebra defined with coefficients in the field of rationals can be imbedded to the resulting algebra or represented as a factor algebra obtained by the sequence $A \rightarrow 1_1 = A/I_1 \rightarrow A_1/I_2...$ where the ideal I_k is defined by $k : th$ relation in A_{k-1}.

Physically the embedding would mean that some field quantities defined in the algebra are restricted to the subalgebra. The representation of algebra B as an iterated factor algebra would mean that some field quantities defined in the algebra are constant inside the ideals I_k of A defined by the relations. For instance, the induced spinor field at space-time surface would have same value for all points of A which differ by an element of the ideal. At the configuration space level, the configuration space spinor field would be constant inside an ideal associated with the algebra of A-valued functions at space-time surfaces.

The units can be interpreted as defining an extension of rationals in C, H, or O. Galois group is defined as automorphisms of the extension mapping the original number field to itself and obviously the transformations $x \rightarrow gxg^{-1}$, where g belongs to the extended number field act as automorphisms. One can regard also the extension by real units as the extended number field and in this case the automorphisms contain also the automorphisms induced by the multiplication of each infinite prime Π_i by a real unit U_i: $\Pi_i \rightarrow \hat{\Pi}_i = U_i \Pi_i$.

The free algebra generated by generalized units and mathematical cognition

One of the deepest questions in theory of consciousness concerns about the space-time correlates of mathematical cognition. Mathematician can imagine endlessly different mathematical structures. Platonist would say that in some sense these structures exist. The claim classical physical worlds correspond to certain 4-surfaces in $M^4_+ \times CP_2$ would leave out all these beautiful mathematical structures unless they have some other realization than the physical one.

The free algebra A generated by the generalized multiplicative units of rationals allows to understand how Platonia is realized at the space-time level. A has no correlate at the level of real physics since the generalized units correspond to real numbers equal to one. This holds true also in quaternionic and octonionic cases since one can require that the units have net quaternionic and octonionic phases equal to one. By its gigantic size A and free algebra character might be able represent all possible algebras in the proposed manner. Also non-associative algebras can be represented.

Algebraic equations are the basic structural building blocks of mathematical thinking. Consider as a simple example the equation $AB = C$. The equations are much more than tautologies since they contain the information at the left hand side about the variables of the algebraic operation giving the outcome on the right hand side. For instance, in the case of multiplication $AB = C$ the information about the factors is present although it is completely lost when the product is evaluated. These equations pop up into our consciousness in some mysterious manner and the question is what are the space-time correlates of these experiences suggested to exist by quantum-classical correspondence.

The algebra of units is an excellent candidate for the sought for correlate of mathematical cognition. I must admit that that it did not occur to me that Leibniz might have been right about his monads! The idealization is however in complete accordance with the idea about the Universe as an algebraic hologram taken to its extreme. One can say that each point represents an equation. The left hand side of the equation corresponds to the element of the free algebra defined by octonionic units. Consider as an example product of powers of $X/\Pi(Q_q)$ representing infinite quaternionic rationals. Equality sign corresponds to the evaluation of this expression by interpreting it as a real quaternionic rational number: real physics does the evaluation automatically. The information about the primes appearing as factors of the result is not however lost at cognitive level. Note that the analogs of quantum states represented by superpositions of the unit elements of the algebra A can be interpreted as equations defining them.

When two points are cobordant?

Topological quantum field theories have led to a dramatic success in the understanding of 3- and 4-dimensional topologies and cobordisms of these manifolds (two n-manifolds are cobordant if there exists an $n+1$-manifold having them as boundaries). In his thought-provoking and highly inspiring article Pierre Cartier [?] poses a question which at first sounds absurd. What might be the the counterpart of cobordism for points? The question is indeed absurd unless the points have some structure.

If one takes seriously the idea that each point of space-time sheet corresponds to a unit defined by an infinite rational, the obvious question is under what conditions there is a continuous line connecting these points with continuity being defined in some generalized sense. In real sense the line is continuous always but in p-adic sense only if all p-adic norms of the two units are identical. Since the p-adic norm of the unit of $Y(n/m) = X/\Pi(n/m)$ is that of $q = n/m$, the norm of two infinite rational numbers is same only if they correspond to the same ordinary rational number.

Suppose that one has

$$
\begin{aligned}
Y_I &= \frac{\prod_i Y(q^I_{1i})}{\prod_i Y(q^I_{2i})} \ , \quad Y_F = \frac{\prod_i Y(q^F_{1i})}{\prod_i Y(q^F_{2i})} \ , \\
q^I_{ki} &= \frac{n^I_{k_i}}{m^I_{k_i}} \ , \qquad q^F_{ki} = \frac{n^F_{k_i}}{m^F_{k_i}} \ ,
\end{aligned}
\tag{7.6.1}
$$

Here $m^{\cdot}_{\cdot}$ representing arithmetic many-fermion state is a square free integer and $n^{\cdot}_{\cdot}$ representing arithmetic many-boson state is an integer having no common factors with $m^{\cdot}_{\cdot}$.

The two units have same p-adic norm in all p-adic number fields if the rational numbers associated with Y_I and Y_F are same:

$$
\frac{\prod_i q^I_{1i}}{\prod_i q^I_{2i}} = \frac{\prod_i q^F_{1i}}{\prod_i q^F_{2i}} \ . \tag{7.6.2}
$$

The logarithm of this condition gives a conservation law of energy encountered in arithmetic quantum field theories, where the energy of state labeled by the prime p is $E_p = log(p)$:

$$
\begin{aligned}
E^I &= \sum_i log(n^I_{1i}) - \sum_i log(n^I_{2i}) - \sum_i log(m^I_{1i}) + \sum_i log(m^I_{2i}) = \\
&= \sum_i log(n^F_{1i}) - \sum_i log(n^F_{2i}) - \sum_i log(m^F_{1i}) + \sum_i log(m^F_{2i}) = E^F \ .
\end{aligned}
\tag{7.6.3}
$$

There are both positive and negative energy particles present in the system. The possibility of negative energies is indeed one of the basic predictions of quantum TGD distinguishing it from standard physics. As one might have expected, Y^I and Y^F represent the initial and final states of a particle reaction and the line connecting the two points represents time evolution giving rise to the particle reaction. In principle one can even localize various steps of the reaction along the line and different lines give different sequences of reaction steps but same overall reaction. This symmetry is highly analogous to the conformal invariance implying that integral in complex plane depends only on the end points of the curve.

Whether the entire four-surface should correspond to the same value of topological energy or whether E can be discontinuous at elementary particle horizons separating space-time sheets and represented by light-like 3-surfaces around wormhole contacts remains an open question. Discontinuity through elementary particle horizons would make possible the arithmetic analogs of poles and cuts of analytic functions since the limiting values of Y from different sides of the horizon are different. Note that the construction generalizes to the quaternionic and octonionic case.

TGD inspired analog for d-algebras

Maxim Kontsevich has done deep work with quantizations interpreted as a deformation of algebraic structures and there are deep connections with this work and braid group [?]. In particular, the Grothendienck-Teichmueller algebra believed to act as automorphisms for the deformation structures acts as automorphisms of the braid group at the limit of infinite number of strands. I must admit that my miserable skills in algebra does not allow to go to the horrendous technicalities but occasionally I have the feeling that I have understood some general ideas related to this work. In his article "Operads and Motives in Deformation Quantization" Kontsevich introduces the notions of operad and d-algebras over operad. Without going to technicalities one can very roughly say that d-algebra is essentially d-dimensional algebraic structure, and that the basic conjecture of Deligne generalized and proved by Kontsevich states in its generalized form that $d+1$-algebras have a natural action in all d-algebras.

In the proposed extension of various rationals a notion resembling that of universal d-algebra to some degree but not equivalent with it emerges naturally. The basic idea is simple.

1. Points correspond to the elements of the assumed to be universal algebra A which in this sense deserves the attribute $d = 0$ algebra. By its universality A should be able to represent any algebra and in this sense it cannot correspond $d = 0$-algebra of Kontsevich defined as a complex, that is a direct sum of vector spaces V_n and possessing d operation $V_n \rightarrow V_{n+1}$, satisfying $d^2 = 0$. Each point of a manifold represents one particular element of 0-algebra and one could loosely say that multiplication of points represents algebraic multiplication. This algebra has various subalgebras, in particular those corresponding to reals, complex numbers and quaternions. One can say that sub-algebra is non-associative, non-commutative, etc.. if its real evaluation has this property.

2. Lines correspond to evolutions for the elements of A which are continuous with respect to real (trivially) and all p-adic number fields. The latter condition is nontrivial and allows to interpret evolution as an evolution conserving number theoretical analog of total energy. Universal 1-group would consist of curves along which one has the analog of group valued field (group being the group of generalized units) having values in the universal 0-group G. The action of the 1-group in 0-group would simply map the element of 0-group at the first end of the curve its value at the second end. Curves define a monoid in an obvious manner. The interpretation as a map to A allows pointwise multiplication of these mappings which generalizes to all values of d.

   One could also consider the generalization of local gauge field so that there would be gauge potential defined in the algebra of units having values on A. This potential would define holonomy group acting on 0-algebra and mapping the element at the first end of the curve to its gauge transformed variant at the second end. In this case also closed curves would define non-trivial elements of the holonomy group. In fact, practically everything is possible since probably any algebra can be represented in the algebra generated by units.

3. Two-dimensional structures correspond to dynamical evolutions of one-dimensional structures. The simplest situation corresponds to 2-cubes with the lines corresponding to the initial and final values of the second coordinate representing initial and final states. One can also consider the possibility that the two-surface is topologically non-trivial containing handles and perhaps even holes. One interpret this cognitive evolutions represents 1-dimensional flow so that the initial points travel to final points. Obviously there is symmetry breaking involved since the second coordinate is in the role of time and this defines kind of time orientation for the surface.

4. The generalization to 4- and higher dimensional cases is obvious. One just uses d-manifolds with edges and uses their time evolution to define $d+1$-manifolds with edges. Universal 3-algebra is especially interesting from the point of view of braid groups and in this case the maps between initial and final elements of 2-algebra could be interpreted as braid operations if the paths of the elements along 3-surface are entangled. For instance field lines of Kähler gauge potential or of magnetic field could define this kind of braiding.

5. The d-evolutions define a monoid since one can glue two d-evolutions together if the outcome of the first evolution equals to the initial state of the second evolution. $d+1$-algebra also acts naturally in d-algebra in the sense that the time evolution $f(A \rightarrow B)$ assigns to the d-algebra

valued initial state A a d-algebra valued final state and one can define the multiplication as $f(A \to B)C = B$ for $A = C$, otherwise the action gives zero. If time evolutions correspond to standard cubes one gets more interesting structure in this manner since the cubes differing by time translation can be identified and the product is always non-vanishing.

6. It should be possible to define generalizations of homotopy groups to what might be called "cognitive" homotopy groups. Effectively the target manifold would be replaced by the tensor product of an ordinary manifold and some algebraic structure represented in A. All kinds of "cognitive" homotopy groups would result when the image is cognitively non-contractible. Also homology groups could be defined by generalizing singular complex consisting of cubes with cubes having the hierarchical decomposition into time evolutions of time evolutions of... in some sub-algebraic structure of A. If one restricts time evolutions to sub-algebraic structures one obtains all kinds of homologies. For instance, associativity reduces 3-evolutions to paths in rational $SU(3)$ and since $SU(3)$ just like any Lie group has non-trivial 3-homology, one obtains nontrivial "cognitive" homology for 3-surfaces with non-trivial 3-homology.

The following heuristic arguments are inspired by the proposed vision about algebraic cognition and the conjecture that Grothendienck-Teichmueller group acts as automorphisms of Feynman diagrammatics relating equivalent quantum field theories to each other.

1. The operations of $d+1$-algebra realized as time evolution of d-algebra elements suggests an interpretation as cognitive counterparts for sequences of algebraic manipulations in d-algebra which themselves become elements of d+1 algebra. At the level of paths of points the sequences of algebraic operations correspond to transitions in which the number of infinite primes defining an infinite rational can change in discrete steps but is subject to the topological energy conservation guaranteing the p-adic continuity of the process for all primes. Different paths connecting a and b represent different but equivalent manipulations sequences.

   For instance, at $d = 2$ level one has a pile of these processes and this in principle makes it possible an abstraction to algebraic rules involved with the process by a pile of examples. Higher values of d in turn make possible further abstractions bringing in additional parameters to the system. All kinds of algebraic processes can be represented in this manner. For instance, multiplication table can be represented as paths assigning to an the initial state product of elements a and b represented as infinite rationals and to the final state their product ab represented as single infinite rational. Representation is of course always approximate unless the algebra is finite. All kinds abstract rules such as various commutative diagrams, division of algebra by ideal by choosing one representative from each equivalence class of A/I as end point of the path, etc... can be represented in this manner.

2. There is also second manner to represent algebraic rules. Entanglement is a purely algebraic notion and it is possible to entangle the many-particle states formed as products of infinite rationals representing inputs of an algebraic operation A with the outcomes of A represented in the same manner such that the entanglement is consistent with the rule.

3. There is nice analogy between Feynman diagrams and sequences of algebraic manipulations. Multiplication ab corresponds to a map $A \otimes A \to A$ is analogous to a fusion of elementary particles since the product indeed conserves the number theoretical energy. Co-algebra operations are time reversals of algebra operations in this evolution. Co-multiplication Δ assigns to $a \in A$ an element in $A \otimes A$ via algebra homomorphism and corresponds to a decay of initial state particle to two final state particles. It defines co-multiplication assign to $a \otimes b \in A \otimes A$ an element of $A \otimes A \to A \otimes A \otimes A$ and corresponds to a scattering of elementary particles with the emission of a third particle. Hence a sequence of algebraic manipulations is like a Feynman diagram involving both multiplications and co-multiplications and thus containing also loops. When particle creation and annihilation are absent, particle number is conserved and the process represents algebra endomorphism $A \to A$. Otherwise a more general operation is in question. This analogy inspires the question whether particle reactions could serve as a blood and flesh representation for $d = 4$ algebras.

4. The dimension $d = 4$ is maximal dimension of single space-time evolution representing an algebraic operation (unless one allows the possibility that space-time and imbedding space dimensions are come as multiples of four and 8). Higher dimensions can be effectively achieved only if several space-time sheets are used defining $4n$-dimensional configuration space. This could reflect some deep fact about algebras in general and also relate to the fact that 3- and 4-dimensional manifolds are the most interesting ones topologically.

7.6.3 Algebraic Brahman=Atman identity

The proposed view about cognition and intentionality emerges from the notion of infinite primes, which was actually the first genuinely new mathematical idea inspired by TGD inspired consciousness theorizing. Infinite primes, integers, and rationals have a precise number theoretic anatomy. For instance, the simplest infinite primes correspond to the numbers $P_{\pm} = X \pm 1$, where $X = \prod_k p_k$ is the product of all finite primes. Indeed, $P_{\pm} \ mod \ p = 1$ holds true for all finite primes. The construction of infinite primes at the first level of the hierarchy is structurally analogous to the quantization of super-symmetric arithmetic quantum field theory with finite primes playing the role of momenta associated with fermions and bosons. Also the counterparts of bound states emerge. This process can be iterated: at the second level the product of infinite primes constructed at the first level replaces X and so on.

The structural similarity with repeatedly second quantized quantum field theory strongly suggests that physics might in some sense reduce to a number theory for infinite rationals M/N and that second quantization could be followed by further quantizations. As a matter fact, the hierarchy of space-time sheets could realize this endless second quantization geometrically and have also a direct connection with the hierarchy of logics labeled by their order. This could have rather breathtaking implications.

1. One is forced to ask whether this hierarchy corresponds to a hierarchy of realities for which level below corresponds in a literal sense infinitesimals and the level next above to infinity.

2. Second implication is that there is an infinite number of infinite rationals behaving like real units ($M/N \equiv 1$ in real sense) so that space-time points could have infinitely rich number theoretical anatomy not detectable at the level of real physics. Infinite integers would correspond to positive energy many particle states and their inverses (infinitesimals with number theoretic structure) to negative energy many particle states and $M/N \equiv 1$ would be a counterpart for zero energy ontology to which oneness and emptiness are assigned in mysticism.

3. Single space-time point, which is usually regarded as the most primitive and completely irreducible structure of mathematics, would take the role of Platonia of mathematical ideas being able to represent in its number theoretical structure even the quantum state of entire Universe. Algebraic Brahman=Atman identity and algebraic holography would be realized in a rather literal sense.

Number theoretic anatomy of space-time point

This number theoretical anatomy should relate to mathematical consciousness in some manner. For instance, one can ask whether it makes sense to speak about quantum jumps changing the number theoretical anatomy of space-time points and whether these quantum jumps give rise to mathematical ideas. In fact, the identifications of Platonia as spinor fields in WCW on one hand and as the set number theoretical anatomies of point of imbedding space force the conclusion that configuration space spinor fields (recall also the identification as correlates for logical mind) can be realized in terms of the space for number theoretic anatomies of imbedding space points. Therefore quantum jumps would be correspond to changes in the anatomy of the space-time points. Or more precisely, to the changes of the configuration space spinor fields regarded as wave functions in the set of imbedding space points which are equivalent in real sense. Imbedding space would be experiencing genuine number theoretical evolution. The whole physics would reduce to the anatomy of numbers. All mathematical notions which are more than mere human inventions would be imbeddable to the Platonia realized as the number theoretical anatomies of single imbedding space point.

To realize this picture would require that both configuration space and configuration space spinor fields are mappable to the number theoretic anatomies of space-time point. The possibility to map infinite primes to polynomials and vice versa gives support for the possibility to map configuration space or at least the space of maxima of Kähler function defining the counterpart of spin glass energy landscape to the number theoretic anatomy of imbedding space point.

Function spaces provide a natural model for the subspaces of the world of classical worlds. The spaces of rational functions, their extensions, and q-adic completions, provide natural candidates for these function spaces, so that a mapping to real units defined by infinite rationals, their extensions, and q-adic completions emerge naturally. In the same manner Fock states can be mapped to infinite primes and one can see the polynomial-infinite prime correspondence also as an articulation of fermion-boson super-symmetry.

The commutativity requirement for infinite primes implies that infinite primes at n:th level can define rational functions of n complex variables. This relates naturally to the effective 2-dimensionality of TGD in the sense that configuration space geometry involves only data about 2-dimensional partonic surfaces at boundaries of $\delta M^4_\pm \times CP_2$. Allowing non-commutativity one would also obtain 4-D surfaces but algebraic continuation would mean that 2-D data is enough.

Could algebraic Brahman Atman identity represent a physical law?

Just for fun and to test these ideas it is interesting to find whether additional constraints coming from zero energy ontology and finite measurement resolution [**?**] might give allow to realize algebraic Brahman Atman identity as a physical law dictating the number theoretic anatomy of some space-time points from the structure of quantum state of Universe.

The identification of quantum corrections as insertion of zero energy states in time scale below measurement resolution to positive or negative energy part of zero energy state and the identification of number theoretic braid as a space-time correlate for the finite measurement resolution give considerable additional constraints.

1. The fundamental representation space consists of wave functions in the Cartesian power U^8 of space U of real units associated with any point of H. That there are 8 real units rather than one is somewhat disturbing: this point will be discussed below. Real units are ratios of infinite integers having interpretation as positive and negative energy states of a super-symmetric arithmetic QFT at some level of hierarchy of second quantizations. Real units have vanishing net quantum numbers so that only zero energy states defining the basis for configuration space spinor fields should be mapped to them. In the general case quantum superpositions of these basis states should be mapped to the quantum superpositions of real units. The first guess is that real units represent a basis for configuration space spinor fields constructed by applying bosonic and fermionic generators of appropriate super Kac-Moody type algebra to the vacuum state.

2. What can one say about this map bringing in mind Gödel numbering? Each pair of bosonic and corresponding fermionic generator at the lowest level must be mapped to its own finite prime. If this map is specified, the map is fixed at the higher levels of the hierarchy. There exists an infinite number of this kind of correspondences. To achieve some uniqueness, one should have some natural ordering which one might hope to reflect real physics. The irreps of the (non-simple) Lie group involved can be ordered almost uniquely. For simple group this ordering would be with respect to the sum $N = N_F + N_{F,c}$ of the numbers N_F *resp.* $N_{F,c}$ of the fundamental representation *resp.* its conjugate appearing in the minimal tensor product giving the irrep. The generalization to non-simple case should use the sum of the integers N_i for different factors for factor groups. Groups themselves could be ordered by some criterion, say dimension. The states of a given representation could be mapped to subsequent finite primes in an order respecting some natural ordering of the states by the values of quantum numbers from negative to positive (say spin for $SU(2)$ and color isospin and hypercharge for $SU(3)$). This would require the ordering of the Cartesian factors of non-simple group, ordering of quantum numbers for each simple group, and ordering of values of each quantum number from positive to negative.

   The presence of conformal weights brings in an additional complication. One cannot use conformal as a primary orderer since the number of $SO(3) \times SU(3)$ irreps in the super-canonical sector is infinite. The requirement that the probabilities predicted by p-adic thermodynamics are rational numbers or equivalently that there is a length scale cutoff, implies a cutoff in conformal weight. The vision about M-matrix forces to conclude that different values of the total conformal weight n for the quantum state correspond to summands in a direct sum of HFFs. If so, the introduction of the conformal weight would mean for a given summand only the assignment n conformal weights to a given Lie-algebra generator. For each representation of the Lie group one would have n copies ordered with respect to the value of n and mapped to primes in this order.

3. Cognitive representations associated with the points in a subset, call it P, of the discrete intersection of p-adic and real space-time sheets, defining number theoretic braids, would be in question. Large number of partonic surfaces can be involved and only few of them need to contribute to P in the measurement resolution used. The fixing of P means measurement of N positions of H and each point carries fermion or anti-fermion numbers. A more general situation corresponds to plane wave type state obtained as superposition of these states. The condition of rationality or at least algebraicity means that discrete variants of plane waves are in question.

4. By the finiteness of the measurement resolution configuration space spinor field decomposes into a product of two parts or in more general case, to their superposition. The part Ψ_+, which is above measurement resolution, is representable using the information contained by P, coded by the product of second quantized induced spinor field at points of P, and provided by physical experiments. Configuration space "orbital" degrees of freedom should not contribute since these points are fixed in H.

5. The second part of the configuration space spinor field, call it Ψ_-, corresponds to the information below the measurement resolution and assignable with the complement of P and mappable to the structure of real units associated with the points of P. This part has vanishing net quantum numbers and is a superposition over the elements of the basis of CH spinor fields and mapped to a quantum superposition of real units. The representation of Ψ_- as a Schrödinger amplitude in the space of real units could be highly unique. Algebraic holography principle would state that the information below measurement resolution is mapped to a Schrödinger amplitude in space of real units associated with the points of P.

6. This would be also a representation for perceiver-external world duality. The correlation function in which P appears would code for the information appearing in M-matrix representing the laws of physics as seen by conscious entity about external world as an outsider. The quantum superposition of real units would represent the purely subjective information about the rest of the universe. Hence number theoretic Brahman=Atman would correspond also to the original Brahman=Atman. Note that one must perceive external world in order to have the representation of the rest of the Universe.

There is an objection against this picture. One obtains an 8-plet of arithmetic zero energy states rather than one state only. What this strange 8-fold way could mean?

1. The crucial observation is that hyper-finite factor of type II_1 (HFF) creates states for which center of mass degrees of freedom of 3-surface in H are fixed. One should somehow generalize the operators creating local HFF states to fields in H, and an octonionic generalization of conformal field suggests itself. I have indeed proposed a quantum octonionic generalization of HFF extending to an HFF valued field Ψ in 8-D quantum octonionic space with the property that maximal quantum commutative sub-space corresponds to hyper-octonions [?]. This construction raises $X^4 \subset M^8$ and by number theoretic compactification also $X^4 \subset H$ in a unique position since non-associativity of hyper-octonions does not allow to identify the algebra of HFF valued fields in M^8 with HFF itself.

2. The value of Ψ in the space of quantum octonions restricted to a maximal commutative subspace can be expressed in terms of 8 HFF valued coefficients of hyper-octonion units. By the hyper-octonionic generalization of conformal invariance all these 8 coefficients must represent zero energy HFF states. The restriction of Ψ to a given point of P would give a state, which has 8 HFF valued components and Brahman=Atman identity would map these components to U^8

associated with P. One might perhaps say that 8 zero energy states are needed in order to code the information about the H positions of points P.

One-element field realized in terms of real units with number theoretic anatomy

One-element field [?] looks rather self-contradictory notion since 1 and 0 should be represented by same element. The real units expressible as ratios of infinite rationals could however provide a well-defined realization of this notion.

1. The condition that same element represents the neutral element of both sum and product gives strong constraint on one-element field. Consider an algebra formed by reals with sum and product defined in the following manner. Sum, call it $\oplus$, corresponds to the ordinary product $x \times y$ for reals whereas product, call it $\otimes$, is identified as the non-commutative product $x \otimes y = x^y$. $x = 1$ represents both the neutral element (0) of $\oplus$ and the unit of $\otimes$. The sub-algebras generated by 1 and multiple powers $P_n(x) = P_{n-1}(x) \otimes x = x \otimes ... \otimes x$ form commutative sub-algebras of this algebra. When one restricts the consideration to $x = 1$ one obtains one-element field as sub-field which is however trivial since $\oplus$ and $\otimes$ are identical operations in this subset.

2. One can get over this difficulty by keeping the operations $\oplus$ and $\otimes$, by assuming one-element property only with respect to the real and various p-adic norms, and by replacing ordinary real unit 1 with the algebra of real units formed from infinite primes by requiring that the real and various p-adic norms of the resulting numbers are equal to one. As far as real and various p-adic norms are considered, one has commutative one-element field. When number theoretic anatomy is taken into account, the algebra contains infinite number of elements and is non-commutative with respect to the product since the number theoretic anatomies of x^y and y^x are different.

7.6.4 Leaving the world of finite reals and ending up to the ancient Greece

If strong number theoretic vision is accepted, all physical predictions of quantum TGD would be numbers in finite algebraic extensions of rationals. Just the numbers which ancient Greeks were able to construct by the technical means at use! This seems rather paradoxical but conforms also with the hypothesis that the dicrete algebraic intersections of real and p-adic 2-surfaces provide the fundamental cognitive representations.

The proposed construction for infinite primes gives a precise division of infinite primes to classes: the ratios of primes in given class span a *subset of rational numbers*. These classes give much more refined classification of infinities than infinite ordinals or alephs. They would correspond to separate phases in the evolution of consciousness identified as a sequence of quantum jumps defining sequence of primes $\rightarrow p_1 \rightarrow p_2$...... Infinite primes could mean a transition from space-time level to the level of function spaces. Configuration space is example of a space which can be parameterized by a space of functions locally.

The minimal assumption is that infinite primes reflect their presence only in the possibility to multiply the coordinates of imbedding space points by real units formed as ratios of infinite integers. The correspondence between polynomials and infinite primes gives hopes of mapping at least the reduced configuration space consisting of the the maxima of Kähler function to the anatomy of space-time point. Also configuration space spinors and perhaps also the the modes of configuration space spinor fields would allow this kind of map.

One can consider also the possibility that infinite integers and rationals give rise to a hierarchy of imbedding spaces such that given level represents infinitesimals from the point of view of higher levels in hierarchy. Even 'simultaneous' time evolutions of conscious experiences at different aleph levels with completely different time scales (to put it mildly) are possible since the time values around which the contents of conscious experience are possibly located, are determined by the quantum jump: also multi-snapshots containing snapshots also from different aleph levels are possible. Un-integrated conscious experiences with all values of p could be contained in given quantum jump: this would give rise to a hierarchy of conscious beings: the habitants above given level could be called Gods with full reason: those above us would probably call us just 'epsilons' if ready to admit that we exist at all except in non-rigorous formulations of elementary calculus!

Quantum entanglement between subsystems belonging to different aleph levels of infinity would make possible experiences containing information about this finite world and about the higher level worlds, too. Perhaps our brightest mathematical thoughts (at least) could correspond to cognitive space-time sheets of infinite duration glued to cognitive space-time sheets with even more infinite duration whereas the contents of sensory experiences would be located around finite values of geometric time.

7.6.5 Infinite primes and mystic world view

The proposed interpretation deserves some additional comments from the point of consciousness theory.

1. An open problem is whether the finite integer S appearing in the infinite prime is product of only finite or possibly even infinite number of lower level primes at a given level of hierarchy. The proposed physical identification of S indeed allows S to be a product of infinitely many primes. One can allow also M and N appearing in the infinite and infinite part to be contain infinite number of factors. In this manner one obtains a hierarchy of infinite primes expressible in the form

$$\begin{array}{l} P = nY^{r_1} + mS \ \ , \ \ r = 1, 2, ... \\ m = m_0 + P_{r_2}(Y) \ \ , \\ Y = \frac{X}{S} \ \ , \\ S = \prod_i P_i \ \ . \end{array}$$

   Note that this ansatz is in principle of the same general form as the original ansatz $P = nY + mS$. These primes correspond in physical analogy to states containing infinite number of particles.

   If one poses no restrictions on S this implies that that the cardinality for the set of infinite primes at first level would be $c = 2^{alef_0}$ ($alef_0$ is the cardinality of natural numbers). This is the cardinality for *all* subsets of natural numbers equal to the cardinality of reals. At the next level one obtains the cardinality 2^c for *all* subsets of reals, etc....

   If S were always a product of *finite number of primes* and $k(p)$ would differ from zero for finite number of primes only, the cardinality of infinite primes would be $alef_0$ at each level. One could pose the condition that mS is infinitesimal as compared to nX/S. This would guarantee that the ratio of two infinite primes at the same level would be well defined and equal to n_1S_2/n_2S_1. On the other hand, the requirement that all rationals are obtained as ratios of infinite primes requires that no restrictions are posed on $k(p)$: in this case the cardinality coming from possible choices of $r = ms$ is the cardinality of reals at first level.

   The possibility of primes for which also S is finite would mean that the algebra determined by the infinite primes must be generalized. For the primes representing states containing infinite number of bosons and/or fermions it would be be possible to tell how P_1P_2 and P_2P_1 differ and these primes would behave like elements of free algebra. As already found, this kind of free algebra would provide single space-time point with enormous algebraic representative power and analog of Brahman=Atman identity would result.

2. There is no physical subsystem-complement decomposition for the infinite primes of form $X \pm 1$ since fermionic degrees of freedom are not excited at all. Mystic could interpret it as a state of consciousness in which all separations vanish and there is no observer-observed distinction anymore. A state of pure awareness would be in question if bosonic and fermionic excitations represent the contents of consciousness! Since fermionic many particle states identifiable as Boolean statements about basic statements are identified as representation for reflective level of consciousness, $S = 1$ means that the reflective level of consciousness is absent: enlightment as the end of thoughts according to mystics.

   The mystic experiences of oneness ($S = 1!$), of emptiness (the subset of primes defined by S is empty!) and of the absence of all separations (there is no subsystem-complement separation and hence no division between observer and observed) could be related to quantum jumps to this kind of sectors of the configuration space. In super-symmetric interpretation $S = 1$ means that state contains no fermions.

3. There is entire hierarchy of selves corresponding to the hierarchy of infinite primes and the relationship between selves at different levels of the hierarchy is like the relationship between God and human being. Infinite primes at the lowest level would presumably represent elementary particles. This implies a hierarchy for moments of consciousness and it would be un-natural to exclude the existence of higher level 'beings' (one might call them Angels, Gods, etc...).

7.6.6 Infinite primes and evolution

The original argument leading to the notion of infinite primes was simple. Generalized unitarity implies evolution as a gradual increase of the p-adic prime labeling the configuration space sector D_p to which the localization associated with quantum jump occurs. Infinite p-adic primes are forced by the requirement that p-adic prime increases in a statistical sense and that the number of quantum jumps already occurred is infinite (assuming finite number of these quantum jumps and therefore the first quantum jump, one encounters the problem of deciding what was the first configuration space spinor field).

Quantum classical correspondence requires that p-adic evolution of the space-time surface with respect to geometric time repeats in some sense the p-adic evolution by quantum jumps implied by the generalized unitarity [?]. Infinite p-adic primes are in a well defined sense composites of the primes belonging to lower level of infinity and at the bottom of this de-compositional hierarchy are finite primes. This decomposition corresponds to the decomposition of the space-time surface into p-adic regions which in TGD inspired theory of consciousness correspond to selves. Therefore the increase of the composite primes at lower level of infinity induces the increase of the infinite p-adic prime. p-Adic prime can increase in two manners.

1. One can introduce the concept of the p-adic sub-evolution: the evolution of infinite prime P is induced by the sub-evolution of infinite primes belonging to a lower level of infinity being induced by .... being induced by the evolution at the level of finite primes. For instance, the increase of the cell size means increase of the p-adic prime characterizing it: neurons are indeed very large and complicated cells whereas bacteria are small. Sub-evolution occurs both in subjective and geometric sense.

   i) For a given value of geometric time the p-adic prime of a given space-time sheet gradually increases in the evolution by quantum jumps: our geometric past evolves also!

   ii) The p-adic prime characterizing space-time sheet also increases as the geometric time associated with the space-time sheet increases (say during morphogenesis).

   The notion of sub-evolution is in accordance with the "Ontogeny recapitulates phylogeny" principle (ORP): the evolution of organism, now the entire Universe, contains the evolutions of the more primitive organisms as sub-evolutions.

2. Infinite prime increases also when entirely new finite primes emerge in the decomposition of an infinite prime to finite primes. This means that entirely new space-time sheets representing new structures emerge in quantum jumps. The creation of space-time sheets in quantum jumps could correspond to this process. By quantum classical correspondence this process corresponds at the space-time level to phase transitions giving rise to new material space-time sheets with more and more refined effective p-adic effective topology.

7.7 Local zeta functions, Galois groups, and infinite primes

The recent view about TGD leads to some conjectures about Riemann Zeta.

1. Non-trivial zeros should be algebraic numbers.

2. The building blocks in the product decomposition of ζ should be algebraic numbers for non-trivial zeros of zeta.

3. The values of zeta for their combinations with positive imaginary part with positive integer coefficients should be algebraic numbers.

These conjectures are motivated by the findings that Riemann Zeta seems to be associated with critical systems and by the fact that non-trivial zeros of zeta are analogous to complex conformal weights. The necessity to make such a strong conjectures, in particular conjecture c), is an unsatisfactory feature of the theory and one could ask how to modify this picture. Also a clear physical interpretation of Riemann zeta is lacking.

7.7.1 Local zeta functions and Weil conjectures

Riemann Zeta is not the only zeta [**?**, **?**]. There is entire zoo of zeta functions and the natural question is whether some other zeta sharing the basic properties of Riemann zeta having zeros at critical line could be more appropriate in TGD framework.

The so called local zeta functions analogous to the factors $\zeta_p(s) = 1/(1-p^{-s})$ of Riemann Zeta can be used to code algebraic data about say numbers about solutions of algebraic equations reduced to finite fields. The local zeta functions appearing in Weil's conjectures [**?**] associated with finite fields $G(p,k)$ and thus to single prime. The extensions $G(p,nk)$ of this finite field are considered. These local zeta functions code the number for the points of algebraic variety for given value of n. Weil's conjectures also state that if X is a mod p reduction of non-singular complex projective variety then the degree for the polynomial multiplying the product $\zeta(s) \times \zeta(s-1)$ equals to Betti number. Betti number is 2 times genus in 2-D case.

It has been proven that the zetas of Weil are associated with single prime p, they satisfy functional equation, their zeros are at critical lines, and rather remarkably, they are rational functions of p^{-s}. For instance, for elliptic curves zeros are at critical line [**?**].

The general form for the local zeta is $\zeta(s) = exp(G(s))$, where $G = \sum g_n p^{-ns}$, $g_n = N_n/n$, codes for the numbers N_n of points of algebraic variety for n^{th} extension of finite field F with nk elements assuming that F has $k = p^r$ elements. This transformation resembles the relationship $Z = exp(F)$ between partition function and free energy $Z = exp(F)$ in thermodynamics.

The exponential form is motivated by the possibility to factorize the zeta function into a product of zeta functions. Note also that in the situation when N_n approaches constant N_∞, the division of N_n by n gives essentially $1/(1-N_\infty p^{-s})$ and one obtains the factor of Riemann Zeta at a shifted argument $s - log_p(N_\infty)$. The local zeta associated with Riemann Zeta corresponds to $N_n = 1$.

7.7.2 Local zeta functions and TGD

The local zetas are associated with single prime p, they satisfy functional equation, their zeros lie at the critical lines, and they are rational functions of p^{-s}. These features are highly desirable from the TGD point of view.

Why local zeta functions are natural in TGD framework?

In TGD framework modified Dirac equation assigns to a partonic 2-surface a p-adic prime p and inverse of the zeta defines local conformal weight. The intersection of the real and corresponding p-adic parton 2-surface is the set containing the points that one is interested in. Hence local zeta sharing the basic properties of Riemann zeta is highly desirable and natural. In particular, if the local zeta is a rational function then the inverse images of rational points of the geodesic sphere are algebraic numbers. Of course, one might consider a stronger constraint that the inverse image is rational. Note that one must still require that p^{-s} as well as s are algebraic numbers for the zeros of the local zeta (conditions 1) and 2) listed in the beginning) if one wants the number theoretical universality.

Since the modified Dirac operator assigns to a given partonic 2-surface a p-adic prime p, one can ask whether the inverse $\zeta_p^{-1}(z)$ of some kind of local zeta directly coding data about partonic 2-surface could define the generalized eigenvalues of the modified Dirac operator and radial super-canonical conformal weights so that the conjectures about Riemann Zeta would not be needed at all.

The eigenvalues of the modified Dirac operator would in a holographic manner code for information about partonic 2-surface. This kind of algebraic geometric data are absolutely relevant for TGD since U-matrix and probably also S-matrix must be formulated in terms of the data related to the intersection of real and partonic 2-surfaces (number theoretic braids) obeying same algebraic equations and consisting of algebraic points in the appropriate algebraic extension of p-adic numbers. Note that the hierarchy of algebraic extensions of p-adic number fields would give rise to a hierarchy of zetas so that the algebraic extension used would directly reflect itself in the eigenvalue spectrum of the modified Dirac operator and super-canonical conformal weights. This is highly desirable but not achieved if one uses Riemann Zeta.

One must of course leave open the possibility that for real-real transitions the inverse of the zeta defined as a product of the local zetas (very much analogous to Riemann Zeta) defines the conformal weights. This kind of picture would conform with the idea about real physics as a kind of adele formed from p-adic physics.

Finite field hierarchy is not natural in TGD context

That local zeta functions are assigned with a hierarchy of finite field extensions do not look natural in TGD context. The reason is that these extensions are regarded as abstract extensions of $G(p,k)$ as opposed to a large number of algebraic extensions isomorphic with finite fields as abstract number fields and induced from the extensions of p-adic number fields. Sub-field property is clearly highly relevant in TGD framework just as the sub-manifold property is crucial for geometrizing also other interactions than gravitation in TGD framework.

The $O(p^n)$ hierarchy for the p-adic cutoffs would naturally replace the hierarchy of finite fields. This hierarchy is quite different from the hierarchy of finite fields since one expects that the number of solutions becomes constant at the limit of large n and also at the limit of large p so that powers in the function G coding for the numbers of solutions of algebraic equations as function of n should not increase but approach constant N_∞. The possibility to factorize $exp(G)$ to a product $exp(G_0)exp(G_\infty)$ would mean a reduction to a product of a rational function and factor(s) $\zeta_p(s) = 1/(1-p^{-s_1})$ associated with Riemann Zeta with argument s shifted to $s_1 = s - log_p(N_\infty)$.

What data local zetas could code?

The next question is what data the local zeta functions could code.

1. It is not at clear whether it is useful to code global data such as the numbers of points of partonic 2-surface modulo p^n. The notion of number theoretic braid occurring in the proposed approach to S-matrix suggests that the zeta at an algebraic point z of the geodesic sphere S^2 of CP_2 or of light-cone boundary should code purely local data such as the numbers N_n of points which project to z as function of p-adic cutoff p^n. In the generic case this number would be finite for non-vacuum extremals with 2-D S^2 projection. The n^{th} coefficient $g_n = N_n/n$ of the function G_p would code the number N_n of these points in the approximation $O(p^{n+1}) = 0$ for the algebraic equations defining the p-adic counterpart of the partonic 2-surface.

2. In a region of partonic 2-surface where the numbers N_n of these points remain constant, $\zeta(s)$ would have constant functional form and therefore the information in this discrete set of algebraic points would allow to deduce deduce information about the numbers N_n. Both the algebraic points and generalized eigenvalues would carry the algebraic information.

3. A rather fascinating self referentiality would result: the generalized eigen values of the modified Dirac operator expressible in terms of inverse of zeta would code data for a sequence of approximations for the p-adic variant of the partonic 2-surface. This would be natural since second quantized induced spinor fields are correlates for logical thought in TGD inspired theory of consciousness. Even more, the data would be given at points $\zeta(s)$, s a rational value of a super-canonical conformal weight or a value of generalized eigenvalue of modified Dirac operator (which is essentially function $s = \zeta_p^{-1}(z)$ at geodesic sphere of CP_2 or of light-cone boundary).

7.7.3 Galois groups, Jones inclusions, and infinite primes

Langlands program [?, ?] is an attempt to unify mathematics using the idea that all zeta functions and corresponding theta functions could emerge as automorphic functions giving rise to finite-dimensional representations for Galois groups (Galois group is defined as a group of automorphisms of the extension of field F leaving invariant the elements of F). The basic example corresponds to rationals and their extensions. Finite fields $G(p,k)$ and their extensions $G(p,nk)$ represents another example. The largest extension of rationals corresponds to algebraic numbers (algebraically closed set). Although this non-Abelian group is huge and does not exist in the usual sense of the word its finite-dimensional representations in groups $GL(n, Z)$ make sense.

For instance, Edward Witten is working with the idea that geometric variant of Langlands duality could correspond to the dualities discovered in string model framework and be understood in terms of topological version of four-dimensional $N = 4$ super-symmetric YM theory [?]. In particular, Witten assigns surface operators to the 2-D surfaces of 4-D space-time. This brings unavoidably in mind partonic 2-surfaces and TGD as $N = 4$ super-conformal almost topological QFT.

This observation stimulates some ideas about the role of zeta functions in TGD if one takes the vision about physics as a generalized number theory seriously.

Galois groups, Jones inclusions, and quantum measurement theory

The Galois representations appearing in Langlands program could have a concrete physical/cognitive meaning.

1. The Galois groups associated with the extensions of rationals have a natural action on partonic 2-surfaces represented by algebraic equations. Their action would reduce to permutations of roots of the polynomial equations defining the points with a fixed projection to the above mentioned geodesic sphere S^2 of CP_2 or δM^4_+. This makes possible to define modes of induced spinor fields transforming under representations of Galois groups. Galois groups would also have a natural action on configuration space-spinor fields. One can also speak about configuration space spinors invariant under Galois group.

2. Galois groups could be assigned to Jones inclusions having an interpretation in terms of a finite measurement resolution in the sense that the discrete group defining the inclusion leaves invariant the operators generating excitations which are not detectable.

3. The physical interpretation of the finite resolution represented by Galois group would be based on the analogy with particle physics. The field extension K/F implies that the primes (more precisely, prime ideals) of F decompose into products of primes (prime ideals) of K. Physically this corresponds to the decomposition of particle into more elementary constituents, say hadrons into quarks in the improved resolution implied by the extension $F \to K$. The interpretation in terms of cognitive resolution would be that the primes associated with the higher extensions of rationals are not cognizable: in other words, the observed states are singlets under corresponding Galois groups: one has algebraic/cognitive counterpart of color confinement.

4. For instance, the system labeled by an ordinary p-adic prime could decompose to a system which is a composite of Gaussian primes. Interestingly, the biologically highly interesting p-adic length scale range 10 nm-5 μm contains as many as four Gaussian Mersennes ($M_k = (1+i)^k - 1$, $k = 151, 157, 163, 167$), which suggests that the emergence of living matter means an improved cognitive resolution.

Galois groups and infinite primes

In particular, the notion of infinite prime suggests a manner to realize the modular functions as representations of Galois groups. Infinite primes might also provide a new perspective to the concrete realization of Langlands program.

1. The discrete Galois groups associated with various extensions of rationals and involved with modular functions which are in one-one correspondence with zeta functions via Mellin transform defined as $\sum x_n n^{-s} \to \sum x_n z^n$ [?]. Various Galois groups would have a natural action in the space of infinite primes having interpretation as Fock states and more general bound states of an arithmetic quantum field theory.

2. The number theoretic anatomy of space-time points due to the possibility to define infinite number of number theoretically non-equivalent real units using infinite rationals [?] allows the imbedding space points themselves to code holographically various things. Galois groups would have a natural action in the space of real units and thus on the number theoretical anatomy of a point of imbedding space.

3. Since the repeated second quantization of the super-symmetric arithmetic quantum field theory defined by infinite primes gives rise to a huge space of quantum states, the conjecture that the number theoretic anatomy of imbedding space point allows to represent configuration space (the world of classical worlds associated with the light-cone of a given point of H) and configuration space spinor fields emerges naturally [**?**].

4. Since Galois groups G are associated with inclusions of number fields to their extensions, this inclusion could correspond at quantum level to a generalized Jones inclusion $\mathcal{N} \subset \mathcal{M}$ such that G acts as automorphisms of $\mathcal{M}$ and leaves invariant the elements of $\mathcal{N}$. This might be possible if one allows the replacement of complex numbers as coefficient fields of hyper-finite factors of type II_1 with various algebraic extensions of rationals. Quantum measurement theory with a finite measurement resolution defined by Jones inclusion $\mathcal{N} \subset \mathcal{M}$ [**?**] could thus have also a purely number theoretic meaning provided it is possible to define a non-trivial action of various Galois groups on configuration space spinor fields via the imbedding of the configuration space spinors to the space of infinite integers and rationals (analogous to the imbedding of space-time surface to imbedding space).

This picture allows to develop rather fascinating ideas about mathematical structures and their relationship to physical world. For instance, the functional form of a map between two sets the points of the domain and target rather than only its value could be coded in a holographic manner by using the number theoretic anatomy of the points. Modular functions giving rise to generalized zeta functions would emerge in especially natural manner in this framework. Configuration space spinor fields would allow a physical realization of the holographic representations of various maps as quantum states.

7.8 Remarks about correspondence between infinite primes, space-time surfaces, and configuration space spinor fields

The correspondence of CH points with infinite primes and thus with real units can be understood if one assume that the points of CH correspond to infinite rationals via their mapping to hyper-octonion real-analytic rational functions conjectured to define foliations of M^8 to hyper-quaternionic 4-surfaces inducing corresponding foliations of H. The correspondence of CH spinors with the real units identified as infinite rationals with varying number theoretical anatomies is not so obvious. It is good to approach the problem by making questions.

1. How the points of CH and CH spinors at given point of CH correspond to various real units? Configuration space Hamiltonians and their super-counterparts characterize modes of configuration space spinor fields rather than only spinors. Does this mean that only ground states of super-conformal representations, which are expected to correspond elementary particles, correspond to configuration space spinors and are coded by infinite primes?

2. How do CH spinor fields (as opposed to CH spinors) correspond to infinite rationals? Configuration space spinor fields are generated by elements of super-conformal algebra from ground states. Should one code the matrix elements of the operators between ground states and creating zero energy states in terms of time-like entanglement between ground states represented by real units and assigned to the preferred points of H characterizing the tips of future and past light-cones and having also interpretation as arguments of n-point functions?

The argument to be represented is in a nutshell following.

1. CH itself and CH spinors are by super-symmetry characterized by ground states of super-conformal representations and can be mapped to infinite rationals defining real units U_k multiplying the eight preferred H coordinates h^k whereas configuration space spinor fields correspond to discrete analogs of Schrödinger amplitudes in the space whose points have U_k as coordinates. The 8-units correspond to ground states for an 8-fold tensor power of a fundamental super-conformal representation or to a product of representations of this kind.

2. General states are coded by quantum entangled states defined as entangled states of positive and negative energy ground states with entanglement coefficients defined by the product of operators creating positive and negative energy states represented by the units. Normal ordering prescription makes the mapping unique.

3. The condition that various symmetries have number theoretical correlates leads to rather detailed view about the map of ground states to real units.

4. It seems that quantal generalization of the fundamental associativity and commutativity conditions might be needed.

Before continuing it is perhaps good to represent the most obvious objection against the idea. The correspondence between CH and CH spinors with infinite rationals and their discreteness means that also CH (world of classical worlds) and space of CH spinors should be discrete. First this looks non-sensible but is indeed what one obtains if space-time surfaces correspond to light-like 3-surfaces expressible in terms of algebraic equations involving rational functions with rational coefficients.

7.8.1 How CH and CH spinor fields correspond to infinite rationals?

The basic question is how CH and CH spinor fields on quantum fluctuating degrees of freedom (degrees of freedom for which configuration space metric is non-vanishing) correspond to infinite rationals.

Associativity and commutativity or only their quantum variants?

Associativity and commutativity conditions are absolutely essential notions in quantum TGD and also in the mapping of infinite primes to the space-time sheets. Associativity, guaranteed by hyper-octonion real-analyticity and implying rational infinite primes, seems to be necessary in order to obtain well-defined representations but might be too strong a condition.

Associativity implies hyper-quaternionicity and commutativity requirement in turn leads to rational infinite primes. Since one can decompose rational primes to hyper-quaternionic and even hyper-octonionic primes, one might hope that this could allow to represent states which consist of colored constituents. This representations has however the flavor of a formal trick and the considerations related to concrete representations of infinite primes suggest that the rationality of infinite primes might be a too restrictive condition.

A more radical possibility is that physical states are only quantum associative. This means that they are obtained as quantum superpositions in the space of real units over all possible associations performed for a given product of hyper-octonion primes (for instance, $|A(BC)\rangle + |(AB)C\rangle$). These states would be associative in quantum sense but would not reduce to hyper-quaternionic primes. Also the notion of quantum commutativity makes sense. The fact that mesons are quantum superpositions of quark-antiquark pairs which each corresponds to different pair of hyper-quaternionic primes and are thus not representable classically, suggests that one can require only quantum associativity and quantum commutativity.

How this idea relates to the representation of space-time surfaces in terms of rational functions of hyper-octonionic variable obtained as an image of rational infinite prime? If one replaces the coefficients of the polynomial which complex or more complex rational, hyper-octonion real analyticity is lost and one must consider some manner to map associative quantum state defined as superposition of various associations to single hyper-quaternionic prime.

1. The first approach is based on the assumption that only infinite integers reduce to infinite rational integers in the sense that the corresponding rational function has rational coefficients. This would allow partons as colored partons represented as non-associative constituents of infinite integers and there would be no problems with space-time correlates. It is however not clear whether this kind infinite integers are possible.

2. In the case of non-commutative group one can speak about commutator group and define Abelian group as coset group of these. Could it be that one can speak about associator algebra and define associative algebra by identifying additive associators $A(BC) - (AB)C$ with zero or multiplicative associators $(A(BC))((AB)C)^{-1}$ with unit. Hyper-octonionic primes would be mapped to

something represented by matrices. A good guess for the representation is in terms of 8-D analog of Pauli spin matrices.

Basic assumptions

The following assumptions serve as constraints when one tries to guess the map of quantum states to infinite primes.

1. Free many-particle states correspond to infinite integers and bound states to infinite primes mappable to irreducible polynomials. The numerator/denominator of the infinite rational should correspond to positive/negative energy states of which zero energy states consist of. At higher levels the mapping should be induced from that for the lowest level. Bosonic (fermionic) elementary particles in ground states should correspond to bosonic (fermionic primes). Phase conjugation as a generalization of that for laser beams) would correspond to the replacement of infinite integer with its inverse.

2. Concerning charge conjugation one can imagine several options but the detailed study of the realization of color symmetry leaves only one option. For this option the two singlets $1 \pm ie_7$ and triplet and antitriplet correspond to leptons and quarks with spin and electro-weak spin represented by the moduli space associated with the hyper-octonionic structures. One must leave open the interpretation of the change of the sign of the small part of the infinite prime, which looks excellent candidate for some discrete symmetry (parity perhaps?).

3. Discrete super-canonical and Super Kac-Moody algebras with bosonic and fermionic generators label the states. One should map the ground states of these representations to infinite primes and thus to real units in a natural manner. The requirement that standard model symmetries reduce to number theory serves as a powerful constraint and will be analyzed in detail later.

4. The excited states of various super-conformal representations can be mapped to quantum superpositions of many particle states formed from infinite primes. The operators creating the positive and negative energy parts are unique combinations of the operators of algebra if normal ordering prescription is applied. The matrix elements of these operators between ground states can be calculated. The entangled state formed from ground states with entanglement coefficients represented by these matrix elements gives the representation of the general state. Note that the real units would be associated with different points of H identifiable as arguments of n-point function in S-matrix elements.

How to map ground states of super-conformal representations to infinite primes?

Under the assumptions just stated the problem reduces to that of guessing the detailed form of the map of the ground states of super-conformal representations to primes at the first level of the hierarchy. The mapping of infinite primes to rational functions could provide a clue about how to achieve a natural one-to-one correspondence.

1. The decomposition of the irreducible polynomials in the algebraic extension of rationals gives interpretation in terms of many-particle states labeled by primes in the extension. This brings in Galois groups and their representations. This seems to be something new to present day physics. Note that color group plays the role of Galois group for octonions regarded as extension of reals.

2. Partonic two-surfaces should correspond to infinite primes but in such a manner that an infinite number of infinite primes are mapped to the same partonic 2-surface since given 3-surface should be able to to carry an arbitrary state of super-canonical and super Kac-Moody representation. This is the case since each light-like 3-surface traversing a given partonic 2-surface corresponds to an infinite prime in turn assumed to code for a foliation of hyper-quaternionic or co-hyper-quaternionic surfaces via corresponding rational function of hyper-octonionic variable. Light-like 3-surfaces and corresponding 4-D space-time sheets would thus code for the ground states of super-conformal representations. Quantum classical correspondence would apply to ground states but not to the excited states of super-conformal representations.

3. One should also understand how light-like partonic 3-surfaces are mapped to the number theoretic anatomies of a point of imbedding space. The natural choice for this point would be the preferred point of H defining the tip of the light-cone and the origin of complex coordinates of CP_2 transforming linearly under $U(2) \subset SU(3)$. This choice should be coded as a zero/pole of infinite rational with unit real norm coding for the zero energy states. Zeros would correspond to the positive energy state and poles to the negative energy state.

The treatment of zero modes

There are also zero modes which are absolutely crucial for quantum measurement theory. They entangle with quantum fluctuating degrees of freedom in quantum measurement situation and thus map quantum numbers to positions of pointers. The interior degrees of freedom of space-time interior must correspond to zero modes and they represent space-time correlates for quantum states realized at light-like partonic 3-surfaces.

As long as states associated with zero modes are represented by operators (such as CH Hamiltonians), the same description applies to them as to the representation of excited states of super-conformal representations. The absence of metric in zero modes means that there is no integration measure. The problems are avoided if one assumes that wave functions in zero modes have a discrete locus as suggested already earlier.

According to the argument represented in [?], the quantum fluctuating configuration space degrees of freedom are by definition super-symmetrizable since configuration space gamma matrices correspond to the super counterparts of Hamiltonians in the case of super-canonical algebra. Supersymmetrizability condition means that the Poisson brackets of bosonic Hamiltonians reduce to 1-dimensional integrals over "stringy" curves of partonic 2-surface [?]. This happens for the sub-algebra of super-canonical algebra having vanishing S^2 spin and color charges.

This would mean that zero modes include also the charged Hamiltonians of the super-canonical algebra. This brings in mind induced representations for which one has coset space structure with entire super-canonical group divided by the group generated by neutral super-canonical algebra. The necessary discretization zero modes of freedom suggests a reduction of the representations of isometry groups of H and CH to those for discrete subgroups of isometry groups which indeed appear naturally in Jones inclusions.

One must take this suggestion with some grain of salt. The coset construction for Kac-Moody representations allows to consider the possibility of extending the representations to charged Hamiltonians in such a manner that "stringy" commutators are preserved. The generation of Virasoro and Kac-Moody central extension parameters might be seen as the price paid for the stringy commutation relations.

Configuration space spinor fields as discrete Schrödinger amplitudes in the space of number theoretic anatomies?

It would seem that the analog of a complex Schrödinger amplitude in the space of number-theoretic anatomies of a given imbedding space point represented by single point of H and represented as 8-tuples of real units could naturally represent the dependence of CH spinors understood as ground states of super-conformal representations obtained as an 8-fold tensor power of a fundamental representation or product of representations perhaps differing somehow. The open question is why eight of them are needed. The excited states of super-conformal representations would be represented as time entangled states with entanglement between real units associated with the preferred points characterizing the tips future and past directed light-cones.

This picture conforms with the simple idea that infinite primes label the points in the fibers of the spinor field bundle having CH_h, h a preferred point of H characterizing the preferred origin of hyper-octonion structure, as a base space and that physical states correspond to discrete analogs of Schrödinger amplitude in this kind of bundles and product bundles formed from them. These 8-tuples define a number theoretical analog of $U(1)^8$ group in terms of which all number theoretical symmetries are represented.

7.8.2 Can one understand fundamental symmetries number theoretically?

One should understand symmetries number theoretically.

1. The basic idea is that color $SU(3) \subset G_2$ acts as automorphisms of hyper-octonion structure with a preferred imaginary unit and preferred point with respect to which hyper-octonionic power series are developed. $SO(7,1)$ would act as symmetries in the moduli space of hyper-octonion structures. Associativity implies symmetry breaking so that only hyper-quaternionic structures are considered and $SO(3,1) \times SO(4)$ acts as symmetries of the moduli space for these structures.

2. Color group is the analog of Galois group for the extension of reals to octonions and has a natural action on the decompositions of rational infinite primes to hyper-octonionic infinite primes. Color confinement is implied by hyper-quaternionicity of primes implied by associativity necessary to assign space-time surfaces to the infinite rationals. If one assumes only quantum associativity, one should have a generalization of the condition guaranteing color confinement. A possible more general condition is that infinite integers give rise to rational polynomials whereas infinite primes can be non-associative and non-commutative if they appear as constituents of N-particle state. This would predict that free quarks are not possible.

3. Electro-weak symmetries and Lorentz group act in the moduli space of hyper-octonionic structures and their actions deform space-time in H picture. CP_2 parameterizes the moduli space of hyper-quaternionic structures induced from a given hyper-octonionic structure with preferred imaginary unit.

4. Four-momenta correspond to translational degrees of freedom associated with the preferred points of M^4 coded by the infinite rational (tip of the light-cone). Color quantum numbers in cm degrees of freedom can be assigned to the CP_2 projection of the preferred point of H. As a matter fact, the definition of hyper-octonionic structure involves the choice of origin of M^8 giving rise to the preferred point of H.

Automorphisms and the symmetries of moduli space of hyper structures as basic symmetries

Consider now in more detail various symmetries.

1. G_2 acts as automorphisms on octonionic imaginary units and $SU(3)$ respects the choice of preferred imaginary unit. Associativity requires a reduction to hyper-quaternionic primes and implies color confinement. For hyper-quaternionic primes the automorphisms restrict to $SO(3)$ which has right/left action of fermionic hyper-quaternionic primes and adjoint action on bosonic hyper-quaternionc primes. The choice of hyper-quaternionic structure is global as opposed to the local choice of hyper-quaternionic tangent space of space-time surface assigning to a point of $HQ \subset HO$ a point of CP_2. $U(2) \subset SU(3)$ leaves invariant given hyper-quaternionic structure which are thus parameterized by CP_2. Color partial waves can be interpreted as partial waves in this moduli space.

2. The choice of global hyper-octonionic coordinate is dictated only modulo a transformation of $SO(1,7)$ acting as isometries of hyper-octonionic norm and as transformations in moduli space of hyper-octonion structures $SO(7)$ acting leaves invariant the choice of real unit. $SO(1,3) \times SO(4)$ acts in the moduli space of global hyper-quaternionic structures identified as sub-structures of hyper-octonionic structure. The choice of global HO structures involves also choice of origin implying preferred point of H. The M^4 projection of this point corresponds to the tip of light-cone. Since the integers representing physical states must be hyper-quaternionic by associativity conditions, the symmetry breaking ("number theoretic compactification") to $SO(1,3) \times SO(4)$ occurs very naturally. This group acts as spinor rotations in H picture and as isometries in M^8 picture.

3. $SO(1,7)$ allows 3 different 8-dimensional representations (8_v, 8_s, and $\overline{8}_s$). All these representations must decompose under $SU(3)$ as $1+1+3+\overline{3}$ as little exercise with $SO(8)$ triality demonstrates. Under $SO(6) \cong SU(4)$ the decompositions are $1+1+6$ and $4+\overline{4}$ for 8_v and 8_s and its conjugate. Both hyper-octonion spinors and gamma matrices are identified as hyper-octonion units rather than as matrices. It would be natural to assign to bosonic M^8 primes 8_v and to fermionic M^8 primes 8_s and $\overline{8}_s$. One can distinguish between $8_v, 8_s$ and $\overline{8}_s$ for hyper-octonionic units only if one considers the full $SO(1,3) \times SO(4)$ action in the moduli space of hyper-octonionic structures.

Physical interpretation of the decomposition of rational primes to various hyper-primes

Consider now the physical interpretation for the decomposition of rational primes to hyper-complex, hyper-quaternionic, and hyper-octonionic primes. Here one must keep doors open by allowing also the notion of quantum commutativity and quantum associativity so that infinite hyper-octonionic primes would not in general have these properties whereas their images to gamma matrices would define primes of an associative algebra so that a unique space-time representation in terms of hyper-octonionic polynomial would result. Abelianization would produce a generalization of hyper-complex algebra with 7 commuting imaginary units satisfying $e_i^2 = 1$. I have considered earlier also the possibility that hyper-analytic functions of this kind of variable could define space-time surfaces. At this stage one cannot distinguish between this and hyper-octonion real-analytic option.

1. The net quantum numbers of physical states must vanish in zero energy ontology. This is implied by the reduction of infinite rationals to infinite rationals associated with rationals but one must consider also more general options. The vanishing of net quantum numbers could be achieved in many manners. In the most general case the quantum numbers of positive and negative energy state represented by integers in the numerator and denominator of the infinite rational would compensate. If one requires only associativity for infinite primes (or integers) then positive (negative) energy state can correspond to hyper-quaternionic integer and one ends up with H picture and breaking of M^8 symmetries to those of H.

2. Commutativity condition implies a restriction to hyper-complex numbers. The only restriction would be due to fermion number conservation. Bosonic rational primes could be decomposed to fermionic and antifermionic hyper-quaternionic/octonionic primes such that the net fermion number vanishes. Fermionic primes could correspond to neutrinos and antineutrinos.

3. Giving up commutativity condition but requiring that the primes are associative gives hyper-quaternionic primes and color confinement. One obtains two states which possess non-vanishing and opposite color hypercharges equal to $\pm 2/3$. Thus only the interpretation as lepton, antilepton, quark and antiquark with no color isospin is possible. Spin, weak spin, and color would not be manifest since it would correspond to degree of freedom in the moduli space of hyper-quaternionic structures.

4. Hyper-quaternionic primes can be decomposed to hyper-octonionic primes. In the fermionic sector the three quark states consisting of hyper-octonion units would give color singlets as linear combination of hyper-octonion real unit and the preferred imaginary unit. A state analogous to baryon would result. Is this representation just a formal trick or does it have a real physical content must be left open. In TGD framework, color quantum numbers correspond to color partial waves in CP_2 labeling the moduli space of hyper-quaternionic structures associated with a given hyper-octonionic structure. One might hope that the decomposition provides a formal representation of information about these partial waves.

5. Giving up also associativity for single hyper-octonionic prime and requiring only quantum associativity and requiring that only infinite integers reduces to rational infinite integers leads to the most general framework allowing to describe entangled many particle states formed from elementary particles with quantum numbers of quark and lepton and basic gauge bosons. Gauge bosons and would correspond to locally entangled fermion antifermion pairs (as predicted by TGD) represented as locally entangled real units.

Electro-weak and color symmetries

The crucial test for this picture is whether color and electro-weak symmetries can be understood number theoretically.

Electro-weak group acts as transformations in the hyper-quaternionic moduli space inducing left or right actions of fermions which cannot interpreted as $U(2) \subset SU(3)$ automorphisms realized via adjoint action. For bosons one adjoint action results. Therefore color singlet states can possess non-vanishing electro-weak quantum numbers as also spin. For bosonic hyper-quaternionic primes one obtains singlet and triplet and for fermionic primes two doublets. The interpretation in terms of electro-weak gauge bosons and electro-weak doublets seems natural. Spin degrees of freedom are not manifestly visible but correspond to the moduli space resulting by $SL(2,C)$ action on hyper-quaternionic units.

Some more detailed comments about color symmetries are in order.

1. Color group SU(3) corresponds to subgroup of G_2 which acts as a Galois group for the extension of reals to octonions. $SU(3)$ leaves invariant real unit and a preferred octonionic imaginary unit. As noticed 8_v, 8_s and $\overline{8}_s$ decompose in a similar manner under $SU(3)$ and only the action of $SL(2,C) \times SO(4)$ modifying hyper-octonionic structure can distinguish between them.

2. Color group would act as a symmetry group on the composites of hyper-octonionic primes and color confinement in spinorial degrees of freedom would follow automatically from (complex) rationality (and even hyper-quaternionicity) of infinite integers necessitated by associativity. This does not however imply color singlet property in color rotational degrees of freedom in imbedding space. The value of color hypercharge (em charge) assignable to the spinors is the only signature of whether lepton or quark is in question.

7.9 A little crazy speculation about knots and infinite primes

D-dimensional knots correspond to the isotopy equivalence classes of the imbeddings of spheres S^d to S^{d+2}. One can consider also the isotopy equivalence classes of more general manifolds $M^d \subset M^{d+2}$. Knots [?] are very algebraic objects. The product (or sum, I prefer to talk about product) of knots is defined in terms of connected sum. Connected sum quite generally defines a commutative and associative product, and one can decompose any knot into prime knots.

Knots can be mapped to Jones polynomials $J(K)$ (for instance - there are many other polynomials and there are very general mathematical results about them [?]) and the product of knots is mapped to a product of corresponding polynomials. The polynomials assignable to prime knots should be prime in a well-defined sense, and one can indeed define the notion of primeness for polynomials $J(K)$: prime polynomial does not factor to a product of polynomials of lower degree in the extension of rationals considered.

This raises the idea that one could define the notion of zeta function for knots. It would be simply the product of factors $1/(1-J(K)^{-s})$ where K runs over prime knots. The new (to me) but very natural element in the definition would be that ordinary prime is replaced with a polynomial prime. This observation led to the idea that the hierarchy of infinite primes could correspond to the hierarchy of knots in various dimensions and this in turn stimulated quite fascinating speculations.

7.9.1 Do knots correspond to the hierarchy of infinite primes?

A very natural question is whether one could define the counterpart of zeta function for infinite primes. The idea of replacing primes with prime polynomials would resolve the problem since infinite primes can be mapped to polynomials. For some reason this idea however had not occurred to me earlier.

The correspondence of both knots and infinite primes with polynomials inspires the question whether $d=1$-dimensional prime knots might be in correspondence (not necessarily 1-1) with infinite primes. Rational or Gaussian rational infinite primes would be naturally selected: these are also selected by physical considerations as representatives of physical states although quaternionic and octonionic variants of infinite primes can be considered.

If so, knots could correspond to the subset of states of a super-symmetric arithmetic quantum field theory with bosonic single particle states and fermionic states labeled by quaternionic primes.

1. The free Fock states of this QFT are mapped to first order polynomials and irreducible polynomials of higher degree have interpretation as bound states so that the non-decomposability to a product in a given extension of rationals would correspond physically to the non-decomposability into many-particle state. What is fascinating that apparently free arithmetic QFT allows huge number of bound states.

2. Infinite primes form an infinite hierarchy, which corresponds to an infinite hierarchy of second quantizations for infinite primes meaning that n-particle states of the previous level define single particle states of the next level. At space-time level this hierarchy corresponds to a hierarchy

defined by space-time sheets of the topological condensate: space-time sheet containing a galaxy can behave like an elementary particle at the next level of hierarchy.

3. Could this hierarchy have some counterpart for knots?In one realization as polynomials, the polynomials corresponding to infinite prime hierarchy have increasing number of variables. Hence the first thing that comes into my uneducated mind is as the hierarchy defined by the increasing dimension d of knot. All knots of dimension d would in some sense serve as building bricks for prime knots of dimension $d+1$ or possibly $d+2$ (the latter option turns out to be the more plausible one). A canonical construction recipe for knots of higher dimensions should exist.

4. One could also wonder whether the replacement of spherical topologies for d-dimensional knot and $d+2$-dimensional imbedding space with more general topologies could correspond to algebraic extensions at various levels of the hierarchy bringing into the game more general infinite primes. The units of these extensions would correspond to knots which involve in an essential manner the global topology (say knotted non-contractible circles in 3-torus). Since the knots defining the product would in general have topology different from spherical topology the product of knots should be replaced with its category theoretical generalization making higher-dimensional knots a groupoid in which spherical knots would act diagonally leaving the topology of knot invariant. The assignment of d-knots with the notion of n-category, n-groupoid, etc.. by putting d=n is a highly suggestive idea. This is indeed natural since are an outcome of a repeated abstraction process: statements about statements about .....

5. The lowest ($d=1, D=3$) level would be the fundamental one and the rest would be somewhat boring repeated second quantization;-). This is why the dimension $D=3$ (number theoretic braids at light-like 3-surfaces!) would be fundamental for physics.

7.9.2 Further speculations

Some further speculations about the proposed structure of all structures are natural.

1. The possibility that algebraic extensions of infinite primes could allow to describe the refinements related to the varying topologies of knot and imbedding space would mean a deep connection between number theory, manifold topology, sub-manifold topology, and n-category theory.

2. Category theory appears already now in fundamental role in the construction of the generalization of M-matrix unifying the notions of density matrix and S-matrix. Generalization of category to n-category theory and various n-structures would have very direct correspondence with the physics of TGD Universe if one assumes that repeated second quantization makes sense and corresponds to the hierarchical structure of many-sheeted space-time where even galaxy corresponds to elementary fermion or boson at some level of hierarchy.

   This however requires that the unions of light-like 3-surfaces and of their sub-manifolds at different levels of topological condensate are able to represent higher-dimensional manifolds physically albeit not in the standard geometric sense since imbedding space dimension is just 8. This might be possible.

3. As far as physics is considered, the disjoint union of sub-manifolds of dimensions d_1 and d_2 behaves like a d_1+d_2-dimensional Cartesian product of the corresponding manifolds. This is of course used in standard manner in wave mechanics (the configuration space of n-particle system is identified as E^{3n}/S_n with division coming from statistics).

4. If the surfaces have intersection points, one has a union of Cartesian product with punctures (intersection points) and of lower-dimensional manifold corresponding to the intersection points.

5. Note also that by posing symmetries on classical fields one can effectively obtain from a given n-manifold manifolds (and orbifolds) with quotient topologies.

The megalomanic conjecture is that this kind of physical representation of d-knots and their imbedding spaces is possible using many-sheeted space-time. Perhaps even the entire magnificent mathematics of n-manifolds and their sub-manifolds might have a physical representation in terms of sub-manifolds of 8-D $M^4 \times CP_2$ with dimension not higher than space-time dimension $d=4$.

7.9.3 The idea survives the most obvious killer test

All this looks nice and the question is how to give a death blow to all this reckless speculation. Torus knots are an excellent candidate for performing this unpleasant task but the hypothesis survives!

1. Torus knots [?] are labeled by a pair integers (m, n), which are relatively prime. These are prime knots. Torus knots for which one has $m/n = r/s$ are isotopic so that any torus knot is isotopic with a knot for which m and n have no common prime power factors.

2. The simplest infinite primes correspond to free Fock states of the supersymmetric arithmetic QFT and are labeled by pairs (m, n) of integers such that m and n do not have any common prime factors. Thus torus knots would correspond to free Fock states! Note that the prime power $p^{k(p)}$ appearing in m corresponds to $k(p)$-boson state with boson "momentum" p and the corresponding power in n corresponds to fermion state plus $k(p) - 1$ bosons.

3. A further property of torus knots is that (m, n) and (n, m) are isotopic: this would correspond at the level of infinite primes to the symmetry $mX + n \rightarrow nX + m$, X product of all finite primes. Thus infinite primes are in $2 \rightarrow 1$ correspondence with torus knots and the hypothesis survives also this murder attempt. Probably the assignment of orientation to the knot makes the correspondence 1-1 correspondence.

7.9.4 How to realize the representation of the braid hierarchy in many-sheeted space-time?

One can consider a concrete construction of higher-dimensional knots and braids in terms of the many-sheeted space-time concept.

1. The basic observation is that ordinary knots can be constructed as closed braids so that everything reduces to the construction of braids. In particular, any torus knot labeled by (m,n) can be made from a braid with m strands: the braid word in question is $(\sigma_1....\sigma_{m-1})^n$ or by $(m, n) = (n, m)$ equivalence from n strands. The construction of infinite primes suggests that also the notion of d-braid makes sense as a collection of d-braids in $d+2$-space, which move and and define $d+1$-braid in $d+3$ space (the additional dimension being defined by time coordinate).

2. The notion of topological condensate should allow a concrete construction of the pairs of d- and $d + 2$-dimensional manifolds. The 2-D character of the fundamental objects (partons) might indeed make this possible. Also the notion of length scale cutoff fundamental for the notion of topological condensate is a crucial element of the proposed construction.

3. Infinite primes have also interpretation as physical states and the representation in terms of knots would mean a realization of quantum classical correspondence.

The concrete construction would proceed as follows.

1. Consider first the lowest non-trivial level in the hierarchy. One has a collection of 3-D light-like 3-surfaces X^3_i representing ordinary braids. The challenge is to assign to them a 5-D imbedding space in a natural manner. Where do the additional two dimensions come from? The obvious answer is that the new dimensions correspond to the partonic 2-surface X^2 assignable to the $3-D$ lightlike surface X^3 at which these surfaces have suffered topological condensation. The geometric picture is that X^3_i grow like plants from ground defined by X^2 at 7-dimensional $\delta M^4_+ \times CP_2$.

2. The degrees of freedom of X^2 should be combined with the degrees of freedom of X^3_i to form a 5-dimensional space X^5. The natural idea is that one first forms the Cartesian products $X^5_i = X^3_i \times X^2$ and then the desired 5-manifold X^5 as their union by posing suitable additional conditions. Braiding means a translational motion of X^3_i inside X^2 defining braid as the orbit in X^5. It can happen that X^3_i and X^3_j intersect in this process. At these points of the union one must obviously pose some additional conditions. Same applies to intersection of more than two X^3_i.

Finite (p-adic) length scale resolution suggests that all points of the union at which an intersection between two or more light-like 3-surfaces occurs must be regarded as identical. In general the intersections would occur in a 2-d region of X^2 so that the gluing would take place along 5-D regions of X_i^5 and there are therefore good hopes that the resulting 5-D space is indeed a manifold. The imbedding of the surfaces X_i^3 to X^5 would define the braiding.

3. At the next level one would consider the 5-d structures obtained in this manner and allow them to topologically condense at larger 2-D partonic surfaces in the similar manner. The outcome would be a hierarchy consisting of $2n+1$-knots in $2n+3$ spaces. A similar construction applied to partonic surfaces gives a hierarchy of $2n$-knots in $2n+2$-spaces.

4. The notion of length scale cutoff is an essential element of the many-sheeted space-time concept. In the recent context it suggests that d-knots represented as space-time sheets topologically condensed at the larger space-time sheet representing $d+2$-dimensional imbedding space could be also regarded effectively point-like objects (0-knots) and that their d-knottiness and internal topology could be characterized in terms of additional quantum numbers. If so then d-knots could be also regarded as ordinary colored braids and the construction at higher levels would indeed be very much analogous to that for infinite primes.

Acknowledgements

I want to thank Tony Smith and Carlos Castro for useful discussions and references related to quaternions and octonions.

Bibliography

Online books about TGD

[1] M. Pitkänen (2006), *Topological Geometrodynamics: Overview*. http://tgd.wippiespace.com/public_html//tgdview/tgdview.html.

[2] M. Pitkänen (2006), *Quantum Physics as Infinite-Dimensional Geometry*. http://tgd.wippiespace.com/public_html//tgdgeom/tgdgeom.html.

[3] M. Pitkänen (2006), *Physics in Many-Sheeted Space-Time*. http://tgd.wippiespace.com/public_html//tgdclass/tgdclass.html.

[4] M. Pitkänen (2006), *Quantum TGD*. http://tgd.wippiespace.com/public_html//tgdquant/tgdquant.html.

[5] M. Pitkänen (2006), *TGD as a Generalized Number Theory*. http://tgd.wippiespace.com/public_html//tgdnumber/tgdnumber.html.

[6] M. Pitkänen (2006), *p-Adic length Scale Hypothesis and Dark Matter Hierarchy*. http://tgd.wippiespace.com/public_html//paddark/paddark.html.

[7] M. Pitkänen (2006), *TGD and Fringe Physics*. http://tgd.wippiespace.com/public_html//freenergy/freenergy.html.

Online books about TGD inspired theory of consciousness and quantum biology

[8] M. Pitkänen (2006), *Bio-Systems as Self-Organizing Quantum Systems*. http://tgd.wippiespace.com/public_html//bioselforg/bioselforg.html.

[9] M. Pitkänen (2006), *Quantum Hardware of Living Matter*. http://tgd.wippiespace.com/public_html//bioware/bioware.html.

[10] M. Pitkänen (2006), *TGD Inspired Theory of Consciousness*. http://tgd.wippiespace.com/public_html//tgdconsc/tgdconsc.html.

[11] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*. http://tgd.wippiespace.com/public_html//genememe/genememe.html.

[12] M. Pitkänen (2006), *TGD and EEG*. http://tgd.wippiespace.com/public_html//tgdeeg/tgdeeg/tgdeeg.html.

[13] M. Pitkänen (2006), *Bio-Systems as Conscious Holograms*. http://tgd.wippiespace.com/public_html//hologram/hologram.html.

[14] M. Pitkänen (2006), *Magnetospheric Consciousness*. http://tgd.wippiespace.com/public_html//magnconsc/magnconsc.html.

[15] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*. http://tgd.wippiespace.com/public_html//magnconsc/mathconsc.html.

References to the chapters of books

[A9] The chapter *Does TGD Predict the Spectrum of Planck Constants?* of [?].
http://tgd.wippiespace.com/public_html//tgdview/tgdview.html#Planck.

[C1] The chapter *Construction of Quantum Theory: Symmetries* of [?].
http://tgd.wippiespace.com/public_html//tgdquant/tgdquant.html#quthe.

[C2] The chapter *Construction of S-matrix* of [?].
http://tgd.wippiespace.com/public_html//tgdquant/tgdquant.html#smatrix.

[C2] The chapter *Construction of Quantum Theory: S-matrix* of [?].
http://tgd.wippiespace.com/public_html//tgdquant/tgdquant.html#towards.

[C4] The chapter *Is it Possible to Understand Coupling Constant Evolution at Space-Time Level?* of [?].
http://tgd.wippiespace.com/public_html//tgdquant/tgdquant.html#rgflow.

[C6] The chapter *Was von Neumann Right After All* of [?].
http://tgd.wippiespace.com/public_html//tgdquant/tgdquant.html#vNeumann.

[D1] The chapter *Basic Extremals of Kähler Action* of [?].
http://tgd.wippiespace.com/public_html//tgdclass/tgdclass.html#class.

[D3] The chapter *The Relationship Between TGD and GRT* of [?].
http://tgd.wippiespace.com/public_html//tgdclass/tgdclass.html#tgdgrt.

[D5] The chapter *TGD and Cosmology* of [?].
http://tgd.wippiespace.com/public_html//tgdclass/tgdclass.html#cosmo.

[E1] The chapter *TGD as a Generalized Number Theory: p-Adicization Program* of [?].
http://tgd.wippiespace.com/public_html//tgdnumber/tgdnumber.html#visiona.

[E2] The chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [?].
http://tgd.wippiespace.com/public_html//tgdnumber/tgdnumber.html#visionb.

[E4] The chapter *p-Adic Numbers and Generalization of Number Concept* of [?].
http://tgd.wippiespace.com/public_html//tgdnumber/tgdnumber.html#padmat.

[E6] The chapter *Fusion of p-Adic and Real Variants of Quantum TGD to a More General Theory* of [?].
http://tgd.wippiespace.com/public_html//tgdnumber/tgdnumber.html#mblocks.

[E8] The chapter *Riemann Hypothesis and Physics* of [?].
http://tgd.wippiespace.com/public_html//tgdnumber/tgdnumber.html#riema.

[E9] The chapter *Topological Quantum Computation in TGD Universe* of [?].
http://tgd.wippiespace.com/public_html//tgdnumber/tgdnumber.html#tqc.

[F1] The chapter *Elementary Particle Vacuum Functionals* of [?].
http://tgd.wippiespace.com/public_html//paddark/paddark.html#elvafu.

[F2] The chapter *Massless States and Particle Massivation* of [?].
http://tgd.wippiespace.com/public_html//paddark/paddark.html#mless.

[F3] The chapter *p-Adic Particle Massivation: Hadron Masses* of [?].
http://tgd.wippiespace.com/public_html//paddark/paddark.html#padmass2.

[F4] The chapter *p-Adic Particle Massivation: Hadron Masses* of [?].
http://tgd.wippiespace.com/public_html//paddark/paddark.html#padmass3.

[F5] The chapter *p-Adic Particle Massivation: New Physics* of [?].
http://tgd.wippiespace.com/public_html//paddark/paddark.html#padmass4.

[F6] The chapter *Topological Condensation and Evaporation* of [?].
http://tgd.wippiespace.com/public_html//paddark/paddark.html#padaelem.

[F8] The chapter *TGD and Nuclear Physics* of [?].
http://tgd.wippiespace.com/public_html//paddark/paddark.html#padnucl.

[F9] The chapter *Dark Nuclear Physics and Living Matter* of [?].
http://tgd.wippiespace.com/public_html//paddark/paddark.html#exonuclear.

[G2] The chapter *The Notion of Free Energy and Many-Sheeted Space-Time Concept* of [?].
http://tgd.wippiespace.com/public_html//freenergy/freenergy.html#freenergy.

[H2] The chapter *Negentropy Maximization Principle* of [?].
http://tgd.wippiespace.com/public_html//tgdconsc/tgdconsc.html#nmpc.

[J4] The chapter *Quantum Antenna Hypothesis* of [?].
http://tgd.wippiespace.com/public_html//bioware/bioware.html#tubuc.

[J6] The chapter *Coherent Dark Matter and Bio-Systems as Macroscopic Quantum Systems* of [?].
http://tgd.wippiespace.com/public_html//bioware/bioware.html#darkbio.

[K1] The chapter *Time, Spacetime and Consciousness* of [?].
http://tgd.wippiespace.com/public_html//hologram/hologram.html#time.

Articles related to TGD

[16] M. Pitkänen (2006), *Topological Geometrodynamics: an Overall View*,
http://tgd.wippiespace.com/public_html/TGDbrief.pdf.

[17] M. Pitkänen (2006), *TGD Inspired Theory of Consciousness*,
http://tgd.wippiespace.com/public_html/tgdconsc.pdf.

[18] M. Pitkänen (2006), *TGD Inspired Quantum Model of Living Matter*,
http://tgd.wippiespace.com/public_html/quantumbio.pdf.

Mathematics related references

[19] J. Esmonde and M. Ram Murty (1991), *Problems in Algebraic Number Theory*, Springer-Verlag, New York.

[20] T. Smith (1997), *D4-D5-E6 Physics*. Homepage of Tony Smith.
http://galaxy.cau.edu/tsmith/d4d5e6hist.html. The homepage contains a lot of information and ideas about the possible relationship of octonions and quaternions to physics.

[21] J. Daboul and R. Delborough (1999) *Matrix Representations of Octonions and Generalizations*, hep-th/9906065.

[22] J. Schray and C. A. Manogue (1994) *Octonionic representations of Clifford algebras and triality*, hep-th/9407179.

[23] R. Harvey (1990), *Spinors and Calibrations*, Academic Press, New York.

[24] S. S. Abhyankar (1980), *Algebraic Geometry for Scientists and Engineers*, Mathematical Surveys and Monographs, No 35, American Mathematical Society.

[25] N. M. J. Woodhouse(1997), *Geometric Quantization*, Second Edition, Oxford University.

[26] M. Eichler (1966), *Introduction to the theory of algebraic numbers and functions*, Academic Press, New York.

[27] K. E. Smith. L. Kahanpää, P. Kekäläinen and W. Traves (2000), *Invitation to Algebraic Geometry*, Springer Verlag, New York, Inc..

[28] A. Khrennikov (1994), *p-Adic Valued Distributions in Mathematical Physics*, Kluwer Academic Publishers, Dordrecht.

[29] L. Brekke and P. G. O Freund (1993), *p-Adic Numbers and Physics*, Phys. Rep. , vol 233, No 1.

[30] F. Q. Gouvêa (1997), *p-adic Numbers: An Introduction*, Springer.

[31] E. C. Zeeman (ed.)(1977), *Catastrophe Theory*, Addison-Wessley Publishing Company.

[32] K. S. A-K Mostafa (2000) *Ring Division Algebras, Self Duality and Super-symmetry*, Thesis. hep-th/0002155.

[33] P. Goddard and D. Olive (1986), *The Vertex Operator Construction for Non-Simply-Laced Kac-Moody Algebras I,II* in *Topological and Geometrical Methods in Field Theory*, Eds. J. Hietarinta and J. Westerholm. Word Scientific.

[34] Atyiah-Singer index-theorem, `http://en.wikipedia.org/wiki/Atiyah-Singer_index_theorem`.

[35] C. Kassel (1995), *Quantum Groups*, Springer Verlag.

[36] C. Gomez, M. Ruiz-Altaba, G. Sierra (1996), *Quantum Groups and Two-Dimensional Physics*, Cambridge University Press.

[37] A. Connes and D. Kreimer (1998), *Hofp algebras, renormalization, and non-commutative geometry*, Quantum Field Theory; Perspective and Prospective (C. DeWitt-Morette and J.-B.- Zueber, eds.), Kluwer, Dortrecth, 1999, 59-109. CMP 2000:05, Commun. Math. Phys. 199, 203.242(1998). MR 99h:81137.
Ibid (1999), *Renormalization in quantum field theory and the Riemann-Hilbert problem I: the Hopf algebra structure of graphs and the main theorem*, arXiv:hep-th/9912092.
Ibid (2000), *Renormalization in quantum field theory and the Riemann-Hilbert problem II: the β function, diffeomorphisms and the renormalization group*, arXiv:hep-th/0003188.

[38] C. Calude and T. Zamifrescu (1998), *The Typical Number is a Lexicon*, New Zealand Journal of Mathematics, Volu. 27, 7-13.

[39] F.W. Meyerstein (1999), *Is movement an illusion?: Zeno's paradox from a modern point of view.* `http://www.cs.auckland.ac.nz/CDMTCS//researchreports/089walter.pdf`.

[40] A. Robinson (1974), *Nonstandard Analysis*, North-Holland, Amsterdam.

[41] R. Harvey (1990), *Spinors and Calibrations*, Academic Press, New York.

[42] A. L. Besse (1987), *Einstein Manifolds* (Arthur L. Besse is a pseudonym for a group of mathematicians mostly in France), Springer-Verlag.

[43] S.-S. Chern, *On the Existence of a Complex Structure on the Six-Sphere*, (Nakai Instite of Mathematics preprint 2004).

[44] A. Marscakov and A. J. Niemi (2005), *Yang-Mills, Complex Structures and Chern's Last Theorem*, arXiv:hep-th/0501050.

[45] P. Cartier (2001), *A Mad Day's Work: From Grothendienck to Connes and Kontsevich: the Evolution of Concepts of Space and Symmetry*, Bulletin of the American Mathematical Society, Vol 38, No 4, pp. 389-408.

[46] *Zeta function*, `http://en.wikipedia.org/wiki/Zeta_function`.

[47] *A directory of all known zeta functions*,
`http://secamlocal.ex.ac.uk/~mwatkins/zeta/directoryofzetafunctions.htm`.

[48] *Weil conjectures*, http://en.wikipedia.org/wiki/Weil_conjectures.

[49] *Mellin transform*, http://en.wikipedia.org/wiki/Mellin_transform.

[50] *Langlands program*,http://en.wikipedia.org/wiki/Langlands_program.

[51] E. Frenkel (2005), *Lectures on Langlands program and conformal field theory*, arXiv:hep-th/0512172.

[52] A. Kapustin and E. Witten (2006), *Electric-Magnetic Duality And The Geometric Langlands Program*, arXiv:hep-th/060451.

[53] M. El Naschie (2001), Chaos, Solitons & Fractals, vol 12, No 6, pp. 1167-1174.

[54] *Knot theory*, http://en.wikipedia.org/wiki/Knot_theory.

[55] *Knot sum*, http://en.wikipedia.org/wiki/Knot_sum.

[56] *Torus knot*, http://en.wikipedia.org/wiki/Torus_knot.

[57] T. Durt (2004), *A new expression for mutually unbiased bases in prime power dimensions*, http://arxiv.org/abs/quant-ph/0409090.

[58] M. Kontsevich (1999), *Operads and Motives in Deformation Quantization*, arXiv: math.QA/9904055.

[59] John Baez (2007), *This Weeks Finds (Week 259)*, http://math.ucr.edu/home/baez/week259.html.

Appendix A

Appendix

A-1 Basic properties of CP_2

A-1.1 CP_2 as a manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-1.1}$$

Here λ is any nonzero complex number. Note that CP_2 can also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1,2$ the coordinates of Eguchi and Freund [?] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it \ , \\ \xi^2 &= x + iy \ . \end{aligned} \tag{A-1.2}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi + \Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi - \Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \tag{A-1.3}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connec- ted, with Euler number 3, Pontryagin number 3 and second Betti number $b = 1$.

A-1.2 Metric and Kähler structures of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \rightarrow exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i \bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5. The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \tag{A-1.4}$$

where the Hermitian, in fact Kähler, metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2 \partial_a \partial_{\bar{b}} K \ , \tag{A-1.5}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= lnF \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-1.6}$$

The representation of the metric is given by

$$\frac{ds^2}{R^2} = \frac{(dr^2 + r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{F} \ , \tag{A-1.7}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-1.8}$$

The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2 \sum_A e_k^A e_l^A \ , \tag{A-1.9}$$

are given by

$$\begin{aligned} e^0 &= \tfrac{dr}{F} \ , \quad & e^1 &= \tfrac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \tfrac{r\sigma_2}{\sqrt{F}} \ , \quad & e^3 &= \tfrac{r\sigma_3}{F} \ . \end{aligned} \tag{A-1.10}$$

The explicit representations of vierbein vectors are given by

$$\begin{aligned} e^0 &= \tfrac{dr}{F} \ , & e^1 &= \tfrac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\ e^2 &= \tfrac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} \ , & e^3 &= \tfrac{r(d\Psi + cos\Theta d\Phi)}{2F} \ . \end{aligned} \tag{A-1.11}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = dr^2/F^2 + (r^2/4F^2)(d\Psi + cos\Theta d\Phi)^2 + (r^2/4F)(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \tag{A-1.12}$$

The vierbein connection satisfying the defining relation

$$de^A = -V_B^A \wedge e^B \ , \tag{A-1.13}$$

is given by

$$\begin{array}{lllllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r-\frac{1}{r})e^3 \ , & V_{12} & = & (2r+\frac{1}{r})e^3 \ . \end{array} \tag{A-1.14}$$

The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{lllllll} R_{01} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , & R_{23} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , \\ R_{02} & = & e^0\wedge e^2 - e^3\wedge e^1 \ , & R_{31} & = & -e^0\wedge e^2 + e^3\wedge e^1 \ , \\ R_{03} & = & 4e^0\wedge e^3 + 2e^1\wedge e^2 \ , & R_{12} & = & 2e^0\wedge e^3 + 4e^1\wedge e^2 \ . \end{array} \tag{A-1.15}$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$J = -ig_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \tag{A-1.16}$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$J^k_{\ r}J^{rl} = -s^{kl} \ . \tag{A-1.17}$$

The form J is integer valued and by its covariant constancy satisfies free Maxwell equations. Hence it can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling). Locally one has therefore

$$J = dB \ , \tag{A-1.18}$$

where B is the so called Kähler potential, which is not defined globally since J describes magnetic monopole.

It should be noticed that the magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$\begin{array}{lll} B & = & 2re^3 \ , \\ J & = & 2(e^0\wedge e^3 + e^1\wedge e^2) = \frac{r}{F^2}dr\wedge(d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F}sin\Theta d\Theta d\Phi \ . \end{array} \tag{A-1.19}$$

The vielbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1,1).

Useful coordinates for CP_2 are the so called canonical coordinates in which Kähler potential and Kähler form have very simple expressions

$$\begin{array}{lll} B & = & \sum_{k=1,2} P_k dQ_k \ , \\ J & = & \sum_{k=1,2} dP_k\wedge dQ_k \ . \end{array} \tag{A-1.20}$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$\begin{array}{lll} P_1 & = & -\frac{1}{1+r^2} \ , \\ P_2 & = & \frac{r^2cos\Theta}{2(1+r^2)} \ , \\ Q_1 & = & \Psi \ , \\ Q_2 & = & \Phi \ . \end{array} \tag{A-1.21}$$

A-1.3 Spinors in CP_2

CP_2 doesn't allow spinor structure in the conventional sense [?]. However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0,1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallelly propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the nonallowed -1- factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential $exp(i2\Phi)$, where Φ is the the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

A-1.4 Geodesic submanifolds of CP_2

Geodesic submanifolds are defined as submanifolds having common geodesic lines with the imbedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [?] a general characterization of the geodesic submanifolds for an arbitrary symmetric space G/H is given. Geodesic submanifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$[X,[Y,Z]] \in t \ \text{ for } \ X,Y,Z \in t \ . \tag{A-1.22}$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic submanifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Phi = 0) \ .$$

The nonequivalence of these submanifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S^2_I. S^2_{II} is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-2 Identification of the electroweak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [?] and in particular that the right handed neutrinos decouple completely from the electroweak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{A-2.1}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \times \gamma_5$, $1 \times \gamma_5$ and $\gamma_5 \times 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electroweak group: $SO(4) = SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{A-2.2}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{aligned} V_{01} &= -\frac{e^1}{r} \ , & V_{23} &= \frac{e^1}{r} \ , \\ V_{02} &= -\frac{e^2}{r} \ , & V_{31} &= \frac{e^2}{r} \ , \\ V_{03} &= (r - \frac{1}{r})e^3 \ , & V_{12} &= (2r + \frac{1}{r})e^3 \ , \end{aligned} \tag{A-2.3}$$

and

$$B = 2re^3 \ , \tag{A-2.4}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ^0_3 and Σ^1_2 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I^1_L + 2V_{13}I^2_L \ , \tag{A-2.5}$$

where one have defined

$$\begin{aligned} I_L^1 &= \frac{(\Sigma_{01}-\Sigma_{23})}{2} \quad , \\ I_L^2 &= \frac{(\Sigma_{02}-\Sigma_{13})}{2} \quad . \end{aligned} \tag{A-2.6}$$

A_{ch} is clearly left handed so that one can perform the identification

$$W^{\pm} = \frac{2(e^1 \pm ie^2)}{r} \quad , \tag{A-2.7}$$

where $W^{\pm}$ denotes the charged intermediate vector boson.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quanties

$$\begin{aligned} X &= re^3 \quad , \\ Y &= \frac{e^3}{r} \quad , \end{aligned} \tag{A-2.8}$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$\begin{aligned} \bar{\gamma} &= aX + bY \quad , \\ \bar{Z}^0 &= cX + dY \quad , \end{aligned} \tag{A-2.9}$$

where the normalization condition

$$ad - bc = 1 \quad ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$\begin{aligned} A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\ &+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \quad . \end{aligned} \tag{A-2.10}$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5\Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \quad . \tag{A-2.11}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \quad . \tag{A-2.12}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$\begin{aligned} Q_{em} &= \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} \quad , \\ I_L^3 &= \frac{(\Sigma^{12}-\Sigma^{03})}{2} \quad . \end{aligned} \tag{A-2.13}$$

The fields γ and Z^0 are defined via the relations

$$\begin{aligned}\gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX+bY) \ , \\ Z^0 &= 4(a+b)\bar{Z}^0 = 4(X-Y) \ . \end{aligned} \tag{A-2.14}$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \tag{A-2.15}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electroweak interactions.

The determination of the value of Weinberg angle is a dynamical problem. The angle is completely fixed once the YM action is fixed by requiring that action contains no crossterm of type γZ^0. Pure symmetry nonbroken electroweak YM action leads to a definite value for the Weinberg angle. One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+1_+ + n_-1_-) \ , \tag{A-2.16}$$

where one has

$$\begin{aligned}R_{03} &= 2(2e^0\wedge e^3 + e^1\wedge e^2) \ , \\ R_{12} &= 2(e^0\wedge e^3 + 2e^1\wedge e^2) \ , \\ J &= 2(e^0\wedge e^3 + e^1\wedge e^2) \ , \end{aligned} \tag{A-2.17}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-2.18}$$

Evaluating the expressions above one obtains for γ and Z^0 the expressions

$$\begin{aligned}\gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{A-2.19}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \tag{A-2.20}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned}L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} \ , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) \ , \end{aligned} \tag{A-2.21}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 crossterm (this coefficient must vanish) the expresssion

$$\begin{aligned} X &= -\frac{K}{2g^2} + \frac{fp}{18} \ , \\ K &= Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] \ , \end{aligned} \tag{A-2.22}$$

In the general case the value of the coefficient K is given by

$$K = \sum_i \left[-\frac{(18+2n_i^2)sin^2\theta_W}{9} \right] \ , \tag{A-2.23}$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9\sum_i 1}{(fg^2 + 2\sum_i (18+n_i^2))} \ . \tag{A-2.24}$$

In the scenario where both leptons and quarks are elemantary fermions the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9}{(\frac{fg^2}{2} + 28)} \ . \tag{A-2.25}$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is quite close to the typical value 9/24 of GUTs [?].

A-2.1 Discrete symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:
a) Symmetries must be realized as purely geometric transformations.
b) Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [?].

The action of the reflection P on spinors of is given by

$$\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0 \Psi \ . \tag{A-2.26}$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) \ , \\ \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ . \end{aligned} \tag{A-2.27}$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \Psi^\dagger\gamma^2\gamma^0 \otimes 1 \ . \end{aligned} \tag{A-2.28}$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-3 Space-time surfaces with vanishing em, Z^0, Kähler, or W fields

In the sequel it is shown that space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP_2 projection, only vacuum extremals and space-time surfaces for which CP_2 projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy.

A-3.1 Em neutral space-times

Em and Z^0 neutral space-times are especially interesting space-times as far as applications of TGD are considered. Consider first the electromagnetically neutral space-times. Using spherical coordinates (r,Θ,Ψ,Φ) for CP_2, the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2}dr\wedge(d\Psi+cos(\Theta)d\Phi)+\frac{r^2}{2F}sin(\Theta)d\Theta\wedge d\Phi \ , \\ F &= 1+r^2 \ . \end{aligned} \tag{A-3.1}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3+2p)\frac{r}{F^2}dr\wedge(d\Psi+cos(\Theta)d\Phi)+(3+p)\frac{r^2}{2F}sin(\Theta)d\Theta\wedge d\Phi \ , \\ p &= sin^2(\Theta_W) \ , \end{aligned} \tag{A-3.2}$$

where Θ_W denotes Weinberg angle.

The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi \ , \\ (3+2p)\frac{1}{r^2F}(d(r^2)/d\Theta)(k+cos(\Theta)) &+ (3+p)sin(\Theta)=0 \ , \end{aligned} \tag{A-3.3}$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D\left[|\frac{(k+u}{C}|\right]^{\epsilon} \ , \\ u &\equiv cos(\Theta) \ , \ C=k+cos(\Theta_0) \ , \ D=\frac{r_0^2}{1+r_0^2} \ , \ \epsilon=\frac{3+p}{3+2p} \ , \end{aligned} \tag{A-3.4}$$

where C and D are integration constants. $0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u+k| = [(1+r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \leq k+1 \ ,$$

where $sign(x)$ denotes the sign of x.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$\begin{aligned} \Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\ \Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ . \end{aligned} \tag{A-3.5}$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which the electromagnetically neutral imbeddings become ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global imbedding for, say a constant magnetic field. Although global imbedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteing the absence of edges are satisfied.

The vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \tag{A-3.6}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

The expression for the Kähler form and Z^0 field of the electromagnetically neutral space-time surface will be needed in sequel and is given by

$$\begin{aligned} J &= -\frac{p}{3+2p} X du \wedge d\Phi \ , \\ Z^0 &= -\frac{6}{p} J \ . \end{aligned} \tag{A-3.7}$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

The effective form of the CP_2 metric is given by

$$\begin{aligned} ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta}d\Theta^2 + s^{eff}_{\Phi\Phi}d\Phi^2] \ , \\ s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\ s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ , \end{aligned} \tag{A-3.8}$$

and is useful in the construction of electromagnetically neutral imbedding of, say Schwartchild metric. Note however that in general these imbeddings are not extremals of Kähler action.

A-3.2 Space-times with vanishing Z^0 or Kähler fields

The results just derived generalize to the Z^0 neutral case as such. The only modification is the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F}du \wedge d\Phi$ is useful.

Also the generalization to the case of vacuum extremals is straightforward and corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$\begin{aligned}
Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u}du \wedge d\Phi = (k+u)du \wedge d\Phi \ , \\
r &= \sqrt{\frac{X}{1-X}} \ , \ X = D|k+u| \ , \\
\gamma &= -\frac{p}{2}Z^0 \ . \qquad \text{(A-3.9)}
\end{aligned}$$

For vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

For vacuum extremals with vanishing induced Kähler form classical em field γ and Z^0 field satisfy

$$\gamma = -\frac{sin^2(\theta_W)}{2}Z^0 \simeq -\frac{Z^0}{8}$$

for $\sin^2(\theta_W) = .23$.

A-3.3 Induced gauge fields for space-times for which CP_2 projection is a geodesic sphere

For space-time sheets for which CP_2 projection is $r = \infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \ .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

For homologically trivial geodesic sphere a standard representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP_2 projection color rotations and weak symmetries commute.

Bibliography

[TGD] M. Pitkänen (1995) *Topological Geometrodynamics* Internal Report HU-TFT-IR-95-4 (Helsinki University). Summary of Topological Geometrodynamics in book form. http://tgd.wippiespace.com/public_html/tgd.html.

[padTGD] M. Pitkänen (1995), *Topological Geometrodynamics and p-Adic Numbers*. Internal Report HU-TFT-IR-95-5 (Helsinki University).
http://tgd.wippiespace.com/public_html/padtgd.html.

[cbookI] M. Pitkänen (1998) *TGD inspired theory of consciousness with applications to biosystems*. http://tgd.wippiespace.com/public_html/cbook.html.

[cbookII] M. Pitkänen (2001) *Genes, Memes, qualia, and Semitrance*,
http://tgd.wippiespace.com/public_html/cbookII.html.

[1] Björken, J. and Drell, S. (1965): *Relativistic Quantum Fields*. Mc-Graw-Hill, New York.

[2] Eguchi, T., Gilkey, B., Hanson, J. (1980): Phys. Rep. 66, 6.

[3] Helgason, S. (1962): *Differential Geometry and Symmetric Spaces*. New York: Academic Press.

[4] Huang, K. (1982): *Quarks,Leptons & Gauge Fields*. World Scientific.

[5] Pope, C., N. (1980): *Eigenfunctions and Spin^c Structures on CP_2* D.A.M.T.P. preprint.

[6] Zee, A. (1982): *The Unity of Forces in the Universe* World Science Press, Singapore.