

Mathematical Model of Load Distribution in Rolling Bearing

Tatjana Lazovic

Assistant Professor
University of Belgrade
Faculty of Mechanical Engineering

Mileta Ristivojevic

Full Professor
University of Belgrade
Faculty of Mechanical Engineering

Radivoje Mitrovic

Full Professor
University of Belgrade
Faculty of Mechanical Engineering

External load of rolling bearing is transferred from one ring to the other one through the rolling elements. Load distribution between rolling elements is unequal. Degree of load distribution inequality depends on internal geometry of bearing and magnitude of external load. Two boundary load distributions in radially loaded ball bearing were defined and discussed in this paper. These are ideally equal and extremely unequal load distribution. Real load distribution is between these boundary cases. The new mathematical model of load distribution is developed respecting classic rolling bearing theory and by introduction of new, original value defined as load distribution factor. Developed mathematical model includes all main influences on load distribution in rolling bearing (number of rolling elements, internal radial clearance and external load).

Keywords: rolling bearing, load distribution, load distribution factor.

1. INTRODUCTION

The load distribution in machine elements is the first step in the analysis of their operating condition. Because of influence of many various factors, constructional and technological, it is very difficult to determine precisely a real condition – real load distribution without the approximations – hypotheses. The load distribution is basic input in mechanical and mathematical modelling with the purpose of the analysis of machine parts operating conditions from different relevant aspects: durability, stiffness, reliability and stability. Because of that, the modelling of load distribution is a very responsible and difficult task. The existing models of load distribution in rolling bearings [1], gear transmissions, chain transmissions etc. contain many approximations and empirical coefficients, which have no the appropriate physical basis. Load distribution in rolling bearings, which operate in condition of the time limited endurance, has a dominant influence on bearing durability.

The small vibrations, noise and friction losses are required from the rolling bearings. The large operational safety, load carrying capacity, durability, and accuracy of rotation are required, as well. Hence, developed mathematical models of rolling bearings, their technology of manufacturing and assembly, as well as design optimization were improved. The special place at designing and optimization of rolling bearings has load distribution between rolling elements. The mathematical model developed in this paper establishes correlation between elastic deformations of the bearing parts in contact, internal radial clearance and external radial load distribution between rolling elements of the ball bearing. Developed mathematical model can be used for more exact analysis of load distribution between rolling elements, and for more reliable

determination of the bearing carrying capacity, durability and power losses.

2. LOAD DISTRIBUTION FACTOR

By load transfer from the shaft to the housing, the rolling elements engagement is unequal. The degree of inequality of external load transfer depends on a number of factors: character and magnitude of external load, contact stiffness of the bearing parts, accuracy of their sizes and form, as well as internal radial clearance.

Because of the large number of the influence factors, and complexity of simultaneous revealing of their influence on load distribution between rolling elements, the need of introduction of the appropriate physical parameter has emerged. On the basis of this value, it would be possible to analyze load distribution between rolling elements of the bearing.

This physical parameter is factor of load distribution between rolling elements [2]. It shows a degree of participation of each rolling element in external load transfer through the bearing. On the basis of this factor, it is possible to analyze various influences on load distribution in the bearing.

If F_R is external radial load of the bearing (Fig. 1) and if i -rolling element transfers load $F_{r,i}$, which has a direction of external load, then the ratio $F_{r,i}/F_R$ shows a degree of participation of i -rolling element in transfer of external radial load [2]. This ratio represents the factor of load distribution between rolling elements of the bearing (or load distribution factor) $K_{r,i}$:

$$K_{r,i} = \frac{F_{r,i}}{F_R}, \quad i = 0, 1, \dots, z-1 \quad (1)$$

where: $F_{r,i} = F_i \cos(i\gamma)$ (Fig. 1) and z – total number of rolling elements in bearing.

The number of rolling elements, participated in transfer of external radial load depends on magnitude of external load, geometry and stiffness of the contacting bearing parts. The expression (1) can be written with contact function Φ_i [3]:

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Correspondence to: Dr Tatjana Lazovic
Faculty of Mechanical Engineering,
Kraljice Marije 16, 11120 Belgrade 35, Serbia
E-mail: tlazovic@mas.bg.ac.rs

$$K_{r,i} = \frac{F_i \cos(i\gamma)}{F_R} \Phi_i, \quad i = 0, 1, \dots, z-1 \quad (2)$$

where $\Phi_i = \begin{cases} 1, & F_i > 0 \\ 0, & F_i = 0 \end{cases}$

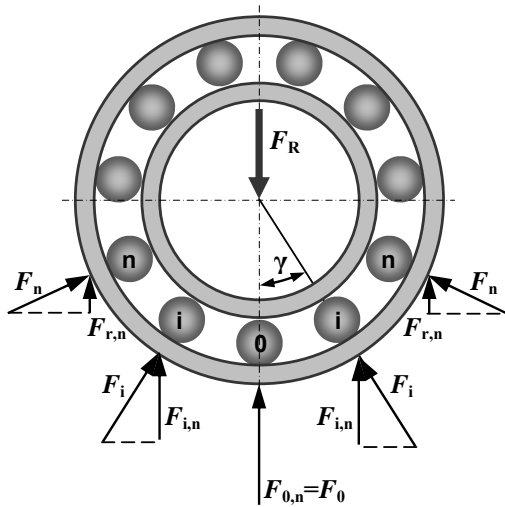


Figure 1. Load distribution between rolling elements of ball bearing

On the basis of the bearing static balance, the expression for external radial load of the bearing F_R can be written in the form:

$$F_R = \sum_{i=1}^{z-1} F_i \cos(i\gamma) \Phi_i. \quad (3)$$

After replacement of expression for external radial load (3) in the equation of the load distribution factor (2), it can be written as follows:

$$K_{r,i} = \frac{F_i \cos(i\gamma)}{\sum_{i=0}^{z-1} F_i \cos(i\gamma) \Phi_i} \Phi_i, \quad i = 0, 1, \dots, z-1. \quad (4)$$

If the bearing is loaded by pure radial load (e.g. non-locating bearing), it is possible to write a condition of contact of rolling elements and rings, on the basis of functions of contact in the form (Fig. 1):

$$\Phi_i = \begin{cases} 1, & i = 0, 1, \dots, n, z-n, \dots, z-1 \\ 0, & n < i < z-n \end{cases} \quad (5)$$

where: $n = \frac{z_s - 1}{2}$ and $z_s = 2 \cdot \text{INT}\left(\frac{z+3}{4}\right) - 1$ – number of rolling elements transmitting external load.

In a case of a radially loaded rolling bearing, the following equations can be written (Fig. 1):

$$\begin{aligned} F_1 &= F_{z-1} \\ F_2 &= F_{z-2} \\ &\dots \\ F_n &= F_{z-n}. \end{aligned} \quad (6)$$

On the basis of the (4) and both (5) and (6), expression for the load distribution factor obtains the following form:

$$K_{r,i} = \frac{F_i \cos(i\gamma)}{F_0 + 2 \sum_{j=1}^n F_j \cos(j\gamma)}, \quad i = 0, 1, \dots, n. \quad (7)$$

3. BOUNDARY LOAD DISTRIBUTION

The boundary values of the load distribution factor are determined by boundary load distribution: ideally equal and extremely unequal (Fig. 2) [2].

If the contacting bearing parts had absolute stiffness, absolute accuracy of dimensions and form, the internal radial clearance equal to zero and external load equally distributed on internal surface of internal ring of the bearing (load q), then radial loads of all rolling elements under central bearing plane are identical (Fig. 3) – ideally equal load distribution:

$$F_0 = F_1 = \dots = F_i = \dots = F_n. \quad (8)$$

On the basis of (7) and (8), the expression for the load distribution factor in a case of ideally equal load distribution can be written as follows:

$$K_{r,i} = \frac{\cos(i\gamma)}{1 + 2 \sum_{j=1}^n \cos(j\gamma)}. \quad (9)$$

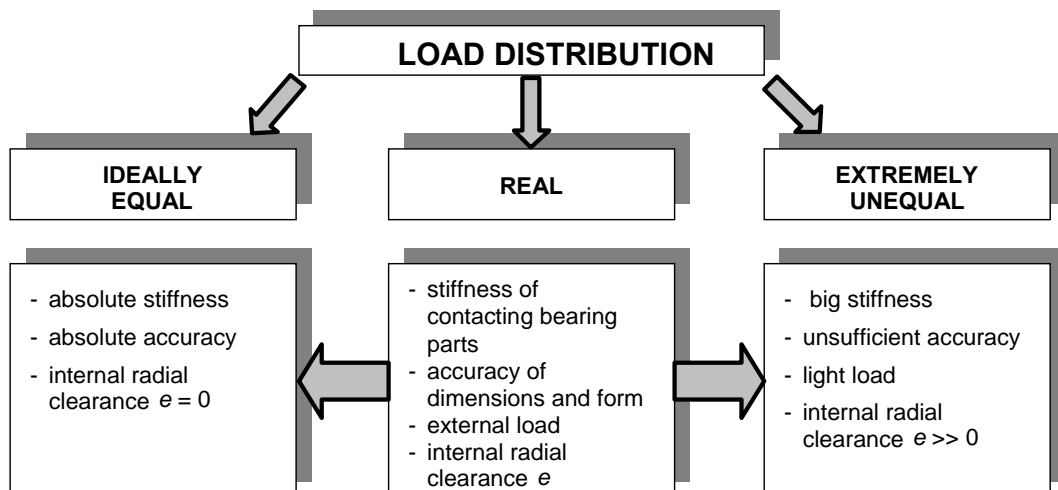


Figure 2. Influence parameters of load distribution in rolling bearing

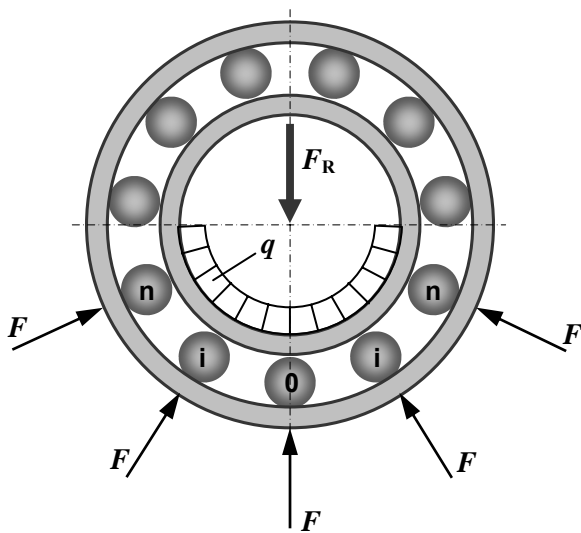


Figure 3. Ideally equal load distribution

Hence, in a case of ideally equal load distribution, the degree of participation of rolling elements in transfer of external radial load depends only on total number of rolling elements. Influence of total number of rolling elements on a degree of participation of each rolling element in transfer of external radial load in a case of ideally equal load distribution is shown in Figure 4.

The degree of participation of the most loaded rolling element in transfer of external radial load decreases with increase of number of rolling elements in the loaded zone and tends to value $1/z_s$. After entry in the loaded zone, the degree of participation of a considered rolling element in load transfer is increased. It results in unloading the most loaded rolling element, as well as the other rolling elements in the loaded zone.

It is theoretically possible to make such combination of the influencing factors of load distribution between rolling elements (increased internal radial clearance, small load, deviation of the form and sizes of contacting bearing parts, which can not compensate their elastic deformations) that complete external radial load is transferred only by one rolling element ("0"-rolling element), in the direction of external load. In this case, it is extremely unequal load distribution, and load of

"0"-rolling element is equal to external load $F_0 = F_R$. The load distribution factor for this rolling element is $F_{r,0} = 1$, and for all other rolling elements, following equations can be written:

$$F_1 = F_2 = \dots = F_i = \dots = F_{z-1} = 0$$

$$K_{r,1} = K_{r,2} = \dots = K_{r,i} = \dots = K_{r,z-1} = 0.$$

Between the described boundary load distributions, ideally equal and extremely unequal, there is a real load distribution.

4. REAL LOAD DISTRIBUTION

Load of a rolling element (Fig. 1) can be determined on the basis of expression [1]:

$$F_i = C_\delta \delta_i^{3/2}, \quad (10)$$

where C_δ – constant value, dependent on bearing internal geometry (diameters of rolling elements and raceways), Young's module of elasticity and Poisson's coefficient of the contacting bearing parts material.

If in expression for the load distribution factor (7) normal load in contact of the contacting bearing parts is replaced by (10), the load distribution factor can be presented in the form:

$$K_{r,i} = \frac{\delta_i^{3/2} \cos(i\gamma)}{\delta_0^{3/2} + 2 \sum_{j=1}^n \delta_j^{3/2} \cos(j\gamma)}, \quad i = 0, 1, \dots, n. \quad (11)$$

Expression representing connection between contact deformations on a place of i -rolling element contact with raceways and internal radial clearance is given in [3,4]:

$$\delta_i = \frac{e}{2} (\cos(i\gamma) - 1) + \delta_0 \cos(i\gamma), \quad i = 0, 1, \dots, n. \quad (12)$$

Introducing (12) in (11) and after appropriate mathematical transformations, the expression for the load distribution factor can be written in the form:

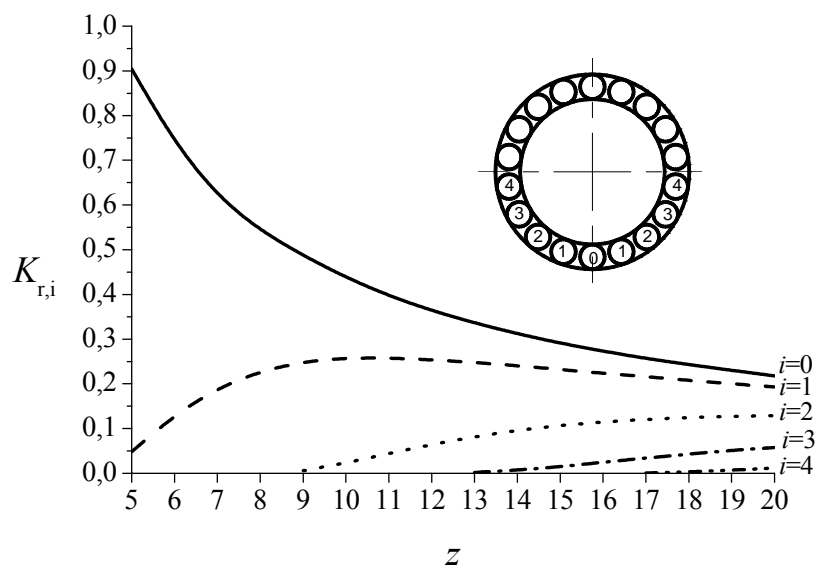


Figure 4. Load distribution factor vs. total number of rolling elements in a case of ideally equal load distribution

$$K_{r,i} = \frac{\left(1 - \left(1 + \frac{e}{2\delta_0}\right)(1 - \cos(i\gamma))\right)^{3/2} \cos(i\gamma)}{1 + 2 \sum_{j=1}^n \left(1 - \left(1 + \frac{e}{2\delta_0}\right)(1 - \cos(j\gamma))\right)^{3/2} \cos(j\gamma)},$$

$i = 0, 1, \dots, n.$ (13)

On the basis of this expression it can be concluded that the load distribution factor of radially loaded ball bearing depends on:

- total number of rolling elements z in the bearing,
- internal radial clearance e and
- contact deformation on a place of the most loaded, “0”-rolling element δ_0 caused by acting of external load.

Dependence of the load distribution factor of radially loaded ball bearing with an internal radial clearance on total number of rolling elements, for various values of the relation of internal radial clearance and contact deformation of the most loaded rolling element $e/2\delta_0$ is shown by the diagram in Figure 5. The relation $e/2\delta_0$ is relative clearance (half internal radial clearance in relation to contact deformation of the contacting bearing parts on a place of the most loaded rolling element). On the basis of analysis of diagram in Figure 5 can be concluded:

- curves of the load distribution factor with increase of total number of rolling elements ($z \rightarrow \infty$) aspire to ideally equal load distribution (in the case of the bearing with a zero-clearance, this value is $1/z_s$, in the case of the bearing with an positive clearance it is $1/z_s'$ and $z_s' < z_s$);
- the equal participation of “0”-rolling element and any i -rolling element (in both sides of loaded zone) in load transfer occurs at a minimum level of the load distribution factor in the case of the bearing with zero-clearance;
- with increase of internal radial clearance, the load distribution factor at which there is equal participation of rolling elements in load transfer is increased;

- with increase of total number of rolling elements in the bearing, load of “0”-rolling element decreases, i -rolling elements take a part of load on and its load increases up to a certain value and after that decreases, due to entry of one more ($i+1$)-rolling element in both sides of the loaded zone;
- the curves of the load distribution factor for “0”-rolling element of the bearing with a radial clearance do not cross a curve of load distribution in a case of the bearing without a radial clearance, that is load of “0”-rolling element is constantly increased with increase of an internal clearance:

$$K_{r,0}(e > 0) > K_{r,0}(e = 0), \quad \forall z;$$

- the curve of the load distribution factor of i -rolling element ($i \neq 0$) at a certain value of an internal radial clearance, which depends on total number of rolling elements and position of a considered rolling element in the bearing, crosses a curve of the load distribution factor for a zero radial clearance, that is load of i -rolling element is not changed with change of an internal radial clearance (x-point in Figure 5);
- in the x-point considered rolling element transfers a part of external load determined by load distribution for zero radial clearance and does not participate in load redistribution;
- up to the x-point, with increase of clearance load of “0”-rolling element is increased, and i -rolling element is unloaded:

$$K_{r,i}(e > 0) < K_{r,i}(e = 0), \quad z < z_x;$$

- with increase of number of rolling elements, behind the x-point, with increase of a clearance ($i+1$)-rolling element is unloaded, and i -rolling element is loaded additionally:

$$K_{r,i}(e > 0) > K_{r,i}(e = 0), \quad z > z_x;$$

Dependence of the load distribution factor on the relative radial clearance $e/2\delta_0$, for various values of total number of rolling elements in the bearing is shown in

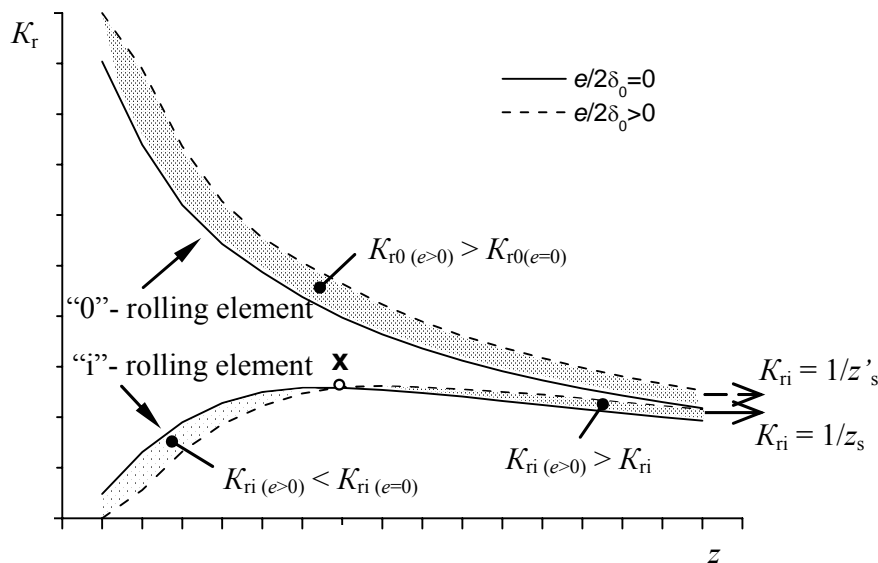


Figure 5. Load distribution factor vs. total number of rolling elements in a case of real load distribution

Figure 6. On the basis of this diagram, it can be concluded that the gradient of change of the load distribution factor has the greatest value in the case of the minimal number of rolling elements in the loaded zone because of load redistribution between fewer number of active rolling elements, and decreases with increase of number of rolling elements participating in external load transfer. With increase of the relative radial clearance $e/2\delta_0$ rolling elements located right under the bearing central plane are unloaded. Then, external load is redistributed between other rolling elements. Due to reduction of the loaded zone, the gradient of change of the load distribution factor of rolling elements remained in the loaded zone is increased. At certain values of the relation $e/2\delta_0$ all rolling elements, except “0”-rolling element, are unloaded down to occurrence of extremely unequal load distribution ($K_{r,0} = 1$) unload. In this case, external radial load is transferred only by one rolling element, that is $K_{r,i} = 1,0$ (A, B, C and D points on the diagram in Figure 6). With increase of the relative clearance, the inequality of load distribution between rolling elements is increased. It is shown by increase of the load distribution factor of the most loaded “0”-rolling element and appropriate changing, decrease or increase, of load distribution factor of other rolling elements in the loaded zone.

5. REDUCED LOAD DISTRIBUTION FACTOR

For consideration of stress and deformation conditions in rolling element-raceway contact, the normal load is relevant. Because of that it is necessary to find the simple mechanism and mathematical model for determination of this load.

The expression (1) for the load distribution factor can be written in the form:

$$\frac{K_{r,i}}{\cos(i\gamma)} = \frac{F_i}{F_R} = k_{r,i} \quad (14)$$

In this expression, $k_{r,i}$ is dimensionless load parameter. It is determined and analyzed in [5] and represents the relation of normal load in i -that rolling element-raceways contact and external radial load. It can be determined as the load distribution factor $K_{r,i}$ corrected by function of a position angle of considered i -rolling element. Thus, parameter $k_{r,i}$ is the load distribution factor reduced to a direction of a position vector of i -rolling element, i.e. to direction of normal load F_i in i -rolling element-raceways contact. Hence, parameter $k_{r,i}$ can be named as reduced factor of radial load distribution between rolling elements of the bearing (or reduced load distribution factor).

If the value of the reduced load distribution factor is known, then on the basis of external radial load magnitude can be determined normal load in rolling element-raceways contact:

$$F_i = k_{r,i} F_R \quad (15)$$

The reduced load distribution factor $k_{r,i}$ for various conditions (internal radial clearance and external load) can be determined on the basis of the load distribution factor $K_{r,i}$. Expression for reduced load distribution factor according to (13) and (14) can be written as follows:

$$k_{r,i} = \frac{\left(1 - \left(1 + \frac{e}{2\delta_0}\right)(1 - \cos(i\gamma))\right)^{3/2}}{1 + 2 \sum_{j=1}^n \left(1 - \left(1 + \frac{e}{2\delta_0}\right)(1 - \cos(j\gamma))\right)^{3/2} \cos(j\gamma)} \quad (16)$$

$i = 0, 1, \dots, n.$

On the basis of this expression can be concluded that the value of the reduced load distribution factor of the ball bearing loaded by external radial load depends on total number of rolling elements z , internal radial clearance e and contact deformation on the place of the most loaded “0”-rolling element δ_0 caused by external load.

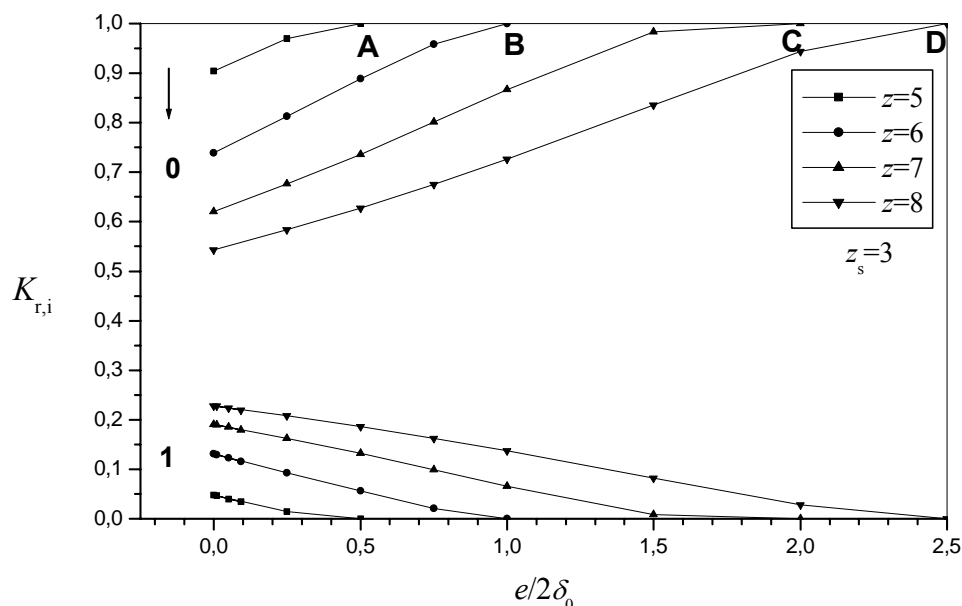


Figure 6. Load distribution factor vs. internal radial clearance

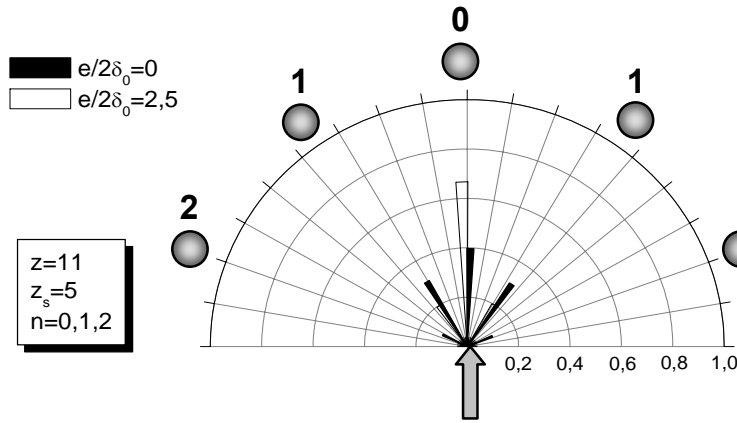


Figure 7. Reduced load distribution factor of bearing with $z=11$ rolling elements

The diagram of the reduced load distribution factor for two relative radial clearances $e/2\delta_0$ of rolling bearing containing $z = 11$ rolling elements is given in Figure 7. Reduced load distribution factor is presented in this diagram as discrete value. By transition from discrete values of the reduced load distribution factor determined by (16) to continuous function can be derived an equation, which can be named as load distribution function. For that purpose, it is necessary to transform (4). On the basis of (4) can be written:

$$k_{r,i} = \frac{F_i}{\sum_{i=0}^{z-1} F_i \cos(i\gamma)}, \quad i = 0, 1, \dots, z-1. \quad (17)$$

By replacement of position angle of each rolling element, expressed by product, of its position number and rolling elements angular distance elements ($i\gamma$) with its position angle (ψ), (17) obtains the form:

$$k_{r\psi} = k_r(\psi) = \frac{F_\psi}{\sum_{\psi=-\pi}^{\psi=\pi} F_\psi \cos \psi}. \quad (18)$$

If it is assumed that in the loaded zone there is a large number of rolling elements ($z_s \rightarrow \infty$) with very small diameter, then the sum in (18) can be replaced by integral:

$$k_{r\psi} = k_r(\psi) = \frac{F_\psi}{\frac{z}{2\pi} \int_{-\psi_0}^{\psi_0} F_\psi \cos \psi d\psi}. \quad (19)$$

On the basis of (19) and (10) for normal load in rolling element-raceway contact, the expression for the reduced load distribution factor as a function of contact deformations in rolling element-raceways contacts is obtained:

$$k_{r\psi} = k_r(\psi) = \frac{\delta_\psi^{3/2}}{\frac{z}{2\pi} \int_{-\psi_0}^{\psi_0} \delta_\psi^{3/2} \cos \psi d\psi}. \quad (20)$$

By introduction of (12), which represents the connection between contact deformation of a rolling

element and internal radial clearance, to (20), and after appropriate mathematical transformations, the expression for the reduced load distribution factor can be written as follows:

$$k_{r\psi} = k_r(\psi) = \frac{\left(1 - \left(1 + \frac{e}{2\delta_0}\right)(1 - \cos \psi)\right)^{3/2}}{\frac{z}{2\pi} \int_{-\psi_0}^{\psi_0} \left(1 - \left(1 + \frac{e}{2\delta_0}\right)(1 - \cos \psi)\right)^{3/2} \cos \psi d\psi}. \quad (21)$$

The expression for the reduced load distribution factor (21) represents load distribution function. The diagram of load distribution function is shown in Figure 8. This diagram represents dependence of the reduced load distribution factor on loaded zone angle (loaded zone width) and relative radial clearance.

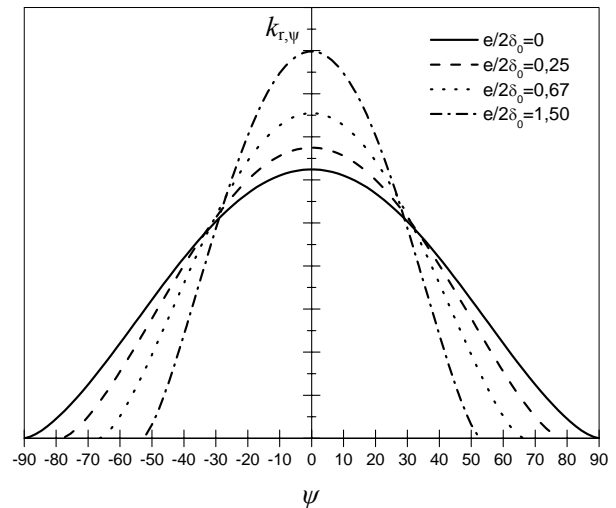


Figure 8. Load distribution function

By analysis of load distribution function it can be concluded, that the loaded zone has two characteristic areas. One has approximate width $\psi \approx \pm (25...35)^\circ$ in relation to a position of "0"-rolling element. Load of rolling elements in this area has the least value at zero radial clearance and is increased with increase of a clearance. The rolling elements in other area of the loaded zone are loaded maximally at zero-clearance. With increase of clearance, rolling elements in this area of the loaded zone are partially or completely unloaded.

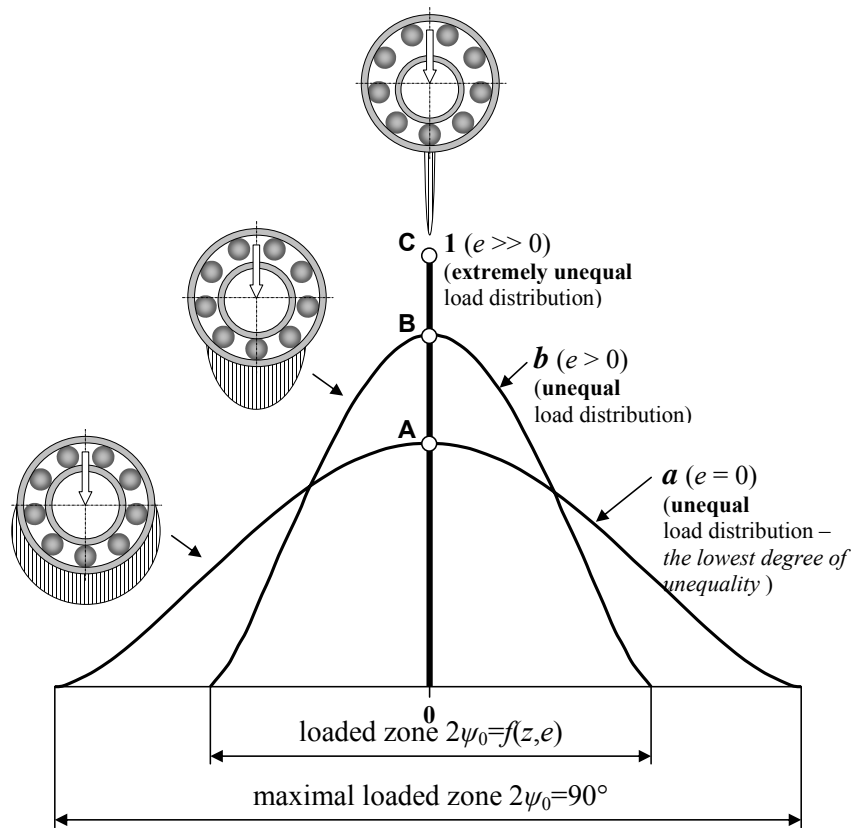


Figure 9. Influence of internal radial clearance on the load distribution between rolling elements

The general diagram of load distribution function with all characteristic points is shown in Figure 9.

Load distribution between rolling elements of the bearing loaded by external radial load is unequal. A degree of inequality of load distribution is the least in the case of the bearing with zero-radial clearance, when all rolling elements located under the bearing central plane participate in load transfer (*a* curve in Figure 9). Then the greatest part of load transfers “0”-rolling element (A point), and the least part of load is carried by rolling elements located directly under the bearing central plane. The loaded zone has maximal width $2\psi_0 = 180^\circ$.

With increase of internal radial clearance, inequality of load distribution between rolling elements is increased (*b* curve in Figure 9). The loaded zone is narrowed ($2\psi_0 < 180^\circ$), and load of the most loaded rolling element (B point) and rolling elements near to it is increased. The rolling elements near to the central plane are partially or completely unload.

At the very large values of a radial clearance and small magnitude of external load, in load transfer the one rolling element participates. It is a boundary case of load distribution – extremely unequal load distribution (point C in Figure 9).

If the external radial load of the bearing F_R is known, then by the load distribution function (21) can be determined load of a rolling element in any position in loaded zone, defined by its angular coordinate ψ :

$$F_\psi = k_{r\psi} F_R, \quad (22)$$

i.e.

$$F_\psi = F(\psi) = \frac{\left(1 - \left(1 + \frac{e}{2\delta_0}\right)(1 - \cos\psi)\right)^{\frac{3}{2}}}{\frac{z}{2\pi} \int_{-\psi_0}^{\psi_0} \left(1 - \left(1 + \frac{e}{2\delta_0}\right)(1 - \cos\psi)\right)^{\frac{3}{2}} \cos\psi} \cdot F_R. \quad (23)$$

Expression (23) shows how normal load in the rolling element-raceway contact depends on position of considered rolling element in loaded zone ψ , total number of rolling elements in the bearing z , relative internal radial clearance $e/2\delta_0$ and magnitude of external radial load F_R . This load function based on load distribution function can be used for more exact determination of operational ball bearing characteristics dependent on load distribution, e.g. lubricant film thickness, load carrying capacity, life, vibrations level etc.

6. CONCLUSION

Mathematical model of load distribution between rolling elements in ball bearing is developed in this paper. It is based on load distribution function, i.e. reduced load distribution factor. This factor is original value which considers all relevant influences on the load distribution in rolling bearing: bearing internal geometry (dimensions of rolling elements and raceways, total number of rolling elements, internal radial clearance) and the most important operational condition – external load of the bearing. Hence, all mentioned influences on bearing performances (load carrying capacity, life, vibration etc.) can be analysed by load distribution

function. Developed mathematical model can be applied only for deep groove ball bearings and partially for cylindrical roller bearings [4]. This is limitation of this mathematical model and there is a need to develop mathematical model of load distribution in other types of rolling bearings.

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РАСПОДЕЛА ОПТЕРЕЋЕЊА У КОТРЉАЈНОМ ЛЕЖАЈУ

Татјана Лазовић, Милета Ристивојевић, Радивоје
Митровић

Спољашње оптерећење котрљајног лежаја се са једног прстена на други преноси преко котрљајних тела. При томе је расподела оптерећења на котрљајна тела неравномерна. Степен неравномерности расподеле оптерећења зависи од унутрашње геометрије лежаја и интензитета спољашњег оптерећења. У овом раду су дефинисана и разматрана два гранична случаја расподеле оптерећења код кугличног котрљајног лежаја оптерећеног спољашњим радијалним оптерећењем. То су идеално равномерна и изразито неравномерна расподела оптерећења. Стварна расподела оптерећења је између ова два гранична случаја. Нови математички модел расподеле оптерећења је развијен на основу класичне теорије котрљајних лежаја и увођењем нове оригиналне величине, дефинисане као фактор расподеле оптерећења. Развијени математички модел обухвата све поменуте релевантне утицаје на расподелу оптерећења у котрљајном лежају (број котрљајних тела у лежају, унутрашњи радијални зазор и спољашње оптерећење).