

# Mathematical Modeling and Computer Simulation

**Daniel P. Maki**

Indiana University

**Maynard Thompson**

Indiana University

**THOMSON**



**BROOKS/COLE**

---

Australia • Canada • Mexico • Singapore • Spain  
United Kingdom • United States

# Contents

## CHAPTER 1

### Basic Principles

1

- 1.0 Overview of the Uses of the Term *Model* 1
- 1.1 The Process of Constructing Mathematical Models 4
- 1.2 Types of Mathematical Models and Some Practical Aspects of Model Building 7
  - Deterministic versus Stochastic Models 8
  - Implementing a Model 9
- 1.3 A Classic Example 10
  - Ptolemy 10
  - Copernicus and Kepler 11
  - Newton 12
- 1.4 Axiom Systems and Models 13
  - Axioms 14
  - Axiom Systems 15
  - Models and Formal Model Building 16
- 1.5 Simulation Models 20
- 1.6 Practical Aspects of Model Building 20
  - Intuitive Evaluations 21
  - Statistics for the Model-Building Process 21

## CHAPTER 2

### Model Building: Selected Case Studies

25

- 2.0 Introduction 25
- 2.1 Mendelian Genetics 25
  - Some Observations 26
  - A Real Model 27
  - A Mathematical Model 28
- 2.2 Models for Growth Processes 36
  - Choices for Growth Models 37
  - Discrete Logistic Model 42
- 2.3 Social Choice 50
- 2.4 Moving Mobile Homes 63
  - The Situation 63
  - A Real Model 64
  - A Mathematical Model 65
- 2.5 A Stratified Population Model 70
- 2.6 Simulation Models in Athletics, Marketing,  
and Population Studies 78
- 2.7 Waiting in Line Again! 86
  - The Setting 86
  - Assumptions Used in the Model 87
  - Comments on the Exponential Distribution 88
  - Predictions 89
  - Comments on Queues with Exponential Arrivals and Service 89
  - Remarks about Other Distributions 90
- 2.8 Estimating Parameters and Testing Hypotheses 93
  - Maximum Likelihood 93
  - Minimum Discrepancy 95
  - Testing Hypotheses and Comparing Models 97

**CHAPTER 3****Markov Chains and Related Stochastic Models****106**

- 3.0 Introduction 106
- 3.1 The Setting and Some Examples 106
  - Animal Ranges 106
  - The Effects of Group Structure on Small-Group Decision Making 110
- 3.2 Basic Properties of Markov Chains 115
  - The Markov Assumption 116
  - State Vectors 118
  - Multistep Transitions and the Sequence of State Vectors 119
- 3.3 Classification of Markov Chains and the Long-Range Behavior of Regular Markov Chains 125
- 3.4 Absorbing Chains and Applications to Ergodic Chains 135
  - Applications of Absorbing Chains to Ergodic Chains 141
  - Chapter Appendix: Mathematical Details 148

**CHAPTER 4****Simulation Models****153**

- 4.0 Introduction 153
- 4.1 The Simulation Process 153
- 4.2 Generating Values of Discrete Random Variables 164
  - Random Numbers Distributed as a Discrete Random Variable 166
  - A Simulation 169
- 4.3 Discrete Event Simulation 171
  - Simulating Markov Chains 178
- 4.4 Generating Values of Continuous Random Variables 185
  - Summary of the Method 188

- 4.5 Applications and Validation of Simulation Modeling 195
  - Estimating Customer Flow in a Retail Store 195
  - Meeting Demands in a Fitness Center 198
  - Modeling the Spread of a Communicable Disease 201
  - Verification and Validation of Simulation Models and Interpretation of Output 207

## CHAPTER 5

### Linear Programming Models

212

- 5.0 Introduction 212
- 5.1 Formulation of Linear Programming Problems 213
  - A Diet Problem 213
  - A Resource Allocation Problem 215
  - A Transportation Problem 217
- 5.2 Linear Programming Problems and Duality 223
  - Complementary Slackness 228
  - General Linear Programming Problems 229
- 5.3 Duality, Sensitivity, and Uncertainty 234
  - Changes in **A**, **b**, and **c** 237
- 5.4 An Example of Integer Programming: A Job Assignment Problem 245
  - The Problem 246
  - A Real Model 246
  - Comments on the Model and Its History 247
- 5.5 Network and Flows 252
  - Definitions and Notation 253
  - Maximal Flows and Minimal Cuts 256
  - Integer Transportation Problems 258
  - The Assignment Problem Revisited 260

<b>APPENDIX</b>
-----------------

## **Addendum for Students and Teachers on Projects and Presentations**

**266**

- A.0 Introduction 266**
- A.1 The Role of Projects and the Types Useful in Learning Model Building 266**
- A.2 Examples of Projects 268**
  - Locating a Community College Campus (Weekend Project, 72 hours) 269
  - Construction of an Earthen Dam (Weekend Project, 72 hours) 271
  - Allocating Teachers (90-minute, In-Class Project) 272
  - A Pipeline Flow Problem (75-minute, In-Class Project) 273
  - An Irrigation Problem (90-minute, In-Class Project) 273
  - An Evacuation Plan for an Elementary School (75-minute, In-Class Project) 275
  - A Vaccination Problem (Long-Term, Several Weeks) 275
  - A Credit Union Scheduling Problem (Long-Term, Several Weeks) 276
- A.3 Reports and Presentations 276**
- A.4 Evaluating Project Reports 277**
- A.5 Sources of Projects 278**
  - Client-Driven Projects 278
  - Self-Initiated or Instructor-Initiated Projects 279
  - The Mathematical Contest in Modeling 280

**Index 281**