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Mathematical Modeling and Simulation of Refrigerating Compressors

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MATHEMATICAL MODELING AND SIMULATION OF REFRIGERATING COMPRESSORS

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INTRODUCTION

Mathematical modeling is the most practical way of studying the basic behavior of cycle performance, the relative losses in various components and the interaction of their performance characteristics. Standard science and engineering formulations are applied to describe mathematically the basic processes occurring in the compressor. Mathematical modeling is not an end in itself but is a step towards simulation and optimization. Simulation is the calculation of operating variables (pressures, temperatures, energy and fluid flow rates) for a system operating in a steady state such that all energy and mass balances, all equations of state of working substances and performance characteristics are satisfied. Simulation could also be defined as the prediction of performance with given inputs or simultaneous solution of performance characteristics. Simulation is used when it is not possible or uneconomical to observe the real system.

The compressor is one of the five essential parts of the compression refrigerating system along with the condenser, expansion valve (or its equivalent), evaporator and the interconnected piping. The various problem areas associated with mathematical modeling and simulation are as follows: thermodynamic analysis and modeling, heat transfer within the compressor, mass transfer through the valves, flow forces on valves and piston and numerous other design considerations. The major progress in compressor modeling has taken place in the last decade, and is still in the development stage, whereas the simulation of internal combustion engines is in a very advanced stage. The close parallel between these two machines is helpful for mathematical formulation of compressor processes except that combustion process is not occurring in the compressor cylinder. The compressor valve dynamics is also different from internal combustion engine case.

The purpose of the present paper is to supplement the reported knowledge[1] on analysis, modeling and simulation of refrigerating compressors particularly in the field of its thermodynamics and heat transfer. A mathematical model has been developed which includes the formulation of thermodynamic processes in the cylinder working space, heat transfer in the valve passages and within the cylinder, mass transfer through the valves, valve dynamics and kinematics of the compressor.

The physical model, to which the mathematical equations will be applied, is that of a single cylinder reciprocating compressor as outlined in Fig. 1, the model may be divided into three interconnected systems: (i) cylinder working space with valves and piston (ii) a suction chamber with a length of suction line and (iii) a discharge chamber with a length of discharge line. Gas pulsations effects in suction and discharge lines have not been investigated in the present study and the intake and exhaust processes are assumed to take place at constant pressures. Hence, the focus of attention here is only on cylinder processes and events.

THERMODYNAMIC ANALYSIS AND MODELING

The thermodynamic processes describe the successive states of refrigerant as it flows through the suction valve, undergoes compression in the cylinder, exhausts through the discharge valve (Fig. 2) and at the same time heat is transferred to and from the refrigerant. In addition to this, thermodynamic behavior is influenced by piston friction, pressure drop across the valves, and oil in the refrigerant etc.

First Law Analysis

The control volume (Fig. 3) consists of the cylinder working space and is bounded by the cylinder walls and the piston. The mass influx is through the suction valve and mass efflux is through the discharge valve. Since four basic processes are occurring in a cycle, the control volume will be as follows:

(1) Suction: Unsteady flow, control volume with only one flow boundary as it is a filling process (Fig 4-a), mass flow rate depends on the pressure difference between suction pressure and cylinder pressure,

heat transfer to the refrigerant vapor in control volume.

(2) Compression: Control volume with no flow boundary i.e. closed system (Fig. 4-b), work into the control volume, heat transfer to and from the control volume.

(3) Discharge: Unsteady flow, control volume with only one flow boundary as it is an emptying process (Fig. 4-c), mass flow rate depends on difference between cylinder pressure and discharge pressure; heat transfer from the control volume.

(4) Re-expansion: Same as compression except that the work transfer is now from the control volume to the surroundings, (Fig. 4-d).

The control volume can be considered to be an open system with suction valve as one flow boundary and discharge valve as another boundary, with both work and heat transfer across the boundary. The assumptions made here for analysis are that the flow is one dimensional, gas follows perfect gas law relationship and uniform cylinder properties at any instant of time. First law of thermodynamics, in its rate form[3] is

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dm_d}{dt} e_{fd} - \frac{dm_s}{dt} e_{fs} + \frac{d}{dt}(me) \quad (1)$$

Where subscripts f, s and d designate flow, suction and discharge respectively, Q is heat flux, m is mass of gas, e is energy and W is the work done.

Ignoring the change in kinetic and potential energies

$$e_f = h = c_p T \quad (2)$$

$$e = u = c_v T \quad (3)$$

where h is enthalpy, u is internal energy, T is temperature, C_p and C_v are specific heats at constant pressure and constant volume respectively.

$$\frac{C_p}{C_v} = k \quad (4)$$

where k is adiabatic constant. From (1), (2), (3), & (4) we get

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dm_d}{dt} k C_v T_d - \frac{dm_s}{dt} k C_v T_s + m C_v \frac{dT}{dt} + C_v T \frac{dm}{dt} \quad (5)$$

Rate of change of mass, $\frac{dm}{dt}$ in the cylinder is given as

$$\text{Suction:} \quad \frac{dm}{dt} = + \frac{dm_s}{dt}$$

$$\text{Discharge:} \quad \frac{dm}{dt} = - \frac{dm_d}{dt} \quad (6)$$

$$\text{Compression \& re-expansion:} \quad \frac{dm}{dt} = 0$$

Ignoring useful work lost because of the friction, the reversible compression work is given by

$$dW = p\,dV \quad (7)$$

where p is pressure and V is the volume.

Equation (7) in rate form is,

$$\frac{dW}{dt} = p \frac{dV}{dt} \quad (8)$$

From perfect gas relationship

$$pV = mRT \quad (9)$$

where R is gas constant. Using (5), (8) & (9), we get

$$m C_v \frac{dT}{dt} + \frac{mRT}{V} \frac{dV}{dt} + k C_v T_d \frac{dm_d}{dt} - k C_v T_s \frac{dm_s}{dt} + C_v T \frac{dm}{dt} - \frac{dQ}{dt} = 0 \quad (10)$$

Table 1 Thermodynamic Equations for Compressor Cylinder

Suction	$m C_v \frac{dT}{dt} + \frac{mRT}{V} \frac{dV}{dt} + \frac{dm_s}{dt}(C_v T - k C_v T_s) - \frac{dQ}{dt} = 0$
Compression and Re-expansion	$m C_v \frac{dT}{dt} + \frac{mRT}{V} \frac{dV}{dt} - \frac{dQ}{dt} = 0$
Discharge	$m C_v \frac{dT}{dt} + \frac{mRT}{V} \frac{dV}{dt} + \frac{dm_d}{dt}(k C_v T_d - C_v T) - \frac{dQ}{dt} = 0$

In equation (10), at present, the unknown quantities are $T(t)$, $m(t)$, $V(t)$, $\frac{dm_s}{dt}$, $\frac{dm_d}{dt}$ and $\frac{dQ}{dt}$. From valve flow model $\frac{dm_s}{dt}$, $\frac{dm_d}{dt}$ and $m(t)$ can be determined, from kinematics model $V(t)$ can be obtained and from heat transfer relationship, $\frac{dQ}{dt}$ can be evaluated. As we have assumed the suction and discharge processes to be constant, both T_d and T_s are known. The only unknown left if $T(t)$ which can be determined by solving equation (10). The pressures inside the cylinder, $p(t)$, can be obtained by using equation (9)

$$p(t) = \frac{m(t)}{V(t)} R\,T(t) \quad (11)$$

Second Law Analysis

As First Law does not make any distinction between heat and work and also since only a part of energy is doing useful work, Second Law of Thermodynamics effects have to be included. Available energy analysis provides a good understanding of all thermodynamic irreversibilities and limitations on work-heat transfer.

The most important deviations from the ideal compression process in a refrigeration cylinder result from the following; (a) throttling or wire drawing during passage through valves and (b) periodic heat transfer between vapor and cylinder walls. Excessive oil circulation, internal gas leakage, heat less to surroundings, heat gains due to friction and imperfect mechanical action of the valve also cause thermodynamic irreversibilities. However, it is not possible to treat all deviations from the ideal cycle analytically. Actual compression process is shown in Fig. 5. Following are the various deviations from the ideal process.

- 1-2: Wire-drawing at suction valve
- 2-3: Heat transfer to refrigerant vapor from cylinder walls and valve passages
- 3-4: Compression process, heat transfer to vapor from cylinder walls
- 4-5: Compression process, heat transfer from vapor to cylinder walls
- 5-6: Heat transfer from vapor to passages and walls at discharge valve
- 6-7: Wire-drawing at discharge valve

Available Energy Analysis

Available energy supplied (based on unit mass flow rate) to the compressor is the work of compression[2] ΔW

$$\Delta W = \frac{n}{n-1} R (T_5 - T_3) \qquad (12)$$

where n is the polytropic index and is given by,

$$\frac{T_1}{T_7} = \left(\frac{p_1}{p_7}\right)^{\frac{n-1}{n}} \qquad (13)$$

Available energy losses in various processes can be found by computing the change in availability functions. Overall loss of available energy (AL) in a compressor is given as (per unit mass flow rate):

$$AL_{comp} = (b_1 - b_3) + (b_3 - \phi_3) + (\phi_3 - \phi_5) + \Delta W + (\phi_5 - b_5) + (b_5 - b_7) \qquad (14)$$

where b = availability function for steady flow (15-a)
= $h - T_0 s$

and ϕ = availability function for a closed system
= $u + p_0 v - T_0 s$ (15-b)

where o subscript refers to the atmospheric or reference conditions. s is entropy and v is specific volume. Note that $(b_1 - b_3)$ represents change in availability during passage through suction valves, $(b_3 - b_5)$ indicates change from open system to closed system, ΔW is increase in availability of the refrigerant because of the work supplied $(\phi_3 - \phi_5)$ is availability change during compression process, $(\phi_5 - b_5)$ indicates change from closed system to open system and $(b_5 - b_7)$ is change during passage through the discharge valve. The total loss of available energy from (14) is

$$AL_{comp} = T_0 \Delta s \qquad (16)$$

Effectiveness of the compressor is given by

$$\epsilon_{comp} = \left|\frac{\Delta b}{\Delta W}\right| \qquad (17)$$

$$= \left|\frac{\Delta h - T_0 \Delta s}{\Delta W}\right| \qquad (18)$$

thus available energy analysis method is a good analytic tool of evaluating performance of a compressor and its various processes. Availability function at each state indicates not the total energy of the refrigerant but its ability to perform useful work. It can be readily extended to other components of the refrigeration system.

HEAT TRANSFER MODELING

Isentropic compression requires that no heat transfer should occur between the vapor passing through the compressor and the cylinder walls. Realization of this condition would require that the cylinder walls and all surfaces making up the clearance volume follow a temperature-time cycle exactly the same as followed by vapor during the compression and re-expansion. But in actual compressor the cylinder wall-temperature-time cycle follows only the similar pattern but with maxima and minima of the wall lagging the maxima and minima of vapor curve. The wall temperature varies periodically around a mean value which is inbetween the suction and discharge temperature of the vapor. The cylinder receives heat from vapor during part of each cycle (re-expansion, suction, and early part of compression process) and returns heat during a latter period (later part of compression, discharge and early re-expansion) as shown in Fig. 4 and Fig. 5.

It is assumed that heat transfer occurs only due to the conductive and convective effects. Radiative heat transfer effects are ignored here (as opposed to I.C. Engines) because of the low temperatures and small temperature differences encountered.

Heat Transfer in Valve Passages

Since suction and discharge gases flow through very narrow valve passages with small thermal resistance, there is a possibility of significant amount of heat transfer from discharge gas to the suction gas which in turn would decrease the thermodynamic and volumetric efficiencies proportional to the increase in the inlet temperature of the gas. No information about this heat transfer is available in the literature for refrigerating compressors[3]. However, in internal combustion engines, Engh and Chiang[4] have shown it to be significant.

Applying First Law of thermodynamics to the suction valve passage (schematic shown in Fig. 6) yields,

$$\frac{dm_s}{dt} c_p T_{so} + \frac{dQ_{ds}}{dt} = \frac{dm_s}{dt} c_p T_s \quad (19)$$

where T_{so} and T_s are the temperature of the gas going into and out of the suction valve passage, $\frac{dm_s}{dt}$ is mass flow rate. The heat transfer $\frac{dQ_{ds}}{dt}$ from the discharge valve gas to the suction valve gas may be expressed as

$$\frac{dQ_{ds}}{dt} = U A_s (T_d - T_s) \quad (20)$$

where U and A_s are overall heat transfer coefficient and area respectively. U through a plane wall is

$$\frac{1}{U} = \frac{1}{h_d} + \frac{1}{h_s} + \sum \frac{b}{K} \quad (21)$$

where h_d and h_s, the film coefficients in discharge and suction passages respectively, are functions of local Reynolds number and Prandtl number. $\sum b/K$ is the thermal resistance of solid parts, b is wall thickness and K is the thermal conductivity of solid parts. The input to mathematical model are A, b, h_s, h_d and K. For common materials, the value of K is widely reported in the literature. The information regarding convective heat transfer coefficient (h) is virtually non-existant, thus in the absence of available data, it is proposed to correlate here McAdam's equation for turbulent flow in pipe,[5]

$$Nu_i = \frac{h_i D_i}{K_i} = 0.023\, Re_i^{0.8}\, Pr_i^{0.4} \quad i = s, d \quad (22)$$

where subscript i designates suction (s) or discharge (d) passage gas. Nu is Nusselt number, Re is Reynolds number and Pr is Prandtl number.

$$Re_i = \frac{\rho_i v_i D_i}{\mu_i} \; ; \quad i = s, d \quad (23)$$

$$Pr_i = \frac{\mu_i C_{pi}}{K_i} \; ; \quad i = s, d \quad (24)$$

$$v_i = \frac{dm_i}{dt} \Big/ \rho_i A_i \; ; \quad i = s, d \quad (25)$$

$$A_i = \frac{\pi}{4} D_i^2 \; ; \quad i = s, d \quad (26)$$

where ρ, μ & K are gas density, viscosity and thermal conductivity respectively. The valve passage has been assumed to be circular in geometry, of diameter D and cross sectional area A. All the fluid properties in (22)-(24) are evaluated at mean bulk temperature of the fluid. Since Prandtl number (Pr) is generally near unity, equation (22) may be reduced to,

$$Nu_i = \frac{h_i D_i}{K_i} \approx 0.023\, Re_i^{0.8} \quad i = s, d \quad (27)$$

The temperature of the suction gas at the conclusion of heat transfer process will be given by (19) as follows:

$$T_s = T_{so} + \frac{dQ_{ds}}{dt} \Big/ c_p \frac{dm_s}{dt}$$

Heat Transfer in Compressor Cylinder

In the cylinder, periodic heat exchange is taking place between its walls and refrigerant vapor due to considerable temperature variation of the vapor as compared with the almost constant wall temperature and the rate of heat transfer from wall to the vapor during early part of the compression is

$$\frac{dQ}{dt} = h(t) A(t) \left[T_w - T(t)\right] \quad (28)$$

where $A(t)$ is the heat transfer area, $h(t)$ is heat transfer coefficient and T_w is wall temperature. During the latter part of the compression, heat is transferred back from vapor to the wall and is expressed as

$$\frac{dQ}{dt} = h(t) A(t) \left[T(t) - T_w\right] \quad (29)$$

For inclusion in thermodynamic model (10), equation (28) can be used as by control volume convention, incoming heat flux is positive, and outgoing heat is negative, thus (28) would automatically take care of the sign.

Heat transfer area $A(t)$ is,

$$A(t) = A_p + A_{cL} + A_w(t) \quad (30)$$

where A_p = piston area $= \frac{\pi}{4} D^2 \quad (31)$

$$A_{cL} = 4 \frac{V_c}{D} \quad (32)$$

$$A_w(t) = \pi D \cdot X(t) \quad (33)$$

where D is diameter of cylinder, V_c is clearance volume and $X(t)$ is the piston displacement from TDC (expression for $X(t)$ is provided by kinematics model). Thus heat transfer area is

$$A(t) = \frac{\pi}{4} D^2 + 4\frac{V_c}{D} + \pi D \cdot X(t) \quad (34)$$

In table 2, some of the available correlations for heat transfer coefficient $h(t)$ are listed. All the correlations except by Adair et al[11] have been evaluated for internal combustion engines and are based on experimental work. In the past, the investigators (Nusselt[6], Eichelberg[7], etc) concentrated on time average heat transfer coefficients and relations were not expressed in dimensionless form and can therefore be generalized with difficulty. LeFeurvre[10], Adair[11] etc) concentrated on instantaneous heat transfer rates and the correlations are in dimensionless form, as shown below:

$$Nu(t) = a\, Re(t)^b \cdot Pr(t)^c \quad (35)$$

where a, b and c are constants.

Table 2 Heat Transfer Correlation for Cylinder

Investigator	Correlation
Nusselt[6] (1928	$h(t) = 0.0278(1+0.38\, v_p)\left[p^2(t)\, T(t)\right]^{1/3}$
Eichelberg (1939)	$h(t) = 0.0565\, v_p^{1/3}\left[p(t)\, T(t)\right]^{1/2}$ p: psi, T: °R, v_p: fps, h: Btu/hr ft^2 °R
Annand[8] (1963)	$Nu(t) = \frac{h(t) D}{K(t)} = \text{const.}\; P_e(t)^{0.7}$ where $P_e(t) = R_e(t) \cdot P_r(t)$ $Re(t) = \frac{\rho(t)\, v_p\, D}{\mu(t)}$ $\text{const.} = 0.4 - 0.7$
Woschni[9] (1968)	$Nu(t) = \frac{h(t) D}{K(t)} = \text{constant}\; Re(t)^{0.7}$ where $Re(t) = \frac{\rho(t)\,(2.28\, v_p)\, D}{\mu(t)}$
LeFeurvre[10] (1968)	$Nu(t) = \frac{h(t) D}{K(t)} = \text{constant}\; Re(t)^{0.8}\, P_r(t)^{0.33}$ where $Re(t) = \frac{\rho(t)\,(D/2)^2\, \omega_g}{\mu(t)}$
Adair et al[11] (1972)	$Nu(t) = \frac{h(t)\, D_e(t)}{K(t)} = 0.053\; Re(t)^{0.8}\, P_r(t)^{0.6}$ where $Re(t) = \frac{\rho(t)\, D_e^2(t)\, \omega_g(t)}{2\mu(t)}$ $D_e(t) = \frac{6\pi\left(\frac{D}{2}\right)^2 X(t)}{\pi D \cdot X(t) + 2\pi\left(\frac{D}{2}\right)^2}$ $\omega_g(t) = 2\omega\left[1.04 + \cos 2\theta\right]$; $\frac{3\pi}{2} < \theta < \frac{\pi}{2}$ $= \omega\left[1.04 + \cos 2\theta\right]$; $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$

VALVE MODELING

The thermodynamic processes consisting of expansions through the suction and discharge valves result in fluid flow into and out of the cylinder through the valves. Motion is imparted to the valve as a result of pressure inequalities on either side of the valve and it should be able to overcome the restoring forces for movement. Also, valve behavior is influenced by stops. Thus, the valve modeling consists of,

(i) Fluid flow model
(ii) Valve dynamics model

Fluid Flow Model

Fluid flow through the suction or discharge valve can be derived by considering the valve to be an orifice and flow to be isentropic[13]. However the actual velocity will be less than the theoretical velocity and shall never be greater than speed of sound as indicated in Fig. 7. Mass flow rates are.

$$\frac{dm_s}{dt} = A_{fs} p_s \frac{G}{\sqrt{T_s}} \sqrt{\left[\frac{p(t)}{p_s}\right]^{\frac{2}{k}} - \left[\frac{p(t)}{p_s}\right]^{k/k-1}} \quad (36)$$

$$\frac{dm_d}{dt} = A_{fd} \frac{p(t)\, G}{\sqrt{T(t)}} \sqrt{\left[\frac{p_d}{p(t)}\right]^{2/k} - \left[\frac{p_d}{p(t)}\right]^{\frac{k}{k-1}}} \quad (37)$$

where $$G = \sqrt{\frac{2 k g_c}{(k-1) R}} \quad (38)$$

A_{fs} and A_{fd} are the effective suction and discharge flow areas. These can be evaluated either theoretically[14] or experimentally[1]. The possibility of back flow is not taken here as gas pulsations effects have been ignored.

Valve Dynamics Model

Assuming the valve to be single degree of freedom system of effective mass M_i, effective stiffness K_i and effective damping C_i (Fig. 8), the following equations are obtained,

suction:

$$M_s \frac{d^2 q_s}{dt^2} + C_s \frac{dq_s}{dt} + K_s q_s = A_{Fs}\left[p_s - p(t)\right] \quad (39)$$

discharge:

$$M_d \frac{d^2 q_d}{dt^2} + C_d \frac{dq_d}{dt} + K_d q_d = A_{Fd}\left[p(t) - p_d\right] \quad (40)$$

where q_s and q_d are the suction and discharge valve displacements respectively. A_{Fs} and A_{Fd} are effective force areas and can be evaluated experimentally[1] or analytically[14].

Table 3 lists the various conditions for valve model. Note that when valve is moving, both flow and dynamics equations hold but when valve reaches the stop, only flow equation holds.

KINEMATICS MODELING

The kinematics of the compressor can be modeled as slider crank mechanism[15], shown in Fig. 9 . The piston displacement from TDC is,

$$X(t) = R(1 - \cos \omega t) + L\left(1 - \sqrt{1 - \left(\frac{R}{L}\sin \omega t\right)^2}\right)$$

Instantaneous cylinder volume is

$$V(t) = V_c + \frac{\pi D^2}{4} X(t) \quad (42)$$

For constant crank speed, crank angle θ is

$$\theta = \omega t \quad (43)$$

where ω = circular frequency = $\frac{2\pi N}{60}$
N = rpm

SIMULATION

The simulation of the compressor involves many mathematical models which are coupled to each other. For instance, the valve behavior is governed by the pressure differential across it and these pressures are calculated by thermodynamic model which in turn depends upon valve model, heat transfer and kinematics models. Thus, the implementation of simulation requires a simultaneous solution of a large number of differential equations. The solution can be approached only by a digital simulation as the problem is complex and difficult for an analog simulation. For integration of differential equations, any of the following methods could be chosen, (i) Runge-Kutta, (ii) Finite Differences and (iii) Predictor-Corrector.

For a single cylinder refrigerating compressor, a generalized computer simulation program was prepared. Fig. 10 shows the outline of flow chart for the main program. It deals with the thermodynamic model. As simultaneous solutions of several models are required, five subroutines as listed below were incorporated in the main program.

Main program: Thermodynamic model
Subroutines 1: Kinematic model for slider crank mechanism.
2: Mass transfer (through valves) model

Table 3 Valve Model Conditions

Process	Valve Position	Cylinder mass	Valve Displacements	Conditions
Suction	Suction valve moving	$\frac{dm(t)}{dt}=\frac{dm_s}{dt}$	$q_s(t)$ $\frac{dq_s}{dt}, \frac{d^2q_s}{dt^2}$	$0 \lesssim q_s(t) \lesssim \Delta_s$ $p_s \gtrsim p(t)$
	Suction valve at stop	$\frac{dm(t)}{dt}=\frac{dm_s}{dt}$	$q_s=\Delta_s$ $\frac{dq_s}{dt}=\frac{d^2q_s}{dt^2}=0$	$q_s=\Delta_s$ $p_s > p(t)$
Compression and Re-expansion	Both valves are closed	$m(t)=const$	$q_s=0$ $q_d=0$	$\frac{dm(t)}{dt}=0$
Discharge	Discharge valve moving	$\frac{dm(t)}{dt}=-\frac{dm_d}{dt}$	q_d $\frac{dq_d}{dt}, \frac{d^2q_d}{dt^2}$	$0 \lesssim q_d \lesssim \Delta_d$ $p(t) \gtrsim p_d$
	Discharge valve at stop	$\frac{dm(t)}{dt}=-\frac{dm_d}{dt}$	$q_d=\Delta_d$ $\frac{dq_d}{dt}=\frac{d^2q_d}{dt^2}=0$	$q_d=\Delta_d$ $p(t) > p_d$

3: Cylinder heat transfer model
4: Valve dynamics model
5: Valve passage heat transfer model
6: Runge-Kutta solution

The input data required can be classified under the following categories, (i) geometrical description (ii) thermal description (iii) initial conditions (iv) gas properties and (v) experimental informations. The increment is given by crank angle, $\Delta\theta$. The program output provides the time history of the refrigerant as it flows through the compressor by punching out cylinder temperature, pressure and fluid flow rates. Valve displacements are also obtained. From this information, available energy analysis of the compressor cycle can be performed.

CONCLUSION

An attempt has been made in this paper to develop and discuss the various aspects of mathematical modeling and simulation of refrigerating compressors, and especially in the area of thermodynamics and heat transfer as it was felt that the reported simulation models have not paid adequate attention to the basic compressor cycle processes. The authors do not claim it to be a final word on thermodynamic models as more sophisticated and authorative models can be developed. Following modifications or additions could be incorporated in the mathematical models: inclusion of real gas properties[17], gas pulsations in compressor lines[18], effects of leakage and friction etc. Also, at present information regarding heat transfer coefficients in cylinder, valve passages and manifolds is virtually non existent[3]. These additions would make simulation models more precise and realistic.

NOMENCLATURE

- A area
- b available energy functions for steady flow
- C_p, C_v specific heats
- C effective damping
- D diameter

e energy
h enthalpy, heat transfer coefficient
k ratio of specific heats
K thermal conductivity, spring stiffness
L connecting rod length
m mass of gas
M mass of valve
n polytropic index
p pressure
q valve displacement
Q heat transfer
R crank radius, gas constant
s entropy
u internal energy
v velocity
V volume
W work
t time
T temperature
x displacement
θ crank angle
ω circular frequency
Δ maximum valve travel
ϕ availability function for closed system
μ viscosity
ρ density

Subscripts

d discharge
f flow
F force
o initial or steady flow conditions
p piston
s suction
v valve

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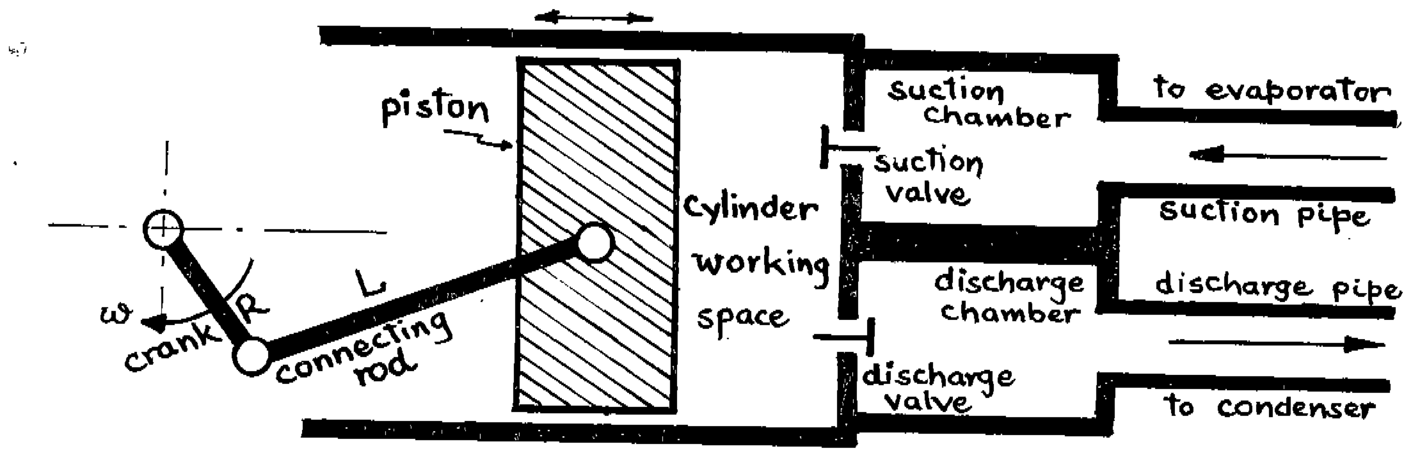
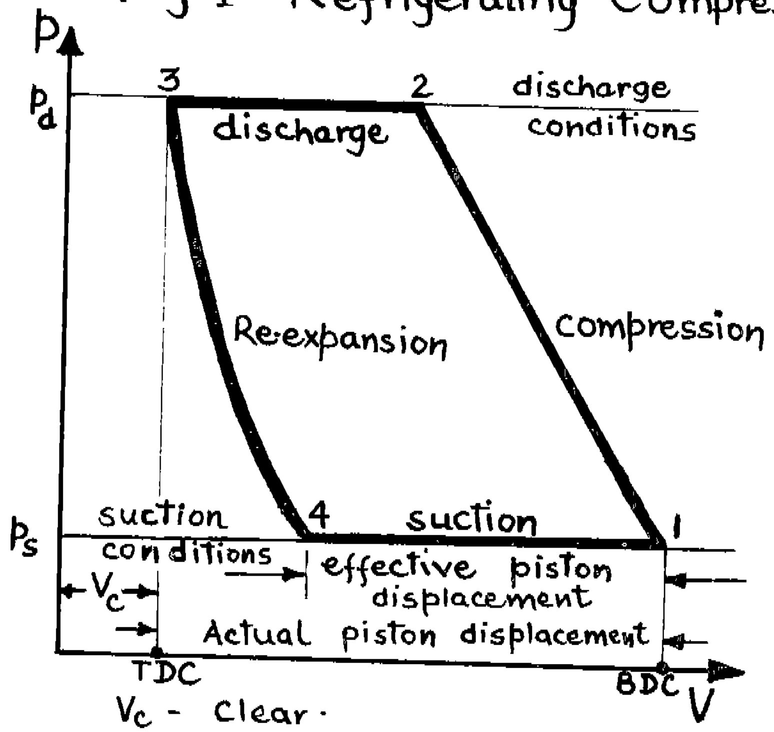


Fig.1 Refrigerating Compressor Physical Model



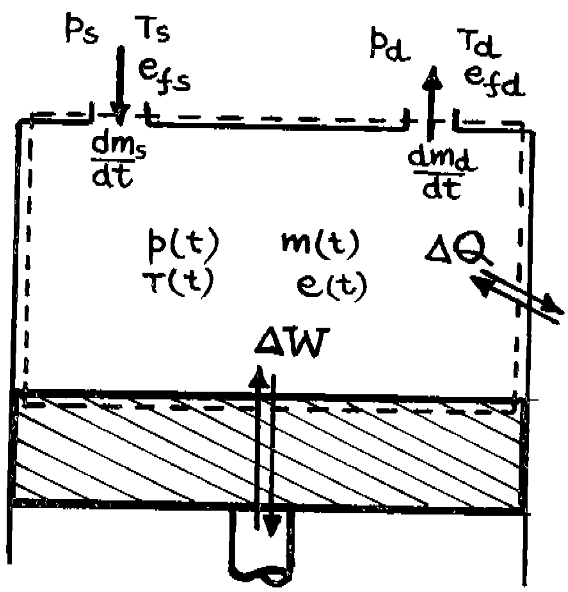
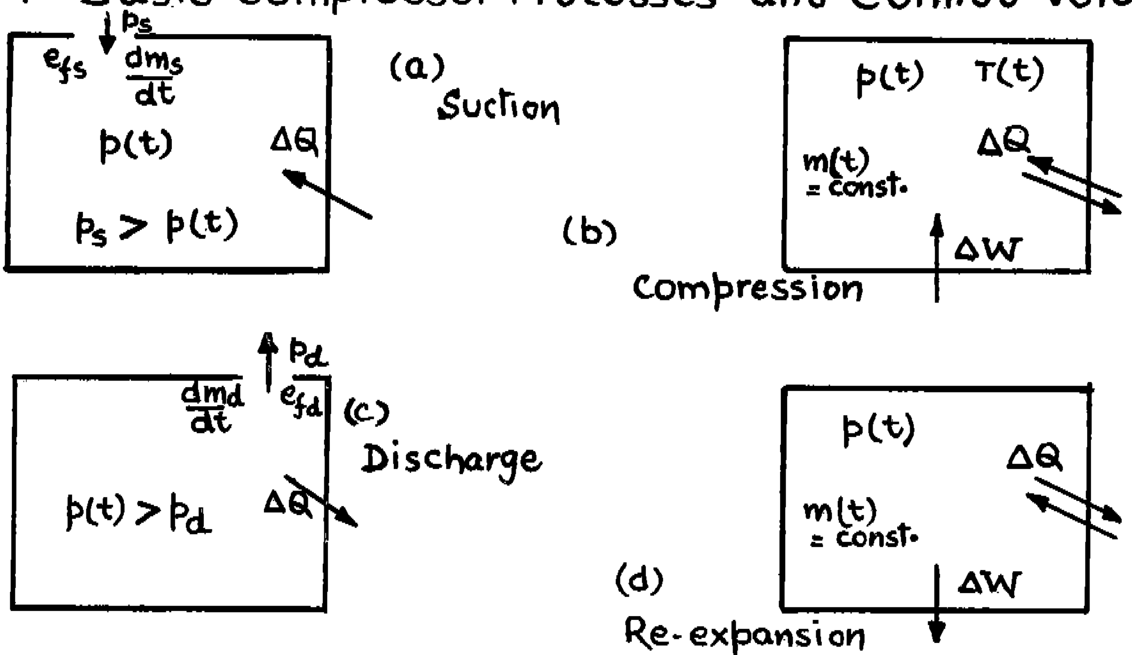


Fig. 3 Control Volume

Fig.2 Ideal p-V Diagram for Compressor

Fig. 4 Basic Compressor Processes and Control Volume



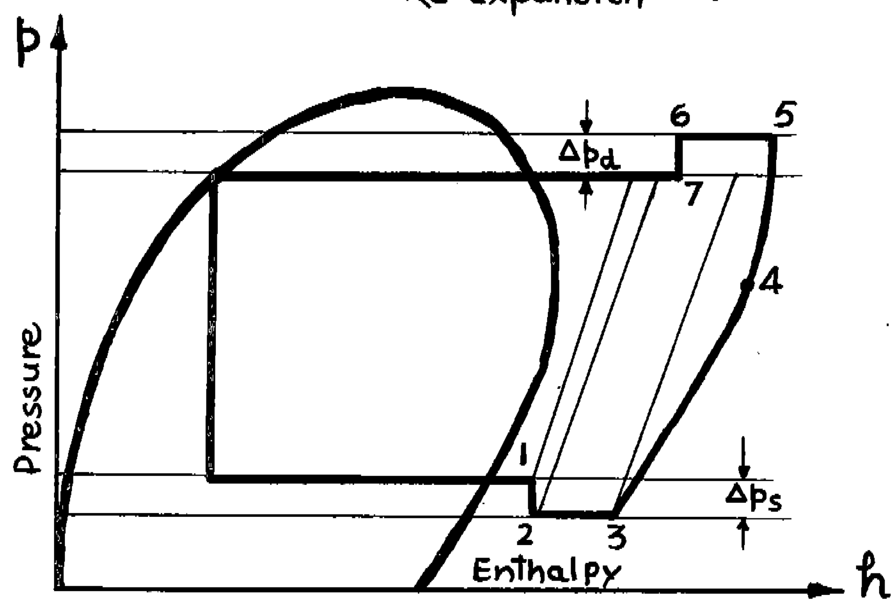


Fig. 5 Refrigeration Cycle Showing Actual Compressor Processes

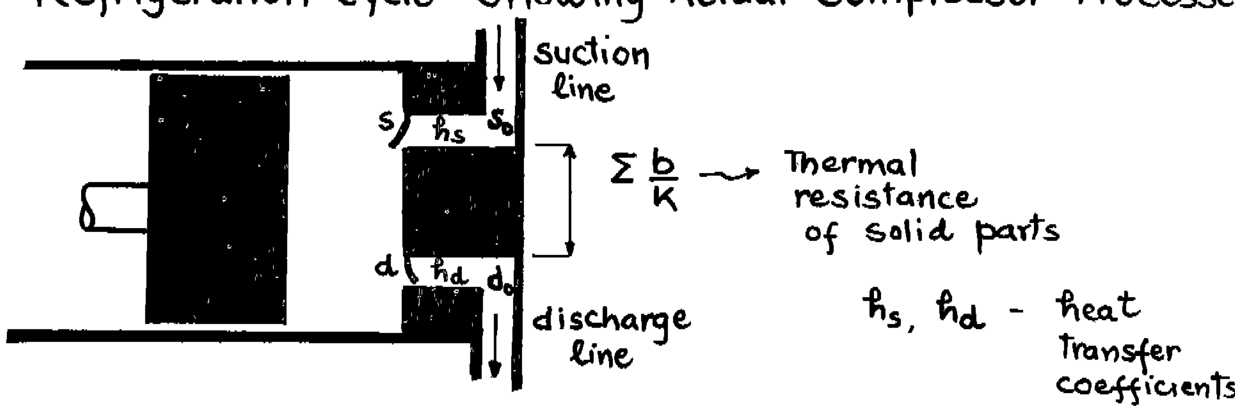
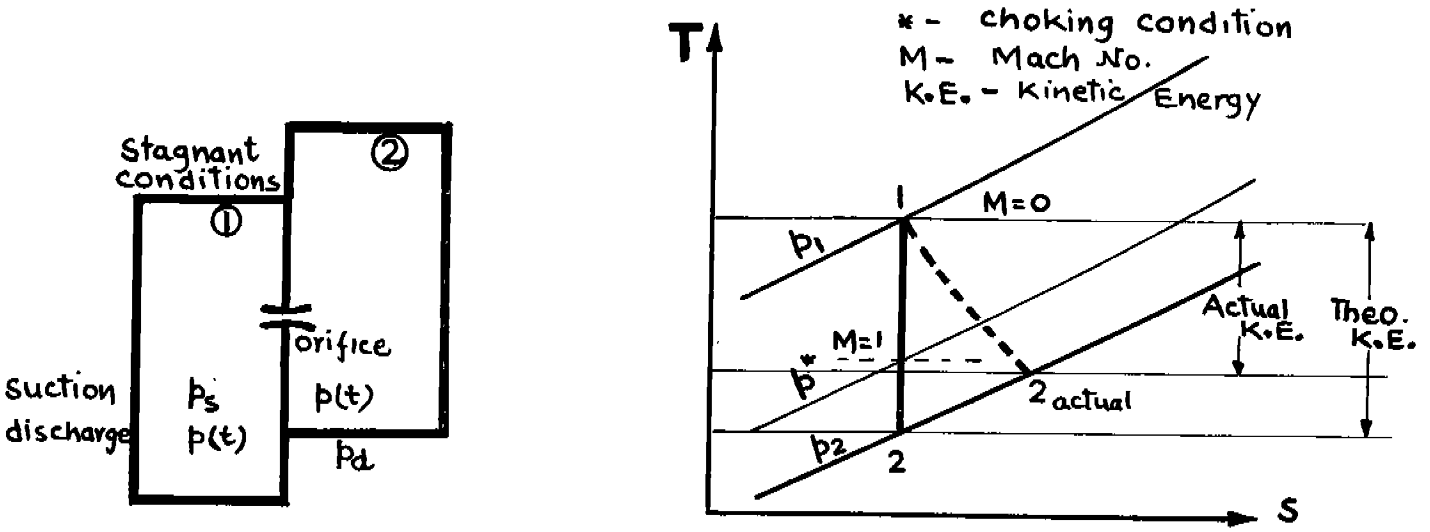


Fig. 6 Heat Transfer in Valve Passages



(a) Flow through an orifice (b) Flow process on temperature-entropy diagram

Fig. 7 Mass Transfer Through Valves

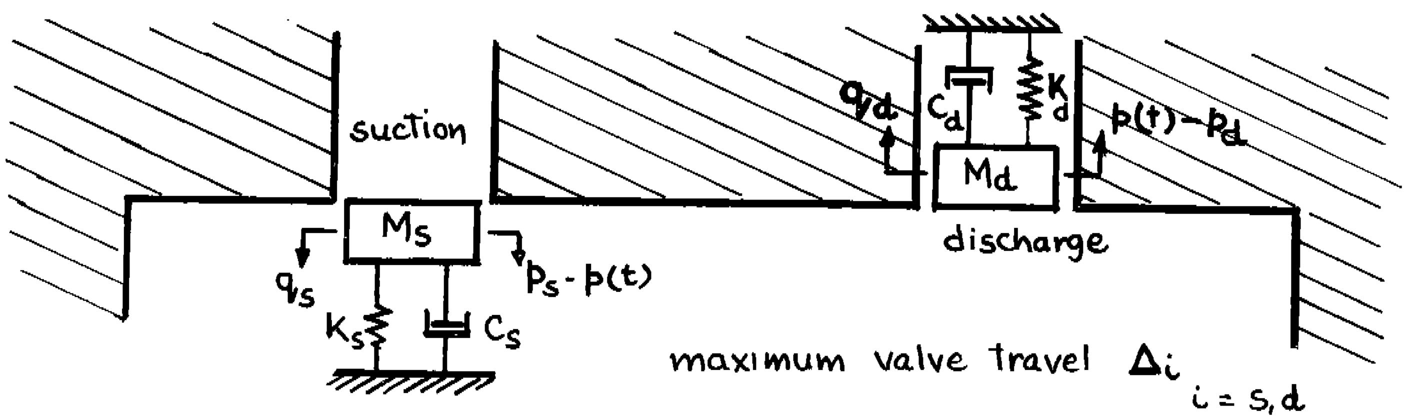
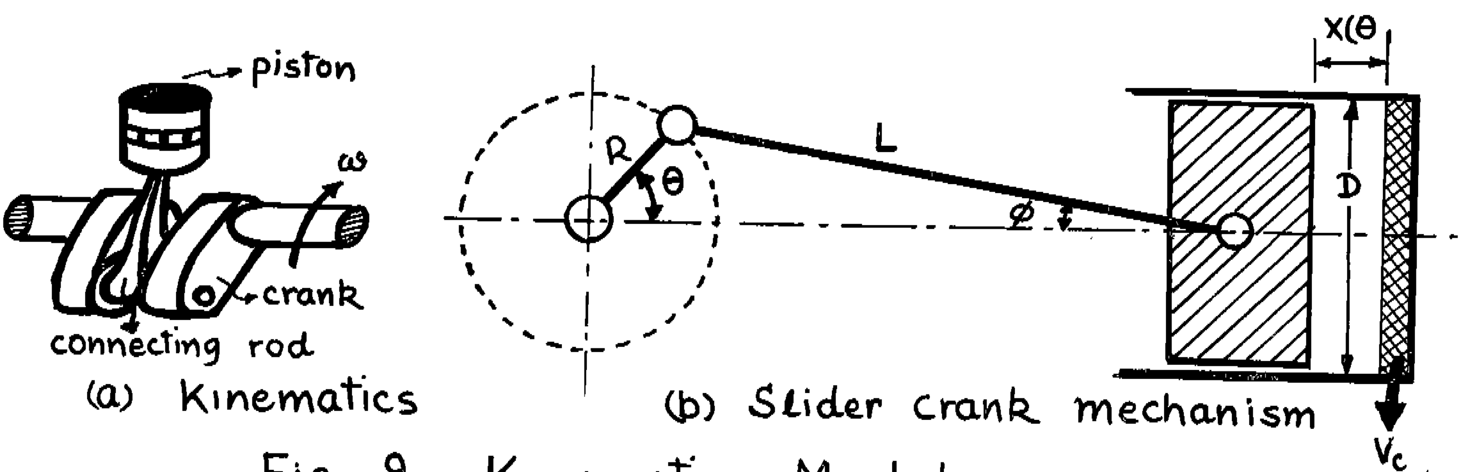


Fig. 8 Valve Dynamics Model



(a) Kinematics (b) Slider crank mechanism

Fig. 9 Kinematics Model

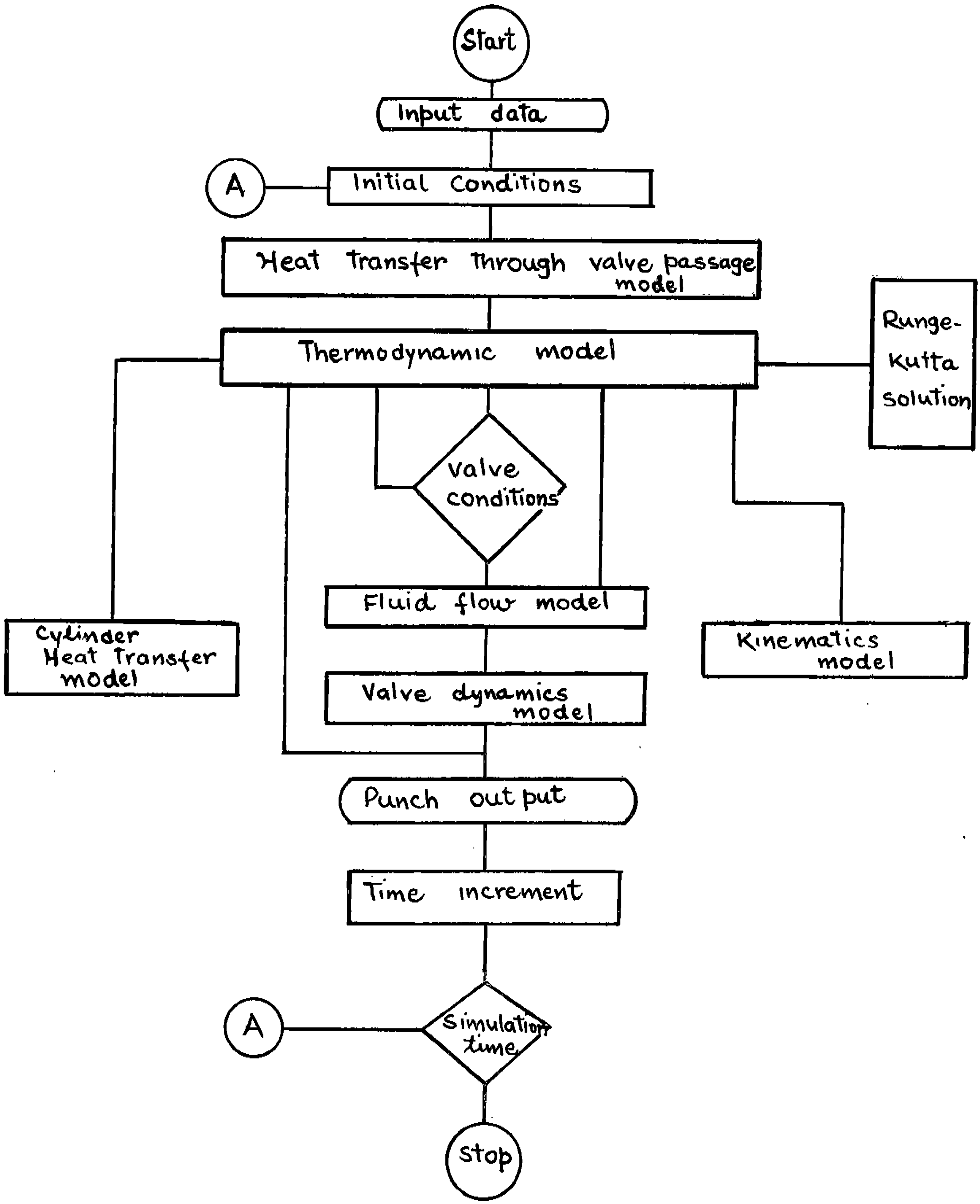


Fig. 10 Outline of Simulation Program