Mathematical modeling of magneto rheological fluid damper in the semi-active suspension system

Cite as: AIP Conference Proceedings **2112**, 020023 (2019); https://doi.org/10.1063/1.5112208
Published Online: 24 June 2019

K. K. Viswanathan, J. S. Tang, Z. A. Aziz, et al.

Mathematical Modeling of Magneto rheological Fluid Damper in the Semi-Active Suspension System

K. K. Viswanathan, [1, 3, b)] J. S. Tang,[2, c)] Z.A. Aziz[3, d)]and P. Sambath[4, a)]

[1] *Department of Mathematics, Kuwait College of Science and Technology, Doha District, Block 4, P.O. Box No. 27235, Safat 13133 Kuwait*
[2]*Department of Mathematics, Faculty of Science, UniversitiTeknologi Malaysia, UTM 81310,Skudai, Johor, Malaysia*
[1, 3]*UTM Centre for Industrial and Applied Mathematics, Faculty of Science, UniversitiTeknologi Malaysia, 81310 JohorBahru, Johor, Malaysia*
[4]*Department of Mathematics, SRM Institute of Science and Technology, SRM Nagar, Kattankulathur, Tamil Nadu 603203, India.*

[a)]Corresponding author: sambathsrm@yahoo.co.in
[b)] visu20@yahoo.com; [c)] tanghoiching@hotmail.com; [d)] zainalaz@ utm.my

Abstract. Ride quality is concerned with the sensation or feel of the passengers in the environment of a moving vehicle. High-speed vehicles including automobiles severely affect the occupants ride comfort and safety due to the vibration. This research aims to simulate the Bouc-Wen model and Modified Bouc-Wen model using suitable parameters taken from literature in MATLAB environment. The system of equations of the model is solved by using the numerical fourth-order Runge-Kutta method. The settlement time required for the suspension system of the vehicle reaches the stabilisation solved by usingBouc-Wen model is almost the same as that of the input of voltage 2V for the Modified Bouc-Wen model. Although the Bouc-Wen model is well suited for the numerical simulation, however it cannot reproduce the experimentally observed roll-off in the yield region for velocities with a small absolute value and an operational sign opposite to the sign of the acceleration. As for the Modified Bouc-Wen model, its magnetic fields strength that depends on the flow of current through the coil of the MRF damper can be varied. After solved this model by using the MATLAB program, voltage 2 V is found to be the best value as observed from the graphical output since the time settlement needed of the vehicle suspension system is the shortest.

INTRODUCTION

In the modern societies, most of the people require high quality of life when they use electronic and automobile which includes the safety and comfort in the moving vehicles. Due to the high demand of the comfort, it is desirable to have high performance of the vehicle suspension system. Perfect automobile suspension system can absorb road shocks produced by the vehicle rapidly and return it to the normal position slowly while maintaining optimal contact between the tire and the surface of the road (Trumper et al., 1996).The vehicle suspension system has two basic functions, that is to keep the car wheels in firm contact with the road and ensuring the comfort of the passengers in the moving car.

The vehicle suspension system is classified into three types, which are passive, semi-active and active suspension system.The magnetorheological fluid (MRF) damper is a device that appears to be particularly promising for suspension protection. The MRF is used to produce a controllable damping forces. The MRF is applied in this research instead of the electrorheological (ER) fluid because the MRF has higher magnitude of yield stress.

This research concentrated on a quarter vehicle model. The quarter vehicle model comprised of one-fourth of the body mass, suspension components and one wheel. The purpose of this research is to review and understanding the existing mathematical models of the MRF damper in the semi-active suspension system for a quarter vehicle model. In addition to that, this research also aims to simulate the Bouc-Wen model and Modified Bouc-Wen model using suitable parameters collected from literature in MATLAB environment.

The 11th National Conference on Mathematical Techniques and Applications
AIP Conf. Proc. 2112, 020023-1–020023-11; https://doi.org/10.1063/1.5112208
Published by AIP Publishing. 978-0-7354-1844-8/$30.00

QUARTER VEHICLE MODEL

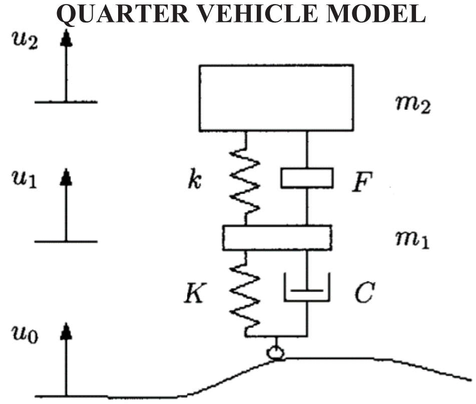

Figure 1. Quarter Vehicle Model with a MRF Damper

The quarter vehicle model with a MRF damper is shown in the Figure 1. By applying Newton's second law ($F = ma$, where F is the force, m is the mass and a is the acceleration), the model can be derived as (Lam et al., 2002a)

$$m_1\ddot{u}_1 = K \cdot (u_0 - u_1) + C \cdot (\dot{u}_0 - \dot{u}_1) + k \cdot (u_2 - u_1) + F \quad (1)$$

$$m_2\ddot{u}_2 = -k \cdot (u_2 - u_1) - F \quad (2)$$

where m_1 is the mass of vehicle axle, m_2 is the mass of thevehicle body, u_0 is the vertical displacement of the road disturbance, u_1 and u_2 are the vertical displacement of the vehicle axle and the vehicle body respectively, k is the spring constant of the damper of the suspension system, K is the spring constant of the tyre, C is the damping coefficient of the tyre and F is the damping force exerted by the MRF damper.

BOUC-WEN MODEL

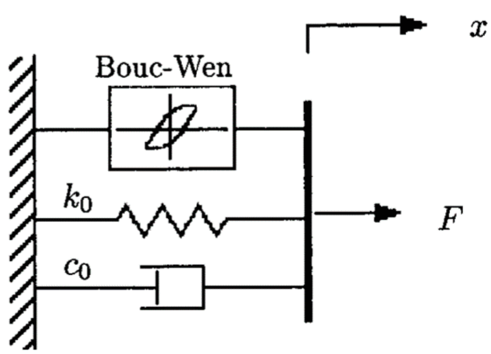

Figure 2. Bouc-Wen Model

The behavior of a MRF damper can be characterized using the Bouc-Wen model suggested by Bouc and presented by Spencer et al. (1996a). Bouc-Wen model is a popular mathematical description for the systems with hysteresis behavior especially the MRF damper. The model's mechanical analogue is shown in Figure 2. The equations for the force generated by the device in this system are given by

$$F(t) = c_0\dot{x} + k_0(x - x_0) + \alpha z \quad (3)$$

where the hysteretic component z satisfies

$$\dot{z} = -\gamma|\dot{x}|z|z|^{n-1} - \beta\dot{x}|z|^n + \delta\dot{x} \quad (4)$$

The initial displacement of the spring is denoted as x_0 which allow for the presence of an accumulator in the MRF damper. c_0 and k_0 are the damping coefficient and the spring constant of the damper of the suspension system respectively. The parameter α is a scaling value for the Bouc-Wen model. The hysteric component z accounts for the time history of the response. γ, β and δ controlled the scale and shape of the hysteresis loop (Ikhouane and Rodellar, 2007).

Although the Bouc-Wen model is well suited for the numerical simulation, however it cannot reproduce the experimentally observed roll-off in the yield region for velocities with a small absolute value and an operational sign opposite to the sign of the acceleration. In order to have better results, Spencher et al.(1996b) proposed an extension of the Bouc-Wen model and named as Modified Bouc-Wen model.

MODIFIED BOUC-WEN MODEL

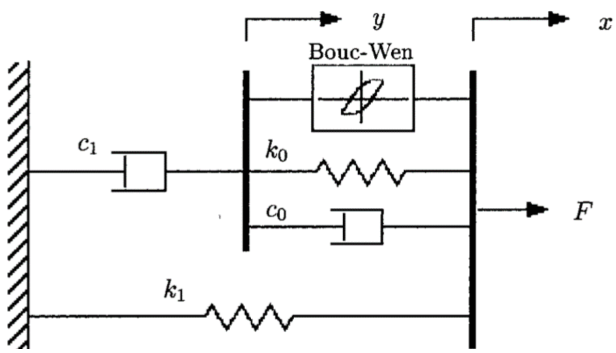

Figure 3. Modified Bouc-Wen Model

Figure 3 shows the analogue of the Modified Bouc-Wen model. This model may give better prediction of the response of the MRF damper in the region of the yield point than the Bouc-Wen model. The resultant force on the rigid bar is zero. The force generated by the device is given as

$$F(t) = \alpha z + c_0(\dot{x} - \dot{y}) + k_0(x - y) + k_1(x - x_0) \tag{5}$$

where

$$\dot{y} = \frac{1}{(c_0 + c_1)}\left[\alpha z + k_0(x - y) + c_0\dot{x}\right] \tag{6}$$

and

$$\dot{z} = -\gamma|\dot{x} - \dot{y}||z|^{n-1} z - \beta(\dot{x} - \dot{y})|z|^n + \delta(\dot{x} - \dot{y}) \tag{7}$$

The initial displacement of the spring is denoted as x_0. c_0 and c_1 are the damping coefficients while k_0 and k_1 are the spring constant of the damper of the suspension system. The parameter α is a scaling value for the Bouc-Wen model. The hysteric component z accounts for the time historical dependence of the response while the spring k_1 and its initial displacement x_0 allow for both the additional stiffness and the force offset produced by the presence of an accumulator (Butz and Von, 2002). The parameter α is a scaling value for the Bouc-Wen model. γ, β and δ controlled the scale and shape of the hysteresis loop.

In recent years, only the case for the constant power input is considered. A model which can represent changing voltage input should be developed in order to fulfil the needs of the people. To obtain a model that is valid for varying and fluctuating magnetic fields strength that depend on the flow of current through the coil of the MRF damper, the dependence of the parameters α, c_0 and c_1 on the applied current must be determined. These parameters are assumed to depend linearly on the voltage, V applied to the current driver. The linear relations are

$$\alpha = \alpha(u) = \alpha_a + \alpha_b u \tag{8}$$

$$c_0 = c_0(u) = c_{0a} + c_{0b}u \quad (9)$$

$$c_1 = c_1(u) = c_{1a} + c_{1b}u \quad (10)$$

where u is the output and given by the following dynamic equation

$$\dot{u} = -\eta(u - V) \quad (11)$$

where η is the filter time constant and V is the voltage applied to the current driver. Equations (8) - (10)are necessary tomodel the dynamics involved in reaching rheological fluid equilibrium and in driving the electromagnet in the MRF due to the change of the magnetic field. The rate of reaching equilibrium is governed by η (Lam et al., 2002b).

ALGORITHM OF OBTAINING THE SOLUTION

The algorithm of obtaining the solution is shown as below.

Step 1: First, solve the Bouc-Wen model and Modified Bouc-Wen model to get the value of force by using the MATLAB program.
Step 2: Next, substitute the obtained value of force into the quarter vehicle model in the MATLAB program.
Step 3: Then, the analysis of the model from 5 aspects, which are the variation of the vertical displacement and vertical velocity of the vehicle body with respect to time, the disparity of the vertical displacement and vertical velocity of the vehicle axle against time and the diversity of the damping force of the vehicle versus time are obtained from the MATLAB output.
Step 4: Finally, the comparison of the graphical output for the Bouc-Wen model and Modified Bouc-Wen model are analysed.

METHOD OF SOLUTION

The numerical Fourth-order Runge-Kutta method intends to increase accuracy to get better approximated solution. This means that the aim of this method is to achieve higher accuracy and to find explicit method of higher order. The algorithm for this method is shown as below.

For solving $\dfrac{dx}{dt} = f(t, x, y)$, $\dfrac{dy}{dt} = g(t, x, y)$ subject to the initial conditions $x(t_0) = x_0$ and $y(t_0) = y_0$ over an interval, I .

Step 1: First divide the interval $t_0 \le t \le t_f$ into n sub-intervals using the equally spaced points, where t_0 is initial value of time and t_f is the final value of time.

$$t_1 = t_0 + h\,,\; t_2 = t_1 + h\,,\; \ldots,\; t_n = t_{n-1} + h = t_f \text{, where } h = \Delta t$$

Step 2: Next, assume that the k^{th} estimates, x_k and y_k have been computed and calculate the following slope estimates in the order listed.

$$M_1 = f(t_k, x_k, y_k)$$

$$L_1 = g(t_k, x_k, y_k)$$

$$M_2 = f\left(t_k + \frac{h}{2}, x_k + \frac{hM_1}{2}, y_k + \frac{hL_1}{2}\right)$$

$$L_2 = g\left(t_k + \frac{h}{2}, x_k + \frac{hM_1}{2}, y_k + \frac{hL_1}{2}\right)$$

$$M_3 = f\left(t_k + \frac{h}{2}, x_k + \frac{hM_2}{2}, y_k + \frac{hL_2}{2}\right)$$

$$L_3 = g\left(t_k + \frac{h}{2}, x_k + \frac{hM_2}{2}, y_k + \frac{hL_2}{2}\right)$$

$$M_4 = f(t_k + h, x_k + hM_3, y_k + hL_3)$$

$$L_4 = g(t_k + h, x_k + hM_3, y_k + hL_3)$$

Step 3: Calculate the next estimates, $(k+1)^{th}$ to the solution.

$$x_{k+1} = x_k + \frac{h}{6}(M_1 + 2M_2 + 2M_3 + M_4)$$

$$y_{k+1} = y_k + \frac{h}{6}(L_1 + 2L_2 + 2L_3 + L_4)$$

Step 4: Repeat Step 2 and Step 3 for $k = 0,1,2,\dots,n-1$.

RESULTS AND DISCUSSION

The numerical computation of the Bouc-Wen model and Modified Bouc-Wen model are computed by using the MATLAB. The system of equations of the model is solved by using the ode 45 built-in function (fourth-order Runge-Kutta method) available in the MATLAB program.

Bouc-Wen Model

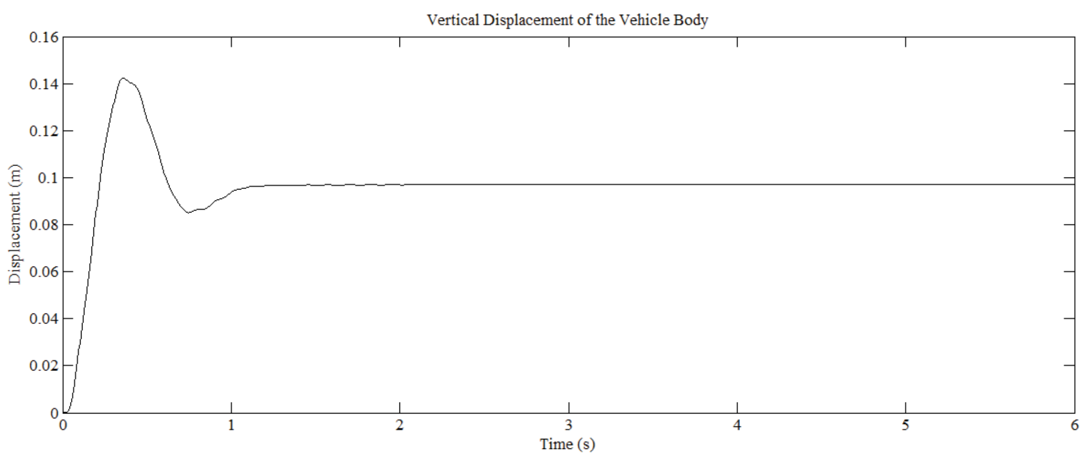

Figure 4. Vertical Displacement of the Vehicle Body versus Time

The vertical displacement of the vehicle body versus time is represented in Figure 4. From Figure 4, we can see that the highest displacement occurs at t=0.3. The displacement of the vehicle body increases rapidly up to 0.14 m for the range 0<t<0.3. Then it significantly decreases to 0.08 m for the range 0.3<t<0.7. As for 0.7<t<1.1, there is slow variation in the increment from 0.08 m to 0.1 m and reaches the stability at the initial displacement 0.1 m after t=1.1.

Figure 5 shows the graph of the vertical displacement of the vehicle axle versus time. The displacement of the first oscillation is 0.17 m and its oscillation keeps reducing as the time goes by. The oscillation of the vehicle axle's vertical displacement is damping and reaches its stability from t=3 onwards.

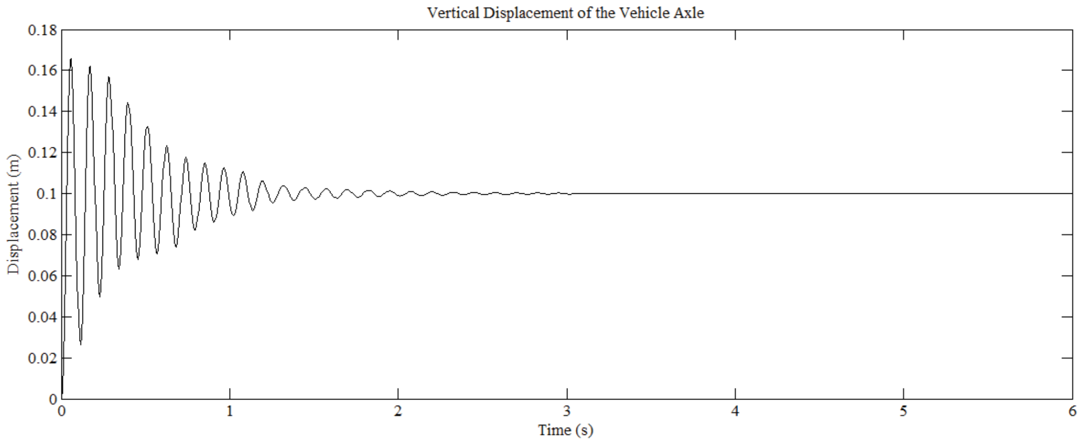

Figure 5. Vertical Displacement of the Vehicle Axle versus Time

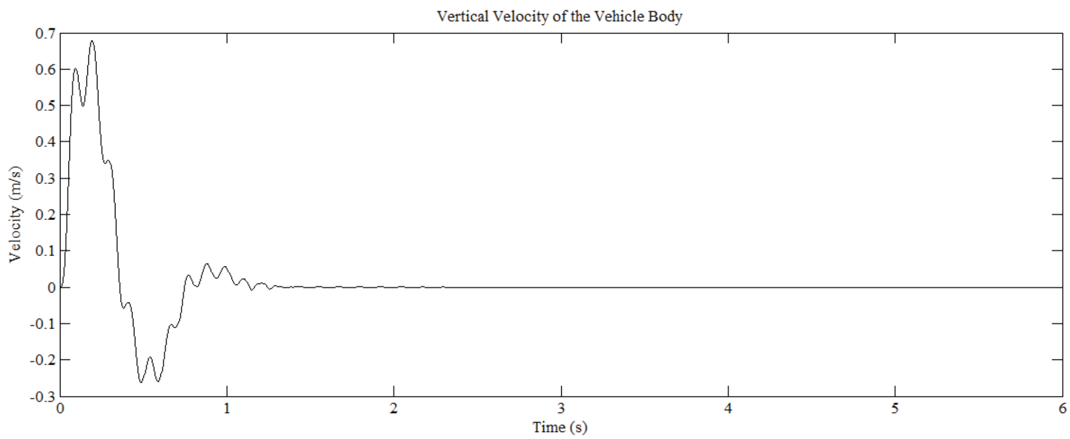

Figure 6. Vertical Velocity of the Vehicle Body versus Time

Figure 6 depicts the graph of the vertical velocity of the vehicle body versus time. The highest velocity of the vehicle body occurs between 0.6 and 0.7. The vehicle body reaches its equilibrium state from t=1.3 afterwards.

The vertical velocity of the vehicle axle versus time is described in Figure 7. The velocity of the vehicle axle records the value between 4 and 5 as the highest velocity. The vehicle axle keeps oscillate and reaches its steady state after t=3.

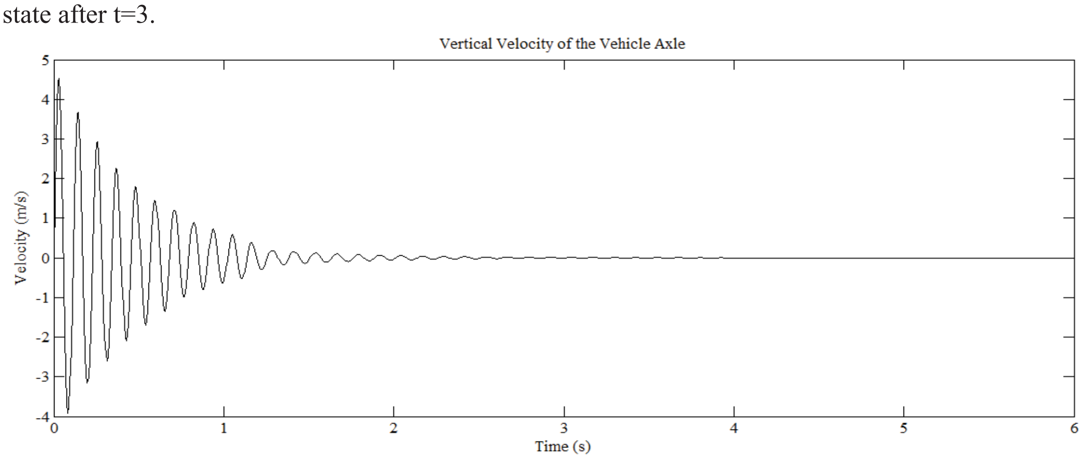

Figure 7. Vertical Velocity of the Vehicle Axle versus Time

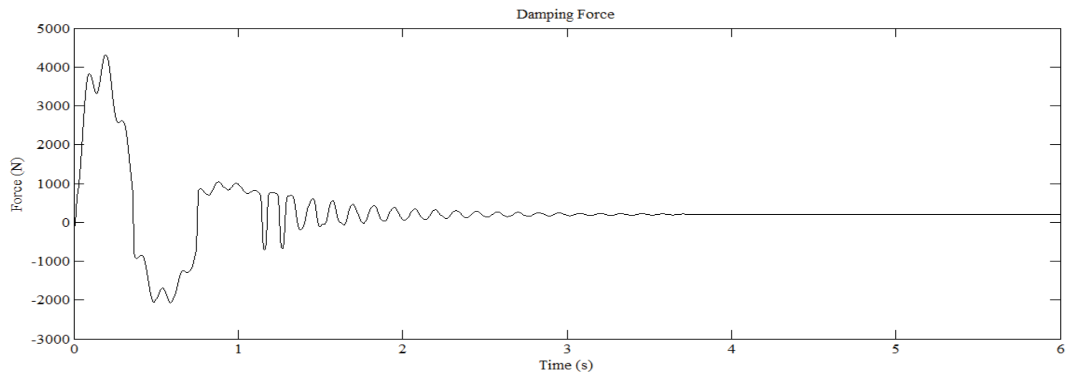

Figure 8. Damping Force of the Vehicle versus Time

Figure 8 indicates the graph of the damping force of the vehicle versus time. From the graph above, the maximum value of damping force is 4300 N. Then it starts to oscillate and then settling time required is t=3.5.

Modified Bouc-Wen Model

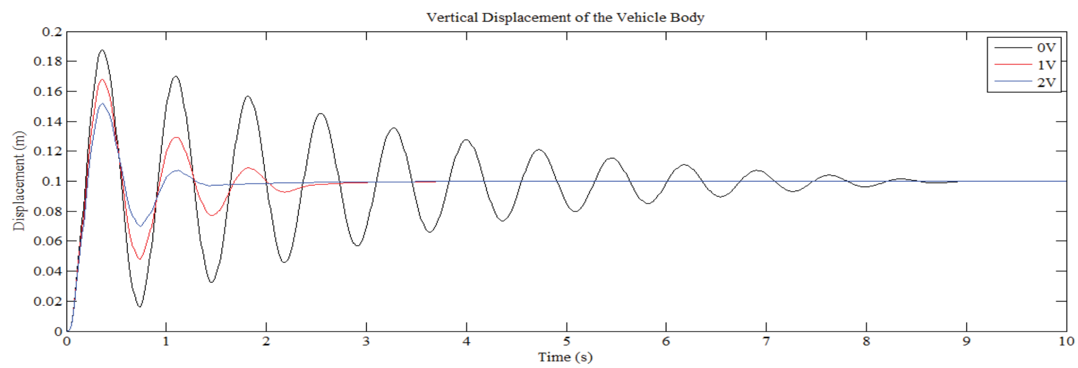

Figure 9. Vertical Displacement of the Vehicle Body versus Time (Combined Graph for Different Voltages)

Figure 9 denotes the combined graph of the vertical displacement of the vehicle body versus time of 0 V, 1 V and 2 V. Based on the figure above, the time settlement for the 0 V, 1 V and 2 V is t=9, t=2.5 and t=1.5 respectively. So, it can be concluded that for the 2 V, the system is the best as it takes the shortest time to stabilize.

Figure 10 demonstrates the combined graph of the vertical displacement of the vehicle axle versus time of 0 V, 1 V, and 2 V. The displacement if the vehicle axle keeps damping and reaches its stability from t=8, t=3 and t=2 onwards for the 0 V, 1 V and 2 V respectively. In conclusion, the time settlement needed for the suspension system back to its origin position is the shortest for the graph that has 2 V as its voltage.

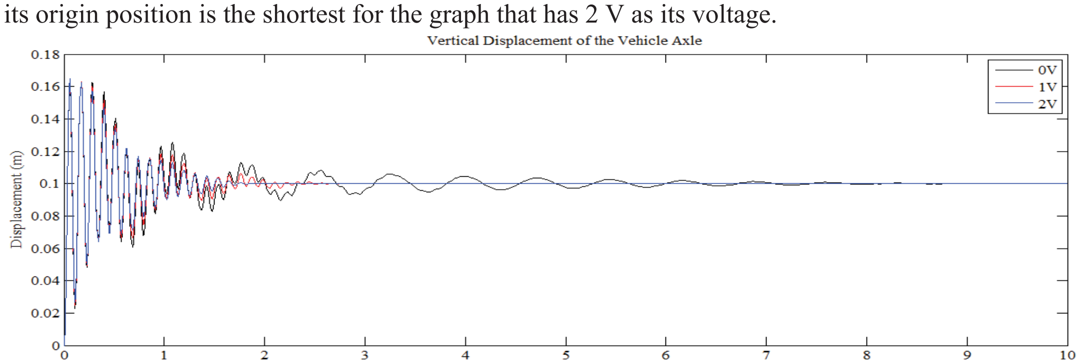

Figure 10. Vertical Displacement of the Vehicle Axle versus Time (Combined Graph forDifferent Voltages)

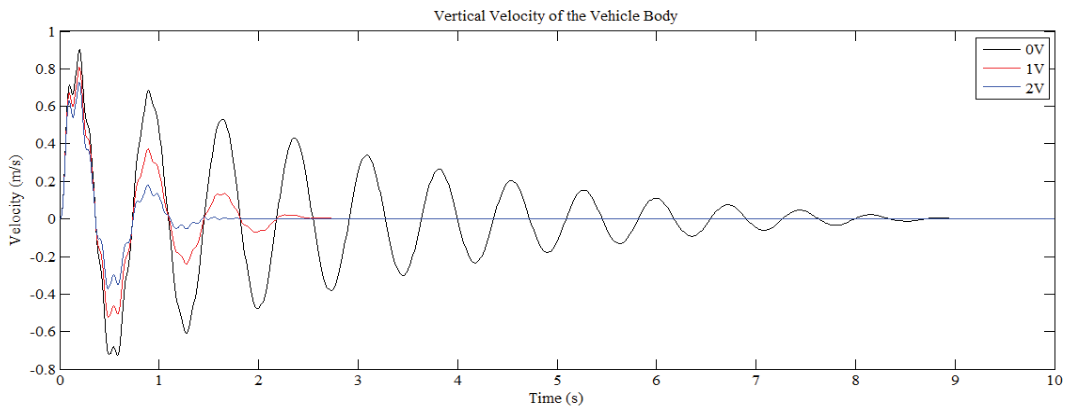

Figure 11. Vertical Velocity of the Vehicle Body versus Time (Combined Graph for Different Voltages)

Figure 11 shows the combined graph of the vertical velocity of the vehicle body versus time of 0 V, 1 V and 2 V. By referring to the graph above, it can be summarised that the velocity of the vehicle body of voltage of 2 V requires least time (t=1.5) to reach its steady state as compared to the velocity of the vehicle body of voltage of 0 V and 1 V which stabilise after t=9 and t=2.5 respectively.

Figure 12 depicts the combined graph of the vertical velocity of the vehicle axle versus time of 0 V, 1 V and 2 V. The velocity of the vehicle axle for 0 V, 1 V and 2 V stabilise after t=5, t=3 and t=2 respectively. Among the three values of voltage investigated in this research, the velocity of the vehicle axle at 2 V has reaches the stabilisation in the shortest time.

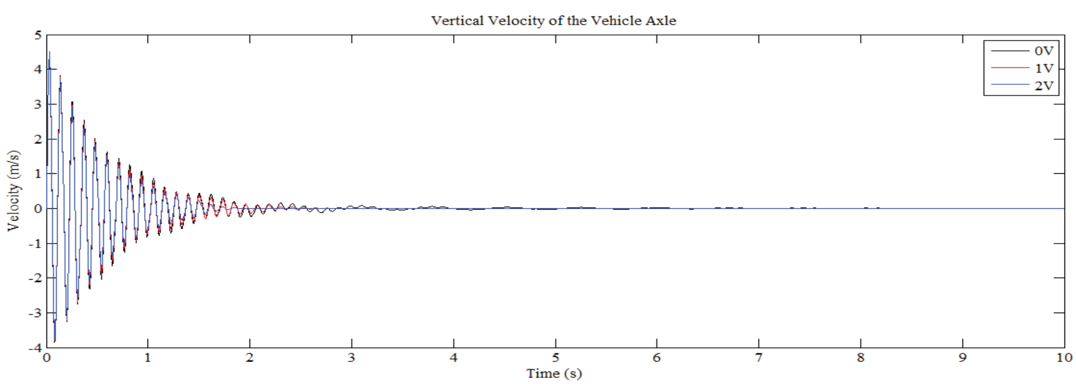

Figure 12. Vertical Velocity of the Vehicle Axle versus Time (Combined Graph forDifferent Voltages)

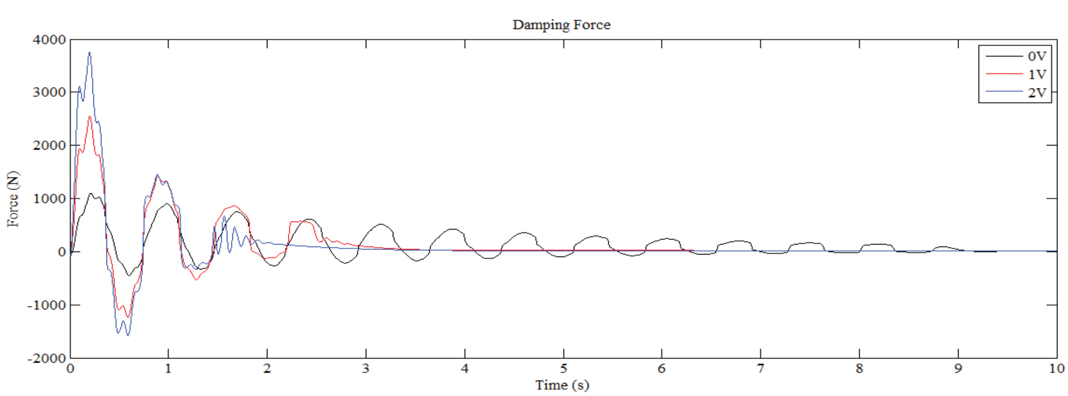

Figure 13. Damping Force of the Vehicle versus Time (Combined Graph forDifferent Voltages)

Figure 13 depicts the combined graph of the damping force of the vehicle versus time of 0 V, 1 V and 2 V. The damping force of the vehicle for 0 V, 1 V and 2 V damps and takes t=9, t=3 and t=2 reach the stabilisation respectively. Although the peak of the damping force for the 2 V is the highest, but it reaches the stabilisation in the shortest time as compared to the others.

COMPARISON BETWEEN THE BOUC-WEN MODEL AND MODIFIED BOUC-WEN MODEL

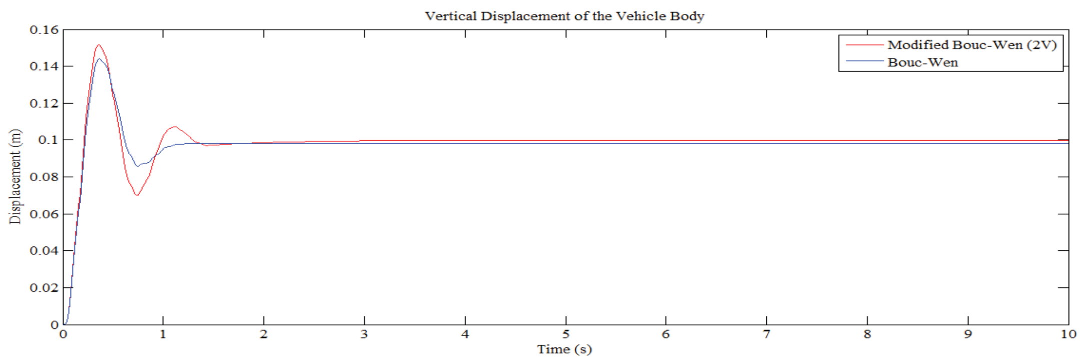

Figure 14. Vertical Displacement of the Vehicle Body versus Time

The vertical displacement of the vehicle body versus time is showed in Figure 14 while Figure 15 presents the vertical displacement of the vehicle axle versus time. As can be seen from Figure 14 and Figure 15, the oscillation of the displacement of the vehicle axle is more fluctuating compared to the displacement of the vehicle body. This is because there is installation of the damper between the vehicle body and its axle in which the damper acts as shock-absorber.

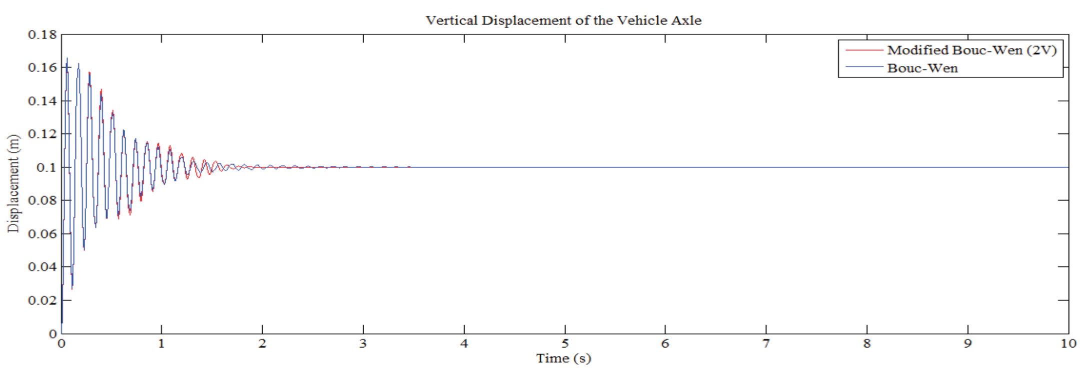

Figure 15. Vertical Displacement of the Vehicle Axle versus Time

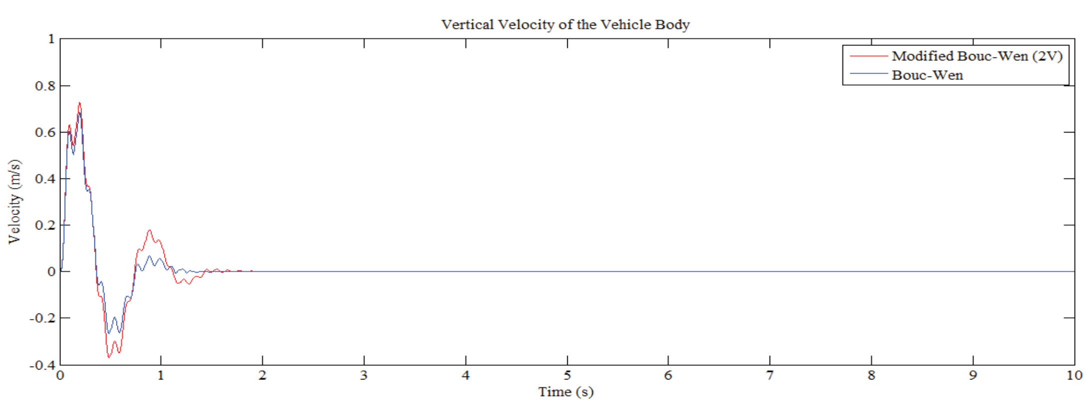

Figure 16. Vertical Velocity of the Vehicle Body versus Time

Figure 16 depicts the vertical velocity of the vehicle body versus time while the vertical velocity of the vehicle axle is described in Figure 17. The range of the velocity of the vehicle body is between -0.4 m/s and 0.8 m/s and for the velocity of the vehicle axle is between -4 m/s and 5 m/s. The range of the velocity for the vehicle axle is so large compared to the velocity of the vehicle body. This is due to installation of the damper between the vehicle body and its axle. The damper will absorb the road shocks produced by the vehicle rapidly and return it to the normal position slowly.

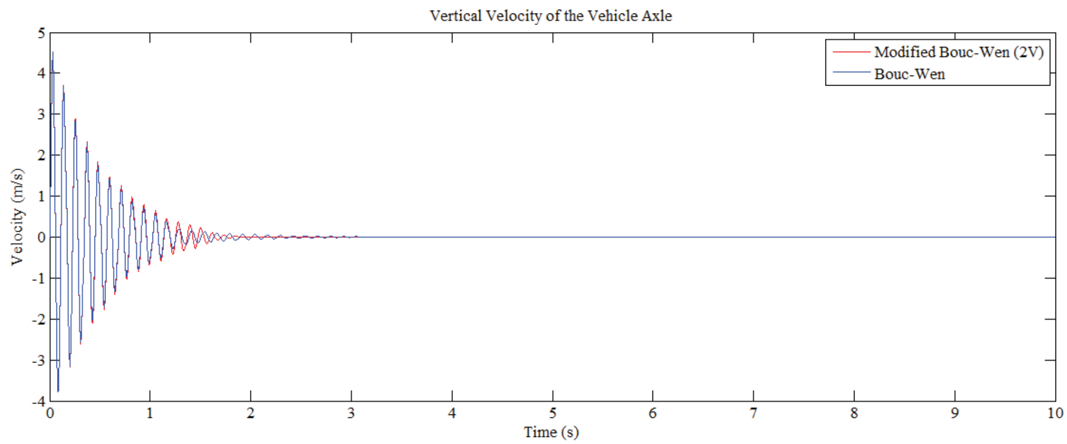

Figure 17. Vertical Velocity of the Vehicle Axle versus Time

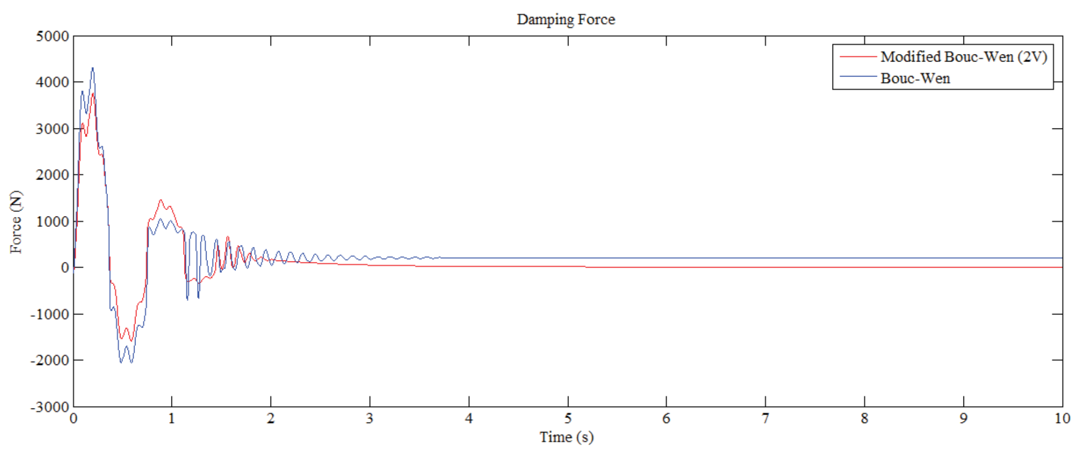

Figure 18. Damping Force of the Vehicle versus Time

Figure 18 indicates the damping force of the vehicle versus time. As can be observed from the Figure above, the damping force of the Modified Bouc-Wen model of 2 V stabilise at the point 0 N but for the damping force of the Bouc-Wen model, it stabilise at the point higher than 0 N.

CONCLUSION AND RECOMMENDATIONS

This study analyses the mathematical model of magnetorheological fluid damper in the semi-active suspension systemusing the two models, Bouc-Wen and Modified Bouc-Wen. The aim is to determine the time settlement of MRF damper in the semi-active suspension system. The system of equations of the model is analysed by using the ode 45 built-in function (fourth-order Runge-Kutta method) in the MATLAB program.

The performance of the MRF damper obtained by using the Modified Bouc-Wen model is better as compared to the Bouc-Wen model in term of time settlement. It can be seen that the time settlement needed for the suspension system back to its origin position for the Bouc-Wen model is almost the same as the voltage of 2 V for the Modified Bouc-Wen model. Although the Bouc-Wen model is well suited for the numerical simulation, however it cannot reproduce the experimentally observed roll-off in the yield region for velocities with a small absolute value and an operational sign opposite to the sign of the acceleration. As for the Modified Bouc-Wen model, its magnetic fields strength that depends on the flow of current through the coil of the MRF damper can be varied. For the comparison between the values of voltage of 0V, 1V and 2V for the Modified Bouc-Wen model, 2 V is found to be the best value as observed from the graphical output since the time settlement needed of the vehicle suspension system back to its original position is the shortest. In a nutshell, it can be summarised that the car system is able to attain its stability within short period of time.

For further research, it is suggested that the design of MR damper for heavy vehicle must include some experimental approach to get the desired parameters. The control theory can be applied to optimize the function of the vehicle suspension system. The study will then help the researchers/designers to identify the desired parameters to meet specific design requirements.

Besides that, the simulation of the semi-active suspension system must take into consideration of the full vehicle model. By simulate using the full vehicle model, vehicle handling and ride comfort performance can be investigated at the same time. The higher degree of freedom in vehicle model, more accurate results could be obtained.

In addition to that, the vehicle suspension system endures much abuse from pot-holes and speed-bumps due to the different road profiles such as sinusoid road profile and "sleeping policeman" road profile. Thus, future study can include the investigation of the different road profiles.

ACKNOWLEDGMENT

This work was supported by the GUP project Vote No: 11H90 , Ministry of Higher Education (MOHE) under Research Management Centre, UniversitiTeknologi Malaysia, Malaysia.

REFERENCES

1. T. Butz and O. Von Stryk, "Modelling and Simulation of Electro- and Magnetorheological Fluid Dampers". *ZAMM*, 82(1), 3 (2002).
2. Takeo.Fukumura, "Car Suspension System". *U.S. Patent* No. 4,746,106.Washington, DC: US. Patent and Trademark Office (1988).
3. F. Ikhouane, and J. Rodellar, Systems with Hysteresis: Analysis, Identification and Control using the Bouc-Wen Model. John Wiley & Sons (2007).
4. H. F. Lam, C. Y. Lai and W. H. Liao, *Automobile Suspension Systems with MR Fluid Dampers. Department of Mechanical and Automation Engineering*.The Chinese University of Hong Kong. Technical Report (2002).
5. K. S. Patil, V. Jagtap, S. Jadhav, A. Bhosale and B. Kedar, *Performance Evaluation of Active Suspension for Passenger Cars Using MATLAB*. In Second National Conference on Recent Developments in Mechanical Engineering. IOSR J Mech Civil Engg. pp. 06-14 (2013).
6. R. S. Sharp, and S.A. Hassan, *The Relative Performance Capabilities of Passive, Active and Semi-Active Car Suspension Systems.* Proceedings of the Institution of Mechanical Engineers, Part D: Journal of automobile engineering, **200(3)**, 219-228 (1986).
7. B. F. Spencer Jr, S. J. Dyke, M. K. Sain and J. D. Carlson, Modelling and Control of Magnetorheological Dampers for Seismic Response Reduction. *Smart materials and structures*, **5(5),** 565 (1996).
8. D. L. Trumper, W. J. Kim and M. E. Williams, Design and Analysis Framework for Linear Permanent-Magnet Machines.*IEEE Transactions onIndustry Applications,* **32(2)**, 371-379 (1996).