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Mathematical modelling and heuristic approaches to the location-routing problem of a cost-effective integrated solid waste management

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Abstract Integrated solid waste management (ISWM) comprises activities and processes to collect, transport, treat, recycle and dispose municipal solid wastes. This paper addresses the ISWM location-routing problem in which different types of municipal solid wastes are factored concurrently into an integrated system with all interrelated facilities. To support a cost-effective ISWM system, the number of locations of the system's components (i.e. transfer stations; recycling, treatment and disposal centres) and truck routing within the system's components need to be optimized. A mixed-integer linear programming (MILP) model is presented to minimise the total cost of the ISWM system including transportation costs and facility establishment costs. To tackle the non-deterministic polynomial-time hardness of the problem, a stepwise heuristic method is proposed within the frames of two meta-heuristic approaches: (i) variable neighbourhood search (VNS) and (ii) a hybrid VNS and simulated annealing algorithm (VNS + SA). A real-life case study from an existing ISWM system in Tehran, Iran is utilized to apply the proposed model and algorithms. Then the presented MILP model is implemented in CPLEX environment to evaluate the effectiveness of the proposed algorithms for multiple test problems in different scales. The results show that, while both proposed algorithms can effectively solve the problem within practical computing time, the proposed hybrid method efficiently has produced near-optimal solutions with gaps of <4%, compared to the exact results. In comparison with the current cost of the existing ISWM system in the study area, the presented MILP model and proposed heuristic methods effectively reduce the total costs by 20–22%.

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1 Introduction

Municipal solid waste (MSW) includes wastes generated from residential and commercial buildings, institutions, public parks, construction and demolition activities, and municipal services (Badran and El-Haggar 2006). While having a diverse range of types based on physical characteristics and chemical contents, in a broad sense, MSW can be divided into three main types: recyclable (e.g. paper), hazardous (e.g. household hazardous waste) and garbage (i.e. non-recyclable and residue) (Slack et al. 2005; Sharholy et al. 2008; Asefi et al. 2015a).

Recently, Solid Waste Management (SWM) imposes a great pressure on the local authorities where almost 20–50% of the available budget of the municipalities in developing countries is now spent on SWM (Sharholy et al. 2008; Lohri et al. 2014; Herva et al. 2014). To tackle the growing issue of SWM, the hierarchy of SWM has been developed since 1970s and has been formed in several versions in the basic order of reduction of waste amount, reusing, recycling or recovery through treatment technologies and finally disposal (Tan et al. 2014). As an extension of SWM hierarchies, Integrated solid waste management (ISWM) is suggested to define a holistic and systematic approach to solid waste management. ISWM is defined as a complete waste reduction, collection, composting, recycling, and disposal system where an efficient ISWM system considers how to reduce, reuse, recycle, dispose, and manage waste to protect human health and the natural environment (EPA 2002b).

ISWM applies a systematic approach to SWM by considering the problem on the whole as an interconnected system of component operations and functions, and by integrating MSW transportation, processing, recycling, resource and energy recovery and disposal technologies (McDougall et al. 2008). To develop an efficient ISWM system, the adoption of Operation Research (OR) techniques as well as mathematical modelling of the system and development of optimization models are necessary requirements as the first step on the road to the concept of ISWM (McDougall et al. 2008; Tan et al. 2014).

To design and/or support effective SWM systems, OR techniques have been widely utilized towards the development of economic-based optimization models for waste streams allocation and collection trucks routes (Chang and Chang 1998; Ghiani et al. 2014; Bing et al. 2016). In the context of SWM, mixed-integer linear programming (MILP) models have been considerably applied with respect to the fact that the frequent decision-making challenge involves finding the system component location(s) and effective routing plans for transportation of wastes among the system components (Tan et al. 2014; Ghiani et al. 2014). While locating the system's facilities and routing wastes among them are challenging issues in development of an effective ISWM system, location-routing problem (LRP) as a widely addressed problem framework in OR, can be effectively utilized in developing optimization models for ISWM. LRP addresses simultaneous optimization of facility location and design of an underling transportation network (Narula 1986; Sbihi and Eglese 2010; Dalfard et al. 2013; Lin et al. 2014). Development of optimization models to effective operation of SWM systems has received a growing attention over the last decades especially for those in developing countries (Chang and Chang 1998). However, the lack of consideration of (1) a complete chain of the ISWM components from generation nodes to landfills, (2) efficient categorisation of MSW types, (3) multiple waste processing technologies and (4) appropriate waste-technology constraints can be mentioned as frequent shortcomings of the majority of the proposed models in the first decade of year 2000, e.g. in the models presented in Badran and El-Haggar (2006), Eiselt (2007) and Dai et al. (2011).

Effective consideration of multiple types of MSW, multiple waste treatment technologies and the consequent waste-to-treatment compatibility constraint in modelling the SWM system has been addressed in a few studies (Bing et al. 2016). Erkut et al. (2008) proposed a MILP model in the frame of LRP to design an ISWM system for the region of Central Macedonia in Greece. While the model effectively integrated all the system's facilities and considered multiple types of MSW and waste-to-treatment compatibility, a single type of landfill was assumed to serve all types of wastes and generated residues from the processing sites. However, in real world scenarios, hazardous waste and residues (e.g. inertised ashes of incineration facilities) have to be sent to distinct disposal centres (Herva et al. 2014; Arena and Di Gregorio 2014). Later, Santibañez-Aguilar et al. (2013) presented a MILP model to optimize supply chain network for a SWM system in the central-west part of Mexico. However, the model cannot be assumed to present an efficient ISWM system due to some shortcomings such as ignorance of transfer stations and landfills in the system. Tan et al. (2014) proposed a MILP model for a utilisation system of MSW in Iskandar, Malaysia. They considered a set of alternative waste treatment technologies and formulated the model in order to maximise the urban profit in the system. However, while addressing multiple types of wastes, the model ignored consideration of waste-to-treatment technology constraints for different types of MSW. Yadav et al. (2016) utilized a MILP model to locate transfer stations for the city of Nashik, India. However, in addition to the formerly discussed frequent negligence of not considering the other system components integrated, considering simplifying assumptions such as single type of waste affect the efficiency of the model. Lack of holistic-based modelling approaches (e.g. lack of integration of all the system components and/or ignoring multiple types of MSW, multiple treatment technologies and waste-to-treatment technology constraints) can be also found in some other studies (Xi et al. 2010; Nga et al. 2013; Shirazi et al. 2016; Sharif et al. 2018).

Without loss of solidarity of the present study to the context of SWM, it should be noticed that more application of LRP can be found in the context of Hazardous/Industrial Waste Management (Alumur and Kara 2007; Samanlioglu 2013; Boyer et al. 2013; Yilmaz et al. 2017; Asgari et al. 2016; Rabbani et al. 2018). However, regarding the fact that they mostly address (i) a single type or multiple types of hazardous waste, (ii) a simplified network of waste flows consisting of generation, treatment and landfill nodes or (iii) single type of landfill and routing consideration specified to hazardous waste only, cannot be assumed as effective practices for developing the ISWM system. For instance, recently, Asgari et al. (2016) proposed a model and a memetic algorithm based on Tabu Search to optimize an obnoxious waste LRP. The model covered multiple types of hazardous wastes and the corresponding compatible treatment technologies. However, the considered three-echelon network (i.e. generation, treatment and disposal nodes) cannot be assumed to be practical for the ISWM system when transfer stations and recycling centres are not addressed and multiple types of MSW (e.g. recyclable and garbage residues) are not considered.

Some recent studies on optimization of the ISWM system have been applied to Tehran, the capital of Iran as a case study where population growth and increasing amount of MSW generation are growing urban management challenges. Harijani et al. (2017) proposed a MILP model to maximise the economic profit of the recycling and disposal network of the SWM in Tehran while minimising its environmental and social destructive impacts. While Harijani et al. (2017) integrated qualitative factors of social impacts of the system to the proposed mathematical by dedicating a distinctive constraint equation, Asefi and Lim (2017) integrated the social impact as a distinct objective function in their proposed MILP model to optimize the ISWM system of Tehran. In another recent studies, Habibi et al. (2017) presented a tri-objective optimization model for site selection and capacity allocation of the ISWM system for Tehran. The model considered waste-technology processing limitations. However, the MSW characterization (classification) was limited to only recyclable and non-recyclable wastes. In spite of significance of consideration of diverse categorisation of MSW types in effectiveness of ISWM optimization models (Asefi and Lim 2017), the poor categorisation of MSW into a limited range of only recyclable and non-recyclable wastes can be also seen in later studies such as the model presented in (Edalatpour et al. 2018) in which a linear optimization model is presented to optimize the ISWM of Tehran. As rare studies which consider complete chain of the ISWM components including treatment centres and distinct hazardous disposal plants, Asefi et al. (2015a, b), and Asefi et al. (2017) presented a conceptual model, MILP model and a simulated annealing (SA) optimization approach in the framework of LRP to minimise the economic cost of an ISWM system targeted in New South Wales, Australia. However, as a simplifying assumption, it was assumed in the studies that MSW are not segregated at source and sent to transfer stations in mixed forms. Table 1 provides the readers with a summary and analysis on recent relevant studies.

Overall, developing an effective ISWM system deals with the inherent complexity arrived from necessity of integration of the many interrelated processes to drive costs down and protect environment (Marshall and Farahbakhsh 2013). This complexity answers why Sbihi and Eglese (2010) said in their presented review: “none of the above mathematical formulations address all components of a complete solid waste management system” and Ghiani et al. (2014) said: “no model in the literature found to capture all different aspects to be considered”. That is, developing an effective ISWM system calls for integration of the system’s all facilities (i.e. transfer station, recycling-, treatment- and disposal centres) in a holistic tailored network which addresses flows of multiple types of MSW in an efficient chain from generation to disposal (Bing et al. 2016). As the analysis of literature in Table 1 demonstrates, the sustainable dominant challenge of the ISWM has been optimizing the economic objective while deterministic environment has been the dominant modelling environment in view of the researches in solid waste management. On the other hand, only a few studies have considered all major components of the ISWM system concurrently; and few have attempted to consider an efficient MSW types categorisation range. For instance, the lack of consideration of treatment centres in modelling the ISWM system is a frequent negligence in the studies while this component plays an important role in protecting environment for real-world scenarios of ISWM by performing significant tasks including rendering some sorts of waste for recovery/compost/disposal (Asefi and Lim 2017). More importantly, to the best of the authors’ knowledge, rare heuristic solution approach (Asefi et al. 2017) has been attempted for an ISWM LRP covering all the mentioned real-world constraints as the one proposed here. However, solving large-scale LRPs with the available computing facilities, when both location and routing problems are considered, is computationally intractable (Alumur and Kara 2007; Samanlioglu 2013).

This study addresses an ISWM LRP with a complete chain of the system’s components, all interrelated flows in the system, efficient MSW types diversification, waste processing technological diversification and the corresponding limitations to investigate a real-world integrated municipal waste management system. The principles of our addressed problem and the presented mathematical model are similar to those of Asefi et al. (2015a) and Asefi et al. (2017) where all the interrelated processing facilities of an ISWM system (i.e. transfer stations, recycling centres, treatment centres and distinct disposal centres for hazardous and

Table 1 Literature analysis

Study	Basic method	Case study	Model's basic assumptions (holism extent)												
			Waste processing technology[a]	Waste segregation at source	Waste-technology compatibility[b]	System Components				MSW types flows in the network					
						Transfer station	Recycling	Treatment	Disposal	Hazardous disposal	Single[c]	Recyclable	Non-recyclable	Garbage[c]	Hazardous[c]
Badran and El-Haggar (2006)	MILP	Port Said, Egypt	Single			✓	✓		✓			✓	✓	✓	✓
Eiselt (2007)	MILP	New Brunswick, Canada	Single			✓			✓		✓				
Erkut et al. (2008)	MILP	North Greece	Multiple			✓	✓	✓	✓		✓				
Xi et al. (2010)	MILP	Beijing, China	Single			✓	✓	✓	✓		✓				
Dai et al. (2011)	MILP	Beijing, China	Single			✓	✓	✓	✓		✓				
Santibañez-Aguilar et al. (2013)	MILP	West Central Mexico	Multiple				✓	✓	✓			✓	✓		
Nga et al. (2013)	MILP	Virtual example	Single					✓	✓			✓	✓		
Tan et al. (2014)	MILP	Iskandar, Malaysia	Multiple			✓	✓	✓	✓			✓	✓		

Table 1 continued

Study	Basic method	Case study	Model's basic assumptions (holism extent)												
			Waste processing technology[a]	Waste segregation at source	Waste-technology compatibility[b]	System Components				MSW types flows in the network					
						Transfer station	Recycling	Treatment	Disposal	Hazardous disposal	Single[c]	Recyclable	Non-recyclable	Garbage[c]	Hazardous[c]
Asefi et al. (2015a)	MILP	NSW, Australia	Multiple		✓	✓	✓	✓	✓	✓		✓		✓	✓
Yadav et al. (2016)	Nonlinear programming	Nashik, India	Single			✓	✓	✓	✓		✓				
Shirazi et al. (2016)	MILP	Tehran, Iran	Single			✓	✓	✓	✓			✓	✓		
Harijani et al. (2017)	MILP	Tehran, Iran	Multiple	✓	✓		✓	✓	✓			✓	✓		
Habibi et al. (2017)	MILP	Tehran, Iran	Multiple	✓		✓	✓		✓			✓	✓		
Asefi and Lim (2017)	MILP	Tehran, Iran	Multiple	✓	✓	✓	✓	✓	✓	✓		✓		✓	✓
Asefi et al. (2017)	MILP +Meta-heuristic (SA)	NSW, Australia	Multiple		✓	✓	✓	✓	✓	✓		✓		✓	✓
Edalatpour et al. (2018)	Linear programming	Tehran, Iran	Single			✓	✓		✓			✓	✓		

Table 1 continued

Study	Basic method	Case study	Model's basic assumptions (holism extent)												
			Waste processing technology^a	Waste segregation at source	Waste-technology compatibility^b	System Components				MSW types flows in the network					
						Transfer station	Recycling	Treatment	Disposal	Hazardous disposal	Single^c	Recyclable	Non-recyclable	Garbage^c	Hazardous^c
Sharif et al. (2018)	Bi-level programming	Tehran, Iran	Single	✓			✓	✓	✓			✓	✓	✓	
Present study	MILP +Hybrid heuristic (VNS + SA)	Tehran, Iran	Multiple	✓	✓	✓	✓	✓	✓	✓		✓		✓	✓

Study	Basic method	Case study	Model's objective(s)			Model's environment		Addressing NP-Hardness of the problem
			Economic	Environmental	Social	Deterministic	Nondeterministic	
Badran and El-Haggar (2006)	MILP	Port Said, Egypt	✓			✓		
Eiselt (2007)	MILP	New Brunswick, Canada	✓			✓		
Erkut et al. (2008)	MILP	North Greece	✓	✓		✓		
Xi et al. (2010)	MILP	Beijing, China	✓				✓	
Dai et al. (2011)	MILP	Beijing, China	✓				✓	

Table 1 continued

Study	Basic method	Case study	Model's objective(s)			Model's environment		Addressing NP-Hardness of the problem
			Economic	Environmental	Social	Deterministic	Nondeterministic	
Santibañez-Aguilar et al. (2013)	MILP	West Central Mexico	✓			✓		
Nga et al. (2013)	MILP	Virtual example	✓			✓		
Tan et al. (2014)	MILP	Iskandar, Malaysia	✓	✓		✓		
Asefi et al. (2015a)	MILP	NSW, Australia	✓			✓		
Yadav et al. (2016)	Nonlinear programming	Nashik, India	✓			✓		
Shirazi et al. (2016)	MILP	Tehran, Iran	✓			✓		
Harijani et al. (2017)	MILP	Tehran, Iran	✓	✓	✓	✓		
Habibi et al. (2017)	MILP	Tehran, Iran	✓	✓	✓		✓	
Asefi and Lim (2017)	MILP	Tehran, Iran	✓	✓	✓	✓		
Asefi et al. (2017)	MILP +Meta-heuristic (SA)	NSW, Australia	✓			✓		✓
Edalatpour et al. (2018)	Linear programming	Tehran, Iran	✓	✓			✓	
Sharif et al. (2018)	Bi-level programming	Tehran, Iran	✓			✓		
Present study	MILP +Hybrid heuristic (VNS+ SA)	Tehran, Iran	✓			✓		✓

[a] Consideration of availability of single/multiple waste processing technology(ies) for a waste processing plant in the model

[b] Consideration of waste-technology constraint for waste processing facilities with multiple technologies in the model

[c] Single: single type flow of MSW without categorisation; Garbage: neither recyclable nor hazardous; Hazardous: Household Hazardous Waste and Waste Electrical and Electronic Equipment Waste

non-hazardous residues) have been factored in the problem network. However, unlike the aforementioned studies, we present a model in this paper by assuming that different types of MSW are sorted separately at generation nodes, which is closer to what is happening in practice.

This paper aims to first formulate a MILP model to formulate the addressed ISWM problem, then, the key contribution of this research is to develop efficient heuristic solution approaches to obtain near-optimal solutions for the problem in large sizes where exact methods are not practical due to the NP-hardness of the problem (Samanlioglu 2013). A novel stepwise heuristic method is therefore developed within the frames of two meta-heuristic approaches: (i) variable neighbourhood search (VNS) and (ii) a hybrid VNS and simulated annealing algorithm (VNS + SA). The effectiveness of the proposed heuristics is compared and validated by examining the existing system of a real-world ISWM case in Tehran.

2 Problem description and formulation

In this section, the characteristics and assumptions of the addressed problem are elaborated with a schematic description. Next, the mathematical model of the addressed problem is presented.

2.1 Municipal solid waste LRP features

- *Multiple waste type*: different types of MSW as recyclable, hazardous and garbage have been concurrently taken as an input to the ISWM system while each has a distinct processing flow among the system's facilities.
- *Waste treatment technology compatibility*: different types of hazardous wastes require distinctly different treatment processes and technologies where a compatible treatment technology must be selected according to waste characteristics (Nema and Gupta 1999). In this paper, three different types of hazardous wastes and their distinct treatment technologies have been considered to represent a more realistic ISWM system. That is, two different treatment technologies Incineration and Physical & Chemical Treatment (PCT) are embedded in the system while hazardous type 1 ($w = 1$) is compatible with incineration technology ($q = 1$), hazardous type 2 ($w = 2$) is compatible with PCT treatment technology ($q = 2$) and hazardous type 3 ($w = 3$) is compatible with the two treatment technologies.
- *Recycling centres*: recycling facilities as an inseparable part of waste management systems have been included in the network of the problem in addition to routing wastes and residues to and from these facilities.
- *Transfer stations*: in practice, sorting and balling the collected wastes into the aforementioned categories (recyclables, garbage and different hazardous waste types) are conducted by some intermediate facilities which are often named transfer stations or screening centres. Intermediate transfer stations play a key role in economising the costs of the waste management systems. A transfer station is a processing site used for the temporary deposition of wastes by collection vehicles. Prior to being loaded into larger vehicles, the wastes are screened, sorted and balled into different sorts (EPA 2002a). Meanwhile, different transportation costs have been considered for small vehicles transporting the wastes from resource nodes to transfer stations, and large vehicles transporting the balled wastes to the other facilities. Generally, the unit cost of transportation is lower for large vehicles compared with smaller vehicles which collect the wastes and transport them to the transfer stations.

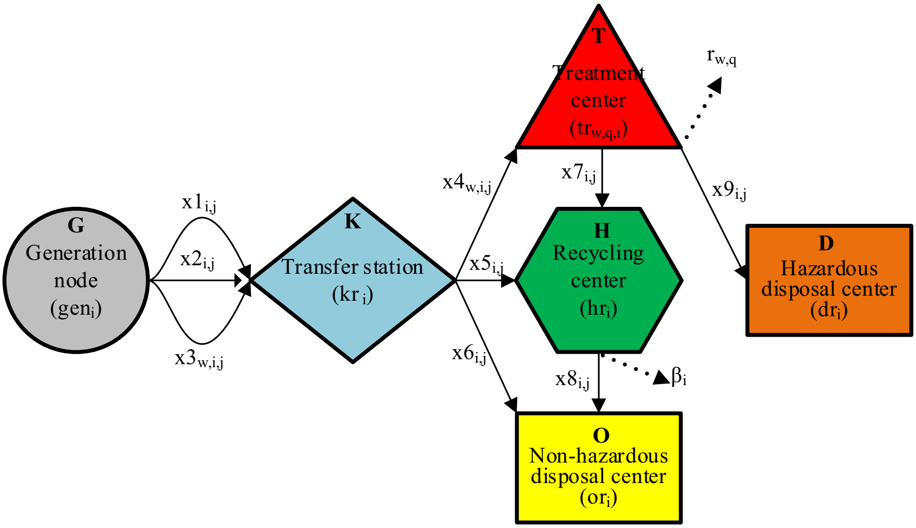


Fig. 1 The MSWM system network

- *Distinct disposal centres*: In a ISWM system as what is assumed here, different types of wastes are considered including hazardous and non-hazardous wastes. Therefore, different disposing processes and disposal centres must be considered in the model. In the real world situation, disposal centres for hazardous wastes are different from non-hazardous disposal centres because much more strict regulations and controls are applied to them (EPA 1996; Arena and Di Gregorio 2014).

2.2 ISWM system network

The addressed problem is schematically displayed in Fig. 1. The diagram starts with transporting garbage, recyclables and hazardous wastes from generation nodes to transfer stations $(x1_{i,j}, x2_{i,j}\text{and}x3_{w,i,j})$. After sorting and balling processes at transfer stations, large balls of recyclables, garbage and different hazardous waste types are shipped to their distinct destination facility by large-class vehicles. That is, recyclable wastes are transported to recycling centres ($x5_{i,j}$); different types of hazardous wastes are transferred to treatment centres with compatible technologies ($x4_{w,i,j}$) and garbage which is neither hazardous nor recyclable is sent directly to non-hazardous disposal centres ($x6_{i,j}$). At the treatment process, based on the type of hazardous wastes and the employed treatment technology, a portion of the waste mass is reduced ($r_{w,q}$) and a portion of the treatment output which is recyclable is transferred to recycling centres ($x7_{i,j}$). The rest which is not recyclable is sent to hazardous disposal centres ($x9_{i,j}$). After conducting recycling processes at recycling centres, the resultant recycled materials are transferred to the market or other manufacturing sectors to be reused (β_i) and the produced residues from recycling centres are sent to non-hazardous disposal centres ($x8_{i,j}$).

The addressed problem can be defined as concurrent facility selection (transfer stations; treatment, recycling, hazardous disposal and non-hazardous disposal centres) from the candidate locations and identification of the routes and amounts of shipment in the entire network to minimise the total cost of transportation and facility establishment.

2.3 Mathematical model

In order to formulate the addressed ISWM LRP a MILP model is proposed while the objective function involves minimising the total cost of transportation and the system's facility establishment in the whole network. The proposed mathematical model aims to simultaneously determine the system's facilities locations and flows of wastes/residues in the entire system. In the following, the problem is mathematically formulated firstly by defining the employed notations.

Sets:

$N = (V, A)$ transportation network of nodes V and arcs A
$G = \{1, \ldots, g\}$ set of waste generation nodes, $G \in V$
$K = \{1, \ldots, k\}$ set of potential transfer station nodes, $K \in V$
$T = \{1, \ldots, t\}$ set of potential treatment nodes, $T \in V$
$D = \{1, \ldots, d\}$ set of potential hazardous disposal nodes, $D \in V$
$O = \{1, \ldots, o\}$ set of potential non-hazardous disposal nodes, $O \in V$
$H = \{1, \ldots, h\}$ set of potential recycling nodes, $H \in V$
$W = \{1, \ldots, w\}$ set of hazardous waste types
$Q = \{1, \ldots, q\}$ set of treatment technologies

Parameters:

$c1_{ij}$ transportation cost per unit of garbage waste on link $(i, j) \in A, i \in G, j \in K$
$c2_{ij}$ transportation cost per unit of recyclable waste on link $(i, j) \in A, i \in G, j \in K$
$c3_{w,ij}$ transportation cost per unit of hazardous waste type $w \in W$ on link $(i, j) \in A, i \in G, j \in K$
c_{ij} transportation cost per unit of hazardous waste on link $(i, j) \in A, i \in K, j \in T$
cz_{ij} transportation cost per unit of hazardous waste residue on link $(i, j) \in A, i \in T, j \in D$
cv_{ij} transportation cost per unit of non-hazardous waste residue on link $(i, j) \in A, i \in H, j \in O$
cr_{ij} transportation cost per unit of recyclable waste on link $(i, j) \in A, i \in K, j \in H$
cl_{ij} transportation cost per unit of recyclable waste residue on link $(i, j) \in A, i \in T, j \in H$
cn_{ij} transportation cost per unit of garbage waste on link $(i, j) \in A, i \in K, j \in O$
fk_i fixed cost of opening a transfer station at node $i \in K$
$fc_{q,i}$ fixed cost of opening a treatment technology $q \in Q$ at node $i \in T$
fd_i fixed cost of opening a hazardous disposal centre at node $i \in D$
fo_i fixed cost of opening a non-hazardous disposal centre at node $i \in O$
fh_i fixed cost of opening a recycling centre at node $i \in H$
$g1_i$ amount of garbage waste generated at generation node $i \in G$
$g2_i$ amount of recyclable waste generated at generation node $i \in G$
$g3_{w,i}$ amount of hazardous waste type $w \in W$ generated at generation node $i \in G$
$r_{w,q}$ proportion of mass reduction of hazardous waste type $w \in W$ treated with technology $q \in Q$
$\alpha_{w,q}$ proportion of hazardous waste type $w \in W$ treated with technology $q \in Q$ to render for recycling
β_i proportion of total waste recycled at node $i \in H$
$tc_{q,i}$ capacity of treatment technology $q \in Q$ at node $i \in T$
rc_i capacity of recycling centre at node $i \in H$
dc_i capacity of hazardous disposal centre at node $i \in D$
oc_i capacity of non-hazardous disposal centre at node $i \in O$
sc_i capacity of transfer station at node $i \in K$

$tc^{m}_{q,i}$ the minimum amount of hazardous waste required to establish treatment technology $q \in Q$ at node $i \in T$

rc^{m}_{i} the minimum amount of recyclable waste required to establish a recycling centre at node $i \in H$

dc^{m}_{i} the minimum amount of hazardous waste residue required to establish a hazardous disposal centre at node $i \in D$

oc^{m}_{i} the minimum amount of garbage and non-hazardous waste residue required to establish a non-hazardous disposal centre at node $i \in O$

sc^{m}_{i} the minimum amount of waste required to establish a transfer station at node $i \in K$

$yn_{w,q}$ 1 if hazardous waste type $w \in W$ is compatible with technology $q \in Q$; or 0 otherwise

Decision variables:

$\boldsymbol{x}1_{i,j}$ amount of garbage waste transported through link $(i, j) \in A, i \in G, j \in K$

$\boldsymbol{x}2_{i,j}$ amount of recyclable waste transported through link $(i, j) \in A, i \in G, j \in K$

$\boldsymbol{x}3_{w,i,j}$ amount of hazardous waste type $w \in W$ transported through link $(i, j) \in A, i \in G, j \in K$

$\boldsymbol{x}4_{w,i,j}$ amount of hazardous waste type $w \in W$ transported through link $(i, j) \in A, i \in K, j \in T$

$\boldsymbol{x}5_{i,j}$ amount of recyclable waste transported through link $(i, j) \in A, i \in K, j \in H$

$\boldsymbol{x}6_{i,j}$ amount of garbage waste transported through link $(i, j) \in A, i \in K, j \in O$

$\boldsymbol{x}7_{i,j}$ amount of treated recyclable waste residue transported through link $(i, j) \in A, i \in T, j \in H$

$\boldsymbol{x}8_{i,j}$ amount of waste residue transported through link $(i, j) \in A, i \in H, j \in O$

$\boldsymbol{x}9_{i,j}$ amount of hazardous waste residue transported through link $(i, j) \in A, i \in T, j \in D$

$\boldsymbol{kr}_i$ amount of waste transferred at node $i \in K$

$\boldsymbol{tr}_{w,q,i}$ amount of hazardous waste type $w \in W$ treated at node $i \in T$ with technology $q \in Q$

$\boldsymbol{dr}_i$ amount of hazardous waste residue disposed at node $i \in D$

$\boldsymbol{or}_i$ amount of non-hazardous waste residue disposed at node $i \in O$

$\boldsymbol{hr}_i$ amount of waste recycled at node $i \in H$

$\boldsymbol{f}_{q,i}$ 1 if treatment technology $q \in Q$ is established at node $i \in T$; or 0 otherwise

$\boldsymbol{dz}_i$ 1 if hazardous disposal centre is established at node $i \in D$; or 0 otherwise

$\boldsymbol{oz}_i$ 1 if non-hazardous disposal centre is established at node $i \in O$; or 0 otherwise

$\boldsymbol{b}_i$ 1 if recycling centre is established at node $i \in H$; or 0 otherwise

$\boldsymbol{a}_i$ 1 if transfer station is established at node $i \in K$; or 0 otherwise.

The objective of the model is to minimise the total cost under the given constraints as follows:

$$\begin{aligned} \mathit{Minimizef}\,(x) = {} & \sum_{i\in G}\sum_{j\in K} c1_{i,j}\boldsymbol{x}1_{i,j} + \sum_{i\in G}\sum_{j\in K} c2_{i,j}\boldsymbol{x}2_{i,j} + \sum_{i\in G}\sum_{j\in K}\sum_{w\in W} c3_{i,j}\boldsymbol{x}3_{w,i,j} \\ & + \sum_{i\in K}\sum_{j\in T}\sum_{w\in W} c_{i,j}\boldsymbol{x}4_{w,i,j} + \sum_{i\in K}\sum_{j\in H} cr_{i,j}\boldsymbol{x}5_{i,j} + \sum_{i\in K}\sum_{j\in O} cn_{i,j}\boldsymbol{x}6_{i,j} + \sum_{i\in T}\sum_{j\in H} cl_{i,j}\boldsymbol{x}7_{i,j} \\ & + \sum_{i\in H}\sum_{j\in O} cv_{i,j}\boldsymbol{x}8_{i,j} + \sum_{i\in T}\sum_{j\in D} cz_{i,j}\boldsymbol{x}9_{i,j} \\ & + \sum_{i\in K} fk_i\boldsymbol{a}_i + \sum_{i\in T}\sum_{q\in Q} fc_{q,i}\boldsymbol{f}_{q,i} + \sum_{i\in H} fh_i\boldsymbol{b}_i + \sum_{i\in D} fd_i\boldsymbol{dz}_i + \sum_{i\in O} fo_i\boldsymbol{oz}_i \end{aligned} \tag{1}$$

$$\mathit{subject\ to}$$

$$g1_i = \sum_{j\in K}\boldsymbol{x}1_{i,j} \quad \forall i \in G \tag{2}$$

$$g2_i = \sum_{j \in K} \boldsymbol{x}2_{i,j} \quad \forall i \in G \tag{3}$$

$$g3_{w,i} = \sum_{j \in K} \boldsymbol{x}3_{w,i,j} \quad \forall w \in W, \quad \forall i \in G \tag{4}$$

$$\sum_{i \in G} \boldsymbol{x}1_{i,j} + \sum_{i \in G} \boldsymbol{x}2_{i,j} + \sum_{i \in G} \sum_{w \in W} \boldsymbol{x}3_{w,i,j} = \boldsymbol{kr}_j \quad \forall j \in K \tag{5}$$

$$\sum_{j \in G} \sum_{w \in W} \boldsymbol{x}3_{w,j,i} = \sum_{j \in T} \boldsymbol{x}4_{w,i,j} \quad \forall w \in W, \quad \forall i \in K \tag{6}$$

$$\sum_{j \in G} \boldsymbol{x}2_{j,i} = \sum_{j \in H} \boldsymbol{x}5_{i,j} \quad \forall i \in K \tag{7}$$

$$\sum_{j \in G} \boldsymbol{x}1_{j,i} = \sum_{j \in O} \boldsymbol{x}6_{i,j} \quad \forall i \in K \tag{8}$$

$$\sum_{i \in K} \boldsymbol{x}4_{w,i,j} = \sum_{q \in Q} \boldsymbol{tr}_{w,q,j} \quad \forall w \in W, \quad \forall j \in T \tag{9}$$

$$\sum_{w \in W} \sum_{q \in Q} \boldsymbol{tr}_{w,q,i} \left(1 - r_{w,q}\right) \left(1 - \alpha_{w,q}\right) = \sum_{j \in D} \boldsymbol{x}9_{i,j} \quad \forall i \in T \tag{10}$$

$$\sum_{w \in W} \sum_{q \in Q} \boldsymbol{tr}_{w,q,i} \left(1 - r_{w,q}\right) \alpha_{w,q} = \sum_{j \in H} \boldsymbol{x}7_{i,j} \quad \forall i \in T \tag{11}$$

$$\sum_{i \in T} \boldsymbol{x}7_{i,j} + \sum_{i \in K} \boldsymbol{x}5_{i,j} = \boldsymbol{hr}_j \quad \forall j \in H \tag{12}$$

$$\boldsymbol{hr}_i \left(1 - \beta_i\right) = \sum_{j \in O} \boldsymbol{x}8_{i,j} \quad \forall i \in H \tag{13}$$

$$\sum_{i \in T} \boldsymbol{x}9_{i,j} = \boldsymbol{dr}_j \quad \forall j \in D \tag{14}$$

$$\sum_{i \in H} \boldsymbol{x}8_{i,j} + \sum_{i \in K} \boldsymbol{x}6_{i,j} = \boldsymbol{or}_j \quad \forall j \in O \tag{15}$$

$$sc_i^m \boldsymbol{a}_i \le \boldsymbol{kr}_i \le sc_i \boldsymbol{a}_i \quad \forall i \in K \tag{16}$$

$$tc_{q,i}^m \boldsymbol{f}_{q,i} \le \sum_{w \in W} \boldsymbol{tr}_{w,q,i} \le tc_{q,i} \boldsymbol{f}_{q,i} \quad \forall q \in Q, \quad \forall i \in T \tag{17}$$

$$rc_i^m \boldsymbol{b}_i \le \boldsymbol{hr}_i \le rc_i \boldsymbol{b}_i \quad \forall i \in H \tag{18}$$

$$dc_i^m \boldsymbol{dz}_i \le \boldsymbol{dr}_i \le dc_i \boldsymbol{dz}_i \quad \forall i \in D \tag{19}$$

$$oc_i^m \boldsymbol{oz}_i \le \boldsymbol{or}_i \le oc_i \boldsymbol{oz}_i \quad \forall i \in O \tag{20}$$

$$\boldsymbol{tr}_{w,q,i} \le tc_{q,i}\, yn_{w,q} \quad \forall w \in W, \quad \forall q \in Q, \forall i \in T \tag{21}$$

$$\left(\boldsymbol{x}1_{i,j}, \boldsymbol{x}2_{i,j}, , \boldsymbol{x}3_{w,i,j}, \boldsymbol{x}4_{w,i,j}, \boldsymbol{x}5_{i,j}, \boldsymbol{x}6_{i,j}, \boldsymbol{x}7_{i,j}, \boldsymbol{x}8_{i,j}, \boldsymbol{x}9_{i,j}, \boldsymbol{kr}_i, \boldsymbol{tr}_{w,q,i}, \boldsymbol{hr}_i, \boldsymbol{or}_i, \boldsymbol{dr}_i\right) \in \left\{\mathbb{R}^{14}\right\}^+ \tag{22}$$

$$\left(\boldsymbol{f}_{q,i}, \boldsymbol{dz}_i, \boldsymbol{oz}_i, \boldsymbol{b}_i, \boldsymbol{a}_i\right) \in \{0, 1\}^5 \tag{23}$$

Equation (1) formulates the objective function of the model to minimise the total cost which is the sum of transportation costs to transport multiple waste types and waste residues among the facilities and the fixed costs to open transfer stations; treatment, recycling, hazardous disposal and non-hazardous disposal centres. The transportation cost per link is calculated by multiplying the unit transportation cost and the amount of shipped wastes. Equations (2), (3) and (4) are the flow balance constraints from generation nodes to transfer stations for garbage, recyclables and hazardous wastes, respectively. Equation (5) ensures that the total amount of the transported waste types to transfer stations undergo sorting and balling processes at these facilities. Equations (6)–(8) indicate the flows of hazardous, recyclable and garbage wastes from transfer stations to treatment, recycling and non-hazardous disposal centres, respectively. Equation (9) indicates that the total hazardous wastes entered into treatment centres have to be treated at these centres. Equations (10) and (11) formulate the flows from treatment centres to hazardous disposal centres and recycling centres, respectively, considering the ratios of recycling and mass reduction associated with different treatment technologies at treatment centres. Equation (12) shows the flows of recyclable residues generated at treatment centres and recyclable wastes sorted at transfer stations from these facilities to recycling centres. Equation (13) indicates the flow of generated residues at recycling centres from these facilities to non-hazardous disposal centres. Equation (14) provides the flow of hazardous waste residues from treatment centres to hazardous disposal centres ensuring that all the entered hazardous residues to these centres have to be disposed there. Equation (15) provides the flow garbage and non-hazardous residues from transfer stations and recycling centres, respectively, to non-hazardous disposal centres. This equation also ensures that the total amount of transported garbage and residues to non-hazardous disposal centres has to be disposed there. Equations (16)–(20) ensure that there must be minimum amounts of different waste types and residues entering the facilities to open the associated centres and provide the capacity limitations of transfer stations; treatment, recycling, hazardous disposal and non-hazardous disposal centres, respectively. Equation (21) indicates the compatibility constraint between the type of hazardous waste and treatment technology. Equations (22) and (23) are formulated to state non-negative and binary variables, respectively.

3 Methodology

The proposed methods to solve the addressed ISWM LRP employ variable neighbourhood search (VNS), simulated annealing (SA) and hybridisation of these two algorithms. The first suggested method utilizes the VNS algorithm in an adaptive heuristic framework and the second proposed approach applies the hybrid VNS + SA algorithm to solve the addressed problem. In the proposed hybrid algorithm, SA is employed within the framework of VNS to expand the space of feasible solutions and to avoid local optimum traps. In the proposed solution approaches, an initial solution is first generated using an adaptive stepwise heuristic method to start the algorithms. After generating the initial solution by subdividing the problem into the multiple phases (i.e. a stepwise approach), the resulted initial solution (S_0) is utilized as the current solution and the proposed algorithms begin to proceed by searching variable neighbourhoods to solve the integrated problem. The approach proposed to solve the addressed ISWM LRP is shown in Fig. 2. The principles of generating an initial solution and the proposed algorithms are elaborated in the next section.

3.1 Generation of an initial solution

The proposed algorithms (VNS and VNS + SA) need an efficient and quality initial solution to start their operations and to reach a near optimal solution. With respect to the complexity of the addressed problem and the importance of an efficient initial solution, a stepwise heuristic approach is developed to generate a quality initial solution. The main idea of the proposed stepwise heuristic is subdividing the complex network of the problem into some separate sub-problems (*phases*), optimizing each phase using the proposed meta-heuristics (VNS and VNS + SA) and then evaluating the fitness of the finalised solution. To do so, the addressed ISWM LRP is divided into seven sub-problems where each sub-problem consists of a location-routing problem for a pair of the facilities in the network (e.g. transfer stations and recycling centres). After fixing the results for a prior pair of the facilities and obtaining the outcomes including the selected facilities' locations, it will be used as the fixed input for the next pair of the facilities (next phase). The order of solving the phases is very challenging

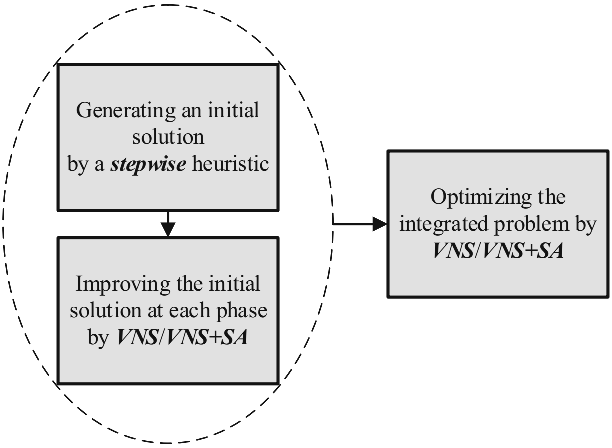


Fig. 2 The utilized approach to solve the considered municipal solid waste LRP

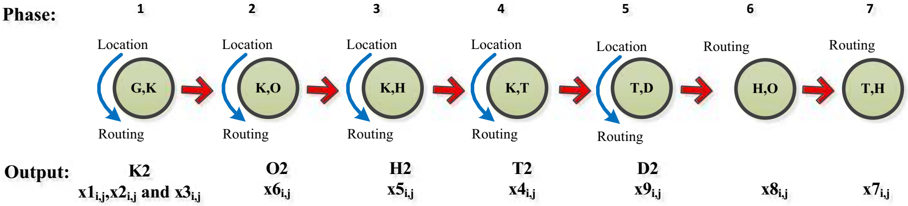


Fig. 3 Phasing the municipal solid waste LRP

because it can highly affect the final results. Here, we set the order of the phases of the problem based on their weight importance in the whole process and consider it as the *phase priority*. That is, a phase of the problem consisting of a pair of the facilities has the higher priority (earlier phase) than the other if the total amount of waste transportation between its facilities is more than the other's. The explained stepwise approach for generating an initial solution is schematically depicted in Fig. 3.

The result of each phase is improved by VNS and VNS + SA algorithms to obtain near optimum solutions separately. The final array resulting by arranging the phases' outcomes into the single structure, makes the final initial solution which then employed again in VNS and VNS + SA to optimize the whole problem in an integrated approach rather than the utilized stepwise method for generating the initial solution.

The considered routing strategy for every candidate solution is a challenging issue. Each candidate solution is generated and/or optimized by two stages: location and routing. The location stage results are improved by searching variable neighbourhoods in the framework of the proposed solution methods. In every step of the proposed algorithms, when a candidate solution consisting of multiple nodes of the facilities' locations is generated, the routes and amounts of all transportation between the nodes are calculated based on the same straightforward strategy of allocating the maximum possible amount of wastes (or residues) from the origin node to its nearest available destination node (to the feasible destination with the lowest cost of transportation). We also applied a *repair strategy* in cases where the problem constraints (e.g. the minimum amount of wastes for establishment and the facilities' capacities) are not addressed. That is, the overly allocated amount of wastes to a facility whose

Table 2 Maximum amount of waste transportation on each link of the network

Link $(f1, f2)$	Total\maximum possible amount of transportation $(U^{f1,f2})$
(G,K)	$U^{G,K} = \sum_{i\in G}\sum_{j\in K} x1_{i,j} + \sum_{i\in G}\sum_{j\in K} x2_{i,j} + \sum_{i\in G}\sum_{j\in K}\sum_{w\in W} x3_{w,i,j}$
(K,T)	$U^{K,T} = \sum_{i\in G}\sum_{j\in K}\sum_{w\in W} x3_{w,i,j}$
(K,H)	$U^{K,H} = \sum_{i\in G}\sum_{j\in K} x2_{i,j}$
(K,O)	$U^{K,O} = \sum_{i\in G}\sum_{j\in K} x1_{i,j}$
(T,D)	$U^{T,D} = \left[\sum_{i\in G}\sum_{j\in K}\sum_{w\in W} x3_{w,i,j}\right]\left[\left(1 - \min_{W\in w,q\in q} \propto_{w,q}\right) \times \left(1 - \min_{w\in W,q\in Q} r_{w,q}\right)\right]$
(T,H)	$U^{T,H} = \left[\sum_{i\in G}\sum_{j\in K}\sum_{w\in W} x3_{w,i,j}\right]\left[\left(1 - \min_{W\in w,q\in q} r_{w,q}\right) \times \left(\max_{w\in W,q\in Q} \alpha_{w,q}\right)\right]$
(H,O)	$U^{H,O} = \left[\sum_{i\in G}\sum_{j\in K} x2_{i,j}\right]\left(1 - \min_{i\in H} \beta_i\right)$

processing capacity is overloaded, is transferred to its nearest available facility of the same type. For the facilities, those allocated wastes are less than their minimum requirements for establishment; transported wastes between the farthest pair of nodes (the nodes with the highest cost of transportation) are selected, then deducted and re-allocated to the required nodes until their minimum requirements are met.

The procedure of the stepwise algorithm to generate an initial solution for the addressed problem is elaborated below.

(1) *Setting the order of phases*

The order of solving the phases of the problem is based on the maximum possible amount of waste transportation between each pair of the facility types. Let F denote the set of generation nodes and all the involved facility types in the network (See Fig. 1) as below; and, $f1$ and $f2$ represent each element (facility type) in F.

$$F = \{G, K, T, D, O, H\}$$

Considering the notations given in Sect. 2.3, the maximum amount of waste transportation on each link of the network is computed and presented in Table 2.

Considering $r_{w,q}$, $\alpha_{w,q}$ and $\beta_i < 1$ and with real-world assumptions as below:

$$\sum_{i\in G}\sum_{j\in K}\sum_{w\in W} x3_{w,i,j} \le \sum_{i\in G}\sum_{j\in K} x2_{i,j} \le \sum_{i\in G}\sum_{j\in K} x1_{i,j} \text{ and } \alpha_{w,q} \le r_{w,q} \le \beta_i$$

The volume order of waste transportation on each link of the network is resulted due to the following relationship:

$$U^{G,K} \ge U^{K,O} \ge U^{K,H} \ge U^{K,T} \ge U^{T,D} \ge U^{H,O} \ge U^{T,H}$$

where $U^{f1,f2}$ denotes the maximum possible amount of transportation between $f1$ and $f2 \in F$.

The resulted order of phasing the problem is depicted in Fig. 3 where the following symbols have been employed in addition to the previously defined notations.

Table 3 Maximum amount of input waste to the facilities

Facility (f)	Input	Total\maximum possible amount of input to the facility (λ^f)
K	G	$\lambda^K = \sum_{i\in G}\sum_{j\in K} x1_{i,j} + \sum_{i\in G}\sum_{j\in K} x2_{i,j} + \sum_{i\in G}\sum_{j\in K}\sum_{w\in W} x3_{w,i,j}$
T	K	$\lambda^T = \sum_{i\in G}\sum_{j\in K}\sum_{w\in W} x3_{w,i,j}$
H	K,T	$\lambda^H = \sum_{i\in G}\sum_{j\in K} x2_{i,j} + \lambda^T\left[\left(1 - \min_{W\in w, q\in q} r_{w,q}\right) \times \left(\max_{w\in W, q\in Q} \alpha_{w,q}\right)\right]$
O	K,H	$\lambda^O = \sum_{i\in G}\sum_{j\in K} x1_{i,j} + \left(\sum_{i\in G}\sum_{j\in K} x2_{i,j}\right)\left(1 - \min_{i\in H}\beta_i\right)$
D	T	$\lambda^D = \left(\sum_{i\in G}\sum_{j\in K}\sum_{w\in W} x3_{w,i,j}\right)\left[\left(1 - \min_{W\in w, q\in q} \propto_{w,q}\right) \times \left(1 - \min_{w\in W, q\in Q} r_{w,q}\right)\right]$

F' is the set of opened (selected) facilities per each type as:

$$F' = \{K2, T2, D2, O2, H2\}$$

$K2$ is a set of selected nodes for transfer stations
$T2$ is a set of selected nodes for treatment centres with technology q
$D2$ is a set of selected nodes for hazardous disposal centres
$O2$ is a set of selected nodes for non-hazardous disposal centres
$H2$ is a set of selected nodes for recycling centres

The order of phases in the proposed stepwise solution approach is set on the basis of the resulted maximum amount of waste transportation between each pair of the facility types ($U^{f1,f2}$) in which the higher $U^{f1,f2}$ has the earlier phase in the stepwise algorithm. The algorithm proceeds at each of the seven phases as a location-routing problem unless the location has been already fixed at the predecessor phases. That is, phases 1–5 include location-routing problems and phases 6 and 7 involve only routing problems. The results at each phase of the problem are mentioned in Fig. 3 as the phase output.

As the definite waste inputs to all the facilities are not known at the earlier phases, we use the maximum possible amounts of input wastes and residues to the facilities (λ^f) as the upper bounds for allocating the routes to and from the facilities. The maximum amounts of input wastes and residues to the facilities are computed and presented in Table 3.

(2) *Selecting a pair of facility types* ($f1$ *and* $f2$)

A pair of facility types, $f1$ and $f2$ are selected as the phase problem with respect to the obtained phase order. Consider $f1$ as the input and $f2$ as the output facility type of the phase.

(3) *Calculating the minimum possible number of the output facility* ($n^{f2}_{\min}$) *to establish*

The minimum number of output facility type at each phase (n^{f2}_{min}) is calculated which satisfies its input upper bound (λ^{f2}).

n^{f2}_{min} is equal to the number of positive elements in the following extension.

$$\sum_{i=0}^{n'_{f2}} \frac{\lambda^{f2} - \left(\sum_{j=0}^{i} cap^{f2}_{j}\right)}{cap^{f2}_{i+1}}; cap^{f2}_{0} = 0$$

where n'_{f2} is the number of candidate nodes for facility type $f2$ and cap_i^{f2} is the processing capacity of facility type $f2$ in candidate node i.

(4) *Setting the number of output facility type* $f2$ *for establishment*

The number of required facilities of $f2$ which is denoted by n^{f2} is set as below.

$$n^{f2} = n_{min}^{f2}$$

(5) *Selecting* n^{f2} *nodes of facility type* $f2$

n^{f2} of candidate locations of facility type $f2$ are selected from among the candidate locations using the *Roulette Wheel* method where the probability of selecting each candidate node (PR_i^{f2}) is calculated as below.

$$PR_i^{f2} = \frac{CO_i^{f2}}{\sum_{i=1}^{n'f2} CO_i^{f2}}$$

here CO_i^{f2} is the establishment cost of facility type $f2$ in candidate node i.

Accordingly, the set of opened centres for facility type $f2$ is determined and denoted by S^{f2} as below.

$$S^{f2} = \left\{ S_1^{f2}, \ldots, S_{n^{f2}}^{f2} \right\}$$

(6) *Checking the feasibility of* S^{f2}

The feasibility of the selected nodes (S^{f2}) is measured by checking if the total capacity of the selected nodes for facility type $f2$ can satisfy the input upper bound of the facility type (λ^{f2}).

$$if \sum_{i=1}^{n^{f2}} cap_i^{f2} \left(i \in S^{f2} \right) \geq \lambda^{f2} GotoStep7; OtherwiseGotoStep5.$$

(7) *Allocating routes from* $f1$ *to* $f2$

Considering the distance matrix of $d(i, j)$ between origin nodes in $f1$ ($i \in f1$) and destination nodes in $f2$ ($j \in S^{f2}$), amounts of waste transportation are allocated from $i \in f1$ to $j \in S^{f2}$ based on the policy of allocating maximum possible amount of waste from the origin node to the shortest available destination ($\min_{\substack{i \in f1 \\ j \in S^{f2}}} d(i, j)$). The facilities' constraints (i.e. processing capacity and minimum requirement for establishment) are checked and *repair strategy* is conducted as necessary.

(8) *Optimizing the solution of the phase*

The resulted array of selected locations for the facilities and transportation matrixes is considered as the current solution of the phase and proceeds to improve by the proposed VNS/VNS + SA algorithms. Then, the cost of obtained solution for the solution array consisting n^{f2} facilities of type $f2$ is calculated as follows:

$$\begin{aligned} Cost_{nf2} = & \sum_{i \in S^{f2}} CO_i^{f2} \\ & + \left[\sum_{i \in f1} \sum_{j \in f2} (amount\ of\ transportation\ on\ (i \in f1, j \in f2) \times d(i, j) \right. \\ & \times unit\ cost\ of\ transportation\ on\ (i \in f1, j \in f2))] \end{aligned}$$

(9) *Testing the higher number of opened centres for facility type* $f2$

As a better solution (lower total cost) can be achieved by opening facilities more than the minimum possible number (n_{min}^{f2}), higher numbers of opened facilities to establish for facility type $f2$ are also evaluated.

$$if\, n^{f2} = n'_{f2} Go\, to\, Step\, 10; otherwise\, n^{f2} = n^{f2} + 1\, and\, Go\, to\, Step\, 5.$$

(10) *Phases* 1–5 *output*

Among all the generated solutions from $= 1{:}n^{f2}$, the solution array which has the minimum cost is reported as the *phase solution* and its associated cost recorded as the *phase cost.*

$$phase\, cost = \min_{i \in n^{f2}} cost_i$$

$$if\, the\, locations\, for\, all\, the\, facility\, types\, in\, F' are\, fixed\, Go\, to\, Step\, 11;\ otherwise\, Go\, to\, Step\, 2.$$

(11) *Phases 6 and 7*

The routes and amounts of waste transportation are allocated from $f1 = H2$ to $f2 = O2$, and from $f1 = T2$ to $f2 = H2$ in phases 6 and 7 respectively. The routing and waste allocation are conducted based on the policy of allocating maximum waste from the origin nodes ($i \in f1$) to the nearest available destination node ($j \in f2$) considering the facilities' constraint and the *repair strategy*. The resulted transportation matrixes form $x8_{i,j}$ and $x7_{i,j}$ as the *phase solution* for phases 6 and 7 respectively. The phase cost for the two last phases are also recorded as below.

$$phase\, cost = \left[\sum_{i \in f1} \sum_{j \in f2} (amount\, of\, transportation\, on\ (i \in f1, j \in f2) \times d\,(i, j) \right.$$
$$\left. \times\, unit\, cost\, of\, transportation\, on\ (i \in f1, j \in f2)) \right]$$

(12) *The generated initial solution*

The final initial solution construction is shaped by forming an array consisting of the selected nodes (locations) through the phases 1–5 and the nine generated transportation matrices ($x1_{i,j}$, $x2_{i,j}$, $x3_{w,i,j}$, $x4_{w,i,j}$, $x5_{i,j}$, $x6_{i,j}$, $x7_{i,j}$, $x8_{i,j}$ and $x9_{i,j}$) through the seven phases. The final cost of initial solution is also calculated by summation of the phases' *phase cost.*

$$Initial\, solution\, cost = \sum_{i=1}^{7} phase\, cost_i$$

3.2 Solution construction and objective evaluation

Programming the proposed algorithms in a computer necessitates a method to display the construction of a solution. The displaying construction of a solution makes the problem to an understandable format for the computer. The displaying construction has to be comprehensive and efficient enough to represent all elements and features of the problem to perform any observation and further analysis. Formulating an efficient displaying construction for candidate solutions plays an important role in performance of the algorithms.

Regarding the complexity of the addressed problem, we formulate a representative solution with 10 components to cover all the solution elements. The first component is an array consisting of the selected nodes (from the candidate locations) for all the facility types. This location array itself consists of five sequential internal arrays to represent the selected nodes for $K2$, $O2$, $H2$, $T2$ and $D2$, respectively. The length of each internal array is equal to the number of selected nodes for each facility type (n^{f2}). The other nine components involve the nine transportation matrixes in the whole network ($x1_{i,j}$, $x2_{i,j}$, $x3_{w,i,j}$, $x4_{w,i,j}$, $x5_{i,j}$, $x6_{i,j}$, $x7_{i,j}$, $x8_{i,j}$ and $x9_{i,j}$).

The displaying construction of a representative solution is exemplified in Fig. 4 where it is assumed that the numbers of selected locations for each type of the facilities (n^{f2}) are 5,3,4,1 and 1 for transfer stations, non-hazardous disposal, recycling, treatment and hazardous disposal centres respectively. Also, seven locations are considered as the generation nodes. The set of generation nodes and the selected nodes for the facilities in the given example are assumed as below.

$$\begin{aligned} G &= \{1, 2, 3, 4, 5, 6, 7\} \\ K2 &= \{2, 3, 5, 7, 9\} \\ O2 &= \{1, 5, 8\} \\ H2 &= \{3, 4, 5, 9\} \\ T2 &= \{3\} \\ D2 &= \{3\} \end{aligned}$$

It is also noticeable that the internal array for treatment centres ($T2$) repeats in the solution array as the number of available treatment technologies. What is more, $x3_{w,i,j}$ and $x4_{w,i,j}$ are considered as multi-dimension matrices in the same dimension as the number of hazardous waste types.

The objective function of a candidate solution is measured by calculating the total cost of the solution including cost of establishment of the selected nodes (facilities) and transportation cost in the network (Eq. 1).

3.3 Neighbouring structures

In the proposed algorithms (VNS and VNS + SA), six neighbouring structures are applied ($l = 1, \ldots, 6$). The utilized neighbouring structures are selected from usually applied structured in location-routing problems. The six neighbouring structures involve the *perturbation* operator where a location position in the solution array is relocated by a new candidate node. The first five neighbouring structures are performed by single *perturbation* (relocation) in a position of $K2$, $O2$, $H2$, $T2$ and $D2$ respectively. That is, a single location node in the array of one of the facility types (e.g. $H2$) is randomly selected, removed from the solution array and relocated by a new location node from the facility's candidates set (e.g. H). To select a new node from the candidate set for relocation, the *Roulette Wheel* method is utilized where the probability of selecting a new node is calculated by dividing its establishment cost by the total cost of establishment of all the candidate nodes. While the first five neighbouring structures improve the *intensification* of the algorithms, the last ($l = 6$) neighboring structure is designed in a way to improve the *diversification* of the algorithm and enhance its performance for escaping from local optimum traps. In this neighbouring structure, a single location node per each of the facility types ($K2$, $O2$, $H2$, $T2$ and $D2$) is selected randomly which results totally in five positions to be removed from the solution array. Then, five new location nodes are selected from the available candidate sets (K, O, H, T and D) via *Roulette Wheel* to be concurrently inserted in the removed positions.

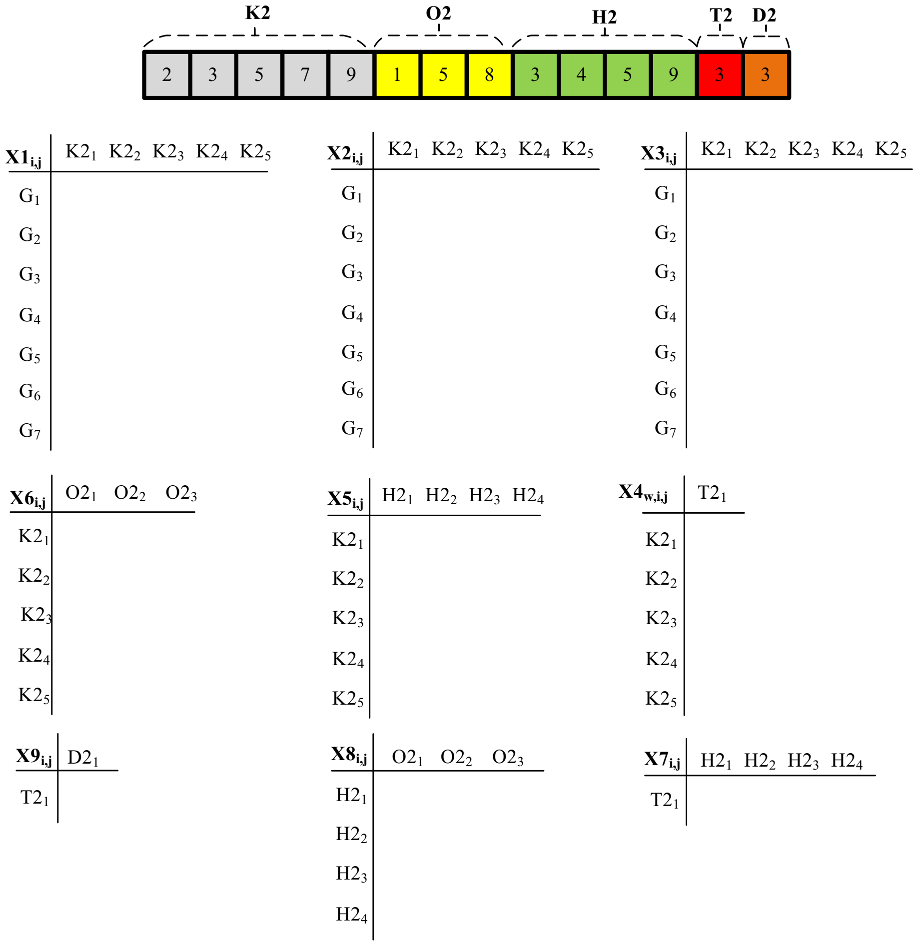


Fig. 4 A sample for a candidate solution and its layers

The six applied neighbouring structures are schematically illustrated in Fig. 5 through the following example. It is also noticeable that for every neighbour solution, the routing part of the solution (nine transportation matrices) is then calculated under the policy of allocating the maximum possible amount of wastes from each resource node to its destination with the least cost of transportation.

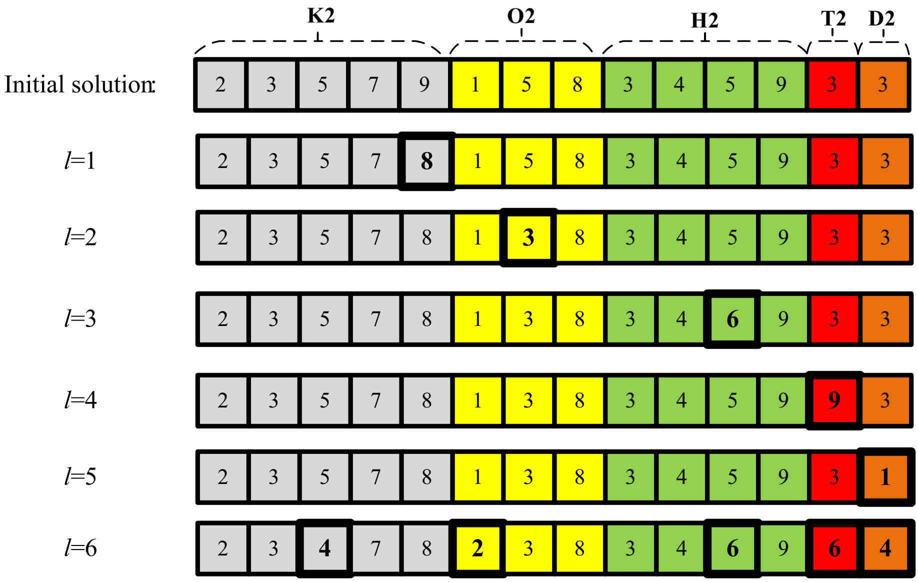
Let$G = K = O = H = T = D = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and the generated initial solution as below.

$$n^{K2} = 5, n^{O2} = 3, n^{H2} = 4, n^{T2} = 1 \text{and}\, n^{D2} = 1;$$

$$K2 = \{2, 3, 5, 7, 9\}, O2 = \{1, 5, 8\}, H2 = \{3, 4, 5, 9\}, T2 = \{3\} \,\text{and} D2 = \{3\}.$$

Suppose the positions 5, 2, 3, 1, 1 and (3, 1, 3, 1, 1) as randomly selected positions for removing and the following sets as new nodes for relocating in the six neighbouring structures respectively.

$$\{8\}, \{3\}, \{6\}, \{9\}, \{1\}, \{4, 2, 6, 6, 4\}$$



	K2					O2			H2				T2	D2
Initial solution:	2	3	5	7	9	1	5	8	3	4	5	9	3	3
l=1	2	3	5	7	**8**	1	5	8	3	4	5	9	3	3
l=2	2	3	5	7	8	1	**3**	8	3	4	5	9	3	3
l=3	2	3	5	7	8	1	3	8	3	4	**6**	9	3	3
l=4	2	3	5	7	8	1	3	8	3	4	5	9	**9**	3
l=5	2	3	5	7	8	1	3	8	3	4	5	9	3	**1**
l=6	2	3	**4**	7	8	**2**	3	8	3	4	**6**	9	**6**	**4**

Fig. 5 The considered six neighbouring structures

The six neighbouring structures are formed for the given example as illustrated in Fig. 5.

3.4 Variable neighbourhood search (VNS)

The variable neighbourhood search (VNS) algorithm was firstly introduced by Mladenović and Hansen in 1997 (Mladenović and Hansen 1997). The main idea of this algorithm is systematically changing in neighbouring structures during the search to avoid being trapped into local optimums. VNS is known as an effective heuristic method for solving different combinatorial optimization problems and has been successfully applied to many location routing problems because of its straightforwardness in implementation in addition to the quality of the obtained results (Cooper 1964; Hansen et al. 2010; Jarboui et al. 2013).

VNS initiates its procedure by generating an initial solution, identifying neighbouring structures and developing a method to search neighbour solutions. The neighbouring structures of the algorithm are defined as $N_l, l = \{1, 2, \ldots, l_{max}\}$ where N_l is the *lth* neighboring structure. Generating high quality initial solution, efficiently identifying neighbouring structures and their performing order and developing an efficient method for performing the local search are key factors of the quality of obtained solutions by VNS. The algorithm begins its procedure using a generated initial solution (S_0) and the main loop of the algorithm repeats until it meets the defined stopping criterion. The main loop of the algorithm consists of two major phases: shaking and local search.

In shaking phase, the algorithm moves from the current solution to a neighbour solution (S') using the *lth* neighboring structure (i.e. the facility type for continuing the algorithm procedure is selected). In the local phase search, the algorithm conducts searching on S' using a local search method to find the local optimum ($S^{'*}$). Then, in order to make a movement decision, the algorithm evaluates the objective value of the obtained solution. If the obtained local optimum ($S^{'*}$) is better than the current solution (S), then, $S^{'*}$ changes with S and searching operation backs to N_1; otherwise, the next neighboring structure (N_{l+1}) is utilized

```
Input: $a\ set\ of\ neighbourhood\ structures\ N_l, l = 1,2,\ldots,l_{max}$
$S = Initial\ solution\ ()$;
Repeat
        $l = 1$;
        While $(l \le l_{max})$
        {
            $S' = Shaking\ (S, N_l)$
            $S'^* = local\ search\ (S')$
            if $f(S'^*) < f(S)$
                $S \leftarrow S'^*$
                $l = 1$ ;
            else
                $l = l + 1$;
        }
Until $Stopping\ criterion\ is\ met$;
Report: $The\ obtained\ solution\ with\ the\ lowest\ f(S)$
```

Fig. 6 The pseudo code of the proposed VNS

for continuing searching operation. The pseudo code of the employed VNS algorithm is presented in Fig. 6 (Mladenović and Hansen 1997).

3.5 Simulated annealing (SA) algorithm

Simulated annealing (SA) is another heuristic method hired in this paper that is one of the well-known meta-heuristic algorithms based on local search. SA is able to avoid local optimums by allocating a low probability to accept less efficient solutions and allow them to proceed through the algorithm. The quality of the obtained results by SA resulted in utilising it in various complex NP-hard problems including LRP (Lin and Kwok 2006; Vincent et al. 2010; Mousavi and Tavakkoli-Moghaddam 2013). SA became popular because of the work by Kirkpatrick (1984). This algorithm is originally inspired from atoms movement in a metal during its annealing. The starting temperature (T_0), final temperature (T_f) and cooling function are key factors in SA which highly affect the final results. In this paper, we set the starting and final temperatures using some primary experiments and consider a geometric cooling function which updates the temperature at each iteration by the equation of $\mathrm{T}_i = \alpha \times \mathrm{T}_{i-1}$ where α represents a positive constant number less than one named *cooling factor*.

The optimization process in SA is a search to find the optimum (or near-optimum) solution(s). The algorithm begins its movement from a random solution (initial solution) and sets the system's temperature as the starting temperature ($\mathrm{T} = \mathrm{T}_0$). At each iteration, a neighbor solution is generated for the current solution and its objective value is compared with the objective value of the current solution. If the neighbour solution results in better fitness, it is changed with the current solution; otherwise, it may still have a chance to change with the current solution with a probability equal to $\exp(-\Delta/KT)$ which is calculated using the *Boltzman* function. The explained mechanism results in enabling the algorithm to stay away from the traps of local optimums. In the above-mentioned equation, Δ is the difference in the objective function between the current and neighbour solutions, K is the Boltzman constant and T is the current temperature. The process of searching neighbor solutions continues until a predefined number of iterations (Max_{It}) is met at each temperature. The algorithm itself continues until it meets the stopping criterion (T_f).

3.6 The proposed hybrid algorithm (VNS + SA)

After describing the principals, the main proposed algorithm for solving the addressed problem is explained in this section. Hybridisation of VNS and SA in different adaptive frameworks has been effectively applied to many combinatorial optimization problems (Blazewicz et al. 2005; Behnamian et al. 2009; Hosny and Mumford 2010; Abbasi et al. 2011; Brito et al. 2012). Here, we employed the SA algorithm within the framework of VNS to improve the algorithm performance by expanding the space of feasible solutions and to avoid the traps of local optimums. Neighbouring structures are developed to provide a possibility for relocation in any type of the facilities. After determining the intended facility type via a shaking phase, local search is conducted by SA.

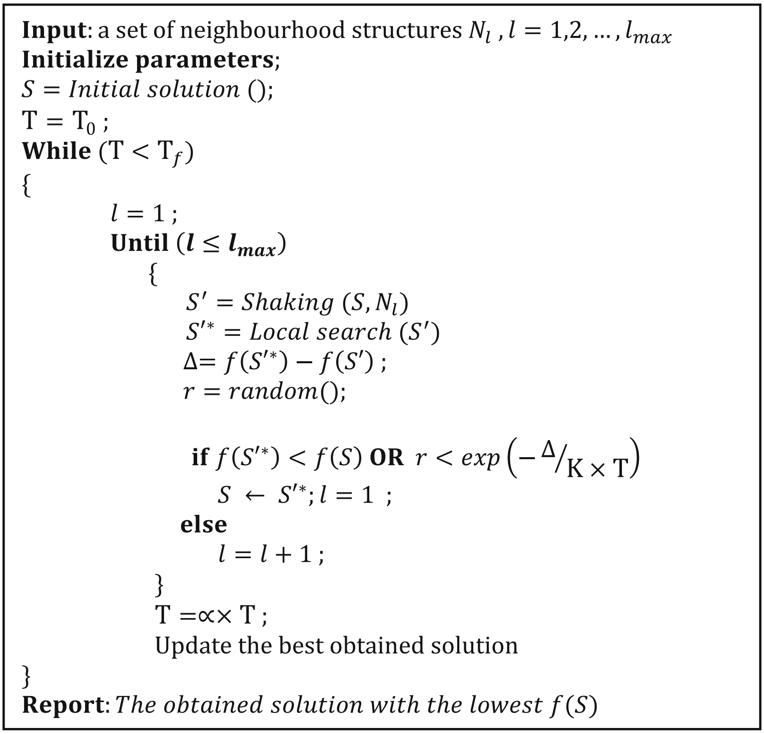
In the beginning, the hybrid algorithm initiates its procedure using an initial solution (S_0) which is generated by the adaptive stepwise heuristic method. The initial solution is set as the current solution of the system ($S = S_0$). To solve the problem, the explained six neighbouring structures are utilized. Initiating the hybrid algorithm is done by firstly neighbouring the structure ($l = 1$). At the shaking phase, the algorithm moves from the current solution (S) to a neighbor solution (S') using the *lth* neighboring structure. Then, at the neighbour search phase, a neighbouring search is conducted on the neighbour solution (S') to find the local optimum one ($S^{'*}$). The neighbor searching operation in the proposed hybrid algorithm is performed by SA using the *lth* neighbor structure as the moving operator to the neighbour position. The length of the Markov chain for local searching with a distinct neighbour structure is considered equal to N sequential repeats without any improvement in the objective function. To make the decision on moving or not, if the objective value for $S^{'*}$ is better than S or a generated random number is less than $\exp(-\Delta/K \times T)$, then: $S = S^{'*}$and $l = 1$; otherwise, the next neighbouring structure is utilized ($l = l + 1$). The loop of the VNS algorithm continues at each temperature until $l \leq l_{max}$ where $l_{max} = 6$ and the temperature is updated by the geometric cooling function ($T = \alpha \times T$). The proposed algorithm continues until it reaches to the final temperature (T_f). The pseudo code of the proposed hybrid algorithm is presented in Fig. 7.

4 Computational results and discussion

In order to validate the proposed algorithms and compare them, a number of test problems are generated and employed for implementation of the proposed algorithms. The results by mixed-integer programming were compared against the solutions obtained by the proposed algorithms for different sizes of problems. The applied approaches for tuning the algorithms' parameters, generating test problems and analysis of experimental results are elaborated in this section.

4.1 Parameter tuning

Efficient tuning of the parameters of the proposed algorithms plays a significant role in the quality of their final results. Therefore, an experimental method is applied to set the parameters at each level to achieve a better performance. Some suggestive values are first considered per parameter and then the value of each parameter (in the order as shown in Table 4) is fixed



```
Input: a set of neighbourhood structures $N_l$ , $l = 1,2, \dots, l_{max}$
Initialize parameters;
$S = Initial\ solution$ ();
$T = T_0$ ;
While ($T < T_f$)
{
        $l = 1$ ;
        Until ($l \leq l_{max}$)
            {
                $S' = Shaking\ (S, N_l)$
                $S'^{*} = Local\ search\ (S')$
                $\Delta = f(S'^{*}) - f(S')$ ;
                $r = random()$;

                if $f(S'^{*}) < f(S)$ OR $r < exp\left(-\Delta / K \times T\right)$
                    $S \leftarrow S'^{*}; l = 1$ ;
                else
                    $l = l + 1$ ;
            }
            $T = \propto \times T$ ;
            Update the best obtained solution
}
Report: The obtained solution with the lowest $f(S)$
```

Fig. 7 Pseudo code of the proposed hybrid algorithm

Table 4 Parameters of the algorithms and their primary suggestive levels

Parameter	Suggestive values
N	$0.5L\,(L-1)\,,L\,(L-1)\,,2L\,(L-1)\,,3L\,(L-1)$
T_0	10, 100, 400, 1000
α	0.9, 0.93, 0.95, 0.97, 0.99
T_f	0.001, 0.01,0.1,1
K	0.1, 0.2, 0.3, 0.5

by keeping the other parameters in their lowest levels with respect to the resulted fitness and running time. After conducting the mentioned experiments on a sample problem, the values of the parameters are fixed as $T_0 = 1000$, $T_f = 0.001$, $\alpha = 0.97$and $K = 0.3$. The number of sequential repeats without any improvement in the objective function (N) is also considered as $L(L-1)$ where L is equal to the length of the array displaying the solution construction.

4.2 Generation of test data: ISWM system in Tehran, Iran

To verify accuracy and performance of the proposed model and solution approach, a series of test problems are designed and solved in different sizes by considering different numbers of candidate location nodes for facility types and generation nodes in the network. To do

Table 5 Generated test problems

Size	Problem ID	Number of generation nodes	Number of candidate nodes for:					
			Transfer stations	Recycling centres	Treatment centre (q = 1)	Treatment centre (q = 2)	Non-hazardous disposal centres	Hazardous disposal centres
Small	S01	14	7	7	6	6	6	6
	S02	16	8	8	7	7	6	6
	S03	18	10	9	8	8	7	7
	S04	22	11	3	1	10	1	1
Medium	M01	23	16	15	13	13	12	12
	M02	24	19	19	16	16	15	15
	M03	25	21	21	19	19	18	18
	M04	26	23	23	22	22	22	22
Large	L01	28	24	24	24	24	24	24
	L02	30	28	28	28	28	27	28
	L03	35	35	35	33	33	33	33
	L04	45	45	45	40	40	40	40

this, data are extracted from a recent real-life case study (Shirazi et al. 2016) and further updated by interviewing the organisation members of Tehran Waste Management Organisation (TWMO). The generated test problems are grouped into three classes: Small, Medium and Large. For each class, four different problems are generated. The generated test problems and numbers of generation nodes and candidate location nodes for each facility type are presented in Table 5.

Tehran metropolitan area is the capital of Iran with a population of more than 8.2 million and is located on a 730 km^2 land. The amount of MSW generated in Tehran is 6629 ton/day (Shirazi et al. 2016). Almost 97% of MSW consists of municipal recyclables and residues and other types of generated waste comprise 1% for hospital waste, 0.6% industrial waste and 0.5% for construction and demolition waste. Approximately 87% of generated MSW of Tehran is disposed in a large 500 ha landfill site located in southern part of the city (Aradkuh Centre), 8.3% is composted and the remaining ($<5\%$) is recovered (Damghani et al. 2008; Abduli et al. 2011; Shirazi et al. 2016).

Tehran is divided into 22 Municipality Districts (MDs) named MD1–MD22 as shown in Fig. 8. The amount of daily MSW generated in each MD varies from 100 to 703 tons as details are reported in (Shirazi et al. 2016). The Municipality of Tehran is responsible for collection, operation and management of MSW in Tehran. MD trucks collect MSW within their district and transport it to 11 transfer stations located in the city. Then, larger vehicles (semi-trailers) transport waste to a centralised waste processing and landfill facility named Aradkuh Centre. The waste transported to Aradkuh Centre, undergo segregation and pre-treatment processes at 10 processing units located in this centre. As the processing units play the role of pre-treatment to render the waste for recovery/compost/dispose, we consider these facilities as the PCT technology in the proposed model. In addition, three recycling facilities, two composting sites and a Material Recovery Facility (MRF), are also centralised in Aradkuh Centre to perform recycling on a fraction (almost 13%) of recyclable waste segregated at 10

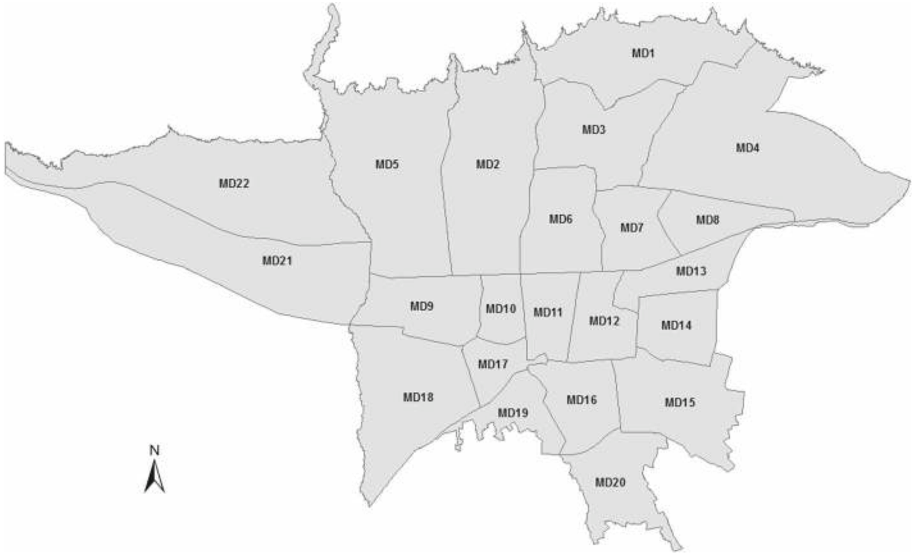


Fig. 8 Tehran and its 22 MDs

processing units (Damghani et al. 2008; Abduli et al. 2011; Shirazi et al. 2016). Recently, a 200 ton/day incineration plant has been established at Aradkuh Centre with Combined Pyrolysis and Gasification (CPG) technology which is employed to treat a fraction (almost 3%) of hazardous wastes and residues and recover electricity. The landfill site with a capacity of 6000 tons/day located in the centre is used to dispose residues of the recycling and treatment sites and segregated residues of 10 processing sites. Moreover, the landfill has a specified cell for disposing hazardous wastes, which is equipped with multiple geosynthetic layers and a particular drainage system for controlling leachates and sanitary disposing hazardous wastes (TWMO 2017). Table 6 presents the list of currently working facilities of ISWM in Tehran and their corresponding data utilized for experimental calculations.

While the actual divisions of the study area in 22 MDs (as shown in Fig. 8) are utilized to generate the test problems in Small class, for the rest of test problems (Medium and Large) which have more than 22 generation nodes and candidate locations, the map of study area is reclassified to equal area polygons in the number of generation nodes by using ArcMap 10.2. To measure the transportation cost, a code was developed to utilize Google Map API to calculate actual distances derived from centroids of districts polygons using ArcMap 10.2. and it is assumed that the average consumption of fuel for a truck is 0.3 litre per km, the cost of fuel is \$0.3 per litre, the unit cost of transportation of wastes from generation sources to transfer stations is 25% higher than that of routes after transfer stations because of lower capacities of transportation vehicles in the first echelon, and to include other related costs of transportation such as driver wage, truck depreciation and insurance, a constant factor two is multiplied by the unit cost of transportation (Boyer et al. 2013; Asefi et al. 2015a). The adjusted values for minimum waste requirement to establish the system components are assumed similar to those in (Asefi et al. 2015a) and fixed costs of the system's components are also extracted from (Abduli et al. 2011; Shirazi et al. 2016; TWMO 2017) (Table 7). Figure 9 illustrates the schematic view of the ISWM system in Tehran within the frame of the proposed model.

Table 6 The currently working components of ISWM in Tehran. Adopted from Shirazi et al. (2016) and TWMO (2017)

Facility: Transfer station		Facility: Processing site (PCT centre)			
Name/ID	Capacity (ton/day)	Name/ID	Capacity (ton/day)	Name/ID	Capacity (ton/day)
				Facility: Recycling centre	
Darabad	1299	M1–M2	1000	Compost site #1	2200
Zanjan	1037	M3	500	Compost site #2	1800
Banihashem	485	S1–S2	1000	MRF plant	50,000
Hakimiyeh	485	S3	500	*Facility: Incineration treatment centre*	
Kuhak	2227	S4–S5	1000	CPG plant	200
Beyhaghi	864	S6–S7	1000	*Facility: Non-hazardous Landfill*	
Harandi	691	S9	1000	MSW landfill site	60,000
Azadegan	1484	S10	1000	*Facility: Hazardous Landfill*	
Yaran	1484	Cargo	300	Sanitary cell	1600
Jehad	1484	Plant	500	–	–
Shahid Avini	323	–	–	–	–

Table 7 Fixed and cost constraint parameters of the system's components

Facility\parameter	Establishment cost ($M)	Minimum required to establish (ton)
Transfer station	7.15E+00	250
Treatment centre (Physical and Chemical)	1.00E+01	50
Treatment centre (Incineration)	1.25E+01	50
Recycling centre	3.54E+01	250
Hazardous disposal centre	5.50E+00	100
Non-hazardous disposal centre	9.17E+00	500

4.3 Model and the algorithms evaluation

In order to evaluate the performance of suggested algorithms and accuracy of the proposed model, the algorithms were implemented for the 12 generated test problems in MATLAB 8.3.0.532 (R2014a), and the mathematical model is solved for each of them using CPLEX on a machine with Core i7 2.40 GHz processor, 8 GB RAM memory with 18,000 s (5 h) limited solving time.

The obtained results by the proposed algorithms and mixed-integer programming are compared in terms of the quality of the resulted solutions (fitness) and running time. Each

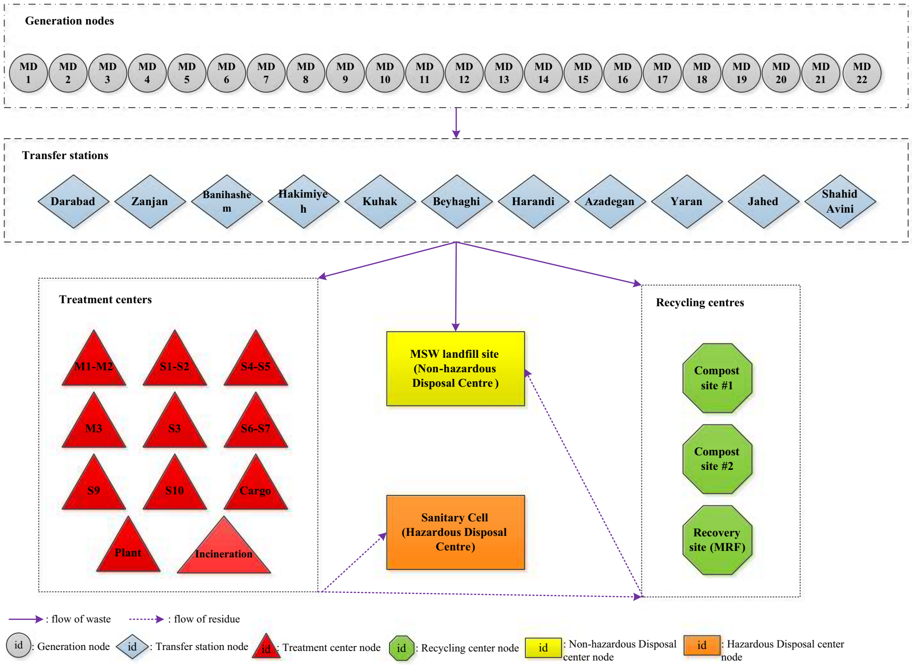


Fig. 9 The ISWM in Tehran

algorithm is run 10 times for each test problem and the best obtained solutions are recorded in Table 8 together with the elapsed computing time. To calculate the optimality gap, regardless of computing time, this formula is used:

$$GAP = \frac{sol' - sol}{sol} \times 100$$

where sol' is the cost of heuristic solution and sol denotes the obtained optimum solution by CPLEX within the set limited time.

According to the obtained results, an exact solution can be achieved only for the two smallest test problems when the computing time is limited to 5 h. However, the proposed hybrid algorithm obtains near-optimal solutions for these two test problems within very short time. For the rest of the test problems, the proposed algorithms resulted in almost 2–5% GAPs compared against mixed-integer programming solutions which indicate the efficiency of the proposed algorithms with respect to the complexity of the addressed problem and its difficulty in large size problems.

As can be seen in Table 8, the proposed hybrid algorithm (VNS + SA) outperformed the VNS algorithm in all test problems in terms of optimality gap and solution cost. However, considering computing time, VNS shows more efficiency compared with the other solution approaches and it resulted in short periods of running time even for large size problems. Overall, the proposed algorithms show efficiency to obtain effective solutions when their running time increases gradually in a linear trend as the size of the problem increases. The convergence of the applied hybrid algorithm for the largest test problem (L04) is depicted in Fig. 10 which resulted in a GAP of 3.8%.

Table 8 Computational results

Problem ID	CPLEX			VNS			VNS + SA		
	Total cost ($M)	Computing time (s)	Status	Total cost ($M)	Computing time (s)	GAP (%)	Total cost ($M)	Computing time (s)	GAP (%)
S01	2.037E+02	1629.19	opt	2.080E+02	0.03	2.1	2.082E+02	19.71	2.2
S02	2.634E+02	14,182.33	opt	2.695E+02	0.03	2.3	2.681E+02	19.02	1.8
S03	2.265E+02	18,000	time	2.331E+02	0.06	2.9	2.317E+02	21.11	2.3
S04	2.420E+02	18,000	time	2.468E+02	0.05	2	2.465E+02	22.32	1.9
M01	3.111E+02	18,000	time	3.202E+02	0.19	2.9	3.205E+02	27.03	3
M02	2.676E+02	18,000	time	2.777E+02	0.19	3.8	2.767E+02	26.42	3.4
M03	2.893E+02	18,000	time	3.000E+02	0.24	3.7	2.985E+02	27.13	3.2
M04	3.003E+02	18,000	time	3.129E+02	0.23	4.2	3.084E+02	28.95	3.3
L01	2.943E+02	18,000	time	3.063E+02	0.28	4.1	3.055E+02	34.36	3.8
L02	3.987E+02	18,000	time	4.183E+02	0.31	4.9	4.123E+02	41.12	3.4
L03	5.309E+02	18,000	time	5.538E+02	0.34	4.3	5.495E+02	42.03	3.5
L04	6.836E+02	18,000	time	7.164E+02	0.37	4.8	7.096E+02	47.63	3.8

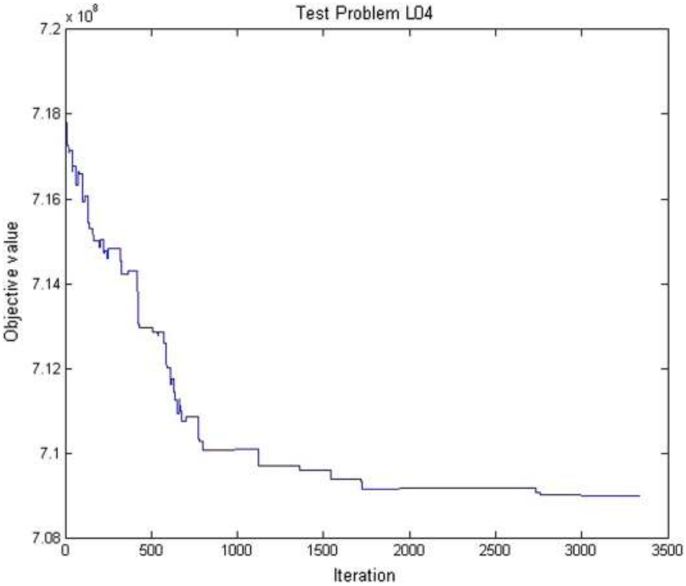


Fig. 10 The convergence of VNS + SA for test problem L04

Furthermore, to evaluate the effectiveness of the proposed approaches compared with the existing system, test problem S04 is designed based on current status of the ISWM system in the studied case. That is, the cost of existing system is resulted by considering the currently working facilities of the ISWM system in Tehran and current routing adopted from (Shirazi et al. 2016) which shows the total cost in \$312 m. Then, the existing facilities are assumed as the candidate sites and the corresponding costs of the system are obtained by using the proposed model and algorithms. The result of the proposed model by CPLEX obtains the total cost of \$242 m which shows 22% reduction in the total cost compared with the cost of current ISWM system in Tehran. Compared to the currently working system, the proposed hybrid algorithm and the VNS also obtained almost 21% reduction in the total cost. The status of the currently working system and the results obtained by the proposed model and algorithms are summarised in Table 9 where the details of routes after transfer stations are eliminated with respect to the fact that the processing sites and landfills are centralised in the same centre (Aradkuh Centre). As it is indicated in Table 9, the proposed methods employ less transfer stations with different locations compared to the existing system, and employ different waste processing plants and new routing plans among the system components compared to the existing system.

Table 9 Status of the existing system and results obtained for Problem S04 Adopted from Shirazi et al. (2016) and TWMO (2017)

Status/method	Opened transfer stations (covered MDs)	Opened treatment centres (q = 1)	Opened treatment centres (q = 2)	Opened recycling centres	Opened non-hazardous disposal centre	Opened hazardous disposal centre	Total cost ($M)
Existing system	Darband (MD1,MD3), Zanjan (MD2,MD10), Banihashem (MD4,MD8), Hakimiyeh (MD4,MD8), Kuhak (MD5,MD21,MD22), Beyhaghi (MD3,MD6,MD7), Harandi (MD11,MD12), Azadegan (MD13,MD14,MD15), Yaran (MD9,MD10,MD17,MD18), Jehad (MD16,MD19), Shahid Avini (MD20)	CPG plant	M1–M2, M3, Plant S1–S2, S9, Cargo, S3, S10, S4–S5, S6–S7	Compost site #1, Compost site #2, MRF plant	MSW landfill site	Sanitary cell	3.120E+02
CPLEX	Azadegan (MD11,MD12,MD15,MD17,MD18,MD19), Jehad (MD8,MD13,MD14,MD15,MD16,MD20), Kuhak(MD1,MD2,MD4,MD5,MD9,MD21,MD22), Yaran(MD1,MD3,MD6,MD7,MD10,MD11)	CPG plant	M1–M2, M3, Plant, S1–S2, S4–S5, S6–S7, S9, S10	Compost site #1, Compost site #2, MRF plant	MSW landfill site	Sanitary cell	2.420E+02
VNS	Azadegan(MD6,MD8,MD13,MD15,MD18,MD20), Jehad(MD9,MD11,MD13,MD15,MD16,MD19), Kuhak (MD2,MD5,MD6,MD12,MD21,MD22), Yaran(MD1,MD3,MD4,MD7,MD10,MD14,MD17)	CPG plant	M1–M2, M3, S1–S2, S3, S4–S5, S6–S7, S9, S10	Compost site #1, Compost site #2, MRF plant	MSW landfill site	Sanitary cell	2.468E+02
VNS+SA	Azadegan(MD8,MD9,MD13,MD14,MD15,MD20), Jehad(MD3,MD6,MD7,MD9,MD11,MD16,MD19), Kuhak (MD2,MD5,MD21,MD22), Yaran(MD1,MD3,MD4,MD10, MD12,MD17,MD18)	CPG plant	M1–M2, S1–S2,S4–S5, S6–S7, S9, S10, Cargo, Plant	Compost site #1, Compost site #2, MRF plant	MSW landfill site	Sanitary cell	2.465E+02

5 Conclusion

In this paper, a location-routing problem is addressed for an ISWM system where multiple types of wastes are factored in concurrently. The problem involves concurrently optimizing the locations of the interrelated facilities of the system together with optimizing the routing wastes and residues within the facilities. Real world constraints such as waste treatment technology compatibility and distinct disposals for hazardous and non-hazardous wastes are taken into account, which provided a more realistic framework for a municipal solid waste location-routing problem consisting of a complex network of all interrelated facilities.

Firstly, a mixed-integer linear programming is proposed to formulate the addressed ISWM location-routing problem. Considering the NP-hardness of the problem and non-efficiency of exact methods for solving large-scale problems, a novel step-wise heuristic method is proposed within the frames of two well-known meta-heuristics (VNS and SA). The computational results show the efficiency of the proposed algorithms to solve the problems within practical spans of time. The experimental results for the test problems in different scales show that the proposed hybrid method (i.e. VNS + SA) obtains near-optimal solutions with gaps of $<4\%$, compared to the results obtained by CPLEX in a limited computing time. However, in terms of computing time, VNS outperformed VNS + SA especially for large-size problems where the exact solution is also unable to achieve within a practical time. The proposed model and algorithms were also successfully applied to a real-life case for an existing ISWM system in Tehran, Iran. Compared to the existing ISWM system in the studied case, the proposed model can effectively reduce 22% of the total cost which consists of the fixed cost of the facilities and the transportation cost. Comparative results justify the effectiveness of the proposed model and algorithms for designing a cost-effective ISWM system.

While the proposed model and methods focus on optimizing the ISWM system for the challenging objective of the economic cost, expansion of the model and methods to a multi-objective optimization framework will be a worthwhile study. That is, factoring other objectives such as minimisation of environmental impacts of the system and social concerns such as Not In My Back Yard (NIMBY) syndrome. Taking even more practical constraints into consideration (e.g. multiple technologies for recycling centres) could also enhance the practicality of the addressed problem. In terms of solution approach, the proposed stepwise heuristic method achieved acceptable results when embedded in frames of SA and VNS, however, applications of other meta-heuristics may obtain even better results. Lastly, as an ISWM system may not always deal with deterministic aspects and parameters (e.g. amount of waste generation), expanding the method to a heuristic solution approach under uncertainty will be another potential future research in this area.

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