Mathematical Models in Contact Mechanics

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Contact phenomena involving deformable bodies arise in industry and everyday life and play important roles in structural and mechanical systems. Owning to the complicated surface physics involved, they lead to new and nonstandard mathematical models. Considerable progress has been achieved recently in modelling and mathematical analysis of phenomena of contact and, as a result, a general Mathematical Theory of Contact Mechanics is currently emerging as a discipline on its own right. Its aim is to provide a sound, clear and rigorous background to the construction of models, their variational analysis as well as their numerical analysis.

In this lecture we give a review of our major results in the study of contact problems involving elastic, viscoelastic and viscoplastic materials. We derive variational formulations of the models and present existence and uniqueness results for the weak solutions. The proofs are based on arguments of elliptic and evolutionary variational inequalities, time-discretization, regularization, compactness and fixed point. Also, we study the behavior of the solutions and prove various convergence results. Moreover, we consider numerical approximations of the models, derive error estimates for both semi-discrete and fully discrete schemes and present numerical simulations for two dimensional test problems.