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Mathematical Models of $M^b/M/1$ Bulk Arrival Queueing System

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Abstract: This paper deals with the study of bulk queueing model with the fixed batch size 'b' and customers arrive to the system with Poisson fashion with the rate λ and are severed exponentially with the rate μ. On formulating the mathematical model, we obtain the expressions for mean waiting time in the queue, mean time spent in the system, mean number of customers/work pieces in the queue and in the system by using generating function method. Some numerical illustrations are also obtained by using MATLAB-7 so as to show the applicability of the model under study.

Keywords: Bulk queue, Poisson, exponential, generating function

1. Introduction

In queueing theory, customers arrive randomly according to a Poisson process and form a queue. In a typical manufacturing situation, the work-pieces arrive at a machine centre in batches and they leave in batches. A batch consists of identical work-pieces that are processed and then transported in batches for further processing. Such a situation can be modeled as queues with bulk arrivals. There is a discipline within the mathematical theory of probability, called a bulk queue (also called batch queue) where customers are served in groups of random size. There has been performed a number of researches on the bulk queueing system. Some of these previous works are briefly discussed here. Maurya [1] studied some significant performance measures of a bulk arrival retrial queueing model with two phase service where first phase service is essential and the next second phase service is optional. Jeeva and Rathnakumari [2] dealt with a mathematical non-linear programming method to construct the membership function of the system characteristics of a M/G/1, bulk arrival queues with server vacations and feedback facility, in which arrival rate, service time, batch size departure probability, and vacation time are all fuzzy numbers. Shinde and Patankar [3] investigated the state dependent bulk service queue with balking, reneging and multiple vacations where (a-1) arrivals waiting in the queue, server will wait for some time (called the change over time), in spite of going for a vacation.

Haridass and Arumuganathan [4] studied the operating characteristics of a $M^x/G/1$queueing system with unreliable server and single vacation is analyzed. The server is subjected to fail, while it is on, and the arrival rate depends on the up and down states of the server. Jain and Bhargava [5] dealt with the analysis of unreliable server bulk arrival retrial queue with two class non-preemptive priority subscribers. The two types of subscribers arrive according to Poisson flow in which priority is assigned to class one, and class two subscribers are of non-priority types. Sikdar et al.[6] contributed to the analysis of a batch arrival single-server queue with renewal input and multiple exponential vacations. Ghimire and Ghimire [7] dealt with the study of M/M/1 queue with heterogeneous arrival and departure with the provision of server vacations and breakdowns. Customer arrive service facilities with poison process and exponential service time distribution. Abolnikov et al. [8] threw the light on the study a class of bulk queueing systems with a compound Poisson input modulated by semi-Markov process, multilevel control service time and queue length dependent, service delay discipline. Pang and Whitt [9] were motivated by large-scale service systems, and considered an infinite-server queue with batch arrivals, where the service times are dependent within each batch. We allow the arrival rate of batches to be time-varying as well as constant. Baruah et al. [10] aimed at studying a queuing model with two stage heterogeneous service where customer arrival in batches and has a single server providing service in two stages, one after the other in succession. Khalaf et al. [11] studied a batch arrival queueing system in which the server may face occasional random breakdowns.

The repair process does not start immediately after a breakdown and there is a delay time waiting for repairs to start. Baruah et al. [12] again studied the behavior of a batch arrival queuing system equipped with a single server providing general arbitrary service to customers with different service rates in two fluctuating modes of service. In addition, the server is subject to random breakdown. As soon as the server faces breakdown, the customer whose service is interrupted comes back to the head of the queue. Khalaf et al. [13] dealt a queuing system with four different main server interruptions and a stand-by server replaces the main server during any potential stop. The main server has five statues in this system where it is either services the customers or it does not work. Chen [14] developed a nonlinear programming approach to derive the membership functions of the steady-state performance measures in bulk arrival queueing systems with varying batch sizes, in that the arrival rate and service rate are fuzzy numbers. Bagyam et al. [15] analyzed bulk arrival general service retrial queueing system where server provides two phases of service-essential and optimal. After each service completion, the server searches for customers in the orbit. Customers may balk or renege at particular times and accidental and active breakdown of the server is considered. Singh et al. [16] investigated a single-server Poisson input queueing model, wherein arrivals of units are in bulk. The arrival rate of the units is state dependent, and service time is arbitrary distributed. It is also assumed that the system is subject to breakdown, and the failed server immediately joins the repair facility which takes constant duration to repair the server.

The rest of the paper is organized as follows: Section 2 presents the mathematical model of the queueing system, Section 3 describes the performance measure of the system, Section 4 illustrates the graphical representation of the model and finally Section 5 concludes the paper.

2. Mathematical Model of the System

The notational convention in this paper is as follows:

λ = arrival rate μ = service rate

ρ = traffic intensity b = batch size

W_s = waiting time in the system W_q = waiting time in the queue

L_s = mean number of work piece in the system

L_q = mean number of work piece in the queue

The steady state transition diagram for our model is shown in figure 1.

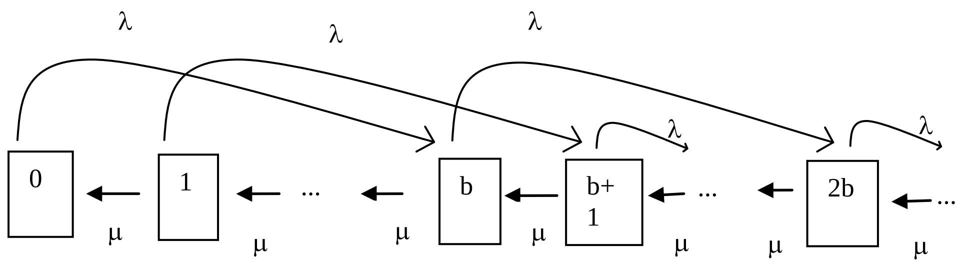

Figure 1: Transition diagram for all the states

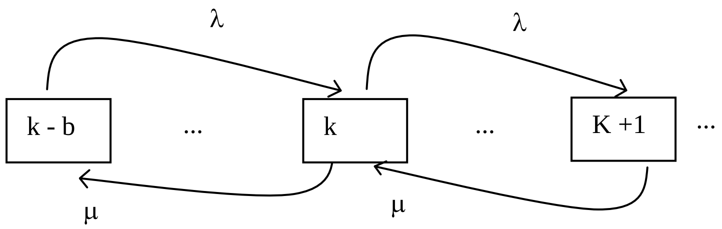

Figure 2: Transition diagram for general states

Let $p_{k\ (k\,=\,0,\,1,\,\ldots\,,\,n)}$ be steady-state probability that there are k customers in the queueing system. The state balance equation can be written with the help of following diagram as follows:

$(\lambda+\mu)\ p_k$ = Flow rate out of k state

$\lambda\ p_{k-b}+\mu\ p_{k+1}$ = Flow rate into k state

At equilibrium, we have

$$(\lambda+\mu)\ p_k = \lambda\ p_{k-b}+\mu\ p_{k+1}\ ;\quad k \geq b \qquad \text{.........(1)}$$

$$(\lambda+\mu)\ p_k = \mu\ p_{k+1}\ ;\quad k < b,\ k = 1, 2,\ldots, b-1 \qquad \text{.........(2)}$$

From (2), $\lambda p_0 = \mu p_1$

i.e.,

$$\mu p_1 - \lambda p_0 = 0; \quad k = 0 \qquad \text{.........(3)}$$

Multiplying (1) by z^k under the summation from b to ∞, multiplying (2) by z^k and sum from k =1 to b - 1 and combining these with (3) we get,

$$\sum_{k=b}^{\infty} \lambda z^k p_{k-b} + \sum_{k=b}^{\infty} \mu z^k p_{k+1} - \sum_{k=b}^{\infty} (\lambda + \mu) z^k p_k + \sum_{k=1}^{b-1} \mu z^k p_{k+1} - \sum_{k=1}^{b-1} (\lambda + \mu) z^k p_k + (\mu p_1 - \lambda p_0) = 0$$

$$\Rightarrow z^b \sum_{k=b}^{\infty} \lambda z^{k-b} p_{k-b} + \sum_{k=1}^{\infty} \mu z^k p_{k+1} - \sum_{k=1}^{\infty} (\lambda + \mu) z^k p_k + (\mu p_1 - \lambda p_0) = 0$$

We define generating function of p_k as:

$$G(z) = \sum_{k=0}^{\infty} z^k p_k = z^0 p_0 + \sum_{k=1}^{\infty} z^k p_k$$

i.e., $G(z) - p_0 = \sum_{k=1}^{\infty} z^k p_k$ ....(4)

$\Rightarrow \lambda z^b \sum_{k=b}^{\infty} z^{k-b} p_{k-b} + \mu z^{-1}\left[\sum_{k=-1}^{\infty} z^{k+1} p_{k+1} - z^0 p_0 - z^1 p_1\right] - \sum_{k=0}^{\infty} (\lambda + \mu) z^k p_k - (\lambda + \mu) p_0 + \mu p_1 - \lambda p_0 = 0$

$$\Rightarrow \lambda z^b G(z) + \mu z^{-1}[G(z) - p_0 - z p_1] - (\lambda + \mu) G(z) + (\lambda + \mu) p_0 + \mu p_1 - \lambda p_0 = 0$$

$$\Rightarrow G(z)[\lambda z^b + \mu z^{-1} - (\lambda + \mu)] + \mu p_0 - \mu z^{-1} p_0 = 0$$

$$\Rightarrow G(z) \frac{\lambda z^{b+1} + \mu - (\lambda + \mu) z}{z} + \mu\left(1 - \frac{1}{z}\right) p_0 = 0$$

$$\Rightarrow G(z) = \frac{\mu(1-z) p_0}{\mu - (\lambda + \mu) z + \lambda z^{b+1}} = \frac{(1-z)\mu p_0}{\mu - \mu z - \lambda z + \lambda z z^b} =$$

$$= \frac{\mu p_0 (1-z)}{(1-z)\mu - \lambda z(1-z)[1 + z + z^2 + \cdots + z^{b-1}} = \frac{\mu p_0}{\mu - \lambda z \sum_{j=0}^{b-1} z^j}$$

$\Rightarrow G(z) = \frac{\mu p_0}{\mu - \lambda \sum_{j=1}^{b} z^j}$ .... (5)

$$G(z)\ (\text{at } z = 1) = \frac{\mu p_0}{\mu - \lambda \sum_{j=1}^{b} (1)^j} = \frac{\mu p_0}{\mu - \lambda b}$$

$$1 = \frac{\mu p_0}{\mu - \lambda b}\ [\text{Since} \sum_0^{\infty} z^k p_k = G(z) \Rightarrow G(z)\ (\text{at } z = 1) = \sum_{k=0}^{\infty} 1.p_k = 1]$$

Since $\rho = \frac{\lambda b}{\mu}$ is the utilization of the server, we have

$$1 = \frac{p_0}{1 - \frac{\lambda b}{\mu}} = \frac{p_0}{1 - \rho}$$

$$\therefore p_0 = 1 - \rho$$

Therefore (5) becomes $G(z) = \frac{\mu(1-\rho)}{\mu - \lambda \sum_{j=1}^{b} z^j}$.

3. Performance Measures of the System

In this section, we calculate the mathematical expressions for mean number of work pieces and mean waiting time in the system. Also, average number work piece and average waiting time of work piece in the queue has been calculated. These four different mathematical expressions of this model are termed as the performance measure of the system and are described as follows:

(i) Mean number of work pieces in the system

$$L_s = \sum_{k=0}^{\infty} k p_k = \frac{d}{dz} G(z) \text{ at } z = 1$$

$$= \frac{d}{dz}\left[\frac{\mu(1-\rho)}{\mu - \lambda \sum_{j=1}^{b} z^j}\right] \text{ at } z = 1$$

$$= \mu(1-\rho) \frac{d}{dz}\left[\mu - \lambda \sum_{j=1}^{b} z^j\right]^{-1} \text{ at } z = 1$$

$$= \mu(1-\rho)(-1)\left[\mu - \lambda \sum_{j=1}^{b} z^j\right]^{-2}.(-\lambda) \sum_{j=1}^{b} j\, z^{j-1}$$

$$= \frac{\mu(1-\rho)\lambda \sum_{j=1}^{b} j\, z^{j-1}}{\left[\mu-\lambda \sum_{j=1}^{b} z^{j}\right]^{2}} \text{ at } z = 1$$

$$= \frac{\lambda\mu(1-\rho)\left(\frac{1+b}{2}\right)}{(\mu-\lambda b)^{2}} = \frac{\rho(1+b)}{2\,(1-\rho)}$$

(ii) Mean waiting time of work piece in the system by Little's result $L_s = \lambda b W_s$

$$W_s = \frac{L_s}{\lambda b} = \frac{1+b}{2\mu\,(1-\rho)}$$

(iii) Average number work piece in the queue is

$$L_q = L_s - \rho = \frac{\rho(b-1+2\rho)}{2(1-\rho)}$$

(iv) Average waiting time of work piece in the queue is

$$W_q = W_s - \frac{1}{\mu} = \frac{1+b}{2\mu\,(1-\rho)} - \frac{1}{\mu} = \frac{b+2\rho-1}{2\mu\,(1-\rho)}$$

4. Numerical Results and Interpretations

The explicit formulae obtained in section 3 have been illustrated numerically with the help of graphs. Fig. 4 and Fig. 5 show that the mean number work pieces in the system increase more rapidly than the increase of mean number of work piece waiting in the queue which can be experienced in the real life situations as well. When traffic intensity increases (ρ), the mean time spent in the system (Ws) and mean waiting time in the queue (Wq) increase with the increase of batch size (b) which is realistic in nature. This has been sketched the Fig. 3 and Fig. 6. Queueing model under study has ubiquitous applications in manufacturing system, transportation system and in assembly line.

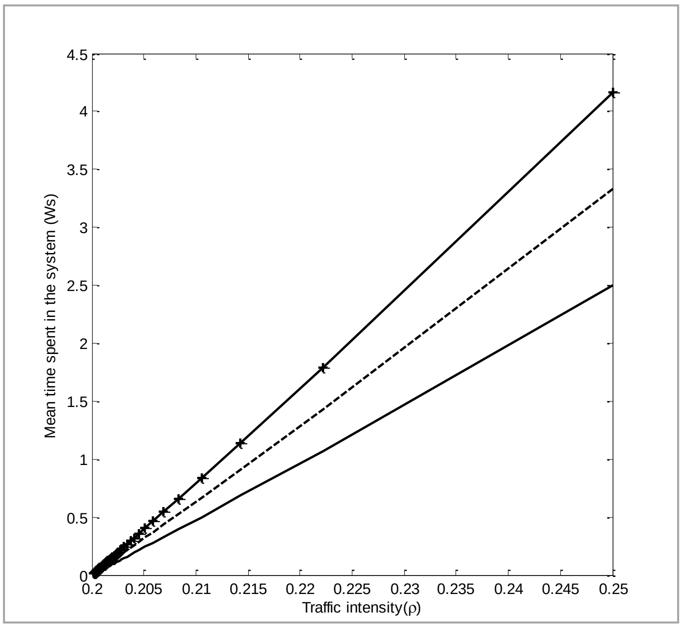

Figure 3: Time spent in system vs. traffic intensity

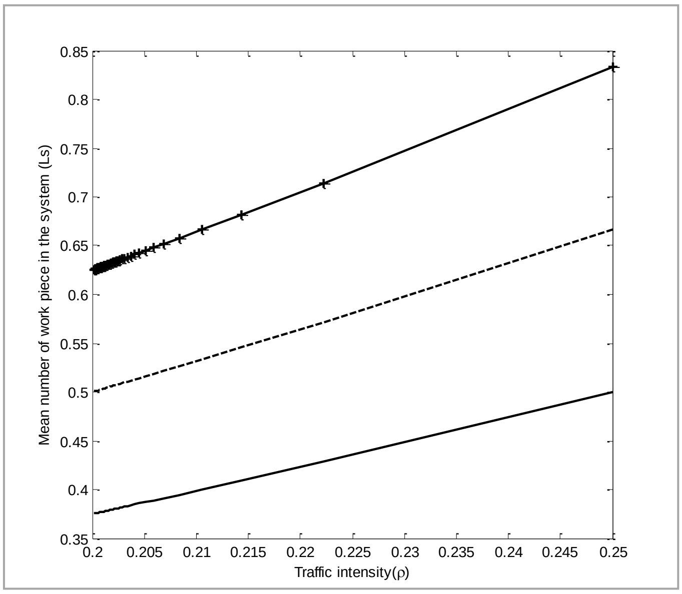

Figure 4: Mean number of work piece in the system vs. traffic intensity

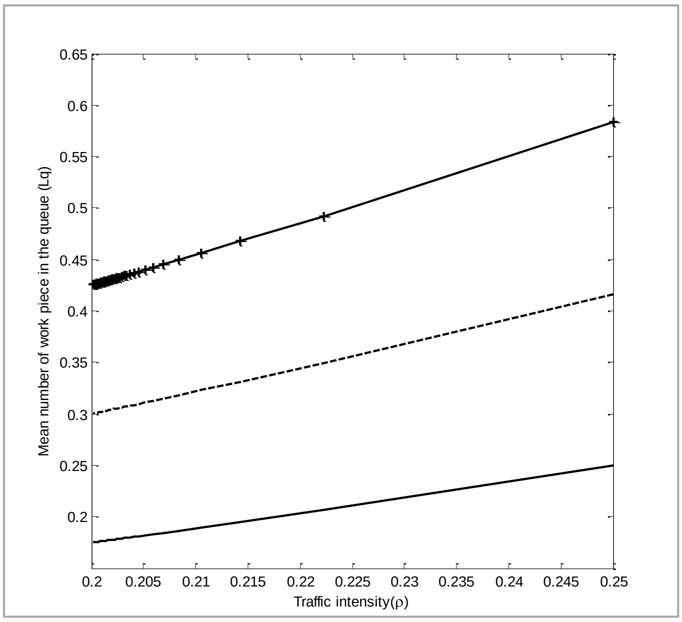

Figure 5: Mean number of work piece in the queue vs. traffic intensity

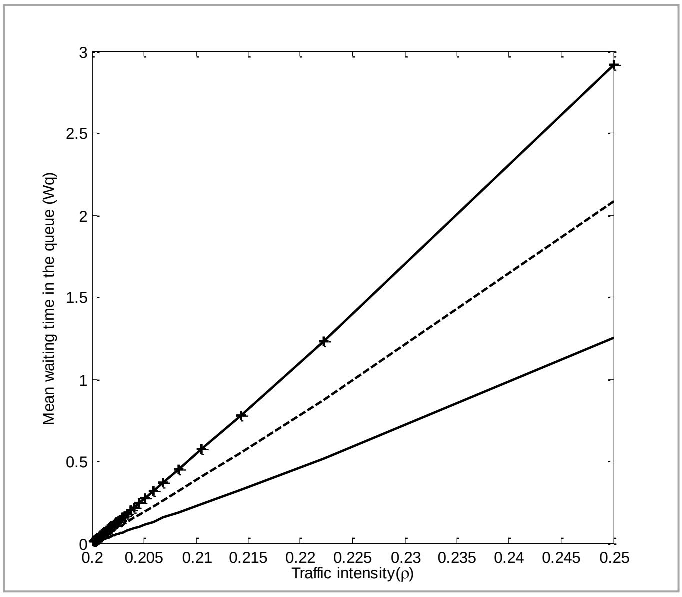

Figure 6: Mean waiting time in the queue vs. traffic intensity

5. Conclusion

In this paper, the explicit formulae obtained have their real life applications such as vehicle dispatching strategies for bulk arrival which consist of some combination of vehicle holding and cancellation strategies. We have obtained some performance measures explicitly by using the probability generating function method which may draw the attention of queueing theorist. The numerical results in our study has been interpreted by using MATLAB-7. These numerical results show that our model has its practical applicability in several real world situations. The queueing model under our study has widespread applications in the manufacturing system, transportation system, assembly line system and in overall supply chain management systems. This model can be studied under the provision of time dependent arrival and service rate which takes the system to more realistic environment. Weighted fair queueing system is another interesting area of study. The departure traffic from a bottleneck is more regular than the arrival traffic. Therefore identifying a workable bottleneck is a crucial point. Our future work will be focused on performance modeling and optimization of some finite buffered queueing systems finding the efficient bottlenecks.

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