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ABSTRACT

This report is the result of a survey of several populations exploring the extent and nature of sex bias in the field of mathematics. Students in grades 2-12, college students concentrating in mathematics, teacher preparation, and other subjects, as well as practicing female mathematicians participated in the survey. Data were analyzed to determine sex differences and apparent trends. Many of the findings replicate or extend those of other studies, some of which are cited in the report (e.g., Carnegie Commission Report on Opportunities for Women in Mathematics). An unexpected result is the apparent agreement between sexes concerning liking for mathematics; this agreement is shown to persist throughout the populations considered. The authors cite testimony from other studies and from women mathematicians concerning the issues involved in motivating women to study mathematics. They conclude that recruitment of women into mathematics courses, especially at the college level, is mandatory. (SD)

MATHEMATICS

AND

S E X

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ERRATA

opposite page 1;	for Ellen Sheff, read Elin Scheff
page 1, line 1;	for rational minded, read rationally minded
page 20, line 14;	for Morawitz, read Morawetz
line 15;	for Taŭsky, read Taussky
page 26, line 19;	for numbers of mathematics, read numbers of mathematicians looking outside mathematics

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MATHEMATICS AND SEX

Our title might well evoke a smile. To a rational minded person it would be hard to imagine two subjects which are less related. Yet it is one of those irrational features of our cultural and social structure that a person's sex is, and has been, very relevant to that individual's opportunities in the world of mathematics.

Some fifty years ago, one of the great mathematicians of all time, Emmy Noether (1882-1935), encountered opposition to her admission to the faculty at the University of Göttingen solely because of her sex. The distinguished mathematician David Hilbert, a professor at Göttingen, was incredulous: "Meine Herren, I do not see that the sex of the candidate is an argument against her admission as a Privatdozent. After all, the Senate is not a bathhouse." Emmy Noether was not admitted (at that time) to the all male faculty at Göttingen, although she delivered many series of advanced lectures on her work, which had to be announced under the name of Hilbert (cf. pp. 142-143 of [18]).

At the time this study was initiated (Fall 1973) the mathematics faculty at the University of California at Santa Barbara consisted of 32 members (all male) while the secretarial staff consisted of 6 members (all female). These curious figures served as our initial stimulus, causing us to wonder about the forces which could bring about such gross sex differences. While few could claim to be totally dispassionate in such matters, we began by trying to put aside our preconceptions and prejudices and to impartially examine these sex differences in mathematics as the curious phenomenon that it truly is. We hope the reader will attempt to do the same in reading this report, which is a compilation of the major findings of a freshmen (oops--freshpeople) seminar on women in mathematics, which met in the Fall quarter of 1973, and ran over into the Winter quarter of 1974, under the auspices of the Mathematics Department of the University of California at Santa Barbara.

ur first impulse was to concentrate on the question of discrimination in employment. Indeed this is the main emphasis of the Department of Health, Education and Welfare and hence also the main emphasis of most affirmative action programs on college campuses. While not denying the existence of such a problem, we have since come to believe that complete non-discrimination in mathematical hiring will have only a minor affect on the percentage of women in the profession. We feel greater emphasis is needed on affirmative action at the academic level, to unblock the beginning end of the pipe-line. As Violet Haney [14] has pointed out, during the past four decades women earned only 4% of the total Ph.D.'s in mathematics. A rough computation indicates that in 1970 "there was available only one female with a Ph.D. in mathematics for every two degree-granting institutions in the United States."

From this point of view the fact that, in the fall of 1973 the UCSB mathematics faculty consisted of thirty-two men is not quite so startling. Indeed consider a weighted coin for which the probability of getting a head is .04 (the relative frequency of women Ph.D.'s). If one flips this coin thirty-two times, the probability of obtaining all tails is about one chance in ten. Considering that a woman professor had recently died and that now (Fall 1974) there are two women on the faculty, we can see that the employment pattern is not particularly unusual, given the very low percentage of women Ph.D.'s. Furthermore the Carnegie Commission on Higher Education (p. 89 of [3]) has indicated that this percentage (of women Ph.D.'s in mathematics) has been steadily decreasing, with only a slight upswing in recent years. (In the period 1920-24, 20% of the mathematics Ph.D.'s were earned by women.)

Thus the extreme sex differences in employment of women mathematicians can be traced rather directly to the enormous sex differences in mathematical training. We have therefore elected to concentrate our attention on sex differences in mathematical education, beginning at a very early age (second grade) and following through to the research mathematician. We have also been concerned (see section 4 below) with the affect of these sex differences in mathematical training on the

participation of women in many other scientific and technological fields requiring some degree of mathematical sophistication. Our studies confirm the hypothesis of the sociologist Lucy Sells [20] that mathematics is a "critical filter" tending to eliminate women from many fields, from chemistry, physics and engineering, to architecture and medicine. This conclusion lends greater import and urgency to this study, and to the need for effective changes in the mathematical training and counseling of women.

The first section of this report deals primarily with student attitudes from the second grade through high school. Our second section examines the attitudes of teachers. Our third section examines sex differences in the extent of mathematical training in the high schools. Here we identify the inadequate mathematics training for women (as compared to that for men) as a major culprit in closing off scientific and technological opportunities for women. In section 4 we go on to examine the situation at the university level, both undergraduate and graduate. Here our results are somewhat special, describing the situation only for the UCSB campus, and then for a rather limited time period. Nevertheless we feel these results are likely to be indicative of sex differences to be found in the mathematics programs of most universities. In section 5 we return for a closer look at the mathematics profession. We conclude, in section 6, with some observations and recommendations.

1. STUDENT ATTITUDES IN ELEMENTARY AND SECONDARY SCHOOL

To obtain some data on this subject, we constructed a simple questionnaire consisting of three questions. The first question was designed to determine which subjects students liked best (and least) and to discover whether there were statistically significant sex differences in these preference patterns. (Here we obtained one of our most unexpected results.) The second question attempted to uncover possible sexist attitudes in the family by finding out whether the person (mother or father) who helped with the homework varied from subject to subject. Here we obtained a rather striking, albeit not unexpected, result. Our third question examined peer group attitudes (who does better—boys or girls?—in various subjects). In the case of all three questions we obtained statistically significant results (significance level .05).

We had a total sample of 1324 questionnairs, distributed near the end of 1973, among grades 4 through 12. Most of the sample came from schools in Southern California, although one portion of the sample came from the East Coast. (The geographical differences were not significant.) We had more than 100 questionnaires for each grade except the fifth, where we had 81.

In the first question we asked the students to rank the four subjects, mathematics, English, science and social studies (mathematics was replaced by arithmetic and English by reading, in the lower grades) on the basis of which they liked the most to the one which they liked the least. There were statistically significant sex differences in the three subjects, English, science and social studies (very striking in the case of English and science, but in all three cases the significance level was less than .01). As one might expect, the boys tended to prefer science and the girls English. Social science got a somewhat better rating from the boys than the girls. What we found totally unexpected—contradicting our preconceptions when designing this question—is that in terms of liking the subject, mathematics was the only subject which

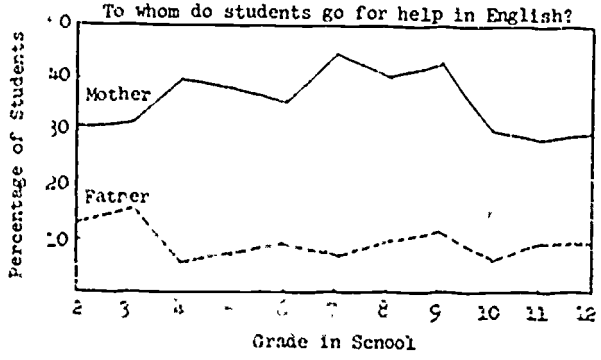
exhibited no sex differences. In fact the preference patterns in mathematics were almost identical for boys and girls, in marked contrast to the three other subjects. (For the statistics buff we mention that we computed the χ^2 - statistic for the data contingency table associated with the following table. Assuming no sex differences, the probability of obtaining a χ^2 - statistic that small (or smaller) is .09. indicating that the preference patterns in mathematics of men and women are remarkably similar.)

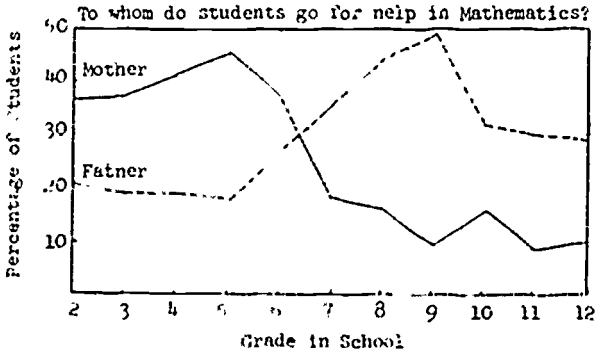
Preference Pattern for Mathematics

Rating for Mathematics	Boys	Girls
Liked best	30%	29%
Liked 2nd best	23%	24%
Liked 3rd best	19%	18%
Liked least	27%	29%

This pattern of no sex difference in mathematics preference pretty much held up through the grades, although for both boys and girls its popularity went down in the high school years. We found this result to be rather surprising and it would be most useful to have additional experimentation to confirm it. The finding indicates that there is nothing intrinsic in arithmetic or mathematics that makes it more appealing or enjoyable to one sex than the other. This is a hopeful sign as other findings, to be discussed later, show that, nevertheless, when mathematics becomes optional (in high school and college) far fewer women take it. We would conjecture that men take more mathematics not for the superficial reason that they like mathematics more than women but because, whether they like it or not, they are aware that such courses are necessary prerequisites to the kinds of future occupations, in medicine, technology or science, they envision for themselves. If this conjecture is true, then we expect that an upgrading of the counseling program in high schools, designed to make women aware of the full range of career opportunities and of the courses they must take in high school in order to keep all their options open, could have a very significant effect.

Our second question to the school children concerned whom they got help from in the various subjects. Here there were no statistically significant sex differences (i.e., between boys and girls responses). For both girls and boys it is true that the mother helps more than the father, except in the higher grades, where the father helps more in mathematics (and, to a lesser extent, in science). The following two graphs, indicating this pattern through the grades, for English and mathematics is quite striking. Beginning in the sixth grade the father becomes the "authority" on mathematics and continues this role through high school. This fact alone must have its subtle influence on a young girl's (or boy's) attitude.





Martha Smith, a mathematician at the University of Texas has written us:

> I once heard a sociological theory to the effect that the crucial factor in whether a young woman became an "academic achiever" was her father's attitude towards her—the attitude most conducive to producing an academic achiever being one which showed approval both of his daughter's "feminine" pursuits and of her intellectual ones. Reflection on my own situation and that of friends who are women and mathematicians doesn't cause me to doubt this theory—e.g., my father seemed equally pleased when I baked an apple pie, accomplished some academic achievement, or helped him saw wood.

Other women mathematicians have also referred, in correspondence with us, to the positive influence of their fathers on their intellectual development. Further Ravenna Helson [8], in a study of highly creative women mathematicians, found that most (2/3) had professional men as fathers and that interviewers judged that they had identified primarily with their fathers.

Of course it is hardly a new psychological insight that our parents have a tremendous influence on our personality development. Yet we feel still more must be done to make parents aware of the great danger of imposing, consciously or unconsciously, sexist roles on their children and the importance of offering their children encouragement and support to aspire to whatever they truly want to be.

The third question in our school questionnaire attempted to examine peer group attitudes by asking the students who they thought did better (boys or girls) in various subjects. In the elementary grades there were very substantial differences between the responses of the boys and the girls. Due undoubtedly to a kind of sexual competition fostered in the lower grades, the boys felt that boys do better—in all subjects and the girls felt that girls do better—in all subjects. However by high school these sharp differences, while still statistically significant, become muted and there emerges an overall peer group attitude concerning competence in various subjects. If we take the top four grades, 9 thru 12, together (sample size 506) we get the following table. The

32% in the table means that 32% of the sample (boys and girls together) feel that boys do better than girls in mathematics.

	Math	English	Science	Social Studies
Boys do better	32%	4%	41%	21%
Girls do better	16%	52%	5%	15%
no difference	52%	44%	48%	64%

We have recently found out that the Stanford Center for Research and Development in Teaching, about the same time we did this study in Southern California, distributed a questionnaire to 1866 high school students in the San Francisco area [cf. [4]). Many of the questions applied to each of the four specific fields of study, mathematics, English, social studies and business/vocational courses. While the primary aim was to discover the roots of student failure, some interesting sex differences relating to student attitudes towards mathematics was uncovered. We quote from an expository article on this study by the sociologist Sanford Dornbusch [4].

> One of our questions had asked the student: when you get a poor grade, which reason do you think usually causes the poor grade? There were four alternative answers: I had bad luck, I didn't work hard enough, the teacher didn't like me, and I'm not good at this subject. Most students gave lack of effort as the reason for receiving a poor grade in every subject. However when it came to math, 26 percent of the females gave lack of ability as the basis for a poor grade as compared to 15 percent of the males. Female students in every ethnic group in San Francisco were more than three times as likely to give, "I'm not good in math" as the basis for a poor grade as "I'm good in math" as the basis for a good grade. In the suburbs, the pattern was similar but the ratio was two-to-one. This pattern was found in no other subject for females and in no subject for males.

These attitudinal patterns (both self-image and peer group) are unfortunately reinforced by those of the teacher. Hence the topic of our next section.

2. TEACHER ATTITUDES

We next asked two questions of a small sample of elementary and high school teachers (24 women and 3 men). For these teachers at least, mathematics was liked better than the other subjects. 44% of the teachers listed it as their favorite subject to teach (from a list of four, mathematics, science, English and social studies) while another third of them listed it as their second most liked subject. Nevertheless their responses to a second question indicated attitudes (concerning sex differences in aptitude and performance with respect to particular subjects) very similar to that obtained for the peer group. 41% of the respondents felt boys did better in science while only one felt girls did better in science. Almost two-thirds (63%) of the respondents felt girls did better in English, while no one felt boys did better. Finally 41% felt boys did better in mathematics while no one felt girls did better.

Given this prevalent attitude (that boys do better in mathematics) among a substantial portion of both peers and teachers, perhaps we should digress here to examine the evidence concerning this question. We tested for sex differences in performance (i.e., class grades) in a number of large elementary mathematics courses at UCSB and in no case did we find any statistically significant sex differences. However there have been other studies (many quite old) that do indicate that boys do better in mathematics (cf. [5], [9], [17]). For example in 1942 there was a study involving a sample of 50,000 students from some 300 schools, where the authors concluded: "Girls have maintained a consistent and, on the whole, significant superiority over boys in the subject tests, save in arithmetic, where small, insignificant gains favor the boys" [22]. On the other hand another more recent study [23] of students in the third and sixth grades contradicts this finding and indicates that girls are in fact better at arithmetic. The Carnegie Commission in a very recent study ([3], pp. 50-51) found that women tend to get better grades in college - in all fields.

In cases where male performance is found to be superior, we may be observing the so-called "Pygmalian effect" in education, according to which the student performs, to some (measurable) extent, in response to the expectations of the teacher. In our small survey for example, almost half the teachers expect their male students to do better in mathematics, while none of them expect the female students to do better.

Psychologists ([14], [21] for example) have measured some sex differences in intellectual functioning, such as space perception, which might be related to aptitude in mathematics. These differences, however, are subtle, being far smaller than differences among individuals of either sex. These minor differences (which may themselves be culturally induced and thus modifiable) might explain some small difference in the percentage of women mathematicians. However we have found nothing in these psychological studies which helps us to explain the enormous sex differences actually found in the mathematics profession.

There is a mathematics course at UCSB designed specifically for future school teachers. We distributed special questionnaires to such a class consisting of 11 men and 64 women. We were discouraged to find that 36% of these future teachers indicated they are indifferent towards mathematics, while another 14% stated they actually dislike or hate it. Thus 50% of these prospective teachers are likely to transmit something less than a positive attitude towards mathematics to their students.

We distributed a similar questionnaire to a number of the large lower division undergraduate courses asking the students to indicate their attitude towards mathematics by marking one of the five possibilities (love it, like it, indifferent to it, dislike it, hate it). We found no statistically significant sex differences in the responses, which tends to confirm, at the college level, the result we obtained for the elementary and high school level: there appears to be no significant sex differences with respect to the liking of matheratics. We also asked the students to indicate what they felt were the major influences determining their attitude. Among those indicating an

extreme attitude (either loving it or hating it) one of the most mentioned factors was a particular teacher they had had in their prior schooling. It is largely for this reason that we were so distressed to find that 40% of the mathematics class for future teachers indicated either a negative or indifferent attitude towards mathematics.

These findings indicate to us that certain adjustments should be considered, both in the training of school teachers and in their employment in the classroom. We need instructors who are competent in mathematics, who love the subject, who enjoy teaching it and who will not project sexist expectations on the students. Our small sample of 75 prospective teachers indicates that about one out of seven either dislike or hate mathematics. This makes it quite likely that an elementary school child will at some point be influenced in a detrimental way by one of his or her teachers. We feel the child should be protected from this possibility. Perhaps those teachers who feel uncomfortable with certain mathematics subject matter and do not enjoy teaching it, should be able to enlist the aid of eager and competent specialists, much as music teachers are currently used. Such specialists would be carefully trained and highly sensitive to the prevailing sexist attitudes concerning mathematics. In particular they would encourage girls to enjoy and to excel in mathematics. If most of these mathematical specialists were women we would have the added advantage that the female students would be presented with a positive role model.

3. THE INADEQUATE MATHEMATICAL TRAINING OF WOMEN IN HIGH SCHOOL

In a perceptive article [19] entitled "Women in Science: why so few?" Alice Rossi has described the many social and psychological influences which restrict women's choice of careers in science. In this section we shall describe another factor which restricts women's choices which, with some effort, may be more quickly corrected than the deep rooted causes of which Alice Rossi speaks. Whether through inadequate or even misleading counseling, or by personal choice, women simply are not taking enough of the optional mathematics courses offered in high school to prepare them to enter college programs in science, engineering or other "hard" disciplines. Without unusual and early efforts to make up this deficiency in their first year of college, all of these career options become effectively closed.

The sociologist Lucy Sells (cf. [20]) shared with us a fact sheet (dated December 1., 1973) which she developed on women in higher education at Berkeley and we quote here two of her points.

> 1. In a systematic random sample of freshmen admitted at Berkeley in Fall, 1972, 57% of the boys had taken four full years of mathematics, including the trigonometry-solid geometry sequence, compared with 8% of the girls. The four year mathematics sequence is required for admission to Mathematics 1A, which in turn is required for majoring in every field at the University except the traditionally female, and hence lower paying, fields of humanities, social sciences, librarianship, social welfare, and education.
>
> 2. Among students earning the Bachelor's degree in the 21 largest letters and science departments, there is a strong and statistically significant relationship between having a one-year college mathematics requirement in the curriculum, and having less than one third of the degrees in the department earned by women.

This second result clearly underlines the tragic consequences of the incredible sex difference uncovered in the first finding. It is for this reason that Lucy Sells refers to mathematics as a "critical filter" in cutting down the percentage of women in many fields other than mathematics. Thus substantial changes in student and teacher

attitudes towards girls taking mathematics, and particularly changes in the academic counseling procedures at the high school level, are likely to have a significant affect on the percentage of women majoring in disciplines which use mathematics as well as in mathematics itself.

We found the result (1 above) of Lucy Sells' concerning the inadequate mathematical training of entering freshwomen at Berkeley so remarkable that we decided to repeat the experiment at UCSB. We took a random sample of the files of freshmen and freshwomen (50 of each) who had been admitted to UCSB in the fall of 1973. Perhaps because this was done a year later (the situation is in great flux) and because UCSB is a rather different campus than Berkeley, we obtained somewhat different results. The percentage of entering men with four years of high school mathematics was considerably smaller (36%) while the percentage of entering women with four years of high school mathematics was somewhat larger (16%). Nevertheless the sex difference is still enormous (and statistically significant) and thus confirms Lucy Sells' findings at Berkeley. (We conjecture that this situation prevails at most colleges throughout the country.) Other voluntary (and hence non-random) samples taken at UCSB also showed a significant sex difference in high school mathematical training.

We mention that the relevance of mathematics to many career options outside of mathematics has also been observed by the Carnegie Commission on Higher Education in its recent report [3] on opportunities for women in higher education. They state (in referring to women on page 64 of the report): "Not only have they preferred fields that lead to traditionally female professions, but they also tended to avoid fields requiring extensive application of mathematical reasoning." We concur completely with the following Carnegie Commission's recommendation:

> Recommendation 7 (page 79 of [3]): Because of the evidence that many women enter college with inadequate mathematical training, special provision should be made to ensure that women desiring to major in fields calling for extensive use of mathematics are encouraged to make up this deficiency in order to enter the fields of their choice.

This brings us to the next stage of our investigation, sex differences in mathematical education after high school.

4. SEX DIFFERENCES IN MATHEMATICAL TRAINING AT THE UNIVERSITY LEVEL

We begin by looking for sex differences in the elementary calculus sequence beginning in the fall of 1971 and extending to the winter quarter of 1973, at UCSB. Math 34AB is the short (two quarter) course while Math 3ABC, 4AB, is the long (five quarter) course. (This long course branches in the second year. The figures for 4A include two other parallel sections, 5A and 4AH. The figures for 4B include 5B.)

We first note that the enrollment of women in these basic courses is disproportionately low. Women comprised only about a third of the class even though women were in the majority in the freshperson class of 1971. This undoubtedly is one of the inevitable corollaries of the deficient mathematical training women receive in high school, which we described in the previous section. Nevertheless we feel that improved counseling and other remedial aids at the college level can significantly improve these enrollment figures.

Enrollment, Fall 1971

	Freshpeople Class	Math 34A	Math 3A
Male	1001 (45%)	133 (63%)	328 (64%)
Female	1228 (55%)	78 (37%)	184 (36%)

We found no statistically significant sex differences in the grades achieved in the various courses. The Carnegie Commission ([3], pp. 50-51) has found that women tend to get better grades in college-in all fields. Nevertheless in every one of the five places (one in the short sequence, four in the long sequence) where a student has an opportunity to drop, the attrition rate was greater for women than men. The attrition rates for 34A → 34B, 3C → 4A and 4A → 4B were particularly large. with the women's attrition rate almost double that of men. This is particularly disconcerting as the original enrollment of women was already disproportionately low. These rather large attrition rates are difficult to interpret as various majors require only certain portions of the complete

seque ce. During the last year both instructors and teaching assistants have been making greater efforts at encouragement, early detection of academic difficulty, and tutorial help, in order to decrease these attrition rates for both sexes. We believe that the many progressive innovations being introduced by UCSB in these calculus sequences will alter these dropout patterns radically. Further we remark that large sections of 3A and 3B are now being taught by a female instructor and we are curious as to whether this fact alone will modify the sex differences in attrition rates observed in this study.

Attrition Rates for Calculus Sequences, beginning Fall 1971

Course	Male	Female	All
34A → 34B	33%	58%	42%
3A → 3[illegible]	33.5%	37.5%	35%
3B → 3C	26%	35%	29%
3C → 4A	30%	51%	36%
4A → 4B	22%	41%	27%
3A → 3C	51%	59%	54%
3A → 4A	66%	80%	71%
3A → 4B	73%	88%	79%

The above table is read as follows: 42% of those enrolled in 34A did not enroll in 34B. 33% of the men dropped out after 34A and 58% of the women dropped out after 34A. We did not actually follow each individual student through the sequence and hence the figures are subject to considerable error due to students transferring to other schools, and other students joining the sequence at a later stage. For this particular sequence, beginning in the fall of the year, we feel these possible errors cannot possibly account for the large attrition rates indicated here. Indeed if one corrected by eliminating students joining the sequence at a later stage, the attrition rates would be even worse than reported here.

The enrollment of women in the honors calculus section (which begins with the second quarter) is considerably lower than in the regular sections. While women comprise about [illegible]% of the second quarter calculus enrollment, they comprise (as an average over the last four years) only about 20% of the honors section enrollment. The author taught this honors section in the winter of 1971 and there were nine men and no women enrolled. The following year the author was again scheduled to teach the honors calculus. This time he visited all the regular sections, described the nature of the honors section and specifically encouraged women to enroll. In the winter of 1972 the enrollment was again nine students, but this time there were six men and three women. We strongly recommend that every mathematics department look into their honors sequence and determine the percentage of female participation. We recently received a letter from Professor Joan Birman telling us of her similar experiences in the honors sequence at Barnard-Columbia (in alphabetical order).

> I learned last year, to my astonishment, that for about four years running the honors calculus course had been all male, in spite of the fact that admission was based on an open competitive examination. This fall, one of the senior mathematics majors and myself made an intensive effort to encourage women to try the exam! The typical answer was, "I know I won't pass it," - to which we replied over and over, "Well, if you try it, at worst you will confirm what you already know, and only an hour of time will have been lost." After three days of such advising, the big day came, the exam was given, and this year the class has five men and five women!

These experiences indicate that we can anticipate considerable change in the enrollment of women in mathematics courses, if only a greater effort is made to encourage them to do so. A tiny survey was conducted, at Stanford, of women majoring in natural sciences, mathematics and engineering [cf. [4]). This highly biased sample reported less encouragement to study mathematics than did any group of Stanford males, even those majoring in history and the humanities.

We next examine the attrition of mathematics majors. The Carnegie Commission [3] had made the following observation:

> When we consider the fields in which students major in college, we shall find that almost as large a percentage of women as of men receive bachelor's degrees in mathematical science, although women are considerably less well represented than men among recipients of degrees in fields calling for the use of mathematics as a tool, such as the natural sciences, engineering and economics.

The statistics on mathematics majors at UCSB do not seem to follow this national pattern. We have followed the class of 72 and the class of 73 and found that women got less than half as many degrees in mathematics as men. Here again we find rather heavy attrition. For example, when the class of 72 began in the fall of 68, about the same percentage of men and women indicated their desire to major in mathematics. (70 women and 63 men indicated their intention to major in mathematics. This represents 4.4% of the women in the class and 4.5% of the men.) These percentages continually decline in the following years, the attrition being higher for women than for men. Finally at graduation time there were only 20 women mathematics majors (1.8% of the women graduates) and 48 men mathematics majors (3.1% of the men graduates). For the class of 73 the pattern is the same. Only 16 women (1.5% of the women graduates) and 49 men (2.8% of the men graduates) received bachelor's degrees in mathematics. Of course the attrition figures may be partially caused by students transferring to other campuses of the UC system as well as the enormous influx of community college students at the beginning of the junior year, some of whom may be ill prepared to then begin a mathematics major at UCSB.

We also note that the attrition rates campus wide have been greater for women than for men. When the class of 72 entered UCSB in 1968, it consisted of 53.4% women. By the time this class was graduated in 1972, it consisted of only 41% women. Similarly the class of 73 started with 53.3% women and ended with only 43.6% women. (Again these figures may be strongly affected by transfers in and out of the university.)

We conclude that it is very much in the university's own interest to invest in subsidized child care centers, better academic and career counseling, and more flexible degree programs (among other things being

recommended by various women's groups and the Carnegie Foundation [41] to try to decrease this sizable attrition among women.

In going from the bachelors degree into a graduate program we once again find that attrition takes a higher toll for women than for men. (At this point we may marvel that there are as many women mathematicians as there are!) For example Lucy Sells [30] has found that "there is a statistically significant drop in the proportion of women earning the bachelors degree in the physical sciences at Berkeley (26%) and the proportion of women applying to graduate school in the physical sciences (11%)." In the fall of 1972, 17% of the applicants to the graduate program in mathematics at UCSB were women, while in 1973 it was 16%. Averaged over these two admission periods, the acceptance rate was 78% for the men and 88% for the women. During the academic years covering 1971 to 1974, women represented 21% of the MA enrollment and 13% of the doctoral enrollment, giving an overall average of 16% in the graduate program. During this same period 35% (8 out of 23) of the masters degrees granted in mathematics went to women while 17% (3 out of 18) of the Ph.D's went to women. During this same period, women were recipients of none of the 15 fellowships awarded. In the fall of 1973 women represented 11.5% (3 out of 26) of the teaching assistants and teaching associates in mathematics.

We feel an effort can and should be made to increase the proportion of women in both the undergraduate and graduate mathematics program. For example, women students at MIT compiled an excellent booklet [1] about the school's opportunities for women and mailed it to 10,000 female high school juniors. This resulted in 1,400 inquiries from women about admission to MIT, nearly four times as many as the year before. The engineering department at Berkeley has also used a special recruiting program, with some success. We recommend that affirmative action funds be used for a similar project on this campus, covering mathematics, science and engineering. As we increase the proportion of women in these areas, our campus could become known as one which has made a particular effort to create a supportive and encouraging learning environment for women in mathematics and the "hard sciences." If such an effort is successful,

the mathematics, science and engineering programs will greatly benefit by the increased enrollments of well motivated students.

In this regard, the dearth of women faculty in mathematics, physics, chemistry and engineering can only have an inhibiting affect on female enrollments in these subjects. The mathematician Martha K. Smith has written us concerning this matter:

> Role models play an often neglected but, to my mind, important role in education. If a young girl never encounters a women in mathematics, it is quite reasonable that she should conclude that mathematics isn't something women do (whether for lack of ability or lack of opportunity). Perhaps more likely, she may not even think of the possibility of a woman mathematician.
>
> I know I felt a certain amount of relief when, the summer after my first year in graduate school, I met a woman mathematician for the first time. There was living evidence that what I wanted to do was not impossible.
>
> To reverse these stereotypes won't be easy, but I think it's possible. An effort should be made to get all the trained women mathematicians possible into positions where they will have an impact on both male and female students and colleagues. Women need to see examples of practicing mathematicians; men need to become accustomed to accepting a woman as an honest-to-goodness colleague rather than a curiosity.

In regard to role models, we recommend that books like [10] and [16] be available in every mathematics reading room, both undergraduate and graduate.

WOMEN IN THE MATHEMATICS PROFESSION

We have been most fortunate that a large number of women mathematicians have corresponded and shared with us their experiences, attitudes and insights. They have enabled us to go beyond statistical averages to see the complexity and variability of human personality and creativity. Women, just like men, have various reasons for being attracted to mathematics, few of them having anything to do with their sex. They also hold diverse opinions regarding the various matters considered in this report. Their suggestions have done much to shape our study. Their ideas permeate this report. We therefore wish to express our deep appreciation to the following women mathematicians who have consulted with us: Joan Birman, Judy Bruckner, Mary Gray, Susan Gerstein, Mary Elizabeth Hamstrom, Eleanor G. Jones, Nancy Kopell, Tilla Milnor, Cathleen Morawitz, Alice Schafer, Marianne Smith, Martha K. Smith, Ann Stehney, Diane Stuebing, Olga Tausky Todd, Karen Uhlenbeck.

Just over half of our respondents indicated they had experienced some form of discrimination in their professional lives. Essentially all of them indicated they had experienced sexist attitudes of some form.

Professor Martha Smith (whom we have already quoted) has put forth the idea that many women are dissuaded from a mathematical career by societal stereotypes:

> Many people on hearing the words "female mathematician" conjure up an image of a six-foot, grey-haired, tweed suited oxford clad woman... This image, of course, doesn't attract the young woman who is continually being bombarded with messages, direct and indirect, to be beautiful, "feminine" and to catch a man.

We have found the notion that women who excel in mathematics are less feminine to be utter nonsense. For example in Ravenna Helson's study [8] of creative women mathematicians, her findings did not "show the creative woman to be more masculine, if one means by this that they might have been expected to score higher on measures of masculinity-

femininity, or dominance, assertiveness, or analytical ability." In another (older) study [11] involving undergraduate students at UCLA Philip Lambert found that among women students "mathematics majors were not only equal to non-mathematics majors in femininity, but significantly more feminine." This finding was sufficiently surprising that Lambert took a second sample, which confirmed the result.

Joan Birman, a professor at Columbia University was influenced earlier in her life by this "unfeminine" stereotype. "Why didn't I study mathematics at age 21? I felt it was not a 'feminine' thing to do. I'm afraid that it seems to me that this is a continuing problem for many young women." As we have just noted, studies indicate that, if anything, women mathematicians tend to be more feminine than the average. We thus feel it is time for society to completely do away with the ridiculous prejudice that a female mathematician is somehow less of a woman because of her intellectual pursuits.

Professor Joan Birman goes on to discuss other misunderstandings and problems of women considering mathematics as a career.

> The female students I know also seem to often misunderstand the nature of mathematics. It appeals to them (I think) because they are systematic, neat, logical and orderly. They do not seem to appreciate the creative, imaginative, esthetic aspect of mathematics, and indeed are often bewildered by it when they first encounter it in advanced courses. I hope this is something which is changing, as more young women become aware of themselves as questioning, growing, thinking people. Conformity is really deadly for mathematics, because it is not possible to learn the subject unless you are constantly questioning whether you understand it. To become a mathematician it is also necessary to a) have a good deal of tolerance for frustration, b) to not be distressed by the distrust or fear other people have of those who are "smart" and c) to not be afraid to be wrong or make mistakes—all of these seem to be problems for women.

In respect to employment discrimination, we feel the mathematical profession is going through some very healthy changes. The nepotism rules, invariably discriminatory against the wife, are rapidly falling away. Under the watchful eye of the Department of Health, Education and

Welfare. academic and industrial institutions are adopting widespread procedures to ensure that hiring patterns are free of sex bias. We strongly support such affirmative action to ensure that all potential female condidates are considered. Such increased recruitment efforts can only enlarge the list of qualified candidates and thus result in the raising of standards. To immediately dispose of a red herring, let us state emphatically that none of us believe a less qualified mathematician should be hired, just because she is female.

While supporting affirmative action we are concerned that most efforts seem to concentrate exclusively at the employment level. We would be most distressed if universities, in fear of losing federal funding, develop programs only in response to HEW threats and emphasize those paper-work procedures designed to protect them in case of HEW audits. We believe our study has established the great need for affirmative action at the academic level - from elementary school through college. It doesn't help much to carefully distribute the few drops from the end of a pipe-line when the real problem is that the pipe is stopped up at a much earlier point. If university administrators are sincere in their desire to open up opportunities for women (and we believe they are) then they must put far more resources into academic affirmative action. even if this is not where the federal government is putting on the pressure. There is no dearth of suggestions in this regard. from information and recruiting efforts at the high school level, subsidized child care centers, improved career counseling, more flexible requirements including external degree programs and part time study, to mention just a few. There are many such recommendations throughout this report.

We are pleased to observe that the "women's movement" is alive and well in the mathematics profession and is continuing to have a very beneficial effect (cf. [2] and [7] for example). More women are appearing as invited speakers at professional meetings. They are being nominated and elected to important positions in the professional societies. Scientific meetings often have special sessions concerned with women in

mathematics. The American Mathematical Society has recently compiled a directory of women mathematicians [1]. There is an active organization concerned with women's rights: The Association for Women in Mathematics (address: Department of Mathematics, The American University, Washington D. C. 20016). Professor Alice Schafer, one of our correspondents, is the president. They issue a regular and lively newsletter, edited by Mary Gray. This association also runs an employment information service which helps women become aware of job opportunities as they develop. We would recommend this organization to any mathematician (male or female) who is concerned with the issues raised in this report.

9. CONCLUSION

We have tried to discuss our results and make some tentative conclusions and recommendations in each of the appropriate sections. The findings of all sections tend to support one basic recommendation of the Carnegie Commission on Higher Education (pp. 56-57 of [3]).

> Recommendation 1: The first priority in the nation's commitment to equal educational opportunity for women should be placed on changing policies in pre-elementary, elementary and secondary school programs that tend to deter women from aspiring to equality with men in their career goals. This will require vigorous pursuit of appropriate policies by state and local boards of education and implementation by school administrators, teachers and counselors.
>
> For example, high school counselors and teachers should encourage women who aspire to professional careers to choose appropriate educational programs. They should also encourage them to pursue mathematical studies throughout high school, because of the increasing importance of mathematics as a background, not only in engineering and the natural sciences, but also in other fields, such as the social sciences and business administration.

The Carnegie Commission goes on to recommend improved career counseling at the university as well.

> Recommendation 5: Not only should colleges and universities take immediate steps to strengthen occupational counseling programs generally in this era of a changing job market for college graduates, but they should also take special steps to strengthen career counseling programs for women. Counselors should be trained to discard outmoded concepts of male and female careers and to encourage women in their abilities and aspirations.

The topic we have taken up is vast and has many ramifications. Our investigation should be looked upon as a pilot study. We hope that this preliminary data will suggest many interesting projects to educators, sociologists and psychologists. We fully recognize that we are trying to measure something that is in great flux. We therefore recommend that

every mathematics department begin to keep annual records of sex differences in enrollment rates, attrition rates, degrees granted and graduate admissions and support. This will help alert the department to areas which need attention as well as to indicate if new policies are having the desired effect. It will be most interesting to observe how these patterns change as our schools and society move away from sexual stereotypes and bias.

We have, for reason of time, necessarily ignored many fascinating and important questions. We have purposely tried not to discuss aspects of sexism and women's rights that do not relate specifically to mathematics. Nevertheless many of these (such as the toys children play with, the games they play, sex roles in movies and television etc.) clearly do have their effect on female attitudes towards mathematics. For an excellent essay covering these more general concerns we refer the reader to the book [illegible] by Nancy Frazier and Myra Sadker entitled Sexism in School and Society.

We would also have been interested in making a study of sexism in mathematics texts, the vast majority of which are written by men. We hope that someone or some group will take up this study and share their findings with us. We have much to learn concerning the way mathematics is presented, both in texts and in lectures and the extent to which women respond (positively or negatively) to it. (Cf. for example, the research study [illegible] on sex differences in mathematics problem appeal as a function of problem context.)

We would like to conclude with a short discussion of certain moral issues which have been lying beneath the surface of this study. These concerns were touched upon in an open and lively interdisciplinary discussion, led by the sociologist Lucy Sells, in the mathematics colloquium series in the fall of 19[illegible]. The question is this: Have we made an implicit assumption that dropping out of a degree program in general, and a mathematics program in particular, is necessarily bad? Given the extremely depressed (and depressing) employment prospects for young mathematicians (both men and women) perhaps those who drop out, or don't

even begin the training process, are in fact making the correct choice, not only in terms of economic consequences, but also in terms of fulfilling their human potential. Since some male mathematicians were somewhat misunderstood when they introduced this question into the discussion, we'd like to quote a woman mathematician, Nancy Kopell of Northeastern University, who has carefully articulated this fundamental issue in a letter to us.

> I don't think the main problem facing potential women mathematicians is that of having so few women already in the field. In fact, although I fully support the women's movement, I don't think it's necessarily desirable to forcefeed mathematics with women. Before you jump on me for being sexist or elitist, I'll try to explain that this reaction comes from feelings about mathematics rather than about women.
>
> There is an agreement among many mathematicians today that there is a kind of crisis in motivation: why be a mathematician? I see this confusion and the search for answers at every level of the profession. It shows up in debates on what to teach undergraduates; it shows up in the numbers of mathematics proper in order to be "relevant." There is a widespread feeling of alienation.
>
> I have my own personal - and tentative - answers to these questions, but that is too long a story for here. The point is, I would not presume to encourage anyone - man or woman - to blindly enter mathematics without facing these issues. However neither would I discourage anyone, especially women. It was apparent that the entry of large numbers of blacks into fields previously closed to them has resulted in enormous cultural change and growth. Perhaps the entry of many women into fields previously "male" will have the same enriching effect. (It is a fantasy of mine that if mathematics had many more women practitioners it would be different, e.g., less authoritarian and formal. Mathematics is cultural, and the product depends on the people who construct it; the people who have formed it in the past have mainly been male.)

We do not claim that the goal is the elimination of all measurable sex differences in all human pursuits. We all have different opinions on the extent to which that would be desirable. But we are all agreed that the many sex differences in mathematical training and attitudes described in this report are <u>not</u> the result of free and informed choice.

If they were then the low enrollments and high attrition rates for women in mathematics would be a matter of less concern. The immorality of these sex differences lies precisely in the fact that they are the result of many subtle (and not so subtle) forces, restrictions, stereotypes, sex roles, parental-peer group-teacher attitudes, and other cultural and psychological constraints which we haven't begun to fully understand. Before we can hope that each individual child and young adult will make these choices freely and wisely we must work towards a society generally (and an academic program specifically) which ensures that the freedom of opportunity to become whatever that individual is truly capable of becoming, is not compromised by such a chance event as the child's sex at birth. We believe that each of us, male or female, who have children or anticipate having children, would desire such a society for them. And those of us who are mathematicians and whose lives have been so greatly enriched as a result would wish that this opportunity be available, regardless of sex, to anyone who finds they have the aptitude, interest and creative and intellectual ability to do it.

Albert Einstein, on the occasion of the death of the mathematician Emmy Noether in 1935, wrote the following lines in a letter to the New York Times (cf. p. 208 of [18]).

> Beneath the effort directed toward the accumulation of worldly goods lie all too frequently the illusion that this is the most substantial and desirable end to be achieved; but there is, fortunately, a minority composed of those who recognize early in their lives that the most beautiful and satisfying experiences open to human kind are not derived from the outside but are bound up with the individual's own feeling, thinking and acting... . However inconspicuously the lives of these individuals run their course, nonetheless, the fruits of their endeavors are the most valuable contributions which one generation can make to its successors.

We believe that we must continue the effort to bring about a sexually unbiased social and intellectual environment where many other Emmy Noether's will be able to flourish and grow. We will all be the beneficiaries of their creative endeavors.

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We would appreciate receiving your comments and views concerning the findings and recommendations of this report.

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