Mathematics Education and the Future: a Long Wave View of Change

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Mathematics education is a concern of the whole community in that the generation of wealth requires a workforce equipped with an appropriate level of scientific abilities. On the other hand sub-standard levels of numeracy contribute to problems of unemployment and hence to a drain on welfare systems. Enquiries such as [1,2] reflect the importance with which mathematics continues to be regarded at national levels.

One problem with mathematics education is its lack of a macro theory. In consequence, while much research and development occurs this tends to be piecemeal and uncoordinated. As evidence of this, from time to time state of the art documents appear [3,4,5,6] which attempt to summarize what has been achieved across a wide variety of contexts and to suggest key areas for renewed effort. But the search for a set of principles to explain the past and to direct future efforts remains unfulfilled. The future is usually viewed within some context of extrapolation from the past and present. This paper is an attempt to contribute to the development of guiding principles for mathematics education by taking a frame of reference outside mathematics education itself. It can be viewed as the development of a scenario that has its basis in the literature of the economic long wave [7,8,9,10,11,12,13]. This paper examines the implications of long wave theory for mathematics education. It addresses some of the traditional issues associated with educational research, interprets some historical events as outcomes rather than causes and points some directions towards which efforts should be directed. In this sense, and from a provocative perspective it attempts to identify priorities for future developments in mathematics education.

1. Cyclic patterns and mathematics education

The waxing and waning of distinct influences on mathematics education has been explicitly noted or implied by a variety of writers. Higginson [14] envisaged the four components of mathematics, psychology, socio-cultural factors and philosophy as combined into a tetrahedral model to which each influence contributed a face. The movement of the "centre of gravity" of the tetrahedron would reflect the changing "weight" exerted by the respective faces. He sees in both the United States and Western Europe the turn of the century (the Perry movement) and the Sputnik era (the new mathematics) as times of content domination, a view shared by Howson [6,15]. Higginson sees the psychological dimension of teaching since 1960 as having undergone an almost complete cycle from didacticism to discovery and back again. Influenced by changes in the social climate variations in philosophical rationale have been seen to vary from intrinsic reward to a largely utilitarian view. In the words of Howson [6]

> Educators in the first two decades of this century faced similar problems to those which we do nowadays. In their case a period of frenzied activity was followed by a period of retrenchment and consolidation.

Robitaille and Dirks [16] observed in summarizing eleven case studies that the type of social and political system operating in a given place at a given time influences "not only the educational system in general, but, in particular the mathematics curriculum". Lucas, in Steen [17], underlines this view when he writes, "the commonly held view that mathematical directions are somehow independent of the stresses acting on them is a naive one". Howson [15] further confirms this thinking by noting the contrast between the sixties when governments welcomed the pleas of educators for mathematics for all and the recent past which has seen disillusionment set in with talk of basics and minimum competencies. Richardson [18] reviewed the history of mathematics curricula in the U.S.A. between 1893 and 1935. He identified the early years of the century as a time galvanised by issues of content which saw, for example, the birth of the College Entrance Examinations Board and the memorable address by E.H. Moore [1902] on the foundations of mathematics. Economically the country was in the process of becoming predominantly industrial and the needs of such a society were clearly influential in shaping curriculum change.

Moving forward in time to 1918, Kilpatrick's project method was identified as influenced by Dewey in its concern for personal engagement in the learning process and the primacy it gave to the interests of the individual, while the publication of the Cardinal Principles of Secondary Education [1918] further emphasized the "rise in concern for individuals and the fall of the discipline concept".

This concern for the pupil found an echo in the sixties as, for example, Begle [3] noted:

> The preparation of special instruction passages, including teacher training, as well as curriculum units, for students doing poorly in mathematics received a great deal of support in the sixties. This

> happened primarily because of the national concern for improved education for minority and disadvantaged students. [page 75]

Reviewing developments in the thirties Richardson noted that school boards reacted then more to local influences — ones that were intimately tied to the economic and social developments of the times. For example, curriculum specialists (like Bobbitt and Charters) advocated a strictly utilitarian approach to the aims and content of instruction based on "what is". Richardson concludes with reference to the low ebb reached by school mathematics as a consequence of the social utility movement. Commenting on the contents of the eleventh yearbook of the NCTM, *Mathematics in the modern world* [1936] he observed that but for the arrival of emerging events, "the future of school mathematics in 1940 was in grave danger of having nothing but a past".

The foregoing discussion, encompassing changes in emphases in mathematics curricula over almost a century, draws attention to cyclical patterns of change that are related to the economic and social climates of the times. Two writers, Sawyer [19] and Shirley [20], have gone beyond anecdotal observation to explicitly identifying the periodic nature of the changes. Sawyer deals mainly with content-based influences while Shirley considers the three components of content and psychological and societal factors. These are seen to rotate in a fixed order, each in turn being the most significant influence on mathematics curricula. The period of the "motion" is seen to be approximately 50 years (Figure 1)

2. The Economic long wave

This section summarizes relevant empirical and theoretical work on the economic long wave that is described extensively in [7,8,9,10,11,12,13,21,22]. The economic long wave is a repeating rise and fall of economic activity usually called the "Kondratieff cycle" after the Russian economist who identified the pattern [21]. From an examination of long term economic time series covering the nineteenth and early twentieth centuries, Kondratieff concluded:

> There is indeed reason to assume the existence of long waves of an average length of about 50 years in the capitalistic economy.

Recently the Dutch economist van Dujin [22] extensively reviewed the evidence and literature and proposed a long wave dating that extends to the present day (Figure 2).

1st Kondratieff	2nd Kondratieff	3rd Kondratieff	4th Kondratieff
prosperity 1783–1793	prosperity 1847–1857	prosperity 1893–1904	prosperity 1949–1958
prosperity 1793–1803	prosperity 1857–1866	prosperity 1904–1913	prosperity 1958–1967
(war 1803–1815)		(war 1913–1921)	
recession 1815–1826	recession 1866–1875	recession 1921–1929	recession 1967–1975
depression 1826–1837	depression 1875–1884	depression 1929–1938	
recovery 1837–1847	recovery 1884–1893	recovery 1938–1949	

Figure 2
Van Dujin [1977] p 563

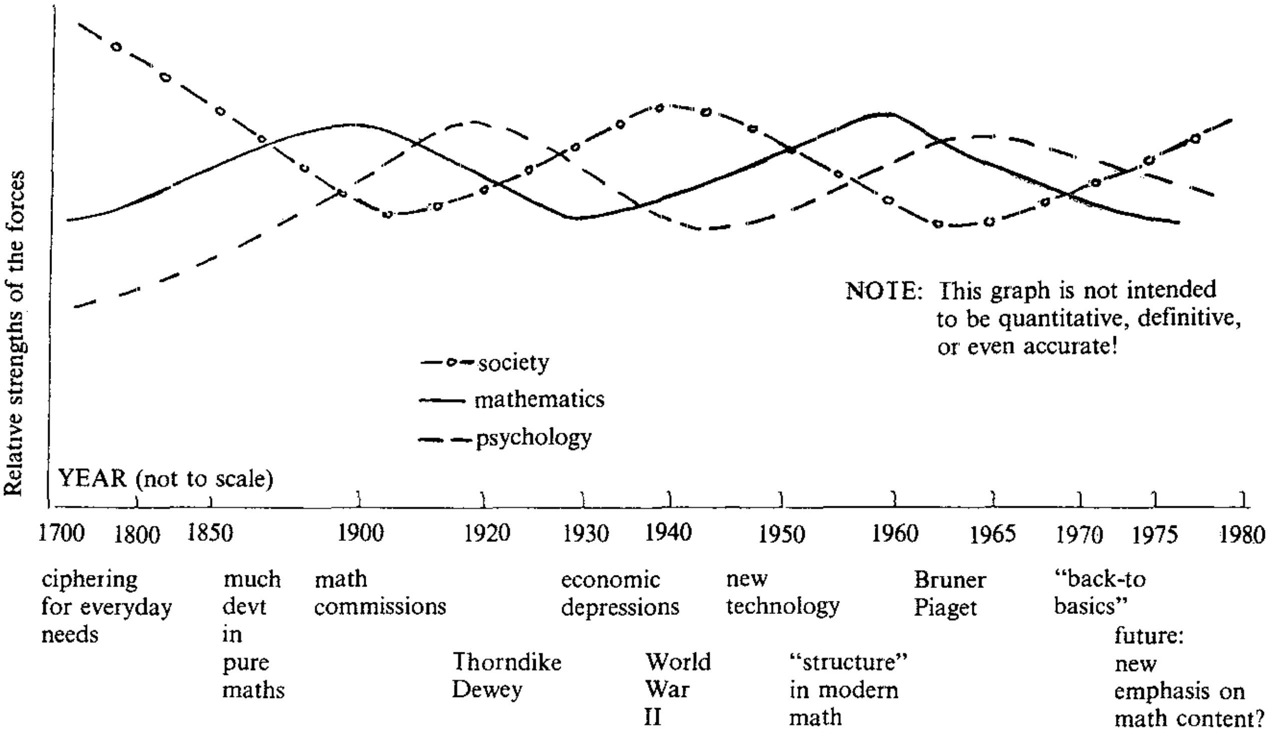

Figure 1 Relative strengths of the influential forces on mathematics curricula. After Shirley [1980]

The phases of prosperity are acknowledged as those in which fundamental innovations cause an acceleration of economic development. Around 1790 these consisted of developments in the textiles and metallurgical sectors together with the application of steam power. Around 1850 the development of railroads created vast investment outlets in the western world while in the late 1890's the basic innovations were electricity, the motor car, and innovations in the chemical sector. The post-war "Kondratieff" has been associated with a whole range of basic innovations such as television, the computer, and synthetic fibres.

The absence of a theory to explain how the internal structure of an economy could produce disturbances covering such long periods of time led to mixed reactions to the long wave hypothesis. However in recent times a modelling of the national economy of the U.S.A. by Forrester [7,8] Forrester, Mass and Ryan [9], Graham and Senge [10], Graham [11], Sterman [12,13], has shown how internal structures can generate a long wave disturbance similar in period and timing to that identified by Kondratieff from the statistics of real world economic behaviour. The simulation captures the major patterns in the development of the economy over almost 200 years. In simplified form the basic theory is described in [8,13] and can be summarized as follows.

The long wave is characterized by successive waves of over-expansion and collapse in the capital-producing sector of the economy. Over-expansion means an increase in the capacity to produce, and in the production of plant, equipment, and goods, beyond the amount needed to replace worn-out units and to provide for long-term growth. Over-expansion is harmful because in the long run production and employment must be cut back below normal to reduce the excess. Production and development then remain depressed while excess physical and financial capital is depreciated over a time scale that is relatively extended due to the long lifetimes of plant and equipment. Once the old stock is worn out investment again becomes worthwhile, triggering the next upswing. As an illustration, Forrester [8] uses the post-World War 2 era as a case history. The strong post-war demand for materials and products led to over-production as the capital sector, driven by self-reinforcing demand, developed a capacity well beyond that needed to sustain and replace physical depreciation. As the orders began to fall the self-reinforcing process reversed its effect. By the late sixties the capital stock had been rebuilt and excess capacity and unemployment began to appear in basic industries. As a result investment in these industries fell, further reducing the need for capital and re-inforcing the decline in investment as the economy moved through the seventies and into the eighties

2.1 Political and social values

There is evidence that political and social values in western nations fluctuate with the period and phasing of the economic climate [23,24]. Independent content analyses of political tracts in the U.S.A. and the U.K. reveal statistically significant 50 year value cycles in both countries which coincided with each other and with the phasing of the long wave. As expressed by Sterman [13],

> During periods of long wave expansion, material wants are satisfied, and social concerns turn to civil liberties, income distribution, and social justice. During the later phases of the expansion foreign policy concerns predominate. As the expansion gives way to decline conservatism grows and political attention returns to material needs. The variation in values is primarily the result of entrainment by the economic cycle. It is quite natural to emphasize material needs during depression periods. People find it easier to be charitable and to extend the rights and privileges of society during good economic times when incomes are rising rather than in times of retrenchment and depression.

2.2 Inventions, innovations and technological change

Relating the long wave to innovation and technological progress requires a distinction between invention (the discovery of a new idea or technical process) and innovation as the first practical application of the invention on a substantial scale. Mensch [25] defines *basic innovations* as

> innovations which produce new markets and industrial branches ... or open new realms of activity in the cultural sphere, in public administration, and in social services. Basic innovations create a *new type of human activity* (emphasis mine)

By contrast *improvement innovations* can be regarded as incremental improvements to existing technologies that do not alter their fundamental natures. Figure 3 from [25] depicts the frequency of basic innovations in Western countries in 22 ten-year periods from 1740-1960.

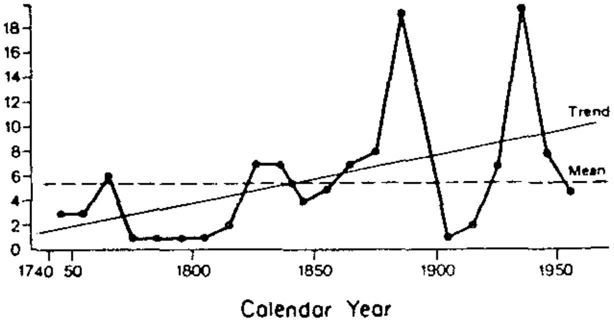

Figure 3
Frequency of Basic Innovations, 1740-1960
Mensch [1979] p. 130

The graph shows that certain periods in history have uniquely favoured basic innovations and the peaks in intensity correspond to troughs in the long wave. Interestingly this feature has been noted by Baursfeld [26] who records that it is in times of deep economical depression that the creative framework and fundamental research are prepared that will "provide the substance of the following period of ascension and flower".

For example the period of the late depression in the 1930's and the early upswing following produced nylon, jet aeroplanes, electronic computation, radar, and television.

Forrester [8], Graham and Senge [10] and Sterman [12,13] argue that the long wave strongly determines the climate for innovation. Each major expansion has grown around a set of mutually supporting basic technologies in which, for example, energy sources determine factory design, transportation and patterns of living are closely related, and communication and business transactions move at the same rate. After an integrated pattern develops, radical inventions do not fit the existing infrastructure and are not able to command financial attention while tried and true opportunities remain available. Hence major (basic) innovations that break from the existing status quo are seen as impractical and uneconomic. Minor improvements become the order of the day as the climate now favours innovations to the existing technology. As the long wave peak unfolds into recession and depression investment falls, capital embodying the obsolete technology of the old wave is not replaced, and the process of using up and wearing out the old technology runs its course. Then with years of stored innovations to draw upon the time is ripe for investment and expansion in a new wave of basic technological development that includes also improved forms of some earlier technologies. Figure 4 from Graham [11] illustrates how time-lags separate an innovation from its invention, i.e. from when the technology first functioned in a laboratory setting. The two numbers shown are respectively the date of the innovation (its first widespread use), and the number of years taken to progress from laboratory invention to commercial use.

Surge in 1870s to 1890s	Surge of 1930s and 1940s
Incandescent light (1867, 79)	Power steering (1930 30)*
Electric locomotive (1979 38)	Radar (1934, 47)
Telephone (1981, 21)	Fluorescent light (1934, 82)
Anesthetics (1883, 52)	Diesel-electric locomotive (1934, 39)
Chemical fertilizers (1885, 45f)	Catalytic refining (1935 20)
Gasoline motor (1886, 26)	Television (1936, 29)
Aluminum (1887, 60)	Nylon (1938, 11)
Rayon (1890 33)0	Automatic Transmission (1939 35)
Antitoxins (1894, 17)	Penicillin (1941, 19)
Refrigeration (1895, 22)	Jet engine (1941 13)

* (Date of innovation, years from date of invention)

Figure 4
Innovations from Previous Surges
Graham [1972] p. 73

The long wave theory implies a shifting climate for innovation with a sharp clustering of basic innovations near a trough in the wave.

2.3 Time scales and the life span

Forrester [8] has responded to doubts about how long wave recycling modes can exist for the order of centuries in spite of major changes in society and technology. In his explanation he links the long wave periodic behaviour to the human life span, pointing out that the policies and structures underpinning economic development have changed very little. The long wave depends on production methods that use capital equipment, on the life of capital equipment and buildings, and is accentuated by factors such as planning time-scales and memories of past failures. These are substantially determined by the length of a human life-time. Long wave behaviour is exhibited by the model even without the impetus provided by accelerating technological change.

> None of these factors that give rise to the long wave depends significantly on faster communications or details of technological change. The policies and industrial structure that generate the long wave capital construction cycle have changed very little since 1800.

3. The mathematics connection

While the changing economic, social and educational climates ascribed to different phases of the long wave are similar to fluctuations noted independently in the mathematics education literature, it is the causal theory of the long wave generation that enables these observations to be regarded with new interest. The long wave theory, being both explanatory and predictive, acts as a lens for examining both past phenomena and future implications.

3.1 Structure-driven versus event-dominated

The education community is traditionally reactive. In responding to the influence of external events after they have occurred educational change tends to be *event-dominated*: e.g. calculators appear in the classroom, a new syllabus changes an examination style. Yet other forces may underlie the circumstances whereby the event in question is but a symptom. Sputnik I was closely linked to the "New Mathematics" movement. Yet while its advent stimulated interest in and support for scientific education it could not be said to have started the revolution, for programs such as UICSM were already in place when Sputnik was launched. In a sense Sputnik was but a symptom — some such event (or collection of less dramatic events) was inevitable as the post-war technological boom gathered momentum. This boom (a long wave expansion) was structure-driven as economies invested in a surge of expansion and development from the ruins of depression and war damage. In England and Australia educational needs of a new growth era were foreshadowed through major education acts passed in the nineteen forties. Some Australian mathematics curricula were re-designed during this period, which also saw the introduction of Ph.D. programs into universities.

A structure-driven perspective differs in important ways from an event-dominated one. The structure-driven view is pro-active and anticipatory rather than re-active. It plans for anticipated changes in climate and future opportunities with the insight that causal theory provides. It will be conscious that its long term view may find it out of step with popular issues of the day (event-stimulated) and hence possibly unpopular with funding sources. A structure-driven view of development would not effectively have abandoned work on the "individual in education" because the climate of practice had moved to social utility. While recognizing that the content spring had ceased to be a dominant force in the twenties and thirties, curriculum work would have continued against the day that the demand for skilled scientists would again emerge — perhaps with happier results than achieved with "New Mathematics".

3.2 The researcher-practitioner tension

The gap between research findings and their application (or applicability) is a continuing problem in education. Viewed through the long wave lens two aspects of this problem can be addressed. Firstly the time needed to articulate and research significant problems means that the "phase" of the wave is likely to be inopportune when the results are ready for injection into practice, e g. the curriculum has changed, the climate sympathetic to a particular teaching style has altered. The physics of the long wave mechanism contains the social and economic analogues of the restoring force of simple harmonic motion, so that the translation of research initiated during earlier conditions is likely to be in opposition to the "forces" currently dominating the educational climate. So the research is perceived to be irrelevant and impractical.

Secondly, the research effort is not sufficiently delineated as to purpose. Most summaries of research, e g. Begle [3], implicitly suggest that the mathematics education community is topic driven and Kilpatrick [27] laments the lack of theory. Knowledge is being advanced on many fronts but it has not yet been discovered how to use the mass of information generated. The profession lacks a macro-theory of change and development. Needed is a framework for research that enables the linking of elements of many strands into a coherent progressive structure. One possible approach is to distinguish between what has been called structure-driven and event-driven research issues. A second approach is to identify components that will together provide the "technology" of a coming basic innovation. This aspect is considered in more detail in the following section. Research driven by structural (e.g. long wave) concepts needs to be long-term and conceptually powerful. Research stimulated by events may be urgent, short-term, and to be effective must relate to current practice. The research summaries provided in works such as [4,5,6] are good bases from which to launch such analyses. This long wave basis for identifying and classifying research questions implies different levels of outcome with different expectations and time-lines for the applicability of results. This clarification is important if researchers and practitioners are not to continue to have inappropriate expectations of each others' work.

3.3 Research and innovation

Within the long wave paradigm the climate for innovation is strongly determined by the phasing of the long wave [11]. As displayed in Figure 4 substantial delays separate the invention of a major technological advance from its widespread introduction as an innovation, and these delays have been attributed to differences in wave phases. This characteristic provides a perspective on another complaint about educational change. Why does it take so long for things to happen? Why are seemingly convincing results so slow to affect practice at the work face? A study of the data in Figure 4 suggests that, contrary to popular opinion, education may not be all that slow to respond relative to industry and commerce. Usually the education community thinks of fundamental changes in teaching techniques, i.e. in terms of radical or basic changes in educational technology that will change the face of teaching. These are the dreams. The innovation theory of the long wave describes how the climate generated by a new set of basic technologies retards the acceptance of later "new" technology that is not easily compatible with the existing infra-structure. It took 47 years for radar to reach practical exploitation and 29 years for television to make a commercial impact from the time of its laboratory invention — times that are at least comparable with those over which substantial educational change is measured. The infra-structure of education is no less resilient than that of business and industry. The implication is that when the periodic opportunities for major transformations do occur the profession needs to be ready and prepared to grasp the opportunity and "get it right". This is where mathematics education has fallen short — unlike the business world it does not seem to develop "basic innovations" to the point where maximum advantage can be taken of new investment opportunities. Missing is the consistent and intense laboratory development of industry. In subscribing to the publication creed of "recent is beautiful", potentially powerful ideas are neglected (or left part-developed) to jump the band-wagon of current fashion and funding.

Distinguishing between research aimed respectively at basic and improvement innovation is not a feature of the mathematics education tradition. One program that appears to make the conceptual distinction without describing it as such is described in Papert [28,29,30]. Papert is sceptical of the ultimate value of research involving the limited presence of computers in classrooms. He likens it to an (imaginary) research of a bygone era in which a new innovation (the pencil) is introduced and its effect noted, then two or three, and so on. Papert's point is that the provision of pencils for all so completely changes the atmosphere and the nature of learning as to render research into their limited use an inefficient and unproductive use of research energy. Papert's microworld concept underpins research against the day when computer power is all-pervasive. His learning experiments are conducted with this universe in mind and as such can be regarded as laboratory research. But the laboratory simulates the real world for which it is intended, i.e. it involves real students in genuine learning situations.

One does not have to agree with Papert to grasp the point. It can be argued that it is worthwhile improving on present methods even though a greater vision lies beyond. But the limitations of improvement innovations to effect quantum changes emphasises the importance of identifying research contexts and groups of problems with the potential to transform the practices of mathematics education as a "basic innovation". This requires an examination of methodologies, research contexts, research problems, and persistent development to the point where maximum opportunity can be taken of favourable conditions for innovation

3.4 The demands of mathematics

If there are implications for research into learning and teaching, what of the *content* of mathematics curricula? Noting the stimulation imparted by past upswings in the early years of the century and during the fifties, the innovation theory suggests that some time in the next twenty years a set of new basic technologies will emerge from conditions of economic run-down to support a new wave of investment and development. This presents the prospect of a world that is as technologically distinct from the present day as the present day is from 1910. Such a concept challenges both conventional post-industrial thinking and notions of the mathematics curriculum. The academic community should be custodians of the longest-term perspectives of its related professions On such an assumption we need to ask what is being done by bodies such as the International Commission on Mathematics Instruction (ICMI). The publication *School mathematics in the 1990's* [31] indicates some current thinking. The booklet raises important questions that are pertinent within the time-line defined in the title. However while some of the issues could be the basis of more futuristic considerations, they reflect an essentially evolutionary view set against an expected context of improvement innovation and associated social change. They need to be supplemented by a longer-term more fundamental conceptualisation of change. Following historical precedent the basic innovations of a future wave will already be substantially present as inventions in today's laboratories. Thus the nature of many of the mathematical requirements of a new wave is potentially accessible. The consideration of an anticipated increase in the importance of discrete mathematics has already begun [32,33]. However in the total context such overtures appear to remain at the level of interesting ideas. The penalty for lack of preparation is vulnerability to opportunism and hasty decision-making when a new surge of development raises consciousness and makes demands on the mathematics component of the curriculum. The new mathematics era is replete with retrospective recognition of mistaken emphases and lost opportunities which may have been avoided had the mathematical community been better prepared for the demands made on its educational expertise. Forrester's inclusion of the human life span as a specific contributor to the period of the long wave finds an echo here Those who look back from long experience of the current wave have much to offer in the realm of planning for evolutionary change and the effects of improvement innovations within the present cycle. However they are not well-equipped to lead preparation for new wave thinking. It is not presently obvious that the mathematical education community has an appropriate blend of both kinds of leadership.

3.5 Mathematics across cultures

School mathematics in the 1990's draws attention to the fact that education systems in developing countries "have arisen as clones of those found in the countries of the colonial powers" The coupling of economies at different stages of development through international trade means that the effects of oscillations in the major economies are imported into less developed economies. These economies have their own needs with respect to skills and mathematical knowledge However coupling also exists between the academic and educational communities of such countries. In long wave theory the economic cycle and the educational and social variations are linked in a cause-effect relationship The response of the one is seen as coupled to the needs and characteristics of the other so that in the major Western economies, for example, educational developments are closely related to their own economic conditions.

However in the less developed countries it is possible for economic drives to be generated from one source (international trade) while educational initiatives are imported from another (curriculum thinking in the advanced nations) Because both effects are imported, and the essential coupling is lacking, there is no guarantee that the needs of the country will be addressed by developments in education Long wave theory interrelates necessary skills with the set of technologies driving the development. It therefore supports the position that developing countries should devise curricula that address their own emerging needs at the appropriate level International curriculum input should be tested against this priority

4. Summary

1. A socio-political climate favourable to fundamental changes in education may occur only about once in a lifetime. A professional community should direct its efforts to take maximum advantage of such an opportunity when it occurs. This involves the critical review of research questions in order to select those that can be developed to contribute most to basic innovation.
2. Expectations for the applicability of research should be reviewed in terms of an innovation climate that is subject to periodic fluctuation
3. The development of hierarchical criteria for the selection and conduct of research can be aided by distinguishing issues that are driven by the structure of long term forces from those that are rendered topical by the happenstance of events.
4. The conceptual distinction between basic and improvement innovation can be employed to help classify the vast amount of unstructured research evidence currently available within mathematics education

5. Long term research and development programs need to be maintained by the profession as a whole in the separate areas of content, learning, and teaching, with practical applications to current life-styles and work contexts. These should not be allowed to wax and wane in circumstantial fashion.
6. A future demand on the mathematics education community is likely to be content-based as a new wave upswing emerges from conditions of economic down-turn. The mathematical needs of this up-turn need to be coherently and urgently addressed.
7. The shifting nature of the social context due to entrainment by the economic cycle warns against the over-interpretation and over-extrapolation of data gathered during any particular time period
8. During transition periods from one wave to the next there is overlap between improvement innovations in the old technology and the basic innovations of the new wave. An educational community needs to discern the difference, both in respect of what they teach and in regard to the "technology" they adopt.
9. Since new wave upswings follow old wave downturns education systems will be met by new and challenging demands at the time they are least equipped to cope with them — at the end of a period of social conservatism associated with a utilitarian view of education as a service to the old technology.

The quality of mathematics education is of foremost concern in both developed and undeveloped countries. If mathematics education is to be other than reactive it requires a set of principles by which to develop and through which to evaluate its purpose and function in a changing world. This paper has served to open a debate by suggesting principles for action based upon a framework set outside mathematics education itself. As such it presents a set of propostions for discussion and contest

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