Mathematics for Physics

A Guided Tour for Graduate Students

An engagingly written account of mathematical tools and ideas, this book provides a graduate-level introduction to the mathematics used in research in physics.

The first half of the book focuses on the traditional mathematical methods of physics: differential and integral equations, Fourier series and the calculus of variations. The second half contains an introduction to more advanced subjects, including differential geometry, topology and complex variables.

The authors' exposition avoids excess rigour whilst explaining subtle but important points often glossed over in more elementary texts. The topics are illustrated at every stage by carefully chosen examples, exercises and problems drawn from realistic physics settings. These make it useful both as a textbook in advanced courses and for self-study. Password-protected solutions to the exercises are available to instructors at www.cambridge.org/9780521854030.

MICHAEL STONE is a Professor in the Department of Physics at the University of Illinois at Urbana-Champaign. He has worked on quantum field theory, superconductivity, the quantum Hall effect and quantum computing.

PAUL GOLDBART is a Professor in the Department of Physics at the University of Illinois at Urbana-Champaign, where he directs the Institute for Condensed Matter Theory. His research ranges widely over the field of condensed matter physics, including soft matter, disordered systems, nanoscience and superconductivity.

MATHEMATICS FOR PHYSICS

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MICHAEL STONE
University of Illinois at Urbana-Champaign

and

PAUL GOLDBART
University of Illinois at Urbana-Champaign

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521854030

First published 2009

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-85403-0 hardback

Cambridge University Press
978-0-521-85403-0 - Mathematics for Physics: A Guided Tour for Graduate Students
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To the memory of Mike's mother, Aileen Stone: $9 \times 9 = 81$.

To Paul's mother and father, Carole and Colin Goldbart.

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Preface

This book is based on a two-semester sequence of courses taught to incoming graduate students at the University of Illinois at Urbana-Champaign, primarily physics students but also some from other branches of the physical sciences. The courses aim to introduce students to some of the mathematical methods and concepts that they will find useful in their research. We have sought to enliven the material by integrating the mathematics with its applications. We therefore provide illustrative examples and problems drawn from physics. Some of these illustrations are classical but many are small parts of contemporary research papers. In the text and at the end of each chapter we provide a collection of exercises and problems suitable for homework assignments. The former are straightforward applications of material presented in the text; the latter are intended to be interesting, and take rather more thought and time.

We devote the first, and longest, part (Chapters 1–9, and the first semester in the classroom) to traditional mathematical methods. We explore the analogy between linear operators acting on function spaces and matrices acting on finite-dimensional spaces, and use the operator language to provide a unified framework for working with ordinary differential equations, partial differential equations and integral equations. The mathematical prerequisites are a sound grasp of undergraduate calculus (including the vector calculus needed for electricity and magnetism courses), elementary linear algebra and competence at complex arithmetic. Fourier sums and integrals, as well as basic ordinary differential equation theory, receive a quick review, but it would help if the reader had some prior experience to build on. Contour integration is not required for this part of the book.

The second part (Chapters 10–14) focuses on modern differential geometry and topology, with an eye to its application to physics. The tools of calculus on manifolds, especially the exterior calculus, are introduced, and used to investigate classical mechanics, electromagnetism and non-abelian gauge fields. The language of homology and cohomology is introduced and is used to investigate the influence of the global topology of a manifold on the fields that live in it and on the solutions of differential equations that constrain these fields.

Chapters 15 and 16 introduce the theory of group representations and their applications to quantum mechanics. Both finite groups and Lie groups are explored.

The last part (Chapters 17–19) explores the theory of complex variables and its applications. Although much of the material is standard, we make use of the exterior calculus, and discuss rather more of the topological aspects of analytic functions than is customary.

A cursory reading of the Contents of the book will show that there is more material here than can be comfortably covered in two semesters. When using the book as the basis for lectures in the classroom, we have found it useful to tailor the presented material to the interests of our students.

Acknowledgments

A great many people have encouraged us along the way:

- Our teachers at the University of Cambridge, the University of California-Los Angeles, and Imperial College London.
- Our students – your questions and enthusiasm have helped shape our understanding and our exposition.
- Our colleagues – faculty and staff – at the University of Illinois at Urbana-Champaign – how fortunate we are to have a community so rich in both accomplishment and collegiality.
- Our friends and family: Kyre and Steve and Ginna; and Jenny, Ollie and Greta – we hope to be more attentive now that this book is done.
- Our editor Simon Capelin at Cambridge University Press – your patience is appreciated.
- The staff of the US National Science Foundation and the US Department of Energy, who have supported our research over the years.

Our sincere thanks to you all.