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Mathematics Student Teachers' Modelling Approaches While Solving the Designed Eşme Rug Problem

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ABSTRACT

The purpose of the study is to analyze the mathematics student teachers' solutions on the Eşme Rug Problem through 7-stage mathematical modelling process. This problem was designed by the researchers by considering the modelling problems' main properties. The study was conducted with twenty one secondary mathematics student teachers. The data were collected from the participants' written solutions related to the problem. To analyze the students' answers through mathematical modelling process, the researchers compiled 7-stage mathematical modelling process from the literature. It was observed that the problem created an appropriate process for mathematical modelling. While examining the solutions of the participants who were informed about this 7-stage mathematical modelling process, it was generally observed that their solution approaches toward the problem decreased while progressing in modelling stages.

Keywords: mathematical modelling, mathematical modelling process, modelling problem, mathematics student teachers

INTRODUCTION

One of the most important goals of mathematics education is to make students understand the value of mathematical modelling (Lingefjärd, 2006; Ministry of National Education [MNE], 2005 National Council of Teachers of Mathematics [NCTM], 2000). In the book of NCTM, Principles and Standards for School Mathematics (2000), it is emphasized that the students should use mathematical models by beginning from pre-school education till the end of the high school. In addition to this, in other countries, such as Germany, Australia, Switzerland etc., mathematical modelling has been gaining more importance and appearing extensively in

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State of the literature

- The modelling problems are non-routine and complex problems that predictions and assumptions are important to be able to interpret real world situations and including the structure revealing the mathematical modelling process.
- Mathematical modelling is a problem solving process including rich cognitive and metacognitive activities that enables the understanding of the relations within the nature of the problems from real world, the assumptions and the relations among assumptions are explained with mathematical models by discovering the factors in problems, the solutions are adapted to the real world by interpreting them with the help of mathematical models.

Contribution of this paper to the literature

- It is seen that the studies examining the mathematical modelling competencies in the solution process of modelling problems are inadequate.
- The data obtained from this study were analyzed by using the detailed cognitive modelling processes of Berry and Houston (1995) and Borromeo Ferri (2007).
- It is important that the student teachers who will implement modelling problems in their future professional life are provided awareness by engaging in solutions of modelling problems and developed their necessary skills for teaching mathematical modelling.
- To enable students to engage in different real world problems to be able to develop their mathematical modelling competencies, the Eşme Rug problem including different solution strategies is thought as an appropriate teaching material to be used in mathematics lessons.

the curriculum from the primary up to the end of high school (Blomhøj & Kjeldsen, 2006; Blum, 2002; Niss, 1989; Skolverket, 2006; Stillman, Brown, Galbraith & Edwards, 2007). In the high school mathematics curriculum in Turkey (MNE, 2005), two of the main goals of the mathematics teaching are expressed as that the students should be able to develop problem solving strategies and to use these strategies to solve real world problems and that the students should able to develop models and to associate the models with verbal and mathematical expressions.

Models are defined as conceptual systems that explain and define mathematical concepts, tools, relations, actions, forms and settings all of which contribute to problem solving situations (Doğan Temur, 2012). Mathematical modelling in which the so-called models are constructed has an important place to conduct mathematics education according to its purposes as seen in our national mathematics curricula. Nevertheless, the mathematical modelling problems which enable students to use modelling in understanding and interpreting the real world situations, and to develop their modelling skills have not an adequate place in the national mathematics curriculum (Hıdıroğlu, Tekin & Bukova Güzel, 2010).

Mathematical modelling is defined as translation of real world problems into mathematical problems, formulating mathematical models necessary for solving problems and interpretation of the results (Berry & Nyman, 1998 cited in Bukova-Güzel, 2011). Heyman (2003) defines mathematical modelling as the application of mathematics into the real world; highlights its relation to the real world and again describes it as an easy way of presenting this relation (cited in Peter-Koop, 2004). Yanagimoto (2005) discussed mathematical modelling as not just a process of solving a real world problem using mathematics but applying "mathematics which is useful in society".

Mathematical modelling requires students interpreting a real world situation, putting this situation into mathematical terms in a way that they can understand, interpreting the data in the problem, choosing the related data, identifying the operations leading to new data and creating meaningful representation (Lesh & Doerr, 2003). According to Doerr (1997), students criticize cognitive models and their self-perceptions, transfer their models considering assumptions, etc., and go back to the problem situation if necessary in the stages of the mathematical modelling process.

When examining the definitions of modelling in the so-called literature, researchers impress generally on real world problems and problem solving process. English and Watters (2004), and Verschaffel, De Corte, and Borghart (1997) state that the usage of word problems in lessons remain incapable to enable students to reach the basic objectives, and this kind of word problems do not give sufficient experiences to students to solve real life problems. The mathematical modelling enabling the implementation of mathematics in real world situations is generally perceived as a multi-digit or circular problem solving process which uses mathematics to discuss real world phenomenon (Ärlebäck, 2009).

LITERATURE REVIEW

Mathematical modelling problems are one of the instruments to enable students to engage in real world situations. Modelling problems are open-ended and non-standard problems requiring the students to make assumptions about the problem situation, estimate relevant quantities before engaging in simple calculations and contain complex process (Ärlebäck, 2009). These problems also require guesses to acquire the necessary information in the problem and besides these they can be solved in different approaches (Taplin, 2007). The modelling problems are real world problems not given enough information, required realistic predictions and assumptions, elaborated calculations, promoting students to use their knowledge and benefit from their experiences (Taplin, 2007).

THE MATHEMATICAL MODELLING PROCESS

When examining the literature regarding mathematical modelling, the existences of different modelling processes are remarkable. Borromeo Ferri (2006) claims that this difference depends on how the researchers understand the modelling and the complexity of the given problem in some situations. In this study, the mathematical modelling process used on the

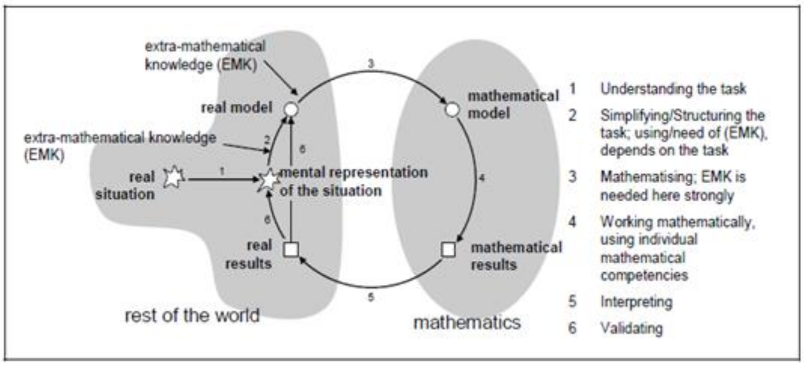


Figure 1. Modelling Cycle under the Cognitive Perspective (Borromeo Ferri, 2007)

purpose of examining the student teachers' solutions about Eşme Rug Problem (ERP) was formed by compiling the mathematical modelling process of Berry & Houston (1995) and the modelling cycle under the cognitive perspective of Borromeo Ferri (2007) (see in **Figure 1**). Berry and Houston (1995) explain mathematical modelling process with six stages as understanding the problem, choosing variables, making assumptions, solving the equations, interpreting the solution, validating the model, and criticizing and improving the model.

When forming the 7-stage modelling process by giving attention to these modelling processes of Berry & Houston (1995) and Borromeo Ferri (2007), the researchers considered the participants' approaches while solving the problem. In this context, some stages are in both researchers' modelling processes and some of them are just in one process. Which approaches are in each stage of modelling process formed by the researchers and why these processes are considered differently from the stages explained as follows:

S_1. Understanding the Problem: The real world problem is defined and the problem is examined by required data for the problem. To elicit the experiences regarding the real world situation and examine the scope of the real world situation, it is necessary to understand the problem. This stage has the same content with the first stage taken in both Berry & Houston and Borromeo Ferri's modelling processes.

S_2. Choosing Variables and Making Assumptions: The variables and the assumptions are identified for the solution of the problem with reference to the real world situation. The variables to be used in the construction of the model are defined in this stage. Because this stage is not in the modelling process improved by Borromeo Ferri, the second and the third stages in the Berry & Houston's modelling process are gathered in this stage.

S_3. Mathematising: It requires transforming the real world into the mathematical world. Mathematical concepts required for solution are identified. Especially these questions should be answered: "Which area of mathematics concern the most appropriate strategy to solve the problem?", "Which concepts elicit the relation among the variables best?". In this stage, the general solution strategy is identified. Because there is no appropriate stage in the modelling

cycle developed by Berry & Houston, the third stage of Borromeo Ferri is discussed in this stage.

S_4. Constructing Mathematical Models and Correlating Them: The mathematical model/s to present or define the real world situation is constructed by using mathematical structures such as graphics, tables, equations etc. in accordance with the assumptions, pre-knowledge and mathematical abilities. Because mathematical model/s which are appropriate to the problem statement are constructed after the mathematising stage, this stage not dealt as a separate stage by either researchers is situated particularly.

S_5. Working Mathematically: The solution of the problem is figured out through developed mathematical model/s. The mathematical results regarding the real world are gained by solving the mathematical model/s. This stage taken part in the modelling cycle of Borromeo Ferri is defined as solving equations in the modelling process of Berry & Houston.

S_6. Interpreting Solutions: The mathematical results obtained from the solution of the problem are analyzed and the solution is expressed and evaluated verbally. The mathematical results are interpreted in the context of the real world situation. This stage is found in the modelling processes of both researchers. But Berry & Houston consider interpreting and validating in the same stage. Because of the thoughts regarding the fact that the validation could not occur in interpreting situations, the researchers distinguished this stage from the validation stage.

S_7. Validating the Model: The data needed for the validation of the model are decided. Whether the model is appropriate for the situation or not is tested by using these data. The model and the results obtained by solving the model are examined. The estimations, measurements and variables are discussed with their ins and outs, and compared with each other toward the strategies. This stage is also in both researchers' modelling processes. Because there is no approaches concerning the criticizing and developing the model in the examinations done by the researchers, the last stage of Berry & Houston is not given.

In this study, the mathematics student teachers' approaches and strategies during the solution of ERP are examined according to the compiled 7-stage mathematical modelling process. With this study, it is thought to make contribution to the so-called literature by examining the student teachers' modelling approaches. The data obtained in our study was analyzed by using cognitive modeling processes of Berry and Houston (1995) and Borromeo Ferri (2007). Thus, the modelling competencies of the high school mathematics student teachers were analyzed in more detail by utilizing these theoretical frameworks. It is of main importance that the student teachers who will implement this kind of problems in their professional life should be provided awareness and developed required skills by being engaging in modelling problems because it will affect the quality of their future instruction. In this context, the purpose of this study is to analyze the mathematics student teachers' solutions on a designed modelling problem named the ERP through the 7-stage mathematical modelling process.

METHOD

The study is designed as a qualitative case study to analyze the mathematics student teachers' solutions on the ERP through 7-stage mathematical modelling process while working individually.

Participants

The research was carried out with twenty one high school mathematics student teachers, twelve female and nine male. The participants were 4th year of their program in the fall term of 2010-2011 and also took Mathematical Modelling Course. The participants' real names were not given in the study, and they were labeled using the codes: P_1, P_2, P_3, …, P_{21}.

In Turkey, the high school mathematics teacher education program enrolled in the participants is a five-year program. In these programs, courses related to subject matter knowledge, general pedagogical knowledge, and pedagogical content knowledge are taught, and apart from this, there are courses related to school-based placement in the last three semesters. One of these courses is an elective course called Mathematical Modelling. The Mathematical Modelling Course's content was about model/modelling, mathematical modelling process, the structure of mathematical modelling problems. Additionally, various applications regarding the solutions of different modelling problems were realized.

The criterion sampling from purposeful sampling methods was used for choosing the participants. The basic insight in this sampling method is that the situations meeting a range of criteria (Yıldırım & Şimşek, 2005) are considered. The so-called implementation was done in the last week of the mathematical modelling course taken by the participants. Accordingly, the criterion sampling is that the participants have the essential knowledge and abilities regarding mathematical modelling.

Instruments

The data were collected from the participants' written solutions of the ERP (see **Figure 2**).

The ERP designed by the researchers was a modelling problem. When the ERP was designed, it was considered that the problem;

- was appropriate to the students' levels and open-ended,
- included the situations that the students could understand in their real world,
- permitted students to discover, interpret and evaluate,
- allowed students to use their experiences, do round calculations and estimate,
- was attractive, clear and understandable for students.

So as to enable students to benefit from the estimations and measurements in the solution of the problem, the researchers took care that the problem statement did not include

The colors of strings used in the carpet from the bottom pattern to outmost pattern and one meter prices of them are given below:

The black string is 10 kr, the yellow one is 20 kr, the green one is 30 kr, the red one is 20 kr, the white one is 40 kr, the dark blue one is 50 kr, the light blue one is 50 kr and cream one is 40 kr.

100 cm^2 section of the carpet is given on the left side and 5 meters string is used in it. In this case how much do these strings cost to weave this carpet?

(1 Turkish Lira equals to 100 Kuruş (kr). But this information was not given to the participants in the problem state.)

Figure 2. The Eşme Rug Problem

the data regarding every variables when designing the problem. For examples, the data concerning the sizes of the rug, the rope amount in different colors used when weaving the rug, the rope amount to be used for the tags, etc. could not be given.

The reason why the ERP was chosen for the real world situation was that Eşme is a distinguished district which is renowned for the precious rugs of its own. Weaving rugs reserves an important place in Turkey. Numerous tourists buy rugs as presents and weaving rugs are mainstay for certain people. Whole ground of these rugs is filled with small and geometrical motifs. Eşme Touristic Rug Culture & Art Festival is organized every year in the last week of June. Another reason why the ERP was designed and used is that this problem enables knowledge regarding both mathematics and geometry.

Data Analysis

In the analysis of the problem solutions, the rubric (see **Table 1**) intended for the 7-stage mathematical modelling process was used. The participants' solutions of the ERP were analyzed through content analysis by using the so-called 7-stage mathematical modelling process. This 7-stage was labeled as S_1, S_2, S_3, S_4, S_5, S_6, S_7 and consist of;

S_1: understanding the problem,
S_2: choosing variables and making assumptions,

S_3: mathematizing,
S_4: constructing mathematical models and correlating them,
S_5: working mathematically,
S_6: interpreting solutions,
S_7: validating the model.

Table 1. The rubric of 7-stage mathematical modelling process

The Stage	Performing no approach (neither true nor false)	Performing partly appropriate approach	Performing appropriate approach
S_1	Not to understand or to understand wrong.	To understand partly but include reasonable mistakes.	To express the problem his own sentences, to determine what is given and asked.
S_2	Not to identify necessary and unnecessary variables for the model, not to make assumptions.	To partly identify necessary and unnecessary variables for the model, not to make enough assumptions.	To present the givens and goals exactly, identify necessary and unnecessary variables for the model, make realistic assumptions.
S_3	Not to express the problem mathematically or express it in a wrong way.	To partly identify the mathematical concepts and symbols needed and express how to use them.	To identify the mathematical concepts and symbols needed and express how to use them exactly.
S_4	Not to construct mathematical model/s or construct wrong model/s.	To construct mathematical model/s but not to correlate it/them exactly.	To construct exactly right mathematical model/s improved for different situations, and correlate it/them.
S_5	Not to solve the constructed model correctly or not to display any solution approaches.	To solve the constructed model but include some mistakes or not to reach the result.	To solve the constructed model in a totally right way and not to include mathematical mistakes.
S_6	Not to deduce mathematical results from the solution or deduce wrong results.	To deduce mathematical results from the solution partly and interpret them incompetently.	To deduce mathematical results from the solution exactly, interpret them, and adapt them to the real world.
S_7	Not to validate the model/s or validate it in a wrong way.	To display approaches when testing the validation of the model/s partly.	To test the validation of the model/s and show its availability for different situations.

The participants' solutions of the ERP were examined by four researchers separately. For the inter reliability, the five participants' solutions were randomly chosen by two researchers. Two researchers' evaluations regarding this five solution papers according to the rubric were compared. In accordance with this comparison, 32 codes matched together and 3 codes not matched were identified. By using the inter-coder reliability formula (Miles & Huberman, 1994) the inter-coder reliability was calculated as 91%. At the end of this calculation, the researchers reached a consensus by comparing their examinations done separately.

The results were presented in **Table 2**. In **Table 2** there are the columns showing performing no approach (neither true nor false), performing partly appropriate approach and

Table 2. The analysis of the participants' approaches according to the stages of the mathematical modelling process

The Stage	Performing no approach (neither true nor false)	Performing partly appropriate approach	Performing appropriate approach
S_1			P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}
S_2			P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}
S_3		P_5, P_8, P_{10}, P_{21}	P_1, P_2, P_3, P_4, P_6, P_7, P_9, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}
S_4		P_1, P_5, P_7, P_8, P_{10}, P_{15}, P_{20}, P_{21}	P_2, P_3, P_4, P_6, P_9, P_{11}, P_{12}, P_{13}, P_{14}, P_{16}, P_{17}, P_{18}, P_{19}
S_5		P_5, P_7, P_8, P_{10}, P_{11}, P_{12}, P_{15}, P_{20}, P_{21}	P_1, P_2, P_3, P_4, P_6, P_9, P_{13}, P_{14}, P_{16}, P_{17}, P_{18}, P_{19}
S_6	P_5, P_6, P_8, P_9, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}	P_1, P_2, P_3, P_4, P_7, P_{10}, P_{21}	
S_7	P_1, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}	P_2	

performing appropriate approach respectively and the rows including each stages. The approaches regarding the basic stages of the modelling process displayed by each participant in the solution transferred to the tables using the participants' codes toward the three levels in question. Additionally, the results are supported with the extracts taken from the participants' solutions. The Turkish expressions in these extracts are presented in English.

RESULTS

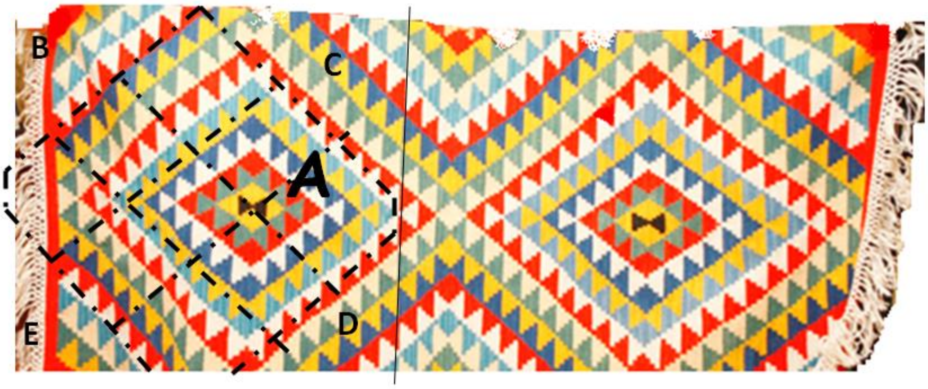
Analysis of the approaches of the participants' solutions regarding the ERP as to 7-stage mathematical modelling process while they were solving the problem individually is given in **Table 2**.

When **Table 2** was examined, it was clearly seen that the participants did not get any trouble in the stage of understanding the problem. The participants expressed the ERP with their own words and put forward the givens and goals for the solution. For example, P_1 expressed what he understood from the problem statement as "*I was asked for calculating the area of the rug and modelling it by using the rug whose extract was given.*" When considered the statement of P_1, it was seen that he thought to reach the area of the rug by the help of the extract given in the problem. P_1 tried to understand the problem by making his first deductions.

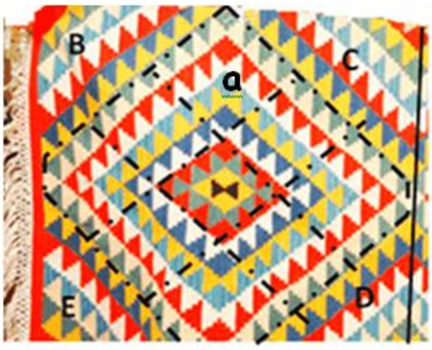
When the participants' written solutions were examined, it was seen that they showed exactly appropriate approaches in the stage of choosing the variables and making assumptions (see **Table 2**). When the participants defined the variables, they presented what is given and asked exactly and identified the variables necessary for the model. The participants also considered the unnecessary data at the beginning of the process but later while continuing the process they recognized that those data were not needed and did not use them. The participants explained the reasons why they identified those variables and provided rational and consistent justification of their assumptions. They simplified the problem by making certain assumptions. For example, P_{12} examined the factors thought important for the problem data and deduced from them: *"The black color used in this rug minimum. Because the rug is symmetric, let's divide the rug into two pieces and interest in only half of it. If I look at the rug cautiously, the colors always form a triangle and I accept that the triangles are the congruent triangles. In addition, the bottom of the rug does not look in the photo but I can estimate this because the shapes are identical. … I accepted the triangles as isosceles triangles. Let's think the base length is equal to the altitude drawn to this base."* He made assumptions such as dealing the half of the rug by dividing it into two pieces and accepted the patterns as equal and isosceles triangle and tried to shape the general solution strategy.

In the stage of mathematising which is the first stage of transition to a mathematical solution, the participants generally showed appropriate approaches (see **Table 2**). They tried to present their mathematical statements by dealing with their approaches in the former stage. A great majority of the participants divided the rug as triangles, quadrangles, squares, etc. and wrote their statements according to these divisions. The variables were expressed with the mathematical symbols to help constructing mathematical models and the basic equalities were presented.

The participants generally supported their geometric statements with mathematical symbols by identifying the smallest pieces given in the extract of the rug with the triangles. However, some participants did not completely give the reason why they showed such approaches while expressing their approaches mathematically (see **Figure 3**). For example, P_8 presented his general solution strategy by dividing the rug differently. But he ignored that the used ropes were in different colors in this process. Therefore, it is thought his approach in this stage is a deficit approach in finding the price of the rug.



I divided the rug into two pieces. A = B+C+D+E



I can see that there is a big square showed by A on the mid-left and B, C, D, E and the square section are formed of triangles. Because the right is the similar, it is showed that the A-area squares are totally four.

Rug=4A

I can count that the a-area sections of the A-area square are nine. Because, A=9a and Rug=4A, Rug=36a.

Figure 3. An extract of P8's solution related to mathematising

In the stage of constructing mathematical models and correlating them, while the eight participants' approaches were partly appropriate, the thirteen participants' approaches were exactly appropriate (see **Table 2**). Some participants caught different patterns in rug designs and different participants formed different models. For example, P_1 figured out a pattern, among the numbers of triangles in the inside out quadrilateral during the model constructing process (see **Figure 4**). P_1 constructed the mathematical model as $(8+4n).2 + (8+4(n-1)).2$ by using the pattern. He constructed this model in an effort to find the numbers of the triangles among the quadrangles drawn by him. He expressed the number of quadrangles inside-out as n. But in the model constructed by him there are 40 triangles for n=1. Hence the model constructed by P_1 does not give real results. If the thought model had been defined as $(8+4(n-2)).2 + (8+4(n-3)).2$, he would have constructed the mathematical model giving the number of triangles between two areas inside-out correctly.

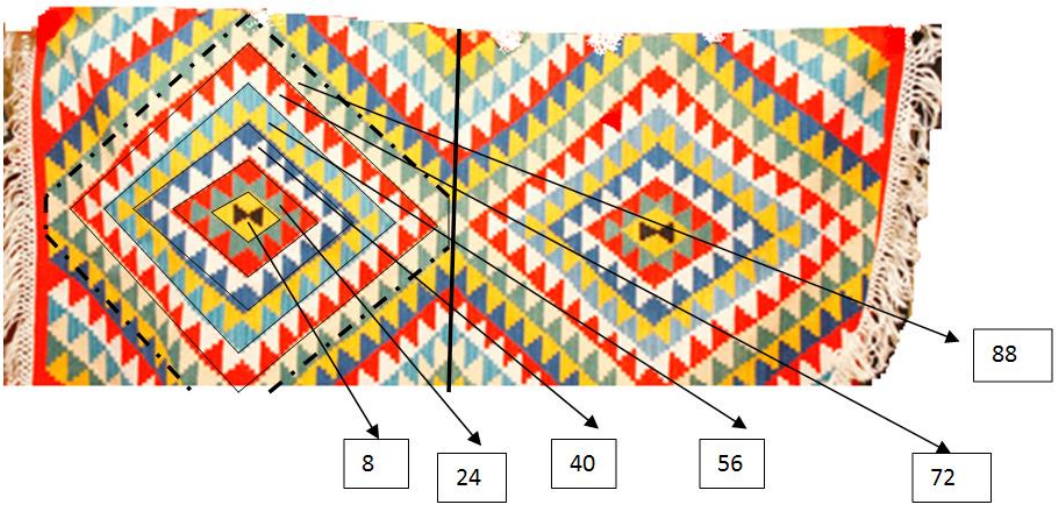
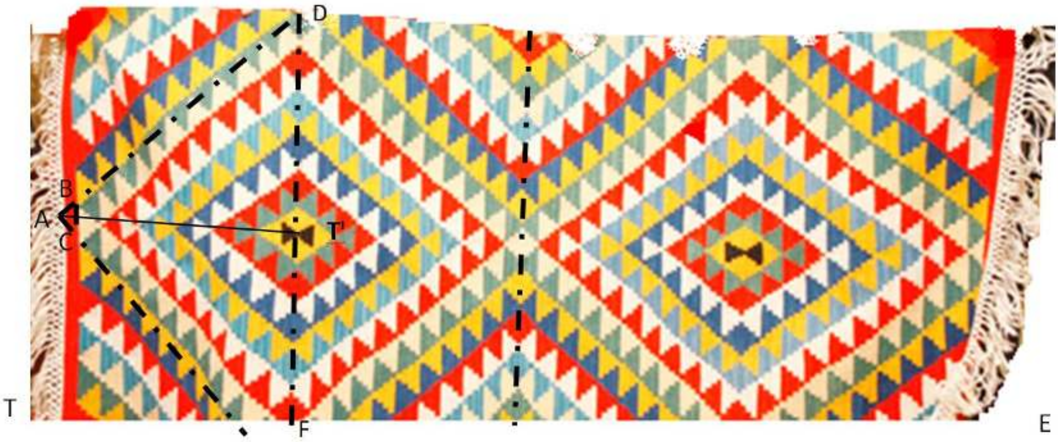


Figure 4. An extract of P_1's solution related to constructing mathematical models and correlating them

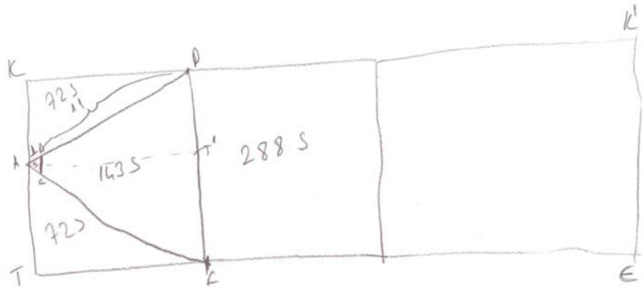
In **Figure 5**, the approach of P_6 was given in which he benefited from the similarity in triangles and tried to find out the numbers of triangles in the rug and took the advantage of the similarity between ΔABC and ΔADF by dividing the rug into pieces. P_6 calculated the ratio between the triangles' areas by using the ratio of similitude between the ΔABC and ΔADF . He posed the relation between the area of the rug and the area of ΔABC accepted by him as a smallest pattern forming the rug.

Some participants (P_5, P_8, P_{10}, P_{15}) tried to get the result by estimating and tried to solve the problem without creating a mathematical model which can reveal the relation between variables. Some participants (P_7, P_{20}, P_{21}) counted the triangle patterns on the rug and tried to reach the solution without creating a model revealing the algebraic rule among the patterns. For example, P_{21} counted the triangles one by one by considering the different colors as seen follows:

9 dark blue+
10 yellow+
11 green+
11 cream+
1 yellow, 1 green, 1 cream, 1 red, 9 white
1 red, 1 dark blue, 1 yellow, 1 green, 1 cream, 1 red, 1 white, 8 light blue, 7 yellow
1 light blue, 1 white, 2 red, 1 dark blue, 1 yellow, 1 green, 1 cream, 1 red, 2 yellow,
2 light blue, 1 yellow, 6 dark blue, 5 white
4 red, 3 green, 2 white, 1 yellow, 2 light blue, 1 white, 2 red, 1 cream, 1 green, 1 yellow,
1 dark blue, 1 red, 1 white, 1 light blue, 1 cream, 1 green



Because $\Delta ABC \sim \Delta ADF$, the similarity rate is $\frac{|AB|}{|AD|} = \frac{1}{12}$



The proportion of the areas is $\frac{A\,(\Delta ABC)}{A(\Delta ADF)} = \frac{1}{144}$.

In other words, if there is one triangle in $\overset{\Delta}{ABC}$, there are 144 triangles in $\overset{\Delta}{ADF}$.

$$\frac{A(AKDT')}{A(KK'ET)} = \frac{1}{8}.$$

There are 1152 triangles in the $KK'ET$ rectangular in total

Figure 5. An extract of P_6's solution related to constructing mathematical models and correlating them

In the stage of working mathematically, the nine participants displayed partly appropriate approaches and the twelve participants displayed appropriate approaches (see **Table 2**). P_2 reached the price of the ropes by considering all of the each color in the rug when solving the model. For instance, for the red color he benefited from the equality of *the cost of the red ropes used=the number of the red triangles x the rope amount used in the triangle (m) x the price of the red rope in one meter* and for each color he did similar solutions. A section related to the solution of P_2 was given as follows:

To find the total cost,		
For the red rope;	$204 \times \frac{5}{32} \times 20 \Rightarrow 637{,}5$ kurus	$\Rightarrow$ 6,375 liras
For the light blue rope;	$108 \times \frac{5}{32} \times 50 \Rightarrow 843{,}75$ kurus	$\Rightarrow$ 8,43 liras
For the green rope;	$140 \times \frac{5}{32} \times 30 \Rightarrow 656{,}25$ kurus	$\Rightarrow$ 6,56 liras
For the dark blue rope;	$116 \times \frac{5}{32} \times 50 \Rightarrow 906{,}25$ kurus	$\Rightarrow$ 9,06 liras
For the white rope;	$92 \times \frac{5}{32} \times 40 \Rightarrow 575$ kurus	$\Rightarrow$ 5,75 liras
For the black rope;	$4\times \frac{5}{32} \times 10 \Rightarrow 6{,}25$ kurus	$\Rightarrow$ 0,06 liras
For the cream rope;	$124 \times \frac{5}{32} \times 40 \Rightarrow 775$ kurus	$\Rightarrow$ 7,75 liras
For the yellow rope;	$78 \times \frac{5}{32} \times 20 \Rightarrow 243{,}75$ kurus	$\Rightarrow$ 2,43 liras
	Total= 46,415 liras.	

Some participants (P_5, P_7, P_8, P_{10}, P_{11}, P_{12}, P_{15}, P_{20}, P_{21}) could not approach effectually in solving the problem mathematically because they made calculation mistakes and got trouble in transformation from lira to kurus and from cm^2 to m^2. For example, P_{12} put up a wrong ratio to find the number of strings in a square centimeter and could not solve the model in a right way. P_{12} hypothesized that the smallest triangle was an isosceles triangle. He accepted that the unequal base of the isosceles triangle and the altitude drawn on this base as x cm and continued the solution. He indicated the area of this triangle as $\frac{x^2}{2}\ cm^2$ in the constructed model. He calculated the number of triangles in the rug for each color separately and then the area of each colored sequences. For example, he found the area for red color in the whole rug as $70 \times \frac{100}{16}\ cm^2 \cong 400\ cm^2$. But afterwards he proportioned in a wrong way and expressed that the amount of the rope to be used in $400\ cm^2$ as $40\ m$, not as $20\ m$ when 5-meter rope was used. Because he repeated the same proportion mistake for other colors, he found the total rope amount used in the rug more than twice that he had to find. P_{12}'s these approaches were seen as follows:

	The total area of colored triangles (cm^2)	The rope amount used in the rug (m) x 1-meter price (kurus) = the price of the rope (kurus)
Red:	$70 \times \frac{100}{16} \cong 400$	$\Rightarrow 40\ .20 = 800$ kurus
Green:	$55 \times \frac{100}{16} \cong 300$	$\Rightarrow 30\ .30 = 900$ kurus
Black:	$4 \times \frac{100}{16} \cong 20$	$\Rightarrow 2\ .10 = 20$ kurus
Yellow:	$160 \times \frac{100}{16} \cong 1000$	$\Rightarrow 100\ .20 = 2000$ kurus

Dark blue:	$140 \times \frac{100}{16} \cong 900$	$\Rightarrow 90.50 = 4500$ kurus
Light blue:	$110 \times \frac{100}{16} \cong 600$	$\Rightarrow 60.50 = 3000$ kurus
Cream:	$120 \times \frac{100}{16} \cong 800$	$\Rightarrow 80.40 = 3200$ kurus
White:	$124 \times \frac{100}{16} \cong 800$	$\Rightarrow 80.40 = 3200$ kurus

It has been seen that the participants could not perform exactly appropriate approaches in the stage of interpreting solutions to the real world (see **Table 2**). A great majority of the participants (14 participants) did not need to interpret solutions to the real world. However, it has been seen that some participants (P_1, P_2, P_3, P_4, P_7, P_{10}, P_{21}), while calculating the cost, considered the amount of the strings used in the red lines surrounding the triangle patterns or in the eaves of the rug. For example, P_3 made assumptions for the strings used in the cream lines and eaves and took their cost into account. His thoughts were as follows: "*All pieces' prices*=$\frac{423}{8}$ *liras. If I hypothesize that 20-meter rope are used for the cream tags in the borders, it will be 8 liras. If I hypothesize 20-meter rope are used in the red band in the borders, it will be 4 liras. The cost of the rug is* $12 + \frac{423}{8} = 64{,}8 \cong 65$ *liras.*"

A great majority of the participants (P_5, P_6, P_8, P_9, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}) tried to find the cost without considering the amount of string in the eaves and lines and ignored the difference between inside out color ordering of small triangles in the areas A_1, A_2, A_3, A_4, A_5, A_6 and the color order of A areas (see **Figure 6**). For this reason, these participants could not approach effectively in interpreting the mathematical results of the model and adapting to the real world.

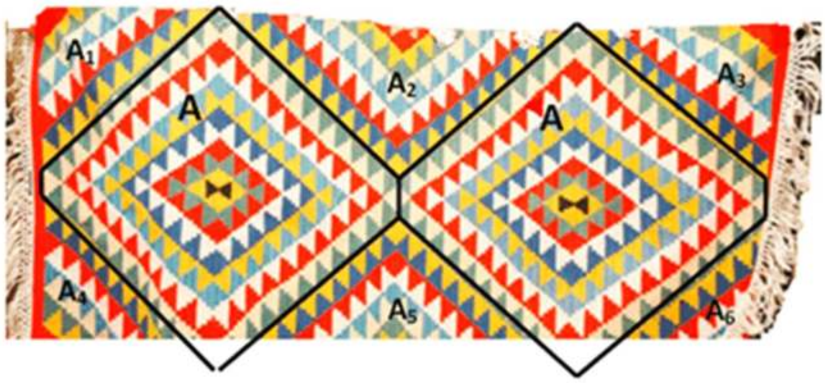


Figure 6. The presentation of the rug cut into small parts

In the stage of validating the model, none of the participants except P_2 showed neither true nor false approach (see **Table 2**). The participants did not test the validity of the models and the results deriving from the models by using appropriate data. P_2 stated that he had hesitations about the functionality of his model and the model was approximately true after reaching the solution as "*I cannot say that the model I constructed is not very functional model.*

Counting pieces one by one can cause a lot of work. But I can say that the value I found is approximately true."

CONCLUSION, DISCUSSION AND IMPLICATIONS

In the study examining the mathematics student teachers' approaches and strategies during the solution of the ERP according to the 7-stage modelling process, it was generally identified that the participants' performed approaches gradually decreased in modelling stages. It was seen that the student teachers divided the rug patterns in different shapes and so this situation caused them to reach the solution by constructing different mathematical models and patterns. In general terms, the student teachers had difficulties in the solution starting from the third stage of mathematising and these difficulties showed increases starting from the stages of constructing mathematical models and correlating them. Similarly, Hıdıroğlu, Tekin and Bukova-Güzel (2010) stated that 11th grade students' success at the very beginning of the stage decreased gradually from the beginning till the end of the process, and were unsuccessful at interpreting and validating stages. According to the results, it is suggested that the student teachers should be encouraged in every stage of the modelling process especially at interpreting and validating stages. It is also suggested to discuss mistakes made by student teachers and how to overcome those mistakes.

The student teachers generally offered a consistent solution when solving the ERP. The chosen strategies were meaningful in the modelling process whereas they did not adequate for the whole modelling process. Peter-Koop (2003, 2004, 2009) emphasized the modeling problems, like Fermi problems, were solved by students meaningfully and rationally and the solution process included modelling process. The participants constructed mathematical models when solving the ERP and used their mathematical and geometric knowledge by considering them occasionally and integrating them when constructing these models.

The results showed that the student teachers did not have difficulty in understanding the problem. In parallel, Peter-Koop (2004) emphasized that problem solvers did not have difficulty in understanding the problem. All of the participants could choose appropriate variables, make reasonable assumptions, and use their knowledge and experiences. To determine the data which would be useful in the solution, Dirks and Edge (1983) stated the students tried to understand the problem adequately identified the assumptions simplifying complex situations and benefited from the previous knowledge and experiences.

In the mathematization stage, some participants directly mathematised the problem by using mathematical symbols while the others preferred using verbal statements. The student teachers determined the variables mathematically besides they benefited from the geometric shapes such as triangle, square, etc. and the mathematical concepts such as function, sequence, etc. These different mathematizations lead up to different mathematical models. Thus it was seen the student teachers constructed different models in the modelling process and correlated them. The thought that there was not just an accurate model to add meaning the real world situations (Yanagimoto, 2005) appeared also in this work. However, it was identified that all models constructed by the student teachers were not sufficient for the model to reflect the situation. The models constructed offered approximate solutions whereas some of them were not functional. It was stated that the modelers determined which models from the others were appropriate for expressing the situation was important (Bukova-Güzel, 2011; Yanagimoto, 2005). In this context, it should be provided that not functional models should be more functional by discussing with the student teachers. The certain student teachers displayed the approaches regarding solving the ERP with verbal statements instead of constructing models in the model construction stage. As a reason for this, it was thought that the so-called student teachers had troubles in constructing models. Similarly, Doerr (1997) emphasized that the students engaging in modelling problems had a great trouble in constructing model initially and they lost time in this phase. Some mathematics student teachers also could not approach effectually in solving the problem mathematically because they made mistakes in calculation and got trouble in turning Turkish Lira to kuruş and cm^2 to m^2. Not realizing these kinds of mistakes in their mathematical solutions prevented them to reach the real price of the rug.

None of the participants desirably interpreted the solutions. Only seven of them partially carried out interpreting solutions, and adapting them to the real world. In stage of validating model partially realized by only one participant, the participants got trouble. The basis of this trouble generally based on for the student teachers to focus on the result. At time the student teachers thought they found the result, and they did not show any approach in the last stage of modelling process. That the participants who were able to carry out the mathematical processes had troubles in this situation resulted in promoting the student teachers for displaying approaches in the interpretation and validation stages. For this, the environments where the student teachers had experiences in the phases such as how the results adapted the real world in the solutions of mathematical modelling problems, which interpretations they made, what kind of validation could be used should be provided. Also it was thought that the students' skills to assess these ideas could be developed by providing the student-teacher and student-student interactions and showing them these different ideas. For this reason, for the following studies, we suggest that collaborative group working should be used because of their possibility of arising a discussion, interpreting and validating the models during the modelling process. Arlebäck (2009) and Peter-Koop (2004) also emphasized this implementation as small study groups will provide rich environments for problems.

Because in the national curriculum, the integration of the mathematical modelling problems was offered on the purpose of bringing students in real world problem solving abilities (MNE, 2005), it was thought that the student teachers should be exposed this kind of problems. It is understood in this study that the ERP gave an opportunity to the cognitive processes of the student teachers and accounted for which modelling competencies the student teachers had and what kind of difficulties they had in the modelling stages. The pattern orders and some color irregularities inside-out in the ERP caused that the student teachers had different assumptions and predictions. This situation enabled to show up different assumptions. When considering the students are needed to engage in different real world problems to develop their mathematical modelling skills (Blum, 2002; Doğan Temur, 2012; Niss, 1989; Peter-Koop, 2004; Skolverket, 2006; Sriraman & Lesh, 2006; Stillman, Brown, Galbraith & Edwards, 2007), it is thought that the rug problem is a convenient problem to be used in instruction as a teaching material.

REFERENCES

Ärlebäck, J. B. (2009). On the use of realistic Fermi problems for introducing mathematical modelling in school. *The Montana Mathematics Enthusiast*, *6*(3), 331- 364.

Ärlebäck, J. B. & Bergsten, C. (2010). On the use of realistic fermi problems in introducing mathematical modelling in upper secondary mathematics. In R. Lesh, P. L. Galbraith, W. Blum & A. Hurford (Eds.), *Modelling students' mathematical modelling competencies*, ICTMA 13, 597-609, Springer.

Beerli, G. (2003). Realitaetsbezug im unterricht mit fermifragen. In H. Henn (Ed.), *Beiträge zum Mathematikunterricht,* (pp. 89-92). Hildesheim: Franzbecker.

Berry, J. & Houston K. (1995). *Mathematical modelling*. Bristol: J. W. Arrowsmith Ltd.

Blomhoj, M. & Kjeldsen, T.H. (2006). Teaching mathematical modelling through project work - Experiences from an in-service course for upper secondary teachers, *Zentralblatt für Didaktik der Mathematik–ZDM*, *38 (2),* 163-177.

Blum, W. (2002). ICMI Study 14: Applications and modelling in mathematics education-Discussion document. *Educational Studies in Mathematics*, *51*(1), 149-171.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process, *Zentralblatt für Didaktik der Mathematik –ZDM*, *38 (2)*, 86-95.

Borromeo Ferri, R. (2007). Personal experiences and extra-mathematical knowledge as an influence factor on modelling routes of pupils. In Pitta-Pantazi D. and Philippou, G. (Eds): *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education,* 2080-2089.

Bukova-Güzel, E. (2011). An examination of pre-service mathematics teachers’ approaches to construct and solve mathematical modelling problems. *Teaching Mathematics and Its Applications*, *30(1),* 19-36.

Dirks, M. K., & Edge, D. R., M. (1983). Problem solving: Enrico Fermi and the bull moose. *School Science and Mathematics*, *83(7),* 25-28.

Doerr, H.M. (1997). Experiment, simulation and analysis: An Integrated instructional approach to the concept of force. *International Journal of Science Education, 19*(3), 265-282.

Doğan Temur, Ö. (2012). Analysis of prospective classroom teachers’ teaching of mathematical modelling and problem solving. *Eurasia Journal of Mathematics, Science & Technology Education, 8(2),* 83-93.

English, L. D. & Watters, J. J. (2004). Mathematical modelling in the early school years. *Mathematics Education Research Journal*, *16(3),* 59-80.

English, L. D. (2006). Mathematical modelling in the primary school: Children’s construction of a consumer guide, *Educational Studies in Mathematics*, *63 (3),* 303-323.

Hıdıroğlu, Ç. N., Tekin, A., & Bukova Güzel, E. (2010). Öğrencilerin modellemede bireysel ve birlikte çalışarak ortaya koydukları yaklaşımlar ve düşünme süreçleri. *9. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi,* 21-26.

Kapur, J. N. (1982). The art of teaching the art of mathematical modelling. *International Journal of Mathematical Education in Science and Technology*, *13(2),* 185-192.

Lee, C., Chen, M., & Chang, W. (2014). Effects of the multiple solutions and question prompts on generalization and justification for non-routine mathematical problem solving in a computer game context. *Eurasia Journal of Mathematics, Science & Technology Education, 10(2),* 89-99.

Lesh, R., & Doerr, H. M. (2003). A modelling perspective on teacher development. In Lesh R. A. & Doerr H. (Eds.): *Beyond constructivism: Models and modelling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum, 3-33.

Lingefjärd, T. (2006). Faces of mathematical modelling. *Zentralblatt für Didaktik der Mathematik-ZDM, 38(2),* 96-112.

Miles, H. B. & Huberman, A. M. (1994). *Qualitative data analysis*. Second Edition. Thousand Oaks, CA:Sage.

Milli Eğitim Bakanlığı. (2006). *Ortaöğretim matematik dersi* öğretim *programı*. Ankara: Yazar.

National Council of Teachers of Mathematics (2000). Principles and standarts for school mathematics: An Overview. *National Council of Teachers of Mathematics*. Reston: Author.

Niss, M. (1989). Aims and scope of applications and modelling in mathematics curricula. In Blum, W. Berry, J. S. Biehler, R. Huntley, I. Kaiser-Messmer G. & Profke L. (Eds): *Applications and modelling in learning and teaching mathematics*. Chichester: Ellis Horwood, 22-31.

Peter-Koop, A. (2003). Modellbildungsprozesse beim bearbeiten von Fermi-Problemen in kleingruppen. In Ruwisch S. & Peter-Koop A. (Eds): *Gute Aufgaben im Mathematikunterricht der Grundschule*. Offenburg: Mildenberger, 111-130.

Peter-Koop, A. (2004). Fermi problems in primary mathematics classrooms: Pupils' interactive modelling processes. In Putt, I. Farragher, R. & McLean M. (Eds): *Mathematics education for the third millenium: Towards 2010* (*Proceedings of the 27th annual conference of the Mathematics Education*. Townsville, Queensland: MERGA, 454-461.

Peter-Koop, A. (2009). Teaching and understanding mathematical modelling through Fermi problem. In Clarke, B. Grevholm B. & Millman R. (Eds): *Tasks in Primary Mathematics Teacher Education*. Springer, 131-146).

Skolverket. (2006). *Förslag till kursplan [Curriculum proposal]*. Stockholm: Liber distribution, 2637-2638.

Sriraman, B., & Lesh, R. (2006). Modelling conceptions revisited. *Zentralblatt für Didaktik der Mathematik - ZDM, 38*(3), 247- 254.

Sriraman, B., & Knott, L. (2009). The Mathematics of estimation: possibilities for interdisciplinary pedagogy and social consciousness. *Interchange: A Quarterly Review of Education*, *40(2),* 205-223.

Stillman, G., Galbraith, P., Brown, J., & Edwards, I. (2007). A Framework for success in implementing mathematical modelling in the secondary classroom. *Mathematics: Essential Research, Essential Practice, 2,* 688- 697.

Taplin, M. (2007). *Teaching values through a problem solving approach to mathematics*. Available from: http://www.mathgoodies.com/articles/teaching_values.html

Verschaffel, L., De Corte, E., & Borghart, I. (1997). Pre-service teachers' conceptions and beliefs about the role of real-world knowledge in mathematical modelling of school word problems. *Learning and Instruction, 7(4),* 339-359.

Yanagimoto, T. (2005). Teaching modelling as an alternative approach to school mathematics. *Teaching Mathematics and Its Applications*, *24(1),* 1-13.

Yıldırım, A., & Şimşek, H. (2005). *Sosyal bilimlerde nitel araştırma yöntemleri*. Seçkin Yayıncılık, Ankara.

http://iserjournals.com/journals/eurasia