

Matrix Perturbation Theory

G. W. Stewart

*Computer Science Department
Institute for Advanced Computer Studies
University of Maryland
College Park, Maryland*

Ji-guang Sun

*Computing Center of the Chinese Academy of Sciences
Beijing, China*



ACADEMIC PRESS, INC.
Harcourt Brace Jovanovich, Publishers
Boston San Diego New York
London Sydney Tokyo Toronto

Contents

Preface	xiii
I Preliminaries	1
1 Notation	1
Notes and References	4
Exercises	5
2 The QR Decomposition — Projections	6
2.1 The QR Decomposition	6
2.2 Hadamard's Inequality	8
2.3 Projections	9
Notes and References	10
Exercises	11
3 Eigenvalues and Eigenvectors	14
3.1 Definitions and Elementary Properties	14
3.2 The Schur Decomposition	17
3.3 The Jordan Canonical Form	20
3.4 Invariant Subspaces	21
3.5 The Field of Values	23
3.6 Sums of Hermitian Matrices	25
Notes and References	26
Exercises	27
4 The Singular Value Decomposition	30
4.1 The Singular Value Decomposition	30
4.2 Two Inequalities	33

	Notes and References	34
	Exercises	36
5	Pairs of Subspaces	37
	5.1 The CS Decomposition	37
	5.2 Pairs of Subspaces	40
	5.3 Pairs of Projections	43
	Notes and References	45
	Exercises	45
II	Norms and Metrics	49
1	Vector Norms	50
	1.1 Definition	50
	1.2 Examples	51
	1.3 Equivalence and Limits	53
	1.4 Linear Functionals and Dual Norms	56
	Notes and References	59
	Exercises	60
2	Matrix Norms	64
	2.1 Basic Concepts	64
	2.2 Operator Norms	67
	Notes and References	71
	Exercises	71
3	Unitarily Invariant Norms	74
	3.1 Von Neumann's Theory	74
	3.2 Properties of Unitarily Invariant Norms	79
	3.3 Doubly Stochastic Matrices and Fan's Theorem	81
	Notes and References	87
	Exercises	88
4	Metrics on Subspaces of \mathbf{C}^n	89
	4.1 The Gap	90
	4.2 Unitarily Invariant Metrics	94
	Notes and References	98
	Exercises	99
III	Linear Systems and Least Squares Problems	101
1	The Pseudo-Inverse and Least Squares	102
	1.1 Generalized Inverses and the Pseudo-Inverse	102
	1.2 Projections and Least Squares	106

Notes and References	108
Exercises	109
2 Inverses and Linear Systems	114
2.1 Absolute and Relative Errors	115
2.2 The Inverse Matrix	117
2.3 Linear Systems	124
2.4 Asymptotic Forms and Derivatives	130
Notes and References	132
Exercises	134
3 The Pseudo-Inverse	136
3.1 Projections and Acute Perturbations	137
3.2 General Results	140
3.3 Acute Perturbations	146
3.4 Asymptotic Forms and Derivatives	150
Notes and References	151
Exercises	152
4 Projections	153
Notes and References	155
5 The Linear Least Squares Problem	155
5.1 Perturbation of the Coefficients	156
5.2 The Residual	160
5.3 Backward Perturbations	160
5.4 Asymptotic Forms and Derivatives	162
Notes and References	163
Exercises	163
IV The Perturbation of Eigenvalues	165
1 General Perturbation Theorems	166
1.1 Continuity: Ostrowski–Elsner Theorems	166
1.2 The Bauer–Fike and Henrici Theorems	170
1.3 Residual Bounds	174
Notes and References	176
Exercises	178
2 Gerschgorin Theory: Differentiability	180
2.1 Gerschgorin’s Theorem	181
2.2 Diagonal Similarities	182
Notes and References	186

	Exercises	187
3	Normal and Diagonalizable Matrices	189
	3.1 The Hoffman–Wielandt Theorem	189
	3.2 Diagonalizable Matrices	192
	Notes and References	193
	Exercises	194
4	Hermitian Matrices	196
	4.1 Inertia and Interlacing	196
	4.2 Wielandt’s Theorem and Its Consequences	198
	4.3 Mirsky’s Theorem	203
	4.4 Residual Bounds	205
	4.5 Approximation by a Low-Rank Matrix	208
	Notes and References	209
	Exercises	210
5	Some Further Results	211
	5.1 Non-Hermitian Perturbations	212
	5.2 Similarity Bounds	215
	Notes and References	217
	Exercises	217
V	Invariant Subspaces	219
1	The Theory of Simple Invariant Subspaces	220
	1.1 Definition	220
	1.2 The Operator $T = X \mapsto AX - XB$	222
	1.3 The Spectral Resolution	223
	Notes and References	227
	Exercises	227
2	Perturbation of Invariant Subspaces	229
	2.1 The Approximation Problem	230
	2.2 Perturbation Theorems	236
	2.3 Eigenvectors	240
	2.4 Solution of a Nonlinear Equation	242
	Notes and References	244
	Exercises	245
3	Hermitian Matrices	246
	3.1 The Approximation Theorem	246
	3.2 Generalized Rayleigh Quotients	248

3.3	Direct Bounds	249
3.4	Residual Bounds for Eigenvalues	254
	Notes and References	258
	Exercises	258
4	The Singular Value Decomposition	259
4.1	Two $\sin \Theta$ Theorems	260
4.2	A Perturbation Expansion	263
	Notes and References	266
	Exercises	267
VI	Generalized Eigenvalue Problems	271
1	Background	273
1.1	Matrix Pairs	273
1.2	Triangular and Weierstrass Forms	276
1.3	Definite Pairs	281
1.4	Metrics and Their Limitations	283
	Notes and References	289
	Exercises	290
2	Regular Matrix Pairs	291
2.1	Continuity, First Order Theory	291
2.2	Gerschgorin Theory	294
2.3	Diagonalizable Pairs	300
2.4	Eigenspaces	303
	Notes and References	311
	Exercises	312
3	Definite Matrix Pairs	312
3.1	Eigenvalues of Definite Pairs	313
3.2	Eigenspaces	317
3.3	Direct Bounds	322
	Notes and References	324
	Exercises	324
	References	325
	Notation	347
	Index	351