Maximization of the Minimum Rate by Geometric Programming for Multiple Users in Partial Frequency Reuse Cellular Networks

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Abstract—**We apply constrained optimization techniques to optimally allocate the bandwidth and power to the users in a cellular network. Partial frequency reuse with multiple users in the inner region and the outer region is considered as inter-cell interference mitigation scheme. We show that the maximization of the minimum rate under variable bandwidth and variable power can be transformed into a geometric programming problem and solved efficiently using disciplined convex programming. CVX is used to solve the transformed optimization problem. The optimal solution assigns all available bandwidth to the inner and the outer users and only the satisfactory level of power that the users achieve higher rates that the minimum required rate.**

I. Introduction

Next generation mobile communication systems use Orthogonal Frequency Division Multiple Access (OFDMA) as their modulation scheme in the downlink [1], [2]. Since cell edge users may suffer severely from Inter-Cell Interference (ICI), several schemes have been proposed for ICI mitigation. One of those schemes, which is applied in [3], [4], [5] is Partial Frequency Reuse (PFR). The sum-rate maximization for two users in PFR including also partial bandwidth reallocation to the Full Frequency Reuse (FFR) users whenever cell edge users are idle is done in [6]. The characteristics of the optimal power allocation for two base stations, employing also scheduling schemes, have been studied in [7] under frequency reuse-1. Additionally to the optimal power allocation, the minimum rate constraint per cell has been considered by the authors in [8]. The problem of sum-rate maximization under variable power for two users, using sequential geometric programming is investigated by authors in [9]. In this study the authors have shown by simulations that in a noise dominated case all power is assigned to the users. Differently from [9], the sum-rate maximization for multiple users in PFR under the assumption that equal power is used to all Full Frequency Reuse (FFR) users is studied in [10]. Additionally to the sum-rate maximization power control problem, in [11] the authors also investigate the maximization of the minimum rate for two users. The maximization of the minimum rate for all users in all cells under variable bandwidth and power allocation is non-convex. However, in [10] such problem was found to be transformable in a convex optimization problem under some variable transformation. In [12] the maximization of the minimum rate for Code Division Multiple Access (CDMA) under variable power is shown to be transformable in a Geometric Programming (GP). To the best of our knowledge, there are currently no studies that consider the maximization of the minimum rate under variable bandwidth and variable power for PFR in GP form.

Our contributions can be summarized as follows. In Section II we show the system model including the bandwidth and power allocation scheme for PFR. In Section III we study the maximization of the minimum rate problem for PFR systems considering inter-cell interference and both, power and bandwidth allocation between PFR and FFR users. Without using any approximation in power or bandwidth allocation, we transform the Generalized Geometric Programming (GGP) into a Geometric Programming (GP) [12], which can be solved efficiently using Disciplined Convex Programming (DCP) methods [13]. Furthermore, we present in Section IV simulation results, which shows that all bandwidth is allocated to all users, and the power to the cells is allocated depending on their need for achieving a higher rate than the minimum requirement rate. By simulation results it is confirmed that the optimization problem proposed in Section III is an efficient tool for dimensioning the cell under power and bandwidth constraints while satisfying the minimum requirement for user rate at the cell edge. Conclusions are drawn in Section V.

II. System Model

In our system model we consider N^{in} users that are located in the inner region of the cell (the full frequency reuse region) and $\mathrm{M}^{\mathrm{out}}$ users that are located in the outer region of the cell (the partial frequency reuse region), as indicated in Fig. 1. Based on the users' distance from their own base station, a scheduler decides whether a user is considered an inner user if the user's distance is less than a threshold distance or an outer user if the user's distance is more than that threshold distance. The frequency reuse pattern applied in our system model is shown Fig. 2. Depending on the amount of

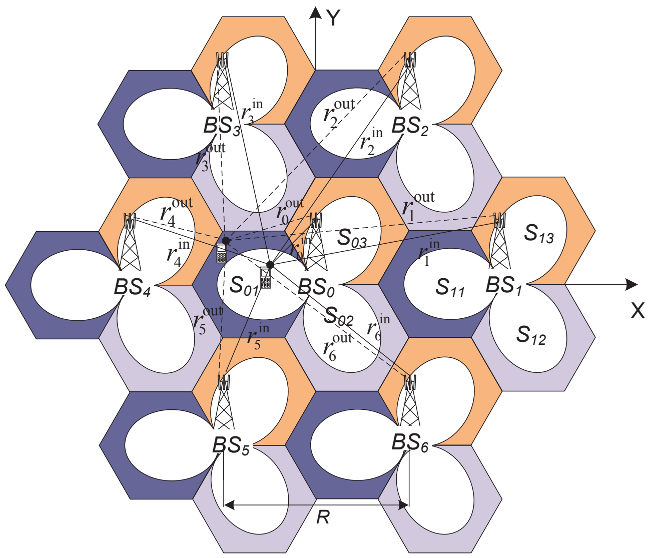

Fig. 1. Partial frequency reuse cell cluster

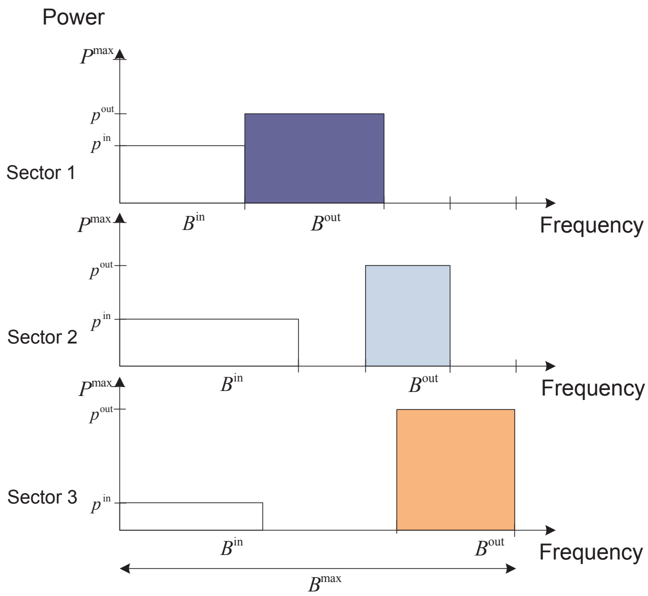

Fig. 2. Frequency reuse pattern for an arbitrary base station

the inner and the outer users and their distance from base station, the bandwidth and power is assigned to the inner and outer regions as indicated in Fig. 2. The different bandwidth and power assignment in the regions for each sector shown in Fig. 2 is an expectation how our optimization problem formulated in Section III will work. The users located in the inner region of the sector, S_{01}, receive power from their own sector antenna and also interference from all sectors of base stations $\mathrm{BS}_k, k = 0...6$. In our system model are not considered more distant base stations but all our results can be easily extended to consider also interference from more distant base stations. The rate achieved by the user n in the inner region of cell S_{01} is given by

$$R_n^{\mathrm{in}} = B_n^{\mathrm{in}} \log_2 \left(1 + \frac{G_{0n}^{\mathrm{in}} p_0^{\mathrm{in}}}{N_0 B_n^{\mathrm{in}} + \sum_{k=1}^{6} G_{kn}^{\mathrm{in}} p_k^{\mathrm{in}}} \right) \quad (1)$$

where B_n^{in} is the bandwidth utilized to the user n in the inner region of cell S_{01} and N_0 is the noise spectral density. The large scale path-loss attenuation including antenna gain, penetration loss, shadowing and fast fading is expressed in the following form [14],

$$G = -[128.1 + 10\alpha \log_{10}(r) + A + L_p + X_\sigma + F] \quad (2)$$

where G is in dB, α the path-loss exponent, r the distance between the mobile station and base station in km, A the sum of user antenna gain and base station antenna gain in dB, L_p the penetration loss in dB, X_σ the log-normal shadowing in dB, and F the fast fading channel coefficient in dB. The antenna gain A is defined by horizontal antenna pattern [14]. The large scale path-loss attenuation of directed channels G_{0n}^{in} is defined by Equation (2). The large scale path-loss attenuation of interference channels G_{kn}^{in} is also defined by Equation (2) except of the fast fading, which is not taken into account here. The transmit power assigned to the users in the inner region is denoted by p_0^{in} and the interference power from the other base stations is denoted by p_k^{in}, $k = 1 \ldots 6$, with k denoting the index of the interfering base stations. The users located in the outer region of the cell receive also interference from non-neighboring sectors of base stations that use the same frequency bands. The transmit power assigned to the users in the outer region is denoted by p_0^{out} and the interference power from the other base stations is denoted by p_k^{out}, $k = 1 \ldots 6$. Thus, the rate achieved by user m in the outer region of cell S_{01} is given by

$$R_m^{\mathrm{out}} = B_m^{\mathrm{out}} \log_2 \left(1 + \frac{G_{0m}^{\mathrm{out}} p_0^{\mathrm{out}}}{N_0 B_m^{\mathrm{out}} + \sum_{k=1}^{6} G_{km}^{\mathrm{out}} p_k^{\mathrm{out}}} \right) \quad (3)$$

where B_m^{out} denotes the bandwidth utilized to the user m in the outer region and G_{0m}^{out} and G_{km}^{out} denote the large scale path-loss attenuations for the direct and the interference channels of the m-th outer user.

III. Power and Bandwidth Allocation Problem in Geometric Programming Form

In this section we study the maximization of the minimum rate for systems employing PFR and performing power and bandwidth allocation. Without relying on any assumptions on power allocations or high SINR approximation, we proof that this problem can be transformed into GP and hence solved efficiently using DCP methods. The problem of maximizing the minimum rate among all users in all cells is written in the following form [10]:

$$\underset{\beta_{cn}^{\text{in}},\beta_{cm}^{\text{out}}\in\mathcal{R}_+,\mathbf{p}\succeq\mathbf{0}}{\text{maximize}} \quad \min\{\beta_{c1}^{\text{in}}t^{\text{in}}\log(2),...,\beta_{c\text{N}^{\text{in}}}^{\text{in}}t^{\text{in}}\log(2), \beta_{c1}^{\text{out}}t^{\text{out}}\log(2),...,\beta_{c\text{M}^{\text{out}}}^{\text{out}}t^{\text{out}}\log(2)\} \tag{4a}$$

subject to

$$t^{\text{in}} \le \log\left(1+\frac{p_c^{\text{in}}}{n_1^{\text{in}}\beta_{c1}^{\text{in}}+\sum_{k\in\mathcal{C}\backslash c} g_{k1}^{\text{in}}p_k^{\text{in}}}\right),\ \forall c\in\mathcal{C}, \tag{4b}$$

$$\vdots$$

$$t^{\text{in}} \le \log\left(1+\frac{p_c^{\text{in}}}{n_{\text{N}^{\text{in}}}^{\text{in}}\beta_{c\text{N}^{\text{in}}}^{\text{in}}+\sum_{k\in\mathcal{C}\backslash c} g_{k\text{N}^{\text{in}}}^{\text{in}}p_k^{\text{in}}}\right),\ \forall c\in\mathcal{C}, \tag{4c}$$

$$t^{\text{out}} \le \log\left(1+\frac{p_c^{\text{out}}}{n_1^{\text{out}}\beta_{c1}^{\text{out}}+\sum_{k\in\mathcal{C}\backslash c} g_{k1}^{\text{out}}p_k^{\text{out}}}\right),\ \forall c\in\mathcal{C}, \tag{4d}$$

$$\vdots$$

$$t^{\text{out}} \le \log\left(1+\frac{p_c^{\text{out}}}{n_{\text{M}^{\text{out}}}^{\text{out}}\beta_{c\text{M}^{\text{out}}}^{\text{out}}+\sum_{k\in\mathcal{C}\backslash c} g_{k\text{M}^{\text{out}}}^{\text{out}}p_k^{\text{out}}}\right),\forall c\in\mathcal{C}, \tag{4e}$$

$$\sum_{n=1}^{\text{N}^{\text{in}}}\beta_{cn}^{\text{in}}+\sum_{m=1}^{\text{M}^{\text{out}}}\beta_{cm}^{\text{out}}\le 1,\forall c\in\mathcal{C} \tag{4f}$$

$$p_c^{\text{in}}+p_c^{\text{out}}\le P_c^{\max},\ \forall c\in\mathcal{C}, \tag{4g}$$

where $\beta_{cn}^{\text{in}} = B_n^{\text{in}}/B^{\max}$ and $\beta_{cm}^{\text{out}} = B_m^{\text{out}}/B^{\max}$ are the normalized inner and outer bandwidths, t^{in} and t^{out} are the normalized [2 p.99] inner and outer user rates, respectively. The subscript c denotes the cell and the calligraphic $\mathcal{C}$ denotes the set of cells. Furthermore, $n_n^{\text{in}} = N_0/G_{0n}^{\text{in}}$ and $g_{kn}^{\text{in}} = G_{kn}^{\text{in}}/G_{0n}^{\text{in}}$ are the normalized noise and the normalized interference channel large scale path-loss attenuations for the inner users, respectively. Similar normalization is considered for the outer users. In GGP optimization problem 4, the constraints 4b-4c show that the normalized inner user rates are constrained by normalized minimum requirement inner user rate. Similarly for the outer users are the constraints 4d-4e. The last two constraints 4f and 4g are the bandwidth and the power constraints.

Proposition 1: The max-min-rate problem (4) can be transformed into GP optimization problem.

Proof: We begin by introducing a variable u which act lower bound [15] in the objective (4a) and by using its inverse in the objective we convert the Generalized Geometric Programming (GGP) into a GP [16]. Applying exponential in both sides of constraints 4b-4e, which notably does not change the optimal variables converts the above optimization problem into a GP and DCP, where CVX can be used to get the optimal power and bandwidth. Now the optimization problem is written in the following form:

$$\underset{\mathbf{p}\succeq\mathbf{0},\beta_{cn}^{\text{in}},\beta_{cm}^{\text{out}},u\in\mathcal{R}_+}{\text{minimize}} \quad \{\frac{1}{u}\} \tag{5a}$$

subject to

$$\beta_{c1}^{\text{in}}t^{\text{in}}\log(2)\ge u \tag{5b}$$

$$\vdots$$

$$\beta_{c\text{N}^{\text{in}}}^{\text{in}}t^{\text{in}}\log(2)\ge u \tag{5c}$$

$$\beta_{c1}^{\text{out}}t^{\text{out}}\log(2)\ge u \tag{5d}$$

$$\vdots$$

$$\beta_{c\text{M}^{\text{out}}}^{\text{out}}t^{\text{out}}\log(2)\ge u \tag{5e}$$

$$\frac{n_1^{\text{in}}\beta_{c1}^{\text{in}}+\sum_{k\in\mathcal{C}\backslash c} g_{k1}^{\text{in}}p_k^{\text{in}}}{p_c^{\text{in}}}\ge e^{(t^{\text{in}}-1)},\ \forall c\in\mathcal{C}, \tag{5f}$$

$$\vdots$$

$$\frac{n_{\text{N}^{\text{in}}}^{\text{in}}\beta_{c\text{N}^{\text{in}}}^{\text{in}}+\sum_{k\in\mathcal{C}\backslash c} g_{k\text{N}^{\text{in}}}^{\text{in}}p_k^{\text{in}}}{p_c^{\text{in}}}\ge e^{(t^{\text{in}}-1)},\ \forall c\in\mathcal{C}, \tag{5g}$$

$$\frac{n_1^{\text{out}}\beta_{c1}^{\text{out}}+\sum_{k\in\mathcal{C}\backslash c} g_{k1}^{\text{out}}p_k^{\text{out}}}{p_c^{\text{out}}}\ge e^{(t^{\text{out}}-1)},\ \forall c\in\mathcal{C}, \tag{5h}$$

$$\vdots$$

$$\frac{n_{\text{M}^{\text{out}}}^{\text{out}}\beta_{c\text{M}^{\text{out}}}^{\text{out}}+\sum_{k\in\mathcal{C}\backslash c} g_{k\text{M}^{\text{out}}}^{\text{out}}p_k^{\text{out}}}{p_c^{\text{out}}}\ge e^{(t^{\text{out}}-1)},\ \forall c\in\mathcal{C}, \tag{5i}$$

$$\sum_{n=1}^{\text{N}^{\text{in}}}\beta_{cn}^{\text{in}}+\sum_{m=1}^{\text{M}^{\text{out}}}\beta_{cm}^{\text{out}}\le 1,\forall c\in\mathcal{C} \tag{5j}$$

$$p_c^{\text{in}}+p_c^{\text{out}}\le P_c^{\max},\ \forall c\in\mathcal{C}, \tag{5k}$$

where the constraints (5b)-(5e) are monomials [16] and posynomials also. In the constraints (5f)-(5i), the expressions in the left side are posynomials since the ratio of posynomial by a monomial is still posynomial [16]. The last two constraints are only posynomials, so the optimization problem 5 is in GP form. ■

IV. Simulation Results

For the simulations we have considered two inner user located in the polar coordinates $(200\,\text{m}, 160^0)$, $(120\,\text{m}, 160^0)$ and two outer users located in polar coordinates $(400\,\text{m}, 160^0)$, $(380\,\text{m}, 160^0)$. The simulation parameters are shown in Table I. By simulations we have noticed that, the amount of bandwidth assigned to inner users is done equally as to the outer users. In the following we show only the bandwidth assigned to inner user 1 and outer user 1. The bandwidth assigned to the inner user one is shown in Fig. 3. From the simulation results shown in Fig. 3 we see that more bandwidth is assigned to the inner user when the requirement for the minimum inner user rate is increased. When the minimum requirement for the outer user rate begins to increase, the bandwidth assignment to the inner user decreases. In order to keep the inner user rate higher than the minimum requirement

TABLE I
SIMULATION PARAMETERS

parameters	value
Maximum base station power $P^{\max}$	40 W
Maximum base station bandwidth $B^{\max}$	20 MHz
Noise spectral density N_0	-174 dBm/Hz
Center frequency f	2.0 GHz
Pathloss exponent α	3.75
Penetration loss L_p	20 dB
Shadowing X_σ	$\mathcal{N}(0, 8)$ dB
Fast Fading F	$\mathcal{CN}(0, 1)$ dB,
Inter base station distance R	700 m
Maximum cell range r	$(2/3)R$ m

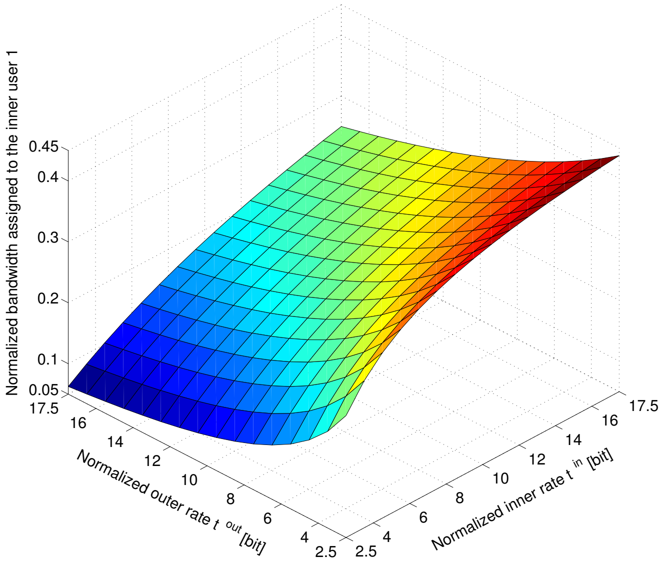

Fig. 3. Assigned normalized bandwidth to the inner user 1 versus normalized inner and outer rates

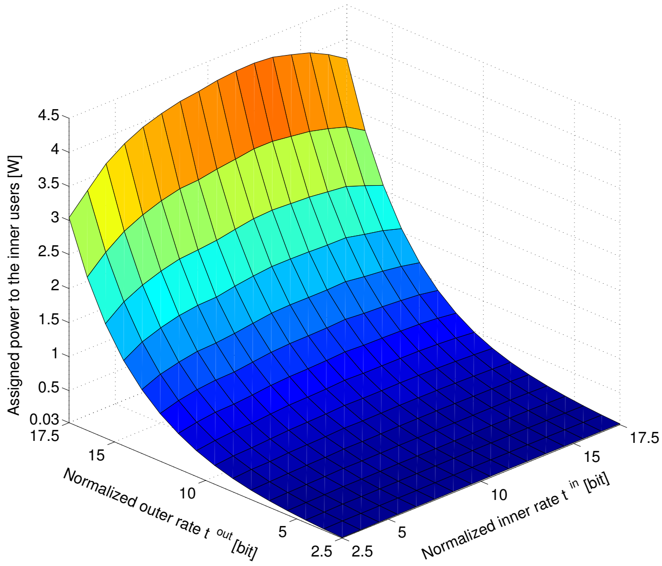

Fig. 4. Assigned power to the inner users

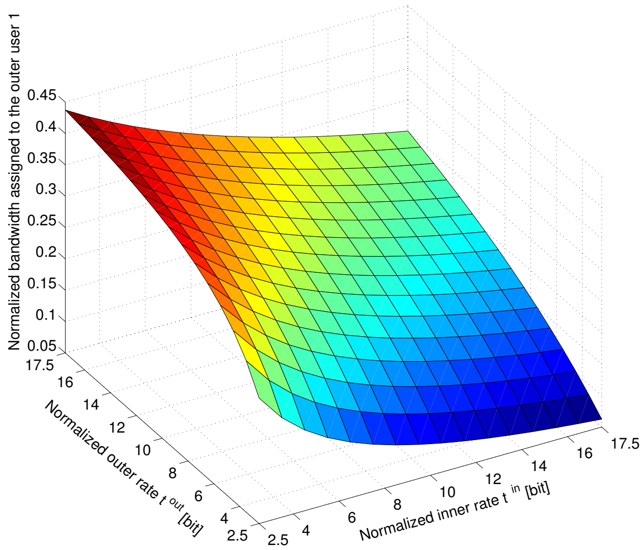

Fig. 5. Assigned normalized bandwidth to the outer user 1 versus normalized inner and outer rates

for the inner user rate, the power assigned to the inner user is increased when the minimum requirement for outer user rate is increased. This is shown on the power assignment to the inner users in Fig. 4. The bandwidth assignment to the outer user 1 is shown in Fig. 5. Looking the simulation results shown in Fig. 5 we see that the bandwidth assignment to the outer user increases when the minimum requirement for outer user rate increases. Because of the requirement for higher minimum inner user rate less bandwidth is assigned to the outer user while at the same time more power is assigned to the outer user. Such increase in the power assignment to the outer users is shown in Fig. 6. Comparing the simulation results for the bandwidth and power assignment to the inner and the outer users we conclude that optimization problem offers a trade off between bandwidth and power assignment. Whenever the bandwidth assignment is decreased, the power assignment is increased in order to achieve higher user rate than the minimum requirement rate. Using Equation 1 we have simulated the rates achieved by the inner user 1 and the inner user 2 which are shown in Fig. 7a and Fig. 7b. The rate achieved by the inner user 2 is higher than the rate achieved by the inner user 1 since the inner user 2 is nearer base station BS_0. Similarly, using Equation 3 are simulated the achievable rates by the outer user 1 and the outer user 2 which are shown in Fig. 7c and 7d. Looking the achievable rates for the outer user 1 and outer user 2 we see that the outer user 2 achieves higher rate than outer user 1, since it is in a shorter distance

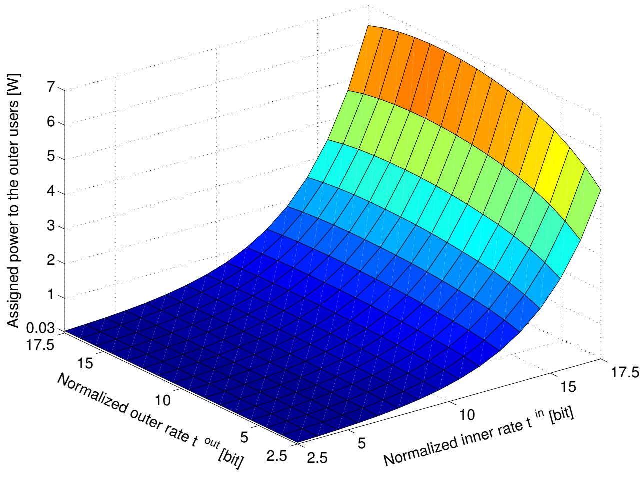

Fig. 6. Assigned power to the outer users

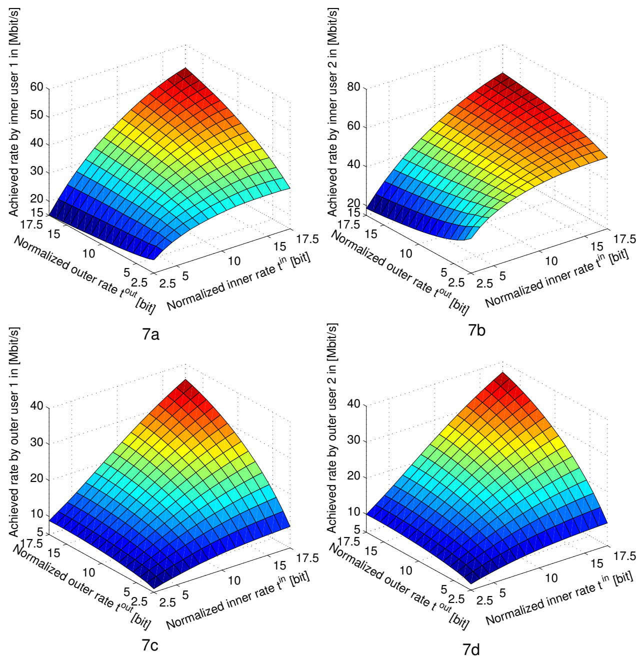

Fig. 7. Transmission rate achieved by each user

from base station BS_0.

V. Conclusions

In this paper we formulated the maximization of the minimum rate problem for partial frequency reuse cellular networks. By applying the known techniques for geometric programming, we transformed the optimization problem into a geometric programming and solved efficiently using disciplined convex programming. From our simulation results we see that the proposed algorithm allocates the whole available bandwidth to the users equally in each region of the cell and only assigns the necessary power to the inner and the outer regions such that users achieve higher rates than the minimum requirement rate. Furthermore, we demonstrated that the maximization of the minimum rate is an efficient algorithm for dimensioning the cell under minimum rate constraint, especially at the cell edge.

Acknowledgments

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