

# Maximizing Network-Lifetime In Large Scale Heterogeneous Wireless Sensor-Actuator Networks: A Near-Optimal Solution

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## ABSTRACT

Delay and energy constraints have a significant impact on the design and operation of wireless sensor-actuator networks. We consider a wireless sensor-actuator network, consisting of large number of sensors and few actuators, in which both energy and delay are *hard* constraints and must be jointly optimized.

In this paper, we propose that each sensor node must transmit its data to only one of the actuators. To maximize the network lifetime and attain minimum end-to-end delays, it is essential to optimally match each sensor node to an actuator and find an optimal routing scheme. We model the actuator selection and optimal flow routing as joint optimization problem, which is NP-hard in general. Therefore, we use a relaxation technique and provide a distributed solution for optimal actuator selection subject to energy and delay constraints. Further, once the destination actuators are fixed, we provide an optimal flow routing solution with the aim of maximizing network lifetime. We then propose to use a delay-energy aware TDMA based MAC protocol in compliance with the routing algorithm. The optimal routing and TDMA MAC schemes together guarantees a near-optimal lifetime. The proposal is validated by means of analysis and ns-2 simulation results.

## Categories and Subject Descriptors

C [Computer Systems Organization]: C.2 COMPUTER-COMMUNICATION NETWORKS; C.2.2 [Network Protocols]: [Routing protocols; Protocol verification]

## General Terms

Design, Performance, Theory

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## Keywords

Wireless sensor-actuator networks, heterogeneity, delay-energy awareness, optimization.

## 1. INTRODUCTION

Distributed systems based on networked sensors and actuators with embedded computation capabilities enable an instrumentation of the physical world at an unprecedented scale and density, thus enabling a new generation of monitoring and control applications. Wireless sensor-actuator networks (SANETs<sup>1</sup>) is an emerging technology that has a wide range of potential applications including environment monitoring, medical systems, robotic exploration, and smart spaces. SANETs are becoming increasingly important in recent years due to their ability to detect and convey real-time, in-situ information for many civilian and military applications. Such networks consist of large number of distributed sensor and few actuator nodes that organize themselves into a multihop wireless network. Each sensor node has one or more sensors (including multimedia, e.g., video and audio, or scalar data, e.g., temperature, pressure, light, infrared, and magnetometer), embedded processors, and low-power radios, and is normally battery operated. Typically, these nodes coordinate to perform a common task. Whereas, the actuators gather this information and react accordingly.

Sensor-actuator networks have the following unique characteristics:

- Real-time requirement: Depending on the application there may be a need to rapidly respond to sensor input. Examples can be a fire application where actions should be initiated on the even area as soon as possible.
- Coordination: Unlike wireless sensor networks where the central entity (i.e., sink) performs the functions of data collection and coordination, in SANETs, new networking phenomena called sensor-actuator and actuator-actuator coordination may occur. In particular, sensor-actuator coordination provides the transmission of event features from sensors to actuators. After receiving event information, actuators may need to coordinate with each other (depend on the acting application) in order to make decisions on the most appropriate way to perform the actions.

<sup>1</sup>In this work, we will use the term SANET to represent sensor-actuator networks. Also, the acting entities are sometimes referred to as *actors* in the related literature.

In this paper, we propose an optimal actuator selection and flow routing solution with the aim of maximizing the network lifetime. Also, the solution is jointly optimal in terms of minimizing energy consumption per successful transmission and attaining minimum end-to-end delays. We first show that the flow routing and actuator-selection with energy constraints can be modeled as a mixed integer nonlinear programming optimization problem (MINLP). Since MINLP is NP-hard in general, we develop a distributed approach which provides a good approximation of the optimal solution. We use a relaxation technique in order to decide on the delay-optimal actuator and then optimize the flow routing toward this actuator to extend network lifetime. We also propose to use an adaptive TDMA like MAC to avoid the problem of synchronization during flow splitting and to meet the delay constraints in SANETs. The optimal routing and TDMA MAC solution together guarantees a near-optimal lifetime for wireless sensor-actuator networks.

The organization of this paper is as follows. Section 2 highlights some interesting related literature. The problem formulation is presented in Section 3. In Section 4, we describe the network model under consideration in detail. Section 5 details the design criteria of our proposed routing scheme and optimization obtained in this direction. The simulation results are presented in Section 6. In Section 7, we conclude the paper and outline the future directions.

## 2. RELATED LITERATURE

If the mapping between a sensor node and a base station/actuator is given a priori, then the problem of finding optimal flow strategies to extend network lifetime has been well investigated in the past [2, 3]. For a sensor network with multiple sinks, the traffic generated by sensor nodes may be split and sent to different basestations [1, 4]. In [6], a coordination framework for WSAWs is addressed. A new sensor-actor coordination model is proposed, based on an event-driven clustering paradigm in which cluster formation is triggered by an event so that clusters are created on-the-fly to optimally react to the event itself and provide the required reliability with minimum energy expenditure. The optimal solution is determined by mathematical programming and a distributed solution is also proposed. In addition, a new model for actor-actor coordination is introduced for a class of coordination problems in which the area to be acted upon is optimally split among different actors. An auction-based distributed solution of the problem is also presented. We study a closely related but more complex model with additional constraints.

## 3. PROBLEM FORMULATION

In cases, when there are multiple actuators and mapping between the sensors and actuators is not given, the joint problem of finding an optimal actuator and extending network lifetime with minimum end-to-end delay constraints is a challenging and interesting problem. This problem is relevant from both the application's and wireless networking perspectives. From an application requirement perspective, for some real-time multimedia sensing applications (e.g., video surveillance), it is necessary to have all the traffic generated from a source sensor to be routed to the same actuator (albeit that it may follow different routes) so that decoding and processing can be properly completed. For

multimedia traffic such as video, the information contained in different packets from the same source are highly correlated and dependent. If packets generated by a source are split and sent to different actuators, any of these receiving actuators may not be able to decode the video packets properly. From a wireless networking perspective, the actuator chosen as a sink could have a significant impact on the end-to-end delays which is a *hard* constraint [7] for sensor-actuator applications. This is because the end-to-end delays are topology dependent; actuator selection simply based on energy constraints can not guarantee optimal end-to-end delays, and therefore, it should be based on both delay-energy constraints. As a result, there appears to be a compelling need to understand how to perform optimal routing to jointly achieve minimum end-to-end delay routes and optimize network lifetime in delay-energy constrained SANETs.

## 4. NETWORK MODEL

Consider a static 3-tier SANET with  $N$  sensor nodes,  $M$  actuator nodes, and  $B$  basestations as shown in Fig. 1.

### 4.1 Neighborhood Relation Model

Given an  $(N + M + B) \times (N + M + B)$  neighborhood relation matrix  $\mathcal{N}$  that indicates the node pairs for which direct communication is possible. We will assume that  $R$  is a symmetric matrix, i.e., if node  $i$  can transmit to node  $j$ , then  $j$  can also transmit to node  $i$ . For such node pairs, the  $(i, j)^{th}$  entry of the matrix  $\mathcal{N}$  is unity, i.e.,  $N_{ij} = 1$  if node  $i$  and  $j$  can communicate; we will set  $N_{ij} = 0$  if nodes  $i$  and  $j$  can not communicate. For any node  $i$ , we define

$$\mathcal{N}_i = \{j : N_{i,j} = 1\}.$$

which is the set of neighboring nodes of node  $i$ . Similarly, a set of interference nodes (cannot be reached by one-hop) for node  $i$  (from where the transmissions can be heard at node  $i$ ), and is defined as

$$S_i = \{K \notin \mathcal{N}_i \cup \{i\} : \mathcal{N}_{k,j} = 1 \text{ for some } j \in \mathcal{N}_i\}$$

note that  $S_i$  does not include any of the first-hop neighbors of node  $i$ .

### 4.2 Forwarding (Relaying)

The sensor-actuator network is deployed in a remote location. The sensors do the application dependent sensing and transmit their readings to the actuators. The actuators react on the environment based on the readings from the sensors and also forward (relay) this information to the basestations (using long-haul communication). Some in-network aggregation techniques could be applied at this stage if the data is correlated. Since this discussion is application dependent, and therefore, we do not go in its detail. The basestations are further responsible for forwarding (relaying) this information to a sink (this communication can be over satellite links) for remote analysis. Since, there are multiple actuators and basestations in our heterogeneous network, we divide the problem of optimal flow (from a sensor to a sink) at three distinct levels. At level one (sensor-actuator coordination), we investigate the actuator-selection problem and optimal flow routing in order to maximize the network lifetime at this level. At level two (actuator-actuator/basestation

coordination), we study a similar problem of base-station selection and optimal flow routing to maximize network lifetime at level 2. Finally at level three (basestation-sink coordination), we study the problem of optimal flow from basestations to the sink. In this study, we assume that there is sufficient energy available at the sink, and thus, there is no energy constraint for the sink.

### 4.3 Channel Model and Antennas

We assume a simple channel model: a node can decode a transmission successfully iff there is no other interfering transmission. Each sensor node is supported by an omnidirectional antenna. Each actuator is provided with two omnidirectional antennas; one to communicate with the sensor network, and the other to communicate with the network of neighboring actuators/basestations using long-range communications. Similarly, each basestation is also provided with two omnidirectional antennas; one to communicate with the network of actuators/basestations and the other to communicate with the sink (as it might be using a satellite link).

### 4.4 Frequency and MAC

Assume that all nodes share the same frequency band at their respective operating level. Time is assumed to be divided into fixed length slots. All the packets (depending on their operating level) are of same length and the length of a time slot corresponds to the time required to transmit a packet over the underlying wireless channel.

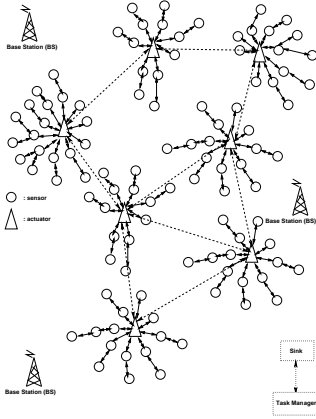


Figure 1: Architecture of SANETs

## 5. DESIGN OF OPTIMAL ROUTING SOLUTION

In the following, we detail several components of our proposed actuator-selection problem and optimal flow routing protocol for SANETs.

### 5.1 Power Consumption Model

For a sensor node, the energy consumption due to wireless communication (i.e receiving and transmitting) is considered the dominant source in power consumption.

The power consumed by a sensor node  $i$  in receiving can be modeled as

$$P_r^i = P_{rx} \sum_{j \in \mathcal{N}_i} f_{j,i} \quad (1)$$

where  $f_{j,i}$  is the rate (*bits/s*) at which node  $j$  is transmitting packets toward node  $i$ . A typical value for the parameter  $P_{rx}$  is  $50 \text{ nJ/bit}$ .

If power consumed to send a packet is given by  $P_{tx}$  (a typical value for this parameter is  $50 \text{ nJ/bit}$  [3]), then the power consumed by a sensor node  $i$  in transmitting its data (both locally originated and forwarded packets) is

$$P_t(i, j) = c_{i,j} \cdot f_{i,j} \quad (2)$$

where  $c_{i,j}$  is the power consumption coefficient for data transmission between sensor  $i$  and  $j$ . And  $f_{i,j}$  is the total flow from sensor  $i$  to sensor  $j$  *bits/s*. Also

$$c_{i,j} = \alpha + \beta \cdot d_{i,j}^m \quad (3)$$

where  $\alpha$  and  $\beta$  are constants,  $d_{i,j}$  is the distance between the sensors  $i$  and  $j$ , and  $m$  is the path loss index. Typical values of  $\alpha$  and  $\beta$  are  $50 \text{ nJ/bit}$  and  $0.0013 \text{ pJ/bit/m}^4$  (for  $m = 4$ ), respectively [3].

### 5.2 Actuator-Selection and Optimal flow Routing

The joint problem of finding an actuator and flow routing to maximize network lifetime at the level 1 (s.t. energy constraints) is non-trivial and interesting for sensor-actuator networks. We define (for details, see Table 1)

$$F_{s,s} = \left\{ f_{s_i, s_j}^{s_k, A_l} : (1 \leq i, j, k \leq N, i \neq j, j \neq k, 1 \leq l \leq M) \right\}$$

$$F_{s,A} = \left\{ f_{s_i, A_l}^{s_k, A_l} : (1 \leq i, k \leq N, 1 \leq l \leq M) \right\}$$

$$F_{s,s_i} = \left\{ f_{s_m, s_i}^{s_k, A_l} : (1 \leq m, k \leq N, m \neq i, k \neq i, 1 \leq l \leq M) \right\}$$

$$F_{s_i, s} = \left\{ f_{s_i, s_r}^{s_k, A_l} : (1 \leq r, k \leq N, r \neq k, r \neq i, 1 \leq l \leq M) \right\}$$

$$F_{s_i, A} = \left\{ f_{s_i, A_l}^{s_k, A_l} : (1 \leq k \leq N, 1 \leq l \leq M) \right\}$$

We denote  $T_{l1}$  as network lifetime at level one (sensor-actuator coordination level), which (in this work) is defined as the time until any sensor drains its energy. Then, we maximize lifetime  $T_{l1}$ , s.t.

$$\sum_{r \neq i} f_{s_i, s_r}^{s_i, A_l} + f_{s_i, A_l}^{s_i, A_l} - g_i \lambda^{s_i, A_l} = 0 \quad (4)$$

$$\sum_{r \neq i, k} f_{s_i, s_r}^{s_k, A_l} + f_{s_i, A_l}^{s_k, A_l} - \sum_{m \neq i, k} f_{s_m, s_i}^{s_k, A_l} = 0 \quad (5)$$

$$\left( \sum_{f_{s_i, s_r}^{s_k, A_l} \in F_{s_i, s}} c_{s_i, s_r} f_{s_i, s_r}^{s_k, A_l} + \sum_{f_{s_i, A_l}^{s_k, A_l} \in F_{s_i, A}} c_{s_i, A_l} f_{s_i, A_l}^{s_k, A_l} + \sum_{f_{s_m, s_i}^{s_k, A_l} \in F_{s, s_i}} p_{rx} f_{s_m, s_i}^{s_k, A_l} \right) T_{l1} \leq e_i \text{ for } (1 \leq i \leq N) \quad (6)$$

**Table 1: Notations**

Symbols	Definitions
$N$	The total number of sensors in the network
$M$	The total number of Actuators in the network
$B$	The total number of BaseStations in the network
$e_i$	Initial energy of a sensor node $i$
$E_i$	Initial energy of an actuator node $i$
$g_i$	The locally generated data rate at sensor $i$
$P_{rx}$	Power consumption coefficient for receiving data
$c_{i,j}$	Power consumption coefficient for transmitting data from sensor $i$ to sensor $j$
$\alpha, \beta$	Two constants terms in power consumption for transmitting data
$d_{i,j}$	The physical distance between two nodes $i$ and $j$
$f_{s_i, s_j}^{s_k, A_l}$ (or $f_{s_i, A_l}^{s_k, A_l}$ )	The flow rate from sensor $i$ to sensor $j$ (or actuator $l$ ) with source and destination being sensor $k$ and actuator $l$
$F_{s,s}$ (or $F_{s,A}$ )	The set of flows from one sensor to another (or Actuator node)
$F_{s, s_i}$	The set of flows coming into sensor $i$
$F_{s_i, s}$ ( $F_{s_i, A}$ )	The set of flows going out of sensor $i$ to other sensors (or Actuator node)
$\lambda^{s_i, A_l}$	If the generated data at sensor $i$ will be transmitted to actuator $l$ , then $\lambda^{s_i, A_l} = 1$ ; otherwise $\lambda^{s_i, A_l} = 0$
$V_{s_i, s_j}^{s_k, A_l}$ (or $V_{s_i, A_l}^{s_k, A_l}$ )	The data volume (in bits) transferred from sensor $i$ to sensor $j$ (or Actuator $l$ ) with source and destination being sensor $k$ and Actuator $l$
$v_{s,s}$ ( $v_{s,A}$ )	The set of volumes from a sensor to another sensor (or an actuator)
$v_{s, s_i}$	The set of incoming volume into sensor $i$
$v_{s_i, s}$ ( $v_{s_i, A}$ )	The set of outgoing volumes from a sensor $i$ to another sensor (or an actuator)
$\mu^{s_i, A_l}$	$= \lambda^{s_i, A_l} T$ in MILP-relax
$f_{A_i, A_j}^{A_k, B_l}$ (or $f_{A_i, B_l}^{A_k, B_l}$ )	The flow rate from actuator $i$ to actuator $j$ (or BaseStation $l$ ) with source and destination being actuator $k$ and BaseStation $l$
$F_{A,A}$ (or $F_{A,B}$ )	The set of flows from one actuator to another (or BaseStation)
$F_{A, A_i}$	The set of flows coming into an actuator $i$
$F_{A_i, A}$ ( $F_{A_i, B}$ )	The set of flows going out of actuator $i$ to other actuators (or BaseStation)
$G_i$	The locally gathered data at actuator $i$ , $G_i = \sum_j g_i$ , ( $1 \leq i \leq N$ )
$\lambda^{A_i, B_l}$	If the data gathered at actuator $i$ will be transmitted to basestation $l$ , then $\lambda^{A_i, B_l} = 1$ ; otherwise $\lambda^{A_i, B_l} = 0$
$V_{B_i, B_j}^{A_k, B_l}$ (or $V_{A_i, B_l}^{A_k, B_l}$ )	The data volume (in bits) transferred from actuator $i$ to actuator $j$ (or BaseStation $l$ ) with source and destination being actuator $k$ and BaseStation $l$
$v_{A,A}$ ( $v_{A,B}$ )	The set of volumes from a actuator to another actuator (or a BaseStation)
$v_{A, A_i}$	The set of incoming volume into actuator $i$
$v_{A_i, A}$ ( $v_{A_i, B}$ )	The set of outgoing volumes from a actuator $i$ to another actuator (or a BaseStation)
$\mu^{A_i, B_l}$	$= \lambda^{A_i, B_l} T$ in MILP-relax

$$\sum_{1 \leq l \leq M} \lambda^{s_i, A_l} = 1 \quad (1 \leq i \leq N) \quad (7)$$

$$Tl1, f_{s_i, s_j}^{s_k, A_l}, f_{s_i, A_l}^{s_k, A_l} \geq 0, \lambda^{s_i, A_l} = 0 \text{ or } 1$$

$$f_{s_i, s_j}^{s_k, A_l} \in F_{s,s}, f_{s_i, A_l}^{s_k, A_l} \in F_{s,A}$$

$$1 \leq i, j, k \leq N, i \neq j, k \neq j, 1 \leq l \leq M$$

Note that  $\lambda^{s_i, A_l}$  is a binary variable used for Actuator selection: if the data stream generated by a sensor  $i$  will be transmitted to actuator  $l$ , then  $\lambda^{s_i, A_l} = 1$ ; otherwise  $\lambda^{s_i, A_l} = 0$ . The set of constraints in eq. (4) to (7) can be interpreted as follows. The set of constraints in (4) focuses on traffic flow generated locally at each sensor  $i$ . They state that, for each sensor  $i$ , if actuator  $l$  is the destination, then the locally generated bit rate (*i.e.*,  $g_i$ ) will be equal to the outgoing data flows from sensor  $i$  to actuator  $l$  via a single

hop (*i.e.*,  $f_{s_i, A_l}^{s_i, A_l}$ ) or multihop (*i.e.*,  $f_{s_i, s_r}^{s_i, A_l}$ ); otherwise, all flows corresponding to the source-destination pair  $(s_i, A_l)$  must be zero. The set of constraints in (5) focus on the traffic that uses sensor  $i$  as a relay node. They state that at each relay sensor  $i$ , the total amount of incoming traffic (*i.e.*,  $\sum_{m \neq i, k} f_{s_m, s_i}^{s_k, A_l}$ ) should be the same as the total amount of outgoing traffic (*i.e.*,  $\sum_{r \neq i, k} f_{s_i, s_r}^{s_k, A_l} + f_{s_i, A_l}^{s_k, A_l}$ ) for each source-destination pair  $(s_i, A_l)$ . The set of constraints in (6) concerns energy consumption at sensor  $i$ . They state that, for each sensor  $i$ , the energy consumption due to transmitting and receiving [see (1) and (2)] over the course of network lifetime should not exceed the initial energy supply  $e_i$ . Note that in (6) both flows generated locally at sensor  $i$  and those flows that use sensor  $i$  as a relay node are included. Finally the remaining set of constraints enforce that sensor  $i$  can only transmit all of its data to one actuator under any

routing protocol, along with the logical restriction on the optimization variables  $\lambda^{s_i, A_l}$ ,  $f_{s_i, s_j}^{s_k, A_l}$ , and  $f_{s_i, A_l}^{s_k, A_l}$ . Note that  $P_{rx}$ ,  $g_i$ ,  $e_i$ ,  $c_{s_i, s_r}$ , and  $c_{s_i, A_l}$  are all constants in this optimization problem.

The formulation of optimal flow routing and actuator selection is a mixed-integer non-linear programming (MINLP) problem, which is, unfortunately, NP-hard in general. We develop an upper bound for our optimal flow routing and actuator selection problem by studying a closely related problem that can be formulated and solved via linear programming. The non-linearity component in the flow routing problem can be removed by multiplying the equations (4)-(7) by  $T_{l1}$  and then use the linear substitutes  $(V_{s_i, s_j}^{s_k, A_l} = T_{l1} \cdot f_{s_i, s_j}^{s_k, A_l})$ ,  $(V_{s_i, A_l}^{s_k, A_l} = T_{l1} \cdot f_{s_i, A_l}^{s_k, A_l})$ , and  $(\mu^{s_i, A_l} = T_{l1} \cdot \lambda^{s_i, A_l})$ .

Therefore, we maximize lifetime  $T_{l1}$ , s.t.

$$\sum_{r \neq i} V_{s_i, s_r}^{s_i, A_l} + V_{s_i, A_l}^{s_i, A_l} - g_i \mu^{s_i, A_l} = 0 \quad (8)$$

$$\sum_{r \neq i, k} V_{s_i, s_r}^{s_k, A_l} + V_{s_i, A_l}^{s_k, A_l} - \sum_{m \neq i, k} V_{s_m, s_i}^{s_k, A_l} = 0 \quad (9)$$

$$\begin{aligned} \sum_{V_{s_i, s_r}^{s_k, A_l} \in v_{s_i, s}} c_{s_i, s_r} V_{s_i, s_r}^{s_k, A_l} + \sum_{V_{s_i, A_l}^{s_k, A_l} \in v_{s_i, A}} c_{s_i, A_l} V_{s_i, A_l}^{s_k, A_l} \\ + \sum_{V_{s_m, s_i}^{s_k, A_l} \in v_{s, s_i}} p_{rx} V_{s_m, s_i}^{s_k, A_l} \leq e_i \text{ for } (1 \leq i \leq N) \end{aligned} \quad (10)$$

$$\sum_{1 \leq l \leq M} \mu^{s_i, A_l} - T_{l1} = 0 \quad (1 \leq i \leq N) \quad (11)$$

$$\begin{aligned} T_{l1}, V_{s_i, s_j}^{s_k, A_l}, V_{s_i, A_l}^{s_k, A_l}, \mu^{s_i, A_l} \geq 0 \\ V_{s_i, s_j}^{s_k, A_l} \in v_{s, s}, V_{s_i, A_l}^{s_k, A_l} \in v_{s, A} \\ 1 \leq i, j, k \leq N, i \neq j, k \neq j, 1 \leq l \leq M \end{aligned}$$

We now have a standard MILP formulation that was transformed directly from the MINLP problem. By their equivalence, the solution of this MILP problem yields an upper bound to the basic flow routing problem. In mathematical optimization problems, Lagrange multipliers, is a method for finding the local extrema of a function of several variables subject to one or more constraints. In this paper, we use this approach to find optimal network lifetime. The method introduces a new unknown scalar variable, the lagrange multiplier, for each constraint and forms a linear combination involving the multipliers as coefficients. The justification for this can be carried out in the standard way as concerns partial differentiation. The objective is to find the conditions, for the implicit lifetime function, so that the derivative in terms of independent variables equals zero. For the mixed-integer component, we have implemented a distributed learning scheme using ns-2 [8] for SANETs, where each sensor can determine their optimal actuator based on the outcome of a cost-function (e.g., min. hop or min. delay routing) [5]. We have considered that there is no kind of mapping available between sensors and actuators at the time of deployment, so the proposed actuator discovery protocol in [5] is based on a purely distributed learning algorithm. It also accounts for incremental network deployments without incurring any extra overhead (mapping obtained by using one-hop neighbor communication). In this fashion, a sensor's destination actuator could be fixed as an outcome of

this cost-function. We now provide the proof for the optimality of network-lifetime.

**Proof:** Maximize lifetime  $T_{l1}$

Subject to

$$\sum_{r \neq i} f_{s_i, s_r}^{s_i, A_l} + f_{s_i, A_l}^{s_i, A_l} - g_i \lambda^{s_i, A_l} = 0 \quad (12)$$

$$\sum_{r \neq i, k} f_{s_i, s_r}^{s_k, A_l} + f_{s_i, A_l}^{s_k, A_l} - \sum_{m \neq i} f_{s_m, s_i}^{s_k, A_l} = 0 \quad (13)$$

$$\left( \begin{aligned} \sum_{f_{s_i, s_r}^{s_k, A_l} \in F_{s_i, s}} c_{s_i, s_r} f_{s_i, s_r}^{s_k, A_l} + \sum_{f_{s_i, A_l}^{s_k, A_l} \in F_{s_i, A}} c_{s_i, A_l} f_{s_i, A_l}^{s_k, A_l} \\ + \sum_{f_{s_m, s_i}^{s_k, A_l} \in F_{s, s_i}} p_{rx} f_{s_m, s_i}^{s_k, A_l} \end{aligned} \right) T_{l1} \leq e_i \quad (14)$$

$$\sum_{1 \leq l \leq M} \lambda^{s_i, A_l} = 1 \quad (15)$$

Eq. (12), (13) and (15) multiply  $T_{l1}$ , then we have the Lagrangian

$$\begin{aligned} L = T_{l1} + \lambda_1 \left( \sum_{r \neq i} V_{s_i, s_r}^{s_i, A_l} + V_{s_i, A_l}^{s_i, A_l} - g_i \mu^{s_i, A_l} \right) \\ + \lambda_2 \left( \sum_{r \neq i, k} V_{s_i, s_r}^{s_k, A_l} + V_{s_i, A_l}^{s_k, A_l} - \sum_{m \neq i} V_{s_m, s_i}^{s_k, A_l} \right) \\ + \lambda_3 \left( \sum_{V_{s_i, s_r}^{s_k, A_l} \in v_{s_i, s}} c_{s_i, s_r} V_{s_i, s_r}^{s_k, A_l} + \sum_{V_{s_i, A_l}^{s_k, A_l} \in v_{s_i, A}} c_{s_i, A_l} V_{s_i, A_l}^{s_k, A_l} \right) \\ + \sum_{V_{s_m, s_i}^{s_k, A_l} \in v_{s, s_i}} p_{rx} V_{s_m, s_i}^{s_k, A_l} - e_i + \lambda_4 \left( \sum_{1 \leq l \leq M} \mu^{s_i, A_l} - T_{l1} \right) \end{aligned}$$

1. derivations

$$\begin{aligned} \nabla T_{l1} L = 1 + \lambda_1 \left( \sum_{r \neq i} f_{s_i, s_r}^{s_i, A_l} + f_{s_i, A_l}^{s_i, A_l} - g_i \lambda^{s_i, A_l} \right) \\ + \lambda_2 \left( \sum_{r \neq i, k} f_{s_i, s_r}^{s_k, A_l} + f_{s_i, A_l}^{s_k, A_l} - \sum_{m \neq i} f_{s_m, s_i}^{s_k, A_l} \right) \\ + \lambda_3 \left( \sum_{f_{s_i, s_r}^{s_k, A_l} \in F_{s_i, s}} c_{s_i, s_r} f_{s_i, s_r}^{s_k, A_l} + \right. \\ \left. \sum_{f_{s_i, A_l}^{s_k, A_l} \in F_{s_i, A}} c_{s_i, A_l} f_{s_i, A_l}^{s_k, A_l} + \sum_{f_{s_m, s_i}^{s_k, A_l} \in F_{s, s_i}} p_{rx} f_{s_m, s_i}^{s_k, A_l} \right) \\ + \lambda_4 \left( \sum_{1 \leq l \leq M} \lambda^{s_i, A_l} - 1 \right) = 0 \end{aligned}$$

$$\nabla_{\lambda_j} L = 0, \quad 1 \leq j \leq 4$$

then we can write

$$1 + \lambda_3 \frac{e_i}{T} = 0$$

and  $\nabla_{\lambda_3} L =$

$$\begin{aligned} & \sum_{V_{s_i, s_r}^{s_k, A_l} \in v_{s_i, s}} c_{s_i, s_r} V_{s_i, s_r}^{s_k, A_l} + \sum_{V_{s_i, A_l}^{s_k, A_l} \in v_{s_i, A}} c_{s_i, A_l} V_{s_i, A_l}^{s_k, A_l} \\ & + \sum_{V_{s_m, s_i}^{s_k, A_l} \in v_{s, s_i}} p_{rx} V_{s_m, s_i}^{s_k, A_l} - e_i = 0 \end{aligned} \quad (16)$$

According to the definition of the sets of volumes, equation (16) can be write as

$$\begin{aligned} & \sum_{l=1}^M \sum_{k=1}^N \sum_{r \neq i, k} c_{s_i, s_r} f_{s_i, s_r}^{s_k, A_l} + \sum_{l=1}^M \sum_{k=1}^N c_{s_i, A_l} f_{s_i, A_l}^{s_k, A_l} \\ & + \sum_{l=1}^M \sum_{k=1, k \neq i}^N \sum_{m \neq i} p_{rx} f_{s_m, s_i}^{s_k, A_l} = \frac{e_i}{T_{l1}} \end{aligned} \quad (17)$$

It is necessary to discuss the summation of the  $k$ . The constraints (12) and (13) are satisfied in the case of  $k = i$  and  $k \neq i$ , respectively. So eq. (17) can be written as

$$\begin{aligned} & \sum_{l=1}^M \left[ \sum_{k \neq i} \left( \sum_{r \neq i, k} c_{s_i, s_r} f_{s_i, s_r}^{s_k, A_l} + c_{s_i, A_l} f_{s_i, A_l}^{s_k, A_l} + \sum_{m \neq i} p_{rx} f_{s_m, s_i}^{s_k, A_l} \right) \right. \\ & \left. + \sum_{r \neq i} c_{s_i, s_r} f_{s_i, s_r}^{s_i, A_l} + c_{s_i, A_l} f_{s_i, A_l}^{s_i, A_l} \right] = \frac{e_i}{T_{l1}} \end{aligned} \quad (18)$$

From eq. (13), we obtain

$$\sum_{m \neq i} f_{s_m, s_i}^{s_k, A_l} = \sum_{r \neq i, k} f_{s_i, s_r}^{s_k, A_l} + f_{s_i, A_l}^{s_k, A_l} \quad (19)$$

and from eq. (12), we have

$$\sum_{r \neq i} f_{s_i, s_r}^{s_i, A_l} = g_i \lambda^{s_i, A_l} - f_{s_i, A_l}^{s_i, A_l} \quad (20)$$

Assume that  $c_{s_i, s_r}$  is constant, then substitute (19) and (20) into (18)

$$\begin{aligned} & \sum_{l=1}^M \left[ \sum_{k \neq i} \left( \sum_{r \neq i, k} (c_{s_i, s_r} + p_{rx}) f_{s_i, s_r}^{s_k, A_l} \right. \right. \\ & \left. \left. + (c_{s_i, A_l} + p_{rx}) f_{s_i, A_l}^{s_k, A_l} \right) + \right. \\ & \left. c_{s_i, s_r} g_i \lambda^{s_i, A_l} + (c_{s_i, A_l} - c_{s_i, s_r}) f_{s_i, A_l}^{s_i, A_l} \right] = \frac{e_i}{T_{l1}} \end{aligned} \quad (21)$$

Therefore, the (21) become

$$\begin{aligned} & \sum_{l=1}^M \sum_{k \neq i} \sum_{r \neq i, k} (c_{s_i, s_r} + p_{rx}) f_{s_i, s_r}^{s_k, A_l} + \\ & \sum_{l=1}^M \sum_{k \neq i} (c_{s_i, A_l} + p_{rx}) f_{s_i, A_l}^{s_k, A_l} + \\ & \sum_{l=1}^M c_{s_i, s_r} g_i \lambda^{s_i, A_l} + \sum_{l=1}^M (c_{s_i, A_l} - c_{s_i, s_r}) f_{s_i, A_l}^{s_i, A_l} = \frac{e_i}{T_{l1}} \end{aligned}$$

then

$$\begin{aligned} T_{l1} & \left[ \sum_{f_{s_i, s_r}^{s_k, A_l} \in F_{s_i, s, k \neq i}} (c_{s_i, s_r} + p_{rx}) f_{s_i, s_r}^{s_k, A_l} + \right. \\ & \sum_{f_{s_i, A_l}^{s_k, A_l} \in F_{s_i, A, k \neq i}} (c_{s_i, A_l} + p_{rx}) f_{s_i, A_l}^{s_k, A_l} \\ & \left. + c_{s_i, s_r} g_i + \sum_{l=1}^M (c_{s_i, A_l} - c_{s_i, s_r}) f_{s_i, A_l}^{s_i, A_l} \right] = e_i \end{aligned} \quad (22)$$

Hence, the optimal network-lifetime with sensor  $i$  having energy  $e_i$  under the given set of constrainte is given by eq. (22)

For the current work, we consider that the cost function is set to min. hop count and the actuators chosen by sensors are optimal in min. hop sense. *An advantage of setting the cost-function to min. hop routing is that the lower-tier (level one) of our heterogeneous network can be organized into clusters, where each cluster is centrally managed by an actuator.* It will also result in the disappearance of the mixed-integer (MI) component from the optimization problem and the resultant is a relaxed linear optimization problem (LP) which is comparatively easier to solve. In this fashion, a sensor can receive its scheduling information by its *mapped* destination-actuator that corresponds to the optimal routing solution, and hence, can result in the *realization* of optimal network lifetime in *practice*. We denote the resulting destination for a sensor via the above mapping as  $d(i)$ . Therefore we have,  $\mu^{s_i, d(i)} = T$ , and  $\mu^{s_i, A_l} = 0$  for  $A_l \neq d(i)$ . Then, we can find a routing solution by replacing  $A_l$  in eq. (8) to eq. (11) by  $d(i)$  (destination actuator for sensor  $i$ , and is thus, not presented here to conserve paper length).

Similarly, the joint problem of finding an optimal basestation and flow routing (to maximize network lifetime at level two) can be modeled the same way as done for level one. We define

$$F_{A,A} = \left\{ f_{A_i, A_j}^{A_k, B_l} : (1 \leq i, j, k \leq M, i \neq j, j \neq k, 1 \leq l \leq B) \right\}$$

$$F_{A,B} = \left\{ f_{A_i, B_l}^{A_k, B_l} : (1 \leq i, k \leq M, 1 \leq l \leq B) \right\}$$

$$F_{A,A_i} = \left\{ f_{A_m, A_i}^{A_k, B_l} : (1 \leq m, k \leq M, m \neq i, k \neq i, 1 \leq l \leq B) \right\}$$

$$F_{A_i, A} = \left\{ f_{A_i, A_r}^{A_k, B_l} : (1 \leq r, k \leq M, r \neq k, r \neq i, 1 \leq l \leq B) \right\}$$

$$F_{A_i, B} = \left\{ f_{A_i, B_l}^{A_k, B_l} : (1 \leq k \leq M, 1 \leq l \leq B) \right\}$$

We maximize lifetime  $T_{l2}$ (lifetime at network level two), s.t.

$$\sum_{r \neq i} f_{A_i, A_r}^{A_k, B_l} + f_{A_i, B_l}^{A_k, B_l} - G_i \lambda^{A_i, B_l} = 0 \quad (23)$$

$$\sum_{r \neq i, k} f_{A_i, A_r}^{A_k, B_l} + f_{A_i, B_l}^{A_k, B_l} - \sum_{m \neq i, k} f_{A_m, A_i}^{A_k, B_l} = 0 \quad (24)$$

$$\left( \sum_{f_{A_i, A_r}^{A_k, B_l} \in F_{A_i, A}} c_{A_i, A_r} f_{A_i, A_r}^{A_k, B_l} + \sum_{f_{A_i, B_l}^{A_k, B_l} \in F_{A_i, B}} c_{A_i, B_l} f_{A_i, B_l}^{A_k, B_l} + \sum_{f_{A_m, A_i}^{A_k, B_l} \in F_{A_m, A_i}} Prx f_{A_m, A_i}^{A_k, B_l} \right) T_{l2} \leq E_i \text{ for } (1 \leq i \leq M) \quad (25)$$

$$\sum_{1 \leq l \leq B} \lambda^{A_i, B_l} = 1 \quad (1 \leq i \leq M) \quad (26)$$

$$T_{l2}, f_{A_i, A_j}^{A_k, B_l}, f_{A_i, B_l}^{A_k, B_l} \geq 0, \lambda^{A_i, B_l} = 0 \text{ or } 1 \\ f_{A_i, A_j}^{A_k, B_l} \in F_{A_i, A}, f_{A_i, B_l}^{A_k, B_l} \in F_{A_i, B}, 1 \leq i, j, k \leq M \\ i \neq j, k \neq j, 1 \leq l \leq B$$

Note that  $\lambda^{A_i, B_l}$  is a binary variable used for basestation selection: if the data stream generated by a sensor  $i$  will be transmitted to actuator  $l$ , then  $\lambda^{A_i, B_l} = 1$ ; otherwise  $\lambda^{A_i, B_l} = 0$ . The set of constraints in eq. (23) to (26) can be interpreted as follows. The set of constraints in (23) focuses on traffic flow generated locally at each actuator  $i$ . They state that, for each actuator  $i$ , if BaseStation  $l$  is the destination, then the locally generated (data gathered by the sensors that belong to actuator  $i$ 's cluster) bit rate (*i.e.*,  $G_i$ ) will be equal to the outgoing data flows from actuator  $i$  to BaseStation  $l$  via a single hop (*i.e.*,  $f_{A_i, B_l}^{A_k, B_l}$ ) or multihop ( $f_{A_i, A_r}^{A_k, B_l}$ ); otherwise, all flows corresponding to the source-destination pair  $(A_i, B_l)$  must be zero. The set of constraints in (24) focus on the traffic that uses actuator  $i$  as a relay node. They state that at each relay actuator  $i$ , the total amount of incoming traffic (*i.e.*,  $\sum_{m \neq i, k} f_{A_m, A_i}^{A_k, B_l}$ ) should be the same as the total amount of outgoing traffic (*i.e.*,  $\sum_{r \neq i, k} f_{A_i, A_r}^{A_k, B_l} + f_{A_i, B_l}^{A_k, B_l}$ ) for each source-destination pair  $(A_i, B_l)$ . The set of constraints in (25) concerns energy consumption at actuator  $i$ . They state that, for each actuator  $i$ , the energy consumption due to transmitting and receiving [see (1) and (2)] over the course of network lifetime should not exceed the initial energy supply  $E_i$ . Note that in (25) both flows generated locally at actuator  $i$  and those flows that use actuator  $i$  as a relay node are included. Finally the remaining set of constraints enforce that actuator  $i$  can only transmit all of its data to one basestation under any routing protocol, along with the logical restriction on the optimization variables  $\lambda^{A_i, B_l}$ ,  $f_{A_i, A_j}^{A_k, B_l}$ , and  $f_{A_i, B_l}^{A_k, B_l}$ . Note that  $Prx$ ,  $G_i$ ,  $E_i$ ,  $c_{A_i, A_r}$ , and  $c_{A_i, B_l}$  are all constants in this optimization problem.

The formulation of optimal flow routing and basestation selection is again a mixed-integer non-linear programming (MINLP) problem. We develop a similar upper bound for flow routing and basestation selection problem that can be formulated and solved via linear programming (similar to the formulation as level one). Here, we only present the modeling of optimal flow routing and basestation selection. An optimization criteria similar to level one can be opted here to solve the system of equations. The non-linearity component in the flow routing problem can be removed by multiplying the equations (23)-(26) by  $T_{l2}$  and then use the linear substitutes  $(V_{A_i, A_j}^{A_k, B_l} = T_{l2} \cdot f_{A_i, A_j}^{A_k, B_l})$ ,  $(V_{A_i, B_l}^{A_k, B_l} = T_{l2} \cdot$

$f_{A_i, B_l}^{A_k, B_l})$ , and  $(\mu^{A_i, B_l} = T_{l2} \cdot \lambda^{A_i, B_l})$ . Then, the MINLP problem can be reformulated into the equivalent MILP problem as shown below.

We maximize lifetime  $T_{l2}$ , s.t. (eq. (23)-(26)).  $T_{l2}$

$$T_{l2}, V_{A_i, A_j}^{A_k, B_l}, V_{A_i, B_l}^{A_k, B_l} \geq 0 \\ V_{A_i, A_j}^{A_k, B_l} \in \mathcal{V}_{A, A}, V_{A_i, B_l}^{A_k, B_l} \in \mathcal{V}_{A, B} \\ 1 \leq i, j, k \leq M, i \neq j, k \neq j, 1 \leq l \leq B$$

Since we have only one sink in the network, the network of basestations can form an aggregation tree toward this common sink and the flow from a basestation can be splitted and send over multiple routes toward the sink. Since we also opt to perform optimization at this network level, the flow problem to extend network lifetime at level three can be written in similar fashion as eq. (8) to eq. (11) with their appropriate subscripts (note that in this case, all the data gathered at different basestations is sent to a common sink and hence, the optimal flow solution formulation will result in an NLP formulation (the mixed integer component disappears due to the existence of a single sink) which can be relaxed using same technique as presented earlier to an equivalent LP formulation), and is therefore, not presented here to conserve paper length.

### 5.3 Medium Access Protocol

Once the optimal actuators are decided for each sensor in the network and optimal flow routing is formulated, then the sensors can be scheduled using a TDMA like MAC protocol that corresponds to the flow solution. The actuators explicitly schedules<sup>2</sup> all the sensors based on its knowledge of the cluster. Another important issue in this regard is that the routing solution obtained by the optimization amounts to flow splitting which require some critical synchronization issues (NP-hard to obtain in ad hoc manner) among neighboring sensor nodes. This is easily solved by a TDMA like MAC as we allow for an actuator to manage its own cluster.

If a random access scheme is used at the MAC layer, then the *experienced* network lifetime can not correspond to the optimal routing solution (22) as the optimal routing solution is only based on the flow coming into-and-out of a sensor node subject to energy-constraints. It also do not take into account the access energy wasted due to *collisions* and *successive retransmissions*. Therefore, the optimal flow routing solution (toward the selected actuator *i.e.*, outcome of a cost-function) and TDMA MAC are *jointly* optimal in extending the network lifetime as optimal flow routing solution is to operate on top of a given MAC layer scheme.

## 6. SIMULATION RESULTS

In this section, we present our ns-2 [8] simulation results demonstrating the performance of our optimal flow routing and TDMA MAC solution. As our analysis is different from the related literature presented in this work, we only compare the results given by the upper bound in eq. (22) (optimal flow solution for the relaxed problem which is independent of MAC) and the simulations performed in ns-2

<sup>2</sup>The details on the TDMA MAC for sensor-actuator networks can be found in [9]. However, since the sensors are scheduled by a TDMA MAC then all packets reach the actuator by a deadline. Since any routing tree may result from this decomposition, the deadline should be achieved for all such possible trees.

on top of a TDMA like MAC. In our simulations, we consider different network sizes (varying the number of sensors and actuators) with randomly deployed topologies. Also, we evaluate the lifetime only at level one as the optimal lifetime solution at other levels can be interpreted in a similar fashion. Further, in the existing network simulators e.g. [8, 10], there are no available means of simulating a heterogeneous network consisting of sensors and actuators (as actuator have different transmission and processing capabilities). Therefore, we post-process our ns-2 based *tcl-scripts* in order to simulate a heterogeneous sensor-actuator network. The simulations are run several times for each network setting and the results presented are averaged over these runs.

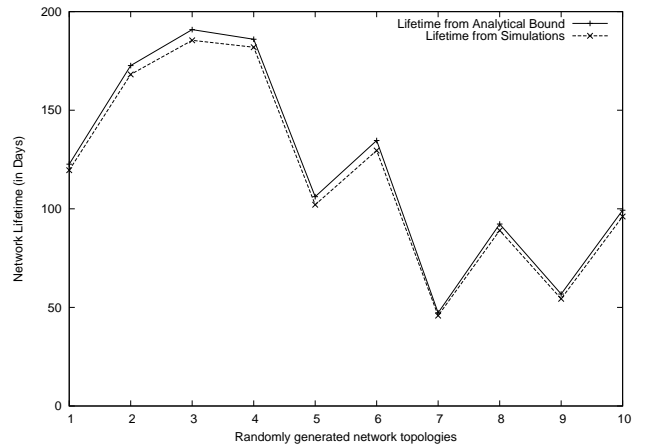
For each network setting, we calculate the upper bound on lifetime provided by eq. (22) through MILP relax. We denote this lifetime as  $T_{optimal}$ . We denote the lifetime obtained with actual ns-2 simulation as  $T_{simulation}$ . The initial energy at sensor  $i$  is randomly generated following a uniform distribution with  $e_i \in [250, 500]$  (kJ). The data generation rate at each sensor  $i$ ,  $g_i$ , is also uniformly distributed within  $[2, 10]$  (kb/s). At simulation start up, the nodes learn the network topology and built routes toward the destination actuators (based on the outcome of a cost-function). This learning process, which depends on the network topology, can take upto 50 – 70 seconds. In this simulation-analysis, actuators are also sensor nodes which have 0 sampling rate<sup>3</sup>, i.e., no actuation is currently performed based on incoming sensor readings.

The results obtained from eq. (22) and simulations using ns-2 are presented in Fig. 2. The slight difference in the lifetime obtained from the simulations is due to the energy expenditure during initial network learning and route discovery toward actuators. The simulated lifetime lies exceptionally close to the analytical bound (for relaxed flow-problem) due to the following reasons: 1) we have built an aggregation tree toward each actuator in the network which is based on the result of optimal flow routing solution. 2) The scheduling information is sent to the sensors by their mapped-actuator nodes which corresponds to the optimal flow solution. 3) The problem of *synchronization* is easily solved as the *transmission schedule* is calculated by the actuator in each cluster. 4) There is no extra energy expenditure as a result of collisions and successive retransmissions. 5) The nodes are sent to sleep mode, when not transmitting, and also no information is expected to arrive from a sensor's downlink tree.

## 7. CONCLUSIONS AND FUTURE WORK

This paper considers a large scale wireless sensor-actuator network with multiple actuators as sinks for data generated by the sensors. Since many applications require to have each source node send all its locally generated data to only one actuator for processing, it is necessary to optimally map each sensor to its actuator. Also considering the fact that the end-to-end delays in wireless sensor-actuator networks is a hard constraint, we jointly optimize the actuator selection and optimal flow routing subject to energy and delay constraints with the global aim of maximizing the network lifetime. We

<sup>3</sup>At run-time, these nodes are modified to actuators (i.e., different communication capabilities compared to sensors) so that an aggregation tree could be built toward these actuators for each cluster.



**Figure 2: Network lifetime under analytical and simulation results**

proposed and evaluated (using a standard network simulator, ns-2) our jointly optimal actuator-selection and routing scheme on top of a TDMA based MAC. we also provide a comparison to the analytical bound. Simulation results show that this approach has near-optimal performance and is practically implementable as compared to earlier analytical studies based only on numerical evaluations.

In future, we will extend our simulation setup to a component based development environment TinyOS [10] and extend the simulator itself to enable development and testing of such heterogeneous wireless sensor-actuator system. We are also working towards a dynamic actuator-binding based routing-protocol for event-driven sensor-actuator applications subject to delay-energy constraints with the aim of extending network-lifetime.

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