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Maximum Likelihood Estimation in the Odd Generalized Exponential-Exponential Distribution

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Abstract

A new distribution, called odd generalized exponential-exponential distribution (OGE-E(ϴ)) is proposed for modeling lifetime data. A comprehensive study of the cumulative distribution function, probability density function, survival and hazard function of the new distribution are presented. Moreover, the maximum likelihood estimation of the parameters of the OGE-E(ϴ) distribution is considered for both simulated and real data sets.

Keywords: Odd generalized distribution, Generalized exponential distribution, Cumulative distribution function, Reliability function, Maximum likelihood estimation

1. Introduction

The skills of proposing generalized classes of distributions has pulled theoretical and applied statisticians due to their flexibility in modeling data in practice. The exponential distribution is perhaps the most widely applied statistical distribution for problems in reliability. However, the generalized exponential distribution (called GE) has lots of different properties and it can be used quite effectively to analyze several skewed lifetime data.

Considered the three-parameter to proposed a generalized of the exponential distribution called Generalized Exponential (GE) distribution fits better than the three-parameter gamma or three-parameter Weibull in some cases and discussed some of different properties of the model (Gupta and Kundu, 1999) [6]. Discussed three-parameter GE distribution for exact expressions for single and product moments of record statistics. In addition, he obtained single and product moments of record statistics (Raqab, 2002) [15]. Introduced a generalization referred to as the beta exponential distribution generated from the logit of a beta random variable. However, they provided a comprehensive treatment of the mathematical properties of the beta exponential distribution and discussed simulation issues, estimation by the methods of moments and maximum likelihood by (Nadarajah and Kotz, 2006) [14]. Used the Bayes estimation of the unknown parameters of the GE distribution under the assumptions of gamma priors on both the shape and scale parameters; Also they considered Bayesian estimation of the parameter based on the idea of Lindley and the Gibbs sampling procedure (Gupta and Kundu, 2007) [7]. Defined a bivariate GE distribution so that the marginal distribution are GE distribution. Furthermore, they used the algorithm to compute the maximum likelihood estimates of the unknown parameters. They observed that the bivariate generalized exponential distribution provides a better fit than the bivariate exponential distribution by (Kundu and Gupta, 2009) [9]. Introduced the Marshall-Olkin approach to introduce an extra shape parameter to the two-parameter GE distribution and observed that the new three-parameter distribution is very flexible, they noticed that the probability density functions can be either a decreasing or an unimodal function. In addition, they established different properties by using the Maximum likelihood method to compute the estimators of the unknown parameters by (Risti´c and Kundu, 2015) [16].

Studied some mathematical properties of the new wider Weibull-G family of distributions and discussed some special models in the new family also derived the properties hold to any distribution in this family. Obtained general explicit expressions for the quantile function, ordinary and incomplete moments, generating function and order statistics and discussed the estimation of the model parameters by maximum likelihood and illustrate the potentiality of the extended family with two applications to real data (Bourguignon et al., 2014) [2]. Studied a new generalization of the exponential Gompertz and generalized exponential distributions called the generalized Gompertz distribution (GGD) which has increasing, constant, decreasing or bathtub curve failure rate depending upon the shape parameter. They found that the GGD is very useful in survival analysis. They derived some statistical properties such as moments, mode, and quantiles. However, they obtained the maximum likelihood estimators of the parameters using a simulations and real data to determine whether the GGD is better than other well-known distributions in modeling lifetime data (EL-Gohary et al., 2013) [5]. Studied the generalized exponentiated moment exponential (GEME) distribution and developed various properties of the distribution. They also presented characterizations of the distribution in terms of conditional expectation as well as based on hazard function of the GEME distribution random variable (Iqbal et al., 2014) [8]. Introduced a new class of distribution called Kumaraswamy generalized exponentiated exponential distribution and calculated the variation of the skewness and kurtosis measures. He derived the likelihood estimators of the parameters and analyzed a real data set (Mohammed, 2014) [15]. Studied a new continuous distribution called exponentiated Kumaraswamy-exponential that extends the exponential distribution and studied several structural properties of the new distribution. They investigated the moments, hazard function, mean deviations and R´enyi entropy. However, they discussed the maximum likelihood estimation of this distribution and an application reveals that the model proposed can be very useful in fitting real data (Rodrigues and Silva, 2015) [17]. Proposed a new family of continuous distributions called the odd generalized exponential family, whose hazard rate function could be increasing, decreasing. It includes as a special case the widely known exponentiated-Weibull distribution and discussed three special models in the family density function can be expressed as a mixture of exponentiated densities based on the same baseline distribution. They derived explicit expressions for the ordinary and incomplete moments, quantile and generating functions, Bonferroni and Lorenz curves, Shannon and Rényi entropies and order statistics for the first time obtained the generating function of the Fréchet distribution and proposed characterizations of the family. They considered the method of maximum likelihood and means of two real lifetime data sets (Tahir *et. al.*, 2015) [18]. Studied a new distribution, called odds generalized exponential-exponential distribution. They obtained some mathematical properties. However, they studied estimation and simulation of the distribution (Maiti and Pramanik, 2015) [11]. Proposed a new lifetime model, called the odd generalized exponential Gompertz distribution. They obtained some of its mathematical properties. However, they studied the maximum likelihood estimation and derived the Fisher's information matrix. Finally, they applied it to a real data set. Introduced a new model called the odd generalized exponential linear failure rate distribution (El-Damcese et al, 2015) [3]. They obtained some statistical properties and discussed the estimation of the model parameters by maximum likelihood by (El-Damcese et al, 2015) [4]. Proposed a new generalization of the Rayleigh distribution called odd generalized exponential Rayleigh distribution and derived its statistical properties. They derived some mathematical properties (Luguterah, 2016) [10]. Studied the new distribution, called odds generalized exponential-Pareto distribution. They derived mathematical properties of the new distribution including estimation, simulation. Finally, they applied it to a real data set (Maiti and Pramanik, 2016) [12].

The core idea of the construction of the odd generalized exponential family (OGE) based on replacing x in the cumulative distribution function (CDF) of the GE model, given by:

$$F_{GE}(x;\alpha,\lambda) = \left(1 - e^{-\lambda x}\right)^{\alpha}; \tag{1}$$

where α and λ are positive parameters; by $G(x;\eta)/\,\overline{G}(x;\eta)$ where $G(x;\eta)$ and $\overline{G}(x;\eta)$ are the CDF and the reliability function of a parent distribution with a parameter vector η. That is, the CDF of the OGE family can be written as:

$$F_{OGE}(x;\alpha,\lambda,\eta) = \left(1 - e^{-\lambda\frac{G(x;\eta)}{\overline{G}(x;\eta)}}\right)^{\alpha}. \tag{2}$$

and the probability density function (PDF) is then given by:

$$f_{OGE}(x;\alpha,\lambda,\xi)=\frac{\alpha\lambda g(x;\eta)}{\overline{G}(x;\eta)}e^{-\lambda\frac{G(x;\eta)}{\overline{G}(x;\eta)}}\left(1-e^{-\lambda\frac{G(x;\eta)}{\overline{G}(x;\eta)}}\right)^{\alpha-1}, \tag{3}$$

where g(x; η) is the baseline PDF (Tahir *et. al*., 2015) [18].

This paper is outlined as follows. In Section 2, the cumulative distribution function, density function, reliability function and hazard function of the odd generalized exponential-exponential OGE-E(Θ) distribution are defined. Section 3 presents the maximum likelihood estimation of the parameters and asymptotic confidence intervals are determined. Section 4 considers the numerical computations of the maximum likelihood estimates of the parameters for both simulated and real data sets. Finally, discussion and conclusions are presented in Section 5.

2. OGE-E(Θ) Distribution

In this section, the construction of the new three parameters distribution called odd generalized exponential-exponential (OGE-E(Θ)) distribution with parameters γ, α, and λ where the vector Θ is defined in the form Θ = (γ, α, λ) is presented.

Definition: A random variable X is said to have the OGE-E(Θ) with parameters γ, α and λ if $G(x;\eta)/\overline{G}(x;\eta)$ where $G(x;\eta)=1-e^{-\gamma x}$ and $\overline{G}(x;\eta)=1-(1-e^{-\gamma x})$, in Equation (2), are the CDF and the reliability function of a exponential distribution with a parameter vector γ That is, the CDF of the OGE-E(Θ) can be written as:

$$F_{OGE-E}(x,\alpha,\lambda,\gamma)=\left(1-e^{e^{-\lambda(\frac{1-e^{-\gamma x}}{e^{-\gamma x}})}}\right)^{\alpha}. \tag{4}$$

$$F_{OGE-E}(x,\alpha,\lambda,\gamma)=\left(1-e^{-\lambda(e^{\gamma x}-1)}\right)^{\alpha}. \tag{5}$$

The corresponding PDF is then given by:

$$f_{OGE-E}(x,\alpha,\lambda,\gamma)=\frac{\alpha\,\lambda\,\gamma}{e^{-\gamma x}}e^{-\lambda(e^{\gamma x}-1)}\left(1-e^{-\lambda(e^{\gamma x}-1)}\right)^{\alpha-1}. \tag{6}$$

The and survival and hazard functions of OGE-E(Θ) function is given by

$$S(x)=1-\left(1-e^{-\lambda(e^{\gamma x}-1)}\right)^{\alpha}, \tag{7}$$

and

$$h(x) = \frac{f(x)}{S(x)} = \frac{\alpha\lambda\gamma e^{-\lambda(e^{\gamma x}-1)+\gamma x}\left(1-e^{-\lambda(e^{\gamma x}-1)}\right)^{\alpha-1}}{1-(1-e^{-\lambda(e^{\gamma x}-1)})^{\alpha}}. \quad (8)$$

Also the reversed hazard function of OGE-E(Θ) is

$$r(x) = \frac{f(x)}{F(x)} = \frac{\alpha\lambda\gamma e^{-\lambda(e^{\gamma x}-1)+\gamma x}}{1-e^{-\lambda(e^{\gamma x}-1)}}. \quad (9)$$

The following Figure 1, Figure 2 and Figure 3 show different shapes of the PDF, CDF and the hazard function, respectively, corresponding to different values of the parameters of the OGE-E(Θ) distribution.

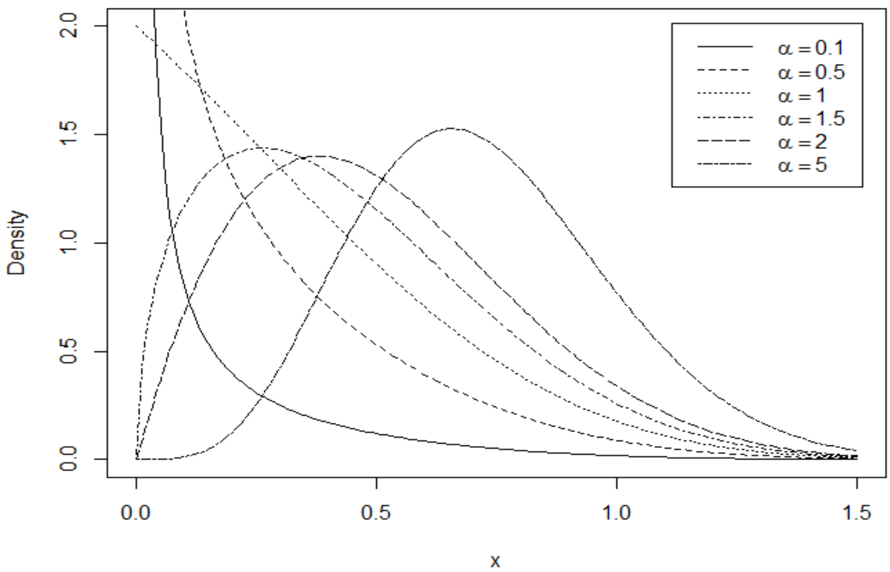

Figure 1: PDF of the OGE-E(Θ) distribution for Θ = ($\boldsymbol{\gamma = 1, \alpha = 0.1, 0.5, 1, 1.5, 2, 5, \ \lambda = 2}$).

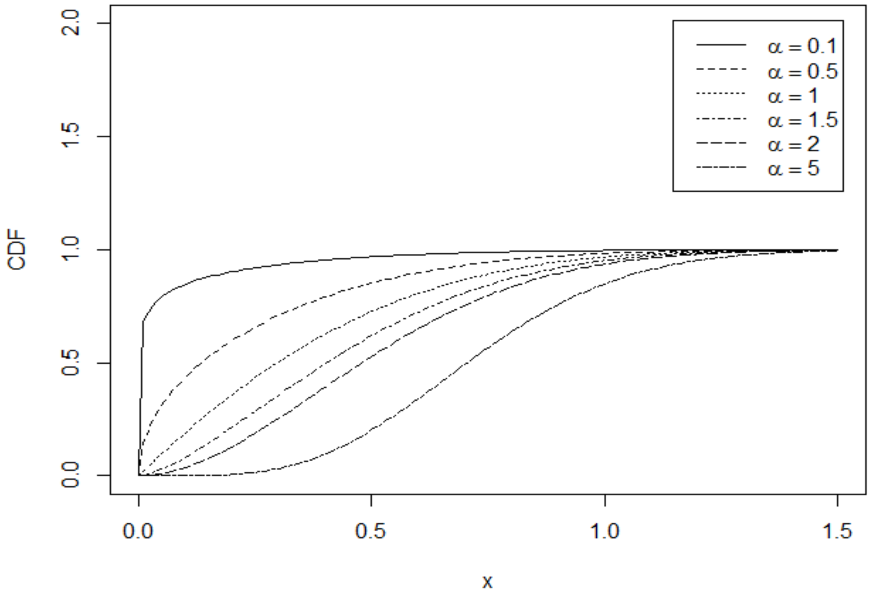

Figure 2: CDF of the OGE-E(ϴ) distribution for ϴ = ($\gamma = 1, \alpha = 0.1, 0.5, 1, 1.5, 2, 5,\ \lambda = 2$)

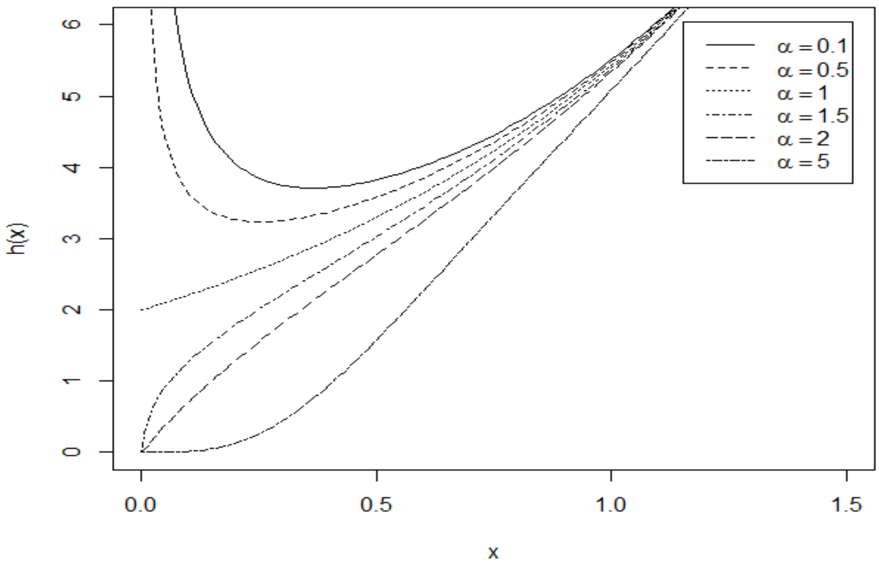

Figure 3: Hazard function of the OGE-E(ϴ) distribution for ϴ = ($\gamma = 1, \alpha = 0.1, 0.5, 1, 1.5, 2, 5,\ \lambda = 2$

3. Maximum Likelihood Estimation

The maximum likelihood estimators (MLE'S) of OGE-E(ϴ) parameters can be obtained as follows, the likelihood function of the OGE-E(ϴ) is

$$\ell = \prod_{i=1}^{n} \alpha\lambda\gamma\, e^{\gamma x} e^{-\lambda(e^{\gamma x}-1)} (1 - e^{-\lambda(e^{\gamma x}-1)})^{\alpha-1}. \tag{10}$$

Then the log-likelihood function is given by

$$L = \quad n\ln(\alpha) + n\ln(\lambda) + n\ln(\gamma) + \gamma \sum_{i=0}^{n} x_i$$
$$-\lambda \sum_{i=0}^{n} (e^{\gamma x_i} - 1) + (\alpha - \ 1) \sum_{i=0}^{n} \ln\left[1 - e^{-\lambda(e^{\gamma x}-1)}\right].$$

Hence, the maximum likelihood equations are

$$\frac{\partial L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \ln\left[1 - e^{-\lambda(e^{\gamma x_i}-1)}\right] = 0, \tag{11}$$

$$\frac{\partial L}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} (e^{\gamma x_i} - 1) + (\alpha - 1) \sum_{i=1}^{n} \frac{e^{\gamma x_i} - 1}{e^{\lambda(e^{\gamma x_i}-1)} - 1} = 0, \tag{12}$$

$$\frac{\partial L}{\partial \gamma} = \frac{n}{\gamma} + \sum_{i=1}^{n} x_i + (\alpha - 1) \sum_{i=1}^{n} \frac{\gamma\lambda e^{\gamma x_i}}{e^{\lambda(e^{\gamma x_i}-1)} - 1} = 0, \tag{13}$$

From equation (11), when assuming the parameters λ and γ are known one can obtain the maximum likelihood estimator of α in a closed form as follow:

$$\hat{\alpha} = \frac{-n}{\sum_{i=1}^{n} \ln\left[1 - e^{-\lambda(e^{\gamma x_i}-1)}\right]}. \tag{14}$$

Otherwise, a statistical software or numerical technique must be applied to solve the likelihood quations (11), (12) and (13) simultaneously in order to obtain the maximum likelihood estimates of the parameters.

It is known that the simplest large sample approach is to assume that the MLEs γ, α and λ are approximately multivariate normal with mean μ and the covariance matrix I_0^{-1}, where I_0^{-1} the inverse of the observed information matrix which defined as follows:

$$I_0^{-1} = E\left\{ -\begin{bmatrix} \frac{\partial^2 L}{\partial \alpha^2} & \frac{\partial^2 L}{\partial \lambda\, \partial \alpha} & \frac{\partial^2 L}{\partial \gamma\, \partial \alpha} \\ \frac{\partial^2 L}{\partial \alpha\, \partial \lambda} & \frac{\partial^2 L}{\partial \lambda^2} & \frac{\partial^2 L}{\partial \gamma\, \partial \lambda} \\ \frac{\partial^2 L}{\partial \alpha\, \partial \gamma} & \frac{\partial^2 L}{\partial \lambda\, \partial \gamma} & \frac{\partial^2 L}{\partial \gamma^2} \end{bmatrix}^{-1} \right\}$$

$$I_0^{-1} = \begin{bmatrix} var(\hat{\alpha}) & cov(\hat{\lambda},\hat{\alpha}) & cov(\hat{\gamma},\hat{\alpha}) \\ cov(\hat{\alpha},\hat{\lambda}) & var(\hat{\lambda}) & cov(\hat{\gamma},\hat{\lambda}) \\ cov(\hat{\alpha},\hat{\gamma}) & cov(\hat{\lambda},\hat{\gamma}) & var(\hat{\gamma}) \end{bmatrix}, \qquad (15)$$

where the second partial derivatives included in I_0^{-1} are given as follows

$$\frac{\partial^2 L}{\partial \alpha^2} = \frac{-n}{\alpha^2},$$

$$\frac{\partial^2 L}{\partial \alpha\, \partial \lambda} = \sum_{i=1}^{n} \frac{e^{\gamma x_i} - 1}{e^{\lambda(e^{\gamma x_i}-1)} - 1},$$

$$\frac{\partial^2 L}{\partial \alpha\, \partial \gamma} = \sum_{i=1}^{n} \frac{\lambda x_i e^{\gamma x_i}}{e^{\lambda(e^{\gamma x_i}-1)} - 1},$$

$$\frac{\partial^2 L}{\partial \lambda^2} = \frac{-n}{\lambda^2} - (\alpha - 1) \sum_{i=1}^{n} \frac{\left((e^{\gamma x_i} - 1)\right)^2 e^{\lambda(e^{\gamma x_i}-1)}}{\left(e^{\lambda(e^{\gamma x_i}-1)} - 1\right)^2},$$

$$\frac{\partial^2 L}{\partial \lambda\, \partial \alpha} = \sum_{i=1}^{n} \frac{e^{\gamma x_i} - 1}{e^{\lambda(e^{\gamma x_i}-1)} - 1},$$

$$\frac{\partial^2 L}{\partial \lambda\, \partial \gamma} = -\sum_{i=1}^{n} x_i e^{\gamma x_i} + (\alpha - 1) \sum_{i=1}^{n} \frac{x_i e^{\gamma x_i}\left(-\lambda\, e^{\gamma x_i + \lambda e^{\gamma x_i} - 1} + (\lambda + 1) e^{\lambda(e^{\gamma x_i}-1)} - 1\right)}{\left(e^{\lambda(e^{\gamma x_i}-1)} - 1\right)^2},$$

$$\frac{\partial^2 L}{\partial \gamma^2} = \frac{-n}{\gamma^2} + (\alpha - 1) \sum_{i=1}^{n} \frac{\lambda e^{\gamma x_i}(-\lambda\, x_i \gamma e^{\gamma x_i + \lambda e^{\gamma x_i} - 1} + x_i \gamma\, e^{\lambda(e^{\gamma x_i}-1)} - x_i \gamma + e^{\lambda(e^{\gamma x_i}-1)} - 1)}{\left(e^{\lambda(e^{\gamma x_i}-1)} - 1\right)^2}$$

$$\frac{\partial^2 L}{\partial \gamma\, \partial \lambda} = (\alpha - 1) \sum_{i=1}^{n} \frac{\lambda e^{\gamma x_i}(-\lambda\, (e^{\gamma x_i} - 1) e^{\lambda(e^{\gamma x_i}-1)} + e^{\lambda(e^{\gamma x_i}-1)} - 1)}{\left(e^{\lambda(e^{\gamma x_i}-1)} - 1\right)^2},$$

$$\frac{\partial^2 L}{\partial \gamma\, \partial \alpha} = \sum_{i=1}^{n} \frac{\gamma \lambda e^{\gamma x_i}}{e^{\lambda(e^{\gamma x_i}-1)} - 1},$$

The asymptotic (1-δ)100% confidence intervals of α, λ, γ are $\hat{\alpha} \pm z_{\frac{\delta}{2}}\sqrt{var(\hat{\alpha})}$, $\hat{\lambda} \pm z_{\frac{\delta}{2}}\sqrt{var(\hat{\lambda})}$, and $\hat{\gamma} \pm z_{\frac{\delta}{2}}\sqrt{var(\hat{\gamma})}$, where $z_{\frac{\delta}{2}}$ is the upper $\frac{\delta}{2}$ percentile of the standard normal distribution

4. Numerical Computations

1. Monte Carlo Simulation Study

In this sub-section, the method of the ML estimation for the parameters of the OGE-E(Θ) distribution is obtained through a Monte Carlo simulation study. The sample size n=100, 150, 300 are considered, and m the number of samples is set to be 1000.

The ML estimates of the parameters γ, α and λ are obtained numerically as the following steps:

1. The ML estimates of parameters γ, α and λ are computed by solving the system of nonlinear equations (11), (12) and (13), simultaneously, by using Newton-Raphson method held in the (nlminb) function in R package.
2. The ML estimates, Var, Bias and MSE's, of the estimates are calculated for each method using the following:
3.

$$\text{Bias}(\hat{\theta}) = \bar{\hat{\theta}} - \theta \tag{16}$$

$$\text{MSE}(\hat{\theta}) = \text{Var}(\hat{\theta}) - (\text{Bias}(\hat{\theta}))^2 \tag{17}$$

where $\Theta = (\gamma; \alpha; \lambda)$ $\hat{\Theta} = (\hat{\gamma}, \hat{\alpha}, \hat{\lambda})$ and $\bar{\hat{\Theta}}$ is the mean of $\hat{\Theta}$ over the m repetitions. Table 1 summarizes the results of the simulation study for different parameters values.

Table 1. Var, Bias and MSE of the ML estimators for different γ, α and λ

n	γ, α, λ	Estimate			Var			Bias			MSE		
		$\hat{\gamma}$	$\hat{\alpha}$	$\hat{\lambda}$	$\hat{\gamma}$	$\hat{\alpha}$	$\hat{\lambda}$	$\hat{\gamma}$	$\hat{\alpha}$	$\hat{\lambda}$	$\hat{\gamma}$	$\hat{\alpha}$	$\hat{\lambda}$
100	0.5,2,1	0.5259	2.0930	1.2871	0.0296	0.3910	1.7223	0.0259	0.0930	0.2871	0.0303	0.3997	1.8048
	0.5, 1.5,1	0.5227	1.5646	1.2626	0.0304	0.1465	1.0621	0.0227	0.0646	0.2626	0.0309	0.1506	1.1311
	1,0.5,1	1.1016	0.5027	1.2047	0.1502	0.0057	1.3501	0.1016	0.0027	0.2047	0.1605	0.0057	1.3921
	1, 2, 1	1.0657	2.0816	1.2711	0.1218	0.3709	2.6177	0.0657	0.0816	0.2711	0.1262	0.3776	2.6912
150	0.5,2,1	0.5124	2.0652	1.1869	0.0196	0.2153	0.7200	0.0124	0.0652	0.1869	0.0197	0.2196	0.7550

Table 1. (Continued): Var, Bias and MSE of the ML estimators for different γ, α a nd λ

	0.5,1.5,1	0.5188	1.5255	1.1328	0.0191	0.0875	0.5095	0.0188	0.0255	0.1328	0.0195	0.0881	0.5272
	1,0.5,1	1.0637	0.5019	1.1480	0.1057	0.0035	0.8194	0.0637	0.0019	0.1480	0.1097	0.0035	0.8413
	1,2,1	1.0262	2.0604	1.1758	0.0809	0.2390	0.6529	0.0262	0.0604	0.1758	0.0816	0.2427	0.6838
300	0.5,2,1	0.5058	2.0364	1.0788	0.0095	0.0986	0.1944	0.0058	0.0364	0.0788	0.0096	0.1000	0.2006
	0.5,1.5,1	0.5079	1.5181	1.0680	0.0092	0.0387	0.1862	0.0079	0.0181	0.0680	0.0093	0.0390	0.1908
	1,0.5,1	1.0244	0.5028	1.0882	0.0575	0.0017	0.2531	0.0244	0.0028	0.0882	0.0581	0.0017	0.2609
	1,2,1	1.0155	2.0293	1.0831	0.0413	0.1014	0.2272	0.0155	0.0293	0.0831	0.0415	0.1023	0.2341

2. Real Data Analysis

A real data set obtained from (Aarset, 1987) [1] are applied to obtain the ML estimates of the parameters of the OGE-E(Θ) distribution. The data set consists of failure times of 50 devices put on life test at time 0, is given in Table 2. The estimates of the parameters are computed and presented in Table 3. In order to compare the ML estimates of the OGE-E(Θ) distribution with the previous corresponding results of other two distributions: Odd Generalized Exponential Gompertz distribution(OGE-G), Odd Generalized Exponential Linear Failure Rate distribution(OGE-LFR) and the Odd Generalized Exponential-Exponential distribution(OGE-E(Θ)). The summarized results are shown in Table 3.

Table 2: Lifetimes of 50 devices (Aarset, 1987) [1]

0.1	0.2	1	1	1	1	1	2	3	6
7	11	12	18	18	18	18	18	21	32
36	40	45	46	47	50	55	60	63	63
67	67	67	67	72	75	79	82	82	83
84	84	84	85	85	85	85	85	86	86

Hence, the (nlminb) function in R package is applied to this real data set in order to compute the ML estimates of the parameters of the OGE-E(ϴ) distribution. In addition, the ML estimates of the parameters of other two distributions: odd generalized exponential Gompertz distribution (OGE-G) with parameters $(\alpha, \beta, \lambda, c)$ and the odd generalized exponential linear failure rate distribution (OGE-LFR) with parameters (α, β, a, b) which have been obtained in [3] and [4], respectively. The comparisons between the computed ML estimates of the three distributions are studied according to the indications AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion). Hence, the summarized results are shown in Table 3.

Table 3: Summarized results of fitting different distributions to the lifetimes of 50 devices of (Aarset, 1987) [1]

Distribution	Estimates	AIC	BIC
OGE-G	$\hat{\alpha}$=0.0400, $\hat{\beta} = 0.1940$, $\hat{\lambda}$=0.000345, $\hat{c}$=0.0780	423.9470	447.5951
OGE-LFR	$\hat{\alpha}$=472.404, $\hat{\beta}$=0.529, $\hat{a}=8.218\times10^{-6}$, $\hat{b}=6.427\times10^{-7}$	473.730	481.378
OGE-E(ϴ)	$\hat{\gamma}$=0.1 $\hat{\alpha}$=0.8820 $\hat{\lambda}$=0.1	14306.45	14312.19

5. Conclusions

1. A new model the so-called odd generalized exponential-exponential (OGE-E(ϴ)) distribution is constructed; its cumulative distribution function, probability density function and survival, hazard functions are presented.

2. The ML estimation of the parameters of the OGE-E(ϴ) distribution is considered, for simulated and real data sets.

3. A comparison according to the indications of the AIC and BIC is held between the three distributions (OGE-E(ϴ), OGE-LFR and OGE-G) providing that the OGE-G distribution to be the best distribution that fits the real data set among others.

References

[1] M. V. Aarset, How to Identify a Bathtub Hazard Rate, *IEEE Transactions On Reliability*, **R-36** (1987), 106-108. https://doi.org/10.1109/tr.1987.5222310

[2] Marcelo Bourguignon Rodrigo B. Silva, Gauss M. Cordeiro, The Weibull-G Family of Probability Distributions, *Journal of Data Science,* **12** (2014), 53-68.

[3] M. A. El-Damcese, Abdelfattah Mustafa, B. S. El-Desouky, M. E. Mustafa, The Odd Generalized Exponential Gompertz Distribution, *Applied Mathematics*, **6** (2015), 2340-2353. https://doi.org/10.4236/am.2015.614206

[4] M. A. El-Damcese, A. Mustafa, B. S. El-Desouky, M. E. Mustafa, The Odd Generalized Exponential Linear Failure Rate Distribution, (2015). *math.ST*, arXiv:1510.06395v1

[5] A. El-Gohary, Ahmad Alshamrani, Adel Naif Al-Otaibi, The generalized Gompertz distribution, *Applied Mathematical Modelling*, **37** (2013), 13-24. https://doi.org/10.1016/j.apm.2011.05.017

[6] R. D. Gupta, D. Kundu, Theory and methods: Generalized exponential distribution, *Australian and New Zealand Journal of Statistics,* **41** (1999), no. 2, 173-188.

[7] R. D. Gupta, D. Kundu, Generalized exponential distribution: Existing results and some recent developments, *Journal of Statistical Planning and Inference,* **137** (2007), 3537 – 3547. https://doi.org/10.1016/j.jspi.2007.03.030

[8] Zafar Iqbal, Syed Anwer Hasnain, Muhammad Salman, Munir Ahmad, Gholamhossein Hamedani, Generalized Exponentiated Moment Exponential Distribution, *Pakistan Journal of Statistics*, **30** (2014), 537-554.

[9] D. Kundu, R. D. Gupta, Bivariate generalized exponential distribution, *Journal of Multivariate Analysis*, **100** (2009), 581-593. https://doi.org/10.1016/j.jmva.2008.06.012

[10] Albert Luguterah, Odd Generalized Exponential Rayleigh Distribution, *Advances and Applications in Statistics*, **48** (2016), 33-48. https://doi.org/10.17654/as048010033

[11] S. Maiti, Odds Generalized Exponential – Exponential Distribution. *Journal of Data Science*, **13** (2015), 733-754.

[12] S. S. Maiti, S. Pramanik, Odds Generalized Exponential-Pareto Distribution: Properties and Application, *Pak. J. Sstat. Oper. Res.*, **12** (2016), 257-279. https://doi.org/10.18187/pjsor.v12i2.849

[13] B. E. Mohammed, Statistical Properties of Kumaraswamy-Generalized Exponentiated Exponential Distribution, *International Journal of Computer Applications*, **94** (2014), 1-8. https://doi.org/10.5120/16328-5602

[14] Saralees Nadarajaha, Samuel Kotz, The beta exponential distribution, *Reliability Engineering and System Safety,* **91** (2006), 689–697. https://doi.org/10.1016/j.ress.2005.05.008

[15] Mohammad Z. Raqab, Inferences for generalized exponential distribution based on record statistics, *Journal of Statistical Planning and Inference*, **104** (2002), 339-350. https://doi.org/10.1016/s0378-3758(01)00246-4

[16] Miroslav M Ristic, Debasis Kundu, Marshall-Olkin generalized exponential distribution, *METRON*, **73** (2015), 317-333. https://doi.org/10.1007/s40300-014-0056-x

[17] Jailson de Araujo Rodrigues, Ana Paula Coelho Madeira Silva, The Exponentiated Kumaraswamy-Exponential Distribution, British Journal of Applied Science & Technology, **10** (2015), 1-12. https://doi.org/10.9734/bjast/2015/16935

[18] Muhammad H Tahir, Gauss M. Cordeiro, Morad Alizadeh, Muhammad Mansoor, Muhammad Zubair, Gholamhossein G. Hamedani, The odd generalized exponential family of distributions with applications, *Journal of Statistical Distributions and Applications*, **2** (2015), 1-28. https://doi.org/10.1186/s40488-014-0024-2

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