Maximum Throughput and Fair Bandwidth Allocation in Multi-Channel Wireless Mesh Networks

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Abstract— **Wireless mesh network is designed as an economical solution for last-mile broadband Internet access. In this paper, we study bandwidth allocation in multi-channel multihop wireless mesh networks. Our optimization goals are to maximize the network throughput and, at the same time, to enhance fairness. First, we formulate and present an Linear Programming (LP) formulation to solve the *Maximum throughput Bandwidth Allocation (MBA)* problem. However, simply maximizing the throughput may lead to a severe bias on bandwidth allocation among wireless mesh nodes. In order to achieve a good tradeoff between fairness and throughput, we consider a simple max-min fairness model which leads to high throughput solutions with guaranteed maximum minimum bandwidth allocation values, and the well-known *Lexicographical Max-Min (LMM)* model. Correspondingly, we formulate the *Max-min guaranteed Maximum throughput Bandwidth Allocation (MMBA)* problem and the *Lexicographical Max-Min Bandwidth Allocation (LMMBA)* problem. For the former one, we present an LP formulation to provide optimal solutions and for the later one, we propose a polynomial time optimal algorithm.**

Index Terms— **Multi-channel wireless mesh network, bandwidth allocation, fairness, throughput maximization, QoS.**

I. INTRODUCTION

A Wireless Mesh Network (WMN) is usually composed of wireless mesh nodes (routers) and mesh clients ([1]). Wireless mesh nodes form the network backbone and provide access for mesh clients. WMN is a promising solution for low-cost last-mile broadband Internet access. Some of the wireless mesh nodes are connected with the wired network via high capacity cable and are called mesh routers with gateway ([1]) or simply *gateway* nodes. In such networks, wireless mesh routers are normally stationary and do not rely on batteries. A high volume of traffic between the wired network and those non-gateway wireless mesh nodes needs to be efficiently delivered on bandwidth limited wireless channels via multihop wireless paths. Moreover, in order to well support major potential applications such as Internet broadband access, it is important to fairly allocate the limited bandwidth to all users. Therefore, rather than power efficiency and mobility, throughput and fairness are the primary concerns in WMNs.

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In multihop wireless networks, it has been shown that wireless interference has a significant impact on network performance ([11], [16]) and using multiple channels instead of a single channel can improve throughput substantially ([24], [25]). The IEEE $802.11b$ standard and $802.11a$ standard ([14], [15]) offer 3 and 12 non-overlapping channels respectively. Different from a single channel network, a multi-channel network allows simultaneous transmissions in a neighborhood as long as they work on different channels. We consider a wireless mesh network in which every node is equipped with one or multiple Network Interface Cards (NICs), each of which can be tuned to one of the available channels. A pair of NICs can communicate with each other if they are tuned to the same channel and are within the transmission range of each other. Carefully assigning multiple non-overlapping channels to NICs can reduce the impact of interference and improve network throughput. We believe that the static channel assignment scheme ([24], [25]) is more suitable for WMNs and future multimedia applications. In this scheme, channels are assigned for each NIC in advance. The computed channel assignment will be kept for a relatively long period of time and will be adjusted only if some nodes fail or new nodes join. The dynamic schemes, i.e., assigning channels for NICs in route discovery and switching channels to serve multiple sessions during packet transmissions, may suffer from deafness problem, synchronization problem or long channel switching delay ([18], [27]). Once a channel assignment is given, the network topology can be easily determined, which will be discussed in Section III. However, channel assignment is not a concern of this paper. As [8], we assume the channel assignment is given here. Previously proposed channel assignment algorithms ([24], [25], [29]) can be applied here to determine the network topology. In [29], we present a simple algorithm to compute channel assignments such that the corresponding network topologies have low interference and are guaranteed to be K-connected. K-connectivity and low topology interference are both favorable for a high throughput bandwidth allocation. So we will use our channel assignment algorithm [29] to determine the network topology in our simulations.

In this paper, we study bandwidth allocation in multi-channel multihop WMNs with the objective of maximizing

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the network throughput and enhancing fairness. Bandwidth allocation is only concerned with the set of non-gateway nodes which are not directly connected with the wired network. Due to the broadcast nature of the wireless medium, transmissions in a common neighborhood will interfere with each other, which makes the bandwidth allocation completely different from that in the wired network. This problem is essentially a network layer routing problem since 802.11 DCF is assumed to be used in the MAC layer for transmission scheduling and how to route traffic from or to the wired network determines how much bandwidth a non-gateway node can obtain. The total bandwidth allocated to non-gateway nodes (throughput) is preferred to be as large as possible and the bandwidth is preferred to be allocated as even as possible for the fairness consideration. To maximize network throughput, we first formulate the *Maximum throughput Bandwidth Allocation (MBA)* problem and present an LP formulation to provide optimal solutions. However, it has been shown by our numerical results that allocating bandwidth according to the solutions of the MBA problem may result in a severe bias among the non-gateway nodes. Therefore, we address the fairness issue based on a simple max-min fairness model which leads to high throughput solutions with guaranteed maximum minimum bandwidth allocation values, and the well-known *Lexicographical Max-Min (LMM)* model. We formulate and present an LP formulation for the *Max-min guaranteed Maximum throughput Bandwidth Allocation (MMBA)* problem. Furthermore, we define and propose an algorithm to optimally solve the *Lexicographical Max-Min Bandwidth Allocation (LMMBA)* problem in polynomial time. To our best knowledge, this is the first paper addressing maximum throughput and fair bandwidth allocation in the context of multi-channel WMNs and proposing LP formulations and a polynomial time algorithm to provide optimal solutions.

The rest of this paper is organized as follows. We discuss related work in Section II. We describe the system model and formally define the problems in Section III. The LP formulations and algorithm for the formulated problems are presented in Section IV. We present numerical results in Section V and conclude the paper in Section VI.

II. Related Work

Multi-channel multihop wireless network has become a very attractive topic recently. The authors in [8] present a new routing metric, namely Expected Transmission Time/Weighted Cumulative ETT (ETT/WCETT), and a corresponding Multi-Radio Link-Quality Source Routing (MR-LQSR) protocol for multi-radio, multihop wireless networks to find a high-throughput path between a source node and a destination node. In [18] and [27], the authors propose algorithms for channel assignment and routing in multi-channel multi-NIC and single-NIC MANET respectively. The most related works are [24], [25], [29]. One of the first 802.11-based multi-channel multihop wireless mesh network architectures is proposed and evaluated in [25]. The authors develop a set of centralized algorithms for channel assignment, bandwidth allocation and routing. They also present distributed channel assignment and routing algorithms utilizing only local traffic load information in a later paper [24]. In [29], we present a channel assignment algorithm which can be used to compute K-connected and low interference network topology, and an optimal QoS routing algorithm which can be used to find routes for connection requests with bandwidth requirements. Besides routing protocols, several link/MAC layer solutions have also been proposed in [2], [26]. In addition, Chandra *et al.* study the problem of placing Internet Transit Access Points (TAPs) in multihop wireless networks in [7]. In [10], the authors develop a reference model and conduct extensive simulations to study the end-to-end performance and fairness in multihop wireless backhaul networks. However, maximum throughput and fair bandwidth allocation in a multi-channel WMN with full consideration for the impact of wireless interference has not been addressed before, which is the subject of study of this paper.

Fairness has been well studied in both network layer and MAC layer. The classical max-min fairness problem ([4]) seeks bandwidth allocation for a set of given routes in a wired network. The problem of computing routes to provide max-min fair bandwidth allocation to a set of connections is much harder. Megiddo in [21] presents a polynomial time optimal algorithm to find LMM fractional flow routing solutions. Extending this work, the authors in [17] address the problem of finding integer flow routing solutions and propose approximation algorithms. In a recent paper [12], Hou *et al.* develop an elegant polynomial time algorithm, Serial LP with Parametric Analysis (SLP-PA), to calculate the LMM rate allocation under a network lifetime constraint in a two-tiered wireless sensor network. As in [12], [17], [21], our bandwidth allocation problem is implicitly coupled with a flow routing problem as well. However, we consider LMM bandwidth allocation under an interference constraint in a multi-channel multihop wireless network, which is different from all previous works. Max-min fair scheduling and fair queuing for TDMA-based or 802.11-based wireless ad hoc networks have also been addressed in [13], [19], [20], [30].

The impact of wireless interference and corresponding interference-aware network solutions have also attracted substantial attention from the networking research community. In their pioneering work [11], Gupta and Kumar show that in a wireless network with n identical nodes, the per-node throughput is $\Theta(1/\sqrt{n \log n})$ $(or\ \Theta(1/\sqrt{n}))$ by assuming random (or optimal) node placement and communication pattern. In [16], the authors model the impact of interference using a conflict graph and derive upper and lower bounds on the optimal network throughput. In [32], the authors present a framework for multihop packet scheduling to achieve maximum throughput with both intra-flow and inter-flow contentions under consideration. Burkhart *et al.* propose topology control algorithms to compute interference-optimal connected subgraphs and spanners in [6]. Along this line, the authors in [22] present algorithms to compute a network topology in a wireless ad hoc network such that the maximum (or average) link (or node) interference of the topology is either minimized or approximately minimized. In [28], we present routing algorithms to compute interference-minimum power bounded single or node-disjoint paths for multihop wireless networks using directional antennas.

III. System Model

We use a similar system model as described in [24], [25], [29]. IEEE 802.11 Distributed Coordination Function (DCF) ([14]) is used as the MAC layer protocol. Like mentioned before, we consider a multi-channel WMN consisting of stationary wireless mesh nodes (routers) with their locations known. We assume that each node transmits at the same fixed transmission power, i.e., there is a fixed transmission range ($r > 0$) and a fixed interference range $R > r$ (which is typically 2 to 3 times of r [24]) associated with every node. There are totally C non-overlapping frequency channels in the system. Each node v is equipped with Q_v NICs, where $1 \leq Q_v \leq C$. Different nodes may have different numbers of NICs. Obviously, if a node has more than one NICs, any two NICs at the same node should be tuned to different channels to make use of network resources efficiently.

A *channel assignment* $\mathcal{A}$ assigns each NIC a channel from $\{1, 2, \ldots, C\}$. $\mathcal{A}(v)$ denotes the set of Q_v different channels assigned to the Q_v NICs at node v. Once a channel assignment $\mathcal{A}$ is given, a network *topology* $G(V, E)$ with n vertices and m edges can be determined as follows: every node $v \in V$ corresponds to a wireless mesh node in the network. There is an undirected edge $e = (u, v; k)$ on channel $\lambda(e) = k$ connecting node u with node v in G if and only if $d(u, v) \leq r$ and $\lambda(e) \in \mathcal{A}(u) \bigcap \mathcal{A}(v)$, where $d(u, v)$ is the Euclidean distance between u and v. However, there is no link among gateway nodes because we assume bandwidth between one gateway node and the wired network is unlimited, i.e., packets never need to be sent from a gateway node to another via wireless links. The topology G may be a *multi-graph*, i.e., a graph with multiple edges between the same pair of nodes, because two or more channels may be shared by a pair of nodes.

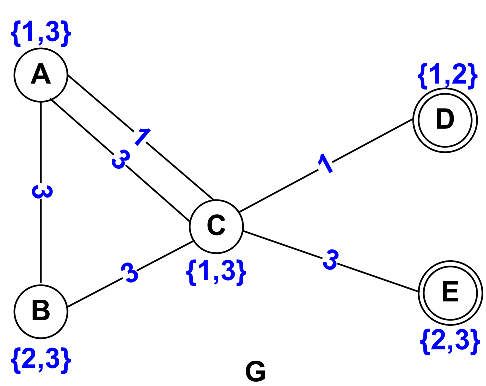

Fig. 1. Network topology G determined by a channel assignment

We use an example in Fig. 1 to illustrate this case. In this example, every node has 2 NICs and there are 3 channels in total. The double circle nodes are gateway nodes. The label associated with each node and each edge indicates the channels assigned to that node and the channel shared by the two end nodes of that edge, respectively. Note that there exist two edges between node A and node C because they share two common channels. Throughout this paper, we will use vertices and nodes interchangeably, as well as edges and links.

Multi-channel and multi-NIC features enable more concurrent transmissions. While one NIC at a node is transmitting or receiving packets on one channel, another NIC at the same node can simultaneously transmit or receive packets on a *different* channel. However, half-duplex operation must be enforced for each NIC to prevent self-interference, i.e., one NIC can only transmit or receive at one time. We can imagine a disk D_u centered at u with radius R (*the interference disk*) associated with each node u because the radio is inherently a broadcast medium and every node transmits at the same fixed transmission power. Without confusion, we will also use D_u to denote the set of nodes covered by this disk. For any pair of wireless nodes u and v, $v \in D_u$ implies $u \in D_v$, and vice versa. Assume that u, v, x, y are wireless nodes such that $d(u, v) \leq r$ and $d(x, y) \leq r$. If node x or y are covered by D_u or D_v (correspondingly, node u or v must be covered by D_x or D_y) and that there is a channel $k \in \mathcal{A}(u) \bigcap \mathcal{A}(v) \bigcap \mathcal{A}(x) \bigcap \mathcal{A}(y)$, then the link $e = (u, v; k)$ on channel $\lambda(e) = k$ interferes with the link $e' = (x, y; k)$ on channel $\lambda(e') = k$, because concurrent transmissions along $(u, v; k)$ and $(x, y; k)$ (using channel k) will lead to collisions. This definition of *link interference* also includes the cases where two links share a common node and the case where they are identical (*the notion that a link interferes with itself is a technical agreement that simplifies the notations and definitions later*). We will use IE_{uv} to denote the set of links in G interfering with link (u, v) (IE_{uv} includes link (u, v)). We adopt a symmetric interference model and model the network using undirected graph because the 802.11 DCF requires the sender to receive a link layer acknowledgment (ACK) message from the receiver for every transmitted data packet.

IV. Problem Definition

Before defining the problems, we will introduce some necessary concepts related to bandwidth allocation. Denote the non-gateway node set as $S = \{s_1, s_2, \ldots, s_N\}$. We use an N-element vector $\mathbf{b} = [b_1, b_2, \ldots, b_N]$ to denote a **bandwidth allocation vector** for nodes in S, where b_i is the bandwidth allocated to node s_i in S. The **throughput** of a bandwidth allocation vector $\mathbf{b}$ is $H(\mathbf{b}) = \sum_{i=1}^{N} b_i$.

When a node s_i is allocated a bandwidth of b_i, node s_i can communicate with the wired network with a total bandwidth of b_i, which includes both traffic from node s_i to the wired network and traffic from the wired network to node s_i. To ease the analysis of the problem and the design of the algorithms, we may imagine that the bandwidth b_i is purely used by traffic from node s_i to the wired network and that there is no traffic from the wired network to node s_i. In this way, we may imagine that the bandwidth b_i is used by a *flow* from node s_i to the wired network, with the understanding that the bandwidth b_i is *physically* used by traffic in both directions. This simplification will not affect the results because of the symmetric interference model.

A bandwidth allocation vector $\mathbf{b}$ is achievable if and only if there exists a corresponding flow allocation vector (which specifies the amount of flow routed through each wireless link) to carry out the desired transmissions. We denote the aggregated flow on each link e by f_e. What we really want to have is a bandwidth allocation for every non-gateway node and a corresponding flow routing solution (which specifies the route and the amount of flow allocated to each link on the route for traffic between each non-gateway node and the wired network), not just the aggregated flow allocation vector. However, once we have the aggregated flow allocation vector, we can easily compute a corresponding flow routing solution, which will be explained in more detail in Section V. We wish to emphasize that the *bandwidth allocation is a node related concept*, while the *flow allocation is a link related concept*. In order to differentiate them, we will call the later one *aggregated **link** flow allocation* and the corresponding vector *aggregated **link** flow allocation vector* in the following.

Definition 1 (Feasible Bandwidth Allocation Vector): An aggregated link flow allocation vector $\mathbf{f} = [f_1, f_2, \ldots, f_m]$ corresponding to a given bandwidth allocation vector $\mathbf{b} = [b_1, b_2, \ldots, b_N]$ assigns an aggregated flow $f_e \geq 0$ for each link $e \in E$ such that at each non-gateway node s_i, the total flow out of [1] node s_i minus the total flow into node s_i is equal to b_i. $\mathbf{f}$ is said to be a **feasible aggregated link flow allocation vector** (corresponding to bandwidth allocation vector $\mathbf{b}$) if for every link $e \in E$, we have $A(e) = (C_e - \sum_{e' \in IE_e} f_{e'}) \geq 0$, where C_e is the capacity of link e. We call $A(e)$ the *residual capacity* on link e. A bandwidth allocation vector is said to be a **feasible bandwidth allocation vector** if there exists a corresponding feasible aggregated link flow allocation.

We note that the above method for residual capacity computation is a worst-case computation. Suppose that links e_1 and e_2 are the links interfering with link e, but do not interfere with each other. Then traffic on e_1 and e_2 may be transmitted simultaneously according to 802.11 DCF. In this case, more than $(C_e - f_{e_1} - f_{e_2})$ traffic can be transmitted along link e. Therefore, $A(e)$ is essentially a lower bound for the actual residual capacity. However, this bound is *tight* because it is achievable in the worst case. We overestimate the interference impact and corresponding bandwidth usage a little bit because we try to provide bandwidth guarantee for users in the WMN, which is very important for those multimedia applications. We try to propose a network layer (routing) solution and depend upon the 802.11 DCF for transmission scheduling. If we allocate bandwidth and routing flows based on such a computation, it is most likely that the bandwidth allocated to each node in S is achievable even though the wireless channel is not reliable and 802.11 DCF is a random access protocol. Similar estimation methods have also been used by several previous QoS routing papers such as [29], [31]. Since neighboring nodes need to periodically exchange maintenance messages, such as HELLO messages, a small amount of bandwidth should be reserved for those routine traffic to prevent them from being destroyed by strong interference. As a result, the link capacity C_e in Definition 1 should be slightly smaller than the *physical* capacity of link e. Now we are ready to define the optimization problems to be studied. Since our ultimate goal is to maximize network throughput, we first formulate a problem which seeks bandwidth allocation and its corresponding aggregated link flow allocation with maximum throughput. Let the network topology $G(V, E)$ and non-gateway node set S be given.

Definition 2 (MBA Problem): The **Maximum throughput Bandwidth Allocation (MBA)** problem seeks a feasible bandwidth allocation vector for all nodes in S along with a corresponding feasible aggregated link flow allocation vector such that the throughput of this bandwidth allocation vector is maximum among all feasible bandwidth allocation vectors.

As discussed before, simply maximizing the throughput may starve some wireless mesh nodes. Therefore, in order to achieve a good bandwidth allocation with regards to both fairness and throughput, we formulate two fair bandwidth allocation problems.

Definition 3 (MMBA Problem): A feasible bandwidth allocation vector $\mathbf{b}$ is said to be a **feasible max-min guaranteed bandwidth allocation vector** if for any other feasible bandwidth allocation vector $\hat{\mathbf{b}}$, we have $\min\{b_i | i \in \{1, 2, \ldots, N\}\} \geq \min\{\hat{b}_i | i \in \{1, 2, \ldots, N\}\}$. The **Max-min guaranteed Maximum throughput Bandwidth Allocation (MMBA)** problem seeks a feasible max-min guaranteed bandwidth allocation vector for all nodes in S along with a corresponding feasible aggregated link flow allocation vector such that the throughput of this bandwidth allocation vector is maximum among all feasible max-min guaranteed bandwidth allocation vectors.

For a bandwidth allocation vector $\mathbf{b} = [b_1, b_2, \ldots, b_N]$, we will use $\mathbf{r} = [r_1, r_2, \ldots, r_N]$ to denote the sorted version of $\mathbf{b}$ such that $r_1 \leq r_2 \leq \ldots \leq r_N$. Similarly, for a bandwidth allocation vectors $\hat{\mathbf{b}}$, we will use $\hat{\mathbf{r}}$ to denote its sorted version.

Definition 4 (LMMBA Problem): A feasible bandwidth allocation vector $\mathbf{b}$ is said to be a **feasible lexicographical max-min bandwidth allocation vector** if for any other feasible bandwidth allocation vector $\hat{\mathbf{b}}$, either $r_i = \hat{r}_i$ for $i = 1, 2, \ldots, N$ or there exists an integer $j \in \{1, 2, \ldots, N\}$ such that $r_i = \hat{r}_i$ for $i < j$ but $r_j > \hat{r}_j$. The **Lexicographical Max-Min Bandwidth Allocation (LMMBA)** problem seeks a feasible lexicographical max-min bandwidth allocation vector for all nodes in S along with a corresponding feasible aggregated link flow allocation vector.

The fairness model behind the MMBA problem is a simple max-min model. The bandwidth allocation vector obtained by solving the MMBA problem is guaranteed to have the maximum minimum bandwidth value, but is not necessarily a feasible LMM bandwidth allocation vector. The LMM is a well used fairness model ([12]) because it is believed to be able to provide a very good tradeoff between throughput and fairness.

V. The Proposed Bandwidth Allocation Schemes

In this section, we present LP formulations for the MBA and MMBA problem, and an LP-based polynomial time optimal algorithm for the LMMBA problem. First, we need to construct an auxiliary directed graph $G'(V', E')$ according to a given

[1] note our discussion in the second paragraph of this section.

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network topology $G(V, E)$. For each node $v \in V$, V' contains Q_v nodes $v^{\lambda_1(v)}, v^{\lambda_2(v)}, \ldots, v^{\lambda_{Q_v}(v)}$, where $\lambda_1(v) < \lambda_2(v) < \cdots < \lambda_{Q_v}(v)$ are the Q_v channels (NICs) constituting the set $\mathcal{A}(v)$. For each non-gateway node $s \in S$, we can pick any of the vertices $s^{\lambda_i(s)} \in V'$ as its corresponding node s' in G'. For simplicity, we can just select the first one, $s^{\lambda_1(s)}$, as s' and we denote the set of such corresponding nodes as S'.

For each $v \in V$ and $1 \leq i < Q_v$, E' contains a directed edge from $v^{\lambda_i(v)}$ to $v^{\lambda_{i+1}(v)}$ and a directed edge from $v^{\lambda_{i+1}(v)}$ to $v^{\lambda_i(v)}$, both with capacity set to ∞. We call all such edges *intra-node edges*. Moreover, V' contains a virtual sink node t. In G', there will be a directed link connecting each gateway node to t whose capacity is set to ∞ as well. Note here that a gateway node in G may also have multiple corresponding gateway nodes in G' as well. We use E'_I to denote the set of intra-node edges and edges connecting all corresponding gateway nodes with node t. For each undirected edge $(u, v; k) \in E$, E' contains two directed edges (u^k, v^k) and (v^k, u^k), both with capacity set to C_{uv}. We call such edges *inter-node edges* and use E'_O to denote the set of inter-node edges. Clearly, $E' = E'_I \bigcup E'_O$. Based on our construction, G' consists of $O(Qn)$ nodes and $O(m + Qn)$ links, where Q is the maximum number of NICs in each node.

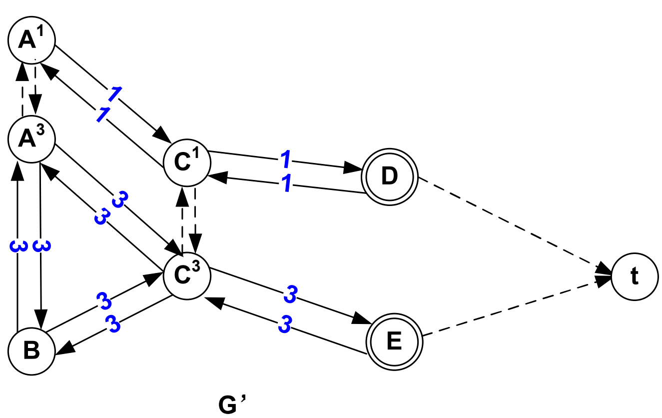

Fig. 2. The auxiliary graph G' constructed from G in Fig. 1

In order to reduce the computational complexity, we can apply a shrinking procedure to eliminate some nodes and intra-node edges in G'. For a node v in G which does not have any incident multi-edge, we can shrink multiple corresponding nodes $(\{v^{\lambda_1(v)}, v^{\lambda_2(v)}, \ldots, v^{\lambda_{Q_v}(v)}\})$ into a single node and remove all corresponding intra-node edges connecting them. Fig. 2 shows the auxiliary directed graph G' constructed from the network topology in Fig. 1. We only have intra-node edges at nodes A^1, A^3, C^1, C^3. All other intra-node edges are shrunk. We therefore drop the superscript at the nodes B^i, D^i and E^i.

Note that the original network topology G is represented as an undirected graph and we construct a directed graph G' to make flow computation easier. Essentially, links (u, v) and (v, u) in G' corresponds to the same edge in G. For each edge (u, v) in G', we associate an aggregated link flow allocation variable, f_{uv}, which indicates the aggregated flow going through link (u, v). Because we are considering symmetric traffic and interference models as mentioned before, *physically*, this link flow value stands for the capacity (bandwidth) reserved for traffic both from u to v and from v to u.

First, we present $LP1$ to compute a maximum throughput bandwidth allocation. Then we present $LP2$ and $LP3$ to solve the MMBA problem. In the following three LP formulations, the variables are b_v for each $v \in S'$ and f_{uv} for each $(u, v) \in E'$.

$LP1$: MBA

$$\text{maximize} \quad \textstyle\sum_{v \in S'} b_v \tag{1}$$

subject to

$$\sum_{w \neq v} f_{vw} - \sum_{u \neq v} f_{uv} = b_v; \qquad \forall v \in S' \tag{2}$$

$$\sum_{w \neq v} f_{vw} - \sum_{u \neq v} f_{uv} = 0; \quad \forall v \in V' \setminus \{S' \textstyle\bigcup \{t\}\} \tag{3}$$

$$\sum_{(x,y) \in IE_{uv}} f_{xy} \leq C_{uv}; \qquad \forall (u, v) \in E'_O \tag{4}$$

$$f_{uv} \geq 0; \qquad \forall (u, v) \in E' \tag{5}$$

$$b_v \geq 0; \qquad \forall v \in S' \tag{6}$$

In the above formulation, IE_{uv} denotes the set of inter-node edges interfering with link $(u, v) \in E'_O$. As far as interference is concerned, no edge in E'_I will be considered. Constraint (2) makes sure that a bandwidth of b_v will be allocated to node v in S'. Constraint (3) is a general flow conservation constraint. It ensures the flow balance at those nodes which are not in S' and not sink node, and guarantees all flows go into sink node t. Based on the aforementioned method for computing residual capacity, Constraint (4) makes sure that the aggregated link flow allocation is feasible. The objective (1) is to maximize the network throughput.

$LP2$: Max-Min

$$\text{maximize} \quad \alpha \tag{7}$$

subject to

$$\sum_{w \neq v} f_{vw} - \sum_{u \neq v} f_{uv} = b_v; \qquad \forall v \in S'$$

$$\sum_{w \neq v} f_{vw} - \sum_{u \neq v} f_{uv} = 0; \quad \forall v \in V' \setminus \{S' \textstyle\bigcup \{t\}\}$$

$$\sum_{(x,y) \in IE_{uv}} f_{xy} \leq C_{uv}; \qquad \forall (u, v) \in E'_O$$

$$f_{uv} \geq 0; \qquad \forall (u, v) \in E'$$

$$b_v \geq \alpha; \qquad \forall v \in S' \tag{8}$$

$LP3(\alpha)$: MMBA

$$\text{maximize} \quad \textstyle\sum_{v \in S'} b_v \tag{9}$$

subject to

$$\sum_{w \neq v} f_{vw} - \sum_{u \neq v} f_{uv} = b_v; \qquad \forall v \in S'$$
$$\sum_{w \neq v} f_{vw} - \sum_{u \neq v} f_{uv} = 0; \quad \forall v \in V' \setminus \{S' \bigcup \{t\}\}$$
$$\sum_{(x,y) \in IE_{uv}} f_{xy} \leq C_{uv}; \qquad \forall (u,v) \in E'_O$$
$$f_{uv} \geq 0; \qquad \forall (u,v) \in E'$$
$$b_v \geq \alpha; \qquad \forall v \in S'$$

By solving $LP2$, we can obtain a maximum minimum bandwidth value α, i.e., we can ensure that for any feasible bandwidth allocation vector $\hat{b}$, we have $\min\{\hat{b}_i | i = 1, 2, \ldots, N\} \leq \alpha = \min\{b_i | i = 1, 2, \ldots, N\}$.

Compared with $LP1$, the objective of $LP3(\alpha)$ is to maximize the network throughput while making sure that each non-gateway node v has a bandwidth allocation of at least α. Therefore, solving $LP2$ to obtain α and then solving $LP3(\alpha)$ can provide a max-min guaranteed maximum throughput bandwidth allocation.

It is more challenging to compute a feasible LMM bandwidth allocation vector and its corresponding feasible aggregated flow allocation vector. We will present a polynomial time algorithm for this problem by solving a sequence of LPs.

Before proceeding with the presentation of the algorithm, we prove an important property–the uniqueness of the feasible lexicographical max-min bandwidth allocation vector.

Lemma 1: Let $\mathbf{b}^1$ and $\mathbf{b}^2$ be two feasible bandwidth allocation vectors which are both lexicographical max-min. Then $b_j^1 = b_j^2$ for $j = 1, 2, \ldots, N$. In other words, the lexicographical max-min bandwidth allocation vector is unique.

PROOF. Let $S_1^1, S_2^1, \ldots, S_K^1$ be a partition of the set $\{1, 2, \ldots, N\}$ such that

a. For any $k \in \{1, \ldots, K\}$ and $i, j \in S_k^1$, we have $b_i^1 = b_j^1$.
b. For any $1 \leq k < k' \leq K$ and $i \in S_k^1, j \in S_{k'}^1$, we have $b_i^1 < b_j^1$.

Similarly, let $S_1^2, S_2^2, \ldots, S_J^2$ be a partition of the set $\{1, 2, \ldots, N\}$ such that

c. For any $k \in \{1, \ldots, J\}$ and $i, j \in S_k^2$, we have $b_i^2 = b_j^2$.
d. For any $1 \leq k < k' \leq K$ and $i \in S_k^2, j \in S_{k'}^2$, we have $b_i^2 < b_j^2$.

Since both $\mathbf{b}^1$ and $\mathbf{b}^2$ are lexicographical max-min bandwidth allocation vectors, we must have

1) $K = J$;
2) $|S_k^1| = |S_k^2|$ for $k = 1, 2, \ldots, K$;
3) $b_i^1 = b_j^2, \ \forall k \in \{1, \ldots, K\}, \ i \in S_k^1, \ j \in S_k^2$.

We need to prove that $S_k^1 = S_k^2$ for $k = 1, 2, \ldots, K$. We will use a simple fact related to *convex set* ([5]).

Since $\mathbf{b}^1$ is a feasible bandwidth allocation vector, there must exist a corresponding feasible aggregated link flow allocation vector $\mathbf{f}^1 = \{\mathbf{f}_{uv}^1 | (u,v) \in E'\}$ so that $\mathbf{b}^1$ and $\mathbf{f}^1$ satisfy the linear constraints (2)–(6). Similarly, since $\mathbf{b}^2$ is a feasible bandwidth allocation vector, there must exist a corresponding feasible link flow allocation vector $\mathbf{f}^2 = \{\mathbf{f}_{uv}^2 | (u,v) \in E'\}$ so that $\mathbf{b}^2$ and $\mathbf{f}^2$ satisfy the linear constraints (2)–(6). Let $\mathbf{b}^3 = \frac{\mathbf{b}^1 + \mathbf{b}^2}{2}$ and $\mathbf{f}^3 = \frac{\mathbf{f}^1 + \mathbf{f}^2}{2}$. Since the set of feasible bandwidth allocation vectors and corresponding aggregated link flow allocation vectors satisfying the set of **linear constraints** (2)–(6) form a **convex set** ([5]), $\mathbf{b}^3$ is a feasible bandwidth allocation vector, with $\mathbf{f}^3$ as a corresponding aggregated link flow allocation vector.

It follows from the definition of $\mathbf{b}^3$ and the partitions $S_1^1, \ldots, S_K^1$ and $S_1^2, \ldots, S_K^2$ (note that $K = J$), we know that

- If i is an index such that $i \in S_k^1 \cap S_k^2$ for some $1 \leq k \leq K$, then $b_i^3 = b_i^1 = b_i^2$.
- If i is an index such that $i \in S_j^1 \cap S_k^2$ for $1 \leq j < k \leq K$, then $b_i^1 < b_i^3 < b_i^2$.

Let α_k be the common value for the elements in S_k^1 for $k = 1, \ldots, K$. From the above facts, we know that

$$b_i^3 \begin{cases} = \alpha_1 & \text{for } i \ \in \ S_1^1 \cap S_1^2 \\ > \alpha_1 & \text{otherwise.} \end{cases} \tag{10}$$

Since $\mathbf{b}^1$ and $\mathbf{b}^2$ are both lexicographical max-min, we must have $S_1^1 = S_1^2$ (otherwise $\mathbf{b}^3$ becomes a better bandwidth allocation vector in the sense of lexicographical max-min).

Suppose we have proved that $S_k^1 = S_k^2$ for $k = 1, 2, ..., K_1$, where $1 < K_1 < K$, and $K_2 = K_1 + 1$. We will have

$$b_i^3 \begin{cases} = \alpha_k & \text{for } i \ \in \ S_k^1 \cap S_k^2, k \in \{1, \ldots, K_1\} \\ = \alpha_{K_2} & \text{for } i \ \in \ S_{K_2}^1 \cap S_{K_2}^2 \\ > \alpha_{K_2} & \text{otherwise.} \end{cases} \tag{11}$$

This proves that we must have $S_{K_2}^1 = S_{K_2}^2$. It follows from the principle of mathematical induction, we know that $S_k^1 = S_k^2$ for $k = 1, 2, \ldots, K$. □

Hou *et al.* in [12] considered LMM rate allocation under a lifetime constraint in wireless sensor networks. They proved the uniqueness of the LMM rate allocation vector for their problem using parametric analysis and duality properties of linear programming ([3]). Our proof of the uniqueness of the LMM bandwidth allocation vector is different and simpler.

Since the feasible lexicographical max-min bandwidth allocation vector is unique, we may denote the partition $S_1^1, S_2^1, \ldots, S_K^1$ of $\{1, 2, \ldots, N\}$ as $S'_1, S'_2, \ldots, S'_K$ and use α_k to denote the lexicographical max-min bandwidth allocation value for nodes in S'_k for $k = 1, 2, \ldots, K$.

The optimal algorithm for the LMMBA problem is formally presented as Algorithm 1. The basic idea of the algorithm is to solve $LP4(k)$ iteratively. For each $k = 1, 2, \ldots, K$, the algorithm computes the bandwidth allocation values (α_k, as defined in the proof of Lemma 1) and identifies the set $S'_k \subseteq S'$ such that $b_i = \alpha_k$ if and only if $i \in S'_k$.

$LP4(k)$: LMM

$$\text{maximize} \quad \delta_k \tag{12}$$

subject to

$$\sum_{w\neq v} f_{vw} - \sum_{u\neq v} f_{uv} = \alpha_{k-1} + \delta_k; \quad \forall v \in S' \setminus \bigcup_{j=0}^{k-1} S'_j \quad (13)$$

$$\sum_{w\neq v} f_{vw} - \sum_{u\neq v} f_{uv} = \alpha_j; \quad \forall v \in S'_j, 1 \leq j < k \quad (14)$$

$$\sum_{w\neq v} f_{vw} - \sum_{u\neq v} f_{uv} = 0; \quad \forall v \in V' \setminus \{S' \bigcup \{t\}\}$$

$$\sum_{(x,y)\in IE_{uv}} f_{xy} \leq C_{uv}; \quad \forall (u,v) \in E'_O$$

$$f_{uv} \geq 0; \quad \forall (u,v) \in E'$$

$LP5(k, \bar{S}, \beta)$

$$\text{maximize} \quad \textstyle\sum_{v\in\bar{S}} \beta_v \quad (15)$$

subject to

$$\sum_{w\neq v} f_{vw} - \sum_{u\neq v} f_{uv} = \alpha_k + \beta_v; \quad \forall v \in \bar{S} \quad (16)$$

$$0 \leq \beta_v \leq \beta; \quad \forall v \in \bar{S} \quad (17)$$

$$\sum_{w\neq v} f_{vw} - \sum_{u\neq v} f_{uv} = \alpha_k; \quad \forall v \in S' \setminus \{\bar{S} \bigcup_{j=0}^{k-1} S'_j\} \quad (18)$$

$$\sum_{w\neq v} f_{vw} - \sum_{u\neq v} f_{uv} = \alpha_j; \quad \forall v \in S'_j, 1 \leq j < k$$

$$\sum_{w\neq v} f_{vw} - \sum_{u\neq v} f_{uv} = 0; \quad \forall v \in V' \setminus \{S' \bigcup \{t\}\}$$

$$\sum_{(x,y)\in IE_{uv}} f_{xy} \leq C_{uv}; \quad \forall (u,v) \in E'_O$$

$$f_{uv} \geq 0; \quad \forall (u,v) \in E'$$

In Algorithm 1, δ_k and f_{uv} for each $(u,v) \in E'$ are variables of $LP4(k)$, and β_v for each $v \in \bar{S}$ and f_{uv} for each $(u,v) \in E'$ are variables of $LP5(k, \bar{S}, \beta)$. The residual graph $G_A(V', E_A)$ is constructed in the following way. The vertex set of G_A is the same as that of G'. For node pair (u,v), if there are edges between u and v in G' and every edge (x,y) in IE_{uv} (remember $(u,v) \in IE_{uv}$) whose residual capacity $A(x,y) = C_{xy} - \sum_{(w,z)\in IE_{xy}} f_{wz} > 0$ based on the aggregated link flow allocation computed by solving $LP4(k)$, then there is an edge (u,v) and a back edge (v,u) in G_A. Otherwise, if $f_{uv} > 0$ (or $f_{vu} > 0$), then there is a back edge (v,u) (or (u,v)) in G_A; if $f_{uv} = 0$ and $f_{vu} = 0$, then there is no edge between vertex u and v in G_A.

Clearly, during the execution of the algorithm, for every node $v \in S'_k$, we must have $b_v = \alpha_k$. However, there may be some node $v \notin \bigcup_{i=1}^{k} S'_i$ such that $b_v = \alpha_k$. Instead of putting every node v with $b_v = \alpha_k$ into $\bar{S}$, we use the residual graph G_A to decrease the cardinality of $\bar{S}$, i.e., reduce the running time. However, we still need to solve $LP5(k, \bar{S}, \beta)$ to determine the actual S'_k. In $LP5(k, \bar{S}, \beta)$, β is a small and tunable parameter. The purpose of having Constraint (17) is to force more nodes out of $\bar{S}$ in each iteration. In this way, we can decrease the running time as well. Note that there is no Constraint (14) when $k = 1$.

Algorithm 1 LMMBA

Step_1 $k := 1$; $\alpha_0 := 0$; $S'_0 := \emptyset$;
Step_2 Solve $LP4(k)$; $\alpha_k := \alpha_{k-1} + \delta_k$;
Step_3 Construct the residual graph $G_A(V', E_A)$ according to the aggregated link flow allocation computed by solving $LP4(k)$;
 $\bar{S} := \emptyset$;
 forall $v \in S' \setminus \bigcup_{j=0}^{k-1} S'_j$
  if there exists no path from v to t in G_A
   $\bar{S} := \bar{S} \bigcup \{v\}$;
  endif
 endforall
Step_4 Solve $LP5(k, \bar{S}, \beta)$; $Y := \emptyset$;
 forall ($v \in \bar{S}$ such that $\beta_v > 0$) **do**
  $Y := Y \bigcup \{v\}$;
 endforall
Step_5 **if** ($Y \neq \emptyset$)
  $\bar{S} := \bar{S} \setminus Y$; **goto Step_4**;
 else
  $S'_k := \bar{S}$;
  if ($\bigcup_{j=0}^{k} S'_j = S'$)
   stop;
  else
   $k := k + 1$; **goto Step_2**;
  endif
 endif

Theorem 1: Algorithm 1 correctly computes the feasible unique lexicographical max-min bandwidth allocation vector $\mathbf{b}$ in polynomial time.

PROOF. When we solve $LP4(1)$, we obtain the max-min bandwidth allocation value α_1, together with a bandwidth allocation vector $\mathbf{b}$ such that $\alpha_1 = \min\{b_i | 1 \leq i \leq N\}$. Note that $i \in S'_1$ implies $b_i = \alpha_1$. However, $b_i = \alpha_1$ does not imply $i \in S'_1$. So we construct the residual graph $G_A(V', E_A)$. If there is a path from v to t in G_A, then we know that the bandwidth allocated to node v can be increased (by a very small, but positive amount) to obtain another feasible bandwidth allocation vector while keeping the bandwidth allocated to all other nodes unchanged. This means that $v \notin S'_1$. However, when there is no path from v to t in G_A, we cannot say for sure that $v \in S'_1$. So we set $\overline{S}$ to include every such node v such that there is no path from v to t in G_A. This means that $S'_1 \subseteq \overline{S}$.

In order to obtain S'_1 from $\overline{S}$, we solve $LP5(k, \overline{S}, \beta)$, which maximizes the sum of the bandwidth values of the nodes in $\overline{S}$, while keeping the bandwidths allocated to nodes not in $\overline{S}$ at α_1. Clearly, $\overline{S}$ is a proper superset of S'_1 if and only if there exists a bandwidth allocation vector b so that

$$b_i \geq \alpha_1, \quad 1 \leq i \leq N, \quad (19)$$

and

$$\sum_{i\in\overline{S}} b_i > |\overline{S}| \times \alpha_1. \quad (20)$$

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This means that when $\overline{S} \neq S'_1$, we can always identify at least one node $v \in \overline{S}$ ($\beta_v > 0$) such that $v \notin S'_1$ and move it out of $\overline{S}$. Clearly, in at most $N - |S'_1|$ iterations, we will have $\overline{S} = S'_1$. This shows that we have computed S'_1 and the bandwidth value for S'_1 (α_1) correctly.

For general k, $1 < k \leq K$, the algorithm first computes a superset $\overline{S}$ of S'_k, and then iteratively remove from $\overline{S}$ the nodes that do not belong to S'_k, eventually we have $\overline{S} = S'_k$. This proves the correctness of the algorithm.

To analyze the time complexity of the algorithm, we note that in the worst case, the running time of the algorithm is dominated by solving a sequence of $LP5(k, \overline{S}, \beta)$ in each iteration. For each value of k, we need to solve $O(N)$ $LP5(k, \overline{S}, \beta)$. This gives an upper bound of $O(N^2)$ $LP5(k, \overline{S}, \beta)$, each of which has $O(m + Qn)$ variables and $O(m + Qn)$ constraints. This completes the proof. □

We notice that Algorithm 1 can compute feasible LMM bandwidth allocations very quickly in practice since there are very efficient algorithms to solve LPs ([3]). Once the bandwidth allocation vector ($\mathbf{b}$) and its corresponding aggregated link flow allocation vector ($\mathbf{f}$) are found, we can obtain a corresponding flow routing solution by using arc-chain decomposition [9] or by simply solving $LP6(\mathbf{r}, \mathbf{f})$, defined in the following. In $LP6(\mathbf{r}, \mathbf{f})$, f^s_{uv} is a node specific flow variable and indicates the flow routed through link (u, v) for traffic between node $s \in S'$ and the sink node t. Since we only want to have a feasible solution, it does not matter what the objective function is. (u_1, v_1) and s_1 can be any link in G' and any node in S' respectively.

$LP6(\mathbf{r}, \mathbf{f})$: Flow Routing Solution

$$\text{maximize} \quad f^{s_1}_{u_1 v_1} \tag{21}$$

subject to

$$\sum_{s \in S'} f^s_{uv} = f_{uv}; \qquad \forall (u, v) \in E' \tag{22}$$

$$\sum_{w \neq s} f^s_{sw} - \sum_{u \neq s} f^s_{us} = b_s; \qquad \forall s \in S' \tag{23}$$

$$\sum_{w \neq v} f^s_{vw} - \sum_{u \neq v} f^s_{uv} = 0; \quad \forall v \in V' \setminus \{S' \bigcup \{t\}\}, \forall s \in S' \tag{24}$$

$$f^s_{uv} \geq 0; \qquad \forall (u, v) \in E', \forall s \in S' \tag{25}$$

VI. Numerical Results

In this section, we use numerical results to illustrate the three proposed bandwidth allocation schemes: Maximum throughput Bandwidth Allocation (MBA), Max-min guaranteed Maximum throughput Bandwidth Allocation (MMBA) and LMM Bandwidth Allocation (LMMBA). We consider static wireless mesh networks with n nodes randomly located in a $900 \times 900m^2$ region. Each node has a fixed transmission range of $250m$ and interference range of $500m$ ([24]). In all simulation scenarios, we use the algorithm in [29] to assign channels for each NIC and require 2-connectivity ($K = 2$) to be preserved for the resulting topologies.

The following system parameters can influence the performance: network size (n), the number of non-gateway nodes (N), the number of available non-overlapping channels (C), the channel capacity (CAP), and the number of NICs (Q_v) in each node. According to IEEE 802.11 specifications, C is set to be 3 (802.11b) in some scenarios and 12 (802.11a) in others. We set the corresponding channel capacity (CAP) to 10.9($Mbps$) (802.11b) and 53.9($Mbps$) (802.11a) respectively, since we simply reserve 0.1 Mbps bandwidth for maintenance message exchange. In all simulations, we set the number of NICs in every node to be 2.

We show the performance of the three schemes in terms of both network throughput and bandwidth allocated to every non-gateway node. The results are presented in Figures 3-10.

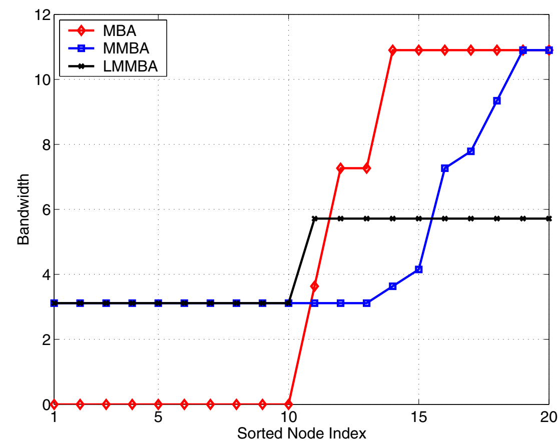

Fig. 3. Bandwidth allocation ($n = 25, N = 20, C = 3, CAP = 10.9$)

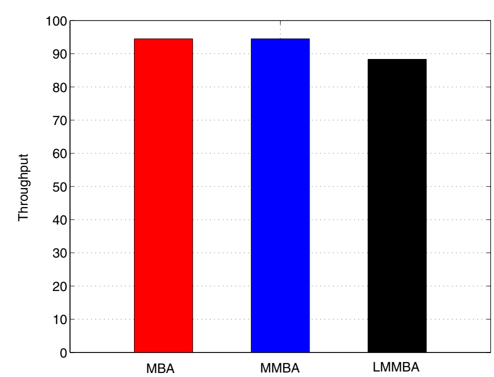

Fig. 4. Network throughput ($n = 25, N = 20, C = 3, CAP = 10.9$)

For each simulation scenario, we use two figures to show the results. The first one illustrates fairness, in which the x-axis is the sorted node index and the y-axis shows the corresponding bandwidth values. The second one is a bar graph showing the network throughput given by these three bandwidth allocation schemes. Our numerical results show that the MBA scheme causes a serve bias on bandwidth allocation. In every simulation scenario, as can be seen from Fig. 3, 5, 7, 9, the bandwidth allocated to some non-gateway nodes are zero while a few other nodes obtain a very large amount of bandwidth.

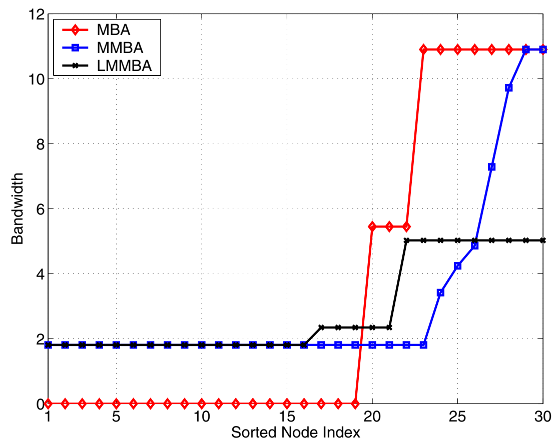

Fig. 5. Bandwidth allocation $(n = 35, N = 30, C = 3, CAP = 10.9)$

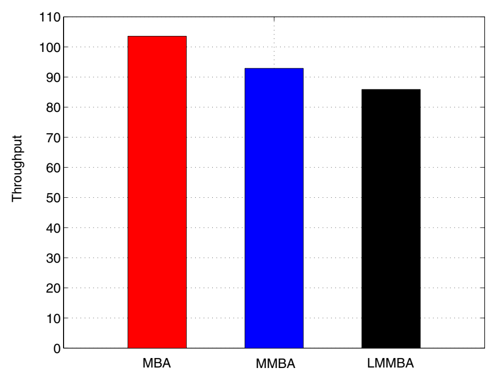

Fig. 6. Network throughput $(n = 35, N = 30, C = 3, CAP = 10.9)$

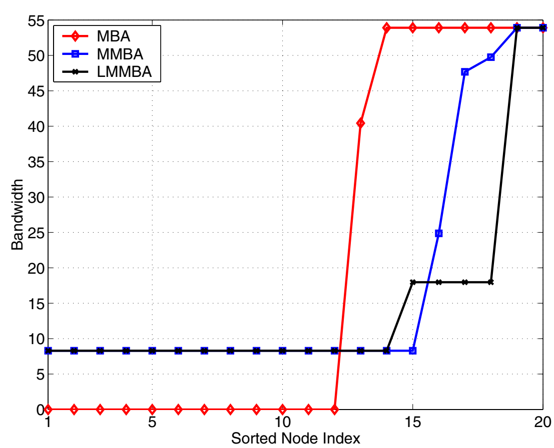

Fig. 7. Bandwidth allocation $(n = 25, N = 20, C = 12, CAP = 53.9)$

That is to say, based on the MBA scheme, some of users will not be able to access the network at all, which is definitely not desirable. Compared with the MBA scheme, the MMBA scheme improves the fairness somehow, without sacrificing the network throughput too much. It guarantees every node be allocated some reasonable amount of bandwidth. For the cases in which the network resources are relatively sparse $(C = 3, CAP = 10.9)$, the throughput provided by the

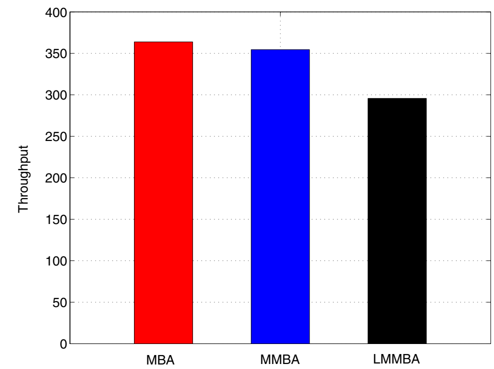

Fig. 8. Network throughput $(n = 25, N = 20, C = 12, CAP = 53.9)$

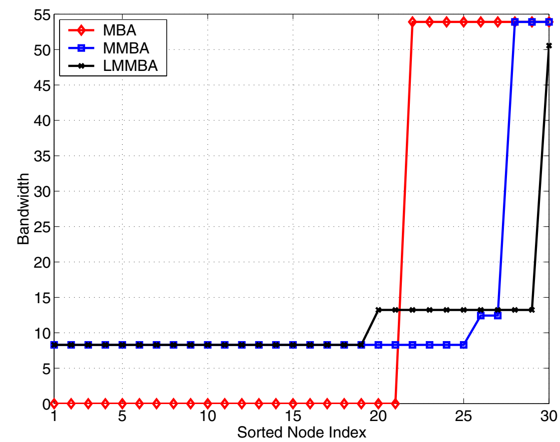

Fig. 9. Bandwidth allocation $(n = 35, N = 30, C = 12, CAP = 53.9)$

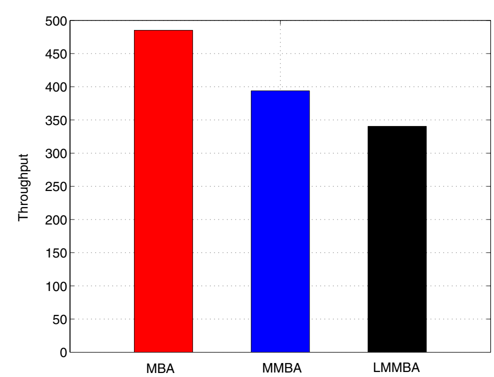

Fig. 10. Network throughput $(n = 35, N = 30, C = 12, CAP = 53.9)$

MMBA scheme is very close to the maximum throughput, which can be seen from Fig. 4 and 6. As expected, among the three bandwidth allocation schemes, the LMMBA scheme achieves fairest bandwidth allocation. If the bandwidth is allocated using the LMMBA scheme, we observe from the results that the nodes are partitioned to several groups (usually the number of groups is very small) and within each group, all nodes are allocate exactly the same amount of bandwidth. The throughput provided by the LMMBA scheme is always smaller than that provided by the other two schemes, especially for cases in which the network resources are relatively rich ($C = 12, CAP = 53.9$).

VII. Conclusions

In this paper, we have studied bandwidth allocation in multi-channel multihop wireless mesh networks with the objective of throughput maximization and fairness enhancement. We have formulated three bandwidth allocation problems, namely, the *Maximum throughput Bandwidth Allocation (MBA)*, the *Max-min guaranteed Maximum throughput Bandwidth Allocation (MMBA)* and the *Lexicographical Max-Min Bandwidth Allocation (LMMBA)* problem. We have presented LP formulations for the MBA and MMBA problems and an LP-based polynomial time algorithm to optimally solve the LMMBA problem.

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