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Mean-Variance Hedging for Discontinuous Semimartingales

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Abstract. Mean-variance hedging is well-known as one of hedging methods for incomplete markets. Our end is leading to mean-variance hedging strategy for incomplete market models whose asset price process is given by a discontinuous semimartingale and whose mean-variance trade-off process is not deterministic. In this paper, on account, we focus on this problem under the following assumptions: (1) the local martingale part of the stock price process is a process with independent increments; (2) a certain condition restricting the number and the size of jumps of the asset price process is satisfied; (3) the mean-variance trade-off process is uniformly bounded; (4) the minimal martingale measure coincides with the variance-optimal martingale measure.

1. Introduction.

The aim of this paper is to lead to *mean-variance hedging* strategy for *incomplete financial market* models whose asset price fluctuation is represented as an RCLL special semimartingale with some assumptions. However, we do not assume that the mean-variance trade-off process is deterministic. Mean-variance hedging is well-known as one of hedging methods for incomplete markets. Mean-variance hedging strategy is a *self-financing* strategy which minimizes, among all self-financing strategies, the expectation of the square of the difference between the value of the strategy at the maturity and the underlying contingent claim.

We consider an incomplete financial market being composed of one riskless asset and d risky assets. Supposed that the maturity is $T > 0$ and, without loss of generality, the price of the riskless asset is 1. Let $(\Omega, \mathcal{F}, P)$ be a completed probability space with a right-continuous filtration $\{\mathcal{F}_t\}_{0 \le t \le T}$ satisfying that $\mathcal{F}_0$ is trivial and contains all null sets of $\mathcal{F}$, and $\mathcal{F}_T = \mathcal{F}$. Let X be an $\mathbf{F}$-adapted RCLL special semimartingale of the space $\mathcal{S}^2_{\mathrm{loc}}(P)$. Assume that the fluctuation of risky assets is described by X. A *contingent claim* is given by an $\mathcal{F}_T$-measurable square integrable random variable H. Moreover, a self-financing strategy is given by an $\mathbf{R}^d$-valued predictable process ϑ such that the stochastic integral $G(\vartheta) := \int \vartheta dX$ is well-defined and a square integrable semimartingale. The process $G(\vartheta)$ means the trading gains induced by a self-financing strategy ϑ. We consider a hedger with initial capital $c \in \mathbf{R}$. Also, we assume that he or she would like to hedge a contingent claim H with a mean-variance objective. Then, the mean-variance hedging strategy is given by the solution to the following

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minimization problem:

$$\text{Minimize } E[(H - c - G_T(\vartheta))^2] \text{ over all self-financing strategies } \vartheta \,.$$

The first important results of mean-variance hedging is given by Duffie and Richardson (1991) and Schweizer (1992). Further, many researchers have studied this problem and published valuable results. As for continuous semimartingale case, Rheinländer and Schweizer (1997) and Gouriéroux, Laurent and Pham (1998) investigated in the case of uniformly bounded *mean-variance trade-off process*. Pham, Rheinländer and Schweizer (1998) (PRS, for short) give another proof for the above result, but they impose an assumption which the *minimal martingale measure* coincides with the *variance-optimal martingale measure*. On the other hand, as for the discontinuous process case, Schweizer (1993,1994) solved this problem under the assumption that the mean-variance trade-off process is deterministic. In addition, Hubalek and Krawczyk (1999) studied this problem for processes with stationary independent increments. Moreover, Schweizer (1999) and Pham (2000) are well-known as famous surveys with respect to the quadratic approaches.

As mentioned above, while mean-variance hedging for continuous processes have been well studied, only a few papers are devoted to the case for discontinuous asset price process models. Moreover, the assumption which the mean-variance trade-off process is deterministic is strong one, so that we would like to except this. Hence, we are under the necessity of solving this problem for discontinuous asset price process whose mean-variance trade-off process is not deterministic. However, it is difficult to solve this problem for entirely general semimartingales.

In this paper, on account, we consider an extension of the result of PRS to discontinuous case under the following four conditions:

1. the local martingale part of the canonical decomposition of the stock price process is a process with independent increments;
2. a certain condition restricting the number and the size of jumps of the asset price process is satisfied;
3. the mean-variance trade-off process is uniformly bounded;
4. the minimal martingale measure coincides with the variance-optimal martingale measure.

Remark that, if the stock price process itself is a process with independent increments, then the mean-variance trade-off process is deterministic by Proposition II.2.29, Corollary II.2.38 and Theorem II.4.15 in Jacod and Shiryaev (1987). Thus, in this model, we can apply the results of Schweizer (1993, 1994). However, our assumption is one related to only the local martingale part. Therefore, the mean-variance trade-off process is not always deterministic. On the other hand, under Condition 1, we can use the representation theorem by Theorem III.4.34 of Jacod and Shiryaev (1987). The truth is that Condition 1 is not necessary except for using the representation theorem. Hence, we can give a proof for jump diffusion models by the same method. Furthermore, Condition 1 do not appear in the proof of the main theorem. We shall use this condition in order to revise slightly Theorem 3.4 of Monat and Stricker

(1995), which is related to the Föllmer-Schweizer decomposition. That is, this condition is not essential. Whereas, Condition 4 is a strong one. Remark that, if the mean-variance trade-off process is deterministic, this condition is satisfied. Even if we impose Condition 4, the results in this paper play crucial role in order to solve the mean-variance hedging problem for general discontinuous cases. We shall state the reason for this fact in Section 5.

We mention an outline of this paper. In Section 2, we define the asset price process and prepare some notations. In Section 3, we state one lemma, two propositions and a main theorem. Furthermore, we give a proof of the main theorem in Section 4. In Section 5, we treat some concluding remarks.

2. Preliminaries.

In this paper, we consider an $\mathbf{R}^d$-valued RCLL stochastic process X adapted to $\mathbf{F}$ as the asset price process. Further, suppose that X is a special semimartingale of the space $\mathcal{S}^2_{\mathrm{loc}}(P)$ and not a quadratic pure jump semimartingale. Thus, there is a unique canonical decomposition of X into a local martingale $M \in \mathcal{M}^2_{0,\mathrm{loc}}(P)$ and a locally natural process A of locally square integrable variation, where $\mathcal{M}^2_{0,\mathrm{loc}}(P)$ is the set of all square integrable P-local martingale starting at 0. We assume that the local martingale M is a process with independent increments. Moreover, let X satisfy the *structure condition* (SC). In other words, X satisfies the following conditions:

(i) there exists an $\mathbf{R}^d$-valued process $\hat{\lambda}$ satisfying

$$A_t = \int_0^t d\langle M\rangle_s \hat{\lambda}_s ,$$

that is, for each $i = 1, \cdots, d$,

$$A^i_t = \sum_{j=1}^d \int_0^t \hat{\lambda}^j_s d\langle M^i, M^j\rangle_s ;$$

(ii) for $0 \le t \le T$,

$$\begin{aligned} \hat{K}_t &:= \int_0^t \hat{\lambda}^{\mathrm{tr}}_s dA_s \\ &= \int_0^t \hat{\lambda}^{\mathrm{tr}}_s d\langle M\rangle_s \hat{\lambda}_s \\ &= \sum_{i=1}^d \sum_{j=1}^d \int_0^t \hat{\lambda}^i_s \hat{\lambda}^j_s d\langle M^i, M^j\rangle_s \\ &< \infty , \end{aligned}$$

uniformly in (t, ω), where tr denotes transposition. The predictable process $\hat{K}$ is said to be the *mean-variance trade-off process*.

Let $\{\mathcal{F}_t^M\}_{0\leq t\leq T}$ be a filtration generated by M. It is natural to assume that this filtration $\{\mathcal{F}_t^M\}_{0\leq t\leq T}$ is a subfiltration of $\{\mathcal{F}_t\}_{0\leq t\leq T}$, because our market is incomplete. Throughout this paper, we assume that a *contingent claim* is an $\mathcal{F}_T^M$-measurable square integrable random variable and the finite variation part A is adapted to $\{\mathcal{F}_t^M\}_{0\leq t\leq T}$. In other words, $\{\mathcal{F}_t^M\}_{0\leq t\leq T}$ is also generated by X.

Next, we prepare some notations and spaces of stochastic processes.

DEFINITION 2.1. For any RCLL process Y, we define the process Y^* by, for $0\leq t\leq T$,

$$Y_t^* := \sup_{0\leq s\leq t} |Y_s|\,.$$

For $p\geq 1$, we denote by $\mathcal{R}^p(P)$ the set of all adapted RCLL process Y such that

$$\|Y\|_{\mathcal{R}^p(P)} := \|Y_T^*\|_{\mathcal{L}^p(P)} < \infty\,.$$

DEFINITION 2.2. For $p\geq 1$, $L^p(M)$ denotes the space of all predictable $\mathbf{R}^d$-valued processes ϑ such that

$$\begin{aligned}\|\vartheta\|_{L^p(M)} &:= \left\| \left(\int_0^T \vartheta_s^{\mathrm{tr}} d\langle M\rangle_s \vartheta_s\right)^{\frac{1}{2}} \right\|_{\mathcal{L}^p(P)} \\ &= \left\| \left\langle \int_0^{\cdot} \vartheta_s dM_s \right\rangle_T^{\frac{1}{2}} \right\|_{\mathcal{L}^p(P)} \\ &< \infty\,.\end{aligned}$$

Moreover, we define the space $\hat{L}^p(M)$ of all predictable $\mathbf{R}^d$-valued processes ϑ such that

$$\|\vartheta\|_{\hat{L}^p(M)} := \left\| \left(\int_0^T \vartheta_s^{\mathrm{tr}} d[M]_s \vartheta_s\right)^{\frac{1}{2}} \right\|_{\mathcal{L}^p(P)} < \infty\,.$$

We denote by $L^p(A)$ the set of all predictable $\mathbf{R}^d$-valued processes ϑ such that

$$\|\vartheta\|_{L^p(A)} := \left\| \int_0^T |\vartheta_s^{\mathrm{tr}} dA_s| \right\|_{\mathcal{L}^p(P)} = \left\| \left| \int_0^{\cdot} \vartheta_s^{\mathrm{tr}} dA_s \right|_T \right\|_{\mathcal{L}^p(P)} < \infty\,.$$

Finally, we define spaces

$$\Theta^p := L^p(M)\cap L^p(A)\,,$$

and seminorms

$$\|\vartheta\|_{\Theta^p} := \|\vartheta\|_{L^p(M)} + \|\vartheta\|_{L^p(A)}\,.$$

In particular, if $p=2$, we abbreviate Θ.

DEFINITION 2.3. Let X be a semimartingale. Thus, there exists at least one decomposition $X = X_0 + \bar{M} + \bar{A}$, where $\bar{M}$ is a local martingale with $\bar{M}_0 = 0$ and $\bar{A}$ is an RCLL finite variation adapted process with $\bar{A}_0 = 0$. For $1 \leq p \leq \infty$, we define the space of all $\mathbf{R}^d$-valued semimartingales such that

$$\begin{aligned}\|X\|_{\mathcal{S}^p(P)} &:= \inf_{X=X_0+\bar{M}+\bar{A}} \left\| [\bar{M}]_T^{\frac{1}{2}} + \int_0^T |d\bar{A}_s| \right\|_{\mathcal{L}^p(P)} \\ &< \infty ,\end{aligned}$$

where the infimum is taken over all possible decompositions $X = X_0 + \bar{M} + \bar{A}$ where $\bar{M}$ is a local martingale starting at 0 and $\bar{A}$ is an RCLL process with paths of finite variation on compacts starting at 0.

In addition to this, for $1 \leq p \leq \infty$, a semimartingale X belongs to the space $\mathcal{S}^p_{\mathrm{loc}}(P)$ if there exists a sequence of stopping times $(T^l)_{l \geq 1}$ increasing to ∞ a.s. such that X^{T^l} belongs to $\mathcal{S}^p(P)$.

As shown in Lemma 2 of Schweizer (1994), Θ is the space of all $\mathbf{R}^d$-valued predictable X-integrable process ϑ such that the stochastic integral

$$G(\vartheta) := \int_0^{\cdot} \vartheta_s dX_s$$

is in the space $\mathcal{S}^2(P)$ of semimartingales. Remark that, since the mean-variance trade-off process $\hat{K}$ is bounded, we have $\Theta = L^2(M)$.

In the rest of this section, we discuss equivalent martingale measures. A probability measure Q is called an equivalent martingale measure if Q is equivalent to P and the asset price process X is a martingale under Q. Since our market is incomplete, there exist infinitely many equivalent martingale measures. We denote by $\mathbf{M}$ the set of all equivalent martingale measures and denote $\mathbf{M}^2 = \{Q \in \mathbf{M};\ \frac{dQ}{dP}$ is square integrable$\}$. We define two important equivalent martingale measures and density processes as follows:

DEFINITION 2.4. (i) A probability measure $\hat{P} \in \mathbf{M}$ is the *minimal martingale measure* if $\hat{P}$ satisfies the following condition:

$$L \in \mathcal{M}^2(P) \quad \text{and} \quad \langle L, M \rangle = 0 \Rightarrow L \in \mathcal{M}(\hat{P}) ,$$

where $\mathcal{M}^2(P)$ means the set of all square integrable P-martingales and $\mathcal{M}(\hat{P})$ means the set of all $\hat{P}$-martingales.

(ii) A probability measure $\tilde{P} \in \mathbf{M}$ is the *variance-optimal martingale measure* if $\tilde{P}$ is the solution to the following minimization problem:

$$\text{Minimize} \quad E\left[\left|\frac{dQ}{dP}\right|^2\right] \quad \text{all over} \quad Q \in \mathbf{M}^2.$$

(iii) For an equivalent martingale measure $Q \in \mathbf{M}$, we define a process Z^Q as

$$Z_t^Q := E\left[\frac{dQ}{dP}\middle|\mathcal{F}_t\right].$$

The process Z^Q is said to be the density process of Q. In particular, we denote by $\hat{Z}$ the density process of the minimal martingale measure.

The minimal martingale measure and the variance-optimal martingale measure are introduced by Föllmer and Schweizer (1990), and Schweizer (1995b) and Delbaen and Schachermayer (1996), respectively. In this paper, we assume that both equivalent martingale measures exist uniquely. The density process of the minimal martingale measure only for one-dimensional jump diffusion models is obtained by Arai (2001). However, if the mean-variance trade-off process is bounded, then we can extend his result to our model, because $\hat{\lambda}$ is in $L^2(M)$. That is, the density process $\hat{Z}$ is given by

$$\hat{Z}_t = \mathcal{E}\left(-\int_0^{\cdot} \hat{\lambda}_s dM_s\right)_t,$$

where, $\mathcal{E}$ means the exponential martingale. Moreover, we can see that the following condition is a necessary and sufficient condition for the existence of the minimal martingale measure as a probability measure:

$$\Delta Y_t > -1 \quad \text{a.s. in } (t, \omega),$$

where $Y = -\int_0^{\cdot} \hat{\lambda}_s dM_s$. Throughout this paper, we impose a slightly stronger assumption that there exists a positive constant $\varepsilon_2 > 0$ such that

$$\Delta Y_t > -1 + \varepsilon_2 \quad \text{a.s. in } (t, \omega). \tag{2.1}$$

Now we assume that the number of jump points of the process Y is finite and the size of the jumps is uniformly bounded. We call this condition the jump condition.

3. Main results.

We mention main results in this section. Note that we postpone the proofs of all results stated in this section to next section.

Firstly, we organize assumptions in detail as follows:

ASSUMPTION. 1. The stock price process X is an $\mathbf{R}^d$-valued $\mathbf{F}$-adapted RCLL special semimartingale of the space $\mathcal{S}^2_{\text{loc}}(P)$ and not a quadratic pure jump one.

2. The local martingale part M is a process with independent increments.

3. The process X satisfies the structure condition (SC).

4. A contingent claim H is an $\mathcal{F}^M_T$-measurable square integrable random variable and the finite variation process A is $\mathcal{F}^M$-adapted.

5. Both the minimal martingale measure $\hat{P}$ and the variance-optimal martingale measure $\tilde{P}$ exist uniquely.

6. (2.1) holds.

7. (The jump condition) The number of jumps of the process $\{-\int_0^t \hat{\lambda}_s dM_s\}_{0\le t\le T}$ is finite and its size is uniformly bounded.

We prepare one lemma and two propositions. The following lemma treat the closedness of Θ^p with $p \geq 2$. The first proposition is one related to a *Föllmer-Schweizer decomposition* of a contingent claim $H \in \mathcal{L}^p(\mathcal{F}_T^M, P)$ with $p > 2$. The other is a problem with respect to integrability of densities related to the minimal martingale measure.

LEMMA 3.1. *Under Conditions* 1, 3 *and* 7 *of Assumption*, *for* $p \geq 2$, *the space* $\hat{L}^p(M)$ *is closed.*

PROPOSITION 3.2. *Under Conditions* 1–4 *of Assumption, every contingent claim* $H \in \mathcal{L}^p(\mathcal{F}_T^M, P)$ *with* $p > 2$ *admits a Föllmer-Schweizer decomposition as*

$$H = H_0 + \int_0^T \xi_s^H dX_s + L_T^H , \tag{3.1}$$

where $H_0 \in \mathbf{R}$, *for every* $2 \leq q < p$, $\xi^H \in L^q(M)$ *and* $L^H \in \mathcal{M}^q(P)$ *strongly orthogonal to* M *with* $E[L_0^H] = 0$.

REMARK. We can extend Corollary 5 of PRS to our model by the same proof as PRS. That is, every $\mathcal{F}_T$-measurable square integrable contingent claim H admits a Föllmer-Schweizer decomposition satisfying $\xi^H \in L^2(M)$ and $L^H \in \mathcal{M}^2(P)$.

PROPOSITION 3.3. *Under Conditions* 1, 3 *and* 5–7 *of Assumption*, *we have*

$$\frac{d\hat{P}}{dP} \in \mathcal{L}^r(P) \quad \text{for every } r < \infty , \tag{3.2}$$

$$\frac{dP}{d\hat{P}} \in \mathcal{L}^r(\hat{P}) \quad \text{for every } r < \infty . \tag{3.3}$$

By the above propositions, the Föllmer-Schweizer decomposition of $\frac{d\hat{P}}{dP}$ is given by

$$\frac{d\hat{P}}{dP} = E[\hat{Z}_T^2] - E[\hat{Z}_T \hat{L}_T] + \int_0^T \hat{\zeta}_s dX_s + \hat{L}_T \tag{3.4}$$

with $\hat{L} \in \mathcal{M}^r(P)$ for every $r < \infty$ and $\hat{\zeta} \in L^r(M)$ for every $r < \infty$.

The main theorem of this paper is as follows:

THEOREM 3.4. *We assume that*

$$\hat{L}_T = 0 \quad \text{in } (3.4) . \tag{3.5}$$

Then, under Assumption, for a contingent claim $H \in \mathcal{L}^{2+\varepsilon}(\mathcal{F}_T^M, P)$ *with* $\varepsilon > 0$, *the solution to minimization problem*

$$\text{Minimize} \;\; E[(H - c - G_T(\vartheta))^2] \quad \text{over all} \;\; \vartheta \in \Theta \,,$$

is given by

$$\xi_t^{(c)} = \xi_t^H - \frac{\hat{\zeta}_t}{\hat{Z}_{t-}^0}\left(\hat{V}_{t-} - c - \int_0^{t-} \xi_s^{(c)} dX_s\right),$$

where

$$\hat{Z}_t^0 := \hat{E}[\hat{Z}_T|\mathcal{F}_t] = E[\hat{Z}_T^2] + \int_0^t \hat{\zeta}_s dX_s \,,$$

$$\hat{V}_t := \hat{E}[H|\mathcal{F}_t] = H_0 + \int_0^t \xi_s^H dX_s + L_t^H \,.$$

REMARK. Under condition (3.5), the minimal martingale measure $\hat{P}$ coincides with the variance-optimal martingale measure. We can prove this fact as follows.

It suffice to prove that, for any $Q \in \mathbf{M}^2$, we have

$$E\left[\frac{d\hat{P}}{dP}\left(\frac{d\hat{P}}{dP} - \frac{dQ}{dP}\right)\right] = 0 \,.$$

By $\hat{L}_T = 0$, the Föllmer-Schweizer decomposition of $\hat{P}$ is given by

$$\frac{d\hat{P}}{dP} = E[\hat{Z}_T^2] + \int_0^T \hat{\zeta}_s dX_s \,.$$

Hence, we obtain

$$E\left[\frac{d\hat{P}}{dP}\frac{dQ}{dP}\right] = E^Q\left[E[\hat{Z}_T^2] + \int_0^T \hat{\zeta}_s dX_s\right] = E\left[\left(\frac{d\hat{P}}{dP}\right)^2\right].$$

In consequence of this, $\hat{P}$ is also the variance-optimal martingale measure under condition (3.5).

REMARK. By Theorem7 of Schweizer (1995a), if $\hat{K}_T$ is deterministic, then (3.5) holds.

REMARK. Wiese (1998) treated the above problem. However, she assume the continuity of L^H. Instead, she do not impose conditions on the stock price process.

4. Proofs.

4.1. Proof of Lemma 3.1. We can prove Proposition 2.1 of Grandits and Krawczyk (1998) in our case. Hence, by the same sort of argument as Theorem 3.1 of Grandits and

Krawczyk (1998), we can show that then there exists a constant C such that for any $\vartheta \in \Theta^p$

$$\|\vartheta\|_{\hat{L}^p(P)} \leq C\|G(\vartheta)\|_{\mathcal{R}^p(P)},$$

where we need to take $\int_0^\cdot \vartheta_s dM_s$ as L in the proof of Theorem 3.1 of Grandits and Krawczyk (1998). Remark that we use the following equation instead of Lemma 3.1 of Grandits and Krawczyk (1998). We denote $Y = \int_0^\cdot \hat{\lambda}_s dM_s$ and fix $p \geq 2$. For any $N \in \mathcal{S}^p(P)$, there exists a constant C_p such that

$$\|[N, Y]_T\|_{\mathcal{L}^p(P)} \leq C_p\|[N]^{\frac{1}{2}}\|_{\mathcal{L}^p(P)}$$

by boundedness of $\hat{K}$ and the jump condition. Consequently, by Banach's closed graph theorem, we obtain the closedness of $\hat{L}^p(P)$.

4.2. Proof of Proposition 3.2. By Galtchouk-Kunita-Watanabe decomposition, if we fix an $\{\mathcal{F}_t^M\}$-predictable process $\vartheta \in L^p(M)$, then we have

$$H - \int_0^T \vartheta_s^{\mathrm{tr}} dA_s = H_0(\vartheta) + \int_0^T \eta_s dM_s + L_T(\vartheta), \tag{4.1}$$

where η is a predictable process and $L(\vartheta)$ is a square integrable P-martingale strongly P-orthogonal to M with $E[L_0^H] = 0$. Since we can prove Corollary 5 of PRS for our model by the same as PRS, it is enough to show that η given in (4.1) is in $L^p(M)$.

Since M is a process with independent increments, by Theorem III.4.34 of Jacod and Shiryaev (1987), we can denote

$$H - \int_0^T \vartheta_s^{\mathrm{tr}} dA_s = E\left[H - \int_0^T \vartheta_s^{\mathrm{tr}} dA_s\right] + \int_0^T \varphi_s^\vartheta dM_s^c + \int_0^T \psi_s^\vartheta dM_s^d,$$

where M^{c} and M^{d} are the continuous and the quadratic pure jump part of M, respectively, and φ^ϑ and ψ^ϑ are $\mathbf{R}^d$-valued predictable processes.

Firstly, we prove that

$$\left\|\left(\int_0^T (\varphi_s^\vartheta)^{\mathrm{tr}} d[M^c]_s \varphi_s^\vartheta\right)^{\frac{1}{2}} + \left(\int_0^T (\psi_s^\vartheta)^{\mathrm{tr}} d[M^d]_s \psi_s^\vartheta\right)^{\frac{1}{2}}\right\|_{\mathcal{L}^p(P)} < \infty. \tag{4.2}$$

Noting that we have

$$\left[\int_0^\cdot \varphi_s^\vartheta dM_s^c, \int_0^\cdot \psi_s^\vartheta dM_s^d\right] = 0$$

and, for semimartingales A and B,

$$[A]^{\frac{1}{2}} + [B]^{\frac{1}{2}} \leq \sqrt{2}[A+B]^{\frac{1}{2}} + 2|[A, B]|^{\frac{1}{2}}.$$

These imply that, by Burkholder-Davis-Gundy's inequality and Doob's inequality,

$$
\begin{aligned}
\text{LHS of (4.2)} &= \left\| \left[\int_0^{\cdot} \varphi_s^{\vartheta} dM_s^c \right]_T^{\frac{1}{2}} + \left[\int_0^{\cdot} \psi_s^{\vartheta} dM_s^d \right]_T^{\frac{1}{2}} \right\|_{\mathcal{L}^p(P)} \\
&\le \sqrt{2} \left\| \left[\int_0^{\cdot} \varphi_s^{\vartheta} dM_s^c + \int_0^{\cdot} \psi_s^{\vartheta} dM_s^d \right]_T^{\frac{1}{2}} \right\|_{\mathcal{L}^p(P)} \\
&\le \text{const.} \left\| \left(\int_0^{\cdot} \varphi_s^{\vartheta} dM_s^c + \int_0^{\cdot} \psi_s^{\vartheta} dM_s^d \right)_T^* \right\|_{\mathcal{L}^p(P)} \\
&\le \text{const.} \left\| \int_0^T \varphi_s^{\vartheta} dM_s^c + \int_0^T \psi_s^{\vartheta} dM_s^d \right\|_{\mathcal{L}^p(P)} \\
&\le \text{const.} \left\| H - \int_0^T \vartheta_s^{\mathrm{tr}} dA_s \right\|_{\mathcal{L}^p(P)} \\
&< \infty\,,
\end{aligned}
$$

from which (4.2) follows.

Next, we claim that

$$
\left\| \left(\int_0^T \eta_s^{\mathrm{tr}} d[M]_s \eta_s \right)^{\frac{1}{2}} + [L(\vartheta)]_T^{\frac{1}{2}} \right\|_{\mathcal{L}^p(P)} < \infty\,. \tag{4.3}
$$

We have

$$
\begin{aligned}
\text{LHS of (4.3)} &\le \left\| \sqrt{2} \left[\int_0^{\cdot} \eta_s dM_s + L(\vartheta) \right]_T^{\frac{1}{2}} + 2 \left| \left[\int_0^{\cdot} \eta_s dM_s, L(\vartheta) \right]_T \right|^{\frac{1}{2}} \right\|_{\mathcal{L}^p(P)} \\
&\le \sqrt{2} \left\| \left[\int_0^{\cdot} \eta_s dM_s + L(\vartheta) \right]_T^{\frac{1}{2}} \right\|_{\mathcal{L}^p(P)} + 2 \left\| \left| \left[\int_0^{\cdot} \eta_s dM_s, L(\vartheta) \right]_T \right|^{\frac{1}{2}} \right\|_{\mathcal{L}^p(P)}.
\end{aligned}
$$

We can see finiteness of the first term by the proof of Lemma 6 of PRS. We shall prove finiteness of the second term. Firstly, since $\eta \in L^2(M) = \Theta$, η is X-integrable. Thus, $\int_0^{\cdot} \eta dM$ is a square integrable P-martingale. Moreover, since $\int_0^{\cdot} \eta dM$ is strongly P-orthogonal to $L(\vartheta)$, $[\int_0^{\cdot} \eta dM, L(\vartheta)]$, denoted by J, is a P-martingale. Also, we denote

$$
J^+ := J \vee 0 = \left[\int_0^{\cdot} \eta dM, L(\vartheta) \right] \vee 0\,,
$$

$$
J^- := -(J \wedge 0) = -\left(\left[\int_0^{\cdot} \eta dM, L(\vartheta) \right] \wedge 0 \right).
$$

That is, $|J| = J^+ + J^-$. On the other hand, we can write $L_T(\vartheta)$ as

$$L_T(\vartheta) = E\left[H - \int_0^T \vartheta_s^{\mathrm{tr}} dA_s\right] - H_0(\vartheta) + \int_0^T (\varphi_s^\vartheta - \eta_s) dM_s^c + \int_0^T (\psi_s^\vartheta - \eta_s) dM_s^d .$$

Also, φ_s^ϑ and ψ_s^ϑ are in $L^2(M)$ by (4.2). Thus, for $0 \le t \le T$, we can represent $L_t(\vartheta)$ as

$$L_t(\vartheta) = E\left[H - \int_0^T \vartheta_s^{\mathrm{tr}} dA_s\right] - H_0(\vartheta) + \int_0^t (\varphi_s^\vartheta - \eta_s) dM_s^c + \int_0^t (\psi_s^\vartheta - \eta_s) dM_s^d .$$

Therefore, we have

$$\begin{aligned} J_T &= \left[\int_0^\cdot \eta_s dM_s, L(\vartheta)\right]_T \\ &= \int_0^T \eta_s^{\mathrm{tr}} d[M^{\mathrm{c}}]_s (\varphi_s^\vartheta - \eta_s) + \int_0^T \eta_s^{\mathrm{tr}} d[M^{\mathrm{d}}]_s (\psi_s^\vartheta - \eta_s) \\ &= \int_0^T \eta_s^{\mathrm{tr}} d[M^{\mathrm{c}}]_s \varphi_s^\vartheta + \int_0^T \eta_s^{\mathrm{tr}} d[M^{\mathrm{d}}]_s \psi_s^\vartheta - \left[\int_0^\cdot \eta_s dM_s\right]_T \\ &\le \int_0^T \eta_s^{\mathrm{tr}} d[M^{\mathrm{c}}]_s \eta_s + d^2 \int_0^T (\varphi_s^\vartheta)^{\mathrm{tr}} d[M^{\mathrm{c}}]_s \varphi_s^\vartheta \\ &\quad + \int_0^T \eta_s^{\mathrm{tr}} d[M^{\mathrm{d}}]_s \eta_s + d^2 \int_0^T (\psi_s^\vartheta)^{\mathrm{tr}} d[M^{\mathrm{d}}]_s \psi_s^\vartheta - \left[\int_0^\cdot \eta_s dM_s\right]_T \\ &= d^2 \int_0^T (\varphi_s^\vartheta)^{\mathrm{tr}} d[M^{\mathrm{c}}]_s \varphi_s^\vartheta + d^2 \int_0^T (\varphi_s^\vartheta)^{\mathrm{tr}} d[M^{\mathrm{d}}]_s \varphi_s^\vartheta . \end{aligned}$$

Hence, by (4.2), we obtain

$$\|J_T^+\|_{\mathcal{L}^p(P)} < \infty . \tag{4.4}$$

Now, we have the following lemma by Doob's maximal lemma:

LEMMA 4.1. *Let Y be a martingale starting at* 0. *Assume that $E[(Y_T^+)^p] < \infty$ for some $p \ge 1$. Then, we have, for each $x > 0$,*

$$P\left(\inf_{0 \le t \le T} Y_t < -x\right) \le x^{-p} E[(Y_T^+)^p] .$$

By Lemma 4.1 and (4.4), we have, for every $q < p$,

$$\begin{aligned} E[|J_T^-|^q] &= \int_0^\infty x^q P(J_T^- \in dx) \\ &= \int_0^\infty P(J_T^- > y^{\frac{1}{q}}) dy \\ &\le \int_0^\infty E[(J_T^+)^p] y^{-\frac{p}{q}} dy \\ &< \infty . \end{aligned}$$

Consequently, (4.3) follows. We can give the rest of the proof by the same as the proof of Lemma 6 of PRS. This completes the proof of Proposition 3.2. □

4.3. Proof of Proposition 3.3. We set

$$Y := -\int \hat{\lambda} dM .$$

Moreover, we denote the continuous part and the quadratic pure jump part of Y by Y^{c} and Y^{d}, respectively. For every $r > 2$ and $r < -1$, we define

$$W_t^r := \sum_{s \le t} ((1 + \Delta Y_s)^r - 1 - r\Delta Y_s) .$$

We have that, for every $r > 2$,

$$\sum_{s \le t} |(1 + \Delta Y_s)^r - 1 - r\Delta Y_s| \tag{4.5}$$

is integrable, by the jump condition.

Thus, by Proposition II.3 of Lepingle et Mémin (1978), we have, for every $r > 2$,

$$\mathcal{E}^r(Y)_t = \mathcal{E}\left\{rY + W^r + \frac{r(r-1)}{2}[Y^{\mathrm{c}}]\right\}_t . \tag{4.6}$$

In order to see that $\mathcal{E}^r(Y)$ is uniformly integrable for every $r > 2$, if we denote by $\hat{Y}$ the content of $\mathcal{E}$ in RHS of (4.6), then, by Théorème II.2 of Lepingle et Mémin (1978), all we have to do is to prove that $\hat{Y}$ is square integrable and $\langle \hat{Y} \rangle$ is bounded. However, by the jump condition, $\hat{K}$ is bounded, from which square integrability of $\hat{Y}$ and boundedness of $\langle \hat{Y} \rangle$ follows, that is, (3.2) follows.

On the other hand, since, for every $r < -1$, (4.5) is integrable by (2.1), $\mathcal{E}^r(Y)$ is represented as (4.6) by Ito's formula. Moreover, we can prove that the content of $\mathcal{E}$ in RHS of (4.6) has square integrability and its conditional quadratic variation process is bounded. Consequently, by the same sort of argument as (3.2), (3.3) follows. □

4.4. Proof of Theorem 3.4.

Step 1. We consider the following stochastic differential equation (SDE):

$$U_t = H_0 - c + L_t^H + \int_0^t \frac{\hat{\zeta}_s}{\hat{Z}_{s-}^0} U_{s-} dX_s . \tag{4.7}$$

By Theorem V.7 of Protter (1990), SDE (4.7) has a unique solution. We define a stochastic process N by

$$U_t = N_t \hat{Z}_t^0 .$$

Remark that, since $\hat{Z}^0$ is strictly positive, N is well-defined.

We claim that N is a $\hat{P}$-local martingale. Remark that we have

$$N_0 = \frac{H_0 - c + L_0^H}{E[\hat{Z}_T^2]} .$$

Since N is a semimartingale, by Theorem V.4 of Protter (1990), $N - N_0$ is prelocally in $\mathcal{S}^\infty(\hat{P})$. In other words, there exists a sequence of stopping times $(\tau^k)_{k\geq 1}$ increasing to ∞ a.s. such that $N^{\tau^k -}$ is a semimartingale of the space $\mathcal{S}^\infty(\hat{P})$. In this subsection, for $k \geq 1$, superscript k means the underlying process stopped at τ^k-. For example, we denote $N^k := N^{\tau^k -}$. Then, for each $k \geq 1$, we denote the canonical decomposition under $\hat{P}$ by

$$N_t^k = N_0 + X_t^{N,k} + A_t^{N,k} ,$$

where $X^{N,k}$ is a $\hat{P}$-local martingale and $A^{N,k}$ is a locally integrable variation process of locally natural under $\hat{P}$. In virtue of VII.98 (c) of Dellacherie and Meyer (1982), the process $A^{N,k}$ is bounded. In addition, by Theorem III.27 of Protter (1990), we obtain that $A^{N,k}$ is predictable. Furthermore, by Galtchouk-Kunita-Watanabe decomposition under $\hat{P}$, we can write

$$N_t^k = N_0 + \int_0^t \xi_s^N dX_s^k + L_t^{N,k} + A_t^{N,k} ,$$

where ξ^N is a predictable process such that $\int \xi^N dX^k$ is a square integrable $\hat{P}$-local martingale, $L^{N,k}$ is a square integrable $\hat{P}$-local martingale strongly $\hat{P}$- orthogonal to X starting at 0. Hence, we have

$$\begin{aligned}(N\hat{Z}^0)_t^k = {} & N_0\hat{Z}_0^0 + \int_0^t N_{s-}\hat{\zeta}_s dX_s^k + \int_0^t \hat{Z}_{s-}^0 \xi_s^N dX_s^k + \int_0^t \hat{Z}_{s-}^0 dL_s^{N,k} \\ & + \int_0^t \hat{Z}_{s-}^0 dA_s^{N,k} + \int_0^t \hat{\zeta}_s^{\mathrm{tr}} d[X]_s^k \xi_s^N + \int_0^t \hat{\zeta}_s^{\mathrm{tr}} d[X, L^{N,k}]_s^k \\ & + \int_0^t \hat{\zeta}_s d[X, A^{N,k}]_s^k . \end{aligned} \tag{4.8}$$

Since each term after the fifth term of RHS is of finite variation, even if each term is a local martingale, then it is either a constant or a quadratic pure jump one. This implies that we can not represent the sum of these terms as a stochastic integral of X, since X is not quadratic pure jump. As compared with (4.7), we obtain

$$\int_0^t \hat{Z}_{s-}^0 \xi_s^N dX_s^k = 0 ,$$

that is, $\xi^N = 0$. Therefore, the sixth term equals to 0. Theorem VIII.19 of Dellacherie and Meyer (1982) yields that $[X, A^{N,k}]$ is a $\hat{P}$-local martingale. On the other hand, since $(N\hat{Z}^0)^k$

is a $\hat{P}$-local martingale, the fifth term need to be a $\hat{P}$-local martingale. In virtue of Lemma of Theorem IV.2 of Protter (1990), the fifth term is locally natural by the $\hat{P}$-integrability of $\hat{Z}^0$ and the boundedness of $\int_0^{\cdot} |dA_s|$. Together with Lemma of Theorem III.6 of Protter (1990), the fifth term is identically zero, and so is $A^{N,k}$. This implies that, for each $k \geq 1$, N^k is a square integrable $\hat{P}$-local martingale strongly $\hat{P}$-orthogonal to X. By diagonal method, we conclude that N is also.

Step 2. We shall prove that, U belongs to, for $\eta < \varepsilon$, $\mathcal{R}^{2+\eta}(P)$ prelocally. Firstly, we denote

$$\hat{X}_t := \int_0^t \frac{\hat{\zeta}_s}{\hat{Z}^0_{s-}} dX_s .$$

By Theorem V.4 of Protter (1990), $\hat{X}$ is a semimartingale prelocally in $\mathcal{S}^p(P)$ for $1 \leq p \leq \infty$. Thus, $\hat{X}_0 = 0$ and, for $1 \leq p \leq \infty$, there exists a sequence of stopping times $(T^l_p)_{l \geq 1}$ increasing to ∞ a.s. such that each $\hat{X}^{T^l_p -}$ belongs to the space $\mathcal{S}^p(P)$. We denote $\hat{X}^l := \hat{X}^{T^l_\infty -}$, for $l \geq 1$.

Now, fix $\eta < \varepsilon$ and $l \geq 1$ arbitrarily. Let $U^{l,0}$ be an $\mathbf{R}^d$-valued RCLL process being in $\mathcal{R}^{2+\eta}(P)$. Moreover, we define, by induction, $\mathbf{R}^d$-valued RCLL processes $U^{l,m}$, for $m \geq 1$, by

$$U^{l,m}_t = H_0 - c + L^H_t + \int_0^t U^{l,m-1}_{s-} d\hat{X}^l_s .$$

By Propositions 3.2, we can see that $L^H \in \mathcal{M}^{2+\eta}(P)$. Hence, by Doob's inequality, $L^H \in \mathcal{R}^{2+\eta}(P)$. Remark that, by the definition of the minimal martingale measure, the second expression for $\hat{V}$ is justified by $L^H \in \mathcal{M}^{2+\eta}(P)$. By Theorems V.2 and V.3 of Protter (1990), we have,

$$\begin{aligned} \|U^{l,m}\|_{\mathcal{R}^{2+\eta}(P)} &\leq H_0 - c + \|L^H\|_{\mathcal{R}^{2+\eta}(P)} + \left\| \int_0^{\cdot} U^{l,m-1}_{s-} d\hat{X}^l_s \right\|_{\mathcal{R}^{2+\eta}(P)} \\ &\leq H_0 - c + \|L^H\|_{\mathcal{R}^{2+\eta}(P)} + c_{2+\eta} \|U^{l,m-1}\|_{\mathcal{R}^{2+\eta}(P)} \|\hat{X}^l\|_{\mathcal{S}^\infty(P)} , \end{aligned}$$

where $c_{2+\eta}$ is a constant depending only $2+\eta$. Since $U^{l,0} \in \mathcal{R}^{2+\eta}(P)$, we obtain $U^{l,m} \in \mathcal{R}^{2+\eta}(P)$ by induction on m.

By the proof of Theorem V.8 of Protter (1990), if let U^l be the solution to the following SDE:

$$U^l_t = H_0 - c + L^H_t + \int_0^t U^l_{s-} d\hat{X}^l_s ,$$

then $U^{l,m}$ converges to U^l prelocally in $\mathcal{R}^{2+\eta}(P)$ as $m \to \infty$. Remark that the proof of Theorem V.8 of Protter (1990) mention only convergence prelocally in $\mathcal{R}^2(P)$, while we can extend his proof to $\mathcal{R}^p(P)$ for every $p > 2$.

Moreover, by Theorem 3.8 of Protter (1978), there exists a subsequence $(l_n)_{n\geq 1}$ such that

$$\lim_{l_n\to\infty} U^{l_n} = U$$

prelocally in $\mathcal{R}^{2+\eta}(P)$, because the $\mathcal{S}^p(P)$ norm is stronger than the $\mathcal{R}^p(P)$ norm, $1 \leq p \leq \infty$. Therefore, we can conclude that U is prelocally in $\mathcal{R}^{2+\eta}(P)$. In other words, for every $\eta < \varepsilon$, there exists a sequence of stopping times $(\tau_\eta^n)_{n\geq 1}$ increasing to ∞ a.s. such that each $U^{\tau_\eta^n -}$ belongs to $\mathcal{R}^{2+\eta}(P)$.

Step 3. Throughout the rest of this proof, superscript n of a process means the process stopped at $\tau_\eta^n -$. Firstly, we have, for each $n \geq 1$ and every $\eta < \varepsilon$,

$$\|N^n\|_{\mathcal{R}^{2+\eta}(P)} = \left\| \frac{U^n}{(\hat{Z}^0)^n} \right\|_{\mathcal{R}^{2+\eta}(P)} \leq \left\| \frac{1}{\hat{Z}^0} \right\|_{\mathcal{R}^r(P)}^{\frac{r}{2+\eta}} \|U^n\|_{\mathcal{R}^{2+\eta'}(P)}^{\frac{2+\eta'}{2+\eta}},$$

where $\eta < \eta' < \varepsilon$ and $r = \frac{(2+\eta)(2+\eta')}{\eta'-\eta}$. On the other hand, by the proof of Lemma 8 of PRS, we have

$$\frac{1}{\hat{Z}^0} \in \mathcal{R}^r(P) \quad \text{for every } r < \infty. \tag{4.9}$$

Thus, we obtain that N^n is in $\mathcal{R}^{2+\eta}(P)$ for each $n \geq 1$ and every $\eta < \varepsilon$. Next, SDE (4.7) and $U \in \mathcal{R}^{2+\eta}(P)$ prelocally imply that $(N_-\hat{\zeta})^n \in \hat{L}^{2+\eta}(M)$. Thus, if we define

$$\xi^{(c)} := \xi^H - N_-\hat{\zeta},$$

then $(\xi^{(c)})^n$ is in $\hat{L}^{2+\eta}(M)$ and in $\Theta^{2+\eta}$, for each $n \geq 1$ and every $\eta < \varepsilon$. However, by Lemma 3.1, we obtain that $\xi^{(c)}$ is in $\hat{L}^{2+\eta}(M)$ and in $\Theta^{2+\eta}$.

On the other hand, for $0 \leq t \leq T$,

$$\begin{aligned} U_t &= H_0 - c + L_t^H + \int_0^t N_{s-}\hat{\zeta}_s dX_s \\ &= H_0 - c + L_t^H + \int_0^t (\xi_s^H - \xi_s^{(c)}) dX_s \\ &= \hat{V}_t - c - \int_0^t \xi_s^{(c)} dX_s. \end{aligned}$$

In particular, when $t = T$, we obtain

$$H - c - G_T(\xi^{(c)}) = U_T = N_T \hat{Z}_T^0 = N_T \frac{d\hat{P}}{dP}.$$

Step 4. We fix $n \geq 1$ and $\vartheta \in \Theta$. In similar way to the proof of PRS, we can see that $N^n G^n(\vartheta)$ is a uniformly integrable $\hat{P}$-martingale. Since, for every $\eta < \varepsilon$, $\xi^H - \xi^{(c)} \in \hat{L}^{2+\eta}(M)$ and $L^H \in \mathcal{M}^{2+\eta}(P)$, U is in $\mathcal{R}^{2+\eta}(P)$. Then, N belongs to $\mathcal{R}^2(\hat{P})$, so that $N^n_T G^n_T(\vartheta)$ is dominated by $\sup_{0\leq t\leq T} N_t G_t(\vartheta) \in \mathcal{L}^1(\hat{P})$. Consequently, by the dominate convergence theorem, we have

$$\begin{aligned}
E[(H - c - G_T(\xi^{(c)}))G_T(\vartheta)] &= E\left[\lim_{n\to\infty}(\hat{V} - c - G(\xi^{(c)}))^n_T G^n_T(\vartheta)\right] \\
&= E\left[\lim_{n\to\infty} U^n_T G^n_T(\vartheta)\right] \\
&= \hat{E}\left[\lim_{n\to\infty} N^n_T G^n_T(\vartheta)\right] \\
&= \lim_{n\to\infty} \hat{E}[N^n_T G^n_T(\vartheta)] \\
&= 0\,.
\end{aligned}$$

By the projection theorem, $\xi^{(c)}$ is optimal. Finally, $\xi^{(c)}$ is represented as

$$\begin{aligned}
\xi^{(c)}_t &= \xi^H_t - N_{t-}\hat{\zeta}_t \\
&= \xi^H_t - \frac{\hat{\zeta}_t}{\hat{Z}^0_{t-}} N_{t-}\hat{Z}^0_{t-} \\
&= \xi^H_t - \frac{\hat{\zeta}_t}{\hat{Z}^0_{t-}}\left(\hat{V}_{t-} - c - \int_0^{t-} \xi^{(c)}_s dX_s\right).
\end{aligned}$$

This complete the proof. □

5. Concluding remarks.

5.1. Comparison with continuous case. In PRS, they assume that the stock price process is a continuous semimartingale. Moreover, Wiese (1998) studied models such that L^H in (3.1) is continuous. In these cases, we can write process N explicitly as follows:

$$N = \frac{H_0 - c + L^H_0}{E[\hat{Z}^2_T]} + \int \frac{1}{\hat{Z}^0_-} dL^H\,.$$

Hence, the seventh term of RHS in (4.8) is 0 in the above both cases. Therefore, by $L^H \in \mathcal{M}^q(P)$ for any $2 \leq q < p$ and (4.9), we can see that $U \in \mathcal{R}^{2+\eta}(P)$ for every $\eta < \varepsilon$.

On the other hand, in our model, we can not write N explicitly, since we know only that the seventh term of RHS in (4.8) is a $\hat{P}$-local martingale. This fact causes the difficulty in estimating of the regularity of U.

In order to extend the results related to mean-variance hedging to general discontinuous case, we need to succeed in proving the following. One is an extension of Theorem 4.1 of

Delbaen et al. (1997) to discontinuous case. The other is the estimation of the regularity of U in SDE (4.7). Our results contribute to success in giving a proof of the second one. In other words, if we obtain the extension of Delbaen et al. (1997), then mean-variance hedging for general discontinuous case will be solved. This is the reason why the result of Theorem 3.4 is important, though we impose some assumptions.

5.2. Two extensions of Theorem 3.4. The assumption that the local martingale M is a process with independent increments is used only in Proposition 3.2. Moreover, we use only Theorem III.4.34 of Jacod and Shiryaev (1987), which is the representation theorem for processes with independent increments. Thus, we can extend our results to classes of processes which has representation property. For example, if the stock price process X is a diffusion with jumps, then Theorem 3.4 holds, where the definition of a diffusion with jumps is given by Definitions III.2.18 and II.2.6 of Jacod and Shiryaev (1987). Hence, we can prove the following theorem easily:

THEOREM 5.1. *Theorem* 3.4 *holds under the following conditions*:
1. *the stock price process X is a diffusion with jumps*;
2. *Assumption except for Condition* 2;

The jump condition in Theorem 5.1 means that the Lévy measure of X is a finite measure and its support is bounded.

On the other hand, all $\mathcal{L}^2(P)$ contingent claims have a Föllmer-Schweizer decomposition (3.1) with $\xi^H \in L^2(M)$ and $L^H \in \mathcal{M}^2(P)$ by Theorem 3.4 of Monat and Stricker (1995). Thus, Proposition 3.2 is a little stronger than it. Therefore, even if we assume that a contingent claim H admits a Föllmer-Schweizer decomposition as (3.1), it will be only a slight assumption. We can rewrite Theorem 3.4 as follows:

THEOREM 5.2. *Theorem* 3.4 *holds under the following conditions*:
1. *the corresponding $\mathcal{F}_T$-measurable contingent claim H admits a Föllmer-Schweizer decomposition as* (3.1);
2. *Conditions* 1, 3 *and* 5–7 *of Assumption*;

REMARK. In Theorem 5.2, we can omit the assumption that a contingent claim H is $\mathcal{F}_T^M$-measurable.

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