Measurement of α_s from Jet Rates at the Z^0 Resonance

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We have determined the strong coupling α_s from measurements of jet rates in hadronic decays of Z^0 bosons collected by the SLD experiment at SLAC. Using six collinear and infrared safe jet algorithms we compared our data with the predictions of QCD calculated up to second order in perturbation theory, and also with resummed calculations. We find $\alpha_s(M_Z^2) = 0.118 \pm 0.002(\text{stat}) \pm 0.003(\text{syst}) \pm 0.010(\text{theory})$, where the dominant uncertainty is from uncalculated higher order contributions.

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The standard model of elementary particles comprises the theory of electroweak interactions and quantum chromodynamics (QCD), the theory of strong interactions. While the former has been tested to high precision [1], accurate tests of QCD have proved more difficult [2], largely due to the complexity of calculating observables to high order in perturbation theory and in the nonperturbative regime. One of the most important tests of QCD has been the measurement of the strong coupling α_s in different hard processes and at different scales, Q^2. Here we present a measurement of $\alpha_s(M_Z^2)$ from the rate of production of multijet final states in hadronic decays of Z^0 bosons produced in e^+e^- annihilations. We employ six collinear and infrared safe jet algorithms to study the uncertainties arising from finite order perturbative QCD calculations, and we also compare our data with recently performed all-orders calculations in the next-to-leading logarithm approximation.

The SLAC Linear Collider (SLC) produces electron-positron annihilation events at the Z^0 resonance which are recorded by the SLC Large Detector (SLD), described elsewhere [3]. The triggers and selection used for hadronic events are described in Ref. [4]. The analysis presented here used charged tracks measured in the central drift chamber and in the vertex detector. Well-measured tracks were required to have (i) a closest approach transverse to the beam axis within 5 cm, and within 10 cm along the axis from the measured interaction point; (ii) a polar angle θ with respect to the beam axis with $|\cos\theta| < 0.80$; and (iii) a minimum momentum transverse to this axis of $p_\perp > 150$ MeV/c. Events well contained in the detector were selected by requiring a minimum of five such tracks, a thrust axis [5] direction with respect to the beam axis, θ_T, within $|\cos\theta_T| < 0.71$, and a charged visible energy greater than 20 GeV. From our 1992 data sample 6476 events survived these cuts. The efficiency for selecting hadronic events satisfying the $|\cos\theta_T|$ cut was estimated to be above 96% and the background to be $(0.3 \pm 0.1)\%$, dominated by $Z^0 \to \tau^+\tau^-$ events. Resulting distributions of single particle and event topology measures were found to be well described by Monte Carlo models of hadronic Z^0 boson decays [6,7] combined with a simulation of the SLD.

In order to define jets we applied several iterative clustering algorithms in which a measure y_{ij}, such as (invariant mass squared)/s, is calculated for all pairs of particles i and j, and the pair with the smallest y_{ij} is combined into a single "particle." This process is repeated until all pairs have y_{ij} exceeding a value y_c, and the jet multiplicity of the event is defined as the number of particles remaining. Various recombination schemes and definitions of y_{ij} have been suggested [8]. We have applied the E, $E0$, P, and $P0$ variations of the JADE algorithm [9] as well as the more recently proposed Durham (D) and Geneva (G) algorithms, all of which are collinear and infrared safe. The n-jet rate $R_n(y_c)$ is defined as the fraction of events classified as n-jet, and the differential 2-jet rate [10] as $D_2(y_c) \equiv [R_2(y_c) - R_2(y_c - \Delta y_c)]/\Delta y_c$. In contrast to R_n, there are no point-to-point correlations in D_2.

The measured jet rates were corrected for the effects of detector acceptance, inefficiency, and resolution, particle interactions and decays within the detector, and bias from the analysis cuts using the SLD Monte Carlo simulation, and a bin-by-bin method [11]. A hadronization correction was also applied, estimated using JETSET 6.3 [6].

The corrected D_2 distributions were derived from the fully corrected jet rates and compared with QCD calculations employing the same jet algorithms, performed up to second order in perturbation theory, which have the general form: $R_3(y_c)=A(y_c)\alpha_s(\mu)+B(y_c,f)\alpha_s^2(\mu)$, and $R_4(y_c)=C(y_c)\alpha_s^2(\mu)$ where $\alpha_s=\alpha_s(\Lambda_{\overline{\rm MS}},\mu)$ [12], $\Lambda_{\overline{\rm MS}}$ is the fundamental scale of strong interactions, ($\overline{\rm MS}$ denotes the modified minimal subtraction scheme), and μ is the renormalization scale, often expressed in terms of the factor $f=\mu^2/s$. Here we have assumed the definition of $\Lambda_{\overline{\rm MS}}$ for five active quark flavors. The explicit dependence of the next-to-leading coefficient B on f is an artifact of the truncation of the perturbation series at finite order. Therefore, if $\Lambda_{\overline{\rm MS}}$ is extracted by fitting these calculations to the data, the variation of f must be taken into account as a contribution to the uncertainty in $\Lambda_{\overline{\rm MS}}$. We used parametrizations [8,13] of the coefficients $A(y_c)$, $B(y_c,f)$, $C(y_c)$ to derive $R_2=1-R_3-R_4$. For each algorithm, $D_2(y_c)$ was calculated and fitted [14] to the fully corrected measured distributions by varying $\Lambda_{\overline{\rm MS}}$ and minimizing χ^2. The fits were restricted to the range of y_c (Table I) for which the measured $R_4<1\%$, since in the second order calculation R_4 was evaluated only at leading order, and $R_{n>4}$ were not considered. The upper y_c fit boundary was chosen to be the kinematic limit for (massless) 3-jet production, $y_c=0.33$.

The fitted $\Lambda_{\overline{\rm MS}}$ values were translated [12] into $\alpha_s(M_Z^2)$. The results are summarized in Fig. 1, where α_s and χ^2 per degree of freedom ($\chi^2_{\rm dof}$) are shown as functions of f in the range $10^{-5}\leq f\leq 10^1$. The number of degrees of freedom varied between 4 and 11 and is given in Table I. Several features are common to all algorithms: (1) α_s depends strongly on f; (2) across a range of f the fit quality is reasonable and $\chi^2_{\rm dof}$ changes slowly; (3) at low f the fits are poor, $\chi^2_{\rm dof}$ changes rapidly, and neither α_s nor its error can be interpreted meaningfully. The boundary between reasonable and poor fits is algorithm dependent. We note also that for some algorithms reasonable fits can be obtained for $f\gg 1$, although such values are beyond the physical scale accessible in e^+e^- annihilation.

TABLE I. $\alpha_s(M_Z^2)$ and errors from $O(\alpha_s^2)$ QCD fits. The experimental systematic error was ± 0.003 for each algorithm.

Algorithm	Fit range $y_c\geq$	Deg. of freedom	$\alpha_s(M_Z^2)$	Errors Stat.	Had.	Scale
D	0.03	11	0.125	0.002	0.003	0.010
E	0.08	7	0.128	0.002	0.002	0.021
$E0$	0.06	8	0.118	0.002	0.003	0.012
P	0.05	9	0.116	0.002	0.003	0.008
$P0$	0.05	9	0.114	0.002	0.003	0.007
G	0.16	4	0.108	0.005	0.005	0.005

Figure 1 contains all of the information from the QCD fits to the data. In order to quote a single value of α_s for each algorithm we adopt the following arbitrary procedure. We consider the range $0.002\leq f\leq 4$. The exact interpretation of μ is renormalization scheme dependent; however, the lower bound corresponds approximately to $\mu\geq m_b$, and restricts μ to the region in which five active quark flavors contribute to $\Lambda_{\overline{\rm MS}}$, in addition to ensuring that the perturbative series for R_3 remains reasonably convergent for all algorithms. This excludes some small scales for which the fit quality is good, but includes the α_s minima for all algorithms except E. The upper bound restricts μ to a reasonable physical region, $\mu\leq 2\sqrt{s}$. Within this range the fit quality is acceptable [Fig. 1(b)], the data show no strong preference for a particular scale, and we take the extrema of the α_s values as the uncertainty from the dependence on f. Table I summarizes the measured α_s, defined as the midpoint between the extrema, the scale uncertainties, and the statistical errors. The large and different scale uncertainties may be interpreted as arising from uncalculated higher order contributions which are different for each algorithm. However, allowing for the scale uncertainties, the six α_s values are in agreement, which is a significant consistency check of QCD.

Experimental systematic errors were investigated [15] by varying the cuts applied to the data and changing parameters in the simulation of the detector over large ranges. In each case, the detector correction factors were reevaluated and the correction and fitting procedures repeated. In addition, the fit ranges were varied by deleting bins at the ends of the y_c regions. None of these effects changed the value of α_s by more than the statistical error. We conservatively estimate the systematic error to be ± 0.003 for each algorithm. Hadronization uncertainties (Table I) were studied by recalculating the hadronization correction factors using JETSET with values of the parton virtuality cutoff Q_0 [6] in the range 0.5 to 2.0 GeV, and

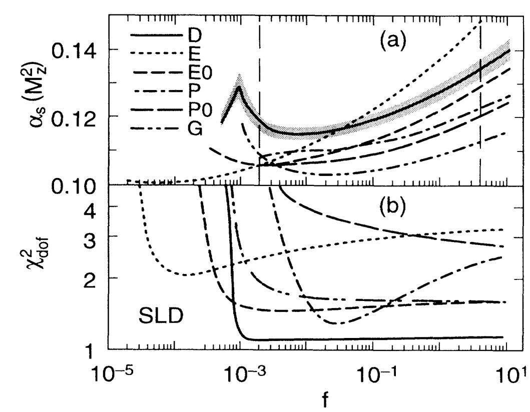

FIG. 1. (a) $\alpha_s(M_Z^2)$ and (b) $\chi^2_{\rm dof}$ from $O(\alpha_s^2)$ QCD fits (see text). The band indicates the size of statistical errors.

by using HERWIG [7], which contains a different hadronization model.

In order to quote a single result we calculated the mean and rms deviation of the six α_s values for each f in the range $0.002 \leq f \leq 4$. We then took the central value of the means in that range as our central result, the rms at the central value as the *algorithm uncertainty*, and the difference between the central value and the extrema as the *scale uncertainty*. This procedure corresponds to the conservative assumption that all six α_s at each f are completely correlated statistically, and yields

$$\alpha_s(M_Z^2) = 0.118 \pm 0.002(\text{stat}) \pm 0.003(\text{syst}) \pm 0.010(\text{theory}),$$

where the theory uncertainty is the sum in quadrature of contributions from hadronization (± 0.003), scale (± 0.009), and algorithm (± 0.003) uncertainties. This result is in good agreement with other measurements of $\alpha_s(M_Z^2)$ [2]. Our theoretical uncertainty is slightly larger than that quoted by some of the experiments at the CERN e^+e^- collider LEP because we considered a wider range of scales and more jet algorithms, and added an additional algorithm uncertainty, which is not normally considered. The scale and algorithm uncertainties are correlated, but we consider the resulting estimate of uncalculated higher order contributions to be realistic.

We have shown that when fitted in the region of y_c where $R_4 < 1\%$ the data show no strong preference for particular scales. A preference for low scales has been reported by other experiments [16], resulting from simultaneous fits of both f and $\Lambda_{\overline{\text{MS}}}$ in ranges of y_c extending to low values around which $R_5 \sim 1\%$. For purposes of comparison we have performed similar fits. With the exception of the G algorithm, for which a value of $f > 10$ is preferred, we found a slight preference for scales $\ll 1$. However, it should be noted that at low y_c R_4 is large, and the perturbation series for R_3 does not converge rapidly if $f \ll 1$. Thus it is not self-consistent to use perturbative formulas with small scales in this domain.

Progress has recently been made in the form of resummed QCD calculations for event shape distributions in e^+e^- annihilation [17]. For the D algorithm these techniques have been used [18] to calculate jet rates at leading and next-to-leading order in $\ln(1/y_c)$, up to all orders in α_s. The resulting all-orders calculation, valid in the region where $\alpha_s \ln(1/y_c) \leq 1$, may be combined with the fixed second order result [19] to yield improved predictions for multijet rates at low y_c. Several matching schemes have been proposed for this combination [20,21], including R matching, $\ln(R)$ matching, and modified $\ln(R)$ matching. For each scheme, D_2 was derived from the recalculated [22] R_2 and fitted to the data in our full range $y_c \geq 0.01$. The resulting $\alpha_s(M_Z^2)$ and χ^2_{dof} values are shown in Fig. 2, labeled "ALEPH scheme," as a function of f. Results from both $\ln(R)$ matching schemes are similar, so we show only the modified

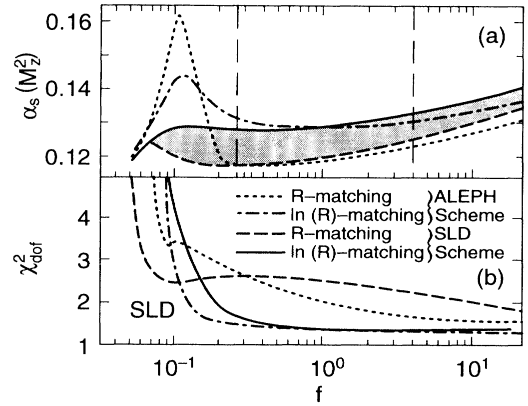

FIG. 2. (a) $\alpha_s(M_Z^2)$ and (b) χ^2_{dof} from fits using resummed calculations for the D algorithm. The band shows the range of uncertainty from higher order effects (see text).

scheme. The behavior is qualitatively similar to the second order result (Fig. 1), although $f \geq 0.1$ is needed to fit the data, and in this range the fitted α_s varies slowly with f, in agreement with other results [20,21]. In contrast with Ref. [21] we find the fit quality to be good.

We found, however, that the resummed calculations yield $R_2 > 1$ in some regions of phase space. This unphysical behavior gives rise to the peak at $f \sim 0.1$ in Fig. 2(a). For $y_c \leq 0.04$ the resummed R_2 remains below unity for $f \geq 0.2$, so we adopted a new procedure, using the matched calculation for $0.01 \leq y_c \leq 0.04$ and the $O(\alpha_s^2)$ calculation for $0.05 \leq y_c \leq 0.33$, giving α_s and χ^2_{dof} labeled "SLD scheme" in Fig. 2. With this procedure we quote a single value of α_s by taking the mean in the range $\frac{1}{4} \leq f \leq 4$, and the difference between the R and $\ln(R)$ matching schemes as the *matching uncertainty*. This range excludes the unphysical $R_2 > 1$ region but includes the full measured variation of α_s up to $f = 4$; it is the same as that considered in Ref. [21] but larger than Ref. [20]. We found

$$\alpha_s(M_Z^2) = 0.126 \pm 0.002(\text{stat}) \pm 0.003(\text{syst}) \pm 0.006(\text{theory}),$$

where the theory uncertainty is the sum in quadrature of contributions from hadronization (± 0.003), scale (± 0.003), and matching (± 0.005) uncertainties. This is in good agreement with the $O(\alpha_s^2)$ result for the D algorithm (Table I); the scale uncertainty is considerably smaller, but there is extra uncertainty from the matching. The latter can be attributed to partly calculated next-to-leading, and uncalculated subleading, logarithmic terms [23]. Nevertheless, for the D algorithm the total theoretical uncertainty is smaller using the resummed calculation than the $O(\alpha_s^2)$ calculation. Further improvement in the accuracy of α_s determinations from jet rates must await better understanding of the remaining higher order contributions.

In conclusion, we have measured $\alpha_s(M_Z^2)$ from jet rates in e^+e^- annihilation using six particle clustering algorithms. The value of α_s is highly correlated with the choice of QCD renormalization scale, but the data show no strong preference for particular scales within wide ranges, unless the fits include regions of phase space where 4-jet production is significant and the perturbative series for the 3-jet rate does not show good convergence. Recent resummed calculations for the Durham algorithm were found to fit the data down to $y_c = 0.01$ for renormalization scales $f > 0.1$, and to yield less variation of α_s in this range. Differences between the schemes used to combine the fixed order and resummed calculations added extra uncertainty, implying that higher order corrections are still not under control, but the total uncertainty was smaller than in the second order case. Since the resummed calculations have been performed only for the Durham algorithm, and yield α_s in good agreement with the fixed order result, we quote the fixed order value based on all six algorithms as our final result: $\alpha_s(M_Z^2) = 0.118 \pm 0.002(\text{stat}) \pm 0.003(\text{syst}) \pm 0.010(\text{theory})$, in good agreement with other measurements [2]. This corresponds to $\Lambda_{\overline{\text{MS}}} \simeq 230 \pm 130$ MeV. The precision is limited by lack of knowledge of higher order contributions.

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