EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN ISR-TH/77-12

MEASUREMENT OF BEAM STABILITY AND COUPLING IMPEDANCE BY RF EXCITATION

by

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Presented at the 1977 Particle Accelerator Conference,
Chicago, March 16-18, 1977

Geneva, Switzerland
March 1977

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1. Summary

The transverse stability limit and coupling impedance of proton beams in the CERN Intersecting Storage Rings have been measured by external excitation of the beam. The impedance values at low frequencies thus obtained are in good agreement with the sum of the skin-effect and the wall-inductance which has been measured previously. The method can be extended to higher frequencies if the beam can be stabilized against low-frequency oscillations, e.g. by feedback.

2. Introduction

It has been known for some time that the transverse stability limit of a particle beam in a vacuum chamber can be obtained from the measurement of the beam response to RF excitation[1]. By the same means one can obtain the coupling impedance of the vacuum chamber[2,3], but the method proposed is very sensitive to measurement errors as it requires analytic continuation of measured data.

In this paper we describe a simpler approach, which has become possible since the distribution of particles over betatron frequencies can be measured with the transverse Schottky scan[4], at least if the amplitude is independent of momentum.

3. Theory

The equation of motion of a single particle can be written[3]

$$\frac{d^2x}{dt^2} + Q_e^2\Omega^2 x + Ax + B\bar{x} = G\, e^{-i\omega t} \qquad (1)$$

where x is the transverse displacement of a particle with betatron frequency $Q_e\Omega$ due to external focussing;

$\bar{x}$ is the transverse displacement of the beam center;

A is the incoherent self-force (divided by mass) acting on a particle when the beam is at rest;

B is the coherent self-force (divided by mass) acting on the beam as a whole;

G is the transverse acceleration due to an external RF field with frequency ω.

The incoherent space charge term can be absorbed in the external focussing term by defining

$$Q_e^2\Omega^2 + A = Q^2\Omega^2 \qquad (2)$$

The equation of motion can be solved for x, and integrated over the distribution of particles in betatron frequency space to obtain the inverse response

$$R(\omega) \equiv \frac{G}{\bar{x}} = B + \frac{1}{I} \qquad (3)$$

$$\text{where } I = \int \frac{f(\nu)d\nu}{Q^2\Omega^2 - (\omega - n\Omega)^2} \qquad (4)$$

is the dispersion integral. If we assume that ω is in the neighbourhood either of the slow-wave $\nu = (n-Q)\Omega$ or the fast wave $\nu = (n+Q)\Omega$, we get approximately

$$I = \pm \frac{1}{2Q_o\Omega_o} \int \frac{f(\nu)d\nu}{\omega - \nu} \qquad (5)$$

where Q_o and Ω_o are the tune and revolution frequency at the center of the beam.

It is convenient to introduce a normalized distribution function $F(\xi)$ with unit area and unit half-spread[5]. We then get a normalized response

$$R^*(\omega) = \frac{R(\omega)}{2Q_o\Omega_o S} = \frac{1}{I^*} - (U' + iV') \qquad (6)$$

$$\text{where } S = |(n - Q_o)\,\Delta\Omega - \Omega_o\Delta Q| \qquad (7)$$

is the halfspread in betatron frequencies,

$$I^* = \pm \int_{-1}^{+1} \frac{F(\xi)d\xi}{\xi - \xi_1} \qquad (8)$$

is the normalized dispersion integral, and

$$\xi_1 = \frac{\omega - (n - Q_o)\Omega_o}{S} \qquad (9)$$

is the normalized frequency deviation. The quantities

$$U' + iV' = \frac{-B}{2Q_o\Omega_o S} \qquad (10)$$

are normalized dispersion relation coefficients[5], related to the original definition by

$$U' + iV' = \frac{U + (1 + i)V}{S} \qquad (11)$$

However, to describe the influence of the surroundings, it is much more appropriate to use the transverse coupling impedance (per unit displacement) defined by

$$Z_T = \frac{-i}{\beta \bar{x} I_o} \int_0^{2\pi R} \left[E + v \times B \right]_T ds \qquad (12)$$

which is practically independent of beam parameters. Comparison with the definition of U and V yields the relation

$$Z_T = -4\pi i \frac{m_o c}{e} \frac{Q\gamma}{I_o} \left[U + (1 + i)V \right] \qquad (13)$$

From equation (6) we get a finite response for vanishing excitation if $R^*(\omega) = 0$, or

$$(U' + iV')_{threshold} = \frac{1}{I^*} \qquad (14)$$

i.e. the inverse of the normalized dispersion integral yields the stability limit.

If we introduce the coupling impedance into equation (6), we get (with j = -i)

$$R^*(\omega) = \frac{1}{I^*} + \frac{e}{4\pi m_o c} \frac{I_o}{Q_o\gamma S}\, jZ_T \qquad (15)$$

A plot of $R^*(\omega)$ in the complex plane thus yields a shifted stability limit. The shift is proportional to the coupling impedance, to the beam current, and inversely proportional to the spread. By changing the beam current of the spread, we can measure the coupling impedance.

4. Experimental Results

We have applied this method to measure the transverse coupling impedance in the ISR. We obtain the absolute calibration for the response from the normalized dispersion integral which is computed from Schottky scan data. We then measure phase and amplitude response of the beam with a swept-frequency generator for one or more values of chromaticity for the same beam (see fig. 1 and 2). After subtraction of the cable delays, we then plot real and imaginary part of the inverse response on a scale chosen to give the same area for the shifted curves (fig. 3) as the calculated one. For low frequencies, the spread is approximately

$S = \frac{1}{2}|Q|\Omega_o \left(\frac{\Delta p}{p}\right)_{full}$, where $Q' = dQ/dp/p$, so equation (15) becomes in mksA-units

$$\Delta R^* = R^*(\omega) - \frac{1}{I^*} = + \frac{0.051}{Q\Omega_o\gamma} \frac{I_o}{(\Delta p/p)_{full}} \frac{jZ_T}{Q'} \qquad (16)$$

For one experiment at the ISR ($\Omega_o = 2.10^6\ s^{-1}$) at 26.6 GeV/c ($\gamma = 28.4$) and at a tune of 8.62, the current was 5.1 A in a full momentum spread of $\frac{\Delta p}{p} = .010$. We thus find

$$Z_T = 18.86j \cdot Q' \cdot \Delta R^*\ (M\Omega/m) \qquad (17)$$

The beam was excited near the n = 9 mode (110 kHz). For increased accuracy, we obtained ΔR^* from the shifts measured with several chromaticities (see fig. 3). On the average we obtained $Q'.\Delta R^* = .405 - .117\ j$, and hence

$$Z_T = (2.19 + 7.63\ j)\ M\Omega/m$$

We can compare this result with the measured high-frequency longitudinal inductive wall impedance[6) $Z_L^{ind}/n = 25\ j\Omega$, to which we add the low-frequency skin-effect. For $n - Q = 0.38$, we find $Z_L^{skin}/n = 7.9(1+j)\Omega$, or a total $Z_L/n = (7.9+32.9\ j)\Omega$.

Converting to transverse impedances[7) with

$$Z_T^{eq} = \frac{2c}{\Omega_o b^2} \frac{Z_L}{n} \qquad (18)$$

with b = 32 mm (60% of ISR 26 mm, 40% 80 mm, average $1/b^2$) we get

$$Z_T^{eq} = (2.31 + 9.64\ j)\ M\Omega/m$$

in quite good agreement with the value obtained directly (the agreement is even better if the more recent value of $Z_L^{ind}/n = 20\Omega$ is used). The inverse response of the slow and fast waves is shown in fig. 4.

Measurements at higher mode numbers have been made up to about 4 MHz (see fig. 5). By using the 50 MHz feedback system to stabilize the lower modes, the range can be extended to over 60 MHz before accuracy is lost.

5. Conclusion

The direct measurement of the transverse coupling impedance seen by a particle beam in a vacuum chamber is possible by measurement of phase and amplitude of the response to a transverse excitation. The method is most easily applied at low frequencies, where it becomes most sensitive when one reduces the chromaticity as much as possible without losing the beam. At higher frequencies, one can use a feedback system to stabilize the beam at lower modes, and thus extend the measurement.

In principle, it should also be possible to excite the beam longitudinally and thus obtain the longitudinal coupling impedance. Experiments in this direction have not been very successful so far because of high noise levels, but are being continued.

Instead of using a swept frequency signal, one can also excite the beam with noise[8) and obtain the response instantaneously as a Nyquist diagram. This method has been tested successfully, and may become an operational facility at the ISR.

References

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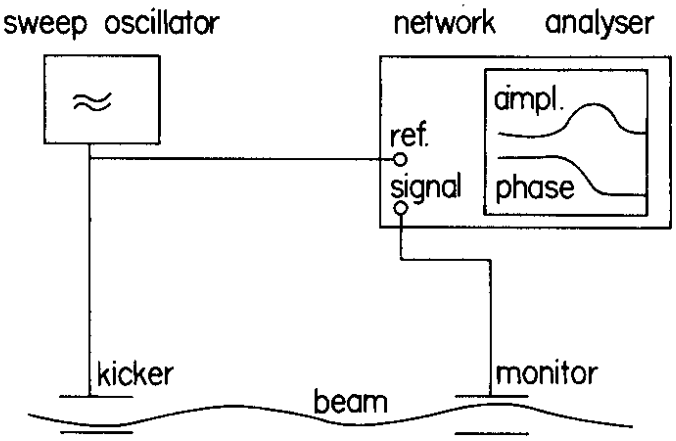

Fig. 1 Block diagram of the experimental set-up

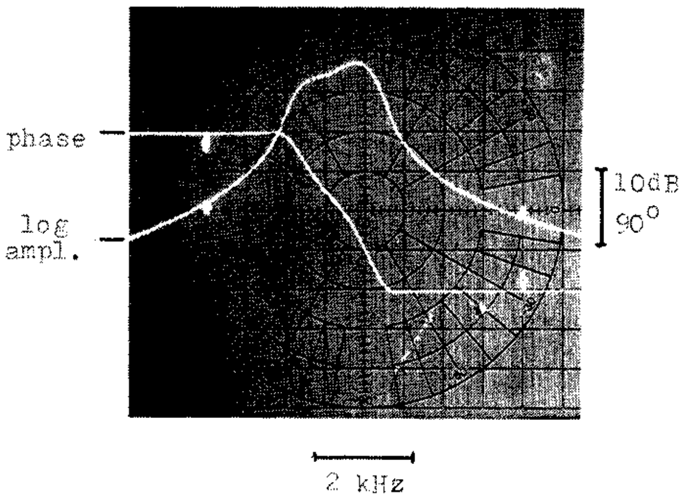

Fig. 2 Phase and amplitude response of a beam vs. frequency

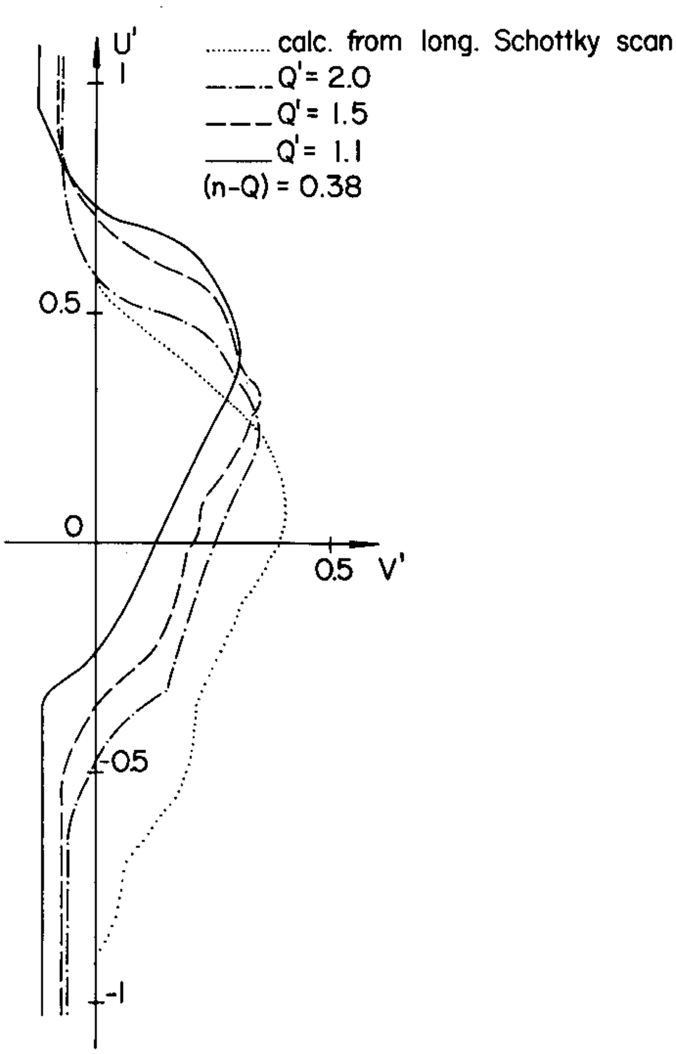

Fig. 3 Plot of the stability limit and of the inverse response for different chromaticities

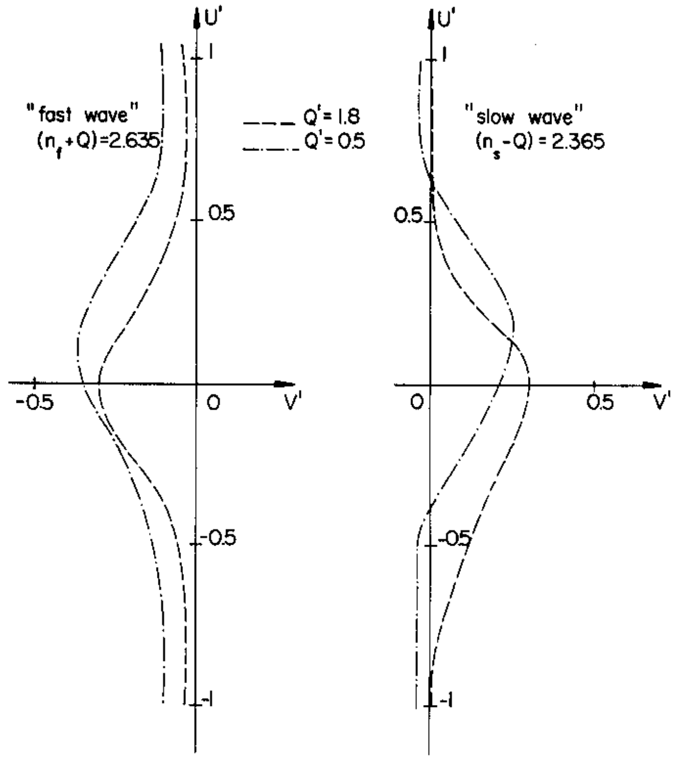

Fig. 4 Plot of the inverse response for the slow and the fast wave

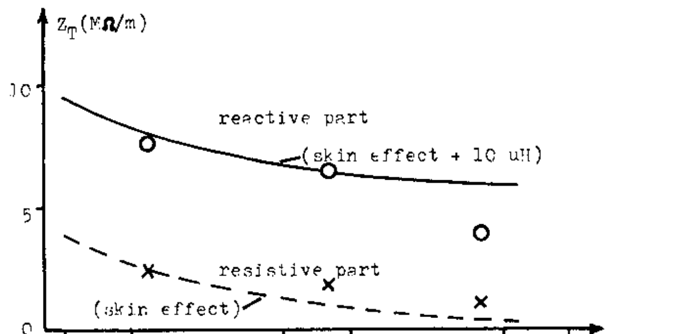

Fig. 5 Preliminary measurement of the transverse coupling impedance vs. frequency