Measurement of Falling Velocity of Individual Snow Crystals

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Abstract

The falling velocity of individual snow crystals was ovserved by means of stroboscopic photographs. At the same time, the shape, dimension and mass of those snow crystals were also measured from the photomicrographs of them. The plane type crystals were classified into six groups, namely hexagonal plates, crystals with sectorlike branches, crystals with broad branches, dendritic crystals, stellar crystals and thick plates. Furthermore, the columnar crystals were also observed.

Consequently, the relationship between the falling velocity and the dimension of particles was obtained for their several major types of snow crystals, and it was found that the relationship was well explained by the computation method which use the drag coefficient determined by the model experiment.

1. Introduction

In the previous paper (Kajikawa, 1971), the author reported the result of model experiment for the falling velocity of snow crystals and presented a computation method for the velocity. On the other hand, measurements of the falling velocity of snow crystals were performed by Nakaya and Terada (1935). They measured the velocity of the dendritic crystals of plane type, the dimensions of which are approximately over 1.5 mm in diameter. However, the falling velocity of snow crystals smaller than those is also important for precipitation phenomena, for example, the formation of aggregate snowflakes and the riming process.

Therefore, the purpose of this study was to observe simultaneously the shape, mass and falling velocity of natural snow crystals, especially those of the smaller sizes. This paper describes the result of the observation and the comparison between the observed fall velocity and the computed velocity with the drag coefficient determined by the model experiment.

The observation was made at the top of Mt. Teine and then the air temperature was about $-7°C$.

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2. Apparatus and method of observation

2.1. Measurement of falling velocity

The apparatus used in this measurement is shown in Fig. 1 and it was placed in a snow igloo. This apparatus is made of two parts, the cylinder as a duct for the falling of snow crystals (the upper part of this figure) and the measuring part (the lower one). The measuring space, which is separated by a shutter with a slit (the elliptical slit with a long axis 2.5 cm and a short one 0.5 cm), was enclosed by heat insulator. At the bottom, it was slightly cooled by dry ice blocks to keep the stability of the air. As a result of this regard, the air temperature of the upper part in the measuring space was always about 2°C higher than the lower part. Moreover, the tiny ice brocks were placed at the bottom to suppress the evaporation of the falling snow crystals.

Two methods of release snow crystals to fall were used in this measurement. In method A, as shown in Fig. 1, the snow crystals received on the black cloth were dropped at the top into the cylinder (90 cm in length) and after an appropriate time the shutter was opened and soon after it was shut again to allow only a few crystals to fall. These fallen crystals were replicated on the slide glass. Method B is, to

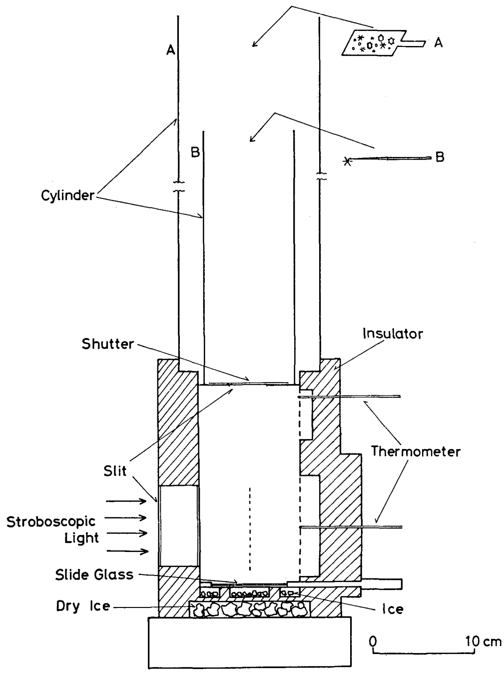

Fig. 1. Apparatus for measurement of falling velocity.

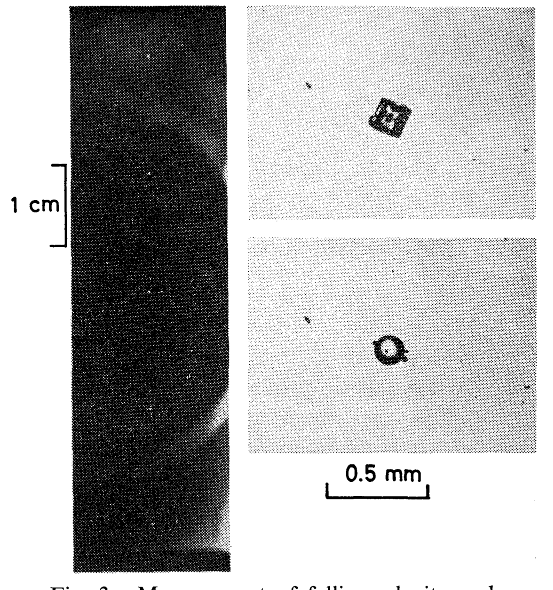

Fig. 3. Measurement of falling velocity and mass of short column.
left: stroboscopic photograph at 1/30 sec intervals.
right: before melting (above) and after melting (below).

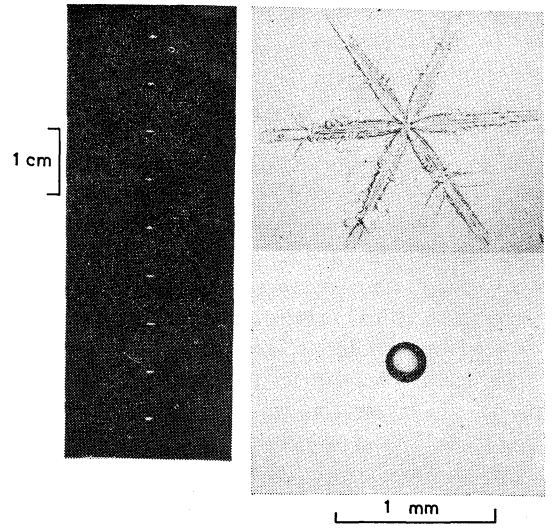

Fig. 2. Measurement of falling velocity and mass of dendritic crystal.
left: stroboscopic photograph at 1/30 sec intervals.
right: before melting (above) and after melting (below).

say, a picking-up method, in which one by one a suitable snow crystal was picked up with a piece of wood from many snow crystals on the black cloth and it was dropped through the slit (the outer cylinder was taken away and the shutter was opened beforehand). These fallen crystals were caught on the slide glass which was covered with silicone oil.

For the measurement of the falling velocity, the measuring volume of space was illuminated for a period as short as possible through a slit located at the side by means of stroboscopic light filtered with heat absorption glass. The camera was placed in the direction perpendicular to the beam of the illumination. Stroboscopic photographs were taken through the double glass plates with the gap of a few centimeters. As a result the falling snow crystals were recorded as the line of dot marks as seen in Fig. 2 and Fig. 3. Because the distances between dots are almost the same, it can be considered that the snow crystals fell with the terminal velocity of their own. (Most of these data were obtained by method B.)

2.2. Measurement of mass and thickness

In method B the ordinary slide glass (7.6×2.6 cm), which was covered thickly with silicone oil, was placed on the slide glass holder and set at the bottom to catch the falling crystals. After

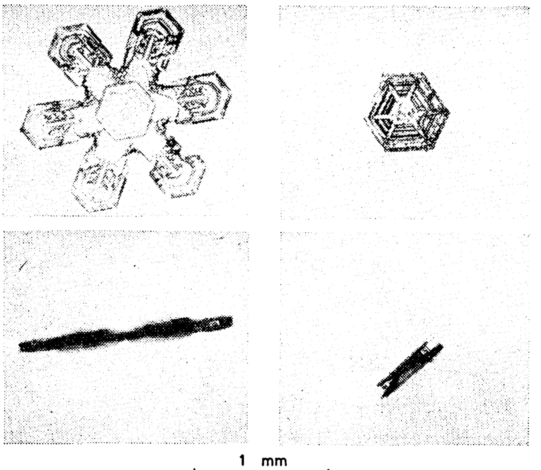

Fig. 4. Measurement of thickness of plane type crystals, the side view photographs (below).

the snow crystals were caught, the holder was taken out of the apparatus and then the crystals were photographed. In order to obtain the accurate value of the mass of snow crystals, the crystals floating in the silicone oil were melted by a electric heater placed under the microscopic stage and then the obtained droplets were photographed to measure the diameters (Fig. 2 and Fig. 3).

Moreover, the thickness of each plane type crystal was measured directly from the photomicrographs of the side views of snow crystals which were erected on the microscopic stage, as shown in Fig. 4.

3. Result and consideraiion

3.1. Mass and thickness of plane type crystals

The equivalent droplet diameters of plane type

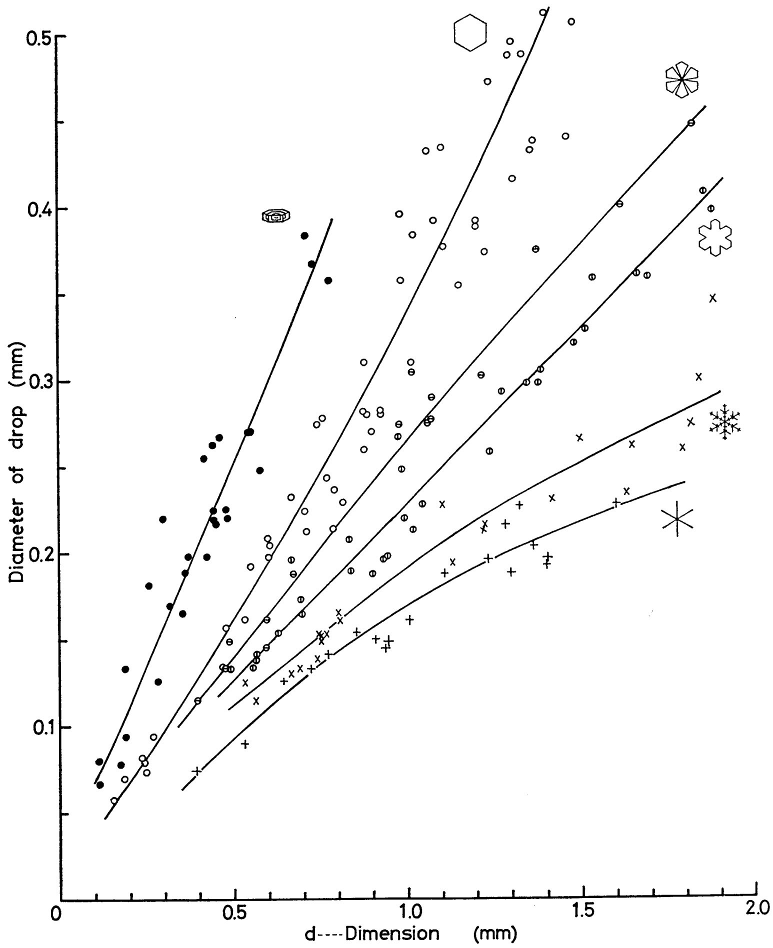

Fig. 5. Equivalent droplet diameter *vs.* dimension (diameter) of plane type crystals (d).

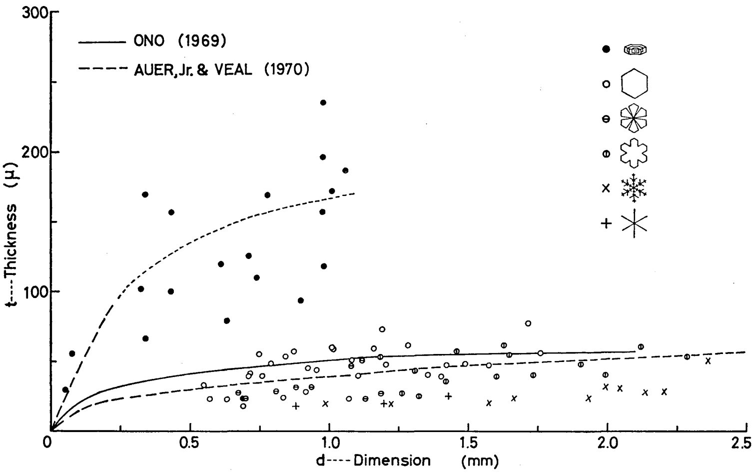

Fig. 6. Thickness (t) *vs.* dimension of plane type crystals (d).

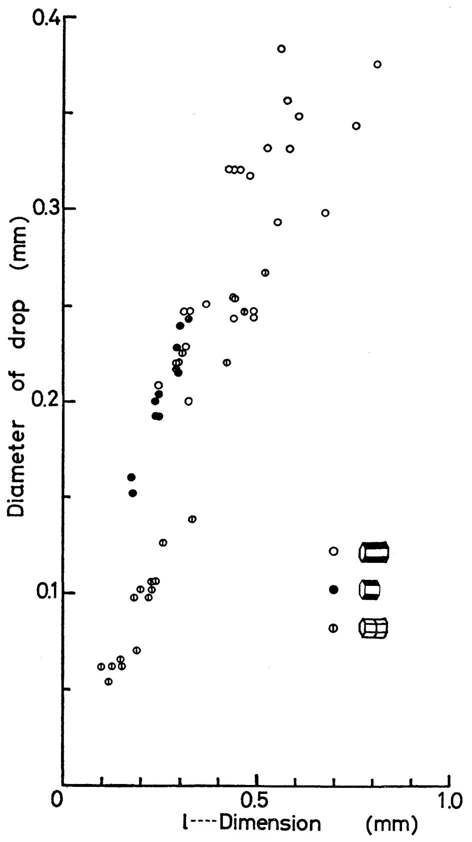

Fig. 7. Equivalent droplet diameter *vs.* dimension (length of c-axis) of columnar crystals (l).

crystals, the six groups of which were classified in the same manner as given by Magono and Lee (1966), are shown in Fig. 5. The solid lines in this figure show the mean values. The masses of crystals can be computed easily from this figure.

The thickness of plane type crystals is shown in Fig. 6. The broken lines in this figure show the values observed by Auer and Veal (1970) and the solid line shows the values observed by Ono (1969), both of which were measured from the plastic replicas of plane type crystals. A dotted line is the mean values of this observation of the thick plates. It can be seen that the values of thick plates are scattered, although those of other shapes generally fit to those obtained by other researchers, except the steller and the dendritic crystals, whose thickness is smaller. Further, it can be considered that the ratio t/d of the plane type crystals except the thick plates is smaller than 0.1, in which t is the thickness and d, the diameter of crystals. In the following computation (which will be described in 3.3), the ratio t/d of 0.1 was used in computing the falling velocity of plane type crystals except the thick plates because the drag coefficient of the circular disk is almost the same so far as the ratio t/d is smaller than 0.1 as described by Kajikawa (1971). On the other hand, the mean values in this figure were used for the ratio t/d of thick plates in computing the falling velocity of them.

3.2. Mass and axial ratio of columnar ctystals

The equivalent droplet diameters of the columnar crystals are shown in Fig. 7. The observed columnar crystals were classified as ordinary solid

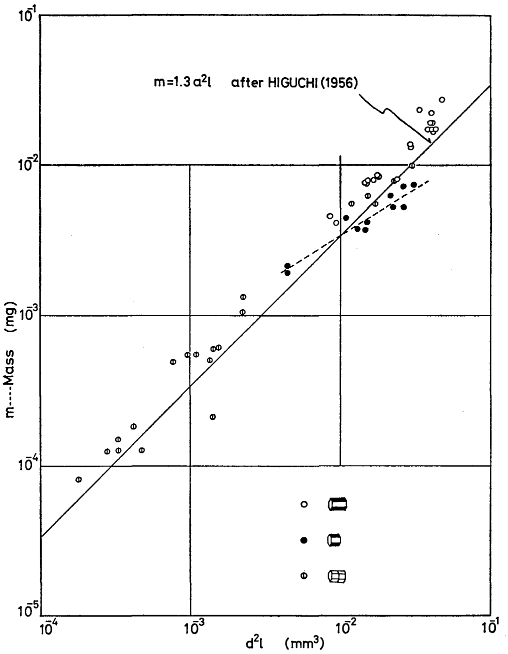

Fig. 8. Mass of columnar crystals *vs.* d^2l.

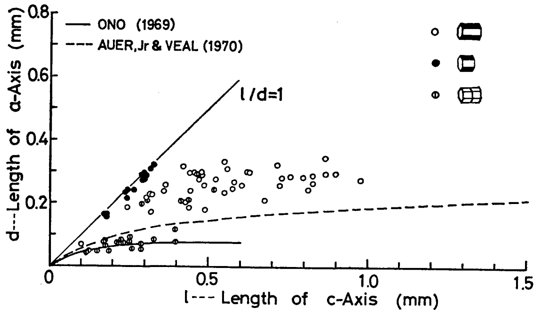

Fig. 9. Length of a-axis (d) *vs.* length of c-axis (l).

columns, hollow columns and short columns (see Fig. 3). It is shown clearly in Fig. 8 that the short column has a tendency different from other shapes. Higuchi obtained an empirical formula for mass of columns, $m=1.3\,a^2l$, in which m is the mass in milligram, a, a half of the diameter (d) and l, the length of the column in millimeter (1956). In comparison with this formula, the mass *vs.* d^2l is shown in this figure and the solid line shows the relation of this formula. It is seen that this relation is a good approximation to the solid columns and the hollow columns but it is not a good approximation to the short columns as indicated by the broken line.

The length of a-axis (d) *vs.* the length of c-axis (l) is shown in Fig. 9. The broken line in this figure shows the mean value observed by Auer and Veal (1970), whereas the solid line shows the one by Ono (1969) and the fine line shows the ratio $l/d=1$. The ratios l/d of the columnar crystals are scattered except the short columns in which the ratio l/d is about 1.

3.3. Falling velocity of plane type crystals

For a plane type crystal falling with horizontal orientation through a stagnant air, drag coefficient (C_d) and Reynolds number (R_e) may be found from the equations,

$$V(\sigma-\rho)g=\frac{1}{2}C_d\rho v^2S$$

$$R_e=\frac{vd}{\nu}$$

where V is the volume of crystal, σ the density of crystal, ρ the density of air, g the gravitational acceleration, v the falling velocity, S the cross sectional area of crystal and ν the kinematic viscosity of air. Neglecting the air density to the crystal density, $C_dR_e^2$ may be written as follows,

$$C_dR_e^2=\frac{16}{3\sqrt{3}}\frac{mg}{\nu^2\rho}$$

where m is the mass of crystal. Accordingly, we can compute the falling velocity by using the $C_dR_e^2-R_e$ diagram (for example, Kajikawa, 1971), the ratio t/d (Fig. 6) and the mass of crystal (Fig. 5).

The observed falling velocities of plane type crystals are shown in Fig. 10, the air temperature and the atmospheric pressure being corrected to -15°C and 1000 mb. The solid lines in this figure show the values computed by using the drag coefficient of circular disk determined from the model experiment. The computed values of the crystals with sectorlike branches and the crystals with broad branches are very similar to the observed values. On the plane dendritic crystals and the stellar crystals, the computed values are slightly smaller than the observed values. The reason of this discrepancy may be ascribed to the difference in the cross sectional area between the models in the experiment and those crystals. Moreover, on the thick plates and the hexagonal plates, the measured values are much more scattered in comparison with the

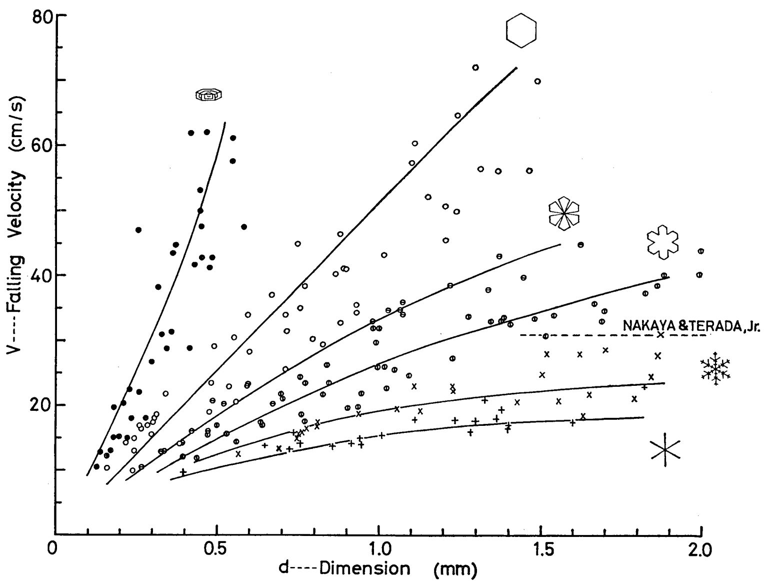

Fig. 10. Falling velocity of plane type crystals.

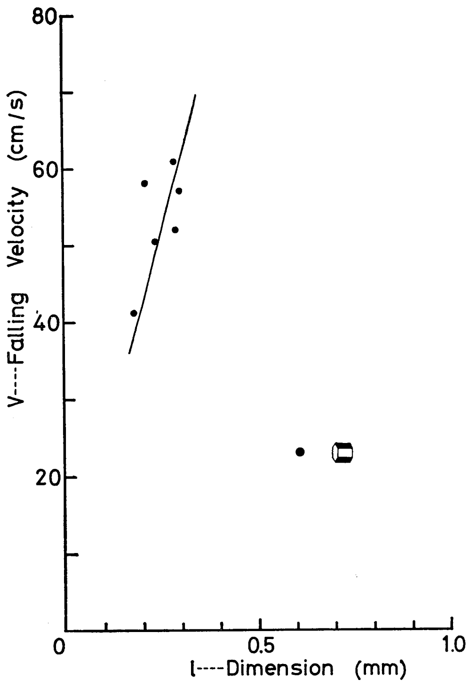

Fig. 11. Falling velocity of short columns.

computed values. In the case of thick plates, it may be considered that the estimated drag coefficients are different from the exact values because the thicknesses of those crystals are very scattered (see Fig. 6). In the case of hexagonal plates, the reason for this difference may be considered that the masses of crystals are scattered, particularly in the region of large crystals, because the crystals looked like the plate with simple extensions and the plate with sectorlike extensions are included in those region of the hexagonal plates (see Fig. 5). The influence of thickness, in the case of hexagonal plates, may be small as described in 3.1.

The broken line in this figure shows the mean value observed by Nakaya and Terada (1935) of plane dendrites and it is slightly larger than those of present observation. The reason for this difference may be attributed to the difference in the structure of the dendrites, namely that in growth of the branches on them.

3.4. Falling velocity of columnar crystals

For a columnar crystal falling with horizontal orientation through a stagnant air, C_d and R_e may be found from the equations,

$$V(\sigma-\rho)g=\frac{1}{2}C_d\rho v^2 dl$$

$$R_e=\frac{vd}{\nu}$$

where the notations are the same as in the equations for the plane type crystal. Neglecting the air density to the crystal density, $C_dR_e^2$ may be written as follows,

$$C_dR_e^2=\frac{2dmg}{\nu^2\rho l}$$

Table 1. Falling velocity of hollow column at 10°C and 1000 mb level

length of c-axis in μ	length of a-axis in μ	equivalent droplet diameter in μ	falling velocity in cm/sec	
			observed value	computed value
313	212	228	58.4	52.5
294	192	202	56.0	51.5

Accordingly, we can compute the falling velocity by using the $C_dR_e^2$-R_e diagram (for example, Kajikawa, 1971), the ratio l/d (Fig. 9) and the mass of crystal (Fig. 8).

The observed falling velocities of the short columns are shown in Fig. 11, the air temperature and the atmospheric pressure being corrected to -10°C and 1000 mb. The solid line in this figure shows the values which were computed by using the drag coefficient of the circular cylinder of the ratio $l/d=1$ obtained from the model experiment. The computed values are very close approximations to the observed values. The falling velocities of the hollow columns are shown in Table 1 and the observed values almost agree with the computed values.

4. Concluding remarks

The relationship between the falling velocity and the dimension of snow crystals was obtained for their several major types, as shown in Figs. 10 and 11. The falling velocity of snow crystals increases gradually with the increase of their dimensions and the remarkable difference of velocity is found on the different shapes at the same dimensions of crystals. The cause for those features may be attributed mainly to the difference of mass, as shown in Fig. 5.

Moreover, it was found that the relationship between the velocity and the dimension was well explained by the computation method which use the drag coefficient determined by the model experiment.

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雪結晶の落下速度の観測

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静止空気中を落下する雪結晶のストロボ写真から，その落下速度が測定された．同時に対応する結晶の大きさと質量も測定された．

板状の結晶については，大きさ 0.15 mm～1.8 mm の範囲で，厚板，角板，扇形，広幅，樹枝，星状の各結晶形に対して，落下速度と大きさとの関係が得られた．他に角柱の測定もなされた．

これらの測定値と，質量，大きさ，形をもとにして，厚い円板と有限円柱の模型実験による抵抗係数を与えて計算した値とを比較した結果，良い一致が得られた．従って，このような計算法は，板状および角柱状の雪結晶の落下速度の計算に適用できることが確かめられた．

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