MEASUREMENT OF FLOW RATE OF PARTICULATE SOLIDS IN SOLID-LIQUID MIXTURES*

MOMPEI SHIRATO, KEISHI GOTOH, KUNIHISA OSASA AND ISAO USAMI
Department of Chemical Engineering, Nagoya University, Nagoya

Although N. Brook has mentioned in his research on flow measurements of particulate solid-liquid mixtures that the flow in vertical pipes should be modified by slip between the particles and the water, the slip effect has not been taken into account.

In this paper, theoretical and experimental methods are presented for measuring the flow rate of particulate solids in solid-liquid mixtures in view of the slip velocity by using vertical connected pipes of two different diameters. It is shown that the slip velocity can be calculated by measuring the difference between the transport concentrations of solids in two pipes resulted from the difference in the particle velocities in each pipe. The velocity of liquid is measured by an electro-magnetic flow meter and the flow rate of particulate solids is calculated theoretically. Reasonable coincidence between theories and experiments are reported.

Two phase flow of solid-liquid mixtures in pipes occurs on a universal scale especially in chemical industries. Most of previous works have been done on homogeneous slurries of extremely fine particles-liquid mixtures which present no special problems other than those associated with viscosities and densities. Although most of the published works on hydraulic conveying of particulate solids are concerned with flow in horizontal and inclined pipes[4], Newitt et al.[5] have demonstrated that there are some features of vertical flow which are of theoretical and practical importance.

Small particulate solids, say, 0.1~3mm in diameter, tend to form a heterogeneous mixture with water, but they can be hydraulically conveyed at an appropriate velocity[1]. Although Brook[1] has reported on flow measurement of particulate solid-liquid mixtures using a venturi-meter of a non-standard type and a vertical counterflow meter and has mentioned that the flow in vertical pipes should be modified by slip between the particles and the water because the particles are usually denser than water, the slip effect has not been taken into account.

The authors are inclined to believe that particulate solid-liquid flow is to be measured more preferably in a vertical pipe rather than in a horizontal pipe in order to avoid sedimentation and blockage at a feasible flow velocity. In this paper, theoretical and experimental methods are presented for obtaining the slip velocity by using vertical connected pipes of two different diameters. It is shown that the slip velocity can be calculated by measuring the difference between the solid concentrations in two pipes resulted from the difference in the particle velocities in each pipe, without taking the mixtures outside from the pipes. The velocity of liquid is measured by an electro-magnetic flow meter and the flow rate of particulate solids is calculated. Reasonable coincidence between theories and experiments are reported.

Basic Concepts

Transport Concentrations in Vertical Connected Pipes of Two Different Diameters: Because particulate solids hydraulically conveyed in a vertical pipe tend to settle, they move with an average velocity u_p, which is lower than the velocity of water u. In view of the tendency of particles to settle, it is apparent that the transport concentration in a vertical pipe, C, is larger than the delivered concentration C_d, and that the velocity of water in the pipe, u, is also larger than the average velocity of the mixture, u_m.

In consideration of the slip velocity $u_s=u-u_p$ in hydraulic conveyance in a vertical pipe, the mass balances of solids and liquid can be written as

$$AC_d u_m \rho_s = ACu_p\rho_s = AC(u-u_s)\rho_s \tag{1}$$

$$A(1-C_d)u_m\rho_l = A(1-C)u\rho_l \tag{2}$$

where A represents the cross sectional area of a pipe, u_s the slip velocity, ρ_s the true density of solids, ρ_l the density of liquid, and C and C_d are the volumetric concentrations. Combining the above equations leads to

$$C = \frac{C_d}{1-(1-C_d)\dfrac{u_s}{u}} \tag{3}$$

In order to develop an expression which relates u_s to the transport concentrations in two vertical pipes of different diameters connected each other as shown in **Fig. 1**, the basic mass balance Eqs. (1) and (2) may be placed in the form

$$W_s = A_1C_1\rho_s(u_1-u_s) = A_2C_2\rho_s(u_2-u_s) \tag{4}$$

$$W_l = A_1(1-C_1)\rho_l u_1 = A_2(1-C_2)\rho_l u_2 \tag{5}$$

for Lines I and II. In Eqs. (4) and (5), u_s in each line is assumed constant under a given condition, and W_s denotes the mass flow rate of solids, W_l the mass flow rate of liquid, and the subscripts 1 and 2 refer to Lines I and II, respectively. Substituting for u_2 from Eq. (5) into Eq. (4) gives

$$u_s = \frac{m(C_1-C_2)}{(1-C_2)(mC_1-C_2)}u_1 \tag{6}$$

where m is the ratio defined by $m=A_1/A_2$. When the

* Received on December 1, 1967

values of C_1, C_2 and u_l are known, the slip velocity u_s can be calculated from Eq. (6).

Total Pressure Difference along Vertical Pipe: It is well known that the electro-magnetic flow meter is available only for measuring the velocity of liquid u_l with no relation to the flow rate of solids in a solid-liquid mixture provided the solids in the mixture are not magnetically active[2]. The transport concentrations, C_1 and C_2 in Eq. (6), can be obtained by measuring the pressure differences mentioned below, or by the methods based upon the electrical properties of a flowing mixture[7] or a γ-ray densimeter.

When a solid-liquid mixture is flowing in a vertical pipe, the pressure difference Δp_m between two points of their distance L may consist of three terms, one of them being the pressure difference Δp_h due to the density of the mixture, one the pressure difference Δp_w due to the pipe-wall friction of liquid, and the other the difference Δp_s due to the additional pressure loss consumed in conveying the particulate solids.

$$\Delta p_m = \Delta p_h + \Delta p_w + \Delta p_s \tag{7}$$

It has been pointed out that the contributions of liquid and solids to the total pressure difference are not independent, since the presence of the solids is to modify the flow pattern[5]. In this paper, however, emphasis is placed on obtaining a mathematical tool without making the analysis unduly complex, and the assumption of an additive expression may be justified as a first approximation[6]. In addition to the assumption mentioned above, the pressure loss due to the friction of solids is neglected in accordance with the previous works[5] of vertical conveyance in the flow regions of Allen (transition region) and Newton (turbulent region).

The energy E_s given to the solids which are hydraulically conveyed at an average velocity u_p $(=u-u_s)$ can be represented by

$$E_s = ALC\frac{g}{g_c}(\rho_s - \rho_l)(u - u_s) \tag{8}$$

for a vertical distance L. By reference to the above definition of Δp_s, the energy consumption E_l of liquid having the average true velocity u due to the additional pressure loss is given by

$$E_l = A\cdot\Delta p_s\cdot u \tag{9}$$

Equating E_s to E_l gives

$$\Delta p_s = LC\frac{g}{g_c}(\rho_s - \rho_l)\left(1 - \frac{u_s}{u}\right) \tag{10}$$

The remaining terms in Eq. (7) may be conventionally written as

$$\Delta p_h = LC\frac{g}{g_c}(\rho_s - \rho_l) \tag{11}$$

$$\Delta p_l = \lambda\frac{L}{D}\frac{\rho_l u^2}{2g_c} \tag{12}$$

where λ is the Fanning's friction factor and D is the diameter of the pipe. Substituting Eqs. (10), (11) and (12) into Eq. (7), one obtains

$$\Delta p_m = \lambda\frac{L}{D}\frac{\rho_l u^2}{2g_c} + LC\frac{g}{g_c}(\rho_s - \rho_l)\left(2 - \frac{u_s}{u}\right) \tag{13}$$

Applying Eq. (13) to flow in Lines I and II and rearranging yields

$$C_1 = \frac{\Delta p_{m1} - \lambda\cdot\frac{L_1}{D_1}\cdot\frac{\rho_l u_1^2}{2g_c}}{L_1\frac{g}{g_c}(\rho_s - \rho_l)\left(2 - \frac{u_s}{u_1}\right)} \tag{14}$$

and

$$C_2 = \frac{\Delta p_{m2} - \lambda\cdot\frac{L_2}{D_2}\cdot\frac{\rho_l u_2^2}{2g_c}}{L_2\frac{g}{g_c}(\rho_s - \rho_l)\left(2 - \frac{u_s}{u_2}\right)} \tag{15}$$

Eqs. (14) and (15) give the relationship between the transport concentration C and the total pressure difference Δp_m measured across the distance L. The values of Δp_{m1}, Δp_{m2}, u_1 and u_2 being known, the slip velocity u_s can be calculated by substitution of Eqs. (14) and (15) into Eq. (6). The value u_s being known, the transport concentrations C_1 and C_2 can be determined on the basis of Eqs. (14) and (15), and the flow rate of particulate solids W_s can be obtained by calculation based upon Eq. (4).

Experimental Procedure and Results

Experimental Equipment The equipment of a close circuit unit schematically shown in Fig. 1 is used for this investigation. The solids-liquid mixture is fed to a centrifugal pump (2) specially designed for hydraulic conveyance of solids from a feed tank (1) with a stirrer. From the pump, it traverses control valves (3) and an electro-magnetic flow meter (4) into measuring Lines, (5) and (6). The flow from the connected test pipes returns to the feed tank (1) via a head hopper (7).

The test line of two connected acrylic pipes is employed, one consisting of a 24.8mm diameter pipe and the other of a 17.1mm diameter pipe, the ratio m $(=A_1/A_2)$ being 2.17. The pressure measurements are performed at 1.6

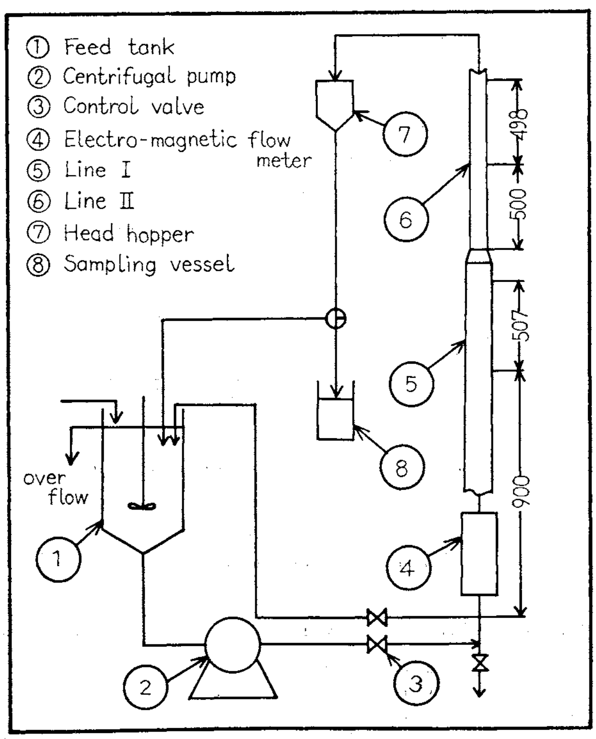

Fig. 1 Schematic view of experimental apparatus

mm in diameter holes drilled in the walls of pipes fitted with traps for solids with two differential carbon tetrachloride-water manometers. For excluding the acceleration effects, the pressure tapping in Line II is installed at a sufficient distance apart from the reducing area section of the test line.

In order to obtain the experimental values of flow rates of solids, the delivered concentrations are measured by diverting the return flow to the feed tank.

Accelerating Distance of Solids The particulate solids in Line I flow in Line II at their final equilibrium velocity u_{p1} $(=u_1-u_s)$. The acceleration distance S necessary for reaching 99.9% of the final equilibrium velocity u_{p2} $(=mu_1-u_s)$ of solids in Line II can be calculated by solving the equation of motion of a particle vertically conveyed[3]. Plots of S for various values of m are shown in **Fig. 2** as a function of u_1/u_s.

In making the experiments for this study, the ratio m is 2.17 and the ratio u_1/u_s ranges from a little more than 1 to 4. Whereas Fig. 2 indicates 30cm of the maximum distance, the sufficient distance of 50cm is used for the apparatus.

Particulate Materials for Experiments Silica sands (Soma Standard Sands) are separated into three classes by successive Tyler Stand. Sieves of 10#, 20#, 24# and 28#. Three fractions and a mixture are used for this study. The properties of the solid materials and the concentration of solids-water mixture investigated are tabulated in **Table 1**.

Experimental Results and Considerations In order to investigate the accuracy for predictions based upon Eqs. (14) and (15), the plots of transport concentrations C_1 and C_2 of Eqs. (14) and (15) based upon data obtained from pressure measurements are shown against C of Eq. (3) determined by direct measurements of delivered concentrations in **Fig. 3**.

Fig. 4 shows the actual results of conveyance and the flow rates of solids predicted by Eq. (4). Where the mean velocity of liquid is not so high compared to the slip velocity of solids, it is apparent that the conventional neglection of slip effects may lead to significant errors in

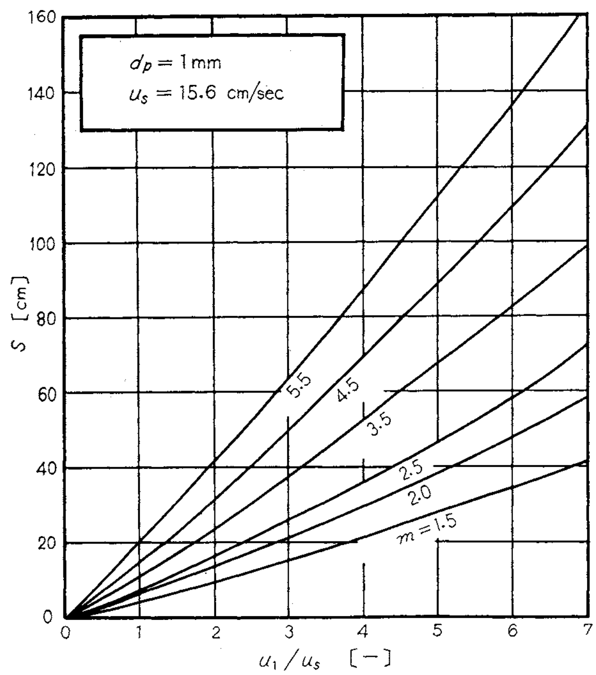

Fig. 2 Accelerating distance S vs. u_1/u_s

Table 1 Experimental meterials

Soma St. Sands: $\rho_s=2.643$ [g-mass/cm^3]

Material	Tyler screen mesh	d_p·avg	Concentration
s-1	−#10 +#16	1.28 mm	0.0101∼0.1198
s-2	−#16 +#20	0.909mm	0.0261∼0.1507
s-3	−#20 +#24	0.764mm	0.0393∼0.1605
s-4	−#10 +#16, −#20 #24	0.989mm	0.0675∼0.1602

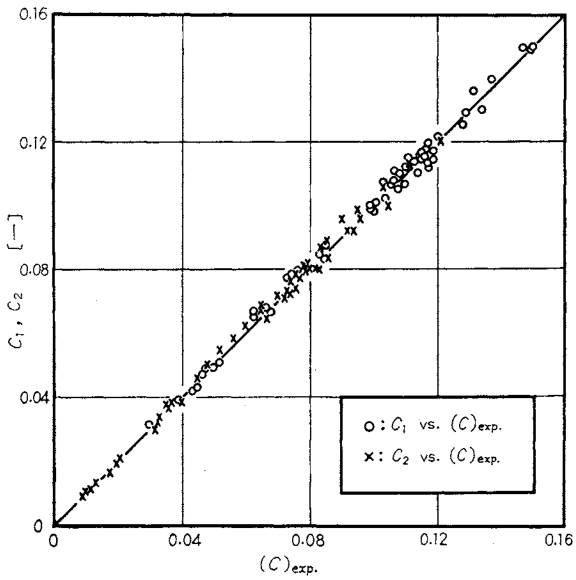

Fig. 3 Transport concentration C_1 & C_2 vs. $(C)_{exp.}$

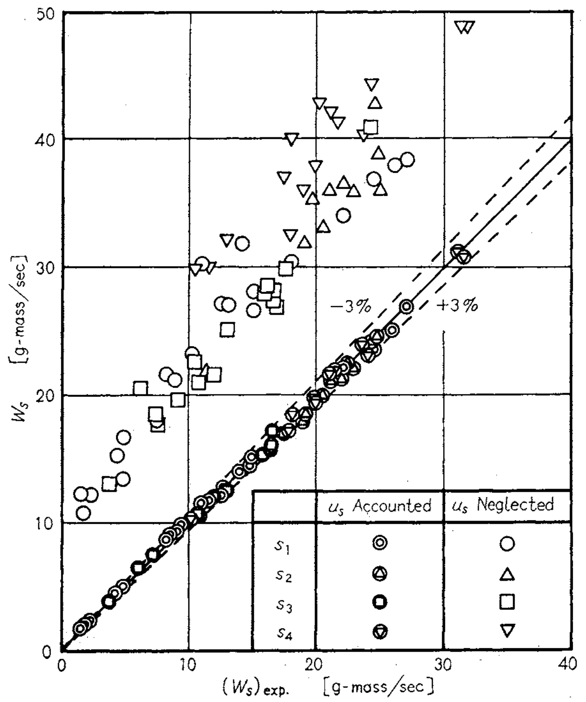

Fig. 4 Mass flow rate W_s vs. $(W_s)_{exp.}$

flow measurements. In view of the slip velocity in the connected pipes, it is possible to make accurate determinations (maximum deviation ±3%) of both the transport concentration and the flow rate of solids.

It should be noted that the accuracy of prediction on the basis of the method presented in this paper would mainly depend upon an appropriate choice of a value u_1/u_s. In **Fig. 5** the transport concentration C_2 in Line II is plotted against the ratio u_1/u_s at the transport concentration C_1 in Line I of 0.03, 0.07 and 0.11, and the ratio m of 2.5. For increasing the accuracy in manometer readings, it could be definitely concluded that the ratio u_1/u_s should be about 1~4 for m of 2.5.

Furthermore, it should be emphasized that the analytical expressions presented in this paper rest on the assumption of approximate constancy of u_s values in Lines I and II. For a single particle hydraulically conveyed up a vertical pipe at low velocities, the slip velocity u_s will be approximately equal to the terminal falling velocity. Although one possible effect of the increased turbulence at higher velocities might be to cause additional slip between the liquid and the particle, Newitt[5] demonstrated by ciné-photographing that no significant slip occurred, the values of u_s being approximately constant. The other important effect of the difference between the transport concentrations C_1 and C_2 may result in unequal values of u_s[6] in each Line. For suspensions of moderate and low concentrations attempted in this paper, however, it can be concluded[6] that no significant difference between u_s in Lines I and II may take place, the most different set of C_1 and C_2 being 0.1605 and 0.1358, respectively. For thick suspensions, the second effect mentioned above may cause a substantial difference in the values of u_s and the assumption of approximate constancy of u_s may not hold.

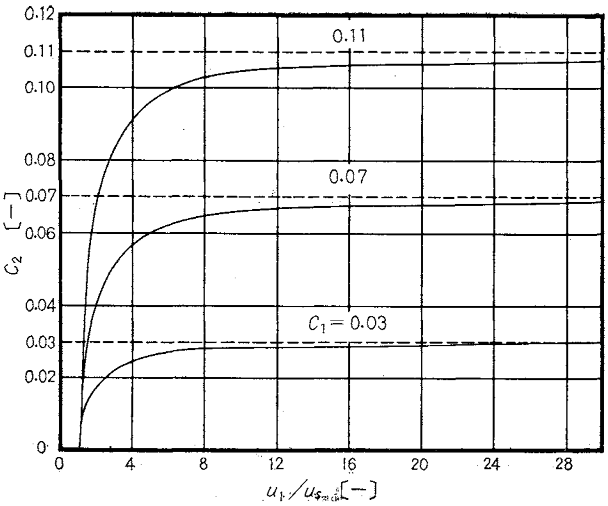

Fig. 5 C_2 vs. u_1/u_s (m=2.5)

Conclusion

In view of the slip velocity, a new method for determining the flow rates of solids hydraulically conveyed is presented. The main parts of the apparatus consist of vertical two pipes connected each other, an electro-magnetic flow meter and pressure manometers.

It has been shown that the flow rates of solids can be calculated theoretically on the basis of data obtained from pressure measurements of connected pipes. Reasonable coincidence (maximum deviation ±3%) between theories and experiments has been reported in the regions of moderate and low concentrations.

Notation

A	= cross sectional area of pipe	[cm²]
C	= transport concentration	[−]
C_d	= delivered concentration	[−]
D	= pipe diameter	[cm]
d_p	= particle diameter	[cm]
E_l	= energy consumption defined by Eq. (9)	[g-force·cm/sec]
E_s	= energy defined by Eq. (8)	[g-force·cm/sec]
g	= acceleration of gravity	[cm/sec²]
g_c	= gravitational conversion factor	[g-mass·cm/g-force·sec²]
L	= pipe length	[cm]
m	= ratio defined by $m=A_1/A_2$	[−]
Δp_h	= pressure drop due to the density of mixture	[g-force/cm²]
Δp_m	= total pressure drop	[g-force/cm²]
Δp_s	= additional pressure drop due to solids	[g-force/cm²]
Δp_w	= pressure drop due to friction of water	[g-force/cm²]
S	= acceleration distance of particle	[cm]
u	= velocity of liquid	[cm/sec]
u_p	= velocity of solids	[cm/sec]
u_m	= velocity of solids-liquid mixture	[cm/sec]
u_s	= slip velocity of solids	[cm/sec]
W_l	= flow rate of liquid	[g-mass/sec]
W_s	= flow rate of solids	[g-mass/sec]
Greek letters		
λ	= Fanning's friction factor	[−]
ρ_l	= density of liquid	[g-mass/cm³]
ρ_s	= true density of solids	[g-mass/cm³]
Subscripts		
1	= refers to Line I in test line	
2	= refers to Line II in test line	

Literature cited

1) Brook, N.: *Proc. Instn Mech. Engrs.*, **176**, 127 (1962)
2) Ito, C.: *Journal of the Central Japan Society of Automatic Control Engineers*, **5**, 35 (1962)
3) Kamei, S., R. Toei and M. Hiraoka: *Kagaku Kōgaku* (*Chem. Eng.*, Japan), **20**, 60 (1956)
4) Newitt, D. M., J. F. Richardson and M. Abbott: *Trans. Instn Chem. Engrs.*, **33**, 93 (1955)
5) Newitt, D. M., J. F. Richardson and B. J. Gliddon: *Trans. Instn Chem. Engrs.*, **39**, 93 (1961)
6) Uematsu, T., et al.: Yusō-Kōgaku Handbook, 327 (1966)
7) Worster, R. C. and D. F. Denny: *Proc. Instn Mech. Engrs.* (London), **169**, 563 (1955)