IOP Publishing — European Journal of Physics

Eur. J. Phys. **34** (2013) 1127–1134 doi:10.1088/0143-0807/34/5/1127

Measurement of fluid velocity development in laminar pipe flow using laser Doppler velocimetry

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Received 30 April 2013, in final form 5 June 2013
Published 21 June 2013
Online at stacks.iop.org/EJP/34/1127

Abstract

In this paper we present a non-intrusive experimental approach for obtaining velocity gradient profiles in a transparent smooth pipe under laminar flow conditions ($Re = 925$) using a laser Doppler velocimeter (LDV). Measurements were taken within the entrance region of the pipe at $l = 300$ mm and $l = 600$ mm from the pipe inlet, in addition to measurements of the fully developed flow at $l = 1800$ mm. The obtained results show how the velocity profile from upstream of the pipe develops into a classical laminar profile downstream, which matches the theoretical profile well. Additionally, a brief summary of historical information about the development of flow measurement techniques, in particular LDV, is provided.

(Some figures may appear in colour only in the online journal)

Introduction

Fluid mechanics is a field of study common to both physical science and engineering education. In addition to the need for strong theoretical foundations, experiments on fluid flows are necessary to complement and verify theoretical knowledge and to validate physical models. Fluid particle velocity is one of the fundamental properties that must be determined to obtain derived quantities such as flow rates, streamlines, pressure, velocity points and time correlations.

While the theory of fluid flow has developed from hydrostatics in Archimedes' day, and then the development of the hydrodynamic theories of inviscid and viscous fluids of the 17th–19th centuries, experimental methods have also been developing continuously. One of the first scientific observations of fluid flow is credited to Leonardo da Vinci, as seen in his many sketches on turbulent motion (Gharib *et al* 2002). The first instrument developed that measured fluid velocities was the Pitot tube, invented by a French civil engineer in the early part of the 18th century who wanted to measure flow rates in the river Seine in Paris

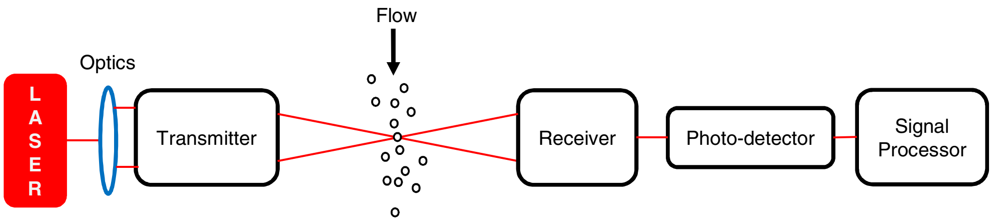


Figure 1. Basic components of an LDV system (dual beam differential mode).

(Pitot 1732). Pitot's design was later improved upon by Darcy in the middle of the 19th century and then later by Prandtl, the famous professor of Göttingen, Germany at the beginning of the 20th century. Another noteworthy technique was the hot wire anemometer developed in the first two decades of the 20th century by King (Comte-Bellot 1976, Fingerson and Freymuth 1996, King 1914). Even though the hot wire anemometer provided kinematic point measurements of flow velocity and also allowed the measurement of highly fluctuating velocity, as seen in turbulent flows, it was an intrusive instrument that required calibration and was not able to sense the direction of the flow. However, its use allowed the development and verification of many hypotheses and theories related to boundary layer and free shear layer flows and turbulence structure.

The possibility of measuring fluid velocity using a laser Doppler velocimeter (LDV, or anemometer) was first demonstrated by Yeh and Cummins (1964) in the USA in the mid-1960s. These efforts were later improved upon by scientists at the University of Minnesota, the Atomic Energy Research Establishment near Harwell, UK and Imperial College London, UK. LDV belongs to the category of optical measurement methods that use a coherent laser source to detect the velocities of a flow seeded with microparticles. It provides punctual velocity information in contrast to planar measurement techniques such as particle image velocimetry. It is used extensively in fluid mechanics research as an essential flow measurement tool. The basic principles of the LDV are based on knowledge from geometrical optics, wave theory, the scattering theory of light, the electronics of photodetectors, and signal processing methods. It thus encompasses three major fields: fluid mechanics, physics and electronics. The purpose of this paper is to present in a succinct way the underlying principles of the LDV technique and use it to demonstrate the use of a simple version of this instrument to measure the development of the velocity profile in a smooth pipe under laminar flow conditions.

Principles of LDV

LDV is a laser-based technique for measuring flow velocity based on the Doppler shift of tracer particles which scatter laser light as they flow through a gaseous or liquid medium (Drain 1980, Durst *et al* 1976). It is among the most reliable methods for flow velocity measurement and general flow characterization in fluid dynamics. One of the advantages of the LDV technique is that unlike hot wire anemometry, it is non-intrusive, provides high spatial and temporal resolution, needs no calibration and is able to resolve the flow direction.

Figure 1 illustrates the basic components of a typical dual beam differential LDV system. It consists of a monochromatic, collimated, and coherent laser source which is split into two laser beams using low-aberration optics that ensures coherence between the two beams. The split beams are then passed through a transmitter, which focuses and intersects the beams at a fixed distance relative to the transmitter. As illustrated in figure 2, this intersection, which is

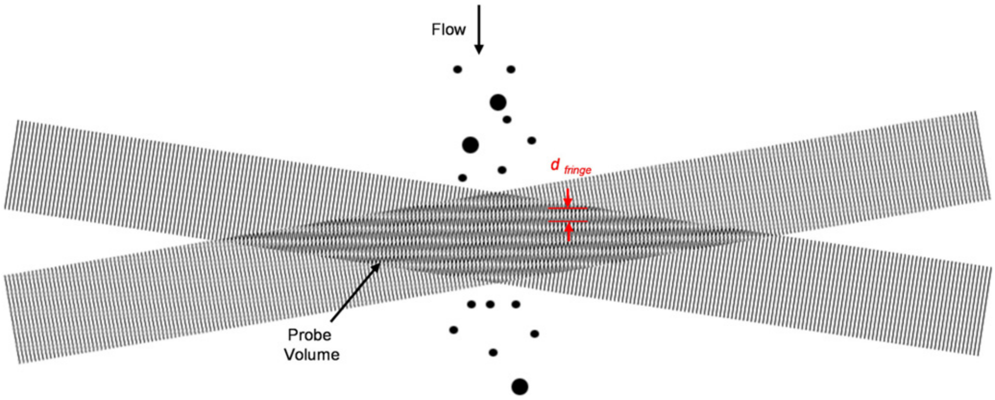
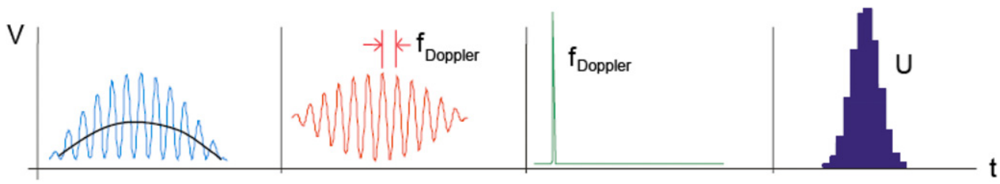


Figure 2. Tracer particles passing through the probe volume with fixed fringe spacing (dual beam differential mode).



(a) *Raw signal and pedestal* (b) *Filtered signal* (c) *FFT of the signal* (d) *Velocity distribution*

Figure 3. A typical LDV signal with corresponding signal analysis. Reproduced with permission from Measurement Science Enterprise, Inc.

referred to as the *measurement volume* or *probe volume*, has a fixed interference pattern and consists of a series of equally-spaced straight fringes (d_{fringe}).

To measure velocity, the flow must first be seeded with neutrally buoyant tracer particles that follow the fluid path. As tracer particles move through the fringes of the probe volume, they illuminate and scatter light which is collected by an optical receiver and projected on the surface of a highly sensitive photodetector. Through the photo-electric phenomenon and by means of an avalanche amplification process, the photodetector outputs a sinusoidal voltage signal representing the intensity of the scattered light. The frequency of the signal is proportional to the velocity of the particle passing normal to the fringes.

The shape of the raw sinusoidal signal (figure 3(a)) is attributed to fact that the light intensity at the edges of the probe volume is weaker than in the middle. In other words, as the tracer particles enter the probe volume they reflect light at lower intensities, and this slowly increases as they reach the middle of the probe volume. Eventually, the intensity decays as the particle exits the probe volume. As shown in figure 3(a), each raw signal consists of two parts, a low frequency component known as the *pedestal*, and a high-frequency component which contains the *Doppler signal*. Modern LDV systems are equipped with signal processors that use band pass filtering to remove the pedestal and a fast Fourier transform is performed on the signal to obtain the frequency of the Doppler signal.

The distance between the fringes is calculated using equation (1) and can be deduced from figure 4 using the basic optics theory of light:

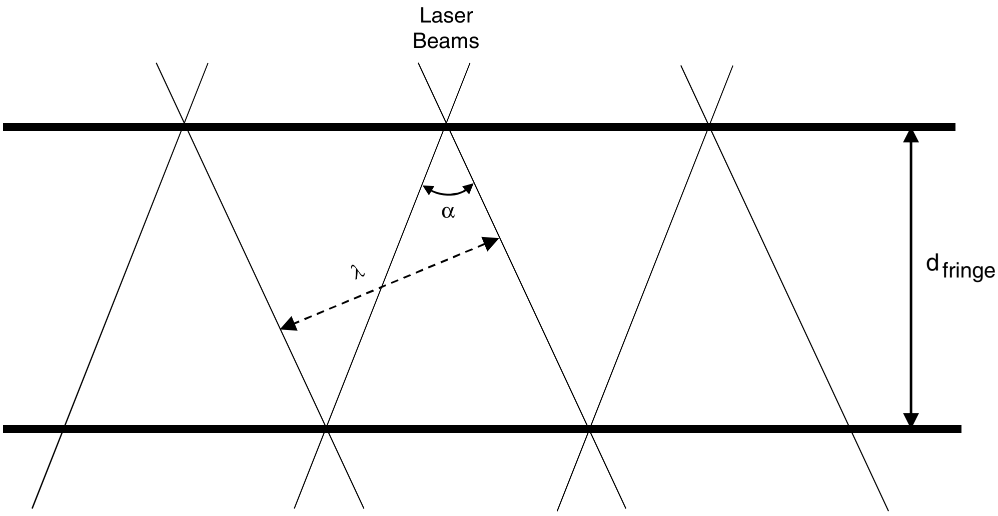


Figure 4. Fringe spacing can be deduced from the laser wave length and angle between the split laser beams.

$$d_{\text{fringe}} = \frac{\lambda}{2\sin\left(\frac{\alpha}{2}\right)}, \tag{1}$$

where λ is the wavelength of the laser light and α is the angle between the split laser beams.

Once the fringe spacing has been determined, velocity of a flow can be calculated using equation (2):

$$U = d_{\text{fringe}} \times f_{\text{Doppler}}, \tag{2}$$

where d_{fringe} is the fringe spacing and f_{Doppler} is the Doppler frequency of the sinusoidal signal.

Depending on the application, LDV systems are supplied with a variety of lasers; however, visible lasers, such as helium–neon (He–Ne), argon–ion (Ar–ion) and diode, are frequently utilized. Some LDV systems have separate transmitter and receiving units, which allows both forward scatter and back scatter measurements. In forward scatter mode, the transmitter and receiver are placed on opposite sides of the flow. This method offers strong signal output due to higher scatter in the direction of the laser beam. However, one of the disadvantages of forward scatter is the difficulty of aligning the transmitter and receiver. In back scatter mode, the transmitter and receiver are placed on the same side, and while this method outputs lower signal levels, it allows for better optical access to the flow. Some LDV systems utilize a combined transmitter and receiver unit, operating in back scatter mode and with the advantage of being permanently aligned for precise measurements.

Pipe flow velocity solution

When a viscous fluid enters a circular pipe, flow characteristics such as pressure distribution and velocity profiles change greatly along the pipe. As illustrated in figure 5, two distinct regions are defined: the *entrance region* and the *fully developed region*. Flow behavior for both regions depends on the Reynolds number, as defined in equation (3):

$$Re = \frac{U_{\text{avg}} \times D}{\nu}, \tag{3}$$

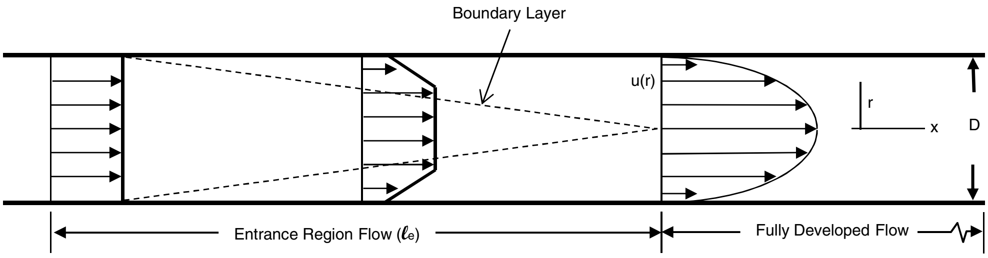


Figure 5. Entrance and fully developed laminar flow regions in a pipe.

where U_{avg} is the average velocity, D is the inner diameter of the pipe, and ν is the kinematic viscosity of the working fluid.

The entrance length (l_e) is defined as the region in which the velocity profile is in constant change due to the growing influence of viscous effects near the wall. Fluid particles near the wall adhere to the inner surface of the wall and thus a boundary layer is produced. The entrance length can be estimated using equations (4) and (5), depending on the Reynolds number and pipe inner diameter (Munson *et al* 2002):

$$l_e = 0.06 \times D \times Re \quad \text{For laminar flows } (Re < 2,000), \tag{4}$$

$$l_e = 4.4 \times D \times (Re)^{1/6} \quad \text{For turbulent flows } (Re > 2,000). \tag{5}$$

Under laminar flow conditions, when the flow reaches the end of the entrance region ($l >= l_e$), the averaged velocity profile takes on a parabolic shape which does not change along the remaining length of the pipe. The profile can be predicted using an analytical solution of steady fully developed laminar pipe flow. In this study fully developed steady laminar flow is considered at the upstream end of the inlet. The Navier–Stokes equations, combined with the continuity equation for incompressible flow are used to determine the analytical solution of the velocity profile for such a pipe flow. This solution is well described in classical fluid mechanics textbooks (Munson *et al* 2002) (equation (6)):

$$u(r) = U_{\max}\left[1 - \left(\frac{2r}{D}\right)^2\right], \tag{6}$$

where $u(r)$ is the velocity in the radial coordinate, $U_{\max}$ is the maximum velocity inside the pipe, r and D are the radius and diameter of the pipe, respectively. Equation (6) shows that the velocity profile for the steady laminar flow has a parabolic shape with a maximum velocity at the center of the pipe, $u(0) = U_{\max}$, and zero velocity near the wall $u(D/2) = 0$ due to the no-slip condition. By definition, the average velocity is one-half of the maximum velocity (equation (7)):

$$U_{avg} = \frac{U_{\max}}{2}. \tag{7}$$

Experimental set-up

To demonstrate the LDV technique to students, a commercial LDV system together with a custom-built experimental set-up constructed by the authors was utilized. A schematic of the experimental set-up is illustrated in figure 6. It consists of a smooth transparent circular acrylic pipe 3000 mm long, with an inner diameter of 24 mm and a wall thickness of 3 mm.

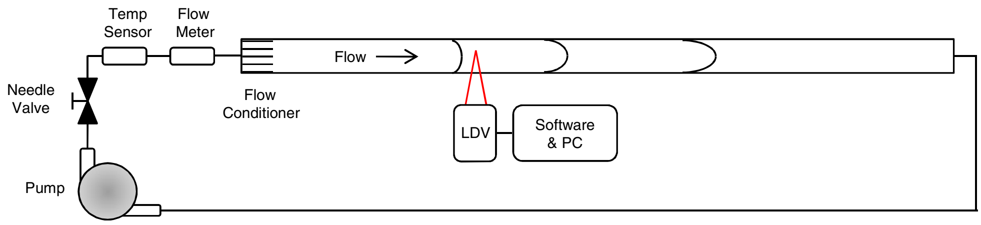


Figure 6. Schematic diagram of the experimental set-up.

Flow was conditioned in the upstream region of the pipe using a series of wire meshes and a flow straightener. A recirculating flow of water was generated using a centrifugal pump. The flow rate was controlled using a needle valve positioned downstream of the pump and measured by a paddle wheel flow meter with an accuracy of $\pm 1\%$. Water temperature was monitored using a K-type naked-bead thermocouple with an accuracy of ± 1 °C. The readings of flow rate and temperature were displayed on two electronic readout indicators.

The set-up utilized the miniLDV system manufactured by Measurement Science Enterprise, Inc. It consists of a combined transmitter/receiver probe, signal processor and Windows-based software for data acquisition and treatment. The unit was equipped with an Ar–ion laser with a wavelength of 658 nm. The probe volume size was 70×70 μm with a fringe spacing of 3.52 μm. The probe volume was factory set at a distance of 90 mm from the transmitter/receiver unit. The unit is capable of measuring flows in the range of 1 mm s^{-1} to 1000 mm s^{-1}, with a repeatability uncertainty of 0.1% and an accuracy of 99.7%. The flow was seeded with polyamide particle tracers with a diameter of 10 μm and density of 1.016 g cm^{-3}. Additionally, an electronically controlled traverse mechanism supplied with the unit was utilized to allow obtaining 1D velocity profiles at various sections of the pipe.

Results and discussion

The purpose of the experiment was to use the LDV technique to illustrate how velocity profile in the upstream of a circular pipe progresses into a classical laminar profile downstream, as well as examining how well the obtained results compare with the theoretical profile. As such, three sets of velocity profiles along the diameter of the pipe were taken: two profiles within the entrance region and one profile within the fully developed region.

Throughout the experiment flow rate fluctuated between 1 and 1.1 liters per minute (LPM), therefore a constant flow rate of $Q = 1.05$ LPM was assumed in our calculations. Assuming a constant inner diameter of $D = 24$ mm along the length of the pipe, the maximum velocity within the fully developed region was estimated to be $U_{max} = 77.4$ mm s^{-1}. Furthermore, water temperature measured $T = 20$ °C and thus a kinematic viscosity of $\nu = 1.004$ mm^2 s^{-1} was assumed. Using equation (3), the corresponding Reynolds number was estimated to be $Re = 925$, which indicates that measurements were taken under laminar flow conditions.

The corresponding entrance length of the pipe was estimated to be $l_e = 1332$ mm, based on equation (4). To study the velocity distribution at both entrance length and fully developed regions, three velocity profiles were obtained: two profiles within the entrance region of the pipe ($l = 300$ mm and $l = 600$ mm) and a third profile within the fully developed region ($l = 1800$ mm). The results are illustrated in figure 7. Each profile was taken along the diameter of the pipe by traversing the LDV on the horizontal plane at 0.5 mm steps starting

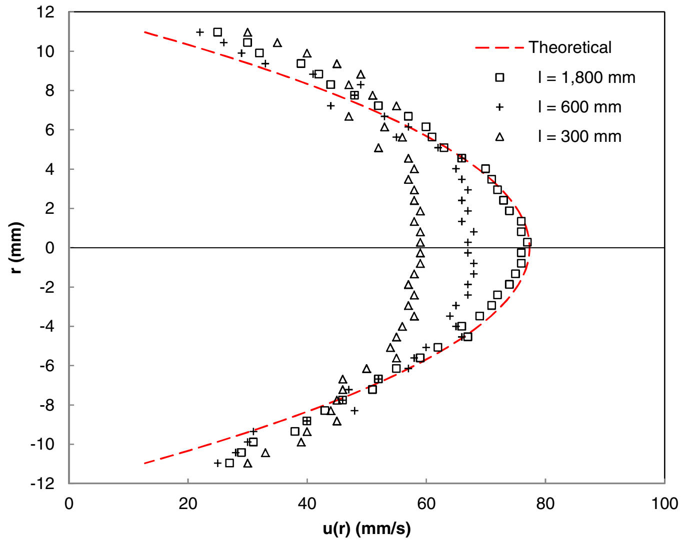


Figure 7. Experimental and theoretical results.

from 1 mm from the pipe wall and up to 1 mm from the adjacent wall. The LDV software was configured such that each collected data point was the average of 20 signal bursts collected by the photodetector. The inability of the LDV system to resolve measurements within 1 mm of the pipe wall is due to both light reflections and low velocity near the wall.

As shown in figure 7, the velocity profile at $l = 300$ mm shows a constant velocity distribution near the center of the pipe. This result shows the fact that the velocity distribution at the inlet is uniform and as flow enters the pipe, this uniform velocity distribution changes due to the presence of the wall friction. The velocity at $l = 600$ mm confirms this phenomenon where the velocity distribution tends to become more parabolic. Far from the inlet region (within the fully developed region) at $l = 1800$ mm, a parabolic shape of the velocity profile is observed. The measured maximum velocity ($U_{max} = 77$ mm s^{-1}) is within 0.5% of the theoretical maximum velocity ($U_{max} = 77.4$ mm s^{-1}). Both experimental results and analytical solution (equation (6)) are plotted in figure 7, which shows good agreement between LDV measurements and analytical solutions.

Conclusions

We presented a brief historical overview of flow measurement techniques, with a particular emphasis on the laser Doppler velocimetry technique, which measures flow velocity based on the Doppler shift of tracer particles that scatter laser light as they flow through a gaseous or liquid medium. The basic principles of laser Doppler velocimetry are based on knowledge from geometrical optics, wave theory, the scattering theory of light, the electronics of photodetectors, and signal processing methods. It thus encompasses three major fields: fluid mechanics, physics and electronics. To show the application of LDV, velocity profile measurements were obtained for a smooth transparent circular pipe under laminar flow conditions ($Re = 925$) within the entrance length and fully developed regions. The obtained results show how the velocity profile develops into a classical parabolic laminar profile as the fluid passes from the entrance region to the fully developed region inside the pipe.

Acknowledgment

The authors would like to thank Measurement Science Enterprise, Inc. for permission to reproduce figure 3.

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