# Measurement of $\pi^{0} \pi^{0}$ production in two-photon collisions 

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The reaction $e^{+} e^{-} \rightarrow e^{+} e^{-} \pi^{0} \pi^{0}$ has been analyzed using $97 \mathrm{pb}^{-1}$ of data taken with the Crystal Ball detector at the DESY $e^{-} e^{+}$storage ring DORIS II at beam energies around 5.3 GeV . For the first time we have measured the cross section for $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ for $\pi^{0} \pi^{0}$ invariant masses ranging from threshold to about 2 GeV . We measure an approximately flat cross section of about 10 nb for $W=m_{\pi^{0} \pi^{0}}<0.8 \mathrm{GeV}$, which is below 0.6 GeV , in good agreement with a theoretical prediction based on an unitarized Born-term model. At higher invariant masses we observe formation of the $f_{2}(1270)$ resonance and a hint of the $f_{0}(975)$. We deduce the following two-photon widths: $\Gamma_{\gamma \gamma}\left(f_{2}(1270)\right)=3.19 \pm 0.16 \pm_{0.28}^{0.29} \mathrm{keV}$ and $\Gamma_{\gamma \gamma}\left(f_{0}(975)\right)<0.53 \mathrm{keV}$ at $90 \%$ C.L. The decay-angular distributions show the $\pi^{0} \pi^{0}$ system to be dominantly spin 0 for $W<0.7 \mathrm{GeV}$ and spin 2 , helicity 2 in the $f_{2}(1270)$ region, with helicity 0 contributing at most $22 \%$ ( $90 \%$ C.L.).

## INTRODUCTION

The production of states with even charge conjugation can be studied in two-photon collisions as they occur at $e^{+} e^{-}$storage rings. Measuring the two-photon couplings of mesons provides information on their charged constituents and thus helps in classifying states with respect to their $q \bar{q}$ content. That is why two-photon physics plays an important role in the search for four-
quark and gluonic states-one of the key issues in hadron spectroscopy.

In the search for such states, the reaction $\gamma \gamma \rightarrow \pi \pi$ is of particular interest since $\pi \pi$ is a likely decay channel for many resonances in the mass region below 2 GeV . A variety of measurements of $\gamma \gamma \rightarrow \pi^{+} \pi^{-}$already exists, showing three $\pi^{+} \pi^{-}$invariant-mass regions with different physical features: Below 1 GeV a large $\pi^{+} \pi^{-}$ continuum and possibly the excitation of a broad scalar
resonance are observed. Between 0.9 and 1.5 GeV the cross section is dominated by the $f_{2}(1270)$ resonance. Scalar resonances such as $f_{0}(975)$ and $f_{0}(1300)$ are also expected to contribute in this region. At higher masses, again, a $\pi^{+} \pi^{-}$continuum is present. Whereas all experiments agree quite well in the $f_{2}(1270)$ region and above, they show considerable differences in the low-mass regime.
(a) DM1/2 data ( $0.3 \leq m_{\pi^{+} \pi^{-}}<0.9 \mathrm{GeV}$ ) show an excess over the Born-term expectation that is attributed to the formation of a broad scalar resonance $f_{0}(700)$ with a two-photon width of $(10 \pm 6) \mathrm{keV}$ (Ref. 1).
(b) PLUTO data ( $0.38 \leq m_{\pi^{+} \pi^{-}}<1.7 \mathrm{GeV}$ ) fall below the Born term at $\pi^{+} \pi^{-}$masses around 0.6 GeV , but are fairly well described elsewhere by a coherent sum of Born-term and $f_{2}(1270)$ amplitudes. ${ }^{2}$
(c) DELCO and TPC/2 $\gamma$ data $\left(\sim 0.5 \mathrm{GeV} \leq m_{\pi^{+}} \pi^{-}\right.$ $<2.0 \mathrm{GeV}$ ) are fairly well described over the whole mass range by the same ansatz that PLUTO used. ${ }^{3,4}$

In the low-mass region, experiments face the problem of separating $\gamma \gamma \rightarrow \pi^{+} \pi^{-}$from the kinematically similar electromagnetic processes $\gamma \gamma \rightarrow e^{+} e^{-}$and $\gamma \gamma \rightarrow \mu^{+} \mu^{-}$. In this region, contrary to the $f_{2}(1270)$ region and above, these signals are large, compared to the $\pi^{+} \pi^{-}$signal, and are difficult to subtract reliably. The analysis of the reaction $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ presented here does not suffer from those backgrounds. Furthermore, the $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ continuum is expected to be much smaller than $\gamma \gamma \rightarrow \pi^{+} \pi^{-}$at lowinvariant masses, so a resonance would be more apparent in the $\pi^{0} \pi^{0}$ channel.

The Born-term approximation for $\gamma \gamma \rightarrow \pi^{+} \pi^{-}$(see, for example, Ref. 5) treats the pions as pointlike particles that do not interact strongly, with the photons directly coupling to their charge. Hence the Born term does not contribute directly to $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$. Extensions to this simple model have been developed that embody the constraints of unitarity and analyticity together with the measured $\pi \pi$ and $K \bar{K}$ phase shifts, while respecting the QED low-energy theorem imposed by gauge invariance; see Refs. 6-8, and references therein.

Following Ref. 8, the amplitude $\mathcal{A}$ for $\gamma \gamma \rightarrow \pi^{+} \pi^{-}$can be written as

$$
\begin{equation*}
\mathcal{A}=\mathcal{B}+\mathcal{C}+\mathcal{D} \tag{1}
\end{equation*}
$$

where $\mathcal{B}$ denotes the Born-term amplitude (i.e., $\pi$ exchange in the $t$ channel), $\mathcal{C}$ denotes exchanges other than $\pi$ (e.g., $\rho$ or $\omega$ exchange), and $\mathscr{D}$ is generated by final-state interactions in which the pions scatter strongly. The last contribution makes the cross section for $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ nonzero, although the magnitude is much smaller than for $\gamma \gamma \rightarrow \pi^{+} \pi^{-}$.

The QED low-energy theorem requires that $\mathcal{A}(s, t) \rightarrow \mathcal{B}(s, t)$ as $s$ approaches the $\pi \pi$ threshold from above. It is expected that $\mathcal{B}$ controls the size of the cross section at $\gamma \gamma$ center-of-mass energies $\sqrt{s}=W<0.6 \mathrm{GeV}$. At higher $W$ values, the effect of all the other exchanges included in $\mathcal{C}$ has to be determined by experiment, pending an exact QCD calculation.

The amplitude $\mathcal{D}$ can be related by unitarity to $\pi \pi \rightarrow \pi \pi$ scattering data. Unitarity tells us that for each
spin $J$ and isospin $I$ :

$$
\begin{equation*}
\operatorname{Im} \mathcal{T}(\pi \pi \rightarrow \pi \pi)=\sum_{i} \rho_{i} \mathcal{T}(\pi \pi \rightarrow i)^{*} \mathcal{T}(i \rightarrow \pi \pi) \tag{2}
\end{equation*}
$$

where the sum extends over all possible intermediate states $i$, and the $\rho_{i}$ are phase-space factors. Below the $K \bar{K}$ threshold and where $4 \pi$ and $6 \pi$ channels can be neglected, the only contributing intermediate state is $\pi \pi$ itself. This yields, for $\gamma \gamma \rightarrow \pi \pi$ :
$\operatorname{Im} \mathcal{A}(\gamma \gamma \rightarrow \pi \pi)$

$$
\begin{equation*}
=\left(\frac{W^{2}-4 m_{\pi}^{2}}{W^{2}}\right)^{1 / 2} \mathcal{A}(\gamma \gamma \rightarrow \pi \pi)^{*} \mathcal{T}(\pi \pi \rightarrow \pi \pi) \tag{3}
\end{equation*}
$$

implying that the phase of $\mathcal{A}$ is constrained to equal the phase of the $\pi \pi \rightarrow \pi \pi$ scattering amplitude with the same quantum numbers. This is expressed by Watson's theorem. ${ }^{9}$ Thus using the measured $\pi \pi$ scattering phase shifts and neglecting $\mathcal{C}$ (i.e., exchanges other than $\pi$ exchange) allows a reliable prediction of the cross section for $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ for $W<0.6 \mathrm{GeV}$. For higher masses, $C$ can no longer be neglected and applying Watson's theorem becomes more complicated since intermediate states other than $\pi \pi$ (e.g., $K \bar{K}, 4 \pi, 6 \pi$, and $\eta \eta$ ) have to be taken into account. In addition, the Born approximation will overestimate the amplitude for the production of the initial pion pair with increasing $W$ because pions are not pointlike. Taking the pion form factor into account could lead to a solution of this problem. ${ }^{5}$

This paper presents an analysis ${ }^{10}$ of the reaction

$$
e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma^{*} \gamma^{*} \rightarrow e^{+} e^{-} \pi^{0} \pi^{0} \rightarrow e^{+} e^{-} \gamma \gamma \gamma \gamma,
$$

which has been used to measure the cross section for $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ over the $\pi^{0} \pi^{0}$ mass range from threshold to about 2 GeV . The data were collected using the Crystal Ball detector at the DESY $e^{+} e^{-}$storage ring DORIS II at an average beam energy of 5.3 GeV . They represent an integrated luminosity of $\mathcal{L}_{e e}=97 \mathrm{pb}^{-1}$ and have been taken with a special low energy threshold trigger. This trigger enables us to observe low-mass final states in which the leptons scatter at very small angles and are not observed in the detector. Therefore, it is necessary to measure the momenta of all particles in the $\pi^{0} \pi^{0}$ final state. An event's two-photon origin is ensured by a cut on the total transverse momentum with respect to the beam axis, also effectively restricting the colliding photons $\gamma^{*}$ to be nearly real ("quasireal").

## DETECTOR AND TRIGGER

The Crystal Ball ${ }^{11,12}$ is a nonmagnetic detector well suited to measure the energies and directions of electromagnetically showering particles. Its main part, the calorimeter, consists of a spherical array of $672 \mathrm{NaI}(\mathrm{Tl})$ crystals with 16 radiation lengths (r.l.) each, covering $93 \%$ of the solid angle. The crystal arrangement is based on the geometry of an icosahedron. Each of its 20 triangular faces ("major triangles") is divided into four "minor triangles," each consisting of nine individual
crystals; see Fig. 1. Each crystal is shaped like a truncated triangular pyramid pointing towards the interaction point and is viewed by a photomultiplier. Two holes, from omitting two groups of 24 crystals each, allow the beam pipe to pass through the calorimeter. The groups of 30 crystals adjacent to each of the two holes are called the "tunnel crystals." The remaining 612 crystals, covering $85 \%$ of $4 \pi$ sr, make up the "main ball." The solidangle coverage of the calorimeter is enlarged to $98 \%$ of $4 \pi$ sr by additional hexagonal $\mathrm{NaI}(\mathrm{Tl})$ end-cap crystals. In this analysis the end caps are used to reject events not fully contained in the ball.

Electromagnetic showers in the ball are measured with an energy resolution of $\sigma_{E} / E=(2.7 \pm 0.2) \% /$ $[E(\mathrm{GeV})]^{1 / 4}$. About $98 \%$ of the shower energy is distributed among a group of 13 neighboring crystals with a characteristic lateral pattern that helps in identifying photons and electrons. Using this pattern, we measure shower directions with a resolution in $\theta$, the polar angle with respect to the beam axis, of $\sigma_{\theta}=3^{\circ}-2^{\circ}$ for energies in the range $50-500 \mathrm{MeV}$. The crystals in the ball are calibrated using large-angle Bhabha-scattering events. We found it necessary to correct the calibration at lower energies. From our studies of the process $\Upsilon(2 S)$ $\rightarrow \pi^{0} \pi^{0} \Upsilon(1 S)$, we derived a one-parameter, nonlinear expression, which gives an energy correction of $+5 \%$ at 100 MeV (Ref. 13).

To detect charged particles, the central cavity of the ball is equipped with four double-layered chambers of proportional tubes that are operated with a (79-20-1)\% Ar- $\mathrm{CO}_{2}-\mathrm{CH}_{4}$ mixture. The chambers surround the beam pipe cylindrically and cover between $98 \%$ (chamber 1) and $78 \%$ (chamber 4 ) of the solid angle. Each chamber has a thickness corresponding to 0.017 r.l., the beam pipe adding another 0.017 r.l.

The trigger used in this analysis is based on fast analog sums of the total energy deposited in the main ball, each of its major triangles, each group of tunnel crystals, and


FIG. 1. Organization of the individual crystals into major and minor triangles, and into top and bottom hemispheres. The shaded area shows one group of tunnel crystals.
on signals from the chambers. Energy sums and chamber signals are subjected to pulse-height discriminators. Accepted events satisfy the following requirements.
(1) Total energy: more than 200 MeV .
(2) Topology: at least one major triangle with more than 40 MeV in each of the six hemispheres that result from dividing the ball by three different planes which contain the beam axis. ${ }^{12}$ Events with badly unbalanced transverse momentum fail this requirement.
(3) Tunnel energy: less than 30 MeV in each group of tunnel crystals.
(4) Chamber veto: no hit in chamber 3. For about $35 \%$ of the data sample, chamber 2 was also required to have no hit.

The energy values quoted above indicate $90 \%$ efficiency for individual trigger elements and $10 \%$ efficiency for the tunnel energy veto. Energy sums for the main ball and each of the major triangles always exclude tunnel crystals.

## EVENT SELECTION AND BACKGROUND SUBTRACTION

All events used in this analysis are first passed through a preliminary selection designed to identify candidates for events originating from the collision of two quasireal photons. Events are required to have a total invariant mass of less than 5 GeV and a total transverse momentum $\left|\sum \mathbf{p}_{t}\right|$ of less than 0.2 GeV . At this stage of the analysis, invariant mass and total transverse momentum are calculated by assigning a momentum vector to each crystal with magnitude equal to the energy detected in that crystal.

Out of the preselected events, candidate events for the four-photon final state have been selected by the following criteria.
(i) There must be exactly four clearly separated clusters of energy of at least 30 MeV each, which are considered as photon candidates.
(ii) All photons must be within $|\cos \theta|<0.8$, where $\theta$ is a photon's polar angle with respect to the beam axis.
(iii) The lateral energy deposition pattern of each photon must be consistent with that expected for an electromagnetically showering particle originating near the $e^{+} e^{-}$interaction point.
(iv) The total energy deposited in the ball must be less than 3.5 GeV . The total energy seen in all end-cap crystals must be less than 50 MeV . These cuts reject $e^{+} e^{-}$ annihilation events and events with additional particles close to the beam.
(v) The energy in the ball that is not associated with the four photon showers ("uncorrelated energy") must not exceed $15 \mathrm{MeV}+0.03 \sum_{i=1}^{4} E_{\text {photon }}^{(i)}$, where for this purpose the photon energy has been calculated as the energy sum in 13 crystals without any further corrections. The cut values are chosen to allow for some low-energy beam related background and to account for the energy leaking out of the group of 13 crystals.
(vi) The total transverse momentum squared $\left|\Sigma \mathbf{p}_{t}\right|^{2}$ now calculated from the momenta of the four photons has to be less than $0.008 \mathrm{GeV}^{2} \times m_{4 \gamma} / \mathrm{GeV}$. A mass-
dependent cut has been chosen to achieve a nearly constant cut efficiency over the whole $4 \gamma$ invariant-mass range. Events failing this cut have been kept to study the $\left|\Sigma \mathbf{p}_{t}\right|^{2}$ distribution.
(vii) There must be no tracks in the chambers.
(viii) Events are removed when there are chamber hits closely matched in the azimuth angle around the beam axis with the direction of a photon. The $\theta$ information is ignored for this cut because the main background comes from beam-gas and beam-wall interactions that did not necessarily originate at the $e^{+} e^{-}$interaction point. Note that the trigger had already required no hit at all in chamber 3.
(ix) The events must satisfy a software trigger filter with thresholds set to energy values where the hardware is fully efficient. These values are typically 5 to 10 MeV above (below for vetoes) the ones quoted in the trigger description.

We then group the four photons into two pairs (there are three combinations in doing so) and make scatter plots of one pair mass versus the other in various intervals of the $4 \gamma$ invariant mass (see Fig. 2). We observe strong peaks from $\pi^{0} \pi^{0}$ events in all intervals and clusters from $\pi^{0} \eta$ events in all intervals above the corresponding threshold. We see no clusters from $\eta \eta$ events. ${ }^{14}$

To select $\pi^{0} \pi^{0}$ events we require at least one combination with

$$
\begin{equation*}
\left(m_{\gamma \gamma}^{(1)}-m_{\pi^{0}}\right)^{2}+\left(m_{\gamma \gamma}^{(2)}-m_{\pi^{0}}\right)^{2}<R^{2}, \tag{4}
\end{equation*}
$$

which is a circular disk around the $\pi^{0} \pi^{0}$ point. The radius $R$ is given by $3 \sigma$ of our expected mass resolution for $\pi^{0}$ s. Using Monte Carlo events, the $\pi^{0}$ mass resolution has been determined as a function of the $4 \gamma$ invariant mass. It rises from $\sigma=7.5 \mathrm{MeV}$ at $m_{4 \gamma}=0.3 \mathrm{GeV}$ to $\sigma=12.5 \mathrm{MeV}$ at $m_{4 \gamma}=1.5 \mathrm{GeV}$. To suppress $\pi^{0} \eta$ and possible $\eta \eta$ events, we perform a $\chi^{2}$ test of the three hypotheses $\pi^{0} \pi^{0}, \pi^{0} \eta$, and $\eta \eta$, and finally classify the events according to the highest confidence level (C.L.). Note that the events are not kinematically fit to the $\pi^{0} \pi^{0}$ hypothesis. For $4 \gamma$ invariant masses below 0.5 GeV , about $15 \%$ of the $\pi^{0} \pi^{0}$ events have two combinations satisfying Eq. (4); for higher $4 \gamma$ invariant masses this fraction is negligible. For the remainder of the analysis, we consider only the most likely combination, the one with the highest C.L. Figure 3 shows the invariant-mass distribution of the selected $1574 \pi^{0} \pi^{0}$ events; it exhibits a dominant peak due to the formation of the $f_{2}(1270)$ resonance.

The $\left|\Sigma \mathbf{p}_{t}\right|^{2}$ distribution of the $\pi^{0} \pi^{0}$ events peaks at zero as expected for two-photon produced events (see below). However, for low $\pi^{0} \pi^{0}$ invariant masses we observe an excess of events with high $\left|\Sigma \mathbf{p}_{t}\right|^{2}$ values. The source of this background is most likely $\pi^{0} \pi^{0}$ production in beam-gas reactions. To study it, we use a sample of data taken with a single $e^{+}$or $e^{-}$beam, or with the $e^{+}$ and $e^{-}$beam separated. These beam-gas data correspond to an integrated luminosity of $6.0 \pm 0.6 \mathrm{pb}^{-1}$. This is the same data sample as used in Ref. 12, where a corresponding background has been observed for the twophoton production of single $\pi^{0}$ s. Applying the selection criteria described above, except for the $\left|\Sigma p_{t}\right|^{2}$ cut, re-


FIG. 2. Scatter plots of one $\gamma \gamma \gamma$ pair mass vs the other (three entries per event) for various intervals of the $4 \gamma$ invariant mass.
sults in $26 \pi^{0} \pi^{0}$ events from the beam-gas sample. Their invariant-mass distribution [Fig. 4(a)] shows that this background contributes only at low $\pi^{0} \pi^{0}$ invariant masses and becomes negligible for $m_{\pi^{0} \pi^{0}}$ above 0.9 GeV . The $\left|\Sigma p_{t}\right|^{2}$ distribution of these events [Figure 4(b)] does not peak at zero but is fairly flat.

Since only three of the $\pi^{0} \pi^{0}$ beam-gas events pass the (mass-dependent) $\left|\boldsymbol{\Sigma} \mathbf{p}_{t}\right|^{2}$ cut, it is not appropriate to use their luminosity scaled invariant-mass distribution to correct Fig. 3 for this background. Instead, we apply the


FIG. 3. Invariant-mass distribution of the selected $\pi^{0} \pi^{0}$ events. The shaded histogram shows the background as determined from fits to the $\left|\Sigma \mathbf{p}_{t}\right|^{2}$ distributions.


FIG. 4. (a) Invariant-mass distribution of $\pi^{0} \pi^{0}$ events from the beam-gas sample. The $\left|\boldsymbol{\Sigma} \mathbf{p}_{t}\right|^{2}$ cut has not been applied to these events. (b) $\left|\Sigma \mathbf{p}_{t}\right|^{2}$ distribution of $\pi^{0} \pi^{0}$ events from the beam-gas sample.
following procedure to the events from the collidingbeam sample. For the $\pi^{0} \pi^{0}$ events in each single mass bin, we fit the $\left|\Sigma \mathbf{p}_{t}\right|^{2}$ distribution to the Monte Carlo expectation ${ }^{15}$ for $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ plus a straight line describing the beam-gas background. (See Fig. 5 for some typical fits.) From the straight line we deduce the number of


FIG. 5. $\left|\Sigma \mathbf{p}_{t}\right|^{2}$ distributions for $\pi^{0} \pi^{0}$ events from the colliding-beam sample in some selected intervals of the $\pi^{0} \pi^{0}$ invariant mass. Also shown are the fits to the distributions with a sum (solid line) of the Monte Carlo expectation for $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ and a straight line (dashed line) describing the beam-gas background.
background events passing the $\left|\boldsymbol{\Sigma} \mathbf{p}_{t}\right|^{2}$ cut. The shaded histogram in Fig. 3 shows the background contribution as a function of the $\pi^{0} \pi^{0}$ invariant mass. This procedure yields $39 \pm 4$ background events for $m_{\pi^{0} \pi^{0}}<0.9 \mathrm{GeV}$. This is in agreement with the expectation of $49 \pm 28$ events we would obtain from the beam-gas sample by scaling the three observed events to the luminosity of the colliding-beam sample.

## MONTE CARLO SIMULATION AND EFFICIENCY DETERMINATION

To obtain the total cross section $\sigma_{\gamma \gamma}$ for $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$, the background-subtracted $\pi^{0} \pi^{0}$ invariant-mass distribution has to be corrected for the $W$-dependent $\gamma \gamma$ luminosity and the detection efficiency, where $W$ is the invariant mass of the $\gamma \gamma$ and $\pi^{0} \pi^{0}$ system.

For each $W$ bin, the $\gamma \gamma$ flux $\mathscr{F}_{\gamma \gamma}(W)$ (i.e., the ratio of the $\gamma \gamma$ luminosity to the $e^{+} e^{-}$luminosity) has been calculated by a numerical integration of the flux formula given in Ref. 15, taking into account only transverse photons.

To determine the detection efficiency, we have generated events of the type $e^{+} e^{-} \rightarrow e^{+} e^{-} \pi^{0} \pi^{0} \rightarrow e^{+} e^{-} \gamma \gamma \gamma \gamma$ using the same formula from Ref. 15. These events have been passed through a detailed detector-simulation program based on the shower-development code EGS 3 (Ref. 16). Some of the selection criteria, especially the cuts on end-cap energy, uncorrelated energy, and tunnel veto energy, are affected by extra energy deposited in the calorimeter by beam-related background. This extra energy was accounted for by using a sample of random background events obtained by triggering on every $10^{7}$ beam crossing, with no other condition. A different background event was superimposed on each Monte Carlo event. The Monte Carlo events were then passed through the same analysis chain as the real data, except for any cuts on chamber information.

The efficiency of the selection criteria based on chamber information varies with chamber performance. It was therefore not calculated by Monte Carlo methods but has been determined using the data itself. From data taken with triggers that had no chamber veto, we select $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ events in the $f_{2}(1270)$ region. The selection uses cuts looser than the ones described above and makes no use of the chamber information. It yields a sample of about $1900 f_{2}(1270)$ events with negligible background. This sample, divided into periods of similar chamber performance, has been used to determine an average efficiency $\epsilon_{\text {ch }}$ of $(69.4 \pm 1.3) \%$ for a $\pi^{0} \pi^{0}$ event to pass the chamber veto and the cuts on hits in the chambers. The inefficiency is due to photons converting in the beam pipe or in the chamber material and to noise hits in the chambers. Monte Carlo studies show that $\epsilon_{\mathrm{ch}}$ is almost constant over the entire $\pi^{0} \pi^{0}$ invariant-mass range. We do not correct for its slight mass dependence but include the effect in the systematic error on $\sigma_{\gamma \gamma}$.

To determine $\sigma_{\gamma \gamma}$ independent of any assumptions on the spin and helicity state of the $\pi^{0} \pi^{0}$ system, we have divided the Monte Carlo-generated events into bins of $\Delta W=0.05 \mathrm{GeV}$ and $\Delta\left|\cos \theta^{*}\right|=0.1$, where $\theta^{*}$ is the an-


FIG. 6. Detection efficiency as a function of $W$ and $\left|\cos \theta^{*}\right|$.


FIG. 7. Cross section for $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ as a function of $W$ for $\left|\cos \theta^{*}\right|<0.8$. The solid line is the result of a fit with $f_{2}(1270)$ (dashed line), $f_{0}(975)$ (dashed-dotted line), and nonresonant continuum contributions (dotted line).

## TOTAL CROSS SECTION FOR $\boldsymbol{\gamma} \boldsymbol{\gamma} \rightarrow \boldsymbol{\pi}^{\boldsymbol{0}} \boldsymbol{\pi}^{\mathbf{0}}$

The total cross section $\sigma_{\gamma \gamma}$ vs $W$ has been calculated by weighting each event with

$$
\left[\mathcal{L}_{e e} \mathcal{F}_{\gamma \gamma}(W) \epsilon_{\mathrm{MC}}\left(W,\left|\cos \theta^{*}\right|\right) \epsilon_{\mathrm{ch}}\right]^{-1}
$$

After subtracting the background fractions determined previously, we obtain the cross section for $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ as a function of $W$ for $\left|\cos \theta^{*}\right|<0.8$; see Fig. 7, and Table I. Between threshold and about $0.8 \mathrm{GeV}, \sigma_{\gamma \gamma}$ is fairly flat and about 10 nb . At higher invariant masses the $f_{2}(1270)$ resonance dominates, and we observe a hint of the $f_{0}(975)$.

The systematic error on $\sigma_{\gamma \gamma}$ is $11 \%$ for $W<0.8 \mathrm{GeV}$ and $7 \%$ for $W>0.8 \mathrm{GeV}$. It includes the following contributions, all added in quadrature: uncertainty in the $e^{+} e^{-}$integrated luminosity ( $\pm 4 \%$ ), uncertainty in the $\gamma \gamma$ flux determination and effect of neglecting a $\rho$-pole form factor ( $\pm 2 \%$ ), variation of cuts, uncertainties in the

TABLE I. Total cross section for $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ for $\left|\cos \theta^{*}\right|<0.8$. The $W$ values listed are the bin centers. The quoted errors are statistical only. A systematic error of $11 \%$ for $W<0.8 \mathrm{GeV}$ and of $7 \%$ for $W>0.8 \mathrm{GeV}$ has to be added.

| $\boldsymbol{W}(\mathrm{GeV})$ | $\sigma\left(\gamma \gamma \rightarrow \pi^{0} \pi^{0}\right)(\mathrm{nb})$ | $\boldsymbol{W}(\mathrm{GeV})$ | $\sigma\left(\gamma \gamma \rightarrow \pi^{0} \pi^{0}\right)(\mathrm{nb})$ |
| :---: | :---: | :---: | :---: |
| 0.275 | $3.8 \pm 1.1$ | 1.075 | $35.4 \pm 4.5$ |
| 0.325 | $3.8 \pm 0.9$ | 1.125 | $47.2 \pm 5.1$ |
| 0.375 | $8.2 \pm 1.6$ | 1.175 | $98.0 \pm 8.3$ |
| 0.425 | $9.5 \pm 1.8$ | 1.225 | $126.3 \pm 12.8$ |
| 0.475 | $8.7 \pm 1.5$ | 1.275 | $133.4 \pm 15.5$ |
| 0.525 | $9.7 \pm 1.7$ | 1.325 | $102.0 \pm 11.3$ |
| 0.575 | $8.6 \pm 1.7$ | 1.375 | $51.6 \pm 8.5$ |
| 0.625 | $12.1 \pm 2.9$ | 1.425 | $47.8 \pm 9.3$ |
| 0.675 | $9.6 \pm 2.0$ | 1.475 | $36.6 \pm 10.2$ |
| 0.725 | $7.5 \pm 2.7$ | 1.525 | $14.1 \pm 6.6$ |
| 0.775 | $9.1 \pm 3.9$ | 1.575 | $13.0 \pm 6.7$ |
| 0.825 | $7.3 \pm 1.4$ | 1.625 | $16.9 \pm 10.5$ |
| 0.875 | $11.9 \pm 2.1$ | 1.675 | $16.6 \pm 13.0$ |
| 0.925 | $21.7 \pm 5.4$ | 1.775 | $15.7 \pm 15.7$ |
| 0.975 | $26.7 \pm 4.7$ | 1.825 | $21.8 \pm 21.8$ |
| 1.025 | $28.3 \pm 3.4$ | 1.925 | $12.1 \pm 12.1$ |

calculation of $\epsilon_{\mathrm{MC}}$ and in the background subtraction ( $\pm 8 \%$ for $W<0.8 \mathrm{GeV}$ and $\pm 4 \%$ for $W>0.8 \mathrm{GeV}$ ), and uncertainties in the calculation of $\epsilon_{\mathrm{ch}}$ and its extrapolation to low $\pi^{0} \pi^{0}$ invariant masses ( $\pm 6 \%$ for $W<0.8 \mathrm{GeV}$ and $\pm 4 \%$ for $W>0.8 \mathrm{GeV}$ ).

The cross section we measure at low $\pi^{0} \pi^{0}$ invariant masses can be compared to the prediction from the unitarized Born-term model described in the Introduction. Using the ansatz $\mathcal{A}=\mathcal{B}+\mathcal{D}$ (i.e., neglecting $\mathcal{C}$ ) and taking the $I=0,2, J=0 \pi \pi$ scattering phase shifts to determine $\mathcal{D}$ results in the prediction ${ }^{8}$ shown by the solid line in Fig. 8. It is in remarkable agreement with the data for $W<0.6 \mathrm{GeV}$. The disagreement at higher masses can be attributed to neglecting $\mathcal{C}$. Attempts to describe the DM1/2 measurement of $\gamma \gamma \rightarrow \pi^{+} \pi^{-}$lead to a predicted cross section for $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ which, close to threshold, is as large as 100 nb (Refs. 8 and 19), in total disagreement with our data. Work is in progress ${ }^{20}$ to apply the model in Ref. 8 up to masses above the $f_{2}(1270)$ and to simultaneously fit $\gamma \gamma \rightarrow \pi^{+} \pi^{-}$and $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ data, which are related by isospin invariance.

The cross section for the formation of a scalar ( $J^{P C}=0^{++}$) or tensor ( $J^{P C}=2^{++}$) resonance $R$ in quasireal two-photon collisions and its subsequent decay into $\pi^{0} \pi^{0}$ can be described by a relativistic Breit-Wigner form:

$$
\begin{align*}
& \sigma_{\gamma \gamma \rightarrow R \rightarrow \pi^{0} \pi^{0}}(W) \\
& \quad=8 \pi(2 J+1)\left[\frac{m}{W}\right)^{2} \frac{\Gamma_{\gamma \gamma} B\left(R \rightarrow \pi^{0} \pi^{0}\right) \Gamma_{\mathrm{tot}}(W)}{\left(W^{2}-m^{2}\right)^{2}+m^{2} \Gamma_{\mathrm{tot}}^{2}(W)}, \tag{5}
\end{align*}
$$

where $J$ denotes the spin of the resonance, $m$ its mass, $\Gamma_{\text {tot }}$ its total width, and $B\left(R \rightarrow \pi^{0} \pi^{0}\right)$ its branching ratio into $\pi^{0} \pi^{0}$. This Breit-Wigner form follows from the helicity-amplitude decomposition given in Ref. 5. In case of the tensor resonance, only the helicity- 2 amplitude is assumed to contribute to the cross section. The $W$ dependence of $\Gamma_{\text {tot }}$ is parametrized according to Ref. 21:

$$
\begin{equation*}
\Gamma_{\mathrm{tot}}(W)=\Gamma_{\mathrm{tot}}(m) \frac{m}{W}\left[\frac{q^{*}}{q_{0}^{*}}\right]^{2 L+1} \frac{D_{L}\left(q_{0}^{*} r\right)}{D_{L}\left(q^{*} r\right)} \tag{6}
\end{equation*}
$$

where $q^{*}(W)$ is the $\pi^{0}$ momentum in the $\gamma \gamma$ c.m.s.,


FIG. 8. Cross section for $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ as a function of $W$ for $W<0.7 \mathrm{GeV}$ and $\left|\cos \theta^{*}\right|<0.8$. Also shown is the prediction (solid line) from the model in Ref. 8.
$q_{0}^{*}=q^{*}(W=m)$, and $L=J$ for a decay into spinless particles. The decay form factors $D_{L}$ are given by Ref. 22:

$$
\begin{equation*}
D_{0}=1 \text { and } D_{2}(W)=9+3\left(q^{*} r\right)^{2}+\left(q^{*} r\right)^{4} \tag{7}
\end{equation*}
$$

and the effective interaction radius $r$ is assumed to be 1 fm.

We fit ${ }^{23}$ the cross section to a sum of three incoherent contributions (solid line in Fig. 7).
(1) A phenomenological curve of the form $\left(W-2 m_{\pi^{0}}\right)^{a} e^{-W / b}$, with free parameters $a$ and $b$ to account for $\pi^{0} \pi^{0}$ continuum production (dotted line). Note that for $W<0.6 \mathrm{GeV}$ the shape of this curve is very similar to the unitarized Born-term model prediction ${ }^{8}$ shown in Figure 8.
(2) A relativistic spin-0 Breit-Wigner form with mass $m=0.975 \mathrm{GeV}$ and total width $\Gamma_{\text {tot }}(m)=0.033 \mathrm{GeV}$ for the $f_{0}(975)$, folded with a Gaussian mass resolution ( $\sigma=0.020 \mathrm{GeV}$ ), with $\Gamma_{\gamma \gamma}\left(f_{0}(975)\right.$ ) as a free parameter (dashed-dotted line).
(3) A relativistic spin-2 Breit-Wigner form for the $f_{2}(1270)$, folded with a Gaussian mass resolution ( $\sigma=0.028 \mathrm{GeV}$ ), with $m, \Gamma_{\text {tot }}(m)$, and $\Gamma_{\gamma \gamma}\left(f_{2}(1270)\right)$ as free parameters (dashed line).

The fit yields, for the $f_{2}(1270)$ (statistical errors only),

$$
\begin{equation*}
m=1.274 \pm 0.006 \mathrm{GeV} \tag{8}
\end{equation*}
$$

and

$$
\Gamma_{\text {tot }}(m)=0.178 \pm 0.015 \mathrm{GeV}
$$

in good agreement with the Particle Data Group (PDG) values. ${ }^{17}$ Our result also agrees with a preliminary JADE measurement ${ }^{24}$ of the reaction $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$. Neither experiment observes a downward shift of the $f_{2}(1270)$ peak as it is measured in the reaction $\gamma \gamma \rightarrow \pi^{+} \pi^{-}$(Ref. 25). A previous measurement from the Crystal Ball at the SLAC $e^{+} e^{-}$storage ring SPEAR (Ref. 26) yielded $m=1.238$ $\pm 0.014 \pm 0.025 \mathrm{GeV}$ (statistical and systematic error) and $\Gamma_{\text {tot }}(m)=0.248 \pm 0.038 \mathrm{GeV}$ (statistical error only). This mass measurement is 1.2 standard deviations (s.d.) away from the PDG value and the shift is not considered to be statistically significant.

In a separate fit the $f_{0}(975)$ mass and total width are also free parameters. This fit yields for the $f_{0}(975)$ (statistical errors only): $m=0.980 \pm 0.028 \mathrm{GeV}$ and $\Gamma_{\text {tot }}(m)=0.050 \pm 0.037 \mathrm{GeV}$, in good agreement with the PDG values. The value of $\Gamma_{\gamma \gamma}\left(f_{0}(975)\right)$ does not change significantly; however, its error increases considerably.

To determine their two-photon widths, we assume isotropy for the $f_{0}(975)$ decay and pure helicity 2 for the $f_{2}(1270)$ decay in correcting for the unmeasured part of the cross section beyond $\left|\cos \theta^{*}\right|=0.8$. The validity of these assumptions will be discussed below. Taking into account the branching ratios ${ }^{17} B\left(f_{2}(1270) \rightarrow \pi^{0} \pi^{0}\right)$ $=\frac{1}{3} \times\left(86_{-1}^{+2}\right) \%$ and $B\left(f_{0}(975) \rightarrow \pi^{0} \pi^{0}\right)=\frac{1}{3} \times(78 \pm 3) \%$, we measure

$$
\begin{align*}
& \Gamma_{\gamma \gamma}\left(f_{2}(1270)\right)=3.19 \pm 0.16 \pm_{0.28}^{0.29} \mathrm{keV} \\
& \Gamma_{\gamma \gamma}\left(f_{0}(975)\right)=0.31 \pm 0.14 \pm 0.09 \mathrm{keV} \tag{9}
\end{align*}
$$

TABLE II. Comparison of our results with other measurements of $\Gamma_{\gamma \gamma}\left(f_{2}(1270)\right)$ and $\Gamma_{\gamma \gamma}\left(f_{0}(975)\right)$. Also shown are the two-photon widths of other tensor and scalar mesons. Statistical and systematic errors have been added in quadrature.

| Particle | $\Gamma_{\gamma \gamma}(\mathrm{keV})$ | Reference |
| :--- | :--- | :---: |
| $f_{2}(1270)$ | $3.19 \pm_{0.32}^{0.33}$ | This experiment |
|  | $2.76 \pm 0.14$ | PDG (Ref. 17) |
| $a_{2}(1320)$ | $0.90 \pm 0.11$ | PDG (Ref. 17) |
|  |  |  |
| $f_{2}^{\prime}(1525)$ | $0.11 \pm 0.04$ | PDG (Ref. 17) |
|  |  |  |
| $f_{0}(975)$ | $0.31 \pm 0.17$ | This experiment |
|  | $0.24 \pm 0.16$ | Mark II (Ref. 29) |
|  | $0.19 \pm_{0.110}^{0.12}$ | Crystal Ball (Ref. 30) |
| $a_{0}(980)$ | $0.29 \pm 0.15$ | JADE (Ref. 24) |

where the errors are statistical and systematic, respectively. This result is obtained with a fixed $f_{0}(975)$ mass and total width in the fit. In addition to the systematic errors on $\sigma_{\gamma \gamma}$ quoted above, the systematic errors on the twophoton widths take into account the following contributions, all added in quadrature: uncertainties in the branching ratios into $\pi^{0} \pi^{0}$ leading to $\pm_{0.04}^{0.08} \mathrm{keV}$ for the $f_{2}$ (1270) and $\pm 0.01 \mathrm{keV}$ for the $f_{0}(975)$; uncertainty in the parametrization of the $f_{2}(1270)$ Breit-Wigner form leading to $\pm 0.15 \mathrm{keV}$ for the $f_{2}(1270)$ and $\pm 0.09 \mathrm{keV}$ for the $f_{0}(975)$. The value of the effective interaction radius $r$ determines the size of the low-mass tail of the
$f_{2}(1270)$, on which the value we obtain for $\Gamma_{\gamma \gamma}\left(f_{0}(975)\right)$ depends crucially. $r$ has been varied between 0.5 fm and 1.5 fm . The uncertainty in the parametrization of the $f_{0}(975)$ itself has a negligible effect.

The value we obtain for $\Gamma_{\gamma \gamma}\left(f_{2}(1270)\right)$ is in good agreement with the average of previous measurements (see Table II). These results, together with the measurements of $\Gamma_{\gamma \gamma}\left(a_{2}(1320)\right)$ and $\Gamma_{\gamma \gamma}\left(f_{2}^{\prime}(1525)\right)$ (see Table II), confirm close to ideal mixing and approximate nonet symmetry in the tensor-meson nonet. ${ }^{27}$

Converting our $f_{0}(975)$ measurement, which has only 2.2 s.d. statistical significance, into an upper limit ${ }^{28}$ yields

$$
\begin{equation*}
\Gamma_{\gamma \gamma}\left(f_{0}(975)\right)<0.53 \mathrm{keV} \text { at } 90 \% \text { C.L. } \tag{10}
\end{equation*}
$$

However, our central value agrees well with a preliminary Mark II measurement. ${ }^{29}$ These measurements, together with results on $\Gamma_{\gamma \gamma}\left(a_{0}(980)\right)$ (see Table II), agree well with an interpretation of both states as four-quark states (see Table III).

Our measurements assume that both the $f_{0}(975)$ and the $f_{2}(1270)$ can be described by Breit-Wigner forms, and that the underlying continuum is smooth. As has been pointed out recently, ${ }^{8}$ this fit ansatz is oversimplified since it neglects the opening of the strongly coupled $K \bar{K}$ channel and ignores the constraints from the $\pi \pi$ phase shifts. However, lacking a procedure that meets these requirements, we use the standard Breit-Wigner approach.

The data presented do not allow us to thoroughly investigate $\pi^{0} \pi^{0}$ production in the mass region well above the $f_{2}(1270)$. Since this analysis aimed for a crosssection measurement down to $\pi^{0} \pi^{0}$ threshold, it used

TABLE III. Some theoretical predictions for the two-photon widths of $f_{0}(975)$ and $a_{0}(980) . \theta_{S}$ denotes the octet-singlet mixing angle in the scalar nonet.

| Model | $\Gamma_{\gamma \gamma}\left(f_{0}(975)\right)(\mathrm{keV})$ | $\Gamma_{\gamma \gamma}\left(a_{0}(980)\right)(\mathrm{keV})$ | Reference |
| :---: | :---: | :---: | :---: |
| $q \bar{q}$ | 5.0 | 1.8 | NRQM |
| $q \bar{q}$ |  | $2.5-3.8$ | Ref. 31 |
| $q \bar{q}$ |  | $0-0.37$ | Ref. 32 |
| $q \bar{q}$ |  |  |  |
| $q \bar{q}$ | 12.8 | $\left.\sin \theta_{S}-\frac{1}{\sqrt{8}} \cos \theta_{S}\right]^{2}$ | Ref. 33 |
| $q \bar{q}$ | 3.0 | 1.1 | Ref. 34 |
| $q^{2} \bar{q}^{2}$ |  | 0.6 | Ref. 35 |
| $K \bar{K}$ molecule | $\sim 0.27$ | $\sim 0.27$ | Ref. 36 |

${ }^{\text {a }}$ In the nonrelativistic quark model (NRQM) one may relate the two-photon widths of scalar and tensor states having identical flavor content and spatial wave functions through a spin and a phase-space factor (Ref. 34):

$$
\Gamma_{\gamma \gamma}\left(q \bar{q}\left(0^{++}\right)\right)=\frac{15}{4}\left(\frac{m\left(0^{++}\right)}{m\left(2^{++}\right)}\right)^{3} \Gamma_{\gamma \gamma}\left(q \bar{q}\left(2^{++}\right)\right) .
$$

Taking the average of $\Gamma_{\gamma \gamma}\left(f_{2}(1270)\right)$ from Table II yields

$$
\Gamma_{\gamma \gamma}\left(f_{0}(975)\right)=5 \mathrm{keV}
$$

Assuming ideal octet-singlet mixing ( $\theta_{S}=35.3^{\circ}$ ) results in the relation

$$
\Gamma_{\gamma \gamma}\left(a_{0}(980)\right)=\frac{9}{25} \Gamma_{\gamma \gamma}\left(f_{0}(975)\right)=1.8 \mathrm{keV}
$$

The factor $\frac{9}{25}$ is the square of the ratio of the mean squared quark charge of the two states.
only data taken with a low energy threshold trigger (about $40 \%$ of the total data sample collected with the Crystal Ball), and the applied selection criteria have low efficiency in the high-mass regime. Yet we can exclude a dramatic rise of the cross section in the region between the $f_{2}(1270)$ and about 2 GeV .

## DIFFERENTIAL CROSS SECTION $\boldsymbol{d} \boldsymbol{\sigma} / \boldsymbol{d}\left|\boldsymbol{\operatorname { c o s }} \boldsymbol{\theta}^{\boldsymbol{*}}\right|$

The extrapolation to the full $\left|\cos \theta^{*}\right|$ range, which we used above in determining $\Gamma_{\gamma \gamma}$, results from investigating the decay angular distributions in the $\gamma \gamma$ c.m.s. Figure 9 shows the background-subtracted differential cross sections $d \sigma / d\left|\cos \theta^{*}\right|$ in various ranges of the $\pi^{0} \pi^{0}$ invariant mass; see also Table IV. (From events with $\left|\Sigma \mathbf{p}_{t}\right|^{2}>0.05 \mathrm{GeV}^{2}$, we find the $\left|\cos \theta^{*}\right|$ distribution of the background to be flat. This has been used in getting the background-subtracted differential cross sections.)

The differential cross sections can be described by a coherent sum of spherical harmonics $Y_{J \lambda}\left(\cos \theta^{*}, \phi^{*}\right)$ with complex coefficients $a_{J \lambda}$, where $Y_{J \lambda}$ is the spherical harmonic for orbital momentum $J$ and helicity $\lambda$ :

$$
\begin{align*}
\frac{d \sigma}{d\left|\cos \theta^{*}\right|}= & 2 \int_{0}^{2 \pi} d \phi^{*}\left|a_{00} Y_{00}+a_{20} Y_{20}+a_{22} Y_{22}\right|^{2} \\
= & 4 \pi\left(\left|a_{00}\right|^{2}\left|Y_{00}\right|^{2}+\left|a_{20}\right|^{2}\left|Y_{20}\right|^{2}+\left|a_{22}\right|^{2}\left|Y_{22}\right|^{2}\right. \\
& \left.\quad+\left|a_{00} \| a_{20}\right| \cos \delta Y_{00} Y_{20}\right) \tag{11}
\end{align*}
$$

Contributions with $J=1$ or $\lambda=1$ have been omitted because of the restrictions for quasireal photons as described by the Landau-Yang theorem. ${ }^{38}$ Allowing for interference between the considered contributions intro-


FIG. 9. Differential cross sections $d \sigma / d\left|\cos \theta^{*}\right|$ in various intervals of the $\pi^{0} \pi^{0}$ invariant mass. The full line is the result of a fit to the sum of $\left|Y_{00}\right|^{2}$ (dotted line), $\left|Y_{20}\right|^{2}$ (dashed-dotted line), and $\left|Y_{22}\right|^{2}$ (dashed line). Note that the fit yields pure $\left|Y_{22}\right|^{2}$ in the highest-mass interval.
duces as an additional degree of freedom the phase angle $\delta$. (There is only one phase angle since the other interference terms vanish after integrating over $\phi^{*}$.) However, it is not possible to determine simultaneously the $a_{J \lambda}$ and $\delta$ from a fit to $d \sigma / d\left|\cos \theta^{*}\right|$. Given this limitation, we follow the standard practice and fit the differential cross sections to a sum (full lines in Fig. 9) of $\left|Y_{00}\right|^{2}$ (dotted lines), $\left|\boldsymbol{Y}_{20}\right|^{2}$ (dashed-dotted lines), and $\left|\boldsymbol{Y}_{22}\right|^{2}$ (dashed lines); see also Table V.

We find the $\pi^{0} \pi^{0}$ system to be dominantly spin 0 for $W<0.7 \mathrm{GeV}$. In the $f_{2}(1270)$ region we measure almost entirely spin 2 , helicity 2 . The spin- 2 , helicity-0 fraction is small over the entire mass range. In the region around the $f_{0}(975)(0.9 \leq W<1.1 \mathrm{GeV})$, we observe a spin-0 contribution of about $40 \%$ that is consistent with the $f_{0}(975)$ contribution seen in the fit to the total cross section.

To further investigate the helicity structure of the $f_{2}(1270)$ and to look for other spin-0 states in the vicinity of the $f_{2}(1270)$, we apply the same fitting procedure to the combined distribution for $1.1 \leq W<1.5 \mathrm{GeV}$ (Fig. 10). We now find pure spin 2, helicity 2 yielding the following upper limits ( $90 \%$ C.L.):

$$
\begin{equation*}
\frac{\text { spin } 0}{\text { total }}<0.19 \text { and } \frac{\text { spin } 2, \text { helicity } 0}{\text { total }}<0.22 . \tag{12}
\end{equation*}
$$

This result confirms the helicity-2 dominance for $f_{2}$ (1270) formation in quasireal two-photon collisions as expected from theory ${ }^{32,39}$ and as already observed by other experiments (see Table VI).

Our upper limit on the spin-0 fraction in the invariant mass range $1.1 \leq W<1.5 \mathrm{GeV}$ has been used to determine upper limits on the two-photon width of the $f_{0}(1300)$ resonance. Assuming $m_{f_{0}}=1.3 \mathrm{GeV}$, we find at $90 \%$ C.L.

$$
\begin{align*}
\Gamma_{\gamma \gamma}\left(f_{0}(1300)\right) & B\left(f_{0}(1300) \rightarrow \pi^{0} \pi^{0}\right) \\
< & \left\{\begin{array}{l}
1.0 \mathrm{keV} \text { for } \Gamma_{\mathrm{tot}}\left(f_{0}(1300)\right)=0.15 \mathrm{GeV} \\
1.3 \mathrm{keV} \text { for } \Gamma_{\mathrm{tot}}\left(f_{0}(1300)\right)=0.3 \mathrm{GeV} \\
1.7 \mathrm{keV} \text { for } \Gamma_{\mathrm{tot}}\left(f_{0}(1300)\right)=0.4 \mathrm{GeV}
\end{array}\right. \tag{13}
\end{align*}
$$

These limits are comparable to a corresponding limit obtained previously by TASSO (Ref. 40) in the $\pi^{+} \pi^{-}$decay channel.

## SUMMARY

We have analyzed the reaction $e^{+} e^{-} \rightarrow e^{+} e^{-} \pi^{0} \pi^{0}$ using $97 \mathrm{pb}^{-1}$ of data taken with a special low energy threshold trigger. For the first time we have measured the cross section for $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ over the $\pi^{0} \pi^{0}$ invariantmass range from threshold to about 2 GeV . We observe an approximately flat cross section of about 10 nb for $W<0.8 \mathrm{GeV}$. For $W<0.6 \mathrm{GeV}$ our measurement is in good agreement with a theoretical prediction based on an unitarized Born-term model, but disagrees with a previous DM1/2 measurement in the isospin related $\pi^{+} \pi^{-}$ channel. At masses above 0.8 GeV the cross section is

TABLE IV. Differential cross sections $d \sigma / d\left|\cos \theta^{*}\right|$ in various $W$ ranges. The $\left|\cos \theta^{*}\right|$ values listed are the bin centers. The quoted errors are statistical only.

| $W$ range ( GeV ) | $\left\|\cos \theta^{*}\right\|$ | $d \sigma / d\left\|\cos \theta^{*}\right\|(\mathrm{nb})$ | $\left\|\cos \theta^{*}\right\|$ | $d \sigma / d\left\|\cos \theta^{*}\right\|(\mathrm{nb})$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.27-0.5 | 0.05 | $9.9 \pm 1.9$ | 0.45 | $7.3 \pm 2.0$ |
|  | 0.15 | $7.8 \pm 1.7$ | 0.55 | $11.6 \pm 2.6$ |
|  | 0.25 | $8.6 \pm 1.9$ | 0.65 | $7.0 \pm 2.4$ |
|  | 0.35 | $6.9 \pm 1.8$ | 0.75 | $8.9 \pm 3.2$ |
| 0.5-0.7 | 0.05 | $14.1 \pm 2.6$ | 0.45 | $10.9 \pm 3.0$ |
|  | 0.15 | $10.8 \pm 2.3$ | 0.55 | $13.4 \pm 4.0$ |
|  | 0.25 | $11.2 \pm 2.5$ | 0.65 | $20.5 \pm 6.3$ |
|  | 0.35 | $14.7 \pm 3.1$ | 0.75 | $4.6 \pm 4.6$ |
| 0.7-0.9 | 0.05 | $15.6 \pm 2.5$ | 0.45 | $10.1 \pm 2.7$ |
|  | 0.15 | $13.5 \pm 2.4$ | 0.55 | $9.6 \pm 3.2$ |
|  | 0.25 | $12.5 \pm 2.4$ | 0.65 | $8.2 \pm 3.8$ |
|  | 0.35 | $5.0 \pm 1.6$ | 0.75 | $15.5 \pm 10.3$ |
| 0.9-1.1 | 0.05 | $38.7 \pm 5.0$ | 0.45 | $33.5 \pm 6.3$ |
|  | 0.15 | $43.5 \pm 5.3$ | 0.55 | $21.0 \pm 5.6$ |
|  | 0.25 | $43.4 \pm 5.7$ | 0.65 | $20.9 \pm 7.9$ |
|  | 0.35 | $51.0 \pm 6.8$ | 0.75 | $27.2 \pm 15.7$ |
| 1.1-1.3 | 0.05 | $208.6 \pm 18.1$ | 0.45 | $96.8 \pm 15.3$ |
|  | 0.15 | $199.1 \pm 18.1$ | 0.55 | $99.5 \pm 17.8$ |
|  | 0.25 | $152.5 \pm 16.1$ | 0.65 | $73.9 \pm 20.7$ |
|  | 0.35 | $141.0 \pm 16.7$ | 0.75 | $39.6 \pm 30.8$ |
| 1.3-1.7 | 0.05 | $72.7 \pm 15.4$ | 0.45 | $46.2 \pm 13.3$ |
|  | 0.15 | $58.2 \pm 11.3$ | 0.55 | $42.5 \pm 13.2$ |
|  | 0.25 | $62.4 \pm 13.9$ | 0.65 | $21.1 \pm 9.9$ |
|  | 0.35 | $69.0 \pm 12.4$ |  |  |
| 1.1-1.5 | 0.05 | $161.3 \pm 12.4$ | 0.45 | $82.5 \pm 11.4$ |
|  | 0.15 | $149.4 \pm 12.2$ | 0.55 | $85.4 \pm 13.4$ |
|  | 0.25 | $115.6 \pm 10.9$ | 0.65 | $57.3 \pm 13.3$ |
|  | 0.35 | $131.3 \pm 12.9$ | 0.75 | $21.4 \pm 16.3$ |

dominated by the excitation of the $f_{2}(1270)$ resonance. We also observe a hint of $f_{0}(975)$ formation. We see no indication for a dramatic rise of the cross section in the mass region between the $f_{2}(1270)$ and about 2 GeV . From a Breit-Wigner fit we measure the $f_{2}(1270)$ mass and total width in agreement with their PDG values and deduce the following two-photon widths:

$$
\begin{aligned}
& \Gamma_{\gamma \gamma}\left(f_{2}(1270)\right)=3.19 \pm 0.16 \pm_{0.28}^{0.29} \mathrm{keV}, \\
& \Gamma_{\gamma \gamma}\left(f_{0}(975)<0.53 \mathrm{keV} \text { at } 90 \% \mathrm{C} . \mathrm{L} .\right.
\end{aligned}
$$

The $f_{2}$ (1270) measurement assumes pure helicity 2. The $f_{0}(975)$ upper limit is derived from measuring $\Gamma_{\gamma \gamma}\left(f_{0}(975)\right)=0.31 \pm 0.14 \pm 0.09 \mathrm{keV}$.

From investigating the decay angular distribution in the $\gamma \gamma$ c.m.s., we find the $\pi^{0} \pi^{0}$ system to be dominantly spin 0 for $W<0.7 \mathrm{GeV}$ and spin 2 , helicity 2 in the $f_{2}(1270)$ region ( $1.1 \leq W<1.5 \mathrm{GeV}$ ). In the latter region, we measure the following $90 \%$-C.L. upper limits:

$$
\frac{\text { spin } 0}{\text { total }}<0.19 \text { and } \frac{\text { spin } 2, \text { helicity } 0}{\text { total }}<0.22 .
$$

TABLE V. Spin- $J$, helicity $-\lambda$ fractions, $a_{J \lambda}$, as determined from the fits to the differential cross sections in various $W$ ranges. Note that the sum of all fractions is normalized to one.

| $W$ Range ( GeV ) | $\left\|a_{00}\right\|^{2}$ | $\left\|a_{20}\right\|^{2}$ | $\left\|a_{22}\right\|^{2}$ |
| :---: | :---: | :---: | :---: |
| 0.27-0.5 | $0.94 \pm_{0.25}^{0.06}$ | $0.06 \pm \pm_{0.06}^{0.19}$ | $0.0 \pm_{0.0}^{0.22}$ |
| 0.5-0.7 | $0.92 \pm \pm_{0.27}^{0.08}$ | $0.0 \pm{ }_{0.0}^{0.15}$ | $0.08 \pm_{0.08}^{0.27}$ |
| 0.7-0.9 | $0.54 \pm_{0.44}^{0.27}$ | $0.22 \pm \pm_{0.12}^{0.15}$ | $0.24 \pm \pm_{0.41}^{0.41}$ |
| 0.9-1.1 | $0.38 \pm_{0.25}^{0.21}$ | $0.0 \pm \pm_{0.0}^{0.22}$ | $0.62 \pm_{0.35}^{0.25}$ |
| 1.1-1.3 | $0.04 \pm{ }_{0.04}^{0.17}$ | $0.18 \pm_{0.13}^{0.15}$ | $0.78 \pm_{0.22}^{0.13}$ |
| 1.3-1.7 | $0.0 \pm{ }_{0.0}^{0.23}$ | $0.0 \pm_{0.0}^{0.15}$ | $1.0 \pm{ }_{0}^{0} 0.02$ |
| 1.1-1.5 | $0.0 \pm \pm_{0.0}^{0.15}$ | $0.0 \pm \pm_{0.0}^{0.18}$ | $1.0 \pm{ }_{0.27}^{0.0}$ |



FIG. 10. Differential cross section $d \sigma / d\left|\cos \theta^{*}\right|$ in the $f_{2}(1270)$ region. Note that the fit yields pure $\left|Y_{22}\right|^{2}$ in this mass region.

This confirms the helicity-2 dominance for $f_{2}(1270)$ formation in quasireal two-photon collisions as expected from theory and as already observed by other experiments.

Using our upper limit on the spin-0 fraction in the $f_{2}(1270)$ region, we deduce the following $90 \%$-C.L. upper limit:

$$
\begin{aligned}
\Gamma_{\gamma \gamma}\left(f_{0}(1300)\right) B\left(f_{0}(1300)\right. & \left.\rightarrow \pi^{0} \pi^{0}\right)<1.7 \mathrm{keV} \\
& \text { for } \Gamma_{\text {tot }}\left(f_{0}(1300)\right)<0.4 \mathrm{GeV} .
\end{aligned}
$$

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TABLE VI. Limits on the spin-2, helicity-0 contribution to $f_{2}(1270)$ formation in two-photon collisions.

| Helicity-0 fraction | Decay channel | Reference |
| :---: | :---: | :---: |
| $<0.22(90 \%$ C.L. $)$ | $\pi^{0} \pi^{0}$ | This experiment |
| $0.11 \pm_{0}^{0}$ 11 | $\pi^{0} \pi^{0}$ | Crystal Ball (Ref. 26) |
| $<0.14(90 \%$ C.L.) | $\pi^{+} \pi^{-}$ | DELCO (Ref. 3) |
| $<0.15(95 \%$ C.L.) | $\pi^{0} \pi^{0}$ | JADE (Ref. 24) |
| $<0.15(95 \%$ C.L.) | $\pi^{+} \pi^{-}$ | Mark II (Ref. 29) |

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FIG. 1. Organization of the individual crystals into major and minor triangles, and into top and bottom hemispheres. The shaded area shows one group of tunnel crystals.

