

Measurement of Pinion Type Cutter Using Coordinate Measuring Machine*

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A method for measuring a pinion type cutter using a coordinate measuring machine is proposed. In this method, the cutting edge profile of the cutter is indirectly measured as the intersecting curve between the flank of tool and the cutting face. Left and right side flanks of the cutter are considered to be the tooth surfaces of two different involute helical gears. The coordinates of many points on the tooth surface and the face are measured using a coordinate measuring machine. The tooth surface and the face are estimated from the measured data by the method of least squares. The cutting edge profile is calculated from the estimated tooth surface and the face. The cutting edge is moved helically and it forms the tooth surface of the actual imaginary gear. A pressure angle of this gear can be calculated. Pressure angle error of the pinion type cutter is that of the imaginary gear.

Key Words: Measurement, Machine Element, Gear, Tool, Pinion Type Cutter, Cutting Edge Profile, Equivalent Helical Gear, Pressure Angle, Coordinate Measuring Machine, Method of Least Squares

1. Introduction

A method for measuring an involute pinion type cutter (pinion cutter for short) is prescribed in JIS B 4356, but a method for measuring its cutting edge profile, which is most important for the cutter, is not yet. The reason is that accurate measurement of the cutting edge profile as an intersecting curve between the side flank of the cutter and the cutting face is very difficult. The method for measuring the pinion cutter in JIS utilizes that of an involute gear and it only regards the side flank of the cutter as a gear tooth surface. A dissatisfaction with the method of mea-

surement of the cutter is on this point.

The theoretical cutting edge profile of the pinion cutter becomes the intersecting curve between the tooth surface of an imaginary gear and the cutting face⁽¹⁾. The side flank of the cutter should be determined from this theoretical cutting edge profile and a side relief angle. Since the side flank is difficult to make, it is substituted for an involute helicoid which is easy to make. However, it seems that the design and manufacturing processes of such a pinion cutter with an involute helicoid are not systematized. Therefore, one of the authors has systematized the design and manufacturing processes of the pinion cutter considering that the involute helicoid is the tooth surface of a conical involute gear⁽²⁾.

When the design and manufacturing of the pinion cutter are systematized, the measurement of the pinion cutter also needs to be systematized. The design and manufacturing of the pinion cutter is complemented by this systematization, and the above-mentioned JIS method will be improved. In this paper, the possibility of the systematized measurement of the pinion cutter using a coordinate measuring machine (CMM) is made clear. In this method, the eccentricity,

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the pitch error, the pressure angle error of the cutter, and the shifting coefficient in order can be measured and the indirect measurements are made.

2. Basic Concept

The side flanks of the pinion type cutter are generally distinguished between a right tooth surface and a reverse tooth surface. In this paper, however, the side flank in the right side and that in the left side as viewed from the cutter center are called a right tooth surface and a left tooth surface, respectively. Although both the left and right tooth surfaces of the ordinary pinion cutter are involute helicoids, the pressure angle and the spiral angle of the left tooth surface are different from those of the right tooth surface, respectively. Therefore, the pinion cutter is considered to be a helical gear with the tooth surface of only one side as shown in Fig. 1 when distinguishing the left and right tooth surfaces of the cutter. These are called equivalent helical gears of the left and right tooth surfaces⁽³⁾. In order to determine the basic dimensions of the equivalent helical gears of the left and right tooth surfaces, the left and right tooth surfaces are measured separately using a CMM and

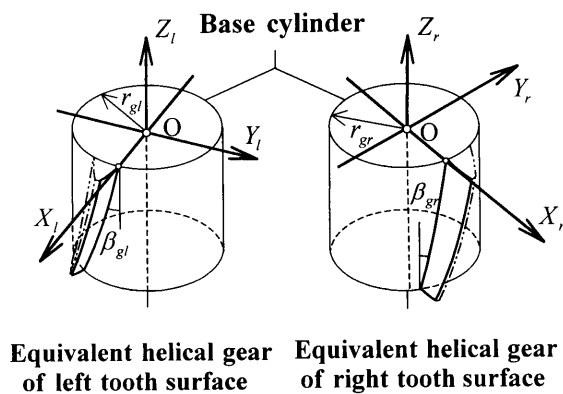


Fig. 1 Equivalent helical gears

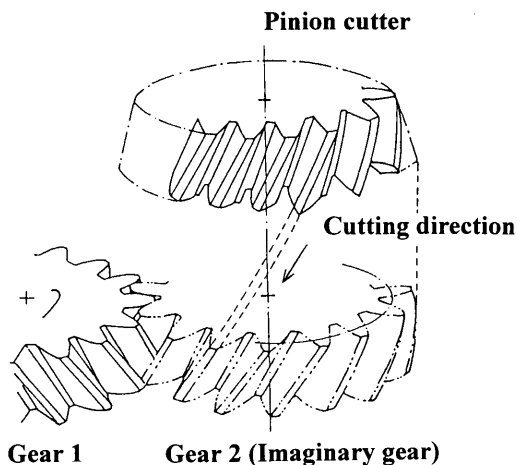


Fig. 2 Gear cutting by pinion cutter

their basic dimensions are estimated by the method presented in a previous paper⁽⁴⁾. Then, the eccentricity and the accumulative pitch error of the helical gear are detected. Also, the planar cutting face of the cutter is measured using a CMM, and the position and the posture of the cutting face are estimated. Furthermore, the cutting edge profile being the intersecting curve between the estimated cutting face and the left or right tooth surface of the equivalent helical gear is calculated. The cutting edge moves helically along a helix of the imaginary gear which the pinion cutter intends to produce (see Fig. 2) and it forms the tooth surface of the imaginary gear. The tooth profile error does not occur but the pressure angle error occurs because the tooth surface of the imaginary gear is the calculated one.

3. Basic Dimensions of Equivalent Helical Gear⁽²⁾

As shown in Fig. 2, gear 1 is the gear cut by the pinion type cutter, and gear 2 is the imaginary gear which meshes with gear 1 and is realized by the pinion cutter. The basic dimensions of this imaginary gear are given as follows:

z_0 : number of teeth (number of teeth of pinion cutter)

m_n : normal module (no error occurs)

α_n : normal pressure angle

β_0 : helix angle on pitch cylinder

Using these dimensions, the following additional dimensions are determined:

$2r_0$: pitch diameter = $m_n z_0 \sec \beta_0$

α_s : transverse pressure angle $\tan \alpha_s = \tan \alpha_n \sec \beta_0$

$2r_g$: base circle diameter = $m_n z_0 \cos \alpha_s \sec \beta_0$

β_g : helix angle on basic cylinder $\tan \beta_g = \tan \beta_0 \times \cos \alpha_s$

The basic dimensions of the equivalent helical gears of the left and right tooth surfaces are determined from these dimensions. First, the basic dimensions of the cutting edge of the pinion cutter are given as follows:

ϵ_l : side relief angle on pitch cylinder of left tooth surface

ϵ_r : side relief angle on pitch cylinder of right tooth surface

γ : rake angle in normal direction

σ : sharpening angle

Then, the helix angles of the equivalent helical gears are calculated as follows:

β_l : helix angle on pitch cylinder of left tooth surface = $\beta_0 + \epsilon_l$

β_r : helix angle on pitch cylinder of right tooth surface = $\beta_0 - \epsilon_r$

Therefore, transverse pressure angles α_{sl} and α_{sr} of the left and right tooth surfaces are represented by

$$\left. \begin{aligned} \tan \alpha_{sl} &= \frac{\tan \alpha_n}{\cos \beta_0} \\ &+ \left\{ \tan \gamma \cos \sigma - \frac{\tan \alpha_n \sin \sigma}{\cos(\beta_0 - \sigma)} \right. \\ &\quad \left. - \tan \gamma \sin \sigma \tan(\beta_0 - \sigma) \right\} (\tan \beta_l - \tan \beta_0) \\ \tan \alpha_{sr} &= \frac{\tan \alpha_n}{\cos \beta_0} \\ &+ \left\{ \tan \gamma \cos \sigma - \frac{\tan \alpha_n \sin \sigma}{\cos(\beta_0 - \sigma)} \right. \\ &\quad \left. - \tan \gamma \sin \sigma \tan(\beta_0 - \sigma) \right\} (\tan \beta_0 - \tan \beta_r) \end{aligned} \right\} (1)$$

The helix angles β_{gl} and β_{gr} on the base cylinders of the left and right tooth surfaces are represented by

$$\left. \begin{aligned} \tan \beta_{gl} &= \tan \beta_l \cos \alpha_{sl} \\ \tan \beta_{gr} &= \tan \beta_r \cos \alpha_{sr} \end{aligned} \right\} (2)$$

The diameters $2r_{gl}$ and $2r_{gr}$ on the base cylinders of the left and right tooth surfaces are represented by

$$\left. \begin{aligned} 2r_{gl} &= m_n z_0 \cos \alpha_{sl} / \cos \beta_l \\ 2r_{gr} &= m_n z_0 \cos \alpha_{sr} / \cos \beta_r \end{aligned} \right\} (3)$$

The dimensions α_{sl} , α_{sr} , β_{gl} , β_{gr} , r_{gl} , and r_{gr} of the equivalent helical gears of the left and right tooth surfaces are determined from Eqs. (1)~(3).

4. Expressions of Tooth Surfaces and Cutting Face

Since the equivalent helical gears of the left and right tooth surfaces differ from each other as mentioned above, the left and right tooth surfaces must be expressed by mathematics separately.

4.1 Right tooth surface of equivalent helical gear

The coordinate system $O-X_r Y_r Z_r$ attached to the gear is set as shown in Fig. 3. In $O-X_r Y_r Z_r$, an intersection point of the equivalent helical gear axis and the bottom surface (small end face) of the cutter is origin O . The straight line that passes through both point O and the starting point of the involute curve on the base cylinder is axis X_r , and the equivalent helical gear axis is axis Z_r (see Fig. 1). Due to the eccentricity, the axis Z_r is different from that of the clamp hole

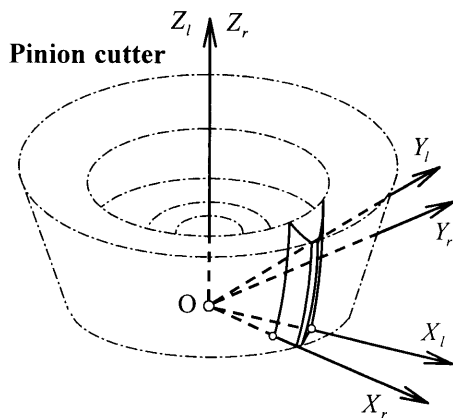


Fig. 3 Coordinate systems of left and right tooth surfaces

of the cutter. The right tooth surface of the equivalent helical gear in $O-X_r Y_r Z_r$ is expressed as a position vector $X_r = (X_r, Y_r, Z_r)^T$:

$$\left. \begin{aligned} X_r(\psi, \theta) &= r_{gr} \{ (\cos \psi + \psi \sin \psi) \cos \theta \\ &\quad - (\sin \psi - \psi \cos \psi) \sin \theta \} \\ Y_r(\psi, \theta) &= r_{gr} \{ (\cos \psi + \psi \sin \psi) \sin \theta \\ &\quad + (\sin \psi - \psi \cos \psi) \cos \theta \} \\ Z_r(\theta) &= r_{gr} \theta \cot \beta_{gr} \end{aligned} \right\} (4)$$

where ψ and θ are the parameters expressing the tooth surface and are variables⁽⁴⁾. The unit normal vector of X_r is $N_r = (N_{xr}, N_{yr}, N_{zr})^T$.

The pinion cutter is set on a CMM. Then, the tooth surface X_r is expressed in the coordinate system $O_m-X_m Y_m Z_m$ of the CMM. Figure 4 shows the relationship between the coordinate systems $O_m-X_m Y_m Z_m$ and $O-X_r Y_r Z_r$. Axis Z_m is vertical and coincides with the axis of the clamp hole of the cutter. However, the origin O is positioned at $D_0 = (X_0, Y_0, 0)^T$ in $O_m-X_m Y_m Z_m$ because of the eccentricity of the pinion cutter. X_0 and Y_0 are unknown. Since we cannot make axis X_m coincide with axis X_r , unknown angle Φ_r between these two axes is defined. Therefore, the right tooth surface X'_r of the equivalent helical gear and its unit normal N'_r in $O_m-X_m Y_m Z_m$ are represented by

$$\left. \begin{aligned} X'_r &= C(\Phi_r) X_r + D_0 \\ N'_r &= C(\Phi_r) N_r \\ C(\Phi_r) &= \begin{bmatrix} \cos \Phi_r & -\sin \Phi_r & 0 \\ \sin \Phi_r & \cos \Phi_r & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ D_0 &= (X_0, Y_0, 0)^T \end{aligned} \right\} (5)$$

4.2 Left tooth surface of equivalent helical gear

The left tooth surface of the equivalent helical gear is expressed as a position vector $X_l = (X_l, Y_l, Z_l)^T$ in the coordinate system $O-X_l Y_l Z_l$ attached to the gear supposing that the gear axis is axis Z_l :

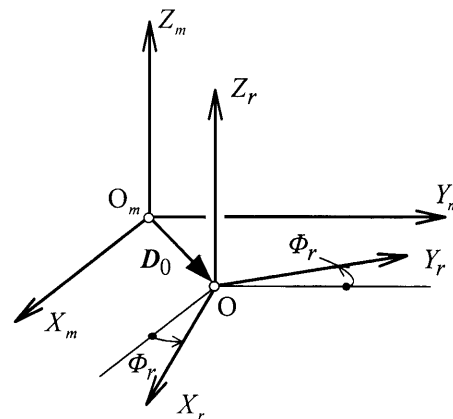


Fig. 4 Relationship between coordinate systems $O_m-X_m Y_m Z_m$ and $O-X_r Y_r Z_r$

$$\left. \begin{aligned} X_l(\psi', \theta') &= r_{gt}\{(\cos \psi' + \psi' \sin \psi') \cos \theta' \\ &\quad - (-\sin \psi' + \psi' \cos \psi') \sin \theta'\} \\ Y_l(\psi', \theta') &= r_{gt}\{(\cos \psi' + \psi' \sin \psi') \sin \theta' \\ &\quad + (-\sin \psi' + \psi' \cos \psi') \cos \theta'\} \\ Z_l(\theta') &= r_{gt} \theta' \cot \beta_{gt} \end{aligned} \right\} \quad (6)$$

where ψ' and θ' are the variable parameters expressing the tooth surface⁽⁴⁾. The unit normal vector of X_l is $N_l = (N_{xl}, N_{yl}, N_{zl})^T$.

Since the equivalent helical gear axis is a rotation axis in the generating process of the left and right tooth surfaces, the gear axis determined from the left tooth surface can be considered to be almost the same as that determined from the right tooth surface. Considering this matter, X_l and N_l are rewritten as X'_l and N'_l in $O_m-X_m Y_m Z_m$, respectively :

$$\left. \begin{aligned} X'_l &= C(\Phi_l) X_l + D_0 \\ N'_l &= C(\Phi_l) N_l \\ C(\Phi_l) &= \begin{bmatrix} \cos \Phi_l & -\sin \Phi_l & 0 \\ \sin \Phi_l & \cos \Phi_l & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ D_0 &= (X_0, Y_0, 0)^T \end{aligned} \right\} \quad (7)$$

4.3 Cutting face

The cutting face of the pinion cutter is inclined by not only sharpening angle σ about axis X but also rake angle γ about axis Y as shown in Fig. 5. The inclination directions of σ and γ cannot be specified in this study because the direction of axis X is unknown. Therefore, first, we consider plane S' which coincides with $X_r Y_r$ plane in the coordinate system $O-X_r Y_r Z_r$ shown in Fig. 3 and set up the coordinate system $O-X_s Y_s Z_s$ on the plane S' in order to express the cutting face by mathematics. Under this situation, $O-X_s Y_s Z_s$ coincides with $O-X_r Y_r Z_r$. The plane S' is expressed as a position vector $S' = (u, v, 0)^T$ supposing that u and v are the variable parameters expressing the plane. Next, after S' is rotated by angle η about axis Y_r , S' is rotated by angle ϕ about axis Z_r and is denoted by S'' as shown in Fig. 6 (a). S'' is assumed to be parallel to cutting face S . S does not generally

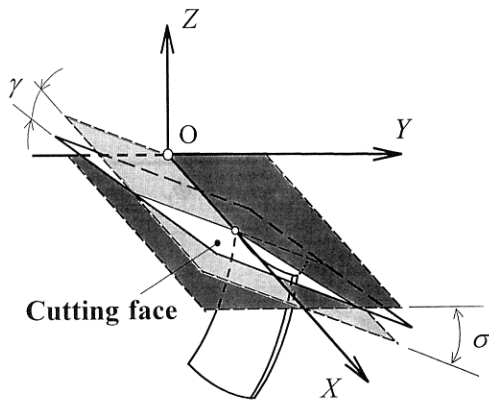
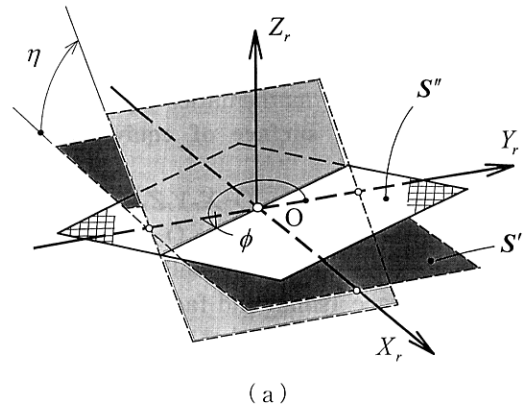


Fig. 5 Cutting face of pinion cutter

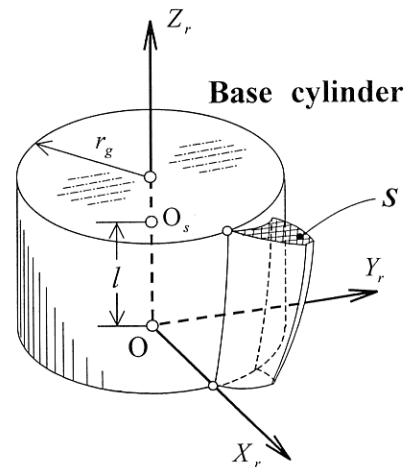
pass through point O [see Fig. 6(b)]. Therefore, an intersection point of the face S and axis Z_r is expressed as O_s , and the distance between O and O_s is expressed as l . The angles ϕ , η and the distance l are unknown at this stage and are determined from the measured data on the cutting face. The cutting face S in $O_m-X_m Y_m Z_m$ is represented by

$$\left. \begin{aligned} S(u, v) &= C(\Phi_r) \left\{ C(\phi) B(\eta) S'(u, v) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \\ &\quad + D_0 \\ B(\eta) &= \begin{bmatrix} \cos \eta & 0 & \sin \eta \\ 0 & 1 & 0 \\ -\sin \eta & 0 & \cos \eta \end{bmatrix} \\ C(\phi) &= \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \right\} \quad (8)$$

where the counterclockwise angles of ϕ and η about the axes Z_r and Y_r are positive, respectively. The unit normal vector of the cutting face S is $n = (n_x, n_y, n_z)^T$.



(a)



(b)

Fig. 6 Cutting face in the coordinate system of CMM

5. Detection Method of Errors

5.1 Eccentricity and pitch error

The eccentricity and accumulative pitch error of the pinion type cutter are detected using the method presented in a previous paper⁽⁴⁾. First, the positions of axis Z_m and origin O_m are determined by the measurement of the cramp hole and datum plane (bottom surface) of the pinion cutter. Second, using the basic dimensions of the equivalent helical gear of the right tooth surface, the eccentricity D_0 is detected from the measured coordinates of one point on every right tooth surface. Third, considering the eccentricity, the accumulative pitch error of the right tooth surface is detected from the coordinates. Finally, the accumulative pitch error of the left tooth surface is detected using the eccentricity D_0 .

5.2 Estimation of tooth surface of equivalent helical gear

After the eccentricity and the accumulate pitch error of the pinion cutter are detected, the tooth surfaces of the equivalent helical gears are estimated. An arbitrary right tooth surface is selected and the coordinates of many points (n points) on the whole tooth surface are measured avoiding the portion of the top corner radius. The relationship between the measured data M_{ri} and the tooth surface X'_r is as follows:

$$\begin{aligned} M_{ri} = & X'_r(\phi_i, \theta_i; \alpha_{sr}, \beta_{gr}, \Phi_r) \\ & + RN'_r(\phi_i, \theta_i; \alpha_{sr}, \beta_{gr}, \Phi_r) \quad i=1, 2, \dots, n \end{aligned} \quad (9)$$

where R is the radius of the spherical probe of the CMM. Equation (9) represents a system of three scalar equations. When ϕ_i and θ_i are determined from two equations in Eq. (9), and are substituted into the rest equation, the residual occurs. Therefore, transverse pressure angle α_{sr} , helix angle β_{gr} on the base cylinder of the tooth surface X'_r , and angle Φ_r are estimated so that the sum of the square of each residual can be minimized, namely, by the method of least squares⁽⁴⁾. When the estimated α_{sr} and β_{gr} are substituted into the first equation of Eq. (5), it becomes the formula of the right tooth surface in $O-X_r Y_r Z_r$. The formula of the tooth surface X'_l is also determined in the same manner as X'_r .

5.3 Estimation of cutting face

When the coordinates of many points on the whole plane corresponding to the cutting face are measured, the measured data M_{si} ($i=1, 2, \dots, n$) can be obtained. On the other hand, considering that the invariable angles, ϕ , η , and the invariable distance l which determine the rake face S in Eq. (8) are unknown and u, v are the variable parameters express-

ing the cutting face, the relationship between M_{si} and S is as follows:

$$M_{si} = S(u_i, v_i; \phi, \eta, l) + Rn(\phi, \eta) \quad (10)$$

ϕ , η , and l are determined by the method of least squares from Eq. (10). When the determined ϕ , η , and l are substituted into Eq. (8), it becomes the formula of the cutting face in $O_m-X_m Y_m Z_m$.

5.4 Pressure angle error of pinion type cutter

After the right and left tooth surfaces X'_r and X'_l of the equivalent helical gears, and a cutting face S are estimated, the cutting edge profile as the intersecting curve between X'_r and S , or between X'_l and S , is calculated. Therefore, the following equations yield:

$$\begin{cases} X'_r(\phi, \theta) - S(u, v) = 0 \\ X'_l(\phi', \theta') - S(u', v') = 0 \end{cases} \quad (11)$$

From Eq. (11), $u = u(\phi)$, $v = v(\phi)$, and $\theta = \theta(\phi)$ for the right tooth surface and $u' = u'(\phi')$, $v' = v'(\phi')$, and $\theta' = \theta'(\phi')$ for the left tooth surface are obtained. The parameters ϕ and ϕ' mean the extensive angles of the involute curves. Therefore, the profile curve of the cutting edge is calculated by varying ϕ or ϕ' . In other words, the right and left cutting edge profiles are represented by $X_{cr} \equiv (X_{cr}, Y_{cr}, Z_{cr})^T = S\{u(\phi), v(\phi)\}$ and $X_{cl} \equiv (X_{cl}, Y_{cl}, Z_{cl})^T = S\{u'(\phi'), v'(\phi')\}$, respectively. The range of ϕ is determined from $r_b < \sqrt{X_{ci}^2(\phi) + Y_{ci}^2(\phi)} < r_k$ supposing that r_b and r_k are the radii of the dedendum circle and the addendum circle of the cutter, respectively.

When the intersecting curve moves helically along a helix of the imaginary gear, a locus of the curve becomes the tooth surface of the imaginary gear. Therefore, the tooth surface of the imaginary gear is a helicoid on calculation, so that the tooth profile error which means the deviated quantity from the ideal tooth surface cannot be detected. Only the pressure angle error of the tooth surface occurs and is detected.

The right tooth surface of the imaginary gear is expressed as a position vector X_k :

$$\begin{aligned} X_k(\phi, \rho) = & C(\rho)X_{cr}(\phi) + \begin{bmatrix} 0 \\ 0 \\ h \end{bmatrix} \rho \\ h = & r_0 \cot \beta_0 \\ C(\rho) = & \begin{bmatrix} \cos \rho & -\sin \rho & 0 \\ \sin \rho & \cos \rho & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (12)$$

where ρ is the rotation angle of the cutting edge about the cutter axis Z_r and h is the reduced pitch of the helical motion. An intersection point of the cutting edge and the pitch cylinder of the cutter is expressed as P_r , and the unit normal vector of X_k at a point P_r is expressed as $n_k = (n_{kx}, n_{ky}, n_{kz})^T$. The unit vector directing from the cutter center O to the point P_r is expressed as $n_g = (n_{gx}, n_{gy}, n_{gz})^T$. n_k and n_g are

projected on the section perpendicular to the axis and are expressed as \mathbf{n}'_k and \mathbf{n}'_g , respectively (see Fig. 7). The angle between \mathbf{n}'_k and \mathbf{n}'_g is expressed as α_k and is determined from the following equations :

$$\left. \begin{aligned} \mathbf{n}'_k \cdot \mathbf{n}'_g &= |\mathbf{n}'_k| \cdot |\mathbf{n}'_g| \cos \alpha_k \\ \mathbf{n}'_k &= (n_{kx}, n_{ky}, 0)^T \\ \mathbf{n}'_g &= (n_{gx}, n_{gy}, 0)^T \end{aligned} \right\} \quad (13)$$

The complementary angle of α_k becomes the transverse pressure angle α_{sr} on the pitch circle of the imaginary gear. Comparing α_{sr} with the designed pressure angle α_s , the pressure angle error $\Delta\alpha_r$ of the right cutting edge profile is detected. The pressure angle error $\Delta\alpha_l$ of the left cutting edge profile is detected in the same manner as $\Delta\alpha_r$.

5.5 Shifting coefficient

The cutting edges produce the right and left tooth surfaces of the imaginary gear in the cutting process. The shifting coefficient x_s of the pinion cutter is determined from the circular tooth thickness on the pitch circle of the imaginary gear. The coordinates of an intersection point P_r or P_l of the right or left cutting edge and the pitch cylinder are expressed as (X_{r0}, Y_{r0}, Z_{r0}) or (X_{l0}, Y_{l0}, Z_{l0}) . Then, the point P_l is projected to plane $Z=Z_{r0}$ along a helix of helix angle β_0 of the imaginary gear. The projected point and its coordinates are expressed as P'_l and (X_d, Y_d, Z_{r0}) , respectively. The rotation angle ρ_d corresponding to X_d, Y_d , and the axial advance $(Z_{l0} - Z_{r0})$ of the helix is determined from the following equations :

$$\left. \begin{aligned} Z_{l0} - Z_{r0} &= r_0 \rho_d / \tan \beta_0 \\ X_d &= X_{l0} \cos \rho_d - Y_{l0} \sin \rho_d \\ Y_d &= X_{l0} \sin \rho_d + Y_{l0} \cos \rho_d \end{aligned} \right\} \quad (14)$$

The circular tooth thickness S_{0s} between two points P'_l and P_r is calculated from the following equations :

$$\left. \begin{aligned} S_{0s} &= 2r_0 \sin^{-1}(L/2r_0) \\ L &= \sqrt{(X_d - X_{r0})^2 + (Y_d - Y_{r0})^2} \end{aligned} \right\} \quad (15)$$

On the other hand, the circular tooth thickness of the imaginary gear whose shifting coefficient is x_s is as follows⁽⁵⁾ :

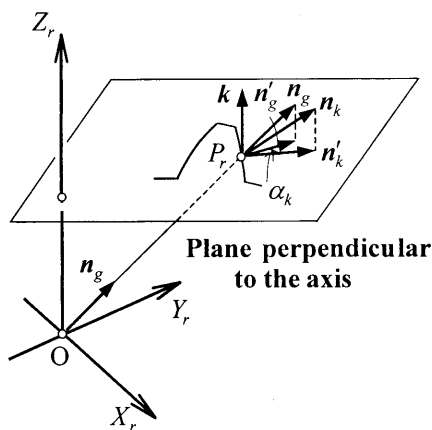


Fig. 7 Relationship between normals of the cutter

$$S_{0s} = (\pi/2 + 2x_s \tan \alpha_s) m_s \quad (16)$$

The shifting coefficient x_s of the pinion cutter is determined from Eqs. (15) and (16).

6. Measurement of Pinion Type Cutter

The basic dimensions of the measured pinion type cutter are shown in Table 1. The basic dimensions of the equivalent helical gears are shown in Table 2. The spherical probe radius of $R=0.997$ mm was used in the measurement. The pinion cutter was set on the CMM once and again changing the date and time, and was measured a multitude time. The results of measurement in three times are shown in Table 3.

6.1 Eccentricity and accumulative pitch error

The coordinates of one point on each right tooth surface of the pinion cutter were measured at random. Then, the eccentricity and the accumulative pitch error of the cutter were detected from the measured data.

6.2 Estimation of tooth surface of equivalent helical gear

The coordinates of twenty-five points on the right or left tooth surface corresponding to an arbitrary cutting edge were measured avoiding the portion of the top corner radius. Then, the basic dimensions of the equivalent helical gear were estimated.

6.3 Estimation of cutting face

After estimating the tooth surfaces of the equivalent helical gears, the coordinates of twenty-five points on the cutting face were measured at random and the cutting face was estimated.

6.4 Pressure angle error of pinion type cutter

After estimating the tooth surfaces of the equivalent helical gears and the cutting face, the pressure angles were calculated and their errors $\Delta\alpha_l$ and $\Delta\alpha_r$ of the pinion cutter were detected.

Table 1 Basic dimensions of pinion cutter

Module	m_n	3 mm
Number of teeth	z_0	40
Pressure angle	α_n	20°
Helix angle	β_0	29° 56' 38"
Side relief angle	ϵ_l, ϵ_r	2°
Sharpening angle	σ	29° 57'
Rake angle	γ	5°

Table 2 Basic dimensions of equivalent helical gears

		Left	Right
Module	m_n	3 mm	3 mm
Number of teeth	z_0	40	40
Pressure angle	α_{sl}, α_{sr}	23° 22' 37"	22° 32' 56"
Basic helix angle	β_{gl}, β_{gr}	29° 47' 13"	26° 6' 1"
Base circle dia.	$2r_{gl}, 2r_{gr}$	127.12 mm	127.90 mm

Table 3 Results of measurement

Number of measurement	1		2		3	
	Results	Δt	Results	Δt	Results	Δt
Eccentricity	3.3 μm		4.1 μm		4.4 μm	
Accumulative pitch error	7.4 μm	1.6 μm	8.6 μm	2.1 μm	10.6 μm	2.0 μm
Pressure angle error of left	$\Delta\alpha_{s,l}$ 0.4 min.	1.3 μm	1.2 min.	1.2 μm	1.6 min.	1.4 μm
and right tooth surface	$\Delta\alpha_{s,r}$ -0.2 min.	1.5 μm	-0.2 min.	1.1 μm	-0.3 min.	1.1 μm
Helix angle error of left	$\Delta\beta_{g,l}$ 0.1 min.	1.3 μm	0.2 min.	1.2 μm	0.3 min.	1.4 μm
and right tooth surface	$\Delta\beta_{g,r}$ 0.9 min.	1.5 μm	1.3 min.	1.1 μm	1.7 min.	1.1 μm
Position and posture of cutting face	ϕ -1.14 rad.		-1.14 rad.		-1.14 rad.	
	η 0.53 rad.	1.0 μm	0.53 rad.	1.1 μm	0.53 rad.	1.1 μm
	l 16.3 mm		16.3 mm		16.3 mm	
Pressure angle error	$\Delta\alpha_l$ 1.1 min.	—	1.2 min.	—	1.1 min.	—
	$\Delta\alpha_r$ -4.3 min.		-4.4 min.		-4.4 min.	
Shifting coefficient of profile x_s	0.004	—	0.004	—	0.003	—

6.5 Shifting coefficient

The shifting coefficient x_s was detected from the coordinates of each one point on the estimated right and left cutting edges.

The experimental measurements of the pinion cutter were carried out. The results of the measurements are shown in Table 3 in order of measurement. The theoretical tooth surface is estimated by the method of least squares so that the surface can fit the measured data best in this study. However, the theoretical tooth surface cannot completely fit each data. Therefore, each data has a small residual. The average of the absolute value of each residual is expressed as Δt (standard deviation) in Table 3 and is called accuracy of fit. From Table 3, Δt of each measurement item is less than 2 μm and the repeatability of the measurements is well. Therefore, it is possible to measure a pinion cutter using a CMM.

7. Conclusions

A method for measuring a pinion type cutter using a coordinate measurement machine was proposed.

Since the side flanks of the involute pinion cutter are considered to be the tooth surfaces of a helical gear, the method of measurement of a helical gear, which the authors has proposed, can be applied to the pinion cutter. In this paper, this application was performed and the pinion cutter was measured as follows. First, the eccentricity and the accumulative pitch error were detected. Second, the left and right tooth surfaces (side flanks of cutting edges) were

estimated. Third, the planar cutting face of the pinion cutter was estimated. Fourth, the curved lines of the cutting edges were calculated as the intersecting curve between this cutting face and the side flank. Fifth, the tooth surfaces of the imaginary gears, which were formed by a helical motion of the cutting edges, were expressed by mathematics. Finally, the pressure angles and the shifting coefficient were calculated and the pressure angle errors were detected. In this method, the indirect measurement of the pinion cutter is made.

The possibility of the measurement of the pinion cutter using a coordinate measuring machine was confirmed by the experimental measurements.

References

- (1) JIS B 1701-1985, (in Japanese).
- (2) Mitome, K., Conical Involute Gear, (2nd, Design and Production System of Involute Pinion Type Cutter), Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 48, No. 430, C (1982), pp. 860-867.
- (3) Mitome, K., Goto, T. and Ueda, T., Tooth Surface Measurement of Conical Involute Gear by CNC Gear-Measuring Machine, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 62, No. 595, C (1996), pp. 1087-1092.
- (4) Tamura, H., Liu, Z. and Kawasaki, K., Measurement of Helical Gear Using Coordinate Measuring Machine, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 63, No. 606, C (1997), pp. 572-578.
- (5) Ishikawa, J., Machine Element, (in Japanese), Vol. 2 (Revised Edition), (1976), p. 175, Corona Publishing Co., Ltd.