Measurement of Polarization Mode Dispersion in Systems Having Polarization Dependent Loss or Gain

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***Abstract*—The principal states of polarization and their propagation characteristics are analyzed for both unitary and nonunitary optical systems in terms of the complex plane representation of polarization. A new method for the estimation of the polarization mode dispersion of the system is proposed together with experimental and simulation results.**

***Index Terms*— Polarization dependent gain, polarization dependent loss, polarization mode dispersion, principal states of polarization.**

I. Introduction

THE PRINCIPAL states of polarization (PSP's) and their differential group delay (DGD) have been extensively used to describe and measure polarization mode dispersion (PMD) in single-mode fibers [1]–[3]. In the pioneering work of Poole and Wagner [4] and in many subsequent studies, the optical system was assumed to have no polarization dependent loss or gain (PDL/G). Such a system is characterized by a unitary Jones transmission matrix that leads to mutually orthogonal PSP's. The generalization of the concept of PSP's to the nonunitary case has been previously discussed in terms of the Jones matrix formulation [5], which forms a common way for the measurement of PMD [6]. Besides the need to measure the state of polarization (SOP) at the output of the optical system the utilization of this method also requires a complete knowledge of the input SOP. In another measurement technique, the Poincare sphere method [1], no knowledge of the input SOP is needed. This method is based on the fact that the first-order frequency dependence of the SOP at the output of the optical system, in the Poincare sphere representation, takes the form of a rotation around a diameter connecting the two PSP's with the DGD as the angular velocity. As shown in [7] the motion of the SOP vector in systems that have nonzero PDL/G is not a pure rotation, therefore, the above description and consequently, the Poincare sphere method of [1], are no longer valid for such systems. In this letter, we propose the use of the complex plane representation of polarization and derive a new differential equation, which governs the frequency dependence of the SOP at the output of a general optical system, not necessarily unitary. This approach forms the basis for a novel measurement technique for the PSP's and PMD of systems with PDL/G. Tested in both an experiment and some simulations, the technique reproduces the results of the Jones matrix method but without the need for precise knowledge of the input SOP's.

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II. Theory

In a linear medium, an input optical field $\vec{E}_i$ with a given polarization, will produce an output field which depends on the optical frequency through $\vec{E}_o(\omega) = \boldsymbol{T}(\omega)\vec{E}_i$, where $\boldsymbol{T}(\omega)$ is a frequency dependent complex Jones matrix [6]. This frequency dependence of the SOP, phase and amplitude of $\vec{E}_o$ is the source of polarization mode dispersion in optical fibers and other linear optical media. Taking the derivative of $\vec{E}_o(\omega)$ with respect to ω (denoted by $\vec{E}_o'(\omega)$) and replacing $\vec{E}_i$ by $\boldsymbol{T}(\omega)^{-1}\vec{E}_o(\omega)$ (a valid replacement for a system which does not contain an ideal polarizer) gives [6]

$$\vec{E}_o'(\omega) = [\boldsymbol{T}'\boldsymbol{T}(\omega)^{-1}]\vec{E}_o \equiv \boldsymbol{N}(\omega)\vec{E}_o. \quad (1)$$

It has been already shown that the PSP's of the system are the eigenvectors of $\boldsymbol{N}$, and their propagation characteristics can be deduced from the respective eigenvalues [6]. We now propose to investigate the frequency dependence of the SOP of $\vec{E}_o$, using the complex plane representation of polarization in which the SOP of a transversely polarized wave having two orthogonal components (E_u, E_v), is represented by a single complex number $\chi = E_v/E_u$ [8]. In this formulation, it is possible to separate the optical field into a unit intensity polarization vector, $\hat{\varepsilon}$, and a complex amplitude $A(\omega)$

$$\vec{E} = \begin{bmatrix} E_u \\ E_v \end{bmatrix} = E_u \begin{bmatrix} 1 \\ \chi \end{bmatrix} = A(\omega)\hat{\varepsilon} \text{ with } A(\omega) = E_u(1+|\chi|^2)^{1/2}; \hat{\varepsilon} = (1+|\chi|^2)^{-(1/2)} \begin{bmatrix} 1 \\ \chi \end{bmatrix}. \quad (2)$$

Differential equations governing the frequency evolution of $\vec{E}_o$ in terms of its new descriptors χ_o and $A_o(\omega)$ can now be derived from (1), giving

$$\chi_o' = -n_{12}\chi_o^2 + (n_{22} - n_{11})\chi_o + n_{21} \quad (3a)$$

$$A_o'(\omega) = [n_{11} + n_{12}\chi_o + [\ln(1+|\chi_o|^2)^{1/2}]']A_o(\omega) \quad (3b)$$

where n_{ij} are the elements of $\boldsymbol{N}$. Equation (3) is valid for both unitary and nonunitary systems. Since the principal states of

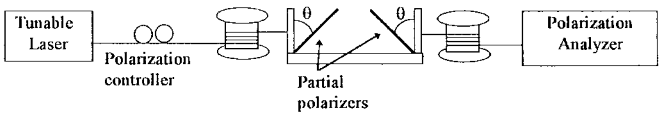

Fig. 1. The experimental setup.

polarization are defined by $\chi'_o = 0$ [6], their complex plane representations directly follow from (3a):

$$\chi_{o\pm} = \frac{1}{2n_{12}}\left[(n_{22} - n_{11}) \pm \sqrt{(n_{22} - n_{11})^2 + 4n_{12}n_{21}}\right] \quad (4)$$

and information about their respective group delays is contained in (3b), which, using the substitution $A_o(\omega) = \sigma_o(\omega)\ \exp[j\varphi_o(\omega)]$, reduces to

$$[\ln\ \sigma_{o\pm}(\omega) + j\varphi_{o\pm}(\omega)]' = n_{11} + n_{12}\chi_{o\pm}. \quad (5)$$

Unlike the unitary case, in general the magnitude $\sigma_o(\omega)$ varies with the frequency. We are more interested in the difference between the propagation characteristics of the two principal states, which are given by

$$\Delta\rho = \Delta(\ln\ \sigma)' + jDGD = \sqrt{(n_{22} - n_{11})^2 + 4n_{12}n_{21}} \quad (6)$$

where $DGD(= \varphi'_{o+} - \varphi'_{o-})$ is the differential group delay [4] and $\Delta(\ln\ \sigma)' = \Delta(\sigma'/\sigma)$ is the frequency derivative of the attenuation/amplification difference between the PSP's.

It is now clear that the frequency evolution of any polarization, (3a), as well as the definitions of the PSP's and their differential propagation characteristics are completely determined by the three complex quantities n_{12}, n_{21}, and $(n_{22} - n_{11})$. Had these quantities been constants, (3a) could have been solved analytically, resulting in $\chi_o(\omega)$ which traverses the complex plane along logarithmic spirals, which reduce to simple circles in the unitary case. In general, however, the n_{ij} are frequency dependent, and the frequency evolution of $\chi_o(\omega)$ can only be determined from a detailed solution of (3a).

III. Measurement Technique

Since the PSP's and their differential propagation characteristics can be calculated from n_{12}, n_{21}, and $(n_{22} - n_{11})$, (4), (6), and since these same quantities relate $\chi_o(\omega)$ to its frequency derivative χ'_o, the problem of identifying the PSP's and measuring the DGD becomes one of best estimating the coefficients of (3a) from a few measurements. Since these coefficients are polarization independent, then for any optical frequency of interest, ω_o, we propose to estimate $n_{12}(\omega_0), n_{21}(\omega_0)$, and $(n_{22}(\omega_0) - n_{11}(\omega_0))$ from measurements of the output SOP $\chi_o(\omega_0)$ and its frequency derivative for a series of $K(\geq 3)$ different input polarizations, the exact SOP's of which need not be determined. The results then form a set of K linear equations for n_{12}, n_{21}, and $(n_{22} - n_{11})$, which can be solved using linear least mean-square parameter estimation. Once n_{12}, n_{21}, and $(n_{22} - n_{11})$ are found, the PSP's and DGD follow from (4) and (6), respectively. Unlike the Poincare sphere method, the proposed technique should work equally well for unitary and nonunitary media. To further examine these observation we performed an experiment and some simulations.

TABLE I
Measured DGD Using Three Different Methods, for Different Values of the PDL

PDL(dB)	DGD (in picoseconds)		
	Complex Plane Method	Jones Matrix Eigenanalysis	Poincare sphere method
0.08	0.2	0.2	0.2
2.5	0.4	0.4	0.7
3.8	0.4	0.4	4.7

IV. Experiment and Simulations

The system under test (Fig. 1) comprised a cascade of a 2.5-km spool of a single-mode fiber, a pair of partial polarizers and another 2.5-km spool of a similar fiber. The partial polarizers were tilted at an angle θ with respect to the optical axis and different values of PDL could be obtained by adjusting θ. Light from a tunable laser source was launched into the system at three frequency points ($\lambda_0 = 1550$ nm, $\Delta\lambda = \pm 0.1$ nm) at three different input SOP's (as determined by the polarization controller), and the output SOP's were measured using a polarization analyzer. The resulting data $\{\chi_{o,k}(\omega_0 \pm \Delta\omega), k = 1 \cdots 3\}$ were used for the estimation of n_{12}, n_{21}, and $(n_{22} - n_{11})$, which were then used to calculate the PSP's and the DGD of the system. $\chi'_o(\omega_0)$ was approximated by $(\chi_o(\omega_0 + \Delta\omega) - \chi_o(\omega_0 - \Delta\omega))/(2\Delta\omega)$ but no knowledge of the input SOP's was required. We have also estimated the DGD using the Poincare sphere method [1], which assumes that as the frequency changes, the SOP rotates on the sphere around a diameter formed by the (orthogonal) PSP's. However, in the presence of nonzero PDL/G, i.e., for nonunitary systems, the PSP's do not form a diameter so that estimating the DGD using the Poincare sphere method could lead to errors. The PDL of the system was measured with an HP-8509B Lightwave Polarization Analyzer, which was also used for the determination of the output SOP's. With the aid of three internal polarizers, this analyzer can also provide an independent estimation of the system DGD, based on the Jones matrix eigenanalysis [6].

Results: Table I shows measurements of the DGD using three different techniques: our proposed complex plane method, the Jones matrix eigenanalysis [6] and the Poincare sphere method [1], for increasing values of the PDL. While the results of the Jones eigenanalysis fully agree with the results of the new method, the estimations made by the Poincare sphere method appear increasingly erroneous, as the PDL departs from zero.

These observations were augmented by a simulation of light propagation through a system, where the PDL was distributed along the whole length of the system. A cascade of 1000 randomly oriented pairs, each made of a waveplate and a partial polarizer, having a relative random orientation was investigated. All waveplates had the same DGD, and the extinction ratio of the partial polarizers was uniformly

TABLE II
VALUES OF THE PARAMETERS OF THE SIMULATIONS FOR FIVE DIFFERENT REALIZATIONS OF THE SYSTEM

	$\Delta\tau_e$ (psec)	η_e	DGD (psec)	$\Delta(\ln\sigma)'$ (psec/rad)	α (deg)	e_{psm}(%)
Unitary	0.0667	1	1.95	0	180	0
Non-Unitary	0.0667	0.99	1.96	0.09	166	4
	0.0667	0.98	1.99	0.17	152	8
	0.0667	0.97	2.05	0.27	138	14
	0.0667	0.96	2.13	0.37	124	23

distributed between 1 and $(\eta(>0)$. Following the procedures of Section III, we estimated not only the DGD but also $\Delta(\ln\ \sigma)'$ (6), the angle (between the two PSP's on the Poincare sphere, as well as the error introduced by the Poincare sphere method. The results are shown in Table II for several realizations of the system. Obviously, in the unitary case $(\eta_e = 1)$ there is no polarization dependent loss ($\Delta(\ln\ \sigma)' = 0$) and the angle between the two orthogonal PSP's is 180° on the Poincare sphere. As η_e decreases, $\Delta(\ln\ \sigma)'$ increases, the angle between the two PSP's is no longer 180°, and estimating the DGD by the Poincare sphere method again leads to relatively large errors.

V. CONCLUSION

The frequency evolution of the output state of polarization was studied in terms of the complex plane representation of polarization, χ, and was found to be governed by a simple first-order nonlinear differential equation. The coefficients of this equation are naturally related to the wavelength (frequency) dependence of the Jones matrix of the medium. Conversely, by measuring the output SOP's for several input polarizations together with their frequency derivatives, the coefficients of this equation can be estimated and the PSP's and PMD determined even for nonunitary media. To avoid higher order effects, the frequency spacing used to calculate the various frequency derivatives should be kept as small as the measurement noise permits. This technique, which was experimentally demonstrated, is somewhat simpler than the Jones matrix eigenanalysis, and can be viewed as a generalization of the Poincare sphere method,[1] which cannot be used for system with nonzero PDL/G.

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[1] It should be noted that in principle the model derived by [7] can be adopted to serve as a basis for a modified version of the Poincare sphere method that would be valid for nonunitary as well as unitary systems.