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Measurement of Semiconductor Laser Gain and Dispersion Curves Utilizing Fourier Transforms of the Emission Spectra

Daniel Hofstetter and Jérôme Faist

***Abstract*— A new technique for the measurement of semiconductor laser gain and dispersion spectra is presented. The technique is based on an analysis of the subthreshold emission spectrum by Fourier transforms. Applications of this method to AlGaInP-based interband laser diodes and mid-infrared intersubband quantum cascade lasers are discussed. A good agreement between the measured dispersion of the refractive index and tabulated values in the literature was found.**

***Index Terms*—Dispersion curves, Fourier transform, gain spectra, semiconductor lasers.**

I. Introduction

THE MEASUREMENT of gain spectra is an important tool for the optimization and characterization of semiconductor lasers. A number of different methods for the determination of the net gain spectrum have been proposed in the literature. The Hakki–Paoli method [1], [2] uses the ratio between the peaks and valleys of individual Fabry–Pérot resonances, while with Cassidy's method [3], one estimates the area under each Fabry–Pérot resonance and compares it with the corresponding area of a smooth, i.e., unstructured spectrum. Finally, Henry's method [4] utilizes the unamplified spontaneous emission spectrum and the relation between the stimulated and spontaneous emission coefficients to obtain the gain spectrum. This technique has the advantage that the gain curve can be measured without relying on the presence of Fabry–Pérot resonances in the spectrum. The Hakki–Paoli technique has the benefit of being a simple method which works very well as long as the cavity has only a moderately high Q-factor. However, as soon as the resonator finesse becomes too high, the method is, due to experimental limitations, not working properly because there occurs no longer an improvement of the fringe contrast [5]. With Cassidy's method, one can determine higher net gain values because the area under the curve continues to diminish as the Q-factor of the cavity increases. A possible limitation is rather on the low Q side where areas of almost equal size have to be compared. In light of these drawbacks, the method presented in this article is superior because the Fourier transform of the emission spectrum (interferogram) takes naturally into account both shape and contrast of the Fabry–Pérot fringes [6], [7]. In addition, it allows a measurement of the refractive index dispersion. Ideally, one starts with the interferogram measured with a Fourier transform spectrometer; this simplifies the use of this new technique in a natural way. However, in order to keep the theory general, we will begin our derivation with the spectrum.

We define the conjugate Fourier variables as wavenumber, $\beta = 1/\lambda$, and distance, d. The minus-i Fourier transform of a function $I(\beta)$ is then given by

$$\overline{I(d)} = \int_{-\infty}^{\infty} I(\beta) e^{-i2\pi\beta d} d\beta. \tag{1}$$

We calculate now the transmission spectrum of a Fabry–Pérot resonator, consisting of a material with constant refractive index, n, and constant absorption index $k = \alpha\lambda/4\pi$ across the wavenumber range of interest. In addition, we assume normal incidence, a wavenumber-independent intensity reflectance of $R = (n-1)^2/(n+1)^2$ at the two mirrors, and a distance L between them. Then the transmission spectrum in the wavenumber domain is given by the Airy-formula [9]

$$I(\beta) = \left(\frac{1-R}{\sqrt{R}}\right)^2 \cdot \frac{b}{1 + b^2 - 2b\cos(4\pi n L \beta)} \tag{2}$$

where the parameter b is defined via $b = Re^{-\alpha L}$. In agreement with earlier publications on this topic [10], we understand that this transmission spectrum is a periodic function and equivalent to an infinite sum of equally displaced Lorentz-shaped peaks. Instead of an infinite sum of Lorentz peaks, it is also possible to write (2) in the wavenumber domain as an infinite series of cosine functions, according to

$$I(\beta) = \left(\frac{1-R}{\sqrt{R}}\right)^2 \cdot \frac{2b}{1-b^2} \cdot \sum_{m=0}^{\infty} b^m \cdot \cos(4\pi n L \beta m). \tag{3}$$

This different representation of (2) in the wavenumber domain is advantageous for the understanding of the following steps. We remark at this point, that the Fourier transform of (2) would be a series of exponentially decaying delta-peaks with a constant harmonic amplitude ratio b. Obviously, the whole derivation shown above becomes slightly more complicated if we allow the material to have wavenumber-dependent gain, $g(\beta)$. b is now a function of wavenumber, according to

$$b = b(\beta) = R \cdot e^{-[\{\alpha - g(\beta)\}L]} \tag{4}$$

The authors are with the Institute of Physics, University of Neuchâtel, CH-2000 Neuchâtel, Switzerland.

and the quantity $\alpha - g(\beta)$ is referred to as net gain. When introducing this new relation into (3), we get in the wavenumber domain

$$I(\beta) = \left(\frac{1-R}{\sqrt{R}}\right)^2 \cdot \frac{2b(\beta)}{1-[b(\beta)]^2} \cdot \sum_{m=0}^{\infty} [b(\beta)]^m \cdot \cos(4\pi n L \beta m). \tag{5}$$

After calculating the Fourier transform of this series, we get a similar result as mentioned before, except for the fact that the former delta peaks are now replaced by peaks whose shapes are given by the Fourier transforms of the corresponding prefactors $[2b(\beta)^{m+1}]/[1-b(\beta)^2]$, and which are obviously different for each harmonic peak pair. Here, we omit the calculation of the Fourier transform of these prefactors and simply call them $W_m(d)$; the transform of (5) now being

$$\overline{I(d)} = \left(\frac{1-R}{\sqrt{R}}\right)^2 \cdot \frac{1}{4nL} \cdot \sum_{m=-\infty}^{\infty} W_m(d) * \delta\left(\frac{d}{2nL} - m\right). \tag{6}$$

If we take the shape-functions of two adjacent harmonic peaks [for instance $W_0(d)$ and $W_1(d)$], inverse-transform them, and divide by each other, then we get the function $b(\beta)$ of (4), according to

$$b(\beta) = \frac{\displaystyle\int_{-\infty}^{\infty} W_{m+1}(d) \cdot e^{i2\pi\beta d} \cdot dd}{\displaystyle\int_{-\infty}^{\infty} W_m(d) \cdot e^{i2\pi\beta d} \cdot dd}. \tag{7}$$

Obviously, the inverse-transforms of $W_0(d)$ and $W_1(d)$ describe the wavenumber-dependent dc- and ac-components of the spectrum, respectively. From $b(\beta)$, we can calculate the net gain by solving (4).

If there is a wavenumber-dependence of both refractive index and absorption, then the above gain analysis works under the following, rather trivial, condition: The broadening of the first harmonic peak due to dispersion and gain must be smaller than the harmonic peak separation. In order to recover the refractive index as a function of wavenumber, one starts from the shape function of the first-harmonic peak, $W_1(d)$, which needs to be inverse-transformed. The amplitude of the resulting complex function describes, as already mentioned, the ac-component of the spectrum, while its phase, $\phi(\beta) = 4\pi n(\beta) L \beta$ is proportional to the refractive index [7]. There are two details which slightly complicate this rather straightforward process. Spectra are usually not known from $\beta_{\min} = 0$ to $\beta_{\max}$. This experimental fact results in an unknown phase constant ϕ_0; this constant can be calculated if one knows one refractive index value within the measured range. For the example shown below, we took this refractive index value from the literature [11]. The other important feature is that the shape function $W_1(d)$ covers only a distance range of $\pm d_0/2$. An inverse-transform of this "cut" function leads thus to an additional factor 2 in the phase.

Fig. 1(top) shows the subthreshold spectrum of an AlGaInP-based laser diode emitting light at 670 nm [12], Fig. 1(center) its net gain curve, and Fig. 1(bottom) the refractive index curve; the latter two were obtained by the procedures described above. The dispersion-corrected refractive index values calculated from Fig. 1(bottom) agree within less than 3% with those calculated from the Fabry–Pérot mode spacing. In Fig. 2, we present the real part of the Fourier transform and, as an inset, the inverse-transformed zeroth- and first-order harmonics.

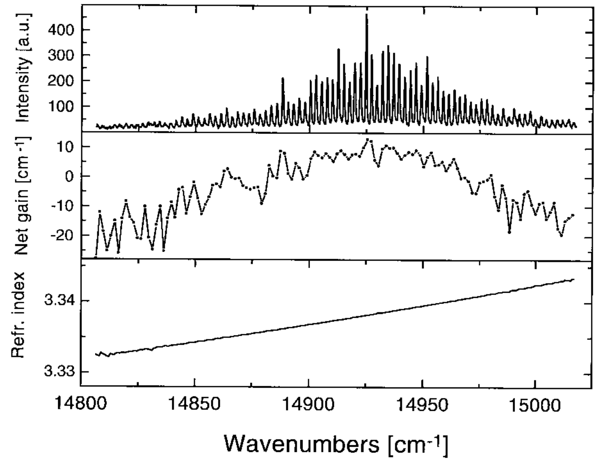

Fig. 1. (Top) High-resolution emission spectrum of a red laser diode. (Center) Corresponding net gain curve for the spectrum shown above obtained using the Fourier transform method. (Bottom) Corresponding refractive index as a function of wavenumber obtained using the Fourier transform method.

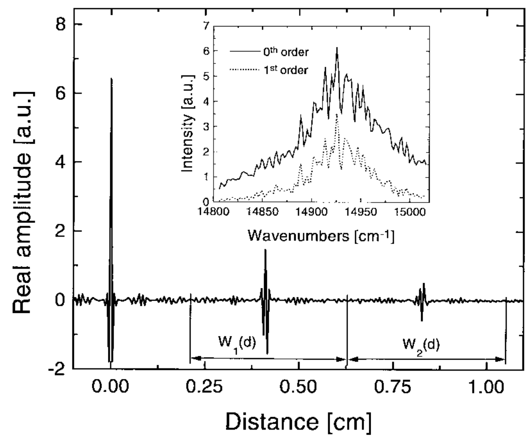

Fig. 2. Real part of the Fourier transformed emission spectrum shown in Fig. 1 (top). The inset shows the inverse transforms of the zeroth- and first-order harmonic peaks.

Three emission spectra of a mid-infrared quantum cascade laser measured at different injection currents are shown in Fig. 3. In Fig. 4, a comparison between Hakki–Paoli and Fourier transform gain curves obtained from the spectra in Fig. 3 is presented. The gain spectra of the two methods agree very well; deviations in the center of the window are on the order of <1 cm^{-1}. Deviations toward the edge of the transform window are due to numerical errors; they can be minimized by using a differently shaped (i.e., not rectangular) window for the Fourier transform. The inset shows the refractive index, which, unlike in an interband laser, decreases with increasing wavenumber. This is due to the fact that there is strong absorption at frequencies around the longitudinal optical phonon; this absorption leads to additional dispersion effects in the material.

An interesting question is how spectrometer resolution, noise, and dc-offsets influence the results of our technique.

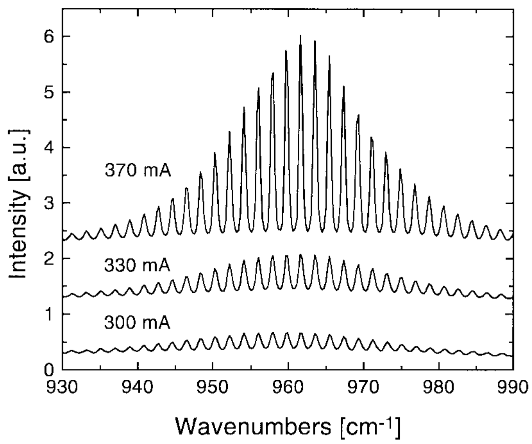

Fig. 3. Subthreshold high-resolution emission spectra of a 10.4-μm mid-infrared quantum cascade laser at three representative injection currents.

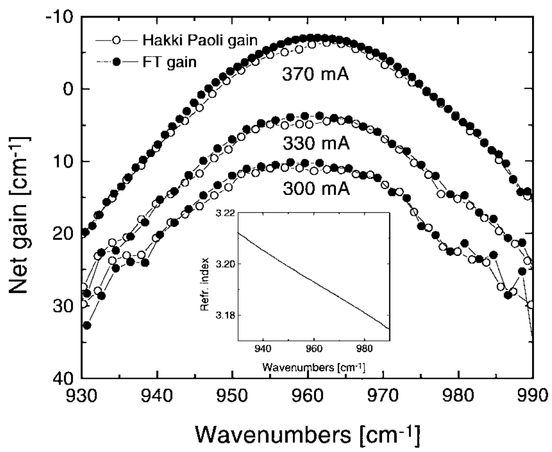

Fig. 4. Comparison between Hakki–Paoli and Fourier transform gain curves for the laser spectra shown in Fig. 3. The inset shows the refractive index dispersion for the same device.

In order to get some insight into these problems, we calculated a Fabry–Pérot mode spectrum, and convoluted it with a triangular line profile, which corresponds to the impulse response of a typical grating spectrometer. Then we added some random noise and a small dc-offset. By doing a series of gain calculations with changing noise levels, dc-offsets, and resolutions, we found that the gain spectrum is influenced by these parameters in the following way. An increase in noise level changes the wavenumber window, within which the gain curve is measured accurately. An increase of the linewidth function results, in general, in a vertical shift of the entire gain curve. Since a worse resolution cuts higher frequencies and thus reduces the peak height of the Fabry–Pérot resonances, the nonideal gain curve exhibits always lower gain values than the ideal one. A small dc-offset, finally, leads to a slight change in the shape of the gain curve. We used a simulated spectrum with a maximal b-parameter of 0.65 in the center. We then assumed a spectrometer impulse response function with the same linewidth as the width of these central Fabry–Pérot resonances. With a cavity length of 500 μm, this leads to a 1-cm^{-1} deviation in the gain spectrum. When using the Hakki–Paoli technique, the fringe visibility of the Fabry–Pérot resonances changed by more than 25%, but the deviation in the gain curve was again only 1.5 cm^{-1}. This shows that the Fourier transform method is at least comparably insensitive to the spectrometer resolution as the Hakki–Paoli technique. In fact, if the spectrometer's impulse response function is known, one can use the convolution theorem to correct the measured interferogram and thus obtain even better results for the gain spectrum.

Concerning the spectrometer resolution, one should outline another important feature. In a Fourier transform spectrometer, the high frequency filter function has a rectangular shape. Therefore, as long as the zeroth- and first-harmonic peaks are within the filter function, the gain curve is not at all affected by the filtering process. For this special case, our method will give much better results than any other gain measurement method which is based on a determination of the fringe contrast.

In conclusion, we have presented a novel method for the measurement of semiconductor laser gain spectra and dispersion curves. The method is based on the Fourier transform of the emission spectrum. A comparison between Hakki–Paoli gain data and data obtained from our novel method showed excellent agreement between the two methods. We also discussed the effects of spectrometer resolution and dispersion on the gain spectrum.

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