

# Measurement of the $B_d^0 - \bar{B}_d^0$ Oscillation Frequency

The L3 Collaboration

## Abstract

Time-dependent  $B^0 - \bar{B}^0$  mixing is studied using about two million hadronic Z decays registered by L3 in 1994 and 1995. For this study three techniques are used. Tagging of the b-quark charge at decay time is performed by identifying leptons from semileptonic B decays. The flavour of the b quark at production time is determined from the charge of the lepton in the opposite hemisphere or by using a jet-charge technique. The proper time of the B-particle decay is obtained by reconstructing the production and decay vertices or by a measurement of the lepton impact parameter. The combined result for the frequency of  $B_d^0$  meson oscillations is

$$\Delta m_d = 0.444 \pm 0.040 \text{ ps}^{-1}.$$

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# Introduction

As in the case of K mesons, oscillations between particle and antiparticle states are expected in the system of neutral B mesons. In the Standard Model [1], the mechanism causing mixing is a second order weak interaction through box diagrams. The flavour eigenstates  $B_d^0$  ( $\bar{b}d$ ) and  $\bar{B}_d^0$  ( $b\bar{d}$ ) are linear combinations of the mass eigenstates  $B_1$  and  $B_2$ . Neglecting effects from CP violation (expected to be small), the probability to find a  $B_d^0$  decaying at proper time  $t$ , provided it was produced as  $\bar{B}_d^0$  at  $t = 0$ , is given by

$$P(\bar{B}_d^0 \rightarrow B_d^0) = \frac{1}{\tau} e^{-\frac{t}{\tau}} \left( \frac{1 - \cos \Delta m_d t}{2} \right)$$

where  $\tau$  is the lifetime of the  $B_d^0$ -meson. A measurement of the oscillation frequency thus gives a direct measurement of the mass difference  $\Delta m_d$  between the two mass eigenstates.

The phenomenon of  $B^0$ - $\bar{B}^0$  mixing is well established by experiment [2–4]. The time dependence of mixing for  $B_d^0$  mesons has been measured at LEP, SLC and at the Tevatron using different techniques [5–10].

We present here a measurement of the  $B_d^0$  oscillation frequency with the L3 detector [11]. Three methods — dilepton decay length, lepton-jet charge and dilepton impact parameter — are used for this study. In the dilepton method we use leptons in opposite hemispheres to tag the flavour of the B particle at production and decay time. In the lepton – jet charge method the lepton tags the state of the B particle at the instant of its decay, and the flavour of the primordial b quark is determined using a jet-charge technique. The signature for mixing is the presence of a same-sign lepton pair or a same-sign lepton – jet charge combination. Proper time is measured by reconstructing secondary vertices or using lepton impact parameter information. We combine the three individual measurements. The event sample corresponds to about two million hadronic Z decays recorded in 1994 and 1995.

## Dilepton decay length method

The published dilepton analysis [8] is extended here to include the data taken in 1995.

Tagging of the b-quark charge at decay time is performed by identifying leptons from semileptonic B decays. The flavour of the b quark at production time is determined from the charge of the lepton in the opposite hemisphere. The selected leptons (electrons or muons) must have a high momentum and a high transverse momentum with respect to the closest jet direction.

To estimate the decay length of a b-hadron candidate we reconstruct the primary and secondary vertex positions. Vertex finding is performed in the  $r$ - $\phi$  plane perpendicular to the beam axis. The direction of the jet containing the lepton is then used to obtain a three-dimensional decay length. To calculate the proper time, one needs to know the momentum of the b-hadron. A constant fraction of the beam energy is used for this momentum estimate. The value  $p_B = 0.85 E_{beam}$  is found to optimize the proper time resolution. The JETSET 7.4 Monte Carlo program [12] is used to generate hadronic Z decays. The detector simulation is performed with a GEANT-based description of the L3 detector [13].

For the fit of the  $B_d^0$ - $\bar{B}_d^0$  oscillation frequency dilepton events with at least one proper time measured are selected. 1490 dilepton events with 1928 secondary vertices fulfill all requirements of lepton identification and vertex reconstruction. There are 630 reconstructed vertices in the like-sign events and 1298 in the unlike-sign ones.

An eight-parameter unbinned maximum-likelihood fit is performed in order to determine  $\Delta m_d$ . A likelihood is assigned to each event proportional to the probability density to find such an event at the measured decay time. Fitted parameters are  $\Delta m_d$  itself and  $f_{\Lambda_b}$ ,  $f_s$ ,  $F_{bcl}$ , where  $f_{\Lambda_b}$  and  $f_s$  are the fractions of b-baryons and  $B_s^0$  mesons in the sample, and  $F_{bcl}$  is the fraction of leptons coming from cascade decays ( $b \rightarrow c \rightarrow \ell$ ,  $b \rightarrow \bar{c} \rightarrow \ell$ ,  $b \rightarrow J \rightarrow \ell^+ \ell^-$ ) in  $b\bar{b}$  events. The other four parameters are the lifetimes of b-hadrons:  $\tau_d$  for  $B_d^0$ ,  $\tau_u$  for  $B^\pm$ ,  $\tau_s$  for  $B_s^0$  and  $\tau_{\Lambda_b}$  for  $\Lambda_b$ . The oscillation frequency  $\Delta m_d$  is a free parameter; for the other seven parameters Gaussian constraints are applied with the central values and errors quoted in Table 1. The value for the  $B_s^0$  fraction in  $b\bar{b}$  events ( $f_s$ ) is fixed to the world average derived from time-integrated mixing measurements [14]. The value for the fraction of b-baryons in  $b\bar{b}$  events is derived from  $\Lambda_c$ -lepton and  $\Lambda$ -lepton correlations [15]. The fractions of  $B_d^0$  and  $B^\pm$  are then obtained through  $f_d = f_u = 0.5 \times (1 - f_{\Lambda_b} - f_s)$ . The central values and uncertainties of the b-hadron lifetimes are taken from [16]. We also impose a Gaussian constraint on the mean lifetime of the b-hadrons,  $\tau_b = f_d \tau_d + f_u \tau_u + f_s \tau_s + f_{\Lambda_b} \tau_{\Lambda_b}$ , requiring it to be compatible with the world average value  $\tau_b = 1.549 \pm 0.02$  ps [16].

The result of the fit is  $\Delta m_d = 0.458 \pm 0.054$  ps<sup>-1</sup>. The error on  $\Delta m_d$  includes the statistical error as well as a contribution from systematic errors of the constrained parameters. To separate out the statistical error we fix all the constrained parameters to their fitted values and perform a one-parameter fit. This gives  $\Delta m_d = 0.458 \pm 0.046$  (stat) ps<sup>-1</sup> where the error is purely statistical. The systematic error on  $\Delta m_d$  is then obtained by subtracting in quadrature the statistical error from the eight-parameter fit error. This gives  $\pm 0.028$  ps<sup>-1</sup> for the systematic error estimate. The individual systematic errors on  $\Delta m_d$  from fitted parameters are estimated by shifting the central value of each constrained parameter by the uncertainty quoted in Table 1 and refitting. The change in  $\Delta m_d$  is taken as systematic error. Due to correlations, the sum in quadrature of the contributions obtained this way is not equal to the total systematic error above (labelled “subtotal” in Table 1). Therefore the individual systematic errors on  $\Delta m_d$  from fitted parameters are scaled to add up in quadrature to 0.028 ps<sup>-1</sup>.

The values of other parameters are the same as in the 1994 data analysis [8]. The contributions to the systematic error on  $\Delta m_d$  are summarized in the Table 1. The fraction of like-sign leptons is plotted in Figure 1 as a function of proper time and compared to the fit result.

The oscillation frequency is found to be

$$\Delta m_d = 0.458 \pm 0.046(\text{stat}) \pm 0.032(\text{syst}) \text{ ps}^{-1}$$

by the dilepton decay length method.

## Lepton – jet charge method

As in the dilepton method described above, the state of the B meson at the instant of its decay is tagged by the lepton that originates from its semileptonic decay. The lepton must pass the selection criteria applied in the dilepton method. The decay time of the b-hadron is also reconstructed in the same way. Although the reconstruction of a secondary vertex in the hemisphere that does not contain the lepton (the *opposite* hemisphere) would not be necessary for this analysis, requiring it increases the b-purity of the sample from 91% to 97%.

The B meson’s state at the time of its production is tagged using jet charges as the spatial distribution of charge in the event tends to reflect the orientation of the primordial quark–antiquark pair. A jet charge is calculated as a weighted sum of charges in a hemisphere defined

by the thrust axis:

$$Q_{jet} = \frac{\sum_i w_i q_i}{\sum_i w_i} \quad \text{with} \quad w_i = C p_L^\kappa, \quad (1)$$

where  $q_i$  is the charge of the  $i$ th track in the hemisphere and its weight,  $w_i$ , is proportional to a power of the longitudinal component of the track's momentum with respect to the thrust axis,  $p_L$ . Coefficient  $C$  is the probability of correct track charge assignment. Since this probability decreases as the track passes nearer to the anode wire plane in the central tracking chamber (TEC),  $C$  depends on the azimuthal angle:  $C = C(\phi)$ . Function  $C(\phi)$  is determined by studying Bhabha events. The power  $\kappa$  is found in Monte Carlo studies by maximizing the probability of reconstructing the correct jet charge. The maximum is found at  $\kappa = 0.4$ . Monte Carlo studies show that in the lepton's hemisphere the value of jet charge depends on whether the B meson changed flavour (mixed event) or not. In order to decrease this correlation, we set  $\kappa = 0$  in the lepton's hemisphere. Thus the tagging variable is  $Q = Q_{opp}^{\kappa=0.4} - Q_{lepton}^{\kappa=0}$ , where  $Q_{opp}^{\kappa=0.4}$  is the jet charge of the *opposite* hemisphere calculated according to equation (1) using  $\kappa = 0.4$ , and  $Q_{lepton}^{\kappa=0}$  is the jet charge of the hemisphere containing the lepton, calculated using  $\kappa = 0$ . Figure 2 shows the distribution of  $Q$  for a b-quark jet and a  $\bar{b}$ -quark jet.

To be included in the jet charge sum calculation, a track must have a momentum in the  $r$ - $\phi$  plane greater than 0.5 GeV, at least 10 TEC hits with a span of at least 20 wires<sup>1)</sup>, and a distance of closest approach to the average position of the  $e^+e^-$  collision point in the  $r$ - $\phi$  plane less than 20 mm.

To increase the tagging power of the method, events with  $|Q| < 0.12$  are rejected. This way the region of low reliability (probability for correct charge reconstruction) is excluded (see figure 2). The reliability is found to be 72% for unmixed events and 67% for mixed events. The number of events passing all selection criteria is 8707.

In order to extract the oscillation frequency, an unbinned maximum-likelihood fit is performed. Each event is assigned a likelihood depending on whether it exhibits a like or unlike signed lepton – jet charge combination as function of the lepton proper time:

$$\begin{aligned} \mathcal{L}_{like}(t) &= \sum_{i \in \{mix, unmix\}} [R^i(t)(1 - P^i) + W^i(t)P^i] + \sum_{i \in \{c, uds\}} f_i w_i Z_i(t), \\ \mathcal{L}_{unlike}(t) &= \sum_{i \in \{mix, unmix\}} [R^i(t)P^i + W^i(t)(1 - P^i)] + \sum_{i \in \{c, uds\}} f_i (1 - w_i) Z_i(t), \end{aligned}$$

where  $P^{mix}$  ( $P^{unmix}$ ) is the reliability of the jet charge reconstruction in b-flavoured events with (without) mixing.  $Z_c(t)$  and  $Z_{uds}(t)$  are the reconstructed decay time distributions of Monte Carlo c-flavoured (fraction  $f_c$ ) and light-quark events (fraction  $f_{uds}$ ), and  $w_c$  and  $w_{uds}$  denote the probabilities of like-sign combination in c-flavoured and light-flavoured event.  $R^{mix}(t)$  and  $R^{unmix}(t)$  are the probabilities to find, at proper time  $t$ , a lepton with the “right” sign of charge (the sign expected from the primordial quark decay without mixing) in a mixed or unmixed event, respectively. They are expressed as

$$\begin{aligned} R^{mix}(t) &= \sum_{j \in \{bl, bcl, bfk\}} F_j w_j M(t), \\ R^{unmix}(t) &= \sum_{j \in \{bl, bcl, bfk\}} F_j (1 - w_j) (D(t) - M(t)). \end{aligned} \quad (2)$$

$F_{bl}$ ,  $F_{bcl}$  and  $F_{bfk}$  are the fractions of the various types of  $b\bar{b}$  events: leptons coming from semileptonic decays of b-hadrons, cascade decays and fake leptons in  $b\bar{b}$  events, respectively.

<sup>1)</sup>The span is the distance between the first and the last hit in units of the wire spacing.

The  $w$  factors ( $w_{bl}$ ,  $w_{bcl}$  and  $w_{bfk}$ ) give the probabilities to find a lepton sign opposite to the one expected if the event resulted from an unmixed B decay. The “wrong” sign probabilities  $W^{mix}(t)$  and  $W^{unmix}(t)$  are obtained from equations 2 by exchanging  $w_j$  and  $(1 - w_j)$ .  $M(t)$  is the probability of a mixed B meson to decay at a reconstructed time  $t$  while  $D(t)$  is that of any b-hadron to decay at a reconstructed time  $t$ :

$$M(t) = \sum_{m \in \{d, s\}} f_m \int_0^\infty \frac{\epsilon(t')}{N_m} \frac{1}{\tau_m} e^{-\frac{t'}{\tau_m}} \frac{1 - \cos \Delta m_m t'}{2} R(t, t') dt' ,$$

$$D(t) = \sum_{n \in \{u, d, s, \Lambda_b\}} f_n \int_0^\infty \frac{\epsilon(t')}{N_n} \frac{1}{\tau_n} e^{-\frac{t'}{\tau_n}} R(t, t') dt' .$$

The resolution function  $R(t, t')$  is described in Reference [8]. The oscillation frequency  $\Delta m_s$  of  $B_s^0$  mesons was fixed to  $10 \text{ ps}^{-1}$ , thus assuming a large mixing in the  $B_s^0$  system.  $\epsilon(t)$  stands for the efficiency of proper time reconstruction and the normalization factors  $N_i$  ( $i \in \{d, s, u, \Lambda_b\}$ ) are defined by the requirements

$$\int_{-\infty}^\infty dt \int_0^\infty dt' \frac{\epsilon(t')}{N_i} \frac{1}{\tau_i} e^{-\frac{t'}{\tau_i}} R(t, t') = 1 . \quad (3)$$

Nine parameters are allowed to vary in the fit to the data sample.  $\Delta m_d$  and  $P^{unmix}$  are free whereas Gaussian constraints are imposed upon  $P^{mix}$ ,  $\tau_d$ ,  $\tau_u$ ,  $\tau_s$ ,  $\tau_{\Lambda_b}$ ,  $f_{\Lambda_b}$  and  $f_s$ . The average b-hadron lifetime is constrained as in the dilepton fit. The central value of  $P^{mix}$  is set to its Monte Carlo value. Its uncertainty is estimated as follows: we fit data with  $Q_{opp}^{\kappa=0.4}$  taken as the tagging variable. This fit yields for the reliability of the jet charge reconstruction in the *opposite* hemisphere  $P_{opp} = 0.634 \pm 0.012$  whereas the Monte Carlo value is  $P_{jet}^{MC} = 0.660 \pm 0.005$ . The difference between them is used as an estimate of the uncertainty on  $P^{mix}$ .

The result of the fit is  $\Delta m_d = 0.437 \pm 0.060 \text{ ps}^{-1}$ . Treating the errors in the same fashion as in the dilepton analysis, we obtain

$$\Delta m_d = 0.437 \pm 0.043 \text{ (stat)} \pm 0.044 \text{ (syst)} \text{ ps}^{-1} .$$

The contributions to the systematic error on  $\Delta m_d$  coming from individual parameters are itemized in Table 2. The like-sign fraction is plotted in Figure 3 as a function of proper time together with the result of the fit.

## Dilepton impact parameter method

A third determination of neutral B mixing is based on measured impact parameters of leptons in dilepton events. In this method the time of decay is measured indirectly and statistically from the impact parameters of the reconstructed lepton trajectories with respect to the beam axis. This method has the advantage of permitting less stringent selection requirements on the b candidates, thereby increasing statistics. Systematic uncertainties, however, are larger using the impact parameter method due to lower b-purity of the selected event sample.

Event selection requirements for the impact parameter method are nearly identical to those used for the decay length method, except that no primary or secondary vertex requirements are imposed on either candidate b-jet. The ratio of the number of same-sign dilepton events to the total number of dilepton events as a function of  $\delta$ , the measured impact parameter, is plotted in Figure 4. The selected event sample consists of 2596 dilepton events.

A simultaneous, binned, maximum-likelihood fit to the like- and opposite-sign impact parameter distributions determines the oscillation frequency  $\Delta m_d$ . Again values are assumed for parameters such as  $f_d$ ,  $f_s$  and  $\Delta m_s$ , and multi-parameter fits are performed to verify the consistency of these assumed parameter values with the observed data. The like-sign and opposite-sign distributions in the data are fitted to predictions from reweighted Monte Carlo events. Reweighting affects composition fractions and right- and wrong-sign probabilities for both signal and background events. Monte Carlo impact parameter resolutions are not used. Instead, the resolutions are determined directly from data, using control samples of hadronic tracks chosen geometrically to have preferentially small true impact parameters in the plane transverse to the beam axis. Reweighting corrections are applied to the control samples to reproduce kinematic, geometrical, and isolation characteristics of the leptons in the dilepton sample before resolution functions are extracted [17].

From the resolution functions and the underlying true impact parameter distributions for hadronic Z decays as obtained from Monte Carlo, one can also predict the expected impact parameter distributions for single-lepton candidates satisfying the electron and muon selection requirements. Comparison of the observed and expected impact parameter distributions shows good qualitative agreement between data and Monte Carlo. As a quantitative test, a fit has been performed to obtain the mean b-hadron lifetime. Values consistent with the L3 measurement of the b lifetime [18] are obtained for single electron and single muon samples in both 1994 and 1995.

Systematic uncertainties arising in the impact parameter method are estimated in a similar way as for the decay length method. Results are summarized in Table 3.

The result for  $\Delta m_d$  using the dilepton impact parameter method is

$$\Delta m_d = 0.472 \pm 0.049(\text{stat}) \pm 0.053(\text{syst}) \text{ ps}^{-1}.$$

## Combination of $\Delta m_d$ results

In order to combine individual results quoted above we measure  $\Delta m_d$  on statistically independent samples. The dilepton decay length event sample is left untouched and the other two event samples are reduced to eliminate statistical overlap. The fit was redone on the two reduced samples and the new central values and statistical errors are used in the combination procedure. A combination of the results is done in a global  $\chi^2$  fit which takes into account correlations from common sources of systematics. The systematic uncertainties from b-hadron fractions, B lifetimes, cascade decay fractions, B momentum and charge confusion ( $w$  factors in Tables 1, 2 and 3) are assumed to be fully correlated. Uncertainties from resolution are also considered correlated for the dilepton decay length and lepton-jet charge methods. The set of input parameters used in the fit for the dilepton decay length and lepton-jet charge methods is the same (Table 1 and 2). Parameter values in the dilepton impact parameter measurement and their errors are adjusted to agree with the ones used in the other methods.

The combined result for  $\Delta m_d$  is then

$$\Delta m_d = 0.444 \pm 0.028 \pm 0.028 \text{ ps}^{-1} = (2.92 \pm 0.18 \pm 0.18) 10^{-4} \text{ eV}.$$

This improves and supersedes the previous L3 result on  $\Delta m_d$ . The result is also consistent with the current world average  $\Delta m_d = 0.474 \pm 0.031 \text{ ps}^{-1}$  [16].

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Parameter	Input / Variation	Fitted value	$\Delta(\Delta m_d)$ [ps <sup>-1</sup> ]
$\Lambda_b$ fraction ( $f_{\Lambda_b}$ )	$0.087 \pm 0.029$	$0.086 \pm 0.027$	$\pm 0.007$
$B_s^0$ fraction ( $f_s$ )	$0.102 \pm 0.016$	$0.104 \pm 0.015$	$\mp 0.015$
cascade decay fraction ( $F_{bcl}$ )	$F_{bcl}^{\text{MC}} (1. \pm 0.15)$	$F_{bcl}^{\text{MC}} (1.04 \pm 0.13)$	$\mp 0.022$
$\tau_d$	$1.56 \pm 0.06$ ps	$1.57 \pm 0.05$ ps	$\mp 0.003$
$\tau_u$	$1.62 \pm 0.06$ ps	$1.64 \pm 0.05$ ps	$\pm 0.006$
$\tau_s$	$1.61 \pm 0.10$ ps	$1.61 \pm 0.10$ ps	$\pm 0.001$
$\tau_{\Lambda_b}$	$1.14 \pm 0.08$ ps	$1.12 \pm 0.08$ ps	$\pm 0.001$
$\tau_b$	$1.549 \pm 0.020$ ps	—	$\mp 0.002$
subtotal	—	—	0.028
fakes fraction ( $F_{bfk}$ )	$\pm 30\%$	—	$\mp 0.007$
$c\bar{c}$ fraction ( $f_c$ )	$\pm 30\%$	—	$\pm 0.001$
uds fraction ( $f_{uds}$ )	$\pm 50\%$	—	$\mp 0.001$
resolution	$\pm 25\%$	—	$\pm 0.007$
boost term	see [8]	—	$\pm 0.006$
$w_{bl}$	$\pm 30\%$	—	$\mp 0.003$
$w_{bcl}$	$\pm 0.05$	—	$\mp 0.007$
$w_{bfk}$	$\pm 0.10$	—	$\mp 0.006$
$w_{udsc}$	$\pm 0.10$	—	$\mp 0.001$
$\Delta m_s$	$3 - 20$ ps <sup>-1</sup>	—	$\mp 0.001$
total			0.032

Table 1: Summary of contributions to the systematic error on  $\Delta m_d$  using the dilepton decay length method.  $F_{bcl}^{\text{MC}}$  is the relative fraction of cascade decays derived from the MC simulation [12, 13].

Parameter	Input / Variation	Fitted value	$\Delta(\Delta m_d)$ [ps <sup>-1</sup> ]
$\mathcal{P}^{unmix}$	free	$0.733 \pm 0.011$	$\pm 0.037$
$\mathcal{P}^{mix}$	$0.670 \pm 0.030$	$0.679 \pm 0.029$	$\mp 0.015$
$\Lambda_b$ fraction ( $f_{\Lambda_b}$ )	$0.087 \pm 0.029$	$0.089 \pm 0.025$	$\pm 0.004$
$B_s^0$ fraction ( $f_s$ )	$0.102 \pm 0.016$	$0.101 \pm 0.016$	$\pm 0.001$
$\tau_d$	$1.56 \pm 0.06$ ps	$1.60 \pm 0.05$ ps	$\mp 0.002$
$\tau_u$	$1.62 \pm 0.06$ ps	$1.64 \pm 0.05$ ps	$\pm 0.004$
$\tau_s$	$1.61 \pm 0.10$ ps	$1.63 \pm 0.10$ ps	$\pm 0.002$
$\tau_{\Lambda_b}$	$1.14 \pm 0.08$ ps	$1.11 \pm 0.08$ ps	$\pm 0.002$
$\tau_b$	$1.549 \pm 0.020$ ps	—	$\mp 0.001$
subtotal	—	—	0.041
cascade decay fraction ( $F_{bcl}$ )	$F_{bcl}^{\text{MC}} (1. \pm 0.15)$	—	$\pm 0.002$
fakes fraction ( $F_{bfk}$ )	$\pm 30\%$	—	$\pm 0.001$
$c\bar{c}$ fraction ( $f_c$ )	$\pm 30\%$	—	$\pm 0.001$
uds fraction ( $f_{uds}$ )	$\pm 50\%$	—	$\pm 0.002$
resolution	$\pm 25\%$	—	$\pm 0.009$
boost term	see [8]	—	$\pm 0.006$
$w_{bl}$	$\pm 30\%$	—	$\pm 0.001$
$w_{bcl}$	$\pm 0.05$	—	$\pm 0.001$
$w_{bfk}$	$\pm 0.10$	—	$\pm 0.001$
$w_c$	$\pm 0.10$	—	$\mp 0.002$
$w_{uds}$	$\pm 0.10$	—	$\pm 0.004$
$\Delta m_s$	$3 - 20$ ps <sup>-1</sup>	—	$+0.004$
total			0.044

Table 2: Summary of contributions to the systematic error on  $\Delta m_d$  using the lepton-jet charge method.  $F_{bcl}^{\text{MC}}$  is the relative fraction of cascade decays derived from the MC simulation [12, 13].

Parameter	Input / Variation	Fitted value	$\Delta(\Delta m_d)$ [ps <sup>-1</sup> ]
$\Lambda_b$ fraction ( $f_{\Lambda_b}$ )	$0.117 \pm 0.034$	$0.112 \pm 0.034$	$\pm 0.015$
$B_s^0$ fraction ( $f_s$ )	$0.102 \pm 0.016$	$0.102 \pm 0.015$	$\mp 0.026$
$b \rightarrow c \rightarrow \ell$ fraction	$0.080 \pm 0.012$	$0.083 \pm 0.011$	$\mp 0.037$
subtotal	–	–	0.048
fakes fraction	$(4.6 \pm 1.3)\%$	–	$\mp 0.011$
$c\bar{c}$ fraction	$(1.5 \pm 0.3)\%$	–	$\pm 0.001$
$b \rightarrow \bar{c} \rightarrow \ell$ fraction	$(1.0 \pm 0.3)\%$	–	$\pm 0.001$
$\tau_d$	$1.56 \pm 0.06$ ps	–	$\mp 0.017$
$\tau_s$	$1.61 \pm 0.10$ ps	–	$\mp 0.002$
$\tau_b$	$1.549 \pm 0.020$ ps	–	$\pm 0.002$
b fragmentation: $\langle x \rangle$	$0.72 \pm 0.02$	–	$\pm 0.002$
resolution function	–	–	$\pm 0.001$
$w_{b\ell}$	$(0.24 \pm 0.02)\%$	–	$\mp 0.001$
$w_{bcl}$	$(0.24 \pm 0.02)\%$	–	$\pm 0.000$
$w_{bkg}$	$(33.7 \pm 3.4)\%$	–	$\mp 0.008$
$w_c$	$(10.0 \pm 3.0)\%$	–	$\mp 0.002$
$\Delta m_s$	$5 - 25$ ps <sup>-1</sup>	–	$- 0.004$ $+ 0.008$
total			0.053

Table 3: Summary of contributions to the systematic error on  $\Delta m_d$  using the dilepton impact parameter method.  $w_{bkg}$  is the  $w$  factor for background from light-quark events and from fake leptons in  $b\bar{b}$  events.

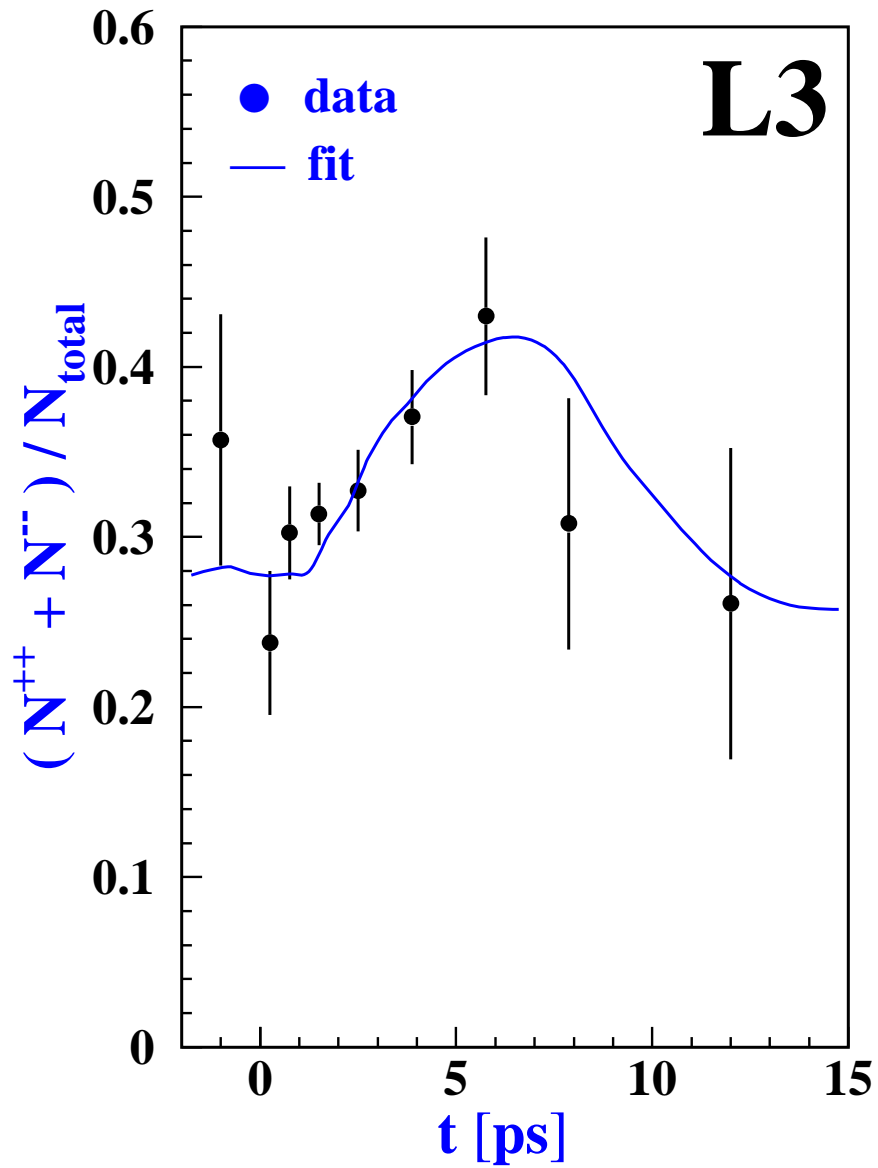


Figure 1: Ratio of the number of same-sign dilepton events to the total number of dilepton events versus the measured proper time compared to the fit result (solid line).

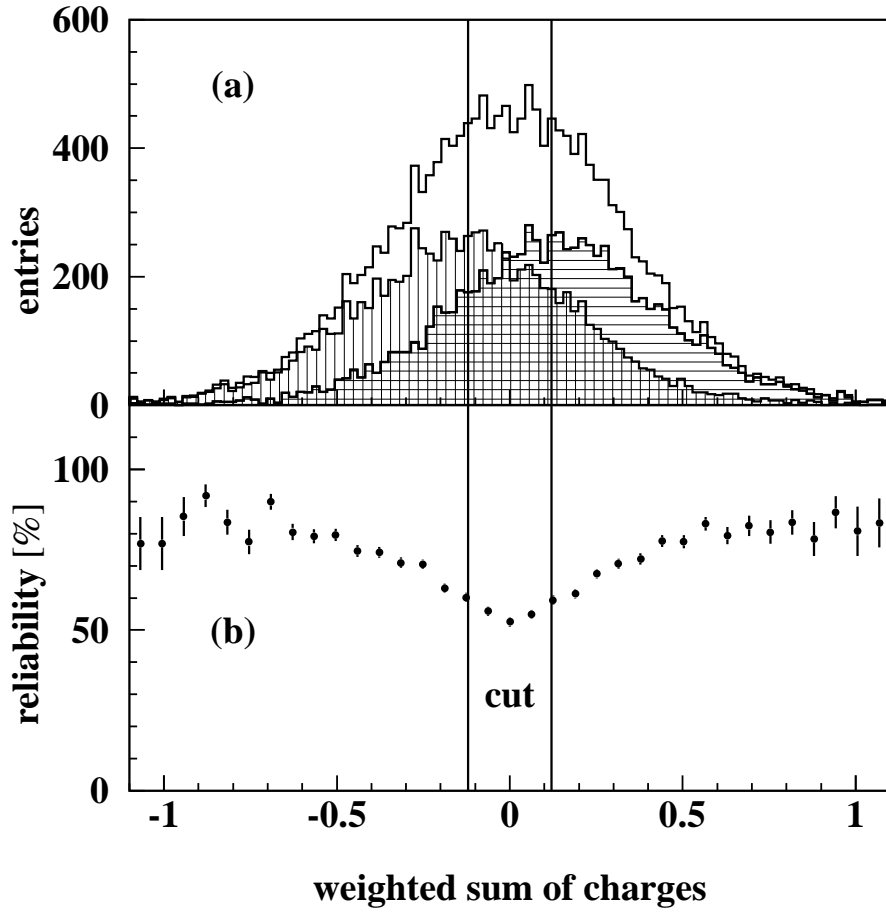


Figure 2: (a)  $Q = Q_{opp}^{\kappa=0.4} - Q_{lepton}^{\kappa=0}$  for a b-quark jet (horizontally hatched histogram) and a  $\bar{b}$ -quark jet (vertically hatched histogram) as determined by Monte Carlo. The unhatched histogram is their sum. (b) The probability of correct jet charge reconstruction (reliability) as a function of  $Q$ . This probability tends to 50% at  $Q = 0$  so the power of separation vanishes there. The two vertical lines delimit the excluded low reliability interval.



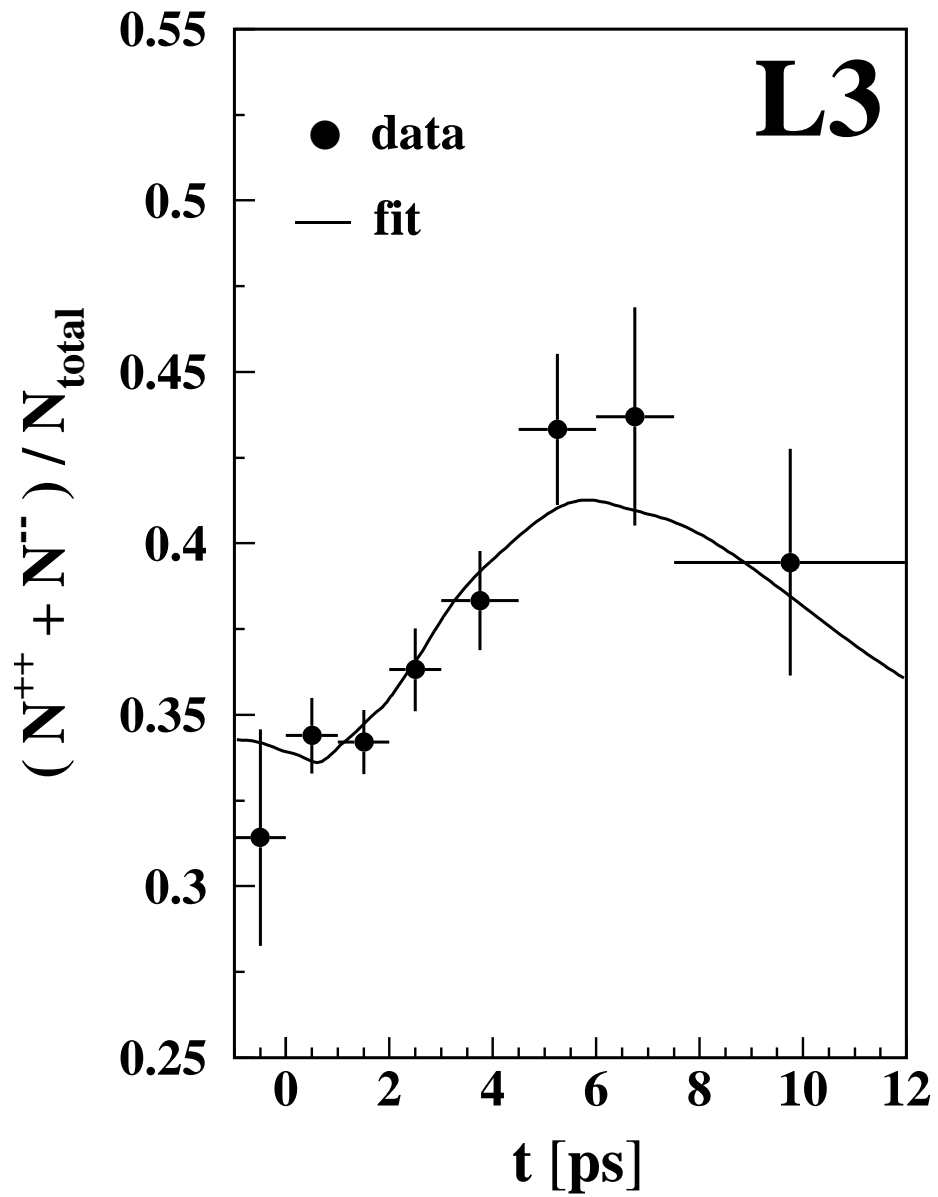


Figure 3: The ratio of the number of like-sign events as determined by jet charge and lepton to the total number of events as a function of the measured proper time. The solid line is the result of the fit.

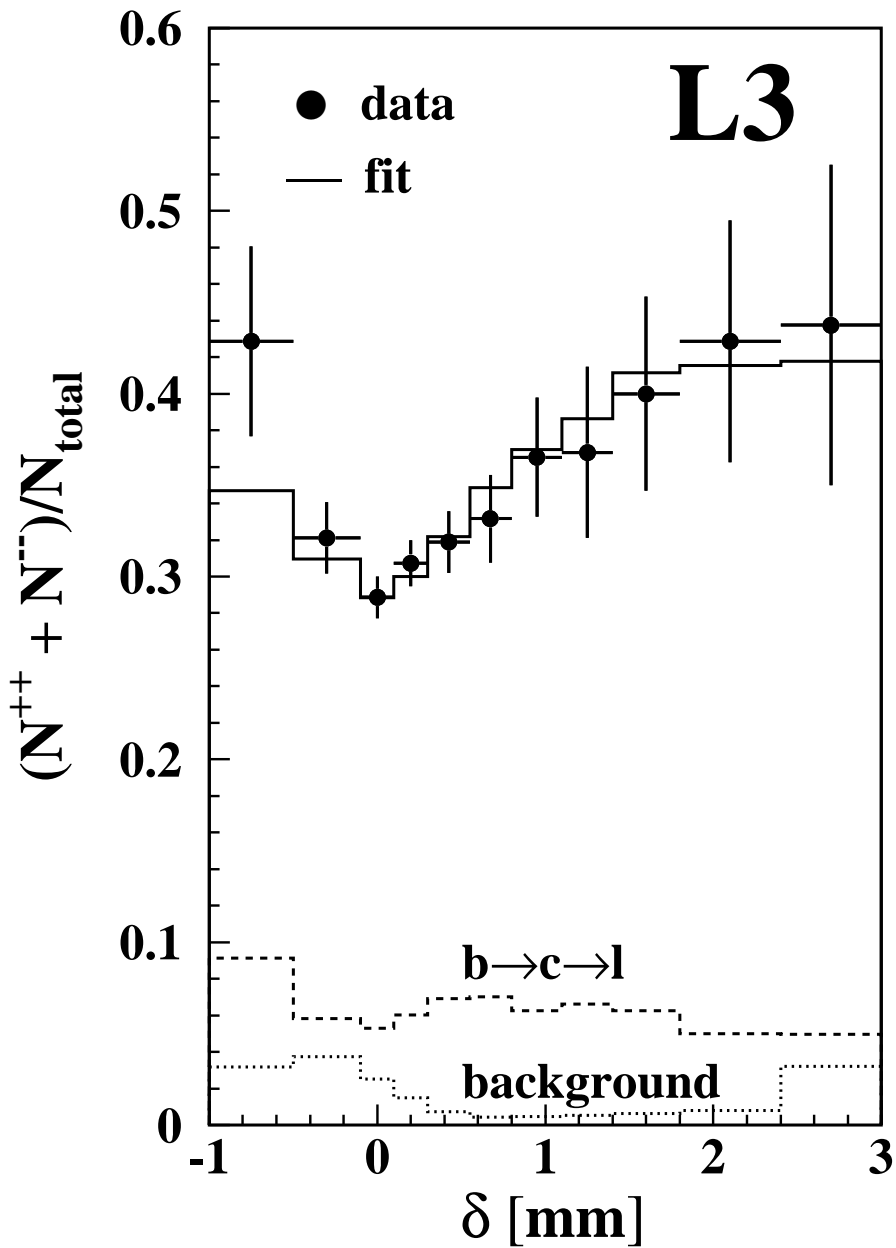


Figure 4: Ratio of the number of same-sign dilepton events to the total number of dilepton events as a function of  $\delta$  (the measured impact parameter) compared to the fit result. The individual contributions to the distribution from cascade decays ( $b \rightarrow c \rightarrow \ell$ ) and background leptons (misidentified hadrons) are shown by dashed and dotted lines, respectively.