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**Measurement of the bowing
parameters in violin playing**

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I. MUSIC ACOUSTICS

A. MEASUREMENT OF THE BOWING PARAMETERS IN VIOLIN PLAYING*

Anders Askenfelt

Abstract

A method is described which allows simultaneous measurement of the bowing parameters in violin playing under normal playing conditions. The measured parameters include bow position, bow velocity, bow-bridge distance, and bow force. The use of the method is illustrated by registrations of typical bowing patterns (sustained notes, scales, crescendo-diminuendo, sforzando and saltellato). The examples are analyzed with a focus on the player's use of the bow-bridge distance and bow velocity in controlling the dynamic level. The correspondence between predicted and observed changes in the dynamic level is discussed. The measurement method is a development of equipment earlier described (Askenfelt, 1986).

Introduction

A method has been previously described (Askenfelt, 1986) which allowed simultaneous measurement of the motion of the violin bow transverse to the strings and the bow force under normal playing conditions. The method has now been developed to include measurement of the distance from the bridge to the contact point between the bow and the string (bow-bridge distance) and the bow velocity.

After a short review of bowing parameters and their influence on the sound produced, the method is described, with focus on the novel measurement of the bow-bridge distance. The use of the method is then illustrated by registrations of typical bowing patterns. In these examples, particular interest is directed towards the player's use of the bow-bridge distance and bow velocity in controlling the dynamic level. Finally, some advantages and drawbacks of the measurement method are discussed, together with some suggestions for future development.

*This article is submitted for publication in J.Acoust.Soc.Am.

I. Bowing parameters

The string player has access to the following four bowing parameters (see Fig. 1a):

- (1) Bow position, the transverse position of the bow in relation to the frog or to the tip.
- (2) Bow velocity, the velocity of the bow transverse to the strings (v_B).
- (3) Bow force, the force between bow and string, normal to the directions of the bow and string.
- (4) Bow-bridge distance, the distance along the string from the bridge to the contact point with the bow (x_B).

Although the player, in principle, is free to vary these parameters independently, only certain combinations will result in a violin tone of a normal quality. The influence of the bowing parameters on the sound produced are reviewed in Askenfelt (1986).

In short, the bow velocity and the bow-bridge distance control the amplitude of the string vibrations. The string amplitude is predicted to vary proportionally to $v_B \cdot (L/x_B)$, where L is the free string length. The bow force is coordinated with the bow velocity and the bow-bridge distance. Increasing the bow velocity or approaching the bridge demands a higher bow force in order to maintain the string vibrations. Theory predicts the lower limit for bow force to increase proportionally to $v_B \cdot (L/x_B)$ (Cremer, 1984).

Further, the bow force in combination with the bow-bridge distance determines the spectral content. A high bow force and a short bow-bridge distance gives a spectrum rich in high harmonics. Finally, the part of the bow being used, reflected by the bow position, is chosen in such way as to facilitate a bowing gesture, or even make certain bowings possible.

Regarding the bow-bridge distance, the player uses a range from very close to the bridge to a position well beyond the end of the fingerboard. Expressed in fractions of the open string length ("standardized" value 327 mm) (Hutchins, 1980, p. 190), this gives the extremes of the bow-bridge distance as approximately 1/50 (6 mm) and 1/5 (65 mm). This span in bow-bridge distance would correspond to a maximum contribution to the dynamic range of 20 dB (cf. Bradley, 1976). Normally a narrower range is used, typically between 1/30 (11 mm) and 1/6 (55 mm), offering a potential shift in dynamic level of 14 dB (Meyer,

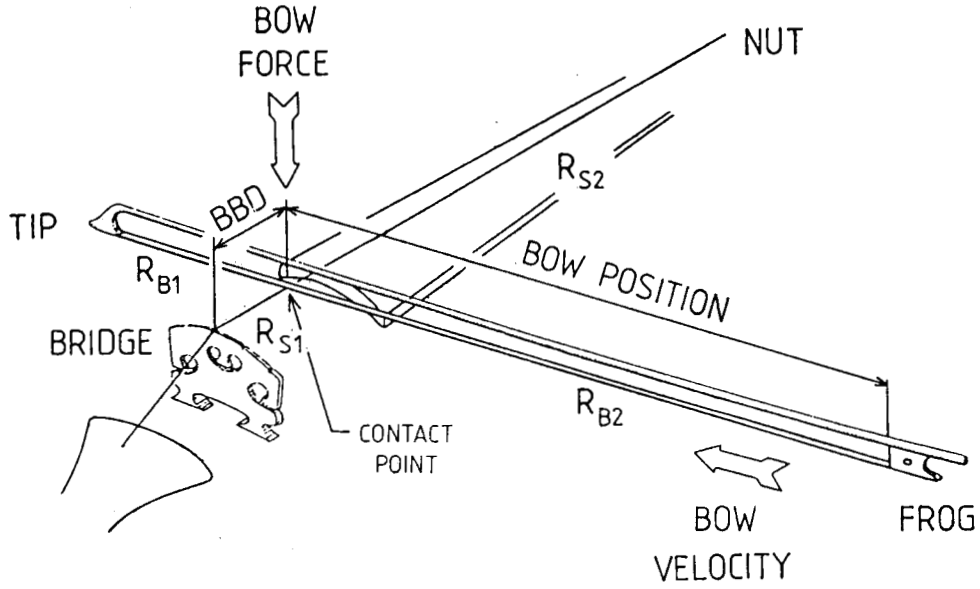


Fig. 1a. View of the violin and bow, designating the bow-bridge distance (BBD) and the bow position.

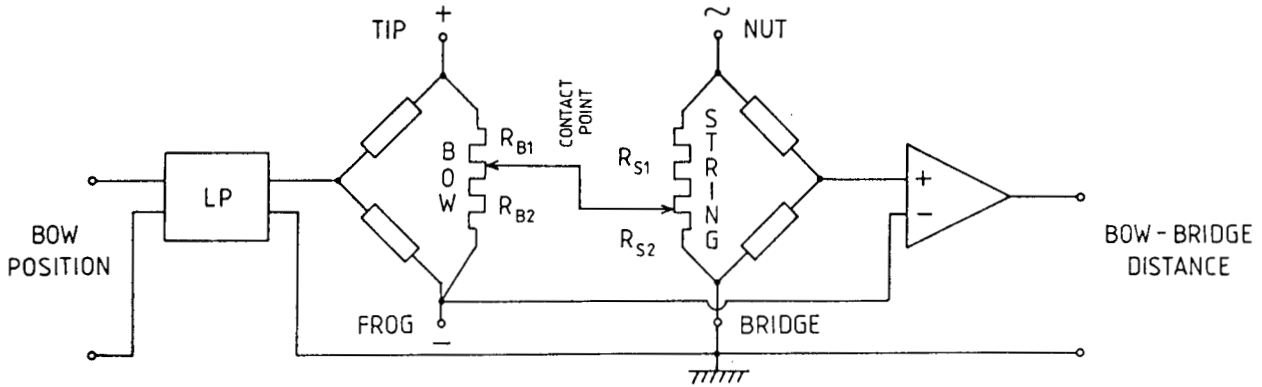


Fig. 1b. Bridge circuits used for the detection of bow position and bow-bridge distance. The "resistors" in the divided resistance wire in the bow hair (R_B) and in the divided string (R_S) are indicated in Fig. 1a. The bow position circuit was fed by a DC-supply and the bow-bridge distance circuit by a sine generator. The output signals were separated with the aid of an AC-coupled differential amplifier and a low pass filter, respectively.

1978, p. 26).

The bow-bridge distance is not explicitly given in the score, except for infrequent calls for the extreme positions *sul ponticello* ("at the bridge") and *sul tasto* ("above the fingerboard"). The reason is that an exact choice of bowing point is in most cases not critical to the sound produced. Due to the relatively large width of the bundle of bow hair (approximately 10 mm) it is, for instance, hardly possible to cancel a set of harmonics by bowing at a certain nodal point (Schelleng, 1973; Cremer, 1984). The choice of bow-bridge distance is thus left to the player, who coordinates the bow-bridge distance with the other bowing parameters with respect to dynamic level, expression marks, etc. An extensive treatment of the guiding principles for the choice of bowing parameters with regard to dynamics, tonal quality, and expression is given in classical works by Flesch (1929; 1931) and more recently by Galamian (1962).

A pedagogically motivated interest in the skilled player's handling of the bow resulted in early experimental studies of bow motion (Hodgson, 1934). At that time, registrations were made of the movements of the player's hand and arm. Small electric bulbs were fastened to selected points on the player and the paths of the light sources were caught on a photographic plate ("cyclegraphs").

II. Method

The following section focuses on the method for measuring the bow-bridge distance, while the measurement of the other bowing parameters will be only briefly described. A thorough description of the measurement methods for these parameters is to be found in Askenfelt (1986).

A. Bow position

The bow position was detected by inserting a thin resistance wire amongst the bow hairs. This wire was divided into two sections by the string, with the ratio of the wire sections determined by the momentary transverse position of the bow. The two wire "resistors" were used in one arm of a Wheatstone bridge (see Fig. 1). The position of the bow was reflected in the output signal from the bridge, with the balanced condition corresponding to the midpoint of the bow in contact with the string. The bow position signal was low-pass filtered (time constant 6 ms) in order to remove noise introduced by the varying electrical contact between the bow wire and the string.

The resolution in bow position on the recorded chart was approximately 10 mm bow movement, corresponding to about 2% of the entire bow hair length (580 mm).

The wire was not mounted in the middle of the bundle of the bow hair, but a few millimeters from the edge facing the fingerboard. The

reason was that the violinist usually tilts the bow-stick towards the fingerboard when playing at soft dynamic levels or in the upper positions, in order to decrease the effective width of the bow hair (Flesch, 1929; Trendelenburg, 1925).

B. Bow velocity

The bow velocity was derived from the bow position signal in an RC-network with a time constant of 10 ms, which allowed a differentiation of the components in the bow motion up to 16 Hz.

C. Bow force

The bow force was measured by means of strain gauges. The bow hair was cut at both ends and glued to thin metal strips fastened to the tip and the frog. Four strain gauges were glued to the strips and connected in a second Wheatstone bridge. The bending of the strips generated a signal reflecting the normal force between bow and string. The bridge was fed by a sine generator (1 kHz). The output was band-pass filtered (time constant 10 ms), amplified, rectified, and smoothed in a low-pass filter with a time constant of 3 ms.

The error in bow force was normally well below 10% but could occasionally reach higher values during unusual combinations of bow force and bow position.

D. Bow-bridge distance

The bow-bridge distance was measured in a third Wheatstone bridge using the same detection principle as for the bow position (see Fig. 1). The string itself was now used as a resistance wire. The bow wire divided the string into two "resistors", with the resistance ratio of the string parts determined by the momentary contact point of the bow along the string. The string "resistors" were used in one arm of the third bridge, the output of which reflected the bow-bridge distance. The bridge was fed by a sine generator (10 kHz). The output was amplified, rectified and smoothed in a low-pass filter with a time constant of 3 ms.

A calibration of the bow-bridge distance signal against measured distance along the string showed a linear relationship within the positioning accuracy of the bow wire (± 0.5 mm), with no observed dependence on bow position. In playing, an error was introduced due to the bow wire which not could be assured to always form a perfect straight line parallel to the bow hair. This error was estimated to approximately ± 1 mm at a maximum. By tightening the bow wire, this error could be reduced, but too high a tension would have a detrimental influence on the playing properties of the bow.

The resolution in the bow-bridge distance on the recorded chart was approximately 1 mm string distance, corresponding to about 2% of the

maximum range used. The values for the bow-bridge distance given in the following refer to the distance from the bridge to the nearest edge of the bow hairs. The width of the bow hair was 3 mm.

E. Considerations on the measurements of the bow-bridge distance

The bridge circuits for the bow position and the bow-bridge distance were inseparably connected by the contact point between bow and string, and consequently their signals were superimposed. Further, this common contact point, relative to which the output signals from both bridges normally were to be measured, was not accessible for direct connection. These circumstances made a modification of the normal bridge circuitry necessary.

- (a) The bow position bridge was connected to a DC-voltage, and the bridge for the bow-bridge distance was fed by a sine generator, in order to be able to separate the signals in the outputs.
- (b) The output from the bridge circuit for the bow-bridge distance was measured via an AC-coupled differential amplifier with one input connected to the frog end of the bow wire. This connection point substituted the inaccessible contact point between bow and string.
- (c) The output from the bow position bridge was measured relative to the string termination at the bridge, which served as another substitution point. The bow position signal was separated from the bow-bridge distance signal in a low pass filter (time constant 6 ms). This same filter also smoothed the bow position signal.

The measuring errors introduced by these modifications of the bridge connections were negligible (<0.1%), due to high input impedances of the low-pass filter and differential amplifier.

A few practical problems were associated with the measurement of the bow-bridge distance. One problem was the low resistance of the strings which made other varying small resistances in the bridge circuit disturbing. Another was the varying contact resistance between the bow wire and the string, which introduced noise in the signals for the bow-bridge distance and the bow position.

The string resistance is determined by the composition of the string. The strings used in the experiments were made from a nylon core wrapped with silver or aluminium.* The resistance of a G-string (silver wrapped) of this set was approximately 3 ohm when mounted on the violin. This was approximately six times higher than the resistance of an all-metal G-string (steel wire wrapped with chrome-steel) from the same

*PRIM: "Synthetic core"

maker. Although the resistance of a synthetic string thus was considerably higher than of a metal string, the resulting low resistance of all four strings in parallel would represent a complication when monitoring playing on any of the strings.

The resistance of a synthetic string depends on the tension. When tension is increased resistance increases, most probably because the turns in the wrapping become slightly separated. The resistance of the synthetic G-string increased from 1 to 3 ohm approximately, when measured unused as compared to strung up on the violin. Consequently, the bow-bridge distance signal had to be calibrated each time the string was tuned.

As the string exhibited a low resistance, it was essential that the contact resistance in the cable connections was constant. If not, the effective supply voltage over the string would fluctuate and hence also the output signal. The best connections were achieved using a copper foil with conductive adhesive*. The cables were soldered to pieces of foil, which were wrapped around the passive parts of the string.

Connections with different types of miniature clamps were also tried but gave too much variation in contact resistance when the violin was moved during playing. Direct connection to a synthetic string by soldering is not possible as the core will break immediately.

The conditions of the contact between string and bow wire demanded continuous attention. After some playing, the bow wire and string accumulated rosin which gave a poor electrical contact. A moderate use of rosin combined with a frequent cleaning of the strings and wire was necessary, in order to keep the noise in the signals for bow position and bow-bridge distance at acceptably low levels.

The conditions were improved significantly by adding some silver powder to the rosin. The silver stuck to the rosin and made it partially conducting without detracting from the playing properties. A commercially available rosin featuring metal ingredients** was tried, but it was found to contain too little metal powder to improve the contact conditions.

Adding silver powder, in combination with smoothing of the bow position signal, was a prerequisite for obtaining a useful bow velocity signal by differentiation of the bow position signal.

* 3M: "Copper foil with conductive acrylic pressure sensitive adhesive" 1181.

**Liebenzeller: "Metall-Kolophonium"

F. Dynamic level

The dynamic level was estimated by measuring the vibration (acceleration) on the top plate, close to the left bridge foot (bass bar side). The correspondence between string displacement and top plate acceleration was checked with the aid of an optical string detector on the bridge. For a given pitch, the difference was in no case more than 1 dB, over a span of more than 30 dB. The resolution in the vibration level on the recorded chart was approximately 0.5 dB.

G. Recording

The five signals representing the vibration level, bow force, bow-bridge distance, bow position and bow velocity, were recorded on two synchronized ink jet plotters, using a paper speed of 25 mm/s.

H. Players' comments

As in the preceding study (Askenfelt, 1986), the players reported that the preparations of the bow and the connections to the bow and the violin did not disturb their playing.

III. Measurements and results

In this section, measurements of the bowing parameters for a selection of bowing patterns are presented. The examples include sustained notes, scales, crescendo-diminuendo, sforzando, and saltellato*. The results are discussed in connection with each example, with focus on the use of the bow-bridge distance and the bow velocity. The examples were collected from the performances of two professional violinists.

A. Sustained notes

1. Dynamic level and bowing parameters

Long notes were played détaché ("separated") on the open G-string in three dynamic levels: forte, mezzo forte, piano (see Fig. 2). The durations of the notes were approximately 2 s (whole notes). The values of the bowing parameters used by the two players are displayed in Fig. 3a-3c, each bar representing an average over approximately 10 s. The maximum and minimum values observed during the notes are also indicated in the figure. The corresponding vibration levels are given in Fig. 3d.

* Measurements of transverse bow motion and bow force for additional bowing patterns can be found in Askenfelt (1986). In both studies, the presented curves are tracings from the recorded charts.

- (a) The bow-bridge distance was decreased with rising dynamic level, from approximately 40 mm in piano to 20 mm in forte. The variation in the bow-bridge distance was rather large, about 10 mm for one of the players, and 20 mm for the second. This second player increased his variation at softer levels, while the other decreased the variation.
- (b) The bow velocity was held about the same for all three dynamic levels, between 0.2-0.3 m/s. Surprisingly, one of the players increased the bow velocity slightly with decreasing dynamic level, reaching the highest velocity at piano level. The variation in velocity was also rather large, exceeding 0.1 m/s, but the shift from one extreme to the other was slow, lasting several seconds. This was in conformity with the changes in the other parameters.
- (c) The bow force was increased continuously with the dynamic level, from approximately 0.5 N in piano to 2 N in forte. The variation in force was rather small, but increased with dynamic level up to approximately 0.5 N in forte.
- (d) The vibration level - The changes in average vibration level from forte to mezzo forte were approximately 5 and 6 dB for the two players (see Fig. 3d). The corresponding changes between mezzo forte and piano were 4 and 3 dB. This gave a total span of about 9 dB between piano and forte for each player. One player was asked to repeat the examples on a second occasion, at which he was instructed to pay special attention to the given dynamic markings. The observed differences were slightly larger in this later experiment, reaching a total span of 10 dB.

The predicted contributions to the changes in dynamic levels from the bow-bridge distance and bow velocity, are included in Fig. 3d. These values indicate that the bow-bridge distance was the main bowing parameter for changing the dynamic level. However, the figure also shows that the players used two different strategies to achieve the level changes. One player (BO) reduced the bow velocity and increased the bow-bridge distance at softer dynamic levels. The second player (SL), on the other hand, continuously increased the bow velocity, but neutralized this increase with larger shifts in the bow-bridge distance.

This latter "reversed" use of the bow velocity in setting the dynamic level may have a simple explanation. As the perceived loudness depends not only on the vibration level but also on the harmonic content, it is possible to obtain a perceptual difference in dynamic level

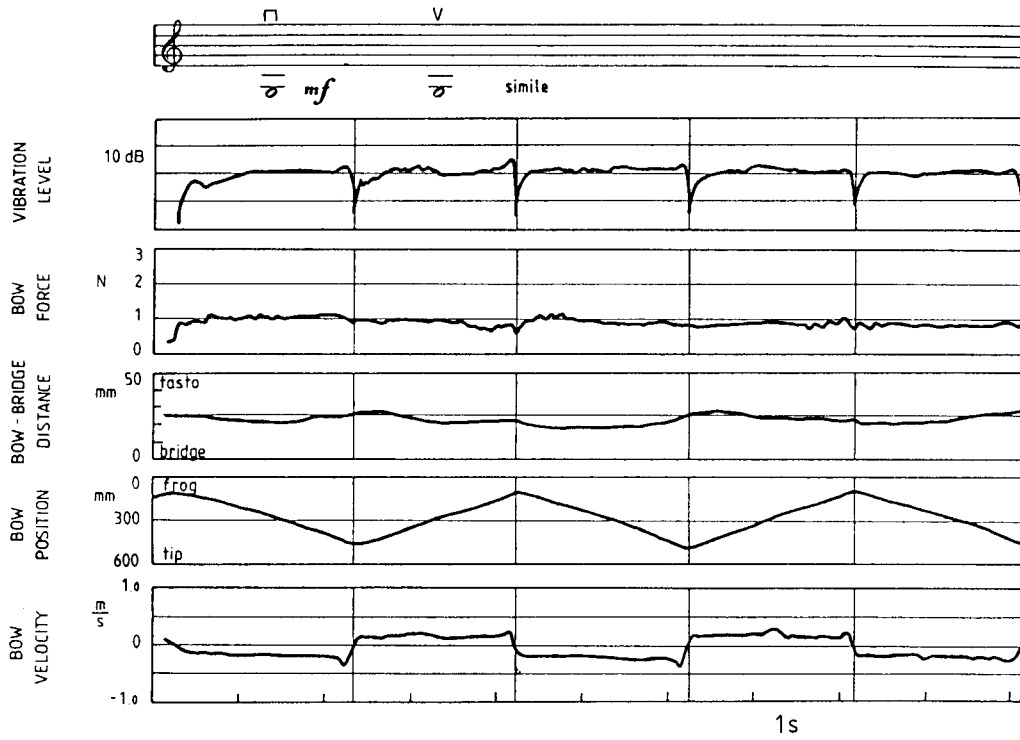
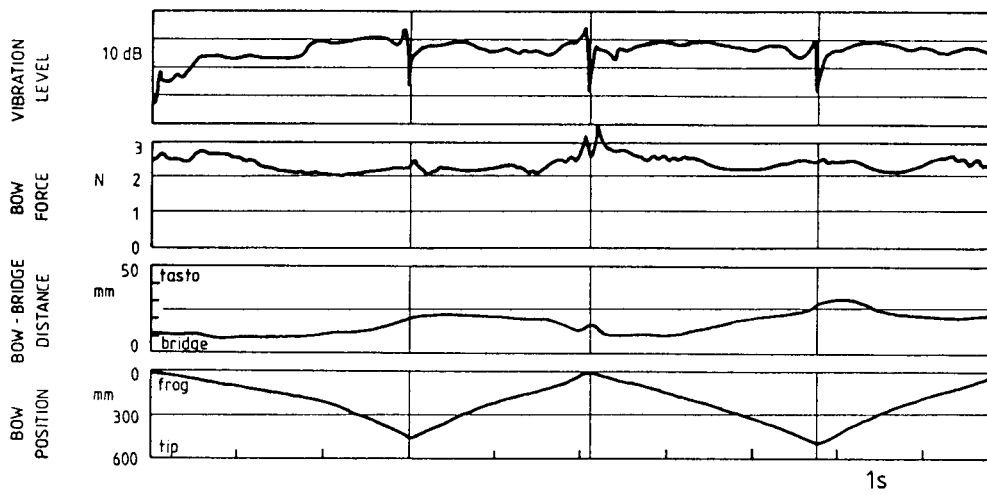
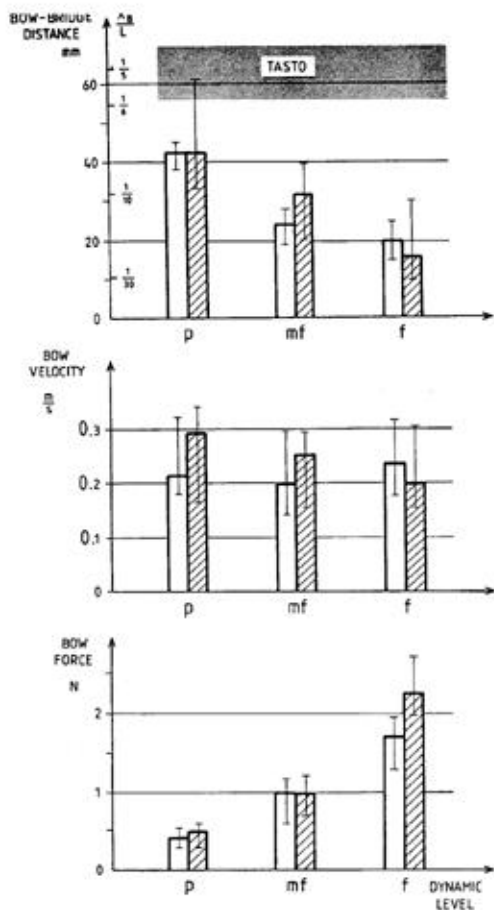


Fig. 2. Registration of detaché bowing on the open G-string.
a. Mezzo forte (player BO).



b. Forte (player SL).



a)

b)

c)

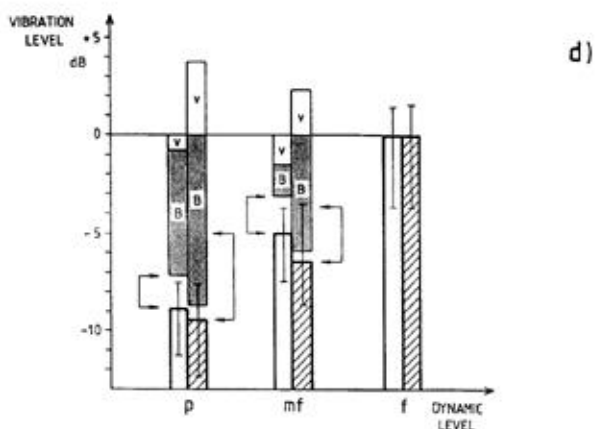


Fig. 3. Values of bowing parameters used by two players in bowing whole notes at three dynamic levels (piano, mezzo forte, forte) together with the corresponding vibration levels. The bars indicate averages over approximately 10 s for the two players (BO unfilled, SL hatched). The maximum and minimum values observed are indicated by the bar line in each bar; the initial changes during the first note and the transient fluctuations in the immediate proximity of the bow changes were excluded.

a. Bow-bridge distance. The distance is alternatively given as a fraction of the string length (x_B/L). The dotted area (tasto) marks the position of the finger-board.

b. Bow velocity.

c. Bow force

d. Vibration level. The predicted changes in vibration due to the changes in bow-bridge distance and bow velocity are marked with B and v, respectively. The discrepancies between the predicted and observed vibration levels are indicated with squared brackets*.

* Due to an unfortunate loss of absolute calibration in a range of ± 2 dB, the vibration levels for player SL have been shifted arbitrarily to give the same level in forte as for the other player.

by changing the relative strength of the higher harmonics. As previously noted, the higher harmonics are attenuated both by a reduction in the bow force as well as an increase in the bow-bridge distance, whereas a decrease in the bow velocity reduces mainly the vibration level.

This particular player seems to have made full use of the dynamic changes induced by the variation in the harmonic content, using both a shorter bow-bridge distance and a higher bow force in forte than the other player, cf. Fig. 3a and 3c. The combination of high force and short bow-bridge distance requires a slower bow velocity, which would explain the observed behavior. Terms commonly used to describe the tone quality associated with such a combination of bowing parameters would be "projecting" or "focusing".

Fig. 3d suggests that the measured changes in vibration level cannot be explained entirely by the changes in the bow-bridge distance and the bow velocity, according to classic theory. Probably, the discrepancies were in part due to the fact that the plotted quantities represented averages over roughly 10 s, and, in fact, never occurred simultaneously. Indeed, the variations in the bow-bridge distance and the bow velocity during the notes were large enough to cover the discrepancies between observed and predicted levels.

However, it is interesting to note that in all four cases the measured shifts in levels were larger than the predicted changes. Further, it can be noted that the largest discrepancy, reaching 4 dB, was found for the player who used the largest changes in bow force. A novel theoretical study by Boutillon & Weinreich (1987) put forward the possibility of an influence of the bow force on the amplitude of the string vibrations, which could be a part of the explanation of the observed differences. On the other hand, earlier experiments with a bowing machine (Müller, 1962) indicated that the vibration amplitude was not influenced by the bow force over a range in force of 14 dB*.

The contrast between forte and piano may seem surprisingly small, but is in accordance with earlier studies (Clark & Luce, 1965) which for the violin reported a typical difference of 14 dB between fortissimo and pianissimo appropriate for "typical orchestra music". Considering the number of dynamic markings to be at least six ((ppp), pp, p, mp, mf, f, ff, (fff)), it is not unreasonable to expect the violinists in this study as well to perform over a range of approximately 15 dB between fortissimo and pianissimo.

The maximum dynamic range of the violin, measured as the difference in level between the softest and loudest possible playing, was found to be 37 dB for the open G-string; however, without full consideration to

* Bradley (1976) examined a smaller force range of 6 dB. He obtained a difference in sound pressure level of approximately 2 dB but interpreted his result as support for the theory that the vibration level is not influenced by the bow force.

the tone quality at the upper limit. This result is in very close agreement with earlier predictions (Bradley, 1986; Meyer, 1978, p. 30). A reduction of the upper level in order to achieve a musically acceptable tone quality resulted in a maximum span of 31 dB, in concordance with measurements earlier reported (Burghauser & Spelda, 1971).

2. Observations on the bowing technique

In spite of the simple nature of this example, several interesting details reflecting the bowing technique can be observed. Some of the details were particularly clear in the registrations of one of the players, cf. Fig. 2a and 2b.

(1) The players moved the bow away from the bridge in down-bows and approached the bridge in up-bows. This motion resulted in bow-bridge distances generally longer at the tip and shorter close to the frog.

According to the players, this change in bow-bridge distance during the bow strokes was intentional in order to avoid a sul ponticello-coloring in the tone quality when approaching the tip. The explanation can probably be assigned to the continuous tradeoff in the supply of the bow force from two different sources during the bow stroke (Trendelenburg, 1925, pp. 71, 95, 263). At bow positions close to the frog, a substantial portion of the necessary bow force is supplied by the weight of the bow. This force must be successively replaced by a down-bearing force from the player's index-finger on the bow stick during the down bow. The pressing of the bow against the string when approaching the tip seems to give a deterioration in tone quality*, unless compensated for by an increase in the bow-bridge distance.

This change in bow-bridge distance during the bow strokes ("crooked", "slanted" bowing), which was observed for both subjects, is at variance with established old violin methods** (Flesch, 1929, Band I, pp. 40, 63; Flesch, 1931, p. 6; Hodgson, 1934, p. 17; Kross, 1904; Kuchler, 1929) but recommended by a modern authority for long notes such as in this example (Galamian, 1962, p. 61).

Irrespective of the recommendations of the violin schools, the motion suggests itself during a full bow stroke, because of the angular motion of the player's arm. However, the less advanced player often finds it difficult to keep the slanted bow motion within limits.

* L. Frydén, personal communication (conservatory of the Swedish Radio).

**Trendelenburg (1925), pp. 32-24, 256, discussing the difference between the artist's and the beginner's use of the slanted bow.

(2) The bow strokes were terminated by a slight increase in bow velocity, preceding the shifts of bowing direction ("bow changes"). This increase in velocity was reflected as a peak in the vibration level preceding the dip in level caused by the bow change. Possibly, this playing manner belongs to the *detaché* character of the performed notes, emphasizing the boundaries between the notes. The pattern was observed to be more pronounced with increasing dynamic level.

(3) During the bow change at the frog, the players made a temporary shift in the bow-bridge distance. Just before the bow change, the bow-bridge distance was increased and afterwards the bow was returned to its initial bow-bridge distance. It was observed that this bowing gesture was accomplished by a circular clockwise motion of the fingers during the bow change.

This motion is a part of the process of controlling the bow-bridge distance during the up-bow and making a non-disturbing bow change (Trendelenburg, 1925, pp. 45-58, 99-102; Kùchler, 1929, p. 28; Flesch, 1929, Band I, pp. 41-43; Galamian, 1962, pp. 48. 86). With the bow at the tip at the beginning of an up-bow, the hand and arm are tilted somewhat in the bowing direction, while the wrist is held essentially straight. Because of the circular motions of the overarm and the forearm during the bow stroke, the direction of the bow would not follow a line in parallel with the bridge, unless compensated for. This compensation is achieved in part by twisting the wrist outwards laterally as the frog is approached. When the frog is reached, the hand may form an angle of 30-40° with the forearm.

Shortly before the bow change, the velocity of the forearm and hand is reduced and eventually brought to a stop, while the fingers extend the up-bow motion the very last distance ("finger-stroke"). This motion is accomplished by straightening the fingers from their normal slightly bent position. Due to the straightening of the fingers, the frog makes a circular motion which is reflected as an increase in the bow-bridge distance. The arm starts the motion in the following down-bow, which brings the bow-bridge distance back to its initial value and successively also resets the finger position.

At the tip, a different technique is used for the bow change, which is not reflected as a change in the bow-bridge distance.

(4) It may seem that the player in Fig. 2b (SL) disposed a potential increase in the bow velocity in forte as he did not use the entire bow length, but avoided approximately the upper fifth of the bow. Probably, the early bow change was made in order to avoid the uncomfortable and even difficult situation of supplying a high bow force close to the tip

(Galamian, 1962, p. 57; Trendelenburg, 1925, p. 95; Frydén, pers.com.).

Also, as discussed in connection with Fig. 3d, this player relied on a short bow-bridge distance and high bow force in producing the forte and could, presumably, not make use of a large increase in bow velocity and correspondingly longer bow strokes. Consequently, the shortened bow stroke probably did not represent a large sacrifice in bow velocity in order to "save bow" for the entire duration of the notes.

(5) One of the players (SL) increased the bow force at bow positions close to the frog and decreased the force towards the tip (see Fig. 2b). This pattern in the force variation is supported by the bow itself (Trendelenburg, 1925, p. 263). The bow acts as a lever which pivots around an axis roughly through the thumb and the middle finger. This means that gravity contributes a considerable amount to the bow force in bow positions close to the frog and much less close to the tip. For a normal violin bow, this difference could be estimated as approximately 3 N, provided that the path of the bow is approximately horizontal as when playing on the G-string. This contribution to the force must be continuously compensated for if the player wants to maintain a constant bow force during the bow stroke (Flesch, 1929, Band I, p. 37; Galamian, 1962, p. 57; Kùchler, 1929, p. 27; Trendelenburg, 1925, p. 263). The compensation is achieved by balancing the bow with the index and little fingers on top of the bow stick on both sides of the pivot point. An example of a careful compensation, resulting in small variations in force is seen in Fig. 2a.

3. Dependence of note durations

Additional data on the use of the bowing parameters for sustained notes in different dynamic levels were collected by letting one player include half and quarter notes in his examples. The tempo was kept the same as for the whole notes, resulting in a succession of durations close to 2, 1, and 0.5 s. The results are summarized in Fig. 4a-4d, which shows the relations between the bowing parameters used for the different note values and vibration levels. The plotted values are averages over the duration of one note, typical for the group of note values it represents.

(a) The bow-bridge distance was decreased with increasing dynamic level for all note values. Further, the bow-bridge distance was varied more between the dynamic levels in the longer note values, approaching a trend for the whole notes of a halving of the bow-bridge distance for each doubling of the vibration level (cf. the line $x \ 0.5/+6 \text{ dB}$). This dependence suggests that the vibration levels for the whole notes were set almost entirely by the bow-bridge distance. In general, the bow-bridge distance was kept

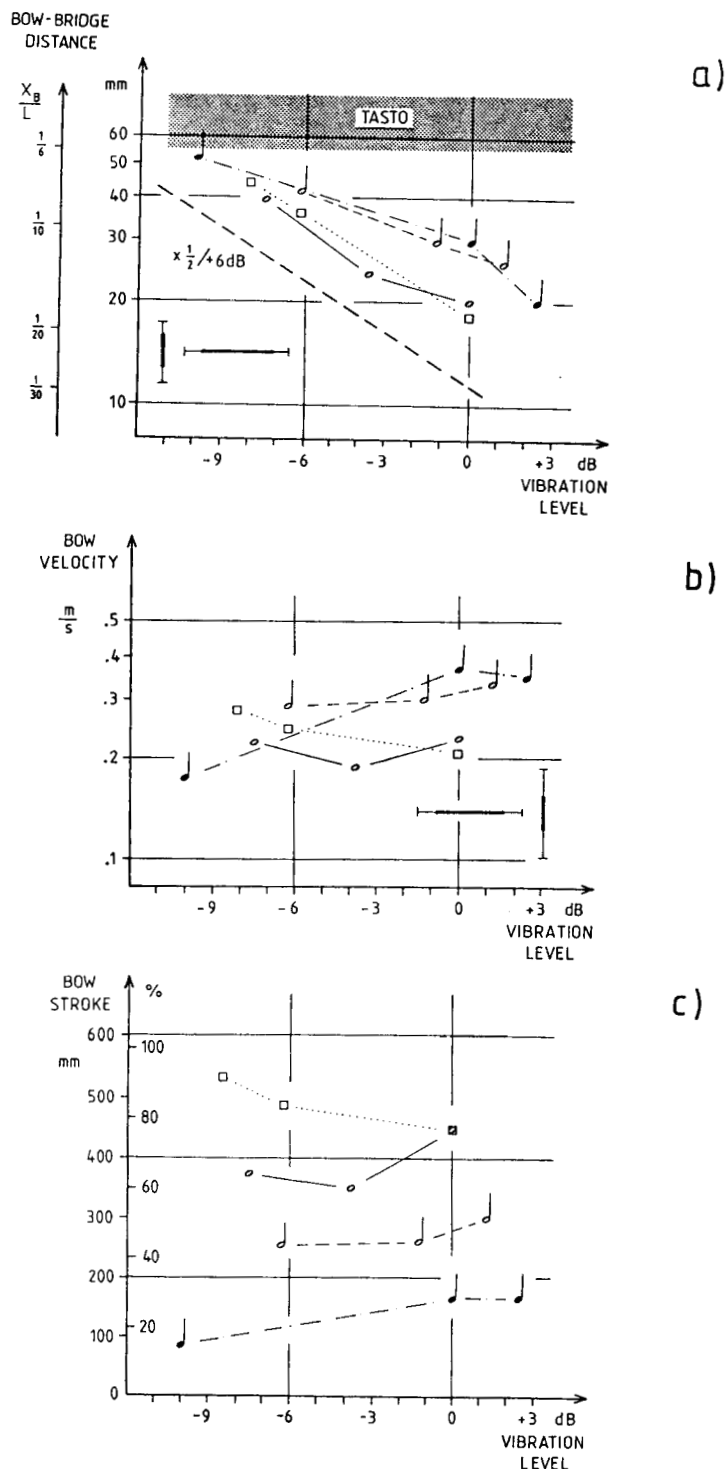
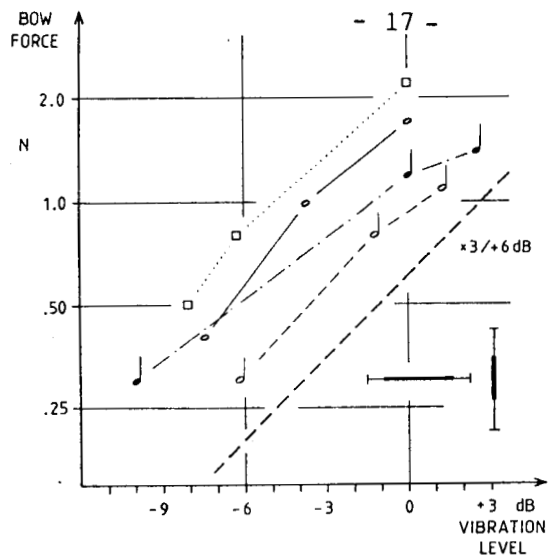


Fig. 4. Values of the bowing parameters for one player performing whole, half, and quarter notes at three dynamic levels (piano, mezzo forte, forte). Values obtained from the other player (SL, whole notes only) are included as a comparison (squares). The data points (notes) represent the average over one note, typical for the group of notes it represents. The horizontal and vertical bar lines give an indication of the variation between the extreme values within a group of notes. The variations were in no case larger than indicated by the thin lines, nor smaller than the broad lines*.

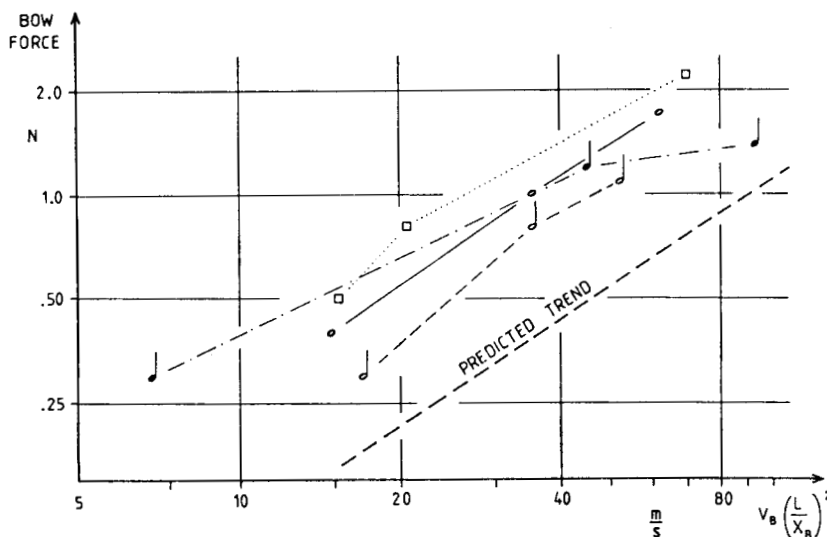
a. Bow-bridge distance. The distance is alternatively given as a fraction of the string length (x_B/L). The dotted area (tasto) marks the position of the finger-board. The broken line marked ($x \frac{1}{2} / +6 \text{ dB}$) represents an inverse proportionality between bow-bridge distance and vibration level.

b. Bow velocity.

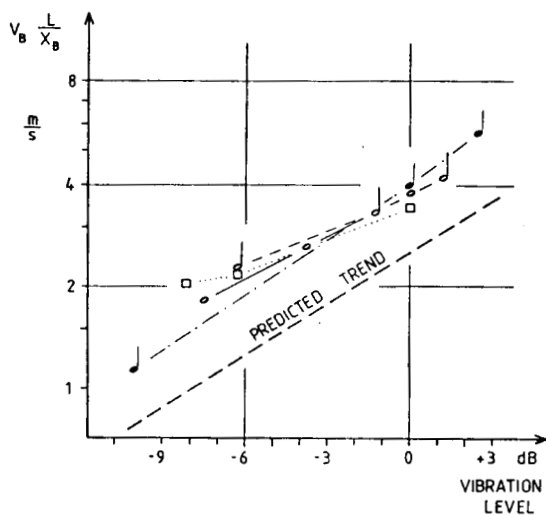
c. Bow stroke, the length of the bow portion used for a note. The values are alternatively given as a percentage of the full length of the bow hair (580 mm).



d)



e)



f)

d. Bow force. The broken line marked $(x3/+6 \text{ dB})$ represents a threefold increase in bow force for each doubling in vibration level.

e. Relations between the bow forces and the corresponding bow velocities and bow-bridge distances expressed as $v_B (L/x_B)^2$. The broken line represents the trend for minimum bow force predicted by classic theory. The line does not indicate the absolute value of the minimum bow force; the vertical position in the figure is arbitrary.

f. Relations between the vibration levels and the corresponding bow velocities and bow-bridge distances expressed as $v_B (L/x_B)$. The broken line represents the trend for increase in vibration level as predicted by classic theory. The line does not indicate the absolute value of the predicted vibration level; the horizontal position in the figure is arbitrary.

shorter the longer the note value (Flesch, 1929, Band I, p. 60).

- (b) The bow velocity showed in general no marked changes with dynamic level. Only for the quarter notes a large change could be observed between piano and mezzo forte. In particular for the whole notes, the tendency was to keep the bow velocity constant regardless of the dynamic level. This left the main control of the dynamic level to the bow-bridge distance, as assumed above. As a rule, the bow velocity was kept lower the longer the note values.

- (c) The bow stroke, i.e., the proportion of the bow length used for a note, was longer the longer the note value. A quarter note was played using only 15% of the bow length (9 cm), while a whole note could require more than 90% (52 cm). As the bow velocity in general was kept higher the shorter the note values, the bow stroke was typically not halved between whole and half notes and half and quarter notes, respectively. The bow stroke increased only slightly with rising dynamic level, because of the generally small changes in the bow velocity.

Fig. 4c also suggests a potential increase in the bow velocity for the louder dynamics, as the entire bow length never was exploited. However, as discussed earlier, the accessible bow length in forte could be assumed to be shorter than the real length, due to the higher bow forces used. This could imply that a major part of the expendable bow length was in fact used by the players for the whole notes in forte, which consequently set an upper limit for the bow velocity.

However, bow velocity was not increased in forte even for the shorter note values, for which the bow length could not have been a limiting factor. This indicates that the choice of bow velocity in setting the dynamic level is not determined solely by the accessible bow length in relation to the duration of the note. There are several other relevant factors, as discussed in connection with Fig. 3d; for example, the desired spectral content.

- (d) The bow force was consistently increased with rising vibration level. The trend was close to a threefold increase in force for each doubling in vibration level (cf. the line $\times 3/+6$ dB). The force was highest for the whole notes, probably reflecting the shorter bow-bridge distances used for these notes.

Theory predicts the minimum bow force required to increase proportionally to $v_B \cdot (L/x_B)^2$, as mentioned earlier. The players followed this dependence rather closely (see Fig. 4e). This behavior implies that they strived to maintain a constant safety factor with

respect to the lower limit in bow force throughout the dynamic range. The safety factor was made a little larger for the whole notes, possibly because they were played with shorter bow-bridge distance and lower bow velocity than the other notes. A short bow-bridge distance and a low velocity is a combination which is sensitive to small accidental changes in any of the bowing parameters.

The vibration level is predicted by theory to be proportional to the ratio $v_B \cdot (L/x_B)$. The measured changes in levels followed this prediction fairly well, when not discriminating between the data points for the different note values (see Fig. 4f). Possibly, a tendency for a steeper increase could be observed for the whole and half notes, which might suggest a dependence on the bow force as earlier discussed.

Curiously, a given dynamic level was performed louder the shorter the duration, with one exception. As an average, the level increased 2 dB for each halving in duration. No satisfactory explanation could be given to this observation.

4. Tradeoff between bow-bridge distance and bow velocity

The above examination of the use of the bowing parameters suggests that several factors determine the tradeoff between the bow-bridge distance and the bow velocity in setting the dynamic level for the sustained notes. Examples of such factors are spectral slope and accessible bow length.

A reason for using mainly the bow-bridge distance to change the dynamic level could be that it gives a more uniform quality to a series of notes in changing dynamics, including the anticipated "normal" increase in harmonic content with rising level. Large changes in the bow velocity will also give the bow changes very different characteristics. Further, a high bow velocity soon becomes uncomfortable to the player, who consequently might prefer to use a shorter bow-bridge distance.

Another factor determining the tradeoff between bow-bridge distance and bow velocity is the deflection of the string. A higher bow velocity requires a higher bow force, which presses the string downwards. When playing far from the bridge, the deflection of the string becomes considerable, which causes the bow to touch the adjacent strings already at a moderate dynamic level. For this reason, the bow has to approach the bridge in forte. If the player tries to avoid the large deflection and increases the bow velocity without increasing the bow force, the tone develops a quality entirely different from normal (flautato, "flute-like") (Trendelenburg, 1925, p. 24).

In summary, for the sustained notes, the bow-bridge distance was found to be the most important parameter for changing the dynamic level, in preference to bow velocity. Examples of factors determining the tradeoff between bow-bridge distance and bow velocity were spectral

slope, accessible bow length, typical changes in tone quality with dynamic level, deflection of the string, and presumably also comfort in playing.

B. Scales, sul tasto, sul ponticello

A one-octave G-major scale was played on the G-string at a mezzo-forte level. A change towards a shorter bow-bridge distance with decreasing free string length could be observed and vice versa. The bow-bridge distance was decreased from about 20 mm for the open string to 10 mm for the octave position, for bow positions close to the frog. As in the earlier examples, the bow-bridge distance generally increased as the bow position approached the tip.

This coordinated variation of the bow-bridge distance with fundamental frequency indicates that the players tried to keep the bow at approximately the same fraction of the free string length throughout the scale. The observation is in accordance with the recommendations of the recognized violin methods (Flesch, 1929, Band I, p. 60; Galamian, 1962, p. 58; Trendelenburg, 1925, p. 77).

The players were also asked to interpret the score instructions sul tasto and sul ponticello, which were performed in a mezzo piano level. Compared to a normal bow-bridge distance of about 25-35 mm, a distance of 55-60 mm was observed in sul tasto, and 1-4 mm in sul ponticello.

An informal check of the players' ability to control the bow-bridge distance was made at the end of the experiments. The players were instructed to maintain as constant a bow-bridge distance as possible during a long bow-stroke. For one player, the total variation was within 6 mm during a down- and up-bow which lasted approximately 5 s, the major changes occurring when approaching the tip. The variation during the steady state part of the bow strokes between the bow changes was less than 2 mm. When the player was asked to close his eyes, the same total variation was observed, but with a slow drift towards the bridge superimposed.

The other player chose a lower bow speed. After an initial adaption to a suitable bow-bridge distance, this player was able to keep the total variation in the bow-bridge distance within less than 2 mm, during a 20 s slow down- and up-bow with open eyes.

In conclusion, the players seem to be able to control the bow-bridge distance very accurately when given a suitable task, such as in this example.

C. Crescendo-diminuendo

Two versions of a crescendo-diminuendo were performed, upbow-downbow and the reversed (see Fig. 5). The upbow-downbow version is preferred because the contribution to the bow force from gravity automatically increases towards the frog in the crescendo, and decreases

towards the tip in the diminuendo (Flesch, 1929, Band I, p. 70; Trendelenburg, 1925, p. 263). This is due to the leverage of the bow, pivoted in the finger grip. A further implication of the leverage is that the accessible bow force, mainly supplied by the player's index finger on the bow stick, is much higher close to the frog than at the tip.

The up-bow crescendo was started with the bow about 35 mm from the bridge, the distance of which was roughly halved during the bow stroke. In combination with an accompanying threefold increase in bow velocity up to 0.3 m/s, a span in vibration level of approximately 20 dB was achieved. In the diminuendo, the bow-bridge distance, the bow velocity, and the bow force were reverted to their initial values. The player coordinated the bow force with the changes in bow-bridge distance and bow velocity, using a high force during the fast parts close to the bridge, reaching approximately 2.5 N at a maximum.

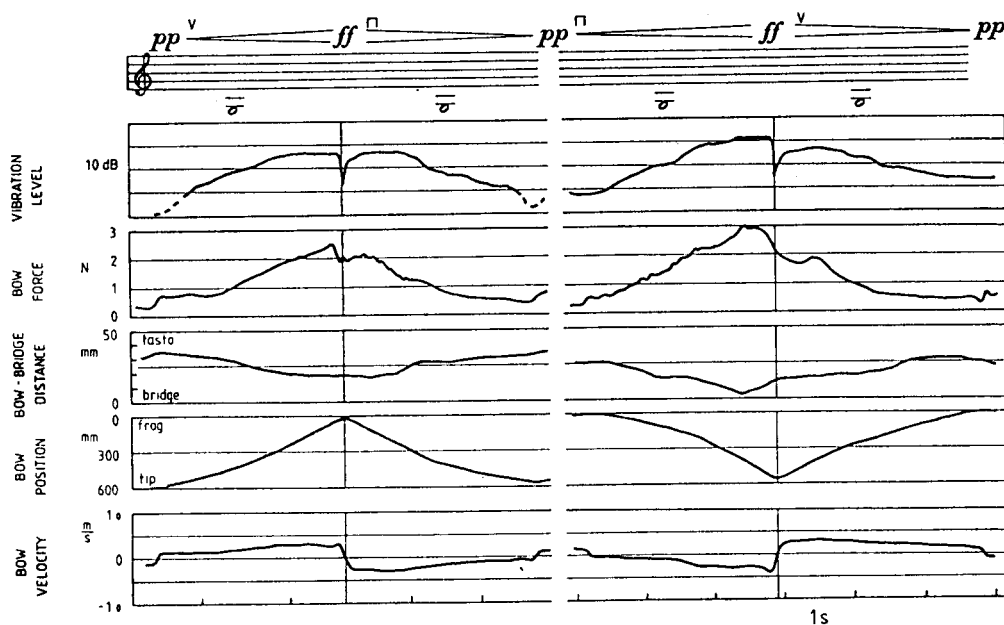


Fig. 5. Crescendo-diminuendo played upbow - downbow (left), and downbow - and upbow (right). The parts in the curve for the vibration level indicated with broken lines represent segments during which it is uncertain if the player maintained the string vibrations properly.

In the reversed version starting with a down-bow, the changes in bow-bridge distance and bow velocity were more pronounced, which resulted in a slightly larger dynamic span of 25 dB. The changes in bow-bridge distance and bow velocity would contribute 13 dB and 12 dB to

this shift, respectively.

Interestingly, in the second version the point of minimum bow-bridge distance (about 4 mm), and a very high vibration level, was reached shortly before the bow change. However, this short bow-bridge distance was not maintained but immediately increased, resulting in a bow-bridge distance of about 15 mm at the actual bow change. Possibly, the player had aimed at a continued short bow-bridge distance at the bow change but perceived a beginning poor control over the string vibrations. For this reason, the bow-bridge distance was increased while a high bow force was maintained, in order to regain a safety margin. The assumption that the bow partially lost control over the string is supported by the fact that the vibration level only changed marginally during the last 10 mm of decrease and increase in the bow-bridge distance before the bow change, although the bow velocity was essentially constant or even increased slightly. After the bow change, the player could not reestablish the same high vibration level, but stayed at approximately the same level as in the preceding version.

An alternative interpretation of the course of events before the bow change could be that the player never aimed at the extreme short bow-bridge distance. When forced to supply a high bow force at a position close to the tip, a too short bow-bridge distance was reached accidentally and immediately corrected.

In summary, for the *crescendi-diminuendi*, the contributions to the span in dynamic level from bow-bridge distance and bow velocity were of the same magnitude. A large total dynamic span of about 25 dB was reached.

D. Sforzando

Repeated half-notes marked *sforzando* ("forced", "stressed") were played (see Fig. 6). The strong initial attack was obtained by using a high bow velocity, reaching about 1.4 m/s, combined with a short bow-bridge distance of approximately 10 mm. The rate of increase in dynamic level was high, approximately 100 dB/s at maximum.

After the attack, the velocity was slowed down to 0.1 m/s and the bow-bridge distance increased to 20 mm. These changes in the bowing parameters would correspond to a difference in level of 29 dB between the attack and the following sustained portion of the note. The change in bow velocity would contribute approximately 23 dB, and the change in bow-bridge distance should add the remaining 6 dB. The observed span in dynamic level was in reasonable agreement with the predicted values.

The decrease in bow-bridge distance was coordinated with the increase in bow velocity during the attack, more successfully so during the first attack in the down-stroke, in which the minimum bow-bridge distance coincides with the maximum in bow velocity. In the second attack, the minimum bow-bridge distance was reached a little too late,

possibly due to a poorer control of the bow-bridge distance in an up-bow on the upper half of the bow ("push" instead of "pull"). This slip in coordination resulted in a loss of roughly 5 dB in peak level, also in approximate agreement with the predicted value.

In both cases, the bow force was well coordinated with the rapid increase in bow velocity, reaching a maximum value of roughly 3 N. The peak in bow force did not coincide exactly with the maximum in bow velocity, but was delivered slightly earlier where the increase in the vibration level was the fastest (Schelleng, 1973, p. 33).

The decay in vibration level was observed to follow two different patterns for the down- and up-bows, respectively. In the up-bow, the initial peak was followed by a sustained portion at a rather constant lower level, as expected according to the changes in the bowing parameters.

In the down-bow, however, the vibration level continued to decay after the attack to a much lower level than in the up-bow, although the bow-bridge distance and bow velocities were essentially identical. This difference between the up- and down-bows probably reflected a difficulty in maintaining a proper string motion, using a low velocity and very low bow force when receding from the frog. The somewhat restricted use of rosin in the experiments, which facilitated the measurements, might have contributed in part to this difficulty.

In summary, for the sforzandi, the contributions to the changes in dynamic level from the bow velocity far exceeded the contributions from the bow-bridge distance. This pronounced use of changes in velocity compared to the earlier examples, was probably necessitated by the demand for fast changes in level, a requirement which could hardly be met by using changes in bow-bridge distance.

E. Saltellato

Repeated sixteenth notes were played saltellato ("hopped"), with a superimposed crescendo-diminuendo (see Fig. 7). The player modulated the dynamic level in part by lengthening the bow strokes to about 40 mm during the crescendo and by shortening the strokes in the diminuendo, thus regulating the bow speed between approximately 0.1 and 0.7 m/s. In addition, the bow-bridge distance was changed from 30 mm to 15 mm, in order to accomplish the dynamic changes. The bow force was modified in accordance with the changes in bow velocity and bow-bridge distance between 1 and 3 N, approximately. The performed dynamic range was about 20 dB, of which the bow-bridge distance should contribute about 6 dB.

The entire example was played on the lower quarter of the bow. However, the player approached the frog in the crescendo and vice versa (Flesch, 1929, Band I, pp. 53-54; Galamian, 1962, p. 77), in order to let the weight of the bow supply a main part of the successively higher bow force in forte. This change in bow position goes hand in hand with

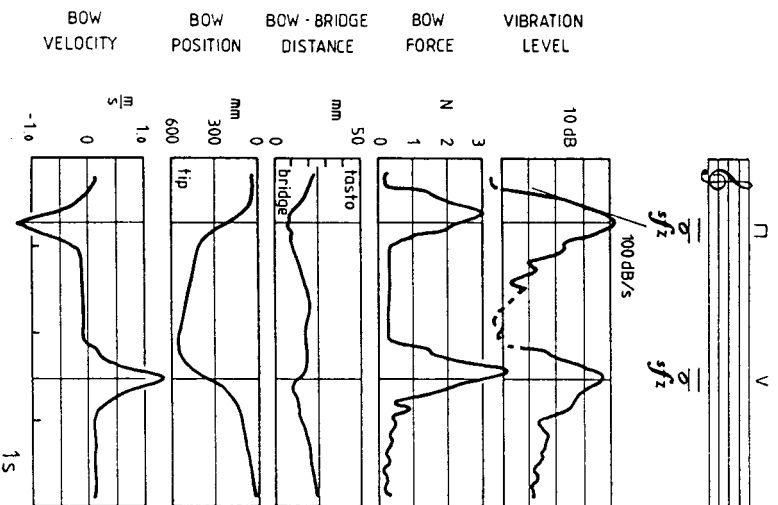


Fig. 6. Sforzando played in a down-bow (left) and in an up-bow (right). The dashed parts in the curve of vibration level represent segments during which it is uncertain if the player properly maintained the string vibrations.

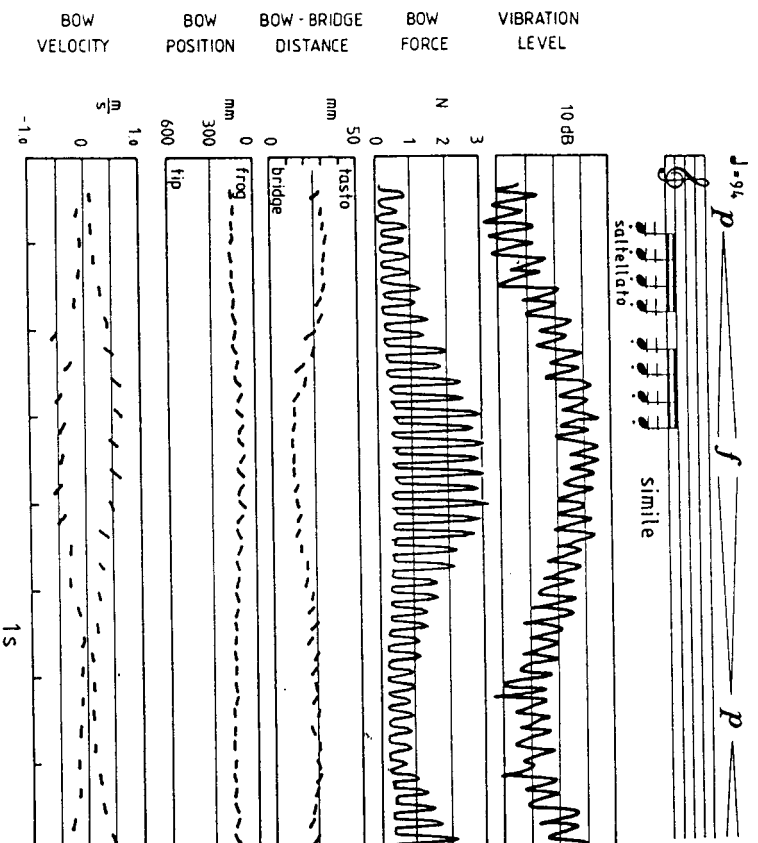


Fig. 7. Sixteenth notes played saltellato ("hopped") with a crescendo-diminuendo superimposed.

the need for a higher stiffness of the bow for higher bow forces, in order to avoid that the bow hair touches the bow stick. The stiffness of the bow increases rapidly towards the frog. The change in average bow position between piano and forte was roughly 90 mm.

Further, as the effective moving mass increases when the pivot point at the frog is approached, a higher stiffness is also desirable in order to help preserving the bouncing frequency of the bow*. The compliance of the string is added to the compliance of the bow, which could be assumed to have influence on the range in bow-bridge distance which could be used for a bouncing bowing pattern.

Most of the notes were played with a slight variation in the bow-bridge distance, approaching the bridge towards the end of the notes. The same pattern was observed both for down- and up-bows, which indicates that the change was not caused by a slanted bowing direction at an angle to the bridge. Possibly, the motion reflected an ambition by the player to make the attacks gentle by starting the note with a slightly longer bow-bridge distance.

Interestingly, the bow force as well as the vibration level in Fig. 7 were often a little higher in the up-bows than in the down-bows. This pattern suggests that the player (BO) superimposed a structure on the flow of repeated notes by putting some stress on every second note. The other player (SL) showed a similar pattern, but consistently applying a higher bow force in the down-bows. This way of structuring the flow may seem more natural, as notes occurring on a beat usually are played down-bow.

In summary, for the saltellato, the contributions to the dynamic span from the changes in the bow velocity were about twice as large as the contributions from the shifts in bow-bridge distance. The change in bow-bridge distance could be assumed to be restricted to a range in which the resulting compliance of the string and bow supports the desired bouncing frequency of the bow.

IV. Discussion

A. Measurement method

The experiments show that the method could be successfully applied for measuring the bowing parameters in violin playing. A main feature is simple detection principles for position and force, which enabled limited preparations of the violin and bow.

* Cf. Trendelenburg (1925), p. 10, which suggests that the bouncing frequency is determined more by resonances in the bow stick than by the springiness of the deflected bow hair ("bow and arrow"-model). Normally, the bouncing frequency tends to decrease towards the frog.

The most disturbing limitation of the method is the relatively high noise level in the signals representing bow position, bow velocity, and bow-bridge distance, especially in connection with low bow forces. This noise sometimes made the interpretation of the recorded signals difficult, in particular the bow velocity signal. As mentioned, the noise was due to the varying contact resistance between the wire in the bow hair and string, and thus intimately connected with the detection principle.

As an alternative to electrical bow position detection, an optical position-measuring method could be considered. One idea would be to direct a pulse-modulated laser beam from an optical fiber along the surface of the bow hair, the beam being reflected against the string and received by a detector at the frog. However, it is doubtful whether the rather thin string could serve as a reliable reflection point. Besides, the string vibrations would probably spread the reflected beam over a large angle, which would make the detection of the reflected signal uncertain. Also, the tilting of the bow earlier described, would impose another difficulty for the method. However, if brought to practical use, the same method could be applied for measuring the bow-bridge distance.

Another alternative would be to use a commercially available position measuring system*. This system features one or several cameras with position-sensitive photodetectors, which register the paths of light sources mounted on the moving parts. With two cameras and a light-emitting diode on the bow plus one on the bridge as reference, it would probably be possible to measure the bow position, the bow velocity, and the bow-bridge distance. However, the player would probably be constrained to very small movements of the instrument. Also, the cost of procuring the equipment would be very high.

Another point which might need some further attention is the bow force measurement. The bow force could be measured either on the bow or on the violin. The present method, using strain gauges on the bow, has several limitations. As discussed in Askenfelt (1986), the bow hair tension influences the calibration, the longitudinal frictional force between bow and string introduces a small measurement error, and the force sensitivity tends to vary with bow position.

However, it is not straightforward to find a better alternative. Considering a bow force measurement on the violin, possible force detectors would be piezoelectric discs, integrated as a part of an assembled bridge, or piezo foil fitted under the feet of the bridge. The response to slowly varying components in the bow force would, however,

* Selspot II. SiTek Laboratories AB, P.O.Box 261, S-433 25 Partille, Sweden.

be poor using piezoelectric detectors, and an absolute calibration would be very laborious.

The high static force from the down-bearing of the strings (roughly 100 N), compared to the small dynamic variations from the bow force (<3 N), excludes the use of miniature load cells. This is unfortunate, as these cells utilize strain gauges and show none of the disadvantages of the piezoelectric detectors.

The discussed limitations of the method suggest further development, but they are not serious enough to prevent its successful use in basic research under laboratory conditions. Monitoring of the bowing parameters is also of interest for pedagogical purposes, but the equipment is hardly suited yet for field use.

An important feature of the present method, besides its simplicity, was that the equipment did not disturb the player. This is an important point. Even if a professional player may perform to his satisfaction under awkward conditions, he might be forced to deviate from normal manners when disturbed by a heavily prepared instrument or bow. Accordingly, the guideline for future development must still be that the performance setting is uncontrived, so that the player is as little aware as possible of being monitored while playing.

B. Bowing parameters

The registrations of the bowing patterns have illustrated the high complexity in the coordination of the bowing parameters for performing a given task. In principle, the player has a wide choice of combinations of bow-bridge distance, bow velocity, and bow force for producing a given dynamic level and tone quality. The dynamic level and tone quality may even be independently controlled to a considerable degree. However, the duration of the notes, the character of the bowing pattern, and the succeeding evolution in dynamic level all put demands on the combination of parameters. These demands do not always converge into one optimal combination. The apparent ease in playing, which is often associated with the performances of great violinists, probably reflects their mastery a wider set of combinations of bowing parameters. Only rarely does the artist have no choice in his bowing.

The experiments suggest that the choice between the bow-bridge distance and the bow velocity as the main parameter for controlling the dynamic level is determined by such factors as the duration of the notes, relative strength of the higher harmonics, speed of the dynamic changes, and comfort in playing. Long sustained notes may demand that the dynamic level be controlled almost entirely by the bow-bridge distance, while fast changes, as in a sforzando, could require the dynamic changes to be performed mainly by means of bow velocity. However, in most cases the players seemed to arrive at a suitable tradeoff between

bow-bridge distance and bow velocity in setting the desired level.

The validity of the results may require a comment. The examples presented in this study were collected from two players only. Although personal differences undoubtedly do exist between players due to bodily structure, temperament, etc., there is, however, little reason to expect that the basic scheme in these two professionals' choice of bowing parameters is not representative of the recognized principles of violin playing today.

The preliminary experiments in this study as well as those in Askenfelt (1986) indicate that there is a wealth of information present in registrations of the violinist's use of the bow. The professional player's bowing seems at no point random, but every detail could be traced either to demands on tone production as given in the score, to the player's interpretation, or to the ease of playing. In this sense, measurements of the bowing parameters offer an open window for studies of the violinist's performance.

V. Summary

The method presented has been shown capable of measuring the complete set of bowing parameters in violin playing (bow position, bow velocity, bow-bridge distance, bow force) without interfering with normal playing conditions. The method is suitable for basic research in violin playing, and also for general studies of music performance.

Typical values of the bow-bridge distance, as observed in the experiments, were 10-20 mm in forte, 20-30 mm in mezzoforte, and 30-50 mm in piano. At special request, the bow-bridge distance was shortened to a few millimeters only (sul ponticello) or increased up to 60 mm (sul tasto). The bow-bridge distance like the other bowing parameters, showed almost no stationary parts, but was changed continuously.

The bow velocity was usually kept within the range 0.2 to 0.4 m/s. Occasionally, the velocity could be brought down to 0.1 m/s or increased up to about 1.5 m/s.

The largest dynamic change obtained by a change in the bow-bridge distance alone was 13 dB*, while the corresponding value for the bow velocity was 23 dB. The largest overall difference in dynamic level which could be produced was 37 dB.

The bow force was increased continuously with increasing dynamic level, essentially following the trend for the lower limit in bow force given by classic theory.

* This observed span was considerably smaller than the predicted maximum span (20 dB). Probably, long sustained notes covering a dynamic range from pianissimo to fortissimo would expose the extremes in bow-bridge distance used in practice for controlling the dynamic level.

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References

- Askenfelt, A. (1986): "Measurement of bow motion and bow force in violin playing", *J. Acoust. Soc. Am.*, 80:4, pp. 1007-1015.
- Boutillon, X. & Weinreich, G. (1987): "Theoretical relationship between bow pressure and vibration amplitude, derived from a sticking-sliding force-velocity characteristic", *J. Acoust. Soc. Am.* Sl 82 GG8 (1987).
- Bradley, J.S. (1976): "Effect of bow force and speed on violin response", *J. Acoust. Soc. Am.* 60, pp. 274-275.
- Burghauser, J. & Spelda, A. (1971): Akustische Grundlagen des Orchestrierens, Gustav Bosse Verlag, Regensburg; cited in J. Meyer (1972): Akustik und Musikalische Aufführungspraxis, Verlag Das Musikinstrument, Frankfurt a. M., p. 63.
- Clark, M. & Luce, D. (1965): "Intensities of orchestral instrument scales played at prescribed dynamic markings", *J. Audio Eng. Soc.* 13, pp. 151-157; cited in B. Patterson (1974): "Musical dynamics", *Sci.Am.* 232, pp. 78-95.
- Cremer, L. (1984): The Physics of the Violin, MIT-Press, Cambridge, MA.
- Flesch, C. (1929): Die Kunst des Violinspiels, Band I & II, C.F. Peters, Leipzig (2nd edition).
- Flesch, C. (1931): Das Klangproblem in Geigenspiel, C.F. Peters, Leipzig.
- Galamian, I. (1962): Principles of Violin Playing & Teaching, Prentice-Hall, NJ.
- Hodgson, P. (1934): Motion Study and Violin Bowing, J.H. Lavender & Co., London; reprinted by American String Teachers Ass., IL., 1953.
- Hutchins, C. (1980): "The new violin family," in (J. Sundberg, ed.) Sound Generation in Winds, Strings and Computers, publ. issued by the Royal Swedish Academy of Music No. 29, Stockholm.
- Kross, E. (1904): Die Kunst der Bogenführung, Verlag C.F. Schmidt, Heilbrunn a. N.
- Küchler, F. (1929): Lehrbuch der Bogenführung, C.F. Peters, Leipzig, p. 10.
- Meyer, J. (1978): Physikalische Aspekte des Geigenspiels, Verlag Instrumentenbau-Zeitschrift, Siegburg, p. 26.

Müller, H.A. (1962): "Untersuchungen zur Physik der Geige," unpublished manuscript, Mittenwald.

Schelleng, J. (1973): "The bowed string and the player", J. Acoust. Soc. Am. 53, 26-41 (1973).

Trendelenburg, W. (1925): Die natürlichen Grundlagen der Kunst des Streichinstrumentspiels, Julius Springer, Berlin, pp. 61-67.