Measurement of the Coherence of a Bose-Einstein Condensate

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We present experimental and theoretical studies of the coherence properties of a Bose-Einstein condensate (BEC) using an interference technique. Two optical standing wave pulses of duration 100 ns and separation Δt are applied to a condensate. Each standing wave phase grating makes small copies of the condensate displaced in momentum space. The quantum mechanical amplitudes of each copy interfere, depending on Δt and on spatial phase variations across the condensate. We find that the behavior of a trapped BEC is consistent with a uniform spatial phase. A released BEC, however, exhibits large phase variation across the condensate.

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Since the first demonstrations of Bose-Einstein condensation in dilute atomic gases [1], there have been many efforts to explore the nature of such condensates [2,3]. The phase properties of Bose-Einstein condensates are of particular interest because they affect how BECs interfere. The characterization of atoms extracted from a BEC as constituting an "atom laser" beam is related to these phase properties. In this Letter we present direct measurements and theoretical calculations of phase variations across a BEC, a property related to spatial coherence. For a trapped, pure BEC one expects the phase to be spatially uniform because the condensate is in a stationary state of the system with no angular momentum. On the other hand, in an incompletely formed BEC, one might expect differences in the phase between different regions of the condensate [4]. A released BEC, composed of atoms with a positive scattering length, develops phase variations as it explosively expands due to the atom-atom (mean-field) interaction [2]. Understanding these phase variations is essential for characterizing condensates as sources of coherent matter waves.

Matter-wave interference between two condensates was reported in 1997 [5] where two condensates initially localized in different regions of a double-well potential were released and allowed to spread and overlap. That experiment, equivalent to a Young's double slit experiment, showed that two independent condensates interfere, as do two separate lasers [6]. Here we describe a novel method of self-interfering a BEC to extract information about its phase.

We measure the spatial coherence of the BEC by creating and interfering two spatially displaced, coherently diffracted "copies" of the original BEC in the same momentum state. An optical standing wave pulse diffracts [7–9] a small fraction of the condensate into momentum states $\pm 2n\hbar\vec{k}$, where n is an integer and $\vec{k} = (2\pi/\lambda)\hat{z}$ is the optical wave vector. For our conditions, a negligible fraction is diffracted into momentum states with $n > 1$. (Because the process is symmetric, the discussion that follows refers to only the $+2\hbar k$ copy.) A second diffraction pulse, applied Δt after the first pulse, creates a second overlapping $2\hbar k$ copy displaced from the first by $\Delta\vec{z} = (2\hbar k\Delta t/m)\hat{z} = 2v_r\Delta t\hat{z}$, with m the atomic mass and v_r the recoil velocity. The amplitudes of the wave functions representing these two copies interfere where they spatially overlap, and this interference affects the total fraction of the BEC diffracted into $2\hbar k$ by the pulse pair. If the interference were completely constructive (destructive), the number of diffracted atoms would be 4 (0) times that diffracted by a single pulse (for well overlapped copies as the output-coupling efficiency per pulse, $\beta \to 0$).

The experiment was performed with a condensate of up to 3×10^6 sodium atoms in the $3^2S_{1/2}$, $F = 1$, $m = -1$ state, without a discernible uncondensed fraction, in a time orbiting potential (TOP) [10] magnetic trap. Our trap [11] has harmonic frequencies along the $\hat{x}$ (vertical), $\hat{y}$, and $\hat{z}$ directions of 14, 20, and 28 Hz, respectively; we measured the asymptotic rms momentum width of the released condensate to be $0.08(1)\hbar k$ [12]. With the magnetic trap either held on (trapped BEC) or shut off (released BEC), we apply a pair of identical optical standing wave pulses (100 ns FWHM) along $\hat{z}$. The optical standing wave is formed from a collimated, retroreflected light beam (peak intensity ≈ 300 mW/cm^2), detuned below the $F = 1 \to F' = 2$ optical transition ($\lambda = 589$ nm) by 600 MHz to reduce spontaneous emission.

In order to describe the interference we write the condensate wave function, normalized to unity, as $\eta(\vec{r}, t) = R(\vec{r}, t)e^{i\varphi(\vec{r},t)}$, where $R(\vec{r}, t)$ and $\varphi(\vec{r}, t)$ are real. When the first pulse is applied at t_0, we create a $2\hbar k$ copy of the condensate $\psi(\vec{r}, t_0) = \epsilon\eta(\vec{r}, t_0)e^{i2kz}$, with $|\epsilon|^2 \equiv \beta \ll 1$. At a time Δt later the copy has moved a distance Δz and thus becomes $\epsilon\eta(\vec{r} - \Delta\vec{z}, t_0)\exp[i2kz - i\omega(\vec{r} - \Delta\vec{r})\Delta t]$, where we have ignored any change in the spatial distribution of the wave packet after application of the first pulse. The phase of atoms diffracted

to $2\hbar k$ evolves faster than that of atoms in the condensate because of the additional kinetic energy and a small mean-field effect due to the change of momentum state [2], $\hbar\omega(\vec{r}) = 4E_r + gN|\eta(\vec{r})|^2$. Here $E_r = \hbar^2k^2/2m$, g is the mean-field coupling constant as in [2], and N is the number of atoms in the condensate. When a second copy is created at $t_0 + \Delta t$, the first copy's phase differs from that of the second copy by $-\langle\omega\rangle\Delta t$. To first order in ϵ the total amplitude in the $2\hbar k$ momentum state after the second pulse is $\Psi(\vec{r}, \Delta t; t_0) = \epsilon e^{i2kz}[\eta(\vec{r}, t_0 + \Delta t) + \eta(\vec{r} - \Delta\vec{z}, t_0)e^{-i\omega(\vec{r}-\Delta\vec{z})\Delta t}]$. The two displaced, diffracted copies include the spatial phase variation $\varphi(\vec{r}, t_0)$ of the original condensate, which affects the interference. Defining the signal as the number of atoms diffracted by the two pulses divided by the maximum number of atoms diffracted (i.e., when $\Delta t = 0$) we obtain

$$S(\Delta t; t_0) = \frac{\int |\Psi(\vec{r}, \Delta t; t_0)|^2\, d^3r}{\int |\Psi(\vec{r}, 0; t_0)|^2\, d^3r} = \frac{1}{2} + \frac{Re\langle\eta(\vec{r}, t_0)|e^{-i\omega(\vec{r}-\Delta\vec{z})\Delta t}|\eta(\vec{r} - \Delta\vec{z}, t_0)\rangle}{2}$$

$$= \frac{1}{2}\{1 + \alpha(\Delta t; t_0)\cos[4E_r\Delta t/\hbar + \theta(\Delta t; t_0)]\}. \quad (1)$$

Here we have introduced a new amplitude α and phase θ [whose functional form depends on $\varphi(\vec{r}, t_0)$]. In addition, we made the assumption $\eta(\vec{r}, t_0 + \Delta t) = \eta(\vec{r}, t_0)$. This is true for a trapped condensate (neglecting the change in the mean field of the parent condensate) and approximately true for a released one because the condensate expands little for our Δt. The signal for a trapped condensate exhibits oscillations at a frequency $\frac{4E_r}{\hbar} + \frac{d\theta}{d\Delta t}$ provided $\frac{d\alpha}{d\Delta t} \ll 2\pi(100\text{ kHz})$. $\frac{d\theta}{d\Delta t}$ adds 0.3 kHz to the $4E_r/h = 100.1$ kHz oscillation frequency [13]. If $\varphi(\vec{r}, t_0)$ is spatially uniform, the decay of the 100 kHz interference oscillations to $S \to \frac{1}{2}$ is mainly determined by the geometrical overlap of the two amplitudes. However, if $\varphi(\vec{r}, t_0)$ is not uniform, the 100 kHz oscillations will apparently decay faster due to dephasing. The decay of the oscillations is a measure of the momentum spread of the condensate. When $\varphi(\vec{r}, t_0)$ is constant the decay is determined mainly by the spatial overlap and the momentum spread is uncertainty-principle limited. Variations in the initial phase imply an additional contribution to the momentum spread because the momentum is proportional to the derivative of the wave function.

Figure 1 shows the real and imaginary amplitudes of trapped (a) and released (b) condensates calculated using a 1D Gross-Pitaevskii (GP) model (3D-GP calculations [14] were used when comparing to data). After the BEC is released, the mean-field potential energy is converted to kinetic energy on a time scale of $1/\omega_z$ (6 ms in our case). The resulting spread in momentum is characterized by a phase that varies quadratically [15] across the condensate.

To experimentally determine $S(\Delta t; t_0)$ we measure the number of diffracted atoms after the diffracted components have become well separated from the original BEC. In principle, only one pair of diffraction pulses would be needed if the output-coupling efficiency were constant. The intensity of the diffraction pulses, however, can vary due to spatial beam inhomogeneity and beam pointing instabilities. In addition, N may vary by $\approx 10\%$ but cannot be measured since the condensate is optically opaque for the on-resonance imaging that allows good measurements of the small number of diffracted atoms. To obtain signals insensitive to these fluctuations we use a four-pulse normalization sequence.

Our four-pulse sequence consists of two pairs separated by ΔT (typically several ms to allow the diffracted atoms enough time to leave the region of the condensate). All pulses are nominally identical and the diffraction efficiency β per momentum state per pulse is chosen to be small ($\beta \approx 0.02$). Figure 2 shows the typical experimental pulse sequence (a) and the resulting wave packets (b). The pulse separation in the first pair is Δt and in the second pair it is $\Delta t + 5\ \mu$s. Since 5 μs is half of an interferogram period, the sum of the atoms diffracted by both pulse pairs is independent of Δt while remaining proportional to β and N. The normalized signal is the number of atoms diffracted by the first set of pulses divided by the number diffracted by both sets of pulses [16], which is equivalent to the signal of Eq. (1). The signal is insensitive to a slowly varying diffraction

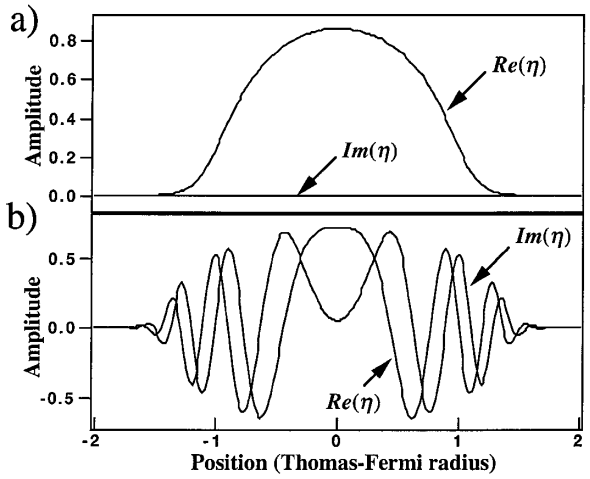

FIG. 1. Theoretical 1D GP calculations showing the evolution of the phase across the condensate for 5×10^5 atoms. The real and imaginary parts of the wave function along the $\hat{z}$ axis are shown for the condensate in the trap (a) and 8 ms after the trap is extinguished (b).

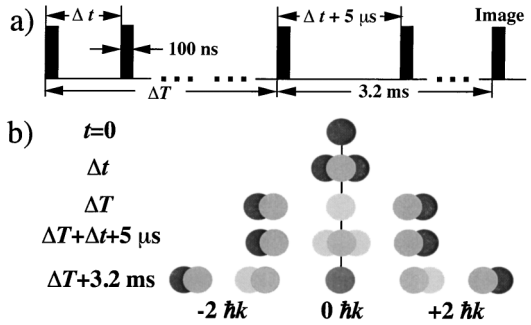

FIG. 2. (a) Timing sequence. The first two diffraction pulses probe the phase evolution of the condensate and the last two pulses are used for normalization. (b) Evolution of the condensate. The overlapping circles represent the interfering amplitudes for scattering atoms into the $\pm 2\hbar k$ momentum states.

efficiency [17] and to number fluctuations in the original BEC. After all wave packets spatially separate, we perform absorption imaging [1] by first optically pumping the atoms to the $F = 2$ ground state and imaging with probe light on the closed $F = 2 \rightarrow F' = 3$ transition. We process the images to obtain optical depth images from which we determine the number of diffracted atoms.

We first study the coherence of a condensate held in the TOP trap [18]. The classical turning point for atoms with momentum $2\hbar k\hat{z}$ at trap center is 260 μm, therefore these atoms can leave the 44 μm spatial extent of the condensate while the trap is held on. Figure 3a shows optical-depth images with no averaging, along with a sinusoidal fit to the signal. The fit yields a frequency of 100.3(3) kHz (when fit to 5 periods), in good agreement with the expected value of 100.4 kHz. The signal is centered about $S = 0.50(1)$ with a peak-to-peak amplitude of only 0.82(1), possibly due to imperfect background subtraction. (Without normalization the signal-to-noise ratio was 7 times worse.) The phase of the 100 kHz interferogram beyond about 100 μs varies from shot-to-shot such that the contrast of the averaged signal is washed out. This may be due, in part, to fluctuations of a few percent in the 100 kHz signal frequency caused by small initial velocities of the condensate [19] (presumably resulting from small time-varying stray magnetic fields). In order to obtain information on the true envelope, we took 30 points at each Δt and used the maximum (minimum) signal as a measure of the upper (lower) envelope. Signal fluctuations due to additive noise simply shift the baseline (typically 5% in our case) of the upper (lower) envelope up (down).

Figure 3b shows the behavior of the signal's upper envelope after subtracting a constant background (we verified that the signal envelope was symmetric within our uncertainty). For this data set there were $1.5(3) \times 10^6$ atoms in the original condensate. The measured fringe decay time from a Gaussian fit [20] is $\tau_{1/2} = 225(40)$ μs, in good agreement with the value $\tau_{1/2} =$

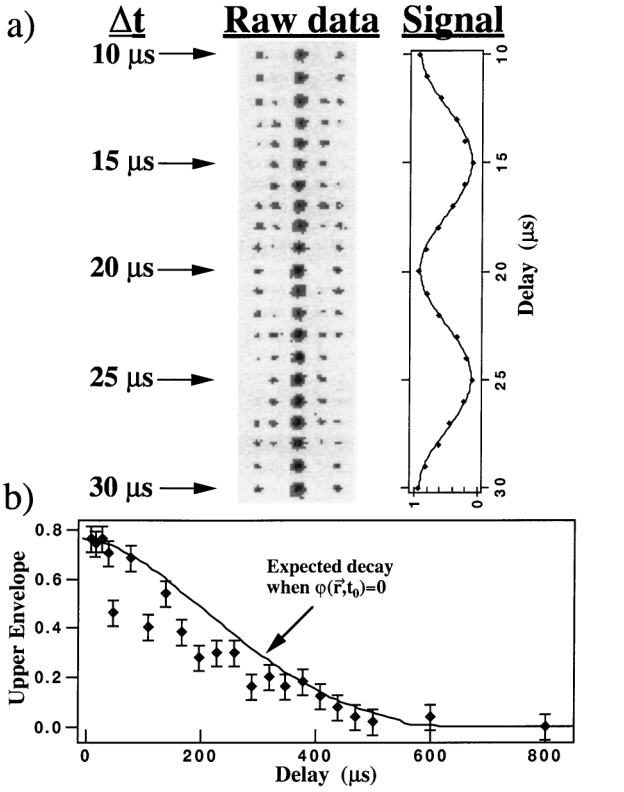

FIG. 3. Two periods of optical depth data are presented as a function of Δt when the trap is held on. The right-hand column of (a) is the resultant normalized signal. The solid line is the result of a fit to 5 periods of these data, yielding a frequency of 100.3(3) kHz. (b) shows the long-time upper envelope of this signal's contrast as well as the theoretical GP envelope (solid line, scaled to the experimental contrast).

275(6) μs from calculations, to be described elsewhere [14]. The calculations are of two types. One is a full 3D time-dependent propagation of the GP equation using a slowly varying-envelope approximation for each momentum component of the wave function. The other is a time-dependent Thomas-Fermi model based on Ref. [15] that gives excellent agreement with the full GP calculations. The theoretical signal decay is mainly due to geometrical overlap, plus a small effect due to the action of the mean field [2] on the ejected part between pulses. The uncertainty in the theoretical value is mainly due to the experimental uncertainty in the number of atoms in the condensate. If we model the initial phase variation as quadratic (a linear phase variation corresponds to a uniform velocity and would not affect the signal envelope), we obtain 0.0038(30) rad/μm^2, which is consistent with zero ($0.6\pi \pm 0.5\pi$ radians at the cloud edge) and with the condensate having a uniform global phase.

In a second series of experiments we released the condensate from the trap before applying the diffraction pulses. As the mean-field energy is converted to kinetic energy, the released condensate develops substantial spatial phase variations $\varphi(\vec{r}, t)$ (see Fig. 1) which lead to a

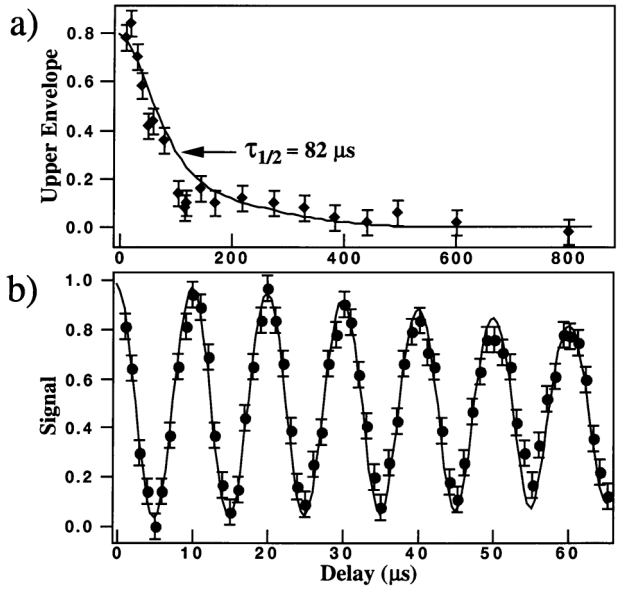

FIG. 4. The upper envelope of the contrast (a) and short-time signal (b) data are presented for a released BEC along with the results of 3D-GP calculations.

decay of signal contrast much faster than that due to the spatial overlap. Figure 4 shows the interference signal and envelope when the first pulse pair is applied 1.2 ms after switching off the trap with $5(1) \times 10^5$ atoms and $\Delta T = 3$ ms. In Fig. 4a the upper envelope of the signal after background subtraction is plotted with theory. The measured decay time $\tau_{1/2} = 65(10)$ μs for the released condensate is in agreement with the theoretical value (solid curve) of 82(3) μs. Figure 4b compares the experimental interferogram data with the theoretical predictions. Data were also taken for long delays ΔT, and the results are in good agreement with theoretical simulations.

The novel interference technique presented here could also be used to study the development of a uniform condensate phase as the BEC forms in the trap. In addition, any uncondensed (thermal) fraction should result in a partial decay of signal contrast at short times, which might be useful for precise measurements of the temperature of a highly degenerate Bose gas. The techniques developed here allow us to study phase properties of atom lasers [21–23]. For example, our results confirm the expectation that the output-coupled pulses forming the continuous beam of our atom laser [22] were in phase. We note that a recent complementary frequency-domain experiment [24] also confirms that a trapped condensate has a uniform phase.

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[12] All uncertainties reported here are 1 standard deviation combined statistical and systematic uncertainties.
[13] The 0.3 kHz is a small average mean-field contribution equal to $\frac{4\mu}{7h}$, where μ is the chemical potential.
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[16] In the absence of any phase evolution between the pulse sets (as in the trap-on case with $\beta \rightarrow 0$), the total fraction of atoms diffracted by both sets of pulses is constant.
[17] For $\beta \rightarrow 0$, the difference between the experimental signal and the ideal two-pulse signal [Eq. (1)] approaches zero if there is no phase evolution in the condensate between the first and second sets of diffraction pulses. However, even for $\beta \neq 0$, it can be shown that there is no difference in the signal envelope.
[18] The micromotion velocity of 1 mm/s in our TOP leads to a negligible modulation of the signal amplitude. The oscillating magnetic potential does not affect the interference because all momentum components experience the same fluctuation.
[19] After forming the condensate with $\nu_z = 360$ Hz, the potential is adiabatically expanded in 4 s to reduce ν_z to 28 Hz. As the trap resonant frequency drops, stray, time-varying magnetic fields at multiples of 60 Hz excite the condensate, producing a small sloshing motion which manifests itself as a random release velocity on the order of $0.02v_r$ after we extinguish the trap.
[20] A Gaussian function was chosen because it fit well with the theoretical results from the GP equation.
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