Measurement of the Earth tides with a MEMS gravimeter

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The ability to measure tiny variations in the local gravitational acceleration allows – amongst 3 other applications – the detection of hidden hydrocarbon reserves, magma build-up before volcanic 4 eruptions, and subterranean tunnels. Several technologies are available that achieve the sensi-5 tivities required for such applications (tens of $\mu \text{Gal}/\sqrt{Hz}$): free-fall gravimeters¹, spring-based 6 gravimeters^{2,3}, superconducting gravimeters⁴, and atom interferometers⁵. All of these devices can 7 observe the Earth Tides⁶; the elastic deformation of the Earth's crust as a result of tidal forces. 8 This is a universally predictable gravitational signal that requires both high sensitivity and high 9 stability over timescales of several days to measure. All present gravimeters, however, have limita-10 tions of excessive cost (> 100 k) and high mass (>8 kg). We have built a microelectromechanical 11 system (MEMS) gravimeter with a sensitivity of 40 $\mu \text{Gal}/\sqrt{Hz}$ in a package size of only a few 12 cubic centimetres. We demonstrate the remarkable stability and sensitivity of our device with a 13 measurement of the Earth tides. Such a measurement has never been undertaken with a MEMS 14 device, and proves the long term stability of our instrument compared to any other MEMS device, 15 making it the first MEMS accelerometer to transition from seismometer to gravimeter. This heralds 16 a transformative step in MEMS accelerometer technology. MEMS accelerometers – found in most 17 smart phones⁷ - can be mass-produced remarkably cheaply, but most are not sensitive enough, 18

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and none have been stable enough to be called a 'gravimeter'. Due to their small size and low cost,
MEMS gravimeters could create a new paradigm in gravity mapping: exploration surveys could
be carried out with drones instead of low-flying aircraft; they could be used for distributed land
surveys in exploration settings, for the monitoring of volcanoes; or built into multi-pixel density
contrast imaging arrays.

Gravimeters can be split into two broad categories: absolute gravimeters and relative gravime-24 ters. Absolute gravimeters measure the gravitational acceleration, q, by timing a mass in free 25 fall over a set distance. Absolute gravimeters are very accurate but are bulky and expensive. 26 The Micro-g Lacoste FG5¹, for example, achieves acceleration sensitivities of 1.6 μ Gal/ \sqrt{Hz} 27 $(1.6 \ \mu \text{Gal}/\sqrt{Hz}$ is an acceleration measurement of 1.6 μ Gal in an integration time of 1 second, 28 where 1 Gal is 1 cm/s²), but it costs over \$100 k and weighs 150 kg. Relative gravimeters make 29 gravity measurements relative to the extension of a spring: the deflection of a mass on a spring 30 will change as q varies. These devices can be made smaller than absolute gravimeters but are in-31 trinsically less stable: the spring constant can change with varying environmental conditions. The 32 Scintrex CG5 relative gravimeter (also costing over \$100 k, but weighing 8 kg) can measure gravity 33 variations down to 2 μ Gal^{2,3} but is much more susceptible to drift than absolute devices. For any 34 mass-on-spring system, increased acceleration sensitivity is achieved by either improving the sen-35 sitivity to displacement, or by minimising the ratio, k/m, between the spring constant, k, and the 36 mass, m. A system in which a mass is suspended from a spring within a rigid housing will respond 37 differently to signals above or below the resonance frequency. In the regime below the resonance 38 there will be a linear relationship between the displacement of the proof mass and the acceleration 39 of the housing. This is the region in which the device can be used as an accelerometer/gravimeter. 40 MEMS devices are microscopic mechanical devices made from semiconductor materials. They 41 have the advantage of being mass-producible, light-weight and cheap. Although mobile phone 42

accelerometers are not very sensitive, some MEMS devices have been developed that reach sensi-43 tivities much better than the 0.23 mGal/ \sqrt{Hz} of the iPhone MEMS device⁷. For example: a device 44 developed by Krishnamoothy et. al.⁸ has a sensitivity of 17 μ Gal/ \sqrt{Hz} ; the SERCEL QuietSieis⁹ 45 has a sensitivity of 15 μ Gal/ \sqrt{Hz} ; and a microseismometer developed by Pike et. al.¹⁰ has a sensi-46 tivity of 2 μ Gal/ \sqrt{Hz} . These devices, however, can only operate as seismometers and do not have 47 a stability sufficient to be classed as gravimeters, which are capable of monitoring low frequency 48 gravimetric signals such as the Earth tides (around 10 μ Hz). Table 1 summarises the characteristics 49 of these MEMS seismometers, the *Scintrex CG5* gravimeter, and our own gravimeter. Figure 11 50 provides a further comparison between our own device, the Pike microseismometer¹⁰, the Scintrex 51 CG5 and two other commercial devices. 52

The Earth tides are an elastic deformation of the Earth's crust caused by the changing rel-53 ative phase of the Sun, the Earth and the $Moon^6$. They produce a small variation in the local 54 gravitational acceleration, the size of which depends also on the latitude and elevation of the mea-55 surement location. Depending on the time of the lunar month, the Earth tides vary in amplitude 56 and frequency, moving between diurnal $(2 \times 10^{-5} \text{ Hz})$ and semi-diurnal $(1 \times 10^{-5} \text{ Hz})$ peaks. Since 57 the Earth tides have a peak signal strength³ of less than 400 μ Gal, and a low frequency oscillation, 58 they are a useful natural signal to demonstrate both the sensitivity and long-term stability of any 59 gravimeter. The Earth tides have never previously been measured with any MEMS device, so a 60 device able to do so will be a transformative step change in the field. 61

⁶² Our device has been designed to have a resonant frequency of under 4 Hz. To achieve such ⁶³ low frequencies a geometric anti-spring system^{11, 12} was chosen. With increasing displacement, ⁶⁴ anti-springs get softer and their resonant frequency gets lower. A geometrical anti-spring requires ⁶⁵ a pair of arched flexures that meet at a constrained central point. In the case of our MEMS device ⁶⁶ they meet at the proof mass. This geometry constrains the motion of the proof mass to the axis

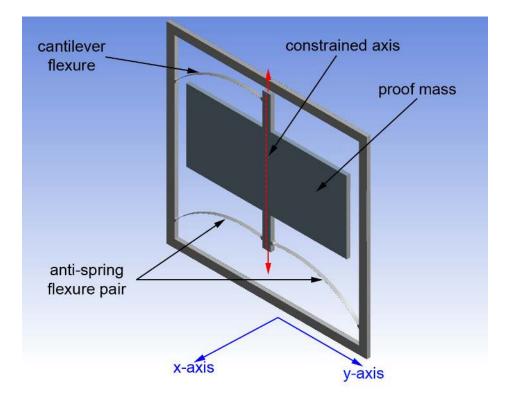


Figure 1

shown in Fig. 1. As the proof mass is pulled away from its un-loaded position the spring constant is 67 lowered. This is in contrast to a Hooke's-law spring, in which the spring constant does not change 68 with increasing displacement. Tilting the MEMS device from horizontal to vertical orientation, 69 pulls the proof mass down, thus lowering the frequency from over 20 Hz when horizontal to 2.3 Hz 70 when vertical. We have opted for a configuration with a pair of anti-spring flexures supporting the 71 lower portion of the proof mass, and a single flexure supporting the top. All of the flexures are 72 only 5 μm wide but 200 μm deep. The three flexure system maintains an anti-spring behaviour 73 as the gravitational loading increases (when the device is tilted from horizontal to vertical). Due 74 to the asymmetry of the design, however, a small level of y-axis tilting occurs. This tilt pulls the 75 system off its constrained axis. When the system reaches its equilibrium, it gains a Hooke's Law 76 behaviour (see methods section and Fig. 5 for further details). We thus have a device which is 77 stable but at a much lower frequency than traditional MEMS devices. A resonant frequency of 78

⁷⁹ 2.3 Hz is the lowest resonant frequency of any reported MEMS device to date. To our knowledge the next lowest resonant frequency reported is 10.2 Hz in a device made by Pike et al.¹³. The fact that the system has a Hooke's law behaviour in its vertical configuration means that it is less sensitive to tilt in the x-axis (see Fig. 1) than would be the case for a normal geometrical anti-spring (see Fig. 12).

The proof mass motion is measured using an optical shadow sensor¹⁴. Here a light emitting diode (LED) illuminates a photodiode with the MEMS device mounted in between. Motion of the proof mass modulates the shadow, generating a change in the current output of the photodiode. This shadow sensor (Fig. 2) achieves a high sensitivity (equating to an acceleration noise floor of $\leq 10 \ \mu Gal$ at the sampling frequency of 0.03 Hz), whilst allowing a large dynamic range of up to $50 \ \mu m$.

Observation of the Earth tides requires stable operation over several days. The main contribu-90 tor to parasitic motion is the varying temperatures of the system. For this reason the 'C'-shaped 91 structure of the shadow sensor was fabricated from fused silica because of its low thermal expan-92 sion coefficient at room temperature $(4.1 \times 10^{-7} K^{-1})^{15}$. Silicon has a significantly larger thermal 93 expansion coefficient $(2.6 \times 10^{-6} K^{-1})^{16}$, but silicon was used to make the MEMS because it is 94 a standard fabrication material in the semiconductor industry, it has high mechanical strength, 95 and its thermal properties are well characterised. The dominant mechanism by which temperature 96 variations affect the gravity measurement is the change in Young's modulus, Y, of the flexures 17,18 . 97 This in turn alters the spring constant of the flexures, resulting in a variation of k, $1/k \ dk/dT$, 98 of $7.88 \times 10^{-6} K^{-1}$. We therefore implemented servo control loops to maintain the temperature 99 of the system to within 1 mK. A 1 mK change in temperature would give an uncertainty in the 100 gravity reading of $\sim 25 \ \mu \text{Gal}$. The primary control loop maintained the temperature of the MEMS 101 device directly, the second controlling the temperature of a copper thermal shield that encased the 102

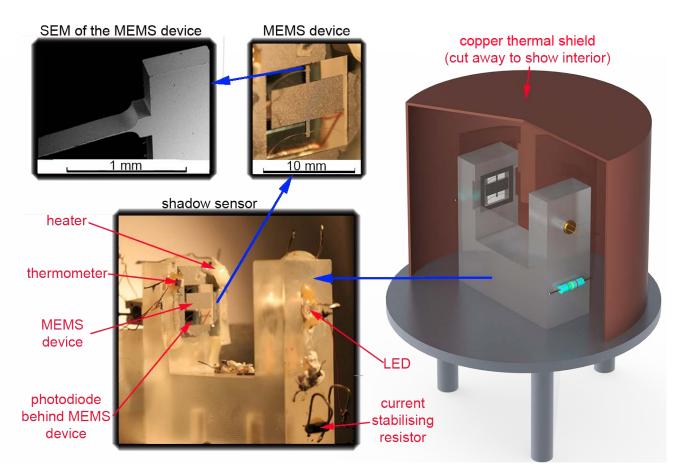


Figure 2

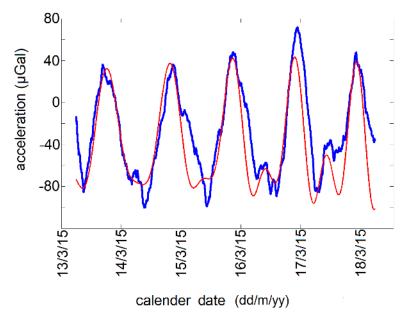


Figure 3

entire shadow sensor (Fig. 2). The MEMS device was placed inside a vacuum system. This was bolted to the floor without an external seismic isolation isolation table, which would be a large and expensive addition.

From December 2014 the system was left in continuous operation whilst the servo control was 106 optimised. Figure 3 demonstrates a data run of five days between the 13/03/15 to the 18/03/15107 in which gravitational acceleration is plotted against time. The blue data demonstrates our exper-108 imental data averaged with a time constant of 240 minutes (the full noise data can be observed 109 in Fig. 6a), together with a data set filtered with a 10 minute time constant (Fig. 6b). The solid 110 red line is a theoretical plot of the Earth tides as should be observed at our location $(55.8719^{\circ} \text{ N},$ 111 4.2875° W), and was plotted using $TSOFT^{19}$. An ocean loading correction is also included in 112 this theoretical plot to account for the effect of nearby tidal waters pressing on the Earth's crust, 113 although the effect is at the level of 5% for our laboratory. There is a strong correlation coefficient, 114 R, of 0.86 between our experimental data and the theory plot. The correlation indicates that 115 this is the first measurement of Earth tides demonstrated by a MEMS device, a landmark result 116

for MEMS gravimetry. This measurement provides a natural calibration for the gravimeter, the results of which allow us to determine that the present sensitivity of the device is $40 \ \mu Gal/\sqrt{Hz}$. We further performed a stability test of the calibration factor for our device by monitoring the tides at two intervals approximately 3 months apart. The calibration remained constant to better than 5 % (Fig. 13).

The noise floor of our device is limited by seismic noise. A theoretical thermal noise floor of under 0.5 μ Gal/ \sqrt{Hz} can be calculated, assuming that losses are due to structural damping²⁰. This calculation is based upon a measurement of the quality factor, Q, of the device under vacuum of ~80 (the relaxation time of the MEMS device is ~11 s). We observe that the Q reduces as the resonant frequency is lowered (Fig. 7). This behaviour is due to the fact that in geometrical antisprings: as the resonant frequency is lowered, the restoring force becomes comparable to internal friction²¹.

To put the sensitivity of our device into context, 40 $\mu Gal/\sqrt{Hz}$ is sufficient in 1 second to 129 detect a tunnel with a cross-sectional area of 2 m^2 and length of 4 m at a depth of 2 m; it could be 130 used to find oil reservoirs of $\geq 50 \ m \times 50 \ m \times 50 \ m$ (with a density contrast of 50%) at a depth of 131 150 m; a change of 45 μ Gal was a 'clear precursor' to a volcanic eruption in the Canary Islands in 132 2011^{22} . It is accepted that intrusion of new magma into a reservoir precedes volcanic eruptions²³; 133 continuous micro gravity measurements around volcanoes are a useful tool in monitoring such 134 events²⁴. The ratio of ground deformation to change in gravity can be used to monitor magma 135 chambers at depths of several km^{25} . 136

In figure 3 a linear drift term has been removed from the data. This drift equates to less than 137 In figure 3 a linear drift term has been removed from the data. This drift equates to less than 138 150 μ Gal per day, a factor of three better than the drift of the *Scintrex CG5* (500 μ Gal per day). 139 Both we and *Scintrex CG5* auto-correct this drift with software. Figure 4 consists of eight subplots 140 demonstrating the drift characteristics on the MEMS device. Figure 4a shows the full-noise tide

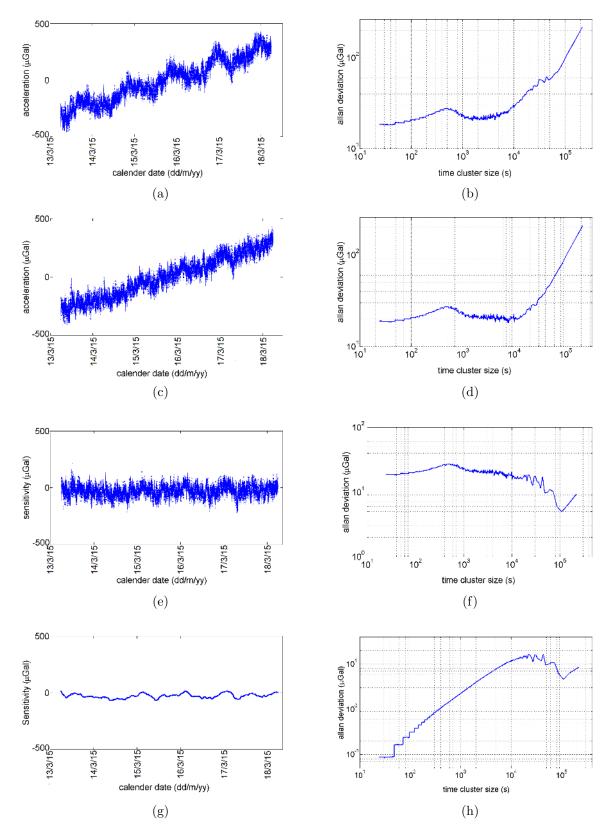


Figure 4

data without a linear drift correction. Figure 4c shows the same data but with the tide signal removed. Figure 4e shows the same data again but with a linear drift correction. Figures 4b, 4d, 4f and 4h show the Allan deviation for the data in figures 4a, 4c, 4e and 4g respectively. Allan deviation is a technique used to measure the variation over the full frequency range of a signal by averaging over increasingly larger time intervals²⁶.

The data analysed in figure 4 spans a frequency range from 10^{-5} Hz to 0.03 Hz (the sampling 146 frequency of this data set, which was used to remove the effect of seismic noise). A second data set 147 was taken at a faster sampling rate to observe the response of the device from 0.03 Hz up to the 148 resonant frequency of 2.3 Hz. Both data sets can be observed in figure 10 in the form of a RMS 149 acceleration sensitivity plot. The Allan deviation for the high frequency series is polluted by the 150 presence of two large signals: the resonant frequency of the device, and the microseismic peak²⁷,²⁸. 151 This deviation plot is not a useful measure of the noise of the device and has therefore not been 152 included in figure 4. Figures 4b and 4d demonstrate the linear drift that the device experiences. 153 Figures 4b, 4d and 4f also demonstrate a small peak at 500 s that is an artefact of the temperature 154 servo. The broad peak that is only visible on the rising edge of Fig. 4b is the tide signal. A 155 comparison between the drift characteristics of our device and some other commercial gravimeters 156 is displayed in figure 11, in which an acceleration power spectral density plot is displayed. 157

This MEMS device, capable of measuring the Earth tides, represents a significant step forward in the field – it is not just an accelerometer, but a gravimeter. Made from a single silicon chip the size of a postage stamp, this sensor has the lowest reported resonant frequency of any MEMS accelerometer (2.3 Hz), is within an order of magnitude of the best acceleration sensitivity of any MEMS device (40 $\mu Gal/\sqrt{Hz}$), and has the best reported stability of any MEMS device. This prototype will enable the development of a new density contrast imaging technology applicable in many industrial, defence, civil, and environmental applications. It has the potential to be inexpensive, mass-produced and lightweight which opens up new markets: it could be flown in drones by oil and gas exploration companies, limiting the need for dangerous low altitude aeroplane flights; it could be used to locate subterranean tunnels; it could be used by building contractors to find underground utilities. Networks of sensors could be operated in unsafe areas for monitoring natural and man-made hazards; for example, on volcanoes or unstable slopes to improve the spatial and temporal resolution of subsurface density changes. This will allow improved hazard forecasting and the reduction of occupational risk to monitoring personnel^{25, 29}.

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- gravimeter and the GWR-C026 superconducting gravimeter in Strasbourg (France) using a
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237 Print figure legends

238	• Figure 1: The MEMS Device. A figure demonstrating the design of the MEMS gravime-
239	ter. The central proof mass is suspended from three flexures: an anti-spring pair at the
240	bottom and a curved cantilever at the top. The anti-spring pair constrain the motion of the
241	proof mass along the red axis. The frequency is lowered by this constraint until the cantilever
242	pushes the motion off-axis, stabilising the MEMS device at a lower frequency.
243	• Figure 2: The Experimental Set-up. A schematic of the MEMS device and the shadow
244	sensor. Both sit on an aluminium plate and are encased in a copper thermal shield. Both
245	the MEMS device and the shield are thermally controlled. At the top left is a photograph
246	and scanning electron microscope (SEM) image of the MEMS device. At the bottom left is
247	a photograph of the MEMS device mounted on the optical shadow sensor with glue holding
248	the heater and thermometer in place.
249	• Figure 3: The Earth Tides. The measurements of the Earth tides obtained from the
250	MEMS device. The data has been averaged with a time constant of 240 minutes. The red
251	line is a theoretical plot calculated with $TSOFT$, including an ocean loading correction. The
252	blue line is the experimental data. The two series have a correlation coefficient of 0.86.
253	• Figure 4: Drift Characteristics. 4a is a full noise time series of the tide measurement. 4b
254	is the Allan Deviation of the series in $4a$. $4c$ is a full noise time series of the tide measurement
255	with the tide signal removed via a regression against the theoretical data from $TSOFT$. 4d
256	is the Allan Deviation of the series in $4c$. $4e$ is a time series of the tide measurement with
257	the tides removed and the linear drift corrected, ${f 4f}$ is the corresponding Allan deviation plot.
258	4g is the same data as 4e but with a 4 hour filter added. 4h is the Allan deviation plot of

this filtered data.

²⁶⁰ Extended data figure legends

Figure 5: Spring Resonant Frequency Behaviour with Tilt The resonant frequency decreases as the MEMS device gets closer to vertical due to the geometrical anti-spring effect. At 88° and and 92° there are minima in the plot. At this point the frequency is constant with tilt and the system displays a Hooke's law behaviour. The resonant frequency of a symmetric anti-spring would reach an instability here. This figure also demonstrates that whilst the instrument is operated at 90° the resonant frequency is 2.3 Hz, it can be lowered to 1.8 - 1.9 Hz by tilting to operate to one of the minima.

- Figure 6: The Earth Tides with Different Filtering. Figure 6a presents measurements of the Earth tides obtained from the MEMS device. This is the raw data output. Figure 6b presents the same data but with a 10 minute filtering time. The red lines are theoretical plots calculated by *TSOFT*. The blue lines are the experimental data.
- Figure 7: Quality Factor Frequency Dependence. We observe a trend of decreasing
 quality factor with decreasing frequency of our device. At low frequencies the internal friction
 of the material becomes the dominant loss mechanism. This trend has been discussed by
 Chin et al.²¹.
- Figure 8: Geometrical Anti-Spring Design. Figures 8a and 8b demonstrate the Hooke's-law behaviour of a straight and curved cantilever respectively. Figures 8c and 8d demonstrate the unstable anti-spring characteristics of a 2 and 4 flexure MEMS device respectively. Figure 8e demonstrates behaviour of a 3 flexure MEMS device (see figure 1).

Whilst a 2 or 4 flexure system reaches an instability with increasing load, a 3 flexure system regains a Hooke's law behaviour. The 3-flexure system behaves as such because it is pushed off its constrained axis by the asymmetry of the design. All of these plots were produced using *Ansys* finite element analysis software.

- Figure 9: Polynomial Drift. This plot demonstrates the drift in the data shortly after the vacuum pump has been turned on. A polynomial component to the drift is clearly visible. Once the vacuum system has settled, however, the drift becomes linear as demonstrated in figure 4b at a level of 150 μ Gal/day.
- Figure 10: MEMS Device RMS Acceleration Sensitivity. Figure 10a demonstrates the RMS acceleration sensitivity in μ Gal, and figure 10b in μ Gal-dB. The tide signal can be observed in both plots at 10⁻⁵ Hz; the peak at 2×10⁻³ Hz is the artefact of the temperature servo discussed earlier; the microseismic peak can be observed 0.1 Hz and 0.2 Hz; and the 2.3 Hz resonant frequency can be observed to the right of the plot. The plot is a composite of two data series because the temporal resolution required to record the higher frequency data would not be possible to maintain at lower frequencies.

• Figure 11: Power Spectral Density Comparison. The red series – plotted using the 295 data from 4g – is our MEMS device, demonstrating its sensitivity in the tidal frequency range. 296 The filtering time means that the sensitivity rolls off above 10^{-4} Hz. The black series is the 297 Scintrex CG5, the blue series is the Micro-g Lacoste gPhone-054, the green series is the SG-298 C026 superconducting gravimeter. The data from these three series are taken from a figure by 299 Riccardi et. al.³⁰ (©Bureau International des Poids et Mesures. Reproduced by permission 300 of IOP Publishing. All rights reserved.). The magenta series is the microseismometer by 301 Pike et. al. (private communication by permission of the author, to be published in the 47th 302

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Lunar and Planetary Science Conference).

304	• Figure 12: Tilt Susceptibility Tests. Figure 12a demonstrates the variation in output
305	of the MEMS device with the x-axis tilt of the sensor plotted on a secondary axis. Figure
306	12b shows the same for the y-axis. There is a x-axis (in-plane MEMS tilt) a tilt sensitivity
307	in this axis of 21.2 μ Gal/arc second, but in the x-axis (out of plane MEMS tilt) the tilt
308	sensitivity of only 0.6 μ Gal/arc second.

• Figure 13: Long Term Reproducibility Tests. Figures 13a and 13b are two data sets separated by approximately 4 months, with no filtering employed. During this period the vacuum chamber was evacuated and vented several times, despite this the calibration factor of the device has not changed by more than 5%.

313 Methods

³¹⁴ MEMS device fabrication

The MEMS device was fabricated from a single chip of 200 μ m thick silicon. The reverse side of the wafer was first coated with 2.5 μ m of PECVD SiO₂. A 100 nm coating of chromium was next deposited on the top surface of the silicon using a thermal evaporator.

The MEMS device pattern was created in a layer of positive photoresist using a g-line pho-318 tolithography process. The mask was a 'halo' design³¹ i.e. instead of etching away all of the 319 unwanted areas of silicon, trenches were used in an outline of the structure, to keep a constant 320 etch rate and profile over all etched areas. The halo was 10 μ m wide. The photoresist pattern 321 was then used as a mask to wet etch the chrome using a nitric acid chrome etchant for 100 s, 322 thus etching the MEMS device proof mass pattern into the chrome. The resist was then removed 323 ultrasonically with acetone and isopropanol, leaving the chrome etch mask in place. A 7 μ m layer 324 of $AZ^{\mathbb{R}}$ -4562 photoresist was then spun onto the back of the sample and used later in making the 325 sample free standing. 326

The sample was fixed to a carrier wafer (chrome side up) using a thin, spun-on layer of 327 $Crystalbond^{\mathbb{R}}$ 509 in solution with acetone. To ensure a good thermal contact the sample was 328 weighted and left on the hotplate at 88° C (just above the melting point of $Crystalbond^{(\mathbb{R})}$) for 329 5 minutes. The sample was next placed in an Oxford Instruments PlasmaPro 100 Estrelas Deep 330 Silicon Etch System, and BoschTM etched³² for 80 minutes using an SF₆, C₄F₈ process optimised 331 for highly anisotropic trenches. This etch was the same depth as the silicon and stopped when it 332 reached the SiO₂ back layer. The *PlasmaPro 100 Estrelas Deep Silicon Etch System* allows control 333 of the gas flow enabling processes to be tuned with negative and positive defined etch profiles. Our 334 spring profiles are vertical to within 0.5° . 335

To remove the sample from the carrier wafer it was heated to 88° C for 5 minutes, and then pushed laterally off the - now fluid - *Crystalbond*[®]. The SiO₂ and $AZ^{\mathbb{R}}$ -4562 layers enabled this to be done without damaging the MEMS device structure. The sample was then turned upside down and placed (not affixed) to a blank piece of silicon. The residual *Crystalbond*[®] and photoresist were removed from the bottom of the sample using an O₂ plasma ash. The sample was exposed to a CF₄/O₂ etchant plasma until all of the SiO₂ was removed, making the sample free standing.

342 Geometrical Anti-Spring Design

Our MEMS device is comprised of a proof mass, suspended from three curved cantilevers/flexures. 343 To better understand the physical characteristics of this system we first discuss these flexures 344 individually. Consider a cantilever, clamped at one end, and free to move at the other. A proof 345 mass mounted on the moving end will oscillate with a frequency that depends on the geometry 346 of the cantilever, and the Young's modulus of the material from which it is made. The proof 347 mass will oscillate along an arc, defined by the length of the flexure. The system will behave as 348 a Hooke's law spring, with a linear relationship between force and displacement. This behaviour 349 can be observed in figure 8a. A curved single cantilever also behaves in the same manner, as seen 350 in figure 8b. 351

To create an anti-spring, one can take two such curved cantilevers and attach them at a central pivot point. A proof mass mounted at this point will no longer be able to trace out an arc as it oscillates. Instead – because of the symmetrical forces applied by the two identical cantilevers – its motion will be constrained along a vertical axis (as presented in figure 1). It is this constraint that forces the spring constant to change as the displacement increases. Instead of observing a linear relationship between force and displacement, a non-linear behaviour is found. This behaviour can be observed in figure 8c. This now means that the spring gets softer with increasing displacement. A four flexure anti-spring system is a simple extension of a two-flexure system. Here, a second pair of cantilevers are placed below the first pair, this allows a non-point source proof mass to be suspended. The behaviour of the spring is still non-linear, and is displayed in figure 8d. The behaviour is identical to that of a two flexure system, except the system can support twice the mass.

Both the two and four anti-spring systems can be used to create oscillators that have low 364 resonant frequencies. When the limits of k/m are pushed to create the lowest resonant frequency 365 possible, however, these systems become unstable. They become unstable because the motion is 366 so well constrained along its vertical axis, the spring gets softer and softer until it can no longer 367 support the weight of the proof mass. This behaviour can be observed in figures 8c and 8d: as the 368 force increases, the displacement increases rapidly. A stable resonant frequency is imperative for 369 a useful relative gravimeter, therefore this instability would create problems if used for the design 370 of a MEMS gravimeter. It would require the use of a closed-loop feedback system. 371

Our MEMS device utilises a novel three-flexure anti-spring system, with one flexure of the upper 372 pair of cantilevers removed (see figure 1). In the first instance, the device behaves as a four-flexure 373 anti-spring: it gets rapidly softer as the displacement of the proof mass increases. The anti-spring 374 behaviour is maintained while the proof mass moves along its vertically constrained axis. The 375 asymmetry of the system, however, means that the device does not stay constrained along the 376 anti-spring constraining axis. The single upper flexure ultimately tilts the proof mass marginally 377 away from the constraining axis. As the motion is pulled from this axis, the anti-spring trend is 378 halted and the device regains a Hooke's law behaviour, where dF/dz = constant. This behaviour 379 can be observed in figure 8e where the gradient of force vs displacement reaches a minimum at 380 z = 0.6. This means that the device assumes a constant spring constant at the maximum stiffness 381 value that we have demonstrated to be stable over many months (as demonstrated by figure 13). 382

383 Optical shadow sensor

The proof mass motion is measured using an optical shadow sensor¹⁴. Using a fused silica 'C'-384 shaped support structure, a red LED (powered at 0.3 mW) was shone onto a split photodiode, 385 with the MEMS device proof mass mounted in between. The change in intensity incident on the 386 photodiode resulting from the motion of the proof mass shadow was then used as a measure of the 387 motion. The split photodiode was made from two 5 mm by 10 mm planar silicon photodiodes, and 388 wired to give a differential output. A split photodiode was used so that at the nominal position of 389 the proof mass the output signal was zero. This allowed maximal amplification without saturation 390 of the measurement instrumentation. The LED signal was modulated (at a frequency of 107 Hz 391 with a 50:50 duty cycle) to reduce the 1/f noise in the output signal. The modulation was carried 392 out by turning the LED on and off with an HP 33120A square wave signal generator. A precision 393 current stabalising resistor (displayed in figure 2 maintained the LED drive current, this was heat 394 sunk to the fused silica 'C'-shaped structure. The current output from the photodiode was first 395 converted into a voltage using an SRS SR570 current-to-voltage converter, band-passed between 396 3 Hz to 100 Hz, and amplified by a factor of 10^6 V/A. This amplified signal was then de-modulated 397 via an analogue lock-in amplifier (Femto LIA-MV-200) referenced from the signal generator. The 398 lock-in amplified the signal with a gain of 10 and undertook readings with a time constant of 399 3 s. This analogue signal was passed through an SRS SR560 low pass filter 0.03 Hz to remove 400 aliasing and filter seismic noise, before being digitised via a 16 bit, 12 dB/octave, analogue-to-401 digital converter (National Instruments M Series 6229) and recorded by a computer with a 24 s 402 time constant. Analogue signals were used to reduce digitisation noise that would have occurred 403 if a digital signal had been amplified by this magnitude. 404

405

The shadow sensor has a readout noise floor of $\leq 10 \ \mu Gal$ at the sampling frequency of 0.03 Hz,

and a dynamic range of ~50 μm . A large dynamic range is required because of the large initial displacement (0.8 mm) of the proof mass when it is tilted to its vertical operating orientation, thus making initial alignment of the MEMS device difficult. Although the maximum peak-to-peak displacement of the proof mass caused by the tides is only 16 nm, the proof mass also oscillates at its resonant frequency by up to 100 nm due to seismic ground motion. A high dynamic range is also useful to measure this signal, which is ultimately removed from the data by averaging with a 0.03 Hz filter in the readout electronics.

413 Temperature control

The control loops used to maintain the temperature of the system were proportional integral 414 derivative (PID) control mechanisms, written in *Labview*. Temperatures were monitored using a 415 four-terminal measurement of small platinum resistors, via two *Keithley 2000* digital multimeters. 416 A four-terminal measurement eradicates contact resistance by driving the thermometer with a 417 current and measuring the voltage across it. This removes the temperature sensitivity of external 418 wires. Low temperature coefficient $Manganin^{(R)}$ wires were used for these connections to minimise 419 parasitic thermal conduction. One platinum resistor was placed on the outer frame of the MEMS 420 device and three were placed equidistantly around the copper shield. Wire wound resistors were 421 used as the heating mechanism to feedback into the system; again, one of these was placed on 422 the MEMS device frame and three around the shield. The output signal to the heaters was sent 423 via a National Instruments (USB 6211) card, and the heaters were powered with non-inverting 424 amplifiers with a capability to power up to 100 mA. All circuitry and instrumentation used to 425 amplify and measure the output signal, and to measure and control the system temperature, were 426 selected for their high thermal stability. This entire configuration was constructed in a vacuum 427 chamber with a pressure of $< 10^{-5}$ mTorr. 428

429 Data analysis

Although PID control was implemented for the MEMS device and the shield, there were other 430 components with variations that could not be actively controlled. Namely the room temperature 431 that coupled into the data via a temperature sensitive lock-in amplifier, and intensity variations of 432 the LED that were monitored using a monitor photodiode. There was also an offset, and a linear 433 drift of under 150 μ Gal per day once the system had been left evacuated for over a week. This drift 434 term is due to stress in the silicon flexures. Like all mechanical systems, application of stress leads 435 to an elasticity which causes creep and drift over long timescales. Our device also shows polynomial 436 drift which decays away approximately one week after evacuating the apparatus. The polynomial 437 drift is likely due to adsorbed water on the surface layer of silicon, and could be mitigated against 438 by baking out the system before evacuation. Figure 9 demonstrates this initial polynomial drift. 439 The data were therefore regressed against the temperature measurements listed above, the drift 440 offset and the intensity. This regression – carried out in *Matlab* with the *mreqq* tool – identified 441 correlations between the output data and these parameters, and removed any resulting correlated 442 trends from the final data. Floor tilt and power variation of the LED were also monitored, but 443 neither had any discernable effect on the signal and were therefore not regressed. 444

The correlation coefficient, R, between the averaged theoretical and experimental tide data was calculated using *Matlab's 'corrcoeff'* function. An R value of 0.86 was produced, for the plot presented in figure 3. To check the level of significance of our experimental data we compared it to the correlation of the noise alone. We created 10,000 random permutations of our data set and calculated the correlation coefficient for each with respect to the theoretical data. This set of Rvalues were plotted as a histogram. This histogram had a distribution with a mean value of zero and a standard deviation of 0.008. The R value from the un-randomised data is 114 σ from this ⁴⁵² distribution suggesting the correlation is real to an extremely high degree of confidence.

Figure 10 is a plot of the RMS acceleration sensitivity of the device over its full spectral 453 range. The tide signal can be observed at 1×10^{-5} Hz. The peak at 10^{-3} Hz is an artefact of 454 the temperature servo. Between 0.1 Hz and 0.2 Hz the micro-seismic peak can be recognised, 455 its presence indicates that the device is also a sensitive seismometer. Past observations – made 456 from Scotland in February to March 2000 – of the microseismic peak²⁸ confirm the validity of 457 our observation. At 2.3 Hz the primary resonant mode of the MEMS device generates a large 458 peak due to excitation from seismic noise. This plot was used to calculate the sensitivity of the 459 MEMS device. To find a sensitivity in $\mu \text{Gal}/\sqrt{Hz}$, it is just necessary to read off the acceleration 460 sensitivity at the point where the data crosses 1 Hz on the horizontal axis. We believe that the 461 value of 40 $\mu \text{Gal}/\sqrt{Hz}$ is an overestimate of the true sensitivity of the device because at 1 Hz the 462 influence of both the primary resonance of the device and the micro-seismic peak are significant. 463

464 Tilt variation

Although tilt did not have an effect on the tide measurement, we are interested to know at what 465 point tilt would become an issue. Figure 12 presents two plots of an experiment used to asses the 466 effect of tilt on our device. Inside the vacuum tank, the MEMS device was mounted vertically and 467 aligned with the tilt sensor. The y-axis of the tilt sensor was aligned with the plane of the MEMS 468 device, with the x-axis perpendicular to this (see Fig. 1). Figure 12a demonstrates the induced 469 tilts of the tank in both x and y axes in arc seconds. Figure 12b demonstrates the corresponding 470 change in the output of the device in μ Gal. There is a strong correlation between the y axis 471 variation and the voltage output, giving a tilt sensitivity in this axis of 21.2 μ Gal/arcsec. There 472 is less sensitivity to the x-axis tilt with a tilt sensitivity of only 0.6 μ Gal/arcsec. 473

The x-axis tilt sensitivity is low because in the vertical configuration, the spring resumes a

Hooke's law response as observed in Fig. 5, for which the x-axis tilt variation is plotted against 475 the resonant frequency (the acceleration sensitivity of the device is proportional to the square of 476 the resonant frequency). Ultimately the spring could be tuned to operate with even less variation 477 with tilt in this axis if it were positioned to operate at one of its minima. Alternatively the flexures 478 could be made marginally thicker to shift the minimum in resonant frequency to 90° , this was not 479 carried out because the device did not show sufficient tilt sensitivity to cause concern. The y-axis 480 variation is larger because the device has a mode of oscillation in which the proof mass tilts and 481 pivots about the upper cantilever flexure. 482

When vertical, the device would need to be levelled with an accuracy limited by the y-axis sensitivity (i.e. less than 2 arc seconds to maintain the current sensitivity) to make repeatable measurements in different locations. This accuracy of levelling is achievable with a simple surveyors bubble level.

487 Temporal Reproducibility Tests

Figure 13 demonstrates two short data sets separated by nearly four months. These were used as a 488 test of the temporal stability of the device. To convert the raw voltage output of the device into a 489 unit of acceleration, a calibration factor was required. By comparing the experimental (blue) data 490 in Fig. 13a with that in 13b we were able to test whether the calibration factor had drifted over 491 time. The same calibration factor has been used to make both of these plots. By averaging the 492 data and changing the calibration factor of Fig. 13b, it was found that a change in the calibration 493 factor of 5% made the fit to the tide theory (red) data noticeably worse. Changes smaller than 494 this were not possible to resolve. We therefore believe that if the calibration factor has changed, 495 it has done so by no more than 5%. During this time period, the vacuum tank was vented and 496 evacuated several times, the MEMS was moved around each time. This is an important feature of 497

⁴⁹⁸ a device that could eventually be used in the field.

499 Applications

MEMS gravimeters have significant industrial applications. Given their small size and low cost, 500 they could be used for down bore-hole exploration in the oil and gas industry³³ and utilised to 501 monitor well drainage. Such devices could also be utilised for environmental monitoring, where 502 networks of sensor arrays could monitor sub surface water levels³⁴, or to determine the location 503 of historic landfill sites. The security industry is an area for which low cost/small form factor 504 gravimeters would also be a transformative technology to detect subterranean tunnels ^{35,36}, or 505 imaging of cargo containers where high spatial resolution via numerous sensors is an advantage³⁷. 506 MEMS gravimeters could also be used in civil engineering. At present in many of the UK victorian 507 cities the placement of utilities is only accurate on maps to within 15 metres of land marks such 508 as trees, fences or buildings. There have been trials of the *Scintrex CG5* and MEMS based arrays 509 would offer an exciting opportunity. Gravimetry is already used in volcanology and can be used 510 to help predict eruptions. Networks of small, low-cost gravimeter arrays could revolutionize the 511 way volcano gravimetry is carried out 22,25,24 . 512

A field prototype is currently being developed at Glasgow that will be the size of a tennis ball and require a power supply of under 1 W. A powerless getter pump will be used to maintain vacuum, both the thermal control and the optical readout will be on-chip; tilt levelling will be included, and all of the read-out and control software will be run on a micro-controller.

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544 Author contributions

545	• R.P.M. led the methodology of the etch process for the MEMS gravimeter and worked with
545	• 10.1 .W. led the methodology of the even process for the WEWB gravineter and worked with
546	G.D.H. on the development of the MEMS gravimeter. With G.D.H., he enhanced the long-
547	term, low-noise stability of the entire system, taking the tide data and performing the com-
548	putational analysis. He led writing the manuscript.
549	• A.S. led the methodology of the MEMS mask fabrication. With R.P.M he took the tide
550	measurements in early 2015 and performed computational analysis of the MEMS gravimeter.
551	• D.J.P. supervised the design of the MEMS device fabrication process and with G.D.H. came
552	up with the concept for a MEMS gravity sensor.
553	• J.H. developed the methodology of utilising geometric anti-springs for the MEMS gravimeter
554	system and provided critical review/commentary on the manuscript.
555	• S.R. was responsible for the resources which were necessary to complete the project and
556	provided critical review/commentary on the manuscript.
557	• G.D.H. had the initial concept of a MEMS gravimeter together with D.J.P. He had oversight
558	of the design, fabrication and testing of the gravimeter, via the supervision of R.P.M and A.S.
559	Together with R.P.M., he characterised and enhanced the low noise performance, resulting

Author information

562	• The research data relevant to this letter are stored on the University of Glasgow's Enlighten
563	Repository, DOI: http://dx.doi.org/10.5525/gla.researchdata.213
564	• Reprints and permissions information is available at www.nature.com/reprints
565	• The authors have no competing financial interests.
566	• Correspondence and requests for material should be addressed to giles.hammond@glasgow.gla.ac.uk
567	or r.middlemiss.1@research.gla.ac.uk.

568 Extended data

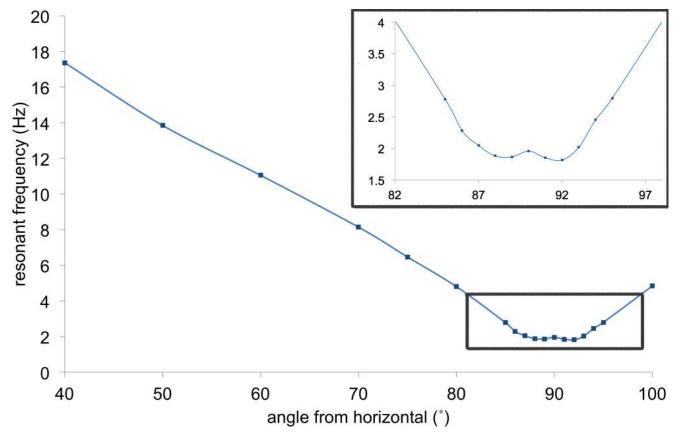


Figure 5

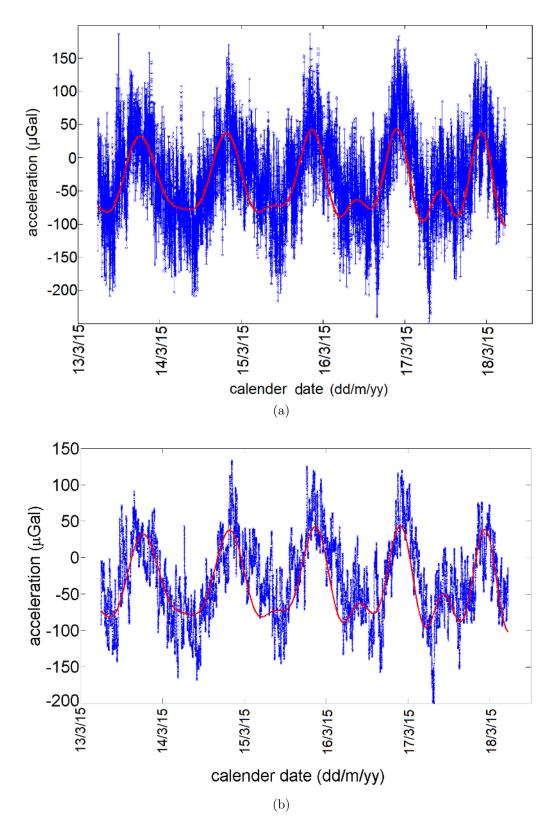


Figure 6

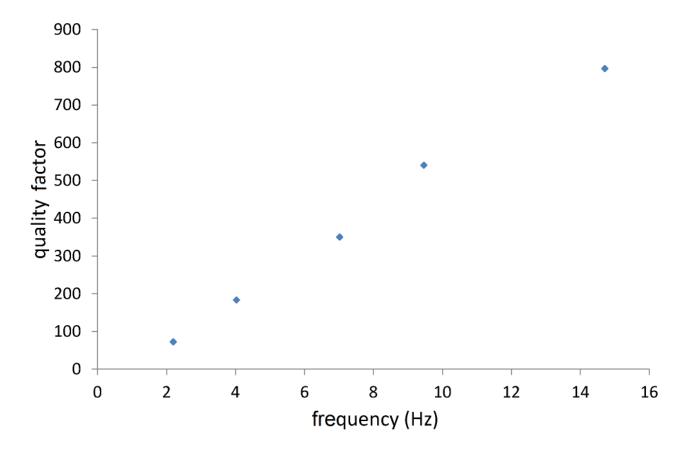


Figure 7

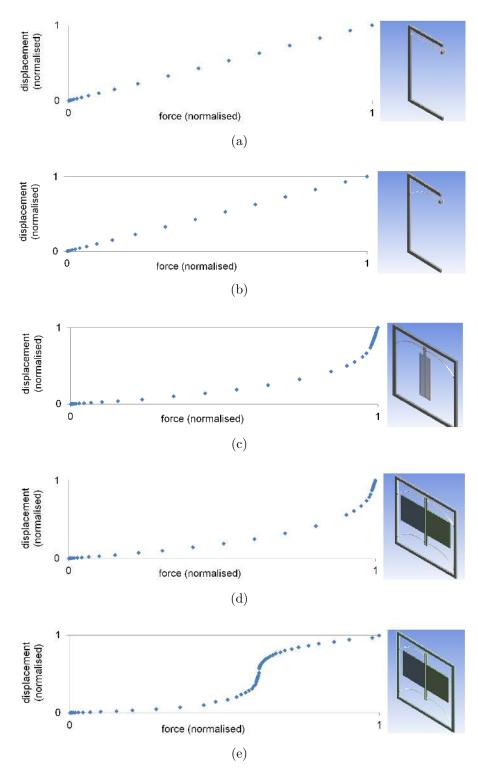
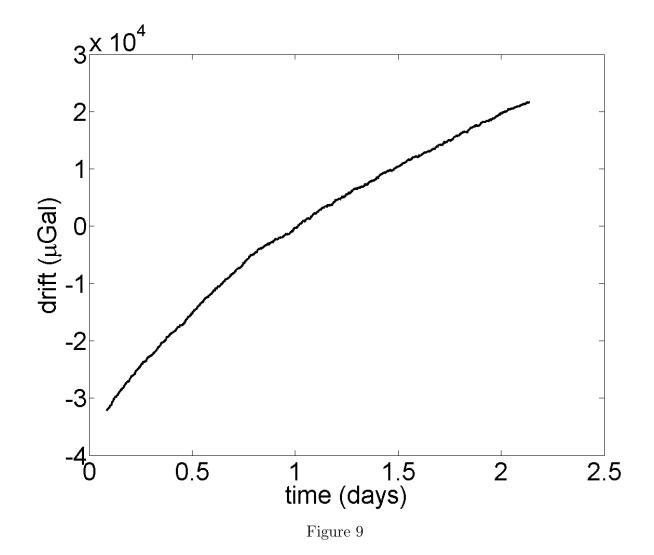


Figure 8



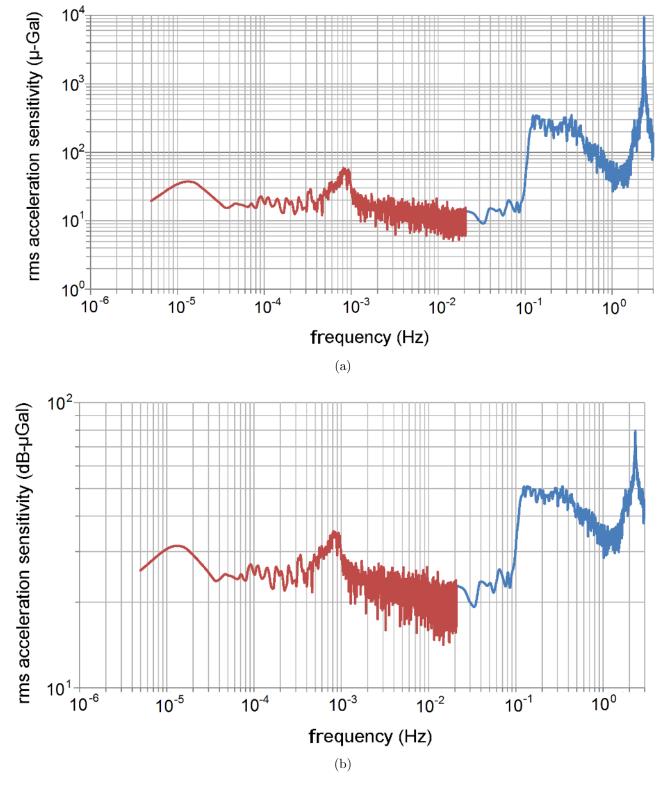


Figure 10

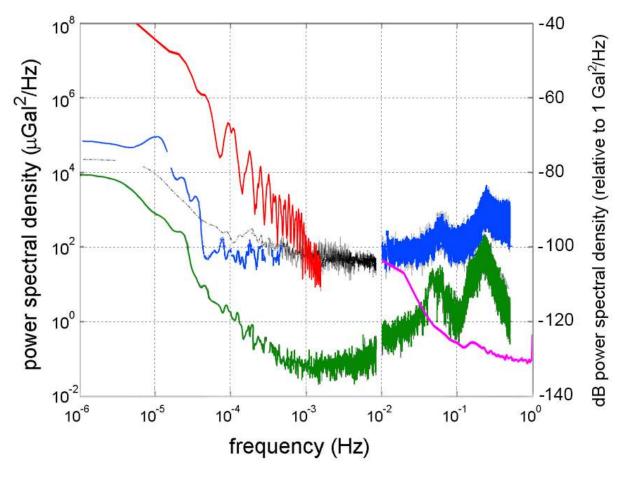
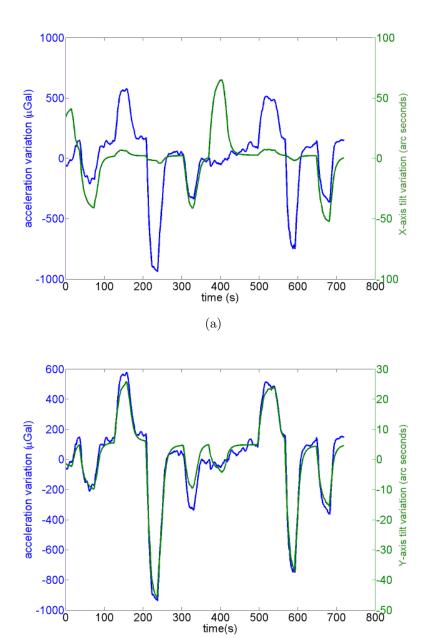


Figure 11



(b)

Figure 12

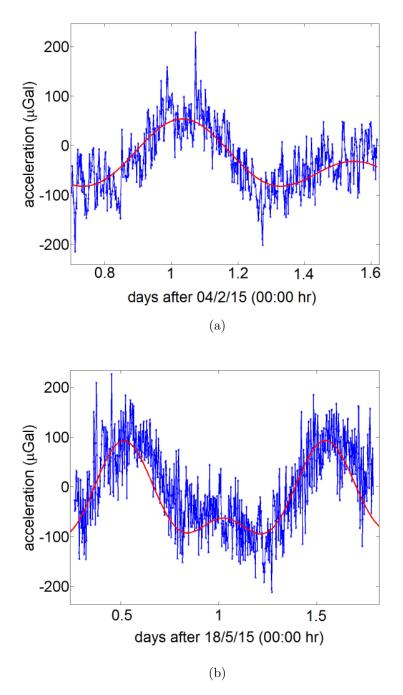


Figure 13

Device	Technology	Sensitivity	Stability in μ Hz	Resonant	Use
		at 1 Hz $$	Regime	Fre-	
				quency	
Scintrex CG5 ^{2,3}	Fused	$2 \ \mu \text{Gal}$	0.5 mGal/day	3 Hz	Gravimetry
	Quartz				
Krishnamoorthy ⁸	MEMS	$17\mu Gal$	N/A	36 Hz	Seismology
Quietseis ⁹	MEMS	$15 \ \mu Gal$	N/A	800 Hz	Seismology
Pike ¹⁰	MEMS	$2 \ \mu \text{Gal}$	N/A	11 Hz	Seismology
Glasgow MEMS	MEMS	$40 \ \mu \text{Gal}$	0.14 mGal/day	2.31 Hz	Gravimetry

 Table 1: Comparison of Acceleration Sensors