Measurement of the Excited-State Lifetime of a Microelectronic Circuit

K.W. Lehnert,[1,*] K. Bladh,[2] L. F. Spietz,[1] D. Gunnarsson,[2] D. I. Schuster,[1] P. Delsing,[2] and R. J. Schoelkopf[1]

[1]*Department of Applied Physics and Physics, Yale University, New Haven, Connecticut 06511*

[2]*Microtechnology Center at Chalmers MC2, Department of Microelectronics and Nanoscience, Chalmers University of Technology and Göteborg University, SE-412 96, Göteborg, Sweden*

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We demonstrate that a continuously measured microelectronic circuit, the Cooper-pair box measured by a radio-frequency single-electron transistor, approximates a quantum two-level system. We extract the Hamiltonian of the circuit through resonant spectroscopy and measure the excited-state lifetime. The lifetime is more than 10^5 times longer than the inverse transition frequency of the two-level system, even though the measurement is active. This lifetime is also comparable to an estimate of the known upper limit, set by spontaneous emission, for this circuit.

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Recently, microelectronic circuits have been coaxed into behaving as quantum two-level systems (TLS) [1–5]. The TLS behavior of circuits is revolutionary because it demonstrates the quantum behavior of a macroscopic degree of freedom composed of many microscopic degrees of freedom. Quantum coherence was believed to be fragile in electrical circuits both because it required the suppression of the dynamics of the microscopic elements in a condensed matter system and because the quantum oscillations of an electric or magnetic degree of freedom would efficiently radiate energy into the electromagnetic environment. Discussed in terms of the Bloch equations [6], familiar from nuclear magnetic resonance, a TLS in a coherent superposition of states has characteristic times T_2 to become an incoherent mixture and T_1 to relax back to its ground state.

In this Letter, we observe that a microelectronic circuit, the Cooper-pair box, may be measured continuously while still behaving approximately as a two-level system. The box is integrated with a radio-frequency single-electron transistor (RF-SET) measurement apparatus, which we operate as weak, continuous measurement of the box's state. Under these conditions we are able to determine the parameters that appear in the box's Hamiltonian, make a worst-case estimate T_2^* of the decoherence time T_2, and measure the excited-state lifetime T_1 of the two-level system. We determine the parameters in the Hamiltonian through a kind of spectroscopy where we observe a resonant change in the box's state when its transition frequency matches a multiple of the frequency of an oscillatory excitation. From the width in frequency of these resonances we can find T_2^* [7]. We stimulate the box into its excited state and measure T_1 directly by exploiting the large measurement bandwidth of the RF-SET to resolve in time the circuit's decay to its ground state. Most remarkably, the value of T_1 that we find while *continuously measuring* the state of the box is comparable to estimates of the excited-state lifetime limited by the quantum fluctuations of the electromagnetic environment. This demonstrates that the Cooper-pair box, when embedded in a circuit for control and measurement, remains well decoupled from other sources of dissipation. Based on the observed noise in the readout and the lifetime, we conclude that RF-SET is a promising qubit readout because a "single-shot" measurement, where the box is observed in its excited state before it has relaxed into its ground state, is possible.

The Cooper-pair box is a microelectronic circuit composed of an isolated superconducting island, attached to a superconducting lead through a tunnel junction. An additional lead, called the gate lead, lies near the island and changes the electrostatic potential of the island with the application of a voltage V_g to the gate lead through the gate capacitance C_g [Fig. 1(a)]. The island's total capacitance C_Σ is small enough to suppress fluctuations of charge on the island. Because the island and the lead are superconducting, all of the electrons form Cooper pairs and participate in the macroscopic quantum ground state of the island. The only degree of freedom is the number of pairs n on the island. Because of the large charging energy $E_C = e^2/2C_\Sigma$, we need consider only two states, a state $|0\rangle$ with no excess Cooper pairs ($n = 0$) and a state $|1\rangle$ with one excess Cooper pair ($n = 1$), as reckoned from electrical neutrality. The Hamiltonian of the Cooper-pair box circuit is

$$\mathbf{H} = -2E_c(1 - 2n_g)\boldsymbol{\sigma}_z - \frac{E_J}{2}\boldsymbol{\sigma}_x, \qquad (1)$$

where $\boldsymbol{\sigma}_z$ and $\boldsymbol{\sigma}_x$ are the Pauli spin matrices and n_g is the total polarization charge applied to the gate electrode, $n_g = C_g V_g/2e - n_{\text{off}}$, in units of a Cooper-pair's charge [10,11]. The offset charge n_{off} accounts for the uncontrolled potential arising from charges nearby the box island. The Josephson energy, $E_J^{\max} = h\Delta/8e^2R_\Sigma$, is the effective tunneling matrix element for Cooper pairs across a junction with resistance R_Σ in a superconductor with BCS gap Δ. The junction is, in fact, a composite of two parallel junctions connected to form a loop with

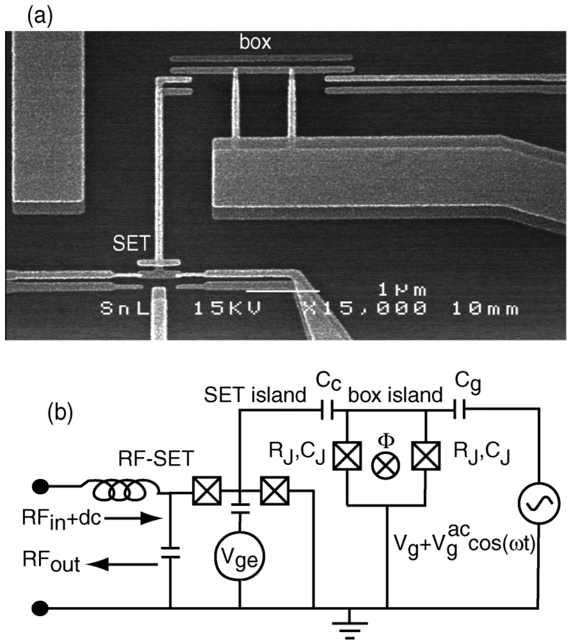

FIG. 1. (a) An SEM micrograph of the Cooper-pair box and SET electrometer. The device is made from an evaporated aluminum film (light gray regions) on an insulating SiO_2 substrate (dark gray regions) by the technique of double angle evaporation [8], which gives the double image. The aluminum has BCS gap $\Delta/k_B = 2.4$ K. (b) A circuit diagram of the box and RF-SET electrometer. The SET gate voltage V_{ge}, the 500 MHz oscillatory bias, and the dc bias (RF_{in}+dc) determine the electrometer's operating point. The charge on the box is inferred from variation in the amount of applied RF power that is reflected (RF_{out}) from the SET electrometer, which is a sensitive function of SET's conductance [9]. The tunnel junctions (crosses in boxes) are characterized by a junction resistance R_J and capacitance C_J, which enter the box's Hamiltonian through $C_\Sigma = C_C + 2C_J + C_g$ and $R_\Sigma = R_J/2$ (see text).

1 $(\mu m)^2$ area (Fig. 1). The effective Josephson energy E_J of the pair of junctions is then tunable with magnetic flux Φ through this loop, as $E_J = E_J^{max}\cos(\pi\Phi/\Phi_0)$, where Φ_0 is the quantum of flux ($h/2e$). Equation (1) is the Hamiltonian of a quasispin 1/2 particle in a fictitious magnetic field that can be decomposed into two orthogonal fields. The z component of this fictitious field which accounts for the box's electrostatic energy, $E_{el}(V_g) = 2E_c(1 - 2n_g)$, is tuned with V_g and the $\boldsymbol{x}$ component, which accounts for the Josephson energy $E_J(\Phi) = E_J^{max}\cos(\pi\Phi/\Phi_0)$, is tuned with Φ [11]. The box is an artificial two-level system and both of the terms in its Hamiltonian are tunable *in situ*.

In the box, states of definite numbers of Cooper pairs on the island are states of definite charge. In order to measure the charge of the Cooper-pair box, we fabricate the box next to a RF-SET [8,9], an exquisitely sensitive electrometer, so that the addition of a Cooper pair to the box's island causes a small fraction ($C_C/C_\Sigma = 3.7\%$) of the Cooper pair's charge to appear as polarization charge on the capacitor C_C that couples the box and the RF-SET (Fig. 1). The electrometer used here had a sensitivity of $4 \times 10^{-5}\ e/\sqrt{\text{Hz}}$ and 10 MHz of measurement bandwidth. Because the RF-SET measures charge, its action can be described as projecting the state of the box into a state of definite Cooper-pair number. In the formal terms of Eq. (1), it measures $Q_{box} = (1 + \langle \boldsymbol{\sigma}_z \rangle)e$ where Q_{box} is further averaged over the measurement time.

We perform spectroscopy by applying a continuous microwave stimulus to the gate of the Cooper-pair box and sweeping n_g to tune the parameters of the TLS and find the resonance condition (Fig. 2). A measurement of Q_{box} vs n_g shows that the box does not remain in its ground state over a range $0.3 < n_g < 0.7$. This behavior is caused by backaction [12,13] generated by currents flowing through RF-SET [14]. We proceed by studying the box in the range of n_g where it does remain in its ground state.

When a 35 GHz microwave signal is applied to the gate, we observe clear evidence that the box is a coherent two-level system. Resonant peaks appear [Fig. 2(b)] in Q_{box} that are sharp and symmetrically spaced about $n_g = 0.5$. The two features, a peak for $n_g < 0.5$ and a dip for

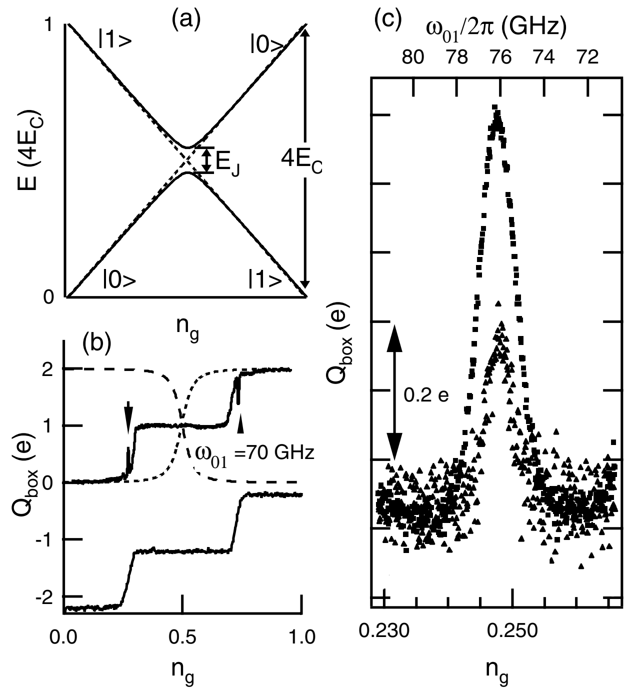

FIG. 2. (a) The ground and excited state energies versus n_g for Eq. (1), with $4E_C = 12E_J$ (solid line) and $E_J = 0$ (dotted lines). Energy eigenstates asymptotically approach charge states ($|1\rangle$ and $|0\rangle$) far from $n_g = 0.5$. (b) Q_{box} vs n_g, calculated for the ground state (dotted line), excited state (dashed line), and measured (solid line) with 35 GHz microwaves applied to the box gate. The arrows indicate resonant peaks. Also shown is Q_{box} measured with no microwaves applied (solid line), with the y axis shifted down by 2.2 e. (c) Two resonant peaks in Q_{box} vs n_g on the bottom axis and vs ω_{01} on the top axis, with $\omega = 38$ GHz and where the larger value of V_g^{ac} (squares) is twice the smaller value (triangles).

$n_g > 0.5$, both correspond to the change in $Q_{\rm box}$ when the box spends some time in the excited state. Because $Q_{\rm box}$ is an average of thousands of repeated measurements, the peak height indicates the probability of finding the box in its excited state [Fig. 2(c)].

The resonant peaks permit a spectroscopic determination of E_C and $E_J^{\max}$. By tuning n_g and Φ while exciting the box with a fixed microwave frequency, we find good agreement between the locations of resonant peaks and the difference between ground-state and excited-state energies $E_{01}(n_g, \Phi) = \hbar\omega_{01}$ expected from Eq. (1). An independent measurement of E_C [15] demonstrates that these peaks occur when the irradiating frequency ω is half ω_{01}, indicating that these peaks correspond to a two-photon transition [16]. At lower frequencies and for single-photon transitions, the peaks would appear at an n_g for which the box does not stay in the ground state while being measured and are therefore not visible. We find a single value for E_C and for $E_J^{\max}$ that account for the location of the resonant peaks at applied frequencies between 32 and 38 GHz giving resonant peaks for ω_{01} between 64 and 76 GHz [Fig. 3(a)]. We are able to extract the parameters of the Hamiltonian, $4E_C/h = 149.1 \pm 0.4$ GHz and $E_J^{\max}/h = 13.0 \pm 0.2$ GHz, which imply $C_\Sigma = 518$ aF and $R_\Sigma = 12.4\,\mathrm{k}\Omega$. Through spectroscopy we have measured the parameters of an electrical circuit that could not have been measured with transport [Fig. 1(b)]. Because these measurements were made at a temperature $T < 40$ mK, they are in the limit $k_B T \ll E_J < E_C$.

Consistent with the behavior of a TLS, the peaks disappear for $\Phi = \Phi_0/2$ when E_J approaches zero. This demonstrates that E_J provides the coupling between the charge states [Fig. 3(b)]. An oscillating gate voltage with amplitude V_g^{ac} adds a term to the Hamiltonian in Eq. (1), which is $(C_g V_g^{ac}/2e)\cos(\omega t)\boldsymbol{\sigma}_z$ and is collinear with the ground state of the quasispin described by Eq. (1) when $E_J = 0$. The microwave excitation therefore applies no torque which could excite the quasispin from its ground state [6].

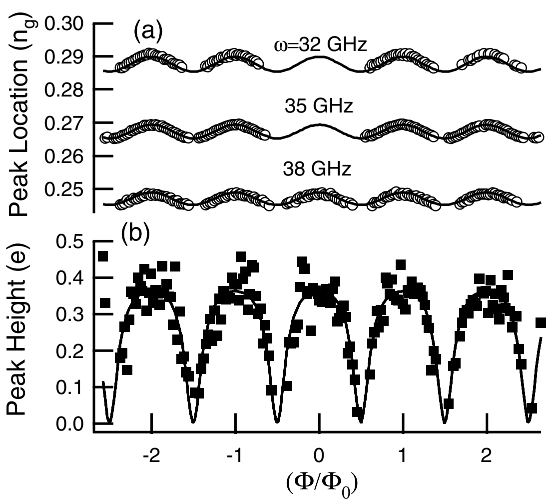

FIG. 3. Resonant spectroscopy of the box versus the two control parameters of the Hamiltonian, V_g and Φ. (a) The locations of resonant peaks (circles) in n_g and Φ, for $\omega = 32$, 35, and 38 GHz and fits (lines), using Eq. (1) for $\omega_{01} = 2\omega = 64$, 70, and 76 GHz to find a single value of E_C and of $E_J^{\max}$. The systematic uncertainty in n_g is represented by the size of the open circle symbols. (b) The height, in electrons, of a 76 GHz resonant peak as a function of Φ (squares) and a guide to the eye (line).

The width of the resonant peaks we observe provides a worst-case estimate of the decoherence time of the two-level system. We express the width of a resonance δn_g as a width in frequency $\delta\omega_{01} = (1/\hbar)(dE_{01}/dn_g)\delta n_g$. In the absence of inhomogenous broadening, the half width at half maximum inferred for zero power is the decoherence rate $1/T_2$ of a TLS [6]. From $\delta\omega_{01}$ measured at the lowest value of V_g^{ac} applied, we estimate a time T_2^* of about 325 ps [7]. The resonant peaks have a Gaussian shape, and $n_{\rm off}$ drifts an amount comparable to δn_g during the 2 min required to complete a measurement. Both observations imply that the width of the peaks expresses not the intrinsic loss of phase coherence due to coupling the TLS to the environment, but rather the degree to which an ensemble of measurements are not identical, due to the well-known $1/f$ noise of single-electron devices [17]. This T_2^* is a worst-case estimate because it is extracted while the system is measured continuously by the RF-SET and because it represents an ensemble average of many single measurements that require about 2 min to complete. Nevertheless, T_2^* is about 150 times longer than $1/\omega_{01}$ [Fig. 2(c)] and is similar to the times found in [18], another Cooper-pair box implementation, as well as [5] a SQUID circuit. Reference [4] demonstrates that this inhomogenous broadening may be overcome by operating the Cooper box at $n_g = 0.5$ where E_{01} is to first order insensitive to fluctuations in $n_{\rm off}$.

In order to measure the excited-state lifetime T_1, we excite the box and then measure the time required to relax back to the ground state. A 38 GHz signal is continuously applied to the gate and the box gate is tuned to $n_g = 0.248$ and $E_J = E_J^{\max}$ so that the microwaves resonantly couple the ground and excited states through a two-photon transition. Abruptly, n_g is then shifted to $n_g = 0.171$ in 30 ns, slowly enough to be adiabatic but much faster than T_1. The microwave excitation then no longer resonantly couples the ground and excited states, and the probability of being in the excited state decays in a time T_1. By averaging many of the transient responses to this stimulus, we find $T_1 = 1.3\ \mu$s (Fig. 4). A similar T_1 was found in [4] for a Cooper-pair box with much smaller E_C and operated at $n_g = 0.5$. The lifetime is a quantity which is insensitive to slow drifts in $n_{\rm off}$ and demonstrates that the TLS, which oscillates $T_1 \times \omega_{01} = 6 \times 10^5$ times before relaxing into its ground state, is well decoupled from all other sources of dissipation.

We can compare this long lifetime with the spontaneous emission rate expected from the quantum fluctuations of a generic electromagnetic environment. Calculating the

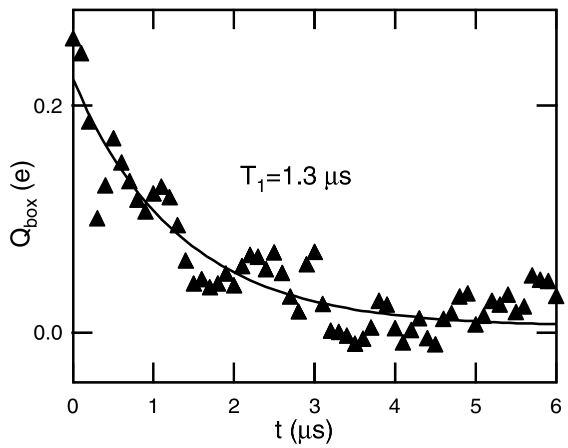

FIG. 4. A determination of the excited-state lifetime of the box. Q_{box} vs time t (triangles), relative to $t = 0$, when n_g is shifted from $= 0.248$ to 0.171 in 30 ns, with 38 GHz microwaves applied. The shift in n_g brings the box out of resonance with the microwave excitation. An exponential fit to the data implies $T_1 = 1.3$ μs (line).

rate using Fermi's golden rule gives

$$\frac{1}{T_1} = \left(\frac{C_g^T}{C_\Sigma}\right)^2 \left(\frac{e}{\hbar}\right)^2 \sin^2(\theta) S_V(\nu_{01} = \omega_{01}/2\pi), \quad (2)$$

where $S_V(\nu) = 2h\nu(\mathrm{Re}(Z_0))$ is the voltage spectral density of the quantum fluctuations of an environment with an impedance Z_0 at frequency ν and $\sin\theta = E_J/\hbar\omega_{01}$ [11]. The quantity C_g^T is the total capacitance of the box to nearby metal traces, including intentional coupling to the gate lead and other unintended capacitive coupling (Fig. 1). We calculate T_1 for a 50 Ω environment to be between 0.25 and 1 μs, extracting $C_g^T = 45 \pm 15$ aF from an electrostatic simulation of the chip layout [11,12]. We do not claim to have demonstrated that the lifetime is limited by spontaneous emission; however, if there are additional relaxation processes, either due to the electrometer or fluctuations of some microscopic degree of freedom in the box, their influence is at most comparable to that of spontaneous emission into a typical ($Z_0 \approx$ 50 Ω) electromagnetic environment.

In these experiments, we demonstrate that a Cooper-pair box is a coherent two-level system with a long excited-state lifetime. With spectroscopy, we determine the box's Hamiltonian and its spontaneous emission rate into a typical environment. We measure an excited-state lifetime of a box that is remarkable for two reasons. First, it shows that a quantum-coherent microelectronic circuit can have a T_1 that approaches the limit set by spontaneous emission of a photon into the electromagnetic environment. Second, it is observed by resolving, on submicrosecond time scales, the decay of the excited-state charge signal while the two-level system is continuously measured. Given the observed electrometer sensitivity of 4×10^{-5} $e/\sqrt{\mathrm{Hz}}$, the excited-state lifetime is long enough that a single measurement can discriminate between the box in its excited state and the box in its ground state. In a coherent superposition of states the box oscillates 6×10^5 times before decaying to the ground state, demonstrating that the circuit is a promising qubit implementation if, as in [4], the sources of inhomogeneous broadening can be overcome.

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*Email address: konrad.lehnert@yale.edu
Group Web site: www.eng.yale.edu/rslab

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