

## Measurement of the $\gamma\gamma^* \rightarrow \eta$ and $\gamma\gamma^* \rightarrow \eta'$ transition form factors

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We study the reactions  $e^+e^- \rightarrow e^+e^-\eta^{(\prime)}$  in the single-tag mode and measure the  $\gamma\gamma^* \rightarrow \eta^{(\prime)}$  transition form factors in the momentum transfer range from 4 to 40 GeV<sup>2</sup>. The analysis is based on 469 fb<sup>-1</sup> of integrated luminosity collected at PEP-II with the BABAR detector at  $e^+e^-$  center-of-mass energies near 10.6 GeV.

PACS numbers: 14.40.Be, 13.40.Gp, 12.38.Qk

## I. INTRODUCTION

In this paper we report results from studies of the  $\gamma^*\gamma \rightarrow P$  transition form factors, where  $P$  is a pseudoscalar meson. In our previous works [1, 2] the two-photon reaction

$$e^+e^- \rightarrow e^+e^-P, \quad (1)$$

illustrated by Fig. 1, was used to measure the  $\pi^0$  and  $\eta_c$  transition form factors. Here this technique is applied to study the  $\eta$  and  $\eta'$  form factors. The transition form factor describes the effect of the strong interaction on the  $\gamma^*\gamma^* \rightarrow P$  transition. It is a function of the photon virtualities  $F(q_1^2, q_2^2)$ . We measure the differential cross sec-

tions for the processes  $e^+e^- \rightarrow e^+e^-\eta^{(\prime)}$  in the single tag mode where one of the outgoing electrons<sup>1</sup> (tagged) is detected while the other (untagged) is scattered at a small angle. The tagged electron emits a highly off-shell photon with the momentum transfer  $q_1^2 \equiv -Q^2 = (p - p')^2$ , where  $p$  and  $p'$  are the four-momenta of the initial and final electrons. The momentum transfer to the untagged electron ( $q_2^2$ ) is near zero. The form factor extracted from the single tag experiment is a function of one of the  $q^2$ 's:  $F(Q^2) \equiv F(-Q^2, 0)$ . To relate the differential cross section  $d\sigma(e^+e^- \rightarrow e^+e^-P)/dQ^2$  to the transition form factor we use formulae equivalent to those for the  $e^+e^- \rightarrow e^+e^-\pi^0$  cross section in Eqs. (2.1) and (4.5) of Ref. [3].

At large momentum transfer, perturbative QCD predicts that the transition form factor can be represented as a convolution of a calculable hard-scattering amplitude for  $\gamma\gamma^* \rightarrow q\bar{q}$  with a nonperturbative meson distribution amplitude (DA)  $\phi_P(x, Q^2)$  [4]. The latter can

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<sup>1</sup> Unless otherwise specified, we use the term “electron” for either an electron or a positron.

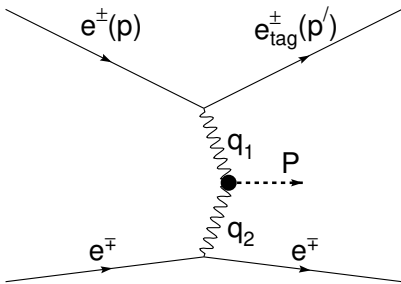


FIG. 1: The diagram for the  $e^+e^- \rightarrow e^+e^-P$  two-photon production process, where  $P$  is a pseudoscalar meson.

be interpreted as the amplitude for the transition of the meson with momentum  $p_M$  into two quarks with momenta  $p_M x$  and  $p_M(1-x)$ . The experimentally derived photon-meson transition form factors can be used to test different models for the DA.

The  $\eta$  and  $\eta'$  transition form factors have been measured in two-photon reactions in several previous experiments [5–9]. The most precise data for the  $\eta^{(\prime)}$  at large  $Q^2$  were obtained by the CLEO experiment [9]. They cover the  $Q^2$  region from 1.5 to about 20  $\text{GeV}^2$ . In this paper we study the  $\eta$  and  $\eta'$  form factors in the  $Q^2$  range from 4 to 40  $\text{GeV}^2$ .

## II. THE BABAR DETECTOR AND DATA SAMPLES

We analyze a data sample corresponding to an integrated luminosity of about  $469 \text{ fb}^{-1}$  recorded with the BABAR detector [10] at the PEP-II asymmetric-energy storage rings at the SLAC National Accelerator Laboratory. At PEP-II, 9-GeV electrons collide with 3.1-GeV positrons to yield a center-of-mass (c.m.) energy near 10.58 GeV (the  $\Upsilon(4S)$  resonance). About 90% of the data used in the present analysis were recorded on-resonance and about 10% were recorded about 40 MeV below the resonance.

Charged-particle tracking is provided by a five-layer silicon vertex tracker (SVT) and a 40-layer drift chamber (DCH), operating in a 1.5-T axial magnetic field. The transverse momentum resolution is 0.47% at 1  $\text{GeV}/c$ . Energies of photons and electrons are measured with a CsI(Tl) electromagnetic calorimeter (EMC) with a resolution of 3% at 1 GeV. Charged-particle identification is provided by specific ionization ( $dE/dx$ ) measurements in the SVT and DCH and by an internally reflecting ring-imaging Cherenkov detector (DIRC). Electron identification also makes use of the shower shape in the EMC and the ratio of shower energy to track momentum. Muons are identified in the instrumented flux return of the solenoid, which consists of iron plates interleaved with either resistive plate chambers or streamer tubes.

Signal  $e^+e^- \rightarrow e^+e^-\eta^{(\prime)}$  and background processes

are simulated with the Monte Carlo (MC) event generator GGResRc [11]. It uses the formula for the differential cross section from Ref. [3] for pseudoscalar meson production and the Budnev-Ginzburg-Meledin-Serbo formalism [12] for the two-meson final states. Because the  $Q^2$  distribution is peaked near zero, the MC events are generated with a restriction on the momentum transfer to one of the electrons:  $Q^2 > 3 \text{ GeV}^2$ . This restriction corresponds to the limit of detector acceptance for the tagged electron. The second electron is required to have momentum transfer  $-q_2^2 < 0.6 \text{ GeV}^2$ . The experimental criteria providing these restrictions for data events will be described in Sec. III. The form factor is fixed to the constant value  $F(0,0)$  in the simulation.

The GGResRc event generator includes next-to-leading-order radiative corrections to the Born cross section calculated according to Ref. [13]. In particular, it generates extra soft photons emitted by the initial- and final-state electrons. The formulae from Ref. [13] are modified to take into account the hadron contribution to the vacuum polarization diagrams. The maximum energy of the photon emitted from the initial state is restricted by the requirement<sup>2</sup>  $E_\gamma^* < 0.05\sqrt{s}$ , where  $\sqrt{s}$  is the  $e^+e^-$  c.m. energy. The generated events are subjected to detailed detector simulation based on GEANT4 [14] and are reconstructed with the software chain used for the experimental data. Temporal variations in the detector performance and beam background conditions are taken into account.

## III. EVENT SELECTION

The decay modes with two charged particles and two photons in the final state,  $\eta' \rightarrow \pi^+\pi^-\eta$ ,  $\eta \rightarrow \gamma\gamma$  and  $\eta \rightarrow \pi^+\pi^-\pi^0$ ,  $\pi^0 \rightarrow \gamma\gamma$ , are used to reconstruct  $\eta'$  and  $\eta$  mesons, respectively.

Events with at least three charged tracks and two photons are selected. Since a significant fraction of signal events contains beam-generated spurious track and photon candidates, one extra track and any number of extra photons are allowed in an event. The tracks corresponding to the charged pions and electron must originate from the interaction region. The identified pion candidates must have polar angles in the range  $0.451 < \theta < 2.400$  rad, while the track identified as an electron must be in the angular range  $0.387 < \theta < 2.400$  rad (0.64–2.69 rad in the  $e^+e^-$  c.m. frame). The angular requirements are needed for good electron and pion identification. To recover electron energy loss due to bremsstrahlung, both internal and in the detector material before the DCH, the energy of any EMC shower close to the electron direction

<sup>2</sup> Throughout this paper an asterisk superscript denotes quantities in the  $e^+e^-$  c.m. frame. In this frame the positive z-axis is defined to coincide with the  $e^-$  beam direction.



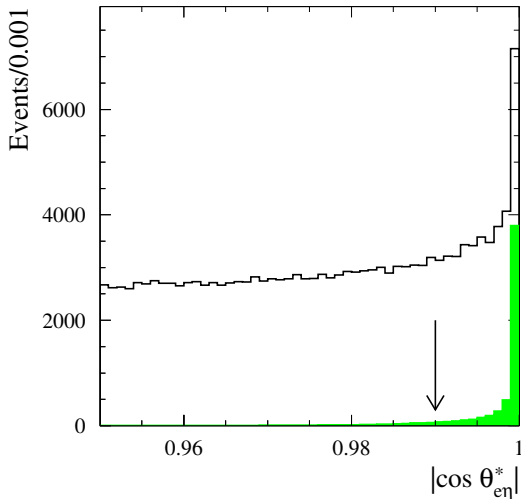


FIG. 2: The  $|\cos \theta_{e\eta}^*|$  distribution for data events. The shaded histogram shows the same distributions for the  $e^+e^- \rightarrow e^+e^-\eta$  simulation. Events with  $|\cos \theta_{e\eta}^*| > 0.99$  (indicated by the arrow) are retained.

is combined with the measured energy of the electron track. The resulting c.m. energy of the electron candidate must be greater than 1 GeV.

The photon candidates are required to have laboratory energies greater than 50 MeV. For the  $e^+e^- \rightarrow e^+e^-\eta'$  selection, two photon candidates are combined to form an  $\eta$  candidate. Their invariant mass is required to be in the range 0.480–0.600 GeV/ $c^2$ . To suppress combinatorial background from spurious photons, the photon helicity angle is required to satisfy the condition  $|\cos \theta_h| < 0.9$ . The helicity angle  $\theta_h$  is defined in the  $\eta$  rest frame as the angle between the decay photon momentum and direction of the boost from the laboratory frame. Each candidate is then fitted with an  $\eta$  mass constraint to improve the precision of its momentum measurement. An  $\eta'$  candidate is formed from a pair of oppositely-charged pion candidates and an  $\eta$  candidate. The  $\eta'$  invariant mass must be in the range 0.920–0.995 GeV/ $c^2$ . The  $\eta'$  candidate is also then fitted with a mass constraint.

Similar selection criteria are used for  $e^+e^- \rightarrow e^+e^-\eta$  candidates. An  $\eta$  candidate is formed from a pair of oppositely charged pion candidates and a  $\pi^0$  candidate, which is a combination of two photons with invariant mass between 0.115 and 0.150 GeV/ $c^2$  and the cosine of the photon helicity angle  $|\cos \theta_h| < 0.9$ . The mass of the  $\eta$  candidate must be in the window 0.48–0.62 GeV/ $c^2$ .

Figure 2 shows the  $|\cos \theta_{e\eta}^*|$  distribution for data events and the  $e^+e^- \rightarrow e^+e^-\eta$  simulation, where  $\theta_{e\eta}^*$  is the polar angle of the momentum vector of the  $e\eta$  system in the  $e^+e^-$  c.m. frame. We require that  $|\cos \theta_{e\eta}^*|$  be greater than 0.99. This condition effectively limits the value of the momentum transfer to the untagged electron ( $q_2^2$ ) and guarantees compliance with the condition  $-q_2^2 < 0.6$  GeV $^2$  used in the MC simulation. The same condition

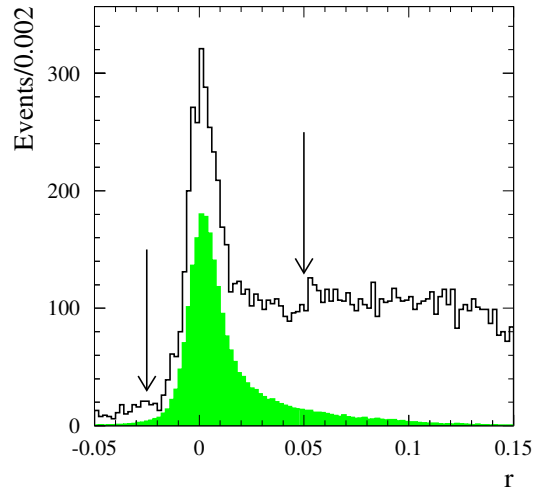


FIG. 3: The  $r$  distributions for  $e^+e^- \rightarrow e^+e^-\eta$  data (solid-line histogram) and signal simulation (shaded histogram). The arrows indicate the region used to select event candidates ( $-0.025 < r < 0.05$ ).

$|\cos \theta_{e\eta'}^*| > 0.99$  is used to select the  $e^+e^- \rightarrow e^+e^-\eta'$  event candidates.

The emission of extra photons by the electrons involved leads to a difference between the measured and actual values of  $Q^2$ . In the case of initial-state radiation (ISR)  $Q_{meas}^2 = Q_{true}^2(1 + r_\gamma)$ , where  $r_\gamma = 2E_\gamma^*/\sqrt{s}$ . To restrict the energy of the ISR photon we use the parameter

$$r = \frac{\sqrt{s} - E_{e\eta^{(\prime)}}^* - |p_{e\eta^{(\prime)}}^*|}{\sqrt{s}}, \quad (2)$$

where  $E_{e\eta^{(\prime)}}^*$  and  $p_{e\eta^{(\prime)}}^*$  are the c.m. energy and momentum of the detected  $e\eta^{(\prime)}$  system. For ISR this parameter coincides with  $r_\gamma$  defined above. The  $r$  distributions for the data  $e^+e^- \rightarrow e^+e^-\eta$  event candidates and signal simulation are shown in Fig. 3. For both processes under study, we select events with  $-0.025 < r < 0.05$ . It should be noted that this condition on  $r$  ensures compliance with the restriction  $r_\gamma < 0.1$  used in the simulation.

For two-photon events with a tagged positron (electron), the momentum of the detected  $e\eta^{(\prime)}$  system in the  $e^+e^-$  c.m. frame has a negative (positive)  $z$ -component, while events resulting from  $e^+e^-$  annihilation are produced symmetrically. To suppress the  $e^+e^-$  annihilation background, event candidates with the wrong sign of the momentum  $z$ -component are removed.

The distributions of the invariant masses of  $\eta$  and  $\eta'$  candidates for data events satisfying the selection criteria described above are shown in Fig. 4. For events with more than one  $e^\pm\eta^{(\prime)}$  candidate (about 5% of the selected events), the candidate with smallest absolute value of the parameter  $r$  is selected. Only events with  $4 < Q^2 < 40$  GeV $^2$  are included in the spectra of Fig. 4. For  $Q^2 < 4$  GeV $^2$  the detection efficiency for single-tag two-photon

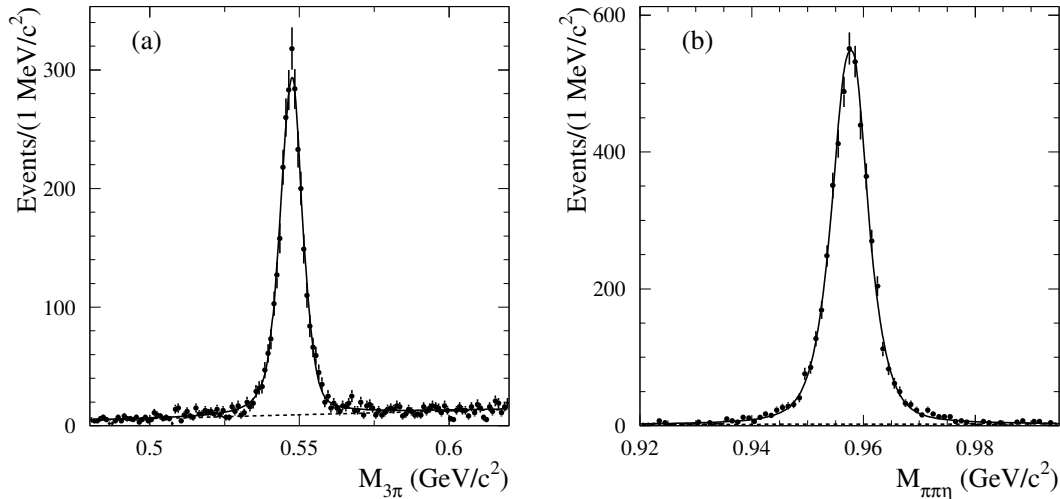


FIG. 4: The (a)  $\pi^+\pi^-\pi^0$  and (b)  $\pi^+\pi^-\eta$  mass spectra for data events with  $4 < Q^2 < 40 \text{ GeV}^2$ . The solid curves are the results of the fits described in Sec. IV. The dashed lines represent non-peaking background.

$\eta$  and  $\eta'$  events is small (see Sec. VI). In the region  $Q^2 > 40 \text{ GeV}^2$  we do not see evidence of  $\eta$  or  $\eta'$  signal over background. About 4350 and 5200 events survive the selection described above for  $\eta$  and  $\eta'$ , respectively.

#### IV. FITTING THE $\pi^+\pi^-\pi^0$ AND $\pi^+\pi^-\eta$ MASS SPECTRA

To determine the number of events containing an  $\eta^{(\prime)}$ , we perform a binned likelihood fit to the spectra shown in Fig. 4 with a sum of signal and background distributions. We describe the signal line shape by the following function:

$$F(x) = A[G(x) \sin^2 \zeta + B(x) \cos^2 \zeta], \quad (3)$$

where

$$G(x) = \exp\left(-\frac{(x-a)^2}{2\sigma^2}\right), \quad (4)$$

$$B(x) = \begin{cases} \frac{(\Gamma_1/2)^{\beta_1}}{(a-x)^{\beta_1} + (\Gamma_1/2)^{\beta_1}} & \text{if } x < a; \\ \frac{(\Gamma_2/2)^{\beta_2}}{(x-a)^{\beta_2} + (\Gamma_2/2)^{\beta_2}} & \text{if } x \geq a, \end{cases} \quad (5)$$

$\zeta$ ,  $a$ ,  $\sigma$ ,  $\Gamma_1$ ,  $\beta_1$ ,  $\Gamma_2$ , and  $\beta_2$  are resolution function parameters, and  $A$  is a normalization factor. The  $B(x)$  term is added to the Gaussian function to describe the asymmetric power-law tails of the detector resolution function. The parameters of the  $\eta^{(\prime)}$  resolution function are determined from the fit to the mass spectrum obtained for simulated signal events weighted to yield the  $Q^2$  dependence observed in data. The mass spectra for simulated events and fitted curves are shown in Fig. 5.

When used in data, the parameters  $\sigma$ ,  $\Gamma_1$ ,  $\Gamma_2$  and  $a$  are modified to account for possible differences between data and simulation in resolution ( $\Delta\sigma$ ) and mass scale calibration ( $\Delta a$ ):

$$\sigma^2 = \begin{cases} \sigma_{\text{MC}}^2 - \Delta\sigma^2 & \text{if } \Delta\sigma < 0; \\ \sigma_{\text{MC}}^2 + \Delta\sigma^2 & \text{if } \Delta\sigma \geq 0, \end{cases} \quad (6)$$

$$\Gamma_i^2 = \begin{cases} \Gamma_{i,\text{MC}}^2 - (2.35\Delta\sigma)^2 & \text{if } \Delta\sigma < 0; \\ \Gamma_{i,\text{MC}}^2 + (2.35\Delta\sigma)^2 & \text{if } \Delta\sigma \geq 0, \end{cases} \quad (7)$$

$$a = a_{\text{MC}} + \Delta a, \quad (8)$$

where the subscript MC indicates the parameter value determined from the fit to the simulated mass spectrum.

The background distribution is described by a linear function. Five parameters are determined in the fit to the measured mass spectrum: the number of  $\eta^{(\prime)}$  events,  $\Delta a$ ,  $\Delta\sigma$ , and two background shape parameters. The fitted curves are shown in Fig. 4. The numbers of  $\eta$  and  $\eta'$  events are found to be  $3060 \pm 70$  and  $5010 \pm 90$ , respectively. The mass shifts are  $\Delta a = 0.25 \pm 0.09 \text{ MeV}/c^2$  for the  $\eta$  and  $\Delta a = -(0.48 \pm 0.06) \text{ MeV}/c^2$  for the  $\eta'$ . To check possible dependence of the mass shift on  $Q^2$ , separate fits are performed for two  $Q^2$  regions:  $4 < Q^2 < 10 \text{ GeV}^2$  and  $10 < Q^2 < 40 \text{ GeV}^2$ . The  $\Delta a$  values obtained for these regions agree with each other both for  $\eta$  and  $\eta'$ . In contrast, the values of  $\Delta\sigma$  are found to be strongly dependent on  $Q^2$ , changing from  $0.9 \pm 0.3 \text{ MeV}/c^2$  for  $4 < Q^2 < 10 \text{ GeV}^2$  to  $-(1.0 \pm 0.6) \text{ MeV}/c^2$  for  $10 < Q^2 < 40 \text{ GeV}^2$ . It should be noted that the mass resolution for  $\eta$  and  $\eta'$  is about  $4 \text{ MeV}/c^2$ . The data-MC difference,  $\Delta\sigma \sim 1 \text{ MeV}/c^2$ , corresponds to a small ( $\sim 3\%$ ) change in the mass resolution when added in quadrature.



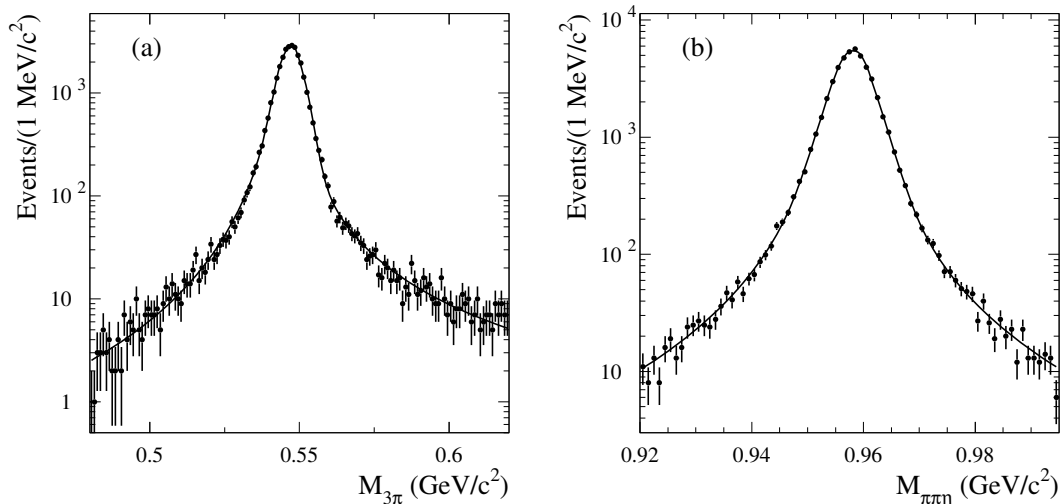


FIG. 5: The  $\pi^+\pi^-\pi^0$  and  $\pi^+\pi^-\eta$  mass spectra for simulated (a)  $e^+e^- \rightarrow e^+e^-\eta$  and (b)  $e^+e^- \rightarrow e^+e^-\eta'$  events, respectively. The curves represent the resolution functions described in the text.

TABLE I: The  $Q^2$  interval, number of detected  $e^+e^- \rightarrow e^+e^-\eta$  signal events ( $N_s$ ), number of peaking-background events ( $N_b$ ), efficiency correction ( $\delta_{\text{total}}$ ), number of signal events corrected for data-MC difference and resolution effects ( $N_{\text{corr}}^{\text{unfolded}}$ ), and detection efficiency obtained from simulation ( $\epsilon$ ). The first and second errors on  $N_s$  and  $N_{\text{corr}}^{\text{unfolded}}$  are statistical and systematic, respectively.

$Q^2$ interval ( $\text{GeV}^2$ )	$N_s$	$N_b$	$\delta_{\text{total}}(\%)$	$N_{\text{corr}}^{\text{unfolded}}$	$\epsilon(\%)$
4–5	$638 \pm 31 \pm 16$	$53 \pm 27$	−1.4	$634 \pm 34 \pm 18$	6.3
5–6	$625 \pm 34 \pm 19$	$89 \pm 34$	−1.6	$641 \pm 38 \pm 22$	13.0
6–8	$622 \pm 36 \pm 23$	$97 \pm 37$	−1.7	$634 \pm 39 \pm 25$	14.7
8–10	$349 \pm 26 \pm 12$	$43 \pm 23$	−2.0	$359 \pm 29 \pm 14$	18.7
10–12	$212 \pm 20 \pm 7$	$15 \pm 16$	−2.3	$224 \pm 22 \pm 8$	22.6
12–14	$104 \pm 14 \pm 4$	$13 \pm 11$	−2.1	$105 \pm 17 \pm 5$	22.9
14–17	$109 \pm 13 \pm 3$	$0.0 \pm 9.2$	−2.0	$116 \pm 15 \pm 4$	22.2
17–20	$40.5 \pm 8.3 \pm 1.2$	$0.7 \pm 5.6$	−2.3	$41.2 \pm 9.5 \pm 1.4$	21.3
20–25	$32.5 \pm 7.4 \pm 0.8$	$0.0 \pm 4.2$	−2.4	$34.4 \pm 8.3 \pm 0.9$	19.6
25–30	$13.7 \pm 5.3 \pm 0.5$	$3.1 \pm 3.5$	−2.7	$14.2 \pm 6.0 \pm 0.6$	18.0
30–40	$13.0 \pm 4.8 \pm 0.3$	$0.5 \pm 3.7$	−2.7	$14.1 \pm 5.3 \pm 0.3$	15.7

A fitting procedure similar to that described above is applied in each of the eleven  $Q^2$  intervals indicated in Table I. The parameters of the mass resolution function are taken from the fit to the mass spectrum for simulated events in the corresponding  $Q^2$  interval. The  $\eta$  and  $\eta'$  masses are fixed to the values obtained from the fit to the spectra of Fig. 4. The  $\Delta\sigma$  parameter is set to zero. Fits with  $\Delta\sigma = 0.9 \text{ MeV}/c^2$  and  $\Delta\sigma = -1.0 \text{ MeV}/c^2$  are also performed. The differences between the results of the fits with zero and non-zero  $\Delta\sigma$  provide an estimate of the systematic uncertainty associated with the data-MC simulation difference in the detector mass resolution.

For the analysis of the  $e^+e^- \rightarrow e^+e^-\eta$  process, the numbers of events containing an  $\eta$  are determined in two regions of the parameter  $r$ :  $-0.025 < r < 0.025$  ( $N_1$ ) and  $0.025 < r < 0.050$  ( $N_2$ ). The  $N_1$  and  $N_2$  values are used to determine the numbers of signal events ( $N_s$ ) and background events peaking at the  $\eta$  mass ( $N_b$ ) as described

in Sec. V. These numbers are listed in Table I. The  $\pi^+\pi^-\pi^0$  mass spectra and fitted curves for three representative  $Q^2$  intervals are shown in Fig. 6. The spectra shown are obtained for the  $-0.025 < r < 0.025$  regions; the  $0.025 < r < 0.050$  regions contain only 10–13% of the signal events and are used mainly to estimate backgrounds.

For the  $e^+e^- \rightarrow e^+e^-\eta'$  process, background is assumed to be small. There is no need to separate events into two  $r$  regions. The  $\pi^+\pi^-\eta$  mass spectra and fitted curves for three representative  $Q^2$  intervals are shown in Fig. 7. The numbers of signal  $\eta'$  events obtained from the fits are listed in Table II.

TABLE II: The  $Q^2$  interval, number of detected  $\eta'$  signal events ( $N_s$ ), number of peaking-background events ( $N_b$ ), efficiency correction ( $\delta_{\text{total}}$ ), number of signal events corrected for data-MC difference and resolution effects ( $N_{\text{corr}}^{\text{unfolded}}$ ), and detection efficiency obtained from simulation ( $\epsilon$ ). The first and second errors on  $N_s$  and  $N_{\text{corr}}^{\text{unfolded}}$  are statistical and systematic, respectively.

$Q^2$ interval (GeV <sup>2</sup> )	$N_s$	$N_b$	$\delta_{\text{total}}(\%)$	$N_{\text{corr}}^{\text{unfolded}}$	$\epsilon(\%)$
4–5	950 ± 32 ± 5	0.0 ± 0.0	−0.4	936 ± 34 ± 6	5.7
5–6	1013 ± 33 ± 6	0.0 ± 0.0	−0.6	1015 ± 36 ± 7	12.5
6–8	1185 ± 36 ± 5	0.0 ± 0.0	−0.7	1207 ± 38 ± 6	14.3
8–10	710 ± 28 ± 3	0.0 ± 0.0	−1.0	716 ± 30 ± 4	19.9
10–12	454 ± 22 ± 4	0.0 ± 0.0	−1.2	467 ± 25 ± 4	26.4
12–14	243 ± 16 ± 1	0.0 ± 0.0	−1.0	250 ± 19 ± 1	28.1
14–17	207 ± 15 ± 2	0.0 ± 0.0	−0.8	214 ± 17 ± 2	28.1
17–20	108 ± 10 ± 1	0.0 ± 0.0	−0.8	112 ± 12 ± 1	26.8
20–25	80.0 ± 9.0 ± 0.1	0.0 ± 0.0	−1.0	82.5 ± 9.9 ± 0.2	26.3
25–30	30.2 ± 5.9 ± 0.2	1.0 ± 1.0	−1.3	31.7 ± 6.7 ± 0.2	25.6
30–40	17.2 ± 5.4 ± 0.1	2.0 ± 1.4	−1.4	18.1 ± 5.8 ± 0.1	22.5

## V. PEAKING BACKGROUND ESTIMATION AND SUBTRACTION

Background containing true  $\eta$  or  $\eta'$  mesons might arise from  $e^+e^-$  annihilation processes, and two-photon processes with higher multiplicity final states than our signal events.

The background from  $e^+e^-$  annihilation can be estimated using events with the wrong sign of the  $e^\pm\eta^{(\prime)}$  momentum  $z$ -component. The numbers of background events from  $e^+e^-$  annihilation in the wrong- and right-sign data samples are expected to be approximately the same, but their  $Q^2$  distributions are quite different. The  $Q^2$  distribution expected for right-sign background events coincides with the  $Q_{ws}^2$  distribution for wrong-sign events, where  $Q_{ws}^2$  is the squared difference between the four-momenta of the detected positron (electron) and the initial electron (positron).

We observe 3 wrong-sign events in the  $\eta'$  data sample, all peaking at the  $\eta'$  mass, and 9 events in the  $\eta$  data sample, 5 of which are in the 0.530–0.565 GeV/ $c^2$  mass window. The contribution from non- $\eta$  events to this mass window is estimated to be 0.3 event. A possible source of these events is the  $e^+e^- \rightarrow X\gamma$  processes, where  $X$  is a hadronic system containing an  $\eta$  or  $\eta'$  meson, for example,  $\pi^+\pi^-\eta'$ , with the photon emitted along the beam axis.

The  $Q_{ws}^2$  distribution for the wrong-sign events is used to estimate of the  $Q^2$  distribution for  $e^+e^-$  annihilation background in the right-sign data sample. For the  $e^+e^- \rightarrow e^+e^-\eta'$  process, the background events are subtracted from the two highest  $Q^2$  intervals (see Table II). For the  $e^+e^- \rightarrow e^+e^-\eta$  process, the  $r$  distribution of the wrong-sign events differs from that for signal events (3 of 5 events have  $r > 0.025$ ). Therefore they are subtracted using the procedure described below.

Other possible sources of peaking background are the two-photon processes  $e^+e^- \rightarrow e^+e^-\eta^{(\prime)}\pi^0$ . For the  $\eta$  selection the additional background comes from the two-photon production of  $\eta'$  mesons followed by the decay

chain  $\eta' \rightarrow \pi^0\pi^0\eta$ ,  $\eta \rightarrow \pi^+\pi^-\pi^0$ . The  $Q^2$  distribution of events from the latter background source is calculated from the  $Q^2$  distribution of the selected  $\eta'$  events. The ratio of the detection efficiencies for the two  $\eta'$  decay modes is obtained from MC simulation. The total number of  $\eta' \rightarrow \pi^0\pi^0\eta$  events in the  $\eta$  data sample is estimated to be  $17 \pm 2$ . The events are concentrated almost entirely in the three lowest  $Q^2$  bins.

To study background contributions from the  $e^+e^- \rightarrow e^+e^-\eta^{(\prime)}\pi^0$  processes, we select events with two extra photons that each have an energy greater than 70 MeV. The distributions of the invariant mass of these extra photons for  $\eta$  and  $\eta'$  events are shown in Fig. 8. The invariant masses of the  $\eta$  and  $\eta'$  candidates are required to be in the mass windows 0.530–0.565 GeV/ $c^2$  and 0.945–0.970 GeV/ $c^2$ , respectively. The spectra are fitted by a sum of the  $\pi^0$  line shape obtained from simulated  $e^+e^- \rightarrow e^+e^-\eta^{(\prime)}\pi^0$  events and a quadratic polynomial. The fitted numbers of events with an extra  $\pi^0$  are  $90 \pm 20$  and  $13 \pm 14$  for the  $\eta$  and  $\eta'$  selections, respectively. It is expected that 8 events with an extra  $\pi^0$  in the  $\eta$  sample arise from two-photon  $\eta'$  production. From MC simulation we estimate that the ratio of the number of  $e^+e^- \rightarrow e^+e^-\eta^{(\prime)}\pi^0$  events with a found extra  $\pi^0$  to the number selected with standard criteria is about 2.5. For the  $e^+e^- \rightarrow e^+e^-\eta'$  process the estimated two-photon background does not exceed 1.6% of the total number of selected  $\eta'$  events at 90% confidence level. This number is considered as a measure of systematic uncertainty due to possible two-photon background for the  $e^+e^- \rightarrow e^+e^-\eta'$  process.

For the  $e^+e^- \rightarrow e^+e^-\eta$  process the two-photon background is about 10% of the total number of selected  $\eta$  events. The distribution of the  $\eta\pi^0$  invariant mass for events with an extra  $\pi^0$  is shown in Fig. 9. The two-photon invariant mass of the  $\pi^0$  candidate is required to be in the 0.115–0.150 GeV/ $c^2$  range. The sidebands, 0.065–0.100 and 0.170–0.205 GeV/ $c^2$ , are used to subtract contamination from non- $\eta\pi^0$  events.

It is known from two-photon measurements in the no-

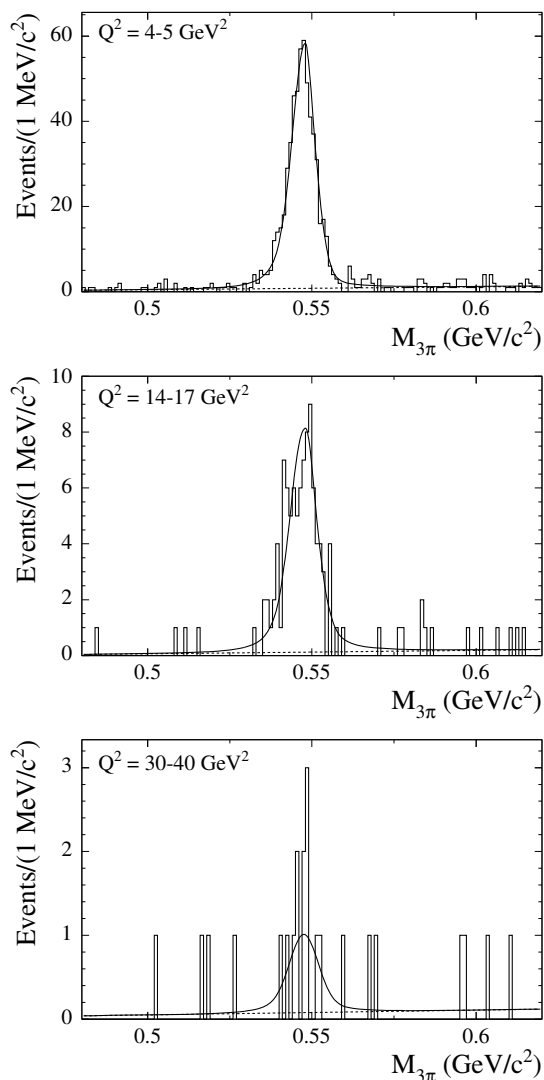


FIG. 6: The  $\pi^+\pi^-\pi^0$  mass spectra for data events with  $-0.025 < r < 0.025$  for three representative  $Q^2$  intervals. The solid curves are the fit results. The dashed lines represent non-peaking background.

tag mode [15] that the  $\eta\pi^0$  final state is produced mainly via  $a_0(980)$  and  $a_2(1320)$  intermediate resonances. Evidence for these two intermediate resonances is seen in the mass spectrum of Fig. 9. An enhancement near the  $\eta\pi^0$  threshold is also observed. Our spectrum differs significantly from the spectrum for the no-tag mode [15], which is dominated by  $a_2(1320)$  production. In the no-tag mode the  $a_2(1320)$  meson is produced predominantly in a helicity-2 state, and thus with an angular distribution proportional to  $\sin^4\theta_\pi$ , where  $\theta_\pi$  is the angle between the  $\pi^0$  direction and the  $\gamma\gamma$  collision axis in the  $\gamma\gamma$  c.m. frame. Our selection criteria favor events with values of  $\theta_\pi$  near zero and hence suppress helicity-2 states.

The distribution of the parameter  $r$  is also sensitive to the contribution of the two-photon background. To obtain the  $r$  distribution, data events are divided into

15  $r$  intervals. For each interval, the fit to the  $\pi^+\pi^-\pi^0$  ( $\pi^+\pi^-\eta$ ) spectra is performed and the number of events containing an  $\eta^{(\prime)}$  is determined. The  $r$  distributions for events in the  $\eta$  and  $\eta'$  data samples are shown in Fig. 10.

For  $\eta'$  events, for which the background is small, the data distribution is compared with the simulated signal distribution normalized to the number of data events. The distributions are in reasonable agreement. The ratio  $R_s$  of the number of events with  $r > 0.025$  to the number with  $r < 0.025$  is found to be  $0.103 \pm 0.006$  in data and  $0.116 \pm 0.002$  in simulation; the 13% difference is taken as a systematic uncertainty on the  $R_s$  value for  $\eta'$  events determined from simulation.

For  $\eta$  events, the data  $r$  distribution is fitted with the sum of the simulated distributions for signal and background  $e^+e^- \rightarrow e^+e^-\eta\pi^0$  and  $e^+e^- \rightarrow e^+e^-\eta' \rightarrow e^+e^-\pi^0\pi^0\eta$  events. The fitted number of background events is  $280 \pm 40$ , in reasonable agreement with the estimate given above based on the number of events with a detected extra  $\pi^0$ . The difference in the  $r$  distributions for signal and background events is used to estimate the background, which is then subtracted for each  $Q^2$  interval. In Sec. IV, we described how the number of events containing an  $\eta$  is determined for two regions of the parameter  $r$ :  $-0.025 < r < 0.025$  ( $N_1$ ) and  $0.025 < r < 0.050$  ( $N_2$ ). The numbers of signal and background events are calculated as follows:

$$N_s = \frac{(1 + R_s)(N_1 R_b - N_2)}{R_b - R_s}, \quad (9)$$

$$N_b = \frac{(1 + R_b)(N_2 - N_1 R_s)}{R_b - R_s}, \quad (10)$$

where  $R_s$  ( $R_b$ ) is the  $N_2/N_1$  ratio obtained from signal (background) MC simulation. The parameter  $R_s$  is found to vary from 0.15 to 0.10 with increasing  $Q^2$ . The systematic uncertainty on  $R_s$  (13%) was estimated above. To calculate  $R_b$  for the  $e^+e^- \rightarrow e^+e^-\eta\pi^0$  process, the simulated background events are reweighted to reproduce the  $\eta\pi^0$  mass spectrum observed in data (Fig. 9). The  $R_b$  value varies from 2.0 to 1.5. The systematic uncertainty on  $R_b$  is estimated from its variation with the  $\eta\pi^0$  mass. The maximum deviation from the value averaged over the  $\eta\pi^0$  spectrum of about 25% is found when we exclude events with mass near the  $\eta\pi^0$  threshold. This deviation is taken as an estimate of the systematic uncertainty on  $R_b$ . The  $r$  distribution for background events from two-photon  $\eta'$  production ( $R_b$  is about 10) differs strongly from the distribution for  $\eta\pi^0$  events. Therefore we first subtract the calculated  $\eta'$  contribution from  $N_1$  and  $N_2$  in each  $Q^2$  interval, and then calculate  $N_s$  assuming that the remaining background comes from the  $e^+e^- \rightarrow e^+e^-\eta\pi^0$  process. The obtained numbers of signal and background events are listed in Table I. The background includes both the  $e^+e^- \rightarrow e^+e^-\eta\pi^0$  and  $e^+e^- \rightarrow e^+e^-\eta'$  contributions. The systematic errors quoted for  $N_s$  are mainly due to the uncertainties on  $R_s$  and  $R_b$ .

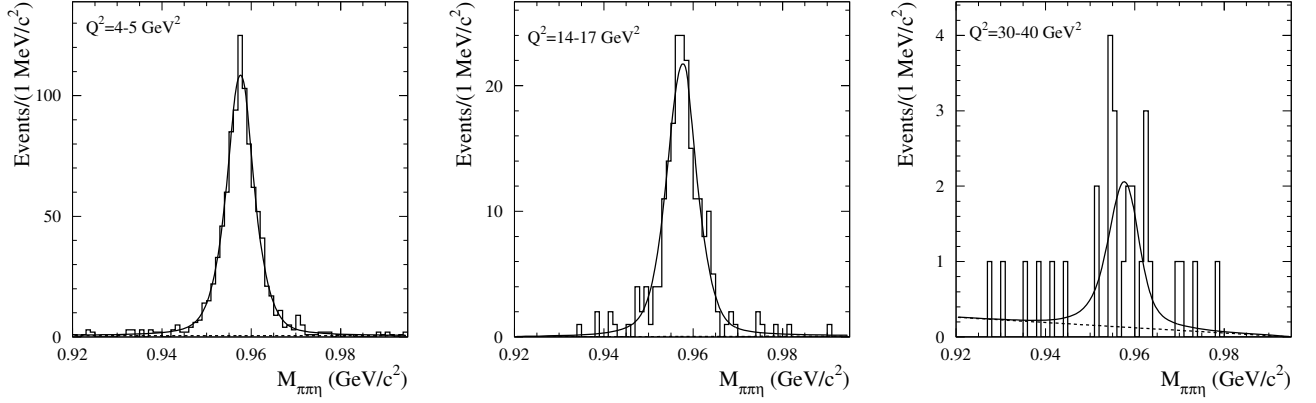


FIG. 7: The  $\pi^+\pi^-\eta$  mass spectra for data events for three representative  $Q^2$  intervals. The solid curves are the fit results. The dashed lines represent non-peaking background.

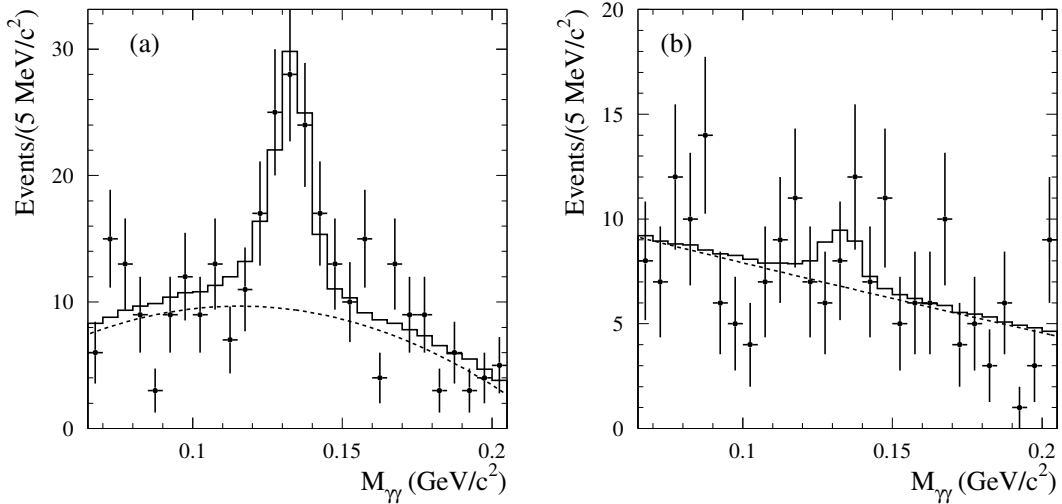


FIG. 8: The two-photon invariant mass spectra for (a)  $\eta$  and (b)  $\eta'$  events with two extra photons. The solid histograms represent the fit results. The dashed curves are the fitted distributions for events without an extra  $\pi^0$ .

## VI. DETECTION EFFICIENCY

The detection efficiency is determined from MC simulation as the ratio of the true  $Q^2$  distributions computed after and before applying the selection criteria. The  $Q^2$  dependencies of the detection efficiencies for both processes under study are shown in Fig. 11. The detector acceptance limits the detection efficiency at small  $Q^2$ . The cross sections are measured in the regions  $Q^2 > 4 \text{ GeV}^2$ , where the detection efficiencies are greater than 5%. The asymmetry of the  $e^+e^-$  collisions at PEP-II leads to different efficiencies for events with electron and positron tags. The  $Q^2$  range from 4 to 6  $\text{GeV}^2$  is measured only with the positron tag.

We study possible sources of systematic uncertainty due to differences between data and MC simulation in detector response. The MC simulation predicts about

a 2.5% loss of signal events, weakly dependent on  $Q^2$ , due to background filters used in the event reconstruction. The filter inefficiency is measured from data using a small fraction of selected events that does not pass the background filters. Combining events from the  $\eta$  and  $\eta'$  samples, we determine the ratio of the inefficiencies in data and MC simulation to be  $1.15 \pm 0.20$ . The error of the ratio is used to estimate the systematic uncertainty for the filter inefficiency:  $0.2 \times 2.5 = 0.5\%$ . The trigger inefficiency obtained using MC simulation is about 1% in the first  $Q^2$  interval (4–5  $\text{GeV}^2$ ) and falls to zero at  $Q^2 > 14 \text{ GeV}^2$ . The limited statistics do not allow us to measure this inefficiency in data. Therefore, the level of the inefficiency observed in the MC simulation is taken as an estimate of the systematic uncertainty due to the trigger inefficiency.

The systematic uncertainty due to a possible difference

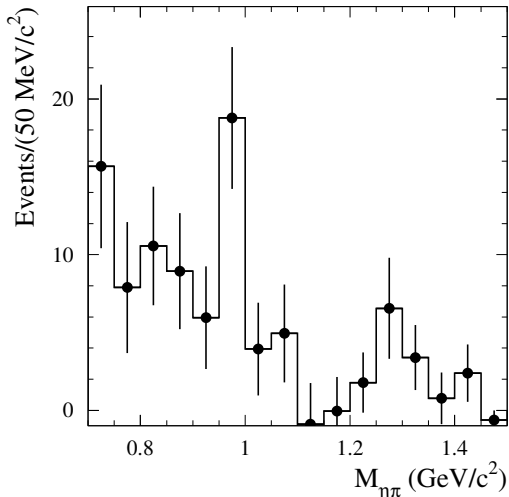


FIG. 9: The distribution of the  $\eta\pi^0$  invariant mass for  $\eta$  events with an extra  $\pi^0$ . The background from non- $\eta\pi^0$  events is subtracted.

between data and simulation in the charged-particle track reconstruction for pions is estimated to be about 0.35% per track, so the total uncertainty is 0.7%. The data-MC simulation difference in the pion identification efficiency is estimated using the identification efficiencies measured for pions in the  $D^{*+} \rightarrow D^0\pi^+$ ,  $D^0 \rightarrow \pi^+K^-$  decay. The ratio of the data and MC identification efficiencies is determined as a function of the pion momentum and polar angle. These functions for positive and negative pions are then convolved with the pion energy and angular distributions for simulated signal events in each  $Q^2$  interval. The resulting  $Q^2$  dependence of the efficiency correction for pion identification is shown in Fig. 12. The systematic uncertainty in the correction does not exceed 0.5%. The difference between the corrections for  $\eta$  and  $\eta'$  events is due to the difference in the pion momentum distributions. The average pion momentum is about 1 GeV/c for  $\eta$  and 0.6 GeV/c for  $\eta'$  events.

The data-MC simulation difference in electron identification is estimated using the identification efficiencies measured for electrons in radiative Bhabha events. The efficiency correction for  $e^+e^- \rightarrow e^+e^-\eta$  events is shown in Fig. 13 as a function of  $Q^2$ . For the  $e^+e^- \rightarrow e^+e^-\eta'$  process the correction is practically the same. The associated systematic uncertainty is estimated to be 0.5%. The systematic uncertainty due to the data-MC simulation difference in the electron track reconstruction is about 0.1%.

The  $\pi^0$  reconstruction efficiency is studied using events from the ISR process  $e^+e^- \rightarrow \gamma\omega$ ,  $\omega \rightarrow \pi^+\pi^-\pi^0$ . These events can be reconstructed and selected without using information related to the  $\pi^0$ . The  $\pi^0$  reconstruction efficiency is computed as the ratio of the number of events with an identified  $\pi^0$  to the total number of reconstructed  $e^+e^- \rightarrow \gamma\omega$  events. The data-MC simulation difference

in the  $\pi^0$  efficiency depends on the  $\pi^0$  momentum and varies from  $(0.7 \pm 1.2)\%$  at momenta below 0.25 GeV/c to  $(-4.2 \pm 1.3)\%$  at 4 GeV/c [1]. The efficiency correction averaged over the  $\pi^0$  spectrum is shown in Fig. 14 as a function of  $Q^2$ . The systematic uncertainty associated with this correction is estimated to be 1%. For  $\eta \rightarrow \gamma\gamma$  decays the efficiency correction is expected to be smaller. The maximum value of the  $\pi^0$  efficiency correction (2%) is conservatively taken as an estimate of systematic uncertainty due to a possible data-MC simulation difference in the  $\eta \rightarrow \gamma\gamma$  reconstruction.

To estimate the effect of the requirement  $-0.025 < r < 0.05$ ,  $\eta'$  events with  $0.05 < r < 0.075$  are studied. We calculate the double ratio minus unity

$$\frac{\Delta\sigma}{\sigma} = \frac{(N_{\text{new}}/N)_{\text{data}}}{(N_{\text{new}}/N)_{\text{MC}}} - 1, \quad (11)$$

where  $N_{\text{new}}$  and  $N$  are the numbers of signal events with the new and standard selection criteria. The ratio is sensitive to the relative change in the measured cross section due to the changes in the selection criteria. We do not observe any significant  $Q^2$  dependence of  $\Delta\sigma/\sigma$ . The average over  $Q^2$  is found to be consistent with zero ( $-0.003 \pm 0.004$ ). We conclude that the simulation reproduces the shape of the  $r$  distribution.

We also study the effect of the  $|\cos\theta_{e\eta^{(\prime)}}^*| > 0.99$  restriction by changing the value to 0.95. The corresponding change of the measured cross section does not depend on  $Q^2$ . The average change in cross section integrating over  $Q^2$  is  $(2.0 \pm 0.4)\%$ . We consider this data-MC simulation difference (2%) as a measure of the systematic uncertainty due to the  $\cos\theta_{e\eta^{(\prime)}}^*$  criterion.

The main sources of systematic uncertainty associated with the detection efficiency are summarized in Table III for both processes under study. The values of the detection efficiency and the total efficiency correction  $\delta_{\text{total}} = \delta_{\pi} + \delta_e + \delta_{\pi^0}$  (the term  $\delta_{\pi^0}$  is only applicable to the  $\eta$  mode) for different  $Q^2$  intervals are listed in Tables I and II. The data distribution is corrected as follows:

$$N_i^{\text{corr}} = N_i / (1 + \delta_{\text{total},i}), \quad (12)$$

where  $N_i$  is the number of signal events in the  $i$ th  $Q^2$  interval.

## VII. CROSS SECTION AND FORM FACTOR

The Born differential cross section for  $e^+e^- \rightarrow e^+e^-\eta^{(\prime)}$  is

$$\frac{d\sigma}{dQ^2} = \frac{(dN/dQ^2)_{\text{corr}}^{\text{unfolded}}}{\epsilon RLB} \quad (13)$$

where  $(dN/dQ^2)_{\text{corr}}^{\text{unfolded}}$  is the mass spectrum corrected for data-MC simulation differences and unfolded for detector resolution effects as explained below,  $L$  is the to-

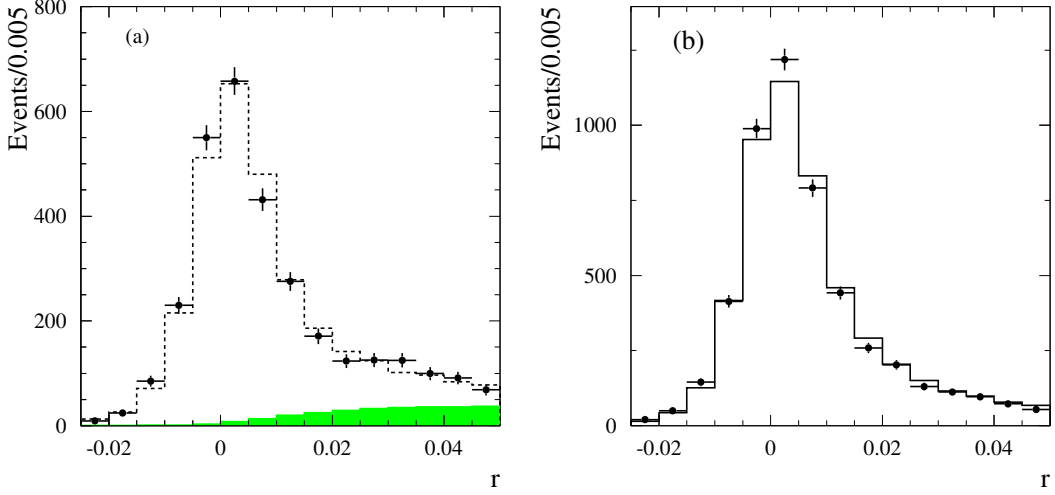


FIG. 10: (a) The  $r$  distribution for data events containing an  $\eta$  (points with error bars). The dashed histogram shows the fit results. The shaded histogram is the fitted background contribution from the processes  $e^+e^- \rightarrow e^+e^-\eta\pi^0$  and  $e^+e^- \rightarrow e^+e^-\eta' \rightarrow e^+e^-\pi^0\pi^0\eta$ . (b) The  $r$  distribution for data events containing an  $\eta'$  (points with error bars). The solid histogram is the simulated distribution for events from the signal  $e^+e^- \rightarrow e^+e^-\eta'$  process normalized to the number of data events.

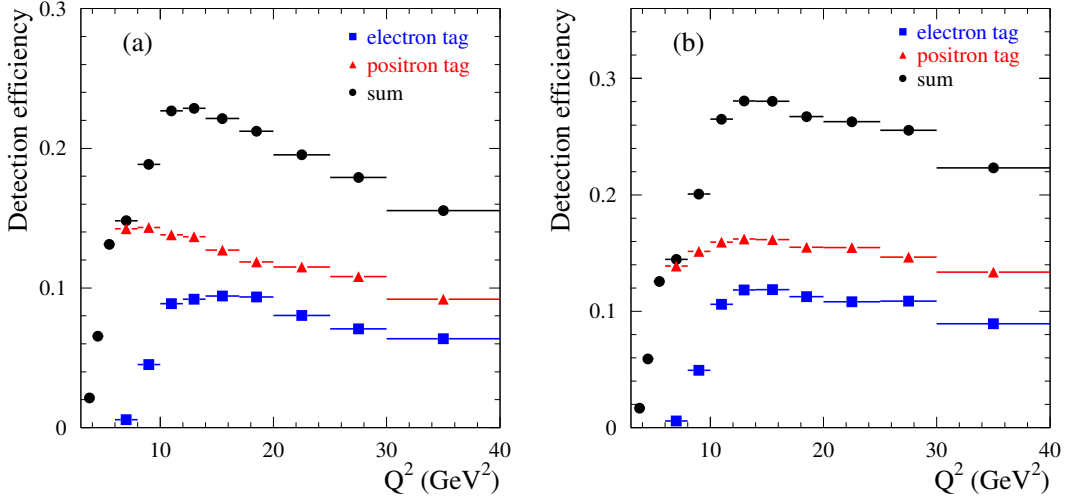


FIG. 11: The detection efficiencies for (a)  $e^+e^- \rightarrow e^+e^-\eta$  with  $\eta \rightarrow \pi^+\pi^-\pi^0$  and  $\pi^0 \rightarrow 2\gamma$  and (b)  $e^+e^- \rightarrow e^+e^-\eta'$  with  $\eta \rightarrow \pi^+\pi^-\eta$  and  $\eta \rightarrow 2\gamma$  as functions of the momentum transfer squared for events with a tagged electron (squares), a tagged positron (triangles), and their sum (circles).

tal integrated luminosity,  $\varepsilon$  is the  $Q^2$ -dependent detection efficiency, and  $R$  is a radiative correction factor accounting for distortion of the  $Q^2$  spectrum due to vacuum polarization effects and the emission of soft photons from the initial-state particles. The factor  $B$  is the product of the branching fractions,  $\mathcal{B}(\eta \rightarrow \pi^+\pi^-\pi^0)\mathcal{B}(\pi^0 \rightarrow \gamma\gamma) = 0.2246 \pm 0.0028$  or  $\mathcal{B}(\eta' \rightarrow \pi^+\pi^-\eta)\mathcal{B}(\eta \rightarrow \gamma\gamma) = 0.1753 \pm 0.0056$  [16].

The radiative correction factor  $R$  is determined using simulation at the generator level, i.e., without detector simulation. The  $Q^2$  spectrum is generated using only

the pure Born amplitude for the  $e^+e^- \rightarrow e^+e^-\eta^{(\prime)}$  process, and then using a model with radiative corrections included. The radiative correction factor, evaluated as the ratio of the second spectrum to the first, varies from 0.994 at  $Q^2 = 4 \text{ GeV}^2$  to 1.002 at  $Q^2 = 40 \text{ GeV}^2$ . The accuracy of the radiative correction calculation is estimated to be 1% [13]. It should be noted that the value of  $R$  depends on the requirement on the extra photon energy. The  $Q^2$  dependence obtained corresponds to the condition  $r = 2E_\gamma^*/\sqrt{s} < 0.1$  imposed in the simulation.

The corrected and unfolded  $Q^2$  distribution



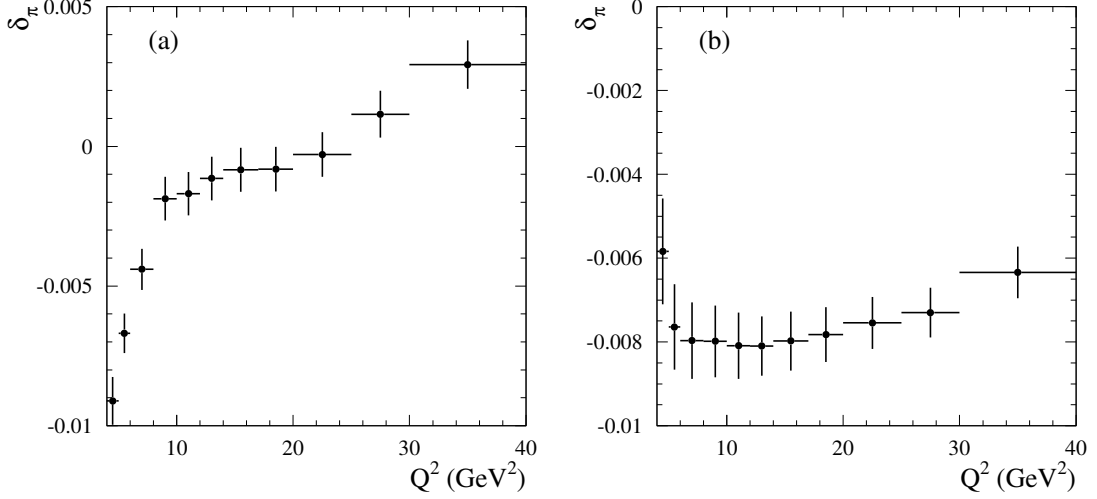


FIG. 12: The correction to the MC-estimated pion identification efficiency  $\delta_\pi$  as a function of  $Q^2$  for (a)  $e^+e^- \rightarrow e^+e^-\eta$  and (b)  $e^+e^- \rightarrow e^+e^-\eta'$ .

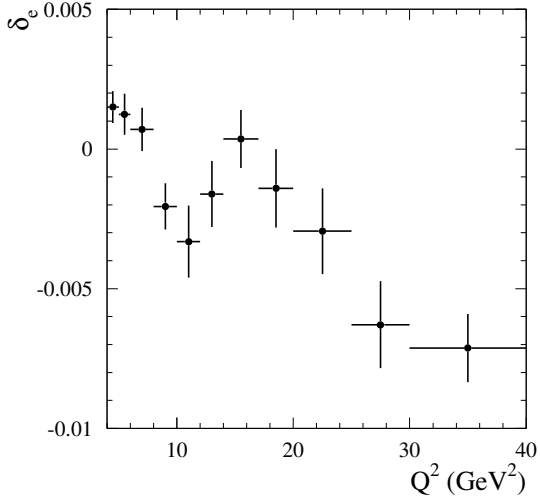


FIG. 13: The correction to the MC-estimated electron identification efficiency  $\delta_e$  as a function of  $Q^2$  for the  $e^+e^- \rightarrow e^+e^-\eta$  process.

TABLE III: The main sources of systematic uncertainty associated with the detection efficiency, and the total efficiency systematic uncertainty for  $e^+e^- \rightarrow e^+e^-\eta$  and  $e^+e^- \rightarrow e^+e^-\eta'$  events.

Source	$\eta$ (%)	$\eta'$ (%)
Trigger, filters	0.7	1.3
Track reconstruction	0.8	0.8
$\pi^\pm$ identification	0.5	0.5
$e^\pm$ identification	0.5	0.5
$\eta, \pi^0 \rightarrow 2\gamma$ reconstruction	1.0	2.0
$ \cos\theta_{e\eta^{(\prime)}}^*  > 0.99$ criterion	2.0	2.0
Total	2.6	3.3

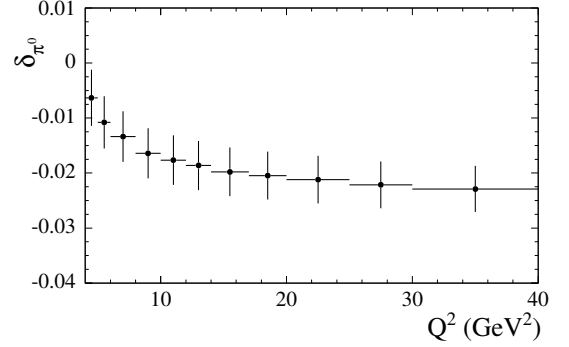


FIG. 14: The correction to the MC-estimated  $\pi^0$  reconstruction efficiency  $\delta_{\pi^0}$  as a function of  $Q^2$  for the  $e^+e^- \rightarrow e^+e^-\eta$  process.

$(dN/dQ^2)_{\text{corr}}^{\text{unfolded}}$  is obtained from the measured distribution by dividing by the efficiency correction factor (see Eq.(12)) and unfolding for the effect of finite  $Q^2$  resolution. Using MC simulation, a migration matrix  $H$  is obtained, which represents the probability that an event with true  $Q^2$  in interval  $j$  is reconstructed in interval  $i$ :

$$\left(\frac{dN}{dQ^2}\right)_i^{\text{rec}} = \sum_j H_{ij} \left(\frac{dN}{dQ^2}\right)_j^{\text{true}}. \quad (14)$$

In the case of extra photon emission,  $Q_{\text{true}}^2$  is calculated as  $-(p-p'-k)^2$ , where  $k$  is the photon four-momentum;  $\varepsilon$  and  $R$  in Eq.(13) are functions of  $Q_{\text{true}}^2$ . As the chosen  $Q^2$  interval width significantly exceeds the resolution for all  $Q^2$ , non-zero elements of the migration matrix lie on and near the diagonal. The values of the diagonal elements are in the range 0.9–0.95. The true  $Q^2$  distribution is obtained by applying the inverse of the migration ma-

trix to the measured distribution. The procedure does not change the shape of the  $Q^2$  distribution significantly, but increases the errors (by about 10%) and their correlations. The number of events ( $N_{\text{corr}}^{\text{unfolded}}$ ) as a function of  $Q^2$  is reported in Tables I and II.

The value of the differential cross section as a function of  $Q^2$  is listed in Tables IV and V. The quoted errors are statistical and systematic. The latter includes only  $Q^2$ -dependent errors: the systematic uncertainty in the number of signal events and the statistical errors on the efficiency correction and MC simulation. The  $Q^2$ -independent systematic error on the  $e^+e^- \rightarrow e^+e^-\eta$  cross section is 3.5%; this results from the uncertainties on the detection efficiency, both systematic (2.6%) and model-dependent (1.5%), the uncertainty in the calculation of the radiative correction factor (1%), and the errors on the integrated luminosity (1%) and the  $\eta$  decay branching fraction (1.2%) [16]. The  $Q^2$ -independent systematic error on the  $e^+e^- \rightarrow e^+e^-\eta'$  cross section is 5.3%. It includes the systematic and model uncertainties on the detection efficiency (3.3% and 1.5%, respectively), the uncertainties on the background subtraction (1.6%) and the radiative correction factor (1%), and the errors on the integrated luminosity (1%) and the  $\eta'$  decay branching fraction (3.2%) [16].

The model dependence of the detection efficiency arises from the unknown cross-section dependence on the momentum transfer to the untagged electron. The MC simulation is performed, and the detection efficiency is determined, with the restriction that the momentum transfer to the untagged electron be greater than  $-0.6$  GeV<sup>2</sup>, so that the cross section is measured for the restricted range  $|q_2^2| < 0.6$  GeV<sup>2</sup>. The actual  $q_2^2$  threshold is determined by the requirement on  $\cos\theta_{e\eta^{(\prime)}}$  and is equal to  $0.38$  GeV<sup>2</sup>. The MC simulation is performed with a  $q_2^2$  independent form factor, which corresponds to the QCD-inspired model  $F(q_1^2, q_2^2) \propto 1/(q_1^2 + q_2^2) \approx 1/q_1^2$  [17]. The event loss due to the  $|q_2^2| < 0.38$  GeV<sup>2</sup> restriction is about 2.5%. The use of the form factor predicted by the vector dominance model  $F(q_2^2) \propto 1/(1 - q_2^2/m_\rho^2)$ , where  $m_\rho$  is  $\rho$  meson mass, leads to a decreased event loss of only 1%. The difference between these efficiencies is considered to be an estimate of the model uncertainty due to the unknown  $q_2^2$  dependence.

Because of the strong nonlinear dependence of the cross section on  $Q^2$ , the effective value of  $Q^2$  corresponding to the measured cross section differs from the center of the  $Q^2$  interval. We parametrize the measured cross section with a smooth function, reweight the  $Q^2$  distribution in simulation to be consistent with data, and calculate the weighted average ( $\overline{Q^2}$ ) for each mass interval. The values of  $\overline{Q^2}$  are listed in Table IV and Table V. The measured differential cross sections for both processes under study are shown in Fig. 15, together with the data reported by the CLEO Collaboration [9] for  $Q^2 > 3.5$  GeV<sup>2</sup>. We average the CLEO results obtained in different  $\eta^{(\prime)}$  decay modes.

To extract the transition form factor, the measured

TABLE IV: The  $Q^2$  interval, the weighted average  $Q^2$  value for the interval ( $\overline{Q^2}$ ), the  $e^+e^- \rightarrow e^+e^-\eta$  cross section ( $d\sigma/dQ^2(\overline{Q^2})$ ), and the product of the  $\gamma\gamma^* \rightarrow \eta$  transition form factor  $F(Q^2)$  and  $\overline{Q^2}$ . The statistical and systematic errors are quoted separately for the cross sections, and are combined for the form factors. In the table we quote the  $Q^2$ -dependent systematic errors. The  $Q^2$ -independent error is 3.5% for the cross section and 2.9% for the form factor.

$Q^2$ interval (GeV <sup>2</sup> )	$\overline{Q^2}$ (GeV <sup>2</sup> )	$d\sigma/dQ^2(\overline{Q^2})$ (fb/GeV <sup>2</sup> )	$\overline{Q^2} F(\overline{Q^2}) $ (MeV)
4–5	4.44	$95.6 \pm 5.1 \pm 3.1$	$142.3 \pm 4.4$
5–6	5.45	$46.6 \pm 2.7 \pm 1.7$	$142.2 \pm 4.9$
6–8	6.84	$20.4 \pm 1.2 \pm 0.8$	$141.5 \pm 5.2$
8–10	8.87	$9.06 \pm 0.72 \pm 0.35$	$150.4 \pm 6.7$
10–12	10.90	$4.67 \pm 0.47 \pm 0.18$	$157.6 \pm 8.5$
12–14	12.91	$2.16 \pm 0.34 \pm 0.10$	$146.4 \pm 12.1$
14–17	15.31	$1.65 \pm 0.22 \pm 0.06$	$178.1 \pm 12.1$
17–20	18.35	$0.61 \pm 0.14 \pm 0.02$	$151.7 \pm 17.8$
20–25	22.14	$0.33 \pm 0.08 \pm 0.01$	$164.5 \pm 20.2$
25–30	27.20	$0.15 \pm 0.06 \pm 0.01$	$166.5 \pm 36.6$
30–40	33.99	$0.085 \pm 0.032 \pm 0.003$	$203.5 \pm 39.0$

TABLE V: The  $Q^2$  interval, the weighted average  $Q^2$  value for the interval ( $\overline{Q^2}$ ), the  $e^+e^- \rightarrow e^+e^-\eta'$  cross section ( $d\sigma/dQ^2(\overline{Q^2})$ ), and the product of the  $\gamma\gamma^* \rightarrow \eta'$  transition form factor  $F(Q^2)$  and  $\overline{Q^2}$ . The statistical and systematic errors are quoted separately for the cross sections, and are combined for the form factors. In the table we quote the  $Q^2$ -dependent systematic errors. The  $Q^2$ -independent error is 5.3% for the cross section and 3.5% for the form factor.

$Q^2$ interval (GeV <sup>2</sup> )	$\overline{Q^2}$ (GeV <sup>2</sup> )	$d\sigma/dQ^2(\overline{Q^2})$ (fb/GeV <sup>2</sup> )	$\overline{Q^2} F(\overline{Q^2}) $ (MeV)
4–5	4.44	$202 \pm 7 \pm 3$	$214.5 \pm 4.3$
5–6	5.45	$99.6 \pm 3.6 \pm 1.4$	$214.0 \pm 4.1$
6–8	6.85	$51.7 \pm 1.6 \pm 0.5$	$231.4 \pm 3.9$
8–10	8.87	$22.1 \pm 0.9 \pm 0.2$	$240.4 \pm 5.2$
10–12	10.90	$10.8 \pm 0.6 \pm 0.1$	$244.4 \pm 6.7$
12–14	12.91	$5.45 \pm 0.41 \pm 0.06$	$236.9 \pm 8.9$
14–17	15.31	$3.10 \pm 0.24 \pm 0.04$	$248.1 \pm 9.9$
17–20	18.35	$1.70 \pm 0.18 \pm 0.02$	$259.0 \pm 13.7$
20–25	22.13	$0.77 \pm 0.09 \pm 0.01$	$254.4 \pm 15.4$
25–30	27.19	$0.30 \pm 0.06 \pm 0.01$	$239.9 \pm 25.7$
30–40	33.98	$0.098 \pm 0.031 \pm 0.002$	$221.9 \pm 35.9$

and calculated cross sections are compared. The simulation uses a constant form factor  $F_{\text{MC}}^2$ . Therefore, the measured form factor is determined from

$$|F(Q^2)|^2 = \frac{(d\sigma/dQ^2)_{\text{data}}}{(d\sigma/dQ^2)_{\text{MC}}} F_{\text{MC}}^2. \quad (15)$$

The calculated cross section  $(d\sigma/dQ^2)_{\text{MC}}$  has a model-dependent uncertainty due to the unknown dependence on the momentum transfer to the untagged electron. The difference between the cross section values calculated with the two form-factor models described above is 4.6% for both  $\eta$  and  $\eta'$ . This difference is considered to be an estimate of the model uncertainty due to the

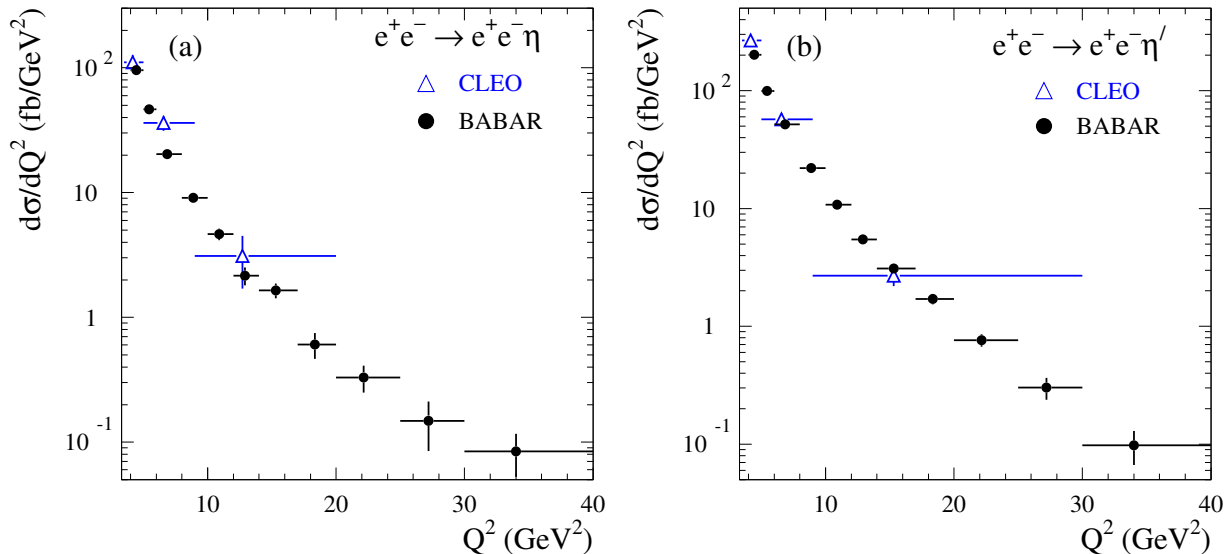


FIG. 15: The differential cross sections for (a)  $e^+e^- \rightarrow e^+e^-\eta$  and (b)  $e^+e^- \rightarrow e^+e^-\eta'$  from the present analysis compared to those from the CLEO experiment [9].

unknown  $q_2^2$  dependence. The values of the form factors obtained, represented in the form  $\overline{Q^2}|F(Q^2)|$ , are listed in Tables IV and V and shown in Fig. 16. For the form factor we quote the combined error, obtained by adding the statistical and  $Q^2$ -dependent systematic uncertainties in quadrature. The  $Q^2$ -independent systematic error is 2.9% for the  $\eta$  and 3.5% for the  $\eta'$  form factor. It includes the uncertainty on the measured differential cross section, and the model-dependent uncertainty due to the unknown  $q_2^2$  dependence.

### VIII. SUMMARY AND DISCUSSION

We have studied the  $e^+e^- \rightarrow e^+e^-\eta$  and  $e^+e^- \rightarrow e^+e^-\eta'$  reactions and measured the differential cross sections ( $d\sigma/dQ^2$ ) and the  $\gamma^*\gamma \rightarrow \eta^{(\prime)}$  transition form factors  $F(Q^2)$  in the momentum transfer range from 4 to 40  $\text{GeV}^2$ . The comparison of our results on the form factors with the most precise previous measurements [9] is shown in Fig. 16. We significantly improve the precision and extend the  $Q^2$  region for form factor measurements. For the  $\eta'$  form factor our results are in good agreement with those reported by the CLEO Collaboration [9]. For the  $\eta$  form factor the agreement is worse. In particular, the CLEO point at  $Q^2 \approx 7 \text{ GeV}^2$  lies higher than our measurements by about 3 standard deviations.

The data for the  $e^+e^- \rightarrow \eta^{(\prime)}\gamma$  reactions are used to determine the transition form factors in the time-like region  $q^2 = s > 0$ . Since the time- and space-like form factors are expected to be similar at high  $Q^2$ , in Fig. 17 we show the results of the high- $Q^2$  time-like measurements together with the space-like data. The form fac-

tors at  $Q^2 = 14.2 \text{ GeV}^2$  are obtained from the values of the  $e^+e^- \rightarrow \eta^{(\prime)}\gamma$  cross sections measured by CLEO [18] near the maximum of the  $\psi(3770)$  resonance. We use the assumption that the contributions of the  $\psi(3770) \rightarrow \eta^{(\prime)}\gamma$  decays to the  $e^+e^- \rightarrow \eta^{(\prime)}\gamma$  cross sections are negligible. It is seen that the measured time- and space-like form factors at  $Q^2 \approx 14 \text{ GeV}^2$  are in agreement both for  $\eta$  and for  $\eta'$ . The *BABAR* measurements of the  $e^+e^- \rightarrow \eta^{(\prime)}\gamma$  cross sections [19] allow us to extend the  $Q^2$  region for the  $\eta$  and  $\eta'$  form factor measurements up to 112  $\text{GeV}^2$ .

In most models for the meson distribution amplitude  $\phi_P(x)$  used for calculation of photon-meson transition form factors, the DA end-point behavior is determined by the factor  $x(1-x)$ . The form factors calculated with such conventional DAs are almost flat for  $Q^2$  values greater than 15  $\text{GeV}^2$  (see, for example, the recent works [20–22] devoted to the  $\gamma\gamma^* \rightarrow \pi^0$  form factor). Some of these models [20] have difficulties in reproducing the  $Q^2$  dependence of the  $\gamma\gamma^* \rightarrow \pi^0$  form factor measured by *BABAR* [1] in the  $Q^2$  range from 4 to 40  $\text{GeV}^2$ . Alternatively, models with a flat DA or a DA that is finite at the end points have been suggested [23–25], which give a logarithmic rise of the product  $Q^2F(Q^2)$  with  $Q^2$  and describe the *BABAR* data reasonably well.

The  $Q^2$  dependencies of the products  $Q^2F(Q^2)$  for  $\eta$  and  $\eta'$  are fit with the function

$$Q^2F(Q^2) = b_l + a_l \ln Q^2(\text{GeV}^2). \quad (16)$$

The results of the fit are shown in Fig. 17. For both  $\eta$  and  $\eta'$  the quality of the fit is acceptable:  $\chi^2/\nu$  is equal to 6.7/10 for  $\eta$  and 14.6/10 for  $\eta'$ , where  $\nu$  is the number of degrees of freedom. The observed rise of the form factors

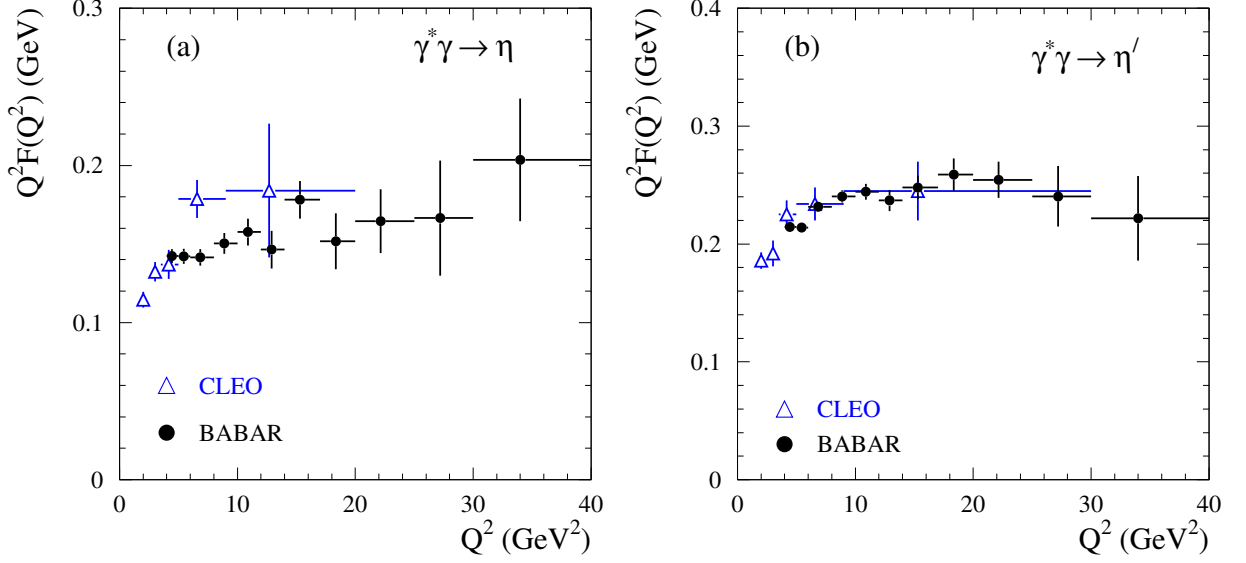


FIG. 16: The transition form factors multiplied by  $Q^2$  for (a)  $\gamma\gamma^* \rightarrow \eta$  and (b)  $\gamma\gamma^* \rightarrow \eta'$ .

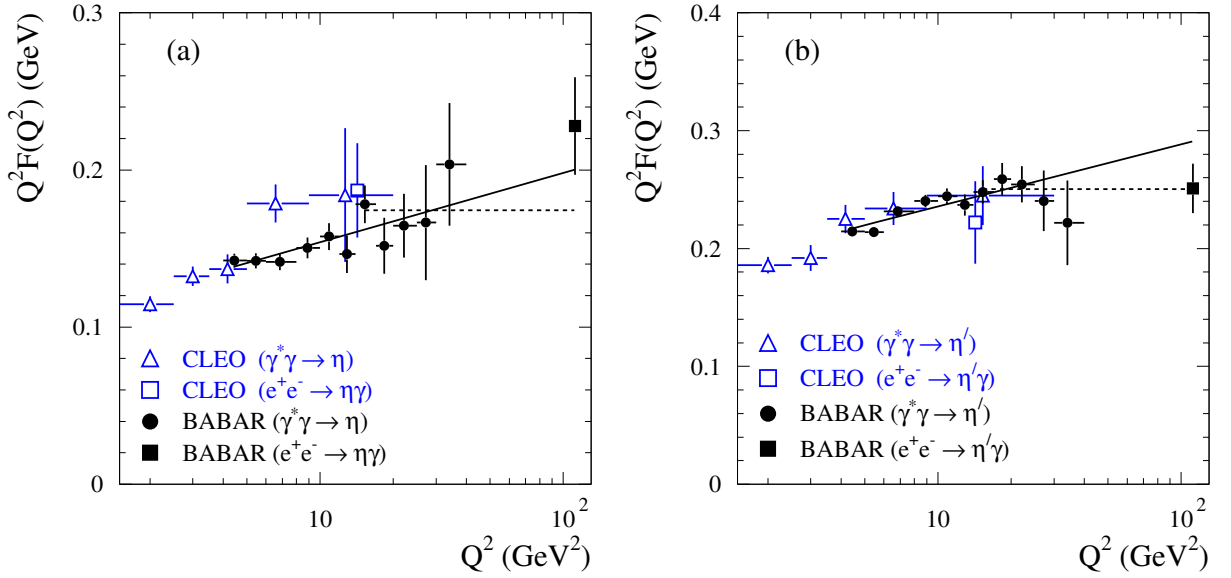


FIG. 17: The transition form factors multiplied by  $Q^2$  for (a)  $\gamma\gamma^* \rightarrow \eta$  and (b)  $\gamma\gamma^* \rightarrow \eta'$ . The solid line shows the result of the fit to *BABAR* data by the function given by Eq. (16). The dashed lines indicate the average form factor values over the data points with  $Q^2 > 14$   $\text{GeV}^2$ .

( $a_l \approx 0.20 \pm 0.05$   $\text{GeV}$ ) is about three times weaker than the corresponding rise of the  $\pi^0$  form factor [24].

The dashed horizontal lines in Fig. 17 show the results of fits assuming  $Q^2 F(Q^2)$  to be constant for  $14 < Q^2 < 112$   $\text{GeV}^2$ . The average values of  $Q^2 F(Q^2)$  in this range are  $0.174 \pm 0.008$   $\text{GeV}$  for  $\eta$  and  $0.251 \pm 0.006$   $\text{GeV}$  for  $\eta'$ . The  $\chi^2/\nu$  for the fits are 5.6/5 for the  $\eta$  and 2.6/5 for the  $\eta'$ . The preferable description for the  $\eta$  form factor is the logarithmic function of Eq. (16), corresponding

to the models with a finite DA at the end points. The  $\eta'$  form factor is better described by the model with a conventional DA, yielding a flat  $Q^2 F(Q^2)$  for  $Q^2 > 15$   $\text{GeV}^2$ .

To compare the measured values of the  $\eta$  and  $\eta'$  form factors with theoretical predictions and data for the  $\pi^0$  form factor we use the description of  $\eta$ - $\eta'$  mixing in the

quark flavor basis [26]:

$$\begin{aligned} |n\rangle &= \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle), \quad |s\rangle = |\bar{s}s\rangle, \\ |\eta\rangle &= \cos\phi|n\rangle - \sin\phi|s\rangle, \\ |\eta'\rangle &= \sin\phi|n\rangle + \cos\phi|s\rangle, \end{aligned} \quad (17)$$

where  $\phi$  is the mixing angle. The  $\eta$  and  $\eta'$  transition form factors are related to the form factors for the  $|n\rangle$  and  $|s\rangle$  states:

$$F_\eta = \cos\phi F_n - \sin\phi F_s, \quad F_{\eta'} = \sin\phi F_n + \cos\phi F_s, \quad (18)$$

which have asymptotic limits for  $Q^2 \rightarrow \infty$  [27] given by

$$Q^2 F_s(Q^2) = \frac{2}{3} f_s, \quad Q^2 F_n(Q^2) = \frac{5\sqrt{2}}{3} f_n, \quad (19)$$

where  $f_n$  and  $f_s$  are the decay constants for the  $|n\rangle$  and  $|s\rangle$  states, respectively. For the  $\pi^0$  form factor, the corresponding asymptotic value is  $\sqrt{2}f_\pi$ . The pion decay constant is determined from leptonic  $\pi$  decays to be  $130.4 \pm 0.2$  MeV [16]. For the  $|n\rangle$  and  $|s\rangle$  states, we use the ‘‘theoretical’’ values from Ref. [26]:  $f_n = f_\pi$  and  $f_s = \sqrt{2f_K^2 - f_\pi^2} \approx 1.36f_\pi$  ( $f_K/f_\pi = 1.193 \pm 0.006$  [16]), which agree to within 10% with the ‘‘phenomenological’’ values [26] extracted from the analysis of experimental data, for example, for the two-photon  $\eta$  and  $\eta'$  decays. The currently accepted value of the mixing angle  $\phi$  is about  $41^\circ$  [28]. Under the assumption that the  $|n\rangle$  and  $\pi^0$  distribution amplitudes are similar to each other, the only difference between the  $|n\rangle$  and  $\pi^0$  form factors is a factor of  $3/5$  that arises from the quark charges. In Fig. 18 the form factor for the  $|n\rangle$ -state multiplied by  $3Q^2/5$  is compared with the measured  $\gamma^*\gamma \rightarrow \pi^0$  form factor [1] and the results of the QCD calculations performed by A. P. Bakulev, S. V. Mikhailov and N. G. Stefanis [29] for the asymptotic DA [30], the Chernyak-Zhitnitsky  $\pi^0$  DA [31], and the  $\pi^0$  DA derived from QCD sum rules with non-local condensates [32]. The horizontal dashed line indicates the asymptotic limit for the  $\pi^0$  form factor.

The  $Q^2$  dependencies of the measured  $|n\rangle$  and  $\pi^0$  form factors are significantly different. This indicates that the distribution amplitudes for the  $|n\rangle$  and  $\pi^0$  are significantly different as well. The data for the  $|n\rangle$  form factor are well described by the model with DA from Ref. [32], while the data for the  $\pi^0$  form factor is reproduced by the models with a significantly wider DA [21, 22] or a flat DA [23–25].

The form factor for the  $|s\rangle$  state is shown in Fig. 19. The dotted curve shows the QCD prediction [29] for the asymptotic DA [30], defined by multiplying the  $\pi^0$  curve in Fig. 18 by a factor of  $(\sqrt{2}/3)f_s/f_\pi$ . The data lie systematically below this prediction. This may indicate, in particular, that the distribution amplitude for the  $|s\rangle$  state is narrower than the asymptotic DA. However, due to the strong sensitivity of the result for the  $|s\rangle$  state to mixing parameters, other interpretations are possible.

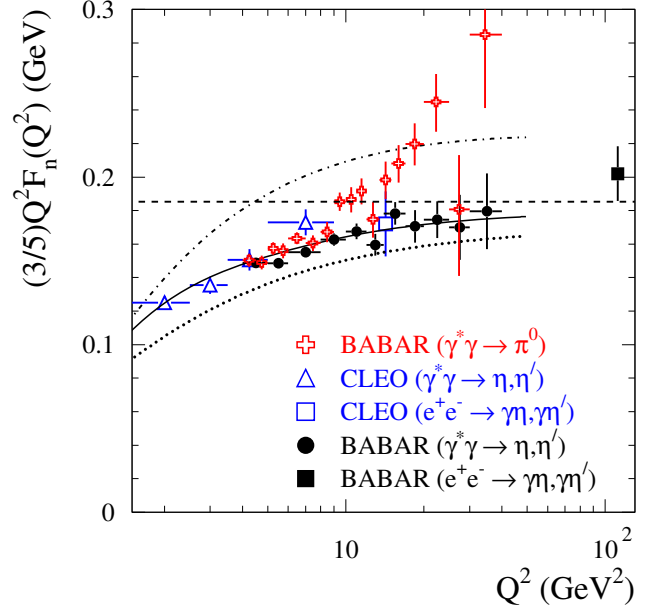


FIG. 18: The  $\gamma\gamma^* \rightarrow |n\rangle$  transition form factor multiplied by  $3Q^2/5$  in comparison with the  $\gamma\gamma^* \rightarrow \pi^0$  transition form factor [1]. The dashed line indicates the asymptotic limit for the  $\pi^0$  form factor. The dotted, dash-dotted, and solid curves show predictions of Ref. [29] for the asymptotic DA [30], the Chernyak-Zhitnitsky  $\pi^0$  DA [31], and the  $\pi^0$  DA from Ref. [32], respectively.

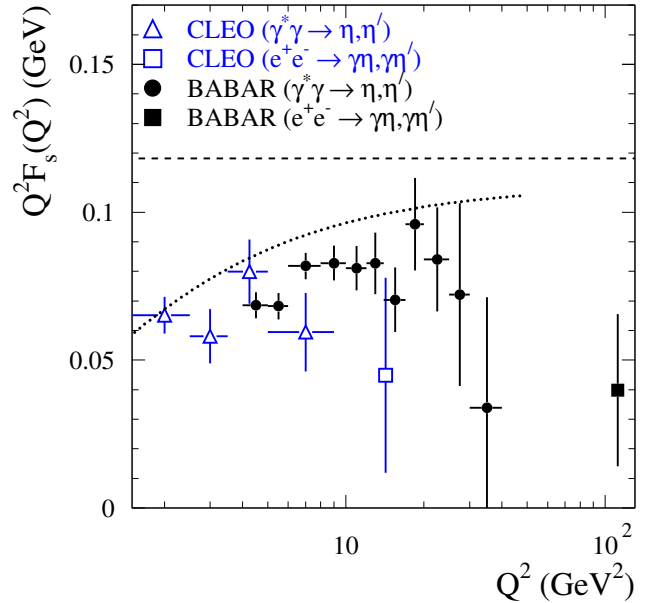


FIG. 19: The  $\gamma\gamma^* \rightarrow |s\rangle$  transition form factor multiplied by  $Q^2$ . The dashed line indicates the asymptotic limit for the form factor. The dotted curve shows the prediction [29] for the asymptotic DA [30].

For example, an admixture of the two-gluon component in the  $\eta'$  meson [33–36] can lead to a significant shift of the values of the  $|s\rangle$  form factor.

### Acknowledgments

We thank V. L. Chernyak for useful discussions. We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The success of this project also relies critically on the expertise and dedication of the computing organizations that support *BABAR*. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by the US

Department of Energy and National Science Foundation, the Natural Sciences and Engineering Research Council (Canada), the Commissariat à l’Energie Atomique and Institut National de Physique Nucléaire et de Physique des Particules (France), the Bundesministerium für Bildung und Forschung and Deutsche Forschungsgemeinschaft (Germany), the Istituto Nazionale di Fisica Nucleare (Italy), the Foundation for Fundamental Research on Matter (The Netherlands), the Research Council of Norway, the Ministry of Education and Science of the Russian Federation, Ministerio de Ciencia e Innovación (Spain), and the Science and Technology Facilities Council (United Kingdom). Individuals have received support from the Marie-Curie IEF program (European Union), the A. P. Sloan Foundation (USA) and the Binational Science Foundation (USA-Israel).

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