# Measurement of the Neutral Current Cross Section and F2 Structure Function for Deep Inelastic $\mathrm{e}^{+} \mathrm{p}$ Scattering at HERA 

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#### Abstract

The cross section and the proton structure function $F_{2}$ for neutral current deep inelastic $e^{+} p$ scattering have been measured with the ZEUS detector at HERA using an integrated luminosity of 30 $\mathrm{pb}^{-1}$. The data were collected in 1996 and 1997 at a centre-of-mass energy of 300 GeV . They cover the kinematic range $2.7<Q^{2}<30000 \mathrm{GeV}^{2}$ and $6 \cdot 10^{-5}<x<0.65$. The variation of $F_{2}$ with $x$ and $Q^{2}$ is well described by next-to-leading-order perturbative QCD as implemented in the DGLAP evolution equations.


[^0]
## 1 Introduction

The measurements of the differential cross-section and $F_{2}$ structure function in neutral current (NC) deep inelastic scattering (DIS) at HERA have been vital in testing the predictions of perturbative QCD and in the determination of the parton densities in the proton at low $x$. In both of these areas of study the accuracy and precision of the measurements are of paramount importance.

[^1]The measurements of the double-differential cross-section $\mathrm{d} \sigma^{e p} / \mathrm{d} x \mathrm{~d} Q^{2}$ and $F_{2}^{e m}$ presented here are based on an $e^{+} p$ DIS event sample twelve times larger than that used in the previous ZEUS publication [1]. This increase, combined with an improved understanding of the detector, results in a significant decrease in both the statistical and systematic uncertainties ${ }^{1}$. The uncertainties are systematics dominated for $Q^{2}<800 \mathrm{GeV}^{2}$.

The outline of the paper is as follows: in Sect. 2, the kinematics and the cross section of $e^{+} p$ NC DIS are discussed; in Sects. 3 and 4, the experimental conditions and the trigger used in the data taking are presented; Sect. 5 explains the method of the reconstruction of the kinematic variables; in Sect. 6, Monte Carlo simulations used in the analyses are described; Sects. 7 and 8 discuss the details of the basic measured quantities; in Sect. 9, the characteristics of the selected events are examined; Sect. 10 presents the results; and Sect. 11 contains the conclusions.

## 2 Kinematics and cross section

The kinematics of deep inelastic scattering, $e(k)+p(P) \rightarrow$ $e\left(k^{\prime}\right)+X$, can be completely described by the negative of the four-momentum-transfer squared, $Q^{2}$, and Bjorken $x$. In the absence of QED radiation,

$$
\begin{aligned}
Q^{2} & =-q^{2}=-\left(k-k^{\prime}\right)^{2}, \\
x & =\frac{Q^{2}}{2 P \cdot q},
\end{aligned}
$$

where $k$ and $P$ are the four-momenta of the incoming particles and $k^{\prime}$ is the four-momentum of the scattered lepton. The fraction of the lepton energy transferred to the proton in its rest frame is $y=(P \cdot q) /(P \cdot k)=Q^{2} /(s x)$, where $s$ is the square of the total centre-of-mass energy of the lepton-proton collision. The invariant mass of the hadronic final state, $W$, is calculated from

$$
W^{2}=Q^{2} \frac{1-x}{x}+m_{p}^{2}
$$

where $m_{p}$ is the proton mass.
The double-differential cross section for inclusive $e^{+} p$ scattering is given in terms of the structure functions $F_{i}$ :

$$
\begin{align*}
\frac{d^{2} \sigma^{e^{+}} p}{d x d Q^{2}}= & \frac{2 \pi \alpha^{2}}{x Q^{4}}\left[Y_{+} F_{2}\left(x, Q^{2}\right)-Y_{-} x F_{3}\left(x, Q^{2}\right)\right. \\
& \left.-y^{2} F_{L}\left(x, Q^{2}\right)\right]\left(1+\delta_{r}\left(x, Q^{2}\right)\right)  \tag{1}\\
= & \frac{d^{2} \sigma_{\text {Born }}^{e^{+} p}}{d x d Q^{2}}\left(1+\delta_{r}\left(x, Q^{2}\right)\right),
\end{align*}
$$

where $Y_{ \pm}=1 \pm(1-y)^{2}$ and $x$ and $Q^{2}$ are defined at the hadronic vertex. In this equation, $F_{L}$ is the longitudinal structure function, $x F_{3}$ is the parity-violating term

[^2]arising from $Z^{0}$ exchange and $\delta_{r}$ is the electroweak radiative correction. The double-differential Born cross section, $d^{2} \sigma_{\text {Born }} / d x d Q^{2}$, is evaluated using $1 / \alpha=137.03599$, $\sin ^{2} \theta_{W}=0.23147$ and $M_{Z}=91.1882 \mathrm{GeV}$ [4] to determine the electroweak parameters [5].

The structure function $F_{2}$ contains contributions from both virtual photon and $Z^{0}$ exchange:

$$
\begin{align*}
F_{2} & =F_{2}^{e m}+\frac{Q^{2}}{\left(Q^{2}+M_{Z}^{2}\right)} F_{2}^{i n t}+\frac{Q^{4}}{\left(Q^{2}+M_{Z}^{2}\right)^{2}} F_{2}^{w k} \\
& =F_{2}^{e m}\left(1+\Delta_{F_{2}}\right) \tag{2}
\end{align*}
$$

where $M_{Z}$ is the mass of the $Z^{0}$ and the contributions to $F_{2}$ from photon exchange $\left(F_{2}^{e m}\right), Z^{0}$ exchange $\left(F_{2}^{w k}\right)$ and the $Z^{0}-\gamma$ interference term $\left(F_{2}^{\text {int }}\right)$ are indicated separately. All three contributions are included in the reduced cross section, $\tilde{\sigma}^{e^{+} p}$, defined as:

$$
\tilde{\sigma}^{e^{+} p}=\left[\frac{2 \pi \alpha^{2}}{x Q^{4}} Y_{+}\right]^{-1} \frac{d^{2} \sigma_{\text {Born }}^{e^{+} p}}{d x d Q^{2}} .
$$

## 3 Experimental conditions

The data presented here were taken in 1996 and 1997 using the ZEUS detector at HERA. In this period HERA operated with $174-177$ bunches of 820 GeV protons and 27.5 GeV positrons, which collided every 96 ns . Additional unpaired positron, unpaired proton and empty bunches were used to determine beam-related backgrounds. The proton bunch length was approximately 11 cm (r.m.s.) while the positron bunch length was negligible in comparison. Variations of the mean interaction position (from run to run) lead to an effective length of the interaction region of 11.5 cm (r.m.s.) centred ${ }^{2}$ around $Z=-0.5 \mathrm{~cm}$. Approximately $6 \%$ of the proton current was contained in satellite bunches, which were shifted by $\pm 4.8 \mathrm{~ns}$ with respect to the nominal bunch crossing time, resulting in a fraction of the $e p$ interactions occurring at $\langle Z\rangle \simeq \pm 72 \mathrm{~cm}$.

A brief outline of the components which are most relevant for this analysis, as illustrated in Fig. 1, is given below. The uranium-scintillator calorimeter (CAL) [6] covers $99.7 \%$ of the total solid angle. It consists of the barrel calorimeter (BCAL), covering the polar angle range $36.7^{\circ}<\theta<129.1^{\circ}$, the forward calorimeter (FCAL), covering $2.6^{\circ}<\theta<36.7^{\circ}$ and the rear calorimeter (RCAL), covering $129.1^{\circ}<\theta<178.5^{\circ}$. The FCAL and RCAL are divided vertically into two halves to allow temporary retraction during beam injection. The CAL is segmented into electromagnetic (EMC) and hadronic (HAC) sections. Each section is further subdivided into cells of typically $5 \times 20 \mathrm{~cm}^{2}\left(10 \times 20 \mathrm{~cm}^{2}\right.$ in the RCAL $)$ for the

[^3]

Fig. 1. Schematic view of the ZEUS detector showing those components important for this analysis

EMC and $20 \times 20 \mathrm{~cm}^{2}$ for the HAC sections. Each cell is viewed by two photomultiplier tubes (PMTs). Under test-beam conditions, the calorimeter has a single particle energy resolution of $\sigma / E=18 \% / \sqrt{E(\mathrm{GeV})}$ for electrons and $\sigma / E=35 \% / \sqrt{E(\mathrm{GeV})}$ for hadrons [6]. The timing resolution of a calorimeter cell is better than 1 ns for energy deposits greater than 4.5 GeV . To minimise the effects of noise coming from the uranium radioactivity, all EMC (HAC) cells with an energy deposit of less than 60 (110) MeV were discarded in the analysis. For the remaining cells with no neighbouring energy deposits, this cut was increased to 100 (160) MeV.

A presampler (PRES) [7] is mounted in front of FCAL and RCAL. It consists of scintillator tiles that detect particles originating from showers in the material between the interaction point and the calorimeter. This information is used to correct the energy of the scattered positron.

The tracking system consists of a central tracking chamber (CTD) [8] and a rear tracking detector (RTD) [9] enclosed in a 1.43 T solenoidal magnetic field. The interaction vertex was measured with a typical resolution along (transverse to) the beam direction of $0.4(0.1) \mathrm{cm}$. The CTD was used to measure track momenta with a resolution of $\sigma\left(p_{t}\right) / p_{t}=0.0058 p_{t} \oplus 0.0065 \oplus 0.0014 / p_{t}\left(p_{t}\right.$ in GeV ) and to extrapolate tracks onto the face of the calorimeter with a resolution of 0.3 cm .

The position of positrons scattered at small angles to the positron-beam direction was measured using the small-angle rear tracking detector (SRTD) [10], which is attached to the front face of the RCAL. The SRTD consists of two planes of scintillator strips, 1 cm wide and 0.5 cm thick, arranged in orthogonal directions and read out via optical fibres and photomultiplier tubes. It covers the region of about $68 \times 68 \mathrm{~cm}^{2}$ in $X$ and $Y$ and is positioned at $Z=-148 \mathrm{~cm}$. A hole of $8 \times 20 \mathrm{~cm}^{2}$ at the centre of the RCAL and SRTD accommodates the beampipe. The SRTD signals resolve single minimum-ionising particles and provide a transverse position resolution of 3 mm . The time resolution is better than 2 ns for a minimum-ionising particle. The RTD was used to determine accurately the position of the SRTD.

The hadron-electron separator (HES) [9] consists of a layer of silicon pad detectors. The rear HES is located in
the RCAL at a depth of 3.3 radiation lengths. Each silicon pad has an area of $28.9 \times 30.5 \mathrm{~mm}^{2}$, providing a spatial resolution of about 9 mm for a single hit pad. If more than one adjacent pad is hit by a shower, a cluster consisting of at most $3 \times 3$ pads around the most energetic pad is considered, giving a positron position resolution of 5 mm .

The luminosity was measured via the bremsstrahlung process, ep $\rightarrow e \gamma p$, using a lead-scintillator calorimeter (LUMI) [11] which detects photons at angles $\leq 0.5 \mathrm{mrad}$ with respect to the positron beam direction. The LUMI photon calorimeter was also used to tag photons from initial-state radiation in DIS events. It was positioned at $Z=-107 \mathrm{~m}$ and had an intrinsic energy resolution of $\sigma / E=18 \% / \sqrt{E(\mathrm{GeV})}$. The carbon-lead filter, placed in front of the calorimeter to shield it from synchrotron radiation, degraded its energy resolution somewhat. The position resolution was 0.2 cm in $X$ and $Y$. In addition, an electromagnetic calorimeter (positron tagger) positioned at $Z=-35 \mathrm{~m}$ was used for tagging positrons scattered at very small angles.

## 4 Trigger

For the 'high- $Q^{2}$ region' $\left(Q^{2} \geq 30 \mathrm{GeV}^{2}\right)$, the data analysed correspond to an integrated luminosity of $30.6 \pm$ $0.5 \mathrm{pb}^{-1}$. For the 'low- $Q^{2}$ region' $\left(Q^{2} \geq 2 \mathrm{GeV}^{2}\right)$, where the positrons are scattered through smaller angles, the data set corresponds to an integrated luminosity of $2.23 \pm$ $0.02 \mathrm{pb}^{-1}$, which was obtained in short dedicated running periods.

Events were selected online by a three-level trigger system [9]. At the first level (FLT) [12], events with a positron candidate were selected by the logical OR of the following conditions:

- total EMC energy deposit in the BCAL greater than 4.8 GeV ;
- total EMC energy deposit in the RCAL, excluding the ring of $20 \times 20 \mathrm{~cm}^{2}$ towers closest to the rear beampipe, greater than 3.4 GeV ;
- an isolated positron condition (ISO-e) in the RCAL. The ISO-e condition requires that the isolated EMC energy deposit be greater than 2.08 GeV and that the HAC energy behind it be either less than 0.95 GeV or no more than one third of the EMC energy. The isolation criterion required all towers adjacent to the EMC energy deposit associated with the positron to have energy deposits less than 2.08 GeV in the EMC and 0.95 GeV in the HAC. Additionally, the total energy deposit in CAL was required to be greater than 0.464 GeV and a signal above threshold was required in the SRTD;
- total transverse energy in the CAL greater than 30 GeV;
- total transverse energy in the CAL greater than 11.6 GeV and at least one track candidate in the CTD;
- total EMC energy deposit in the CAL greater than 10 GeV and at least one track candidate in the CTD.

Backgrounds from protons interacting outside the detector were rejected using the time measurement of the energy deposits in upstream veto counters and in the SRTD.

The trigger efficiency of the FLT for events that passed the offline selection cuts (see Sect.9.1) was greater than $99 \%$, as determined from dedicated trigger studies and from Monte Carlo (MC) simulations.

The second-level trigger (SLT) further reduced the background using the times measured in the calorimeter cells and the summed energies from the calorimeter. Energy and momentum conservation require ep-collisions to conserve $\delta$ :

$$
\begin{equation*}
\delta=E-P_{Z}=\sum_{i} E_{i}\left(1-\cos \theta_{i}\right) \tag{3}
\end{equation*}
$$

In this equation, $E_{i}$ and $\theta_{i}$ are the energies and polar angles of all energy deposits in the CAL. For perfect detector resolution and for fully contained DIS events, $\delta$ will equal twice the positron beam energy ( 55 GeV ). For photoproduction events, where the scattered positron escapes down the beampipe, $\delta$ peaks at much lower values. Proton beam-gas events that originate from inside the detector have energy concentrated in the forward direction and so also have small values of $\delta$.

The events were accepted at the SLT if they satisfied the condition that

$$
\delta_{S L T}=\sum_{i} E_{i}\left(1-\cos \vartheta_{i}\right)>\left(29 \mathrm{GeV}-2 E_{\gamma}\right)
$$

where $\vartheta_{i}$ is the polar angle of the energy deposit with respect to the nominal vertex and $E_{\gamma}$ is the energy deposit measured in the LUMI photon calorimeter.

The full event information was available at the thirdlevel trigger (TLT). Tighter timing cuts as well as algorithms to remove beam-halo muons and cosmic rays were applied. The quantity $\delta_{T L T}$ was determined using the reconstructed vertex and the events were required to have $\delta_{T L T}>\left(30 \mathrm{GeV}-2 E_{\gamma}\right)$. Finally, events were accepted if a scattered positron candidate with energy greater than 4 GeV was found. To ensure full containment of the shower in the calorimeter, events in which the scattered positron was located within 2 cm of the beamhole were rejected.

In total $1,747,944(3,935,516)$ NC DIS candidates satisfied the above low- $Q^{2}$ (high- $Q^{2}$ ) trigger condition.

## 5 Reconstruction of event kinematics

In the determination of the event kinematics, the CAL energy deposits were separated into two categories: those associated with the identified scattered positron, and all other energy deposits. The latter were defined as the hadronic energy. The kinematics of the event was then determined from:

- the energy $\left(E_{e}^{\prime}\right)$ and polar angle $\left(\theta_{e}\right)$ of the scattered positron;
- the hadronic energy expressed in terms of the longitudinal:

$$
\delta_{h}=\sum_{h}\left(E_{h}-P_{Z, h}\right)
$$

and transverse:

$$
P_{T, h}=\sqrt{\left(\sum_{h} P_{X, h}\right)^{2}+\left(\sum_{h} P_{Y, h}\right)^{2}}
$$

components, where the sums run over all energy deposits not associated with the scattered positron.
The hadronic energy flow is characterised by an angle $\gamma_{h}$ defined by

$$
\begin{equation*}
\cos \gamma_{h}=\frac{P_{T, h}^{2}-\delta_{h}^{2}}{P_{T, h}^{2}+\delta_{h}^{2}} \tag{4}
\end{equation*}
$$

In the quark-parton model, the angle $\gamma_{h}$ corresponds to the polar angle of the struck quark.

The following refinements in the calculation of $\gamma_{h}$ were applied in order to optimise the accuracy and precision of the reconstruction:

- the transverse momentum of the hadronic system, $P_{T, h}$, was replaced by the more accurately measured transverse momentum of the scattered positron, $P_{T, e}$;
- at low values of $\delta_{h}$, part of the hadronic system escapes detection through the forward beamhole. A parameterisation, determined from MC simulation, as a function of the measured ratio $P_{T, h} / P_{T, e}, \gamma_{h}$ and $Q^{2}$ was used to correct $\delta_{h}$ :

$$
\begin{equation*}
\delta_{h, \mathrm{cor}}=\delta_{h} \mathcal{C}\left(P_{T, h} / P_{T, e}, \gamma_{h}, Q^{2}\right) \tag{5}
\end{equation*}
$$

- in the determination of $y$ at large values of $\delta_{h}$, the information from the positron provides the most accurate measurement, $y_{e}=1-\left(E_{e}^{\prime} / 2 E_{e}\right)\left(1-\cos \theta_{e}\right)$. However, at low values of $\delta_{h}$, the hadronic information provides a more accurate measurement, $y_{\text {cor }}=$ $\delta_{h, \text { cor }} /\left(2 E_{e}\right)$. The $\Sigma$-correction [13] combines $y_{\text {cor }}$ and $y_{e}$ such that

$$
y_{\Sigma, \mathrm{cor}}=\frac{\delta_{h, \mathrm{cor}} y_{e}+\delta_{e} y_{\mathrm{cor}}}{\delta_{e}+\delta_{h, \mathrm{cor}}}
$$

where $\delta_{e}=E_{e}^{\prime}\left(1-\cos \theta_{e}\right)$;

- finally, $\gamma_{\mathrm{PT}}$ was calculated from:

$$
\begin{equation*}
\cos \gamma_{\mathrm{PT}}=\frac{P_{T, e}^{2}-\delta_{\Sigma, \mathrm{cor}}^{2}}{P_{T, e}^{2}+\delta_{\Sigma, \mathrm{cor}}^{2}} \tag{6}
\end{equation*}
$$

where $\delta_{\Sigma, \mathrm{cor}}=2 E_{e} y_{\Sigma, \mathrm{cor}}$.
For the present analysis, the PT method [1] was used to reconstruct the kinematic variables of the events:

$$
\begin{aligned}
Q_{\mathrm{PT}}^{2} & =4 E_{e}^{2} \frac{\sin \gamma_{\mathrm{PT}}\left(1+\cos \theta_{e}\right)}{\sin \gamma_{\mathrm{PT}}+\sin \theta_{e}-\sin \left(\gamma_{\mathrm{PT}}+\theta_{e}\right)} \\
x_{\mathrm{PT}} & =\frac{E_{e}}{E_{p}} \frac{\sin \gamma_{\mathrm{PT}}+\sin \theta_{e}+\sin \left(\gamma_{\mathrm{PT}}+\theta_{e}\right)}{\sin \gamma_{\mathrm{PT}}+\sin \theta_{e}-\sin \left(\gamma_{\mathrm{PT}}+\theta_{e}\right)} \\
y_{\mathrm{PT}} & =\frac{Q_{\mathrm{PT}}^{2}}{s x_{\mathrm{PT}}} \\
W_{\mathrm{PT}}^{2} & =Q_{\mathrm{PT}}^{2} \frac{1-x_{\mathrm{PT}}}{x_{\mathrm{PT}}}+m_{p}^{2} .
\end{aligned}
$$

This reconstruction method achieves good resolution and minimises biases in $x$ and $Q^{2}$ over the full kinematic range.

## 6 Monte Carlo simulation

Monte Carlo (MC) event simulation was used to correct for detector acceptance and resolution effects. The detector simulation is based on the GEANT program [14] and incorporates the best understanding of the detector and the trigger, including test-beam results. Neutral current DIS events were simulated with radiative effects using the HERACLES 4.5.2 [15] program interfaced to the LEPTO 6.5 [16] MC program by the DJANGO6 2.4 program [17]. HERACLES includes $\mathrm{O}(\alpha)$ leptonic corrections for initialand final-state radiation, and one-loop vertex and propagator corrections. In addition, leading logarithmic effects to order $\mathrm{O}\left(\alpha \alpha_{s}\right)$ are included. The hadronic final state was simulated using the colour-dipole model CDMBGF [18] including all leading-order QCD diagrams as implemented in ARIADNE 4.08 [20] for the QCD cascade and JETSET 7.4 [21] for the hadronisation. The ARIADNE model provides the best description of the characteristics of the DIS non-diffractive hadronic final state [22]. Diffractive events which are characterised by a large rapidity gap in the detector $[23,24]$ were simulated within ARIADNE, which was interfaced to the RAPGAP 2.06/52 [25] program. The latter assumes that the struck quark belongs to a colourless state having only a small fraction of the proton momentum. The diffractive and non-diffractive samples were mixed as a function of $x_{\mathrm{PT}}$ and $Q_{\mathrm{PT}}^{2}$ as determined from the $\eta_{\max }$ distribution in the data, where $\eta_{\max }$ is the pseudorapidity of the most forward energy deposit with more than 400 MeV . The overall normalisation was kept fixed. The CTEQ4D [26] parton density parameterisations were used in the simulations.

The vertex distribution used in the simulation was taken from DIS events having both scattered positrons and hadrons well reconstructed by the CTD. The vertex finding efficiency is well described by the MC simulation, as is shown below (see Sect. 7.1).

The effects of the uranium radioactivity were studied in randomly triggered events. The energy deposits in these events originate entirely from the calorimeter noise. The rate and energy distribution were simulated in the MC program.

Twelve different MC samples with varying $Q^{2}$ ranges were generated with integrated luminosities increasing with $Q^{2}$, from $1.2 \mathrm{pb}^{-1}$ for $Q^{2}>0.5 \mathrm{GeV}^{2}$ to $1500 \mathrm{fb}^{-1}$ for $Q^{2}>40000 \mathrm{GeV}^{2}$. The fluctuations in the final result due to the MC-sample statistics are small.

The main source of background in the data comes from photoproduction interactions that led to the detection of a fake scattered positron. Resolved and direct photoproduction events [27] were simulated using PYTHIA 5.724 [21], with the total cross section given by the ALLM parameterisation [28]. Resolved and direct photoproduction events, corresponding to integrated luminosities of 0.7 and 2.5 $\mathrm{pb}^{-1}$, respectively, were generated with $y>0.36$. Events
with smaller $y$ values do not contribute to the photoproduction background due to the requirement on $\delta$.

## 7 Positron identification and measurement

The positron-identification algorithm is based on a neural network using information from the CAL [29]. The network separates energy deposits in the calorimeter that are due to electromagnetic showers from those that are of hadronic origin by assigning a 'positron-probability' value. If more than one positron is identified, the one with the highest probability is taken to be the scattered positron. Monte Carlo studies show that the efficiency for finding the scattered positron increases from $70 \%$ at 8 GeV to greater than $99 \%$ for energies above 25 GeV . This was checked by using QED-Compton events. The results obtained from the neural network were compared to an alternative positron-finding algorithm [30]. Systematic differences of the order of $2 \%$ were observed at 8 GeV , decreasing to less than $1 \%$ for energies higher than 15 GeV .

### 7.1 Positron angle measurement

The scattering angle of the positron was determined using either its impact position on the CAL inner face and the event vertex or from a reconstructed track in the CTD. For $\theta_{e} \lesssim 150^{\circ}$, the scattered positron registers in the three innermost superlayers of the CTD and the parameters of the reconstructed track, matched to the positron impact position, were used.

For $\theta_{e} \gtrsim 150^{\circ}$, the impact position of the scattered positron on the inner face of the calorimeter was reconstructed using other detector elements. The SRTD information was used, when available; otherwise the HES information was used. If neither HES nor SRTD information was available, then the CAL reconstruction was used:

- SRTD - the positron position in the SRTD was determined from the centres of gravity of the pulse-heights of the strips in the $X$ and $Y$ planes of the SRTD. The measured position resolution was 3 mm . For a subset of these positrons, the position measured by the SRTD was compared to the result from the track measurement. The resolution of the RTD-SRTD matching was 3 mm and systematic deviations were less than 1 mm ;
- HES - the position in the HES was determined from the pulse-height sharing in the pads. The position measurement was checked by matching tracks and positron candidates. The resolution of the track-HES matching was 5 mm with systematic deviations smaller than 1.5 mm ;
- CAL - the position of a shower in the CAL was determined using the relative pulse-height of the left and right PMTs and energy deposits in neighbouring cells. The position resolution on the face of the CAL was about 10 mm . The alignment of the calorimeter was checked by matching tracks and positron candidates in the CAL. The resolution of the track-CAL matching was 10 mm with systematic biases of less than 2 mm .


Fig. 2. Vertex finding efficiency as a function of the hadronic angle $\gamma_{\mathrm{PT}}$, defined in (6)

The coordinates of the event vertex were determined from tracks reconstructed with the CTD. The $Z$ coordinate ( $Z_{\text {vertex }}$ ) was determined on an event-by-event basis. However, since the transverse sizes of the beams were smaller than the resolutions for the $X$ and $Y$ coordinates, the average $X$ and $Y$ vertex positions per beam fill were used. For events not having a tracking vertex, the $Z$ coordinate was set to $Z=0$, the nominal position of the interaction point averaged over all runs.

The probability of finding a vertex depended on the number of particles detected in the tracking detectors and thus mainly on the hadronic angle, $\gamma_{h}$. Figure 2 shows the fraction of events having a reconstructed vertex as a function of $\gamma_{\mathrm{PT}}$. For $\gamma_{\mathrm{PT}}>70^{\circ}$, the vertex efficiency was greater than $95 \%$. In the low- $Q^{2}$ sample, it decreased to about $50 \%$ for events with $\gamma_{\mathrm{PT}} \sim 20^{\circ}$. The MC simulation reproduces the observed behaviour well. The drop in vertex-finding efficiency near $\gamma_{\mathrm{PT}}=174^{\circ}$ results from the combination of the characteristics of the DIS final states and the acceptance of the CTD in the rear direction. At low $\gamma_{\mathrm{PT}}$, the vertex-finding efficiency is higher for high- $Q^{2}$ events, since the scattered positron is detected more often by the CTD.

The $Z$ coordinate of the vertex can also be determined from the measurements of the arrival times of energy deposits in the FCAL. The corresponding resolution is $\sigma_{z}^{\text {timing }}=9 \mathrm{~cm}$. This determination was only used for studies of systematic uncertainties.

For polar angles $17.2^{\circ} \lesssim \theta_{e} \lesssim 150^{\circ}$ and positron momenta $>5 \mathrm{GeV}$, the tracking efficiency was greater than $98 \%$. Thus, when the impact position on the CAL inner face was outside a radius of 80 cm around the beampipe, corresponding to a scattering angle of $150^{\circ}$ for events with $Z=0$, a track was required to match the positron identified in the CAL and the polar angle was taken from the track parameters. A successful match required that the distance between the extrapolated impact point of the track on the face of the CAL and the position determined by the CAL was less than 10 cm .

The resolution of the scattering-angle measurement was $\sigma_{\theta_{e}}^{\mathrm{SRTD}} \simeq 2.0 \mathrm{mrad}$ for positrons reconstructed in the SRTD, $\sigma_{\theta_{e}}^{\mathrm{HES}} \simeq 3.4 \mathrm{mrad}$ for positrons measured in the HES, $\sigma_{\theta_{e}}^{\text {CAL }} \simeq 6.8 \mathrm{mrad}$ for positrons measured in the CAL and $\sigma_{\theta_{e}}^{\text {track }} \simeq 2.6 \mathrm{mrad}$ for positrons with a matched track.

### 7.2 Positron energy determination

The scattered positron loses energy in the passive material in front of the CAL. In the region used for this analysis, this corresponds to about 1.2 radiation lengths except in areas around the rear beampipe, $\theta \gtrsim 170^{\circ}$, and the solenoid support structure, $130^{\circ} \lesssim \theta \lesssim 145^{\circ}$, where it corresponds to 2.0 and 2.5 radiation lengths, respectively. In the analysis, the CAL measurement of the scattered positron energy was corrected for these energy losses.

The correction was determined directly from the data using the following subsamples:

- at low $y$, the scattered-positron energy is kinematically constrained to be close to the positron beam energy and to be primarily a function of the scattering angle. For these events, called kinematic-peak (KP) events, the mean positron energy can be determined from the scattering angle $\left(E_{e}^{\prime} \approx 2 E_{e} /\left(1-\cos \theta_{e}\right)\right)$ to within $0.5 \%$. The KP events were selected by requir$\operatorname{ing} y_{J B}=\delta_{h} /\left(2 E_{e}\right)<0.04$ [31], and provide an energy calibration at $E_{e}^{\prime} \approx E_{e}=27.5 \mathrm{GeV}$ for $\theta \gtrsim 135^{\circ}$;
- the double-angle (DA) method [32] predicts the positron energy from the angular information of the scattered positron and the hadrons, $E_{e}^{\prime}=\left(2 E_{e} \sin \gamma_{h}\right) /$ $\left(\sin \gamma_{h}+\sin \theta_{e}-\sin \left(\gamma_{h}+\theta_{e}\right)\right)$. This relation was used to relate the energy loss in passive material to the SRTD and PRES signal, in the energy range $15 \lesssim E_{e}^{\prime} \lesssim 25$ GeV , assuming no hard QED radiation in the initial state;
- for QED-Compton events ( $e p \rightarrow e \gamma p$ ) observed in the main detector, the energies of the positron and the photon can be predicted precisely from the measurement of their scattering angles since the transverse momentum of the scattered proton is small. QEDCompton events provide a calibration for $5 \lesssim E_{e}^{\prime} \lesssim 20$ GeV and $\theta \gtrsim 160^{\circ}$;
- in the range $17.2^{\circ} \lesssim \theta \lesssim 150^{\circ}$, the momenta of the positrons can be independently determined from the CTD.

For scattered positrons in the RCAL, the correlation between the energy lost in the passive material in front of the calorimeter and the energy deposited in the SRTD or presampler was used to correct the calorimeter energy measurement. The corrections were determined using the KP and DA samples. The QED-Compton events were used as a check on the correction.

The uncertainty in the energy determination, after all corrections, was $2 \%$ at 8 GeV , falling linearly to $1 \%$ at 15 GeV . In the region covered by BCAL and FCAL, $\theta \lesssim$ $130^{\circ}$, the positron cluster energy was corrected based on
material maps implemented in the detector simulation package. For scattered positrons in the BCAL [30] and RCAL, the corrections for non-uniformities due to geometric effects caused by cell and module boundaries were also made. Identical studies were also performed on the event samples generated by the MC simulations. The resulting corrections were applied to the events generated in the MC simulation.

## 8 Hadronic energy determination

Energy deposits coming from the hadronic final state were used to evaluate the quantities $\delta_{h}$ and $P_{T, h}$. These energy deposits, unlike those of the positron, were not corrected for energy loss in the passive material. Instead, a combination of clusters of energy deposits in the CAL and the corresponding tracks measured in the CTD were used in determining the hadronic energy [33]. The selected calorimeter clusters and tracks are referred to as energy-flow objects (EFOs). The use of track information reduces the sensitivity to energy losses in inactive material in front of the CAL. However, the energies of particles for which no track information was available (e.g., because the energy was deposited by a neutral particle) were measured using the CAL alone.

Monte Carlo studies of the calorimeter response indicated that the uncorrected $\gamma_{h}$ calculated with (4) was biased by hadronic energy coming from interactions in material between the primary vertex and the calorimeter or by backsplash from the calorimeter (albedo), primarily from the FCAL. To minimise this bias, neutral clusters with energy below 3 GeV and with polar angles larger than a $\gamma_{\text {max }}, \gamma_{\text {max }}=\gamma_{h}+50^{\circ}$, were removed on an event-by-event basis. The choice of $50^{\circ}$ was derived from the MC simulation by minimising the dependence of $\delta_{h}$ on $\gamma_{\max }$. For the values of $\gamma_{\max }$ used, the biases in $\delta_{h}$ and $P_{T, h}$ were small. The agreement between data and MC simulation of the distributions of removed energies for different $\gamma_{h}$ ranges was also good. After a first pass of cluster removal, the value of $\gamma_{h}$ was re-calculated and the procedure repeated until it converged, typically after two or three iterations. Removing calorimeter clusters in this manner substantially improved the resolution and reduced the bias of the quantities $\delta_{h}$ and $P_{T, h}$ at low values of $\delta_{h}$ (corresponding to small values of $y$ ) and left them largely unchanged for large values of $\delta_{h}$.

The transverse momentum of the positron, $P_{T, e}$, calculated using the positron energy, corrected as described in the previous section, was compared to the $P_{T, h}$ of the hadrons in both the MC simulation and the data. Uncertainties in the determination of the hadronic energy were estimated from this comparison. The Monte Carlo simulation of the mean $P_{T, h} / P_{T, e}$ as a function of $\gamma_{h}$ agrees within $2 \%$ with the data for the entire kinematic range covered in this paper. Based on these comparisons, an uncertainty of $\pm 2 \%$ was assigned to the hadronic-energy measurement, neglecting the uncertainty due to the angular information in the calculation of $P_{T, h}$, which is small.

## 9 Data sample

### 9.1 Event selection

The following cuts, using the energy and angle of the scattered positron, were used both to select NC DIS events and to reject background:

- a positron candidate, defined as described in Sect. 7;
- $E_{e}^{\prime}>8 \mathrm{GeV}$, where $E_{e}^{\prime}$ is the corrected positron energy. This cut ensured high and well understood positron finding efficiency and suppressed background from photoproduction;
- events with an impact point of the scattered positron on the RCAL inside a box of $26 \times 14 \mathrm{~cm}^{2}$ centred around the beampipe were rejected ("box cut"). This ensured that the impact point was at least 2.5 cm away from the edge of the RCAL and therefore guaranteed full containment of the electromagnetic shower in the calorimeter. Events in which the scattered positron traversed the rear cooling pipes around the beampipe were also rejected. In the high- $Q^{2}$ sample, events with an impact point of the scattered positron on the RCAL inside a radius of 28 cm around the beampipe were also rejected;
- a track match for $17.2^{\circ}<\theta_{e} \lesssim 150^{\circ}$. This condition suppressed events from cosmic rays, halo-muons, photoproduction and DIS events in which an electromagnetic shower was falsely identified as the scattered positron;
- to reduce photoproduction background further, isolated positrons were selected by requiring no more than a total of 5 GeV from all calorimeter cells not associated with the scattered positron in an $\eta-\phi$ cone of radius 0.8 centred on the positron.
The hadronic information was used in the following selection criteria:
$-38<\delta<65 \mathrm{GeV}$, where $\delta=\sum_{i}\left(E_{i}-P_{Z i}\right)$. Here the sum runs over both the energy-flow objects from the hadronic system and the energy deposits belonging to the identified positron. This cut removed events with large initial-state radiation and further reduced the background from photoproduction. For events with a scattered positron beyond the tracking acceptance $\left(\theta_{e}<17.2^{\circ}\right)$, the minimum requirement was raised to $\delta>44 \mathrm{GeV}$;
- $P_{T, h} / P_{T, e}>0.3$. For the PT method to yield a reliable measurement of $x$ and $Q^{2}$, the loss of hadronic transverse momentum must be small. In some events, a substantial part of the current jet remained in the forward beampipe. These events, produced at small $\gamma_{h}$, could be falsely reconstructed at large $\gamma_{h}$ due to the CAL noise or backsplash. This cut suppressed such events. Finally, the following cuts were applied:
$-y_{e}<0.95$. This condition removes events where fake positrons are found in the FCAL;
$--50 \mathrm{~cm}<Z_{\text {vertex }}<50 \mathrm{~cm}$. This cut was applied to events with a reconstructed tracking vertex; it suppressed beam-gas background events and the small


## ZEUS



Fig. 3. a The purity. b The acceptance. The analysis binning is shown, together with the limits of the ZEUS acceptance. At the edges of the kinematic plane, the acceptance is lower due to the various selection criteria. The rear beamhole limits the acceptance of events with small angles of the scattered positron to $Q^{2} \gtrsim 2 \mathrm{GeV}^{2}$. For $y \gtrsim 0.7$, the positron-energy cut and for $Q^{2} \gtrsim 1000 \mathrm{GeV}^{2}$ and $y=1$ the kinematic limit of HERA constrains the measurements. The forward beamhole limits the acceptance to $y \gtrsim 0.004$ for events with small angles of the hadronic system; this limit is shown by the line $W=20 \mathrm{GeV}$
fraction of the events in which the vertex position was incorrectly measured. Events without a tracking vertex were accepted and were assigned the mean vertex $Z$ position;
$-W_{\mathrm{PT}}>20 \mathrm{GeV}$. This cut, corresponding to $y \gtrsim 0.004$ for $Q^{2} \lesssim 400 \mathrm{GeV}^{2}$, selected the kinematic region where a reliable measurement could be made. Events below this cut failed the acceptance and purity requirements described in Sect.9.2. At low $W$, the hadronic system is partially lost in the forward beampipe and therefore the measurements are sensitive to the detailed simulation of the beampipe region.

A total of $561,405(1,300,887)$ events passed the low (high) $Q^{2}$ selection cuts.

### 9.2 Binning and resolution

The relative resolution in $x$ improves from approximately $35 \%$ at $y \sim 0.005$, to $\sim 11 \%$ for $y>0.1$. The relative resolution in $Q^{2}$ is $\sim 2 \%$ for $y \sim 0.1$ and deteriorates to approximately $5 \%$ at $y \gtrsim 0.5$ and $\sim 10 \%$ at $y \lesssim 0.01$. For $Q^{2}>400 \mathrm{GeV}^{2}$, the $\widetilde{Q^{2}}$ relative resolution is approximately $2.5 \%$.

The ( $x, Q^{2}$ )-bins used for the determination of the structure function are shown in Fig. 3. The numbers of events in each of the bins are given in Table 1. The bins were chosen commensurate with the resolutions. At large $Q^{2}$ and also at low $y$, larger bin sizes were chosen to obtain adequate statistics in each bin and to minimise bin-to-bin migrations as a result of the non-Gaussian tails. Further-

Table 1. Results for the reduced cross section and the structure function $F_{2}^{e m}$. The relative corrections in the last three columns are defined in Sect. 10.1

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{array}$ | $x$ | $y$ | $N_{\text {data }}$ | $N_{p h p}$ | $\tilde{\sigma}^{e^{+} p}$ | $\begin{gathered} \delta_{\text {stat }} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{s y s} \\ (\%) \end{gathered}$ | $F_{2}^{e m}$ | $\begin{gathered} \Delta_{\text {all }} \\ (\%) \end{gathered}$ | $\begin{gathered} \Delta_{F_{2}} \\ (\%) \end{gathered}$ | $\begin{array}{r} \Delta_{x F_{3}} \\ (\%) \end{array}$ | $\Delta_{F_{L}}$ <br> (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.7 | $6.32 \cdot 10^{-5}$ | 0.474 | 9506 | 769 | 0.832 | 1.7 | 4.7 | 0.869 | -4.3 | 0.0 | 0.0 | -4.3 |
| 2.7 | $1.61 \cdot 10^{-4}$ | 0.186 | 21489 | 229 | 0.740 | 1.0 | 2.4 | 0.744 | -0.6 | 0.0 | 0.0 | -0.6 |
| 2.7 | $4.00 \cdot 10^{-4}$ | 0.075 | 18716 | 47 | 0.649 | 1.1 | 3.9 | 0.650 | -0.1 | 0.0 | 0.0 | -0.1 |
| 2.7 | $6.32 \cdot 10^{-4}$ | 0.047 | 8745 | 13 | 0.607 | 1.5 | 4.3 | 0.607 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2.7 | $2.00 \cdot 10^{-3}$ | 0.015 | 23981 | 6 | 0.477 | 0.9 | 4.2 | 0.477 | 0.0 | 0.0 | 0.0 | 0.0 |
| 3.5 | $6.32 \cdot 10^{-5}$ | 0.614 | 5960 | 1422 | 1.076 | 2.7 | 12.7 | 1.177 | -8.6 | 0.0 | 0.0 | -8.6 |
| 3.5 | $1.02 \cdot 10^{-4}$ | 0.380 | 6859 | 354 | 0.951 | 2.0 | 3.6 | 0.979 | -2.9 | 0.0 | 0.0 | -2.9 |
| 3.5 | $1.61 \cdot 10^{-4}$ | 0.241 | 5818 | 62 | 0.882 | 2.0 | 2.5 | 0.891 | -1.0 | 0.0 | 0.0 | -1.0 |
| 3.5 | $2.53 \cdot 10^{-4}$ | 0.153 | 6258 | 32 | 0.829 | 1.9 | 2.4 | 0.833 | -0.4 | 0.0 | 0.0 | -0.4 |
| 3.5 | $4.00 \cdot 10^{-4}$ | 0.097 | 5676 | 0 | 0.787 | 1.9 | 2.7 | 0.788 | -0.1 | 0.0 | 0.0 | -0.2 |
| 3.5 | $6.32 \cdot 10^{-4}$ | 0.061 | 5619 | 3 | 0.700 | 1.9 | 3.0 | 0.701 | -0.1 | 0.0 | 0.0 | -0.1 |
| 3.5 | $2.00 \cdot 10^{-3}$ | 0.019 | 25800 | 11 | 0.554 | 0.9 | 2.5 | 0.554 | 0.0 | 0.0 | 0.0 | 0.0 |
| 3.5 | $8.00 \cdot 10^{-3}$ | 0.005 | 17413 | 0 | 0.472 | 1.0 | 6.3 | 0.472 | 0.0 | 0.0 | 0.0 | 0.0 |
| 4.5 | $1.02 \cdot 10^{-4}$ | 0.489 | 6886 | 824 | 1.125 | 2.0 | 5.8 | 1.189 | -5.4 | 0.0 | 0.0 | -5.4 |
| 4.5 | $1.61 \cdot 10^{-4}$ | 0.310 | 5020 | 107 | 0.994 | 2.0 | 2.2 | 1.013 | -1.9 | 0.0 | 0.0 | -1.9 |
| 4.5 | $2.53 \cdot 10^{-4}$ | 0.197 | 4476 | 10 | 0.923 | 2.0 | 2.6 | 0.929 | -0.7 | 0.0 | 0.0 | -0.7 |
| 4.5 | $4.00 \cdot 10^{-4}$ | 0.125 | 3877 | 1 | 0.882 | 2.2 | 2.4 | 0.884 | -0.3 | 0.0 | 0.0 | -0.3 |
| 4.5 | $6.32 \cdot 10^{-4}$ | 0.079 | 3708 | 5 | 0.803 | 2.2 | 2.0 | 0.804 | -0.1 | 0.0 | 0.0 | -0.1 |
| 6.5 | $1.02 \cdot 10^{-4}$ | 0.706 | 6501 | 1382 | 1.210 | 2.4 | 10.2 | 1.392 | -13.1 | 0.0 | 0.0 | -13.1 |
| 6.5 | $1.61 \cdot 10^{-4}$ | 0.448 | 9445 | 732 | 1.159 | 1.5 | 3.6 | 1.213 | -4.4 | 0.0 | 0.0 | -4.4 |
| 6.5 | $2.53 \cdot 10^{-4}$ | 0.285 | 9279 | 159 | 1.078 | 1.5 | 1.9 | 1.094 | -1.5 | 0.1 | 0.0 | -1.6 |
| 6.5 | $4.00 \cdot 10^{-4}$ | 0.180 | 7217 | 33 | 0.972 | 1.5 | 1.8 | 0.977 | -0.6 | 0.0 | 0.0 | -0.6 |
| 6.5 | $6.32 \cdot 10^{-4}$ | 0.114 | 6435 | 7 | 0.898 | 1.6 | 3.1 | 0.900 | -0.2 | 0.0 | 0.0 | -0.2 |
| 6.5 | $1.02 \cdot 10^{-3}$ | 0.071 | 6100 | 7 | 0.828 | 1.6 | 2.1 | 0.828 | -0.1 | 0.0 | 0.0 | -0.1 |
| 6.5 | $1.61 \cdot 10^{-3}$ | 0.045 | 4958 | 11 | 0.740 | 1.8 | 2.5 | 0.740 | 0.0 | 0.0 | 0.0 | 0.0 |
| 6.5 | $2.53 \cdot 10^{-3}$ | 0.028 | 9848 | 1 | 0.666 | 1.2 | 1.9 | 0.666 | 0.0 | 0.0 | 0.0 | 0.0 |
| 6.5 | $5.00 \cdot 10^{-3}$ | 0.014 | 17517 | 0 | 0.576 | 0.9 | 4.1 | 0.576 | 0.0 | 0.0 | 0.0 | 0.0 |
| 6.5 | $2.10 \cdot 10^{-2}$ | 0.003 | 11829 | 0 | 0.487 | 1.2 | 8.7 | 0.487 | 0.0 | 0.0 | 0.0 | 0.0 |
| 8.5 | $1.61 \cdot 10^{-4}$ | 0.585 | 5783 | 1219 | 1.212 | 2.5 | 9.9 | 1.323 | -8.4 | 0.0 | 0.0 | -8.3 |
| 8.5 | $2.53 \cdot 10^{-4}$ | 0.372 | 7175 | 321 | 1.188 | 1.6 | 2.7 | 1.223 | -2.8 | 0.0 | 0.0 | -2.9 |
| 8.5 | $4.00 \cdot 10^{-4}$ | 0.236 | 6460 | 111 | 1.067 | 1.6 | 2.0 | 1.078 | -1.0 | 0.0 | 0.0 | -1.0 |
| 8.5 | $6.32 \cdot 10^{-4}$ | 0.149 | 6028 | 24 | 0.973 | 1.6 | 2.8 | 0.977 | -0.4 | 0.0 | 0.0 | -0.4 |
| 8.5 | $1.02 \cdot 10^{-3}$ | 0.092 | 5597 | 16 | 0.875 | 1.6 | 1.6 | 0.877 | -0.1 | 0.0 | 0.0 | -0.1 |
| 8.5 | $1.61 \cdot 10^{-3}$ | 0.059 | 4719 | 8 | 0.813 | 1.8 | 1.6 | 0.813 | 0.0 | 0.0 | 0.0 | -0.1 |
| 10 | $1.61 \cdot 10^{-4}$ | 0.689 | 2222 | 570 | 1.230 | 4.2 | 12.8 | 1.400 | -12.1 | 0.1 | 0.0 | -12.2 |
| 10 | $2.53 \cdot 10^{-4}$ | 0.438 | 4923 | 485 | 1.237 | 2.1 | 4.5 | 1.291 | -4.2 | 0.0 | 0.0 | -4.1 |
| 10 | $4.00 \cdot 10^{-4}$ | 0.277 | 4588 | 125 | 1.099 | 1.9 | 2.3 | 1.115 | -1.4 | 0.0 | 0.0 | -1.4 |
| 10 | $6.32 \cdot 10^{-4}$ | 0.175 | 4665 | 23 | 1.016 | 1.8 | 1.9 | 1.021 | -0.5 | 0.0 | 0.0 | -0.5 |
| 10 | $1.02 \cdot 10^{-3}$ | 0.109 | 4575 | 1 | 0.934 | 1.8 | 1.9 | 0.935 | -0.2 | 0.0 | 0.0 | -0.2 |
| 10 | $1.61 \cdot 10^{-3}$ | 0.069 | 3729 | 4 | 0.851 | 2.1 | 2.0 | 0.852 | -0.1 | 0.0 | 0.0 | -0.1 |
| 10 | $2.53 \cdot 10^{-3}$ | 0.044 | 3879 | 5 | 0.761 | 2.0 | 2.4 | 0.761 | 0.0 | 0.0 | 0.0 | 0.0 |
| 10 | $5.00 \cdot 10^{-3}$ | 0.022 | 11983 | 4 | 0.624 | 1.1 | 2.9 | 0.624 | 0.0 | 0.0 | 0.0 | 0.0 |
| 10 | $2.10 \cdot 10^{-2}$ | 0.005 | 12719 | 0 | 0.503 | 1.1 | 6.8 | 0.503 | 0.0 | 0.0 | 0.0 | 0.0 |
| 12 | $2.53 \cdot 10^{-4}$ | 0.526 | 3380 | 557 | 1.322 | 2.6 | 7.5 | 1.410 | -6.3 | 0.0 | 0.0 | -6.2 |
| 12 | $4.00 \cdot 10^{-4}$ | 0.333 | 3427 | 169 | 1.191 | 2.3 | 2.7 | 1.217 | -2.2 | 0.0 | 0.0 | -2.1 |
| 12 | $6.32 \cdot 10^{-4}$ | 0.211 | 3485 | 32 | 1.070 | 2.1 | 2.3 | 1.078 | -0.7 | 0.0 | 0.0 | -0.8 |
| 12 | $1.02 \cdot 10^{-3}$ | 0.130 | 3402 | 4 | 0.983 | 2.0 | 1.9 | 0.985 | -0.3 | 0.0 | 0.0 | -0.3 |
| 12 | $1.61 \cdot 10^{-3}$ | 0.083 | 2929 | 6 | 0.899 | 2.2 | 1.6 | 0.900 | -0.1 | 0.0 | 0.0 | -0.1 |
| 12 | $2.53 \cdot 10^{-3}$ | 0.053 | 2915 | 1 | 0.793 | 2.1 | 1.9 | 0.794 | 0.0 | 0.0 | 0.0 | 0.0 |
| 15 | $2.53 \cdot 10^{-4}$ | 0.657 | 2479 | 400 | 1.397 | 2.9 | 7.6 | 1.559 | -10.4 | 0.1 | 0.0 | -10.5 |
| 15 | $4.00 \cdot 10^{-4}$ | 0.416 | 3486 | 255 | 1.258 | 2.3 | 3.7 | 1.303 | -3.5 | 0.0 | 0.0 | -3.5 |
| 15 | $6.32 \cdot 10^{-4}$ | 0.263 | 3736 | 79 | 1.137 | 2.0 | 2.2 | 1.151 | -1.2 | 0.0 | 0.0 | -1.2 |
| 15 | $1.02 \cdot 10^{-3}$ | 0.163 | 3779 | 15 | 1.055 | 1.9 | 1.9 | 1.059 | -0.4 | 0.0 | 0.0 | -0.4 |
| 15 | $1.61 \cdot 10^{-3}$ | 0.103 | 3126 | 4 | 0.944 | 2.2 | 1.6 | 0.945 | -0.1 | 0.0 | 0.0 | -0.2 |

Table 1. (continued)

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \\ \hline \end{array}$ | $x$ | $y$ | $N_{\text {data }}$ | $N_{p h p}$ | $\tilde{\sigma}^{e^{+} p}$ | $\begin{gathered} \delta_{\text {stat }} \\ (\%) \\ \hline \end{gathered}$ | $\begin{gathered} \delta_{\text {sys }} \\ (\%) \\ \hline \end{gathered}$ | $F_{2}^{e m}$ | $\begin{gathered} \Delta_{\text {all }} \\ (\%) \\ \hline \end{gathered}$ | $\begin{gathered} \Delta_{F_{2}} \\ (\%) \\ \hline \end{gathered}$ | $\begin{array}{r} \Delta_{x F_{3}} \\ (\%) \\ \hline \end{array}$ | $\begin{gathered} \Delta_{F_{L}} \\ (\%) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 15 | $2.53 \cdot 10^{-3}$ | 0.066 | 3182 | 0 | 0.836 | 2.1 | 1.6 | 0.837 | 0.0 | 0.0 | 0.0 | -0.1 |
| 15 | $5.00 \cdot 10^{-3}$ | 0.033 | 5672 | 3 | 0.706 | 1.5 | 1.5 | 0.706 | 0.0 | 0.0 | 0.0 | 0.0 |
| 15 | $2.10 \cdot 10^{-2}$ | 0.008 | 13289 | 0 | 0.519 | 1.0 | 6.9 | 0.519 | 0.0 | 0.0 | 0.0 | 0.0 |
| 18 | $5.00 \cdot 10^{-4}$ | 0.399 | 4653 | 424 | 1.284 | 1.9 | 4.1 | 1.325 | -3.1 | 0.0 | 0.0 | -3.1 |
| 18 | $8.00 \cdot 10^{-4}$ | 0.249 | 3646 | 53 | 1.157 | 2.2 | 2.4 | 1.169 | -1.0 | 0.0 | 0.0 | -1.0 |
| 18 | $1.30 \cdot 10^{-3}$ | 0.154 | 3571 | 8 | 1.038 | 2.0 | 2.2 | 1.041 | -0.3 | 0.0 | 0.0 | -0.3 |
| 18 | $2.10 \cdot 10^{-3}$ | 0.095 | 2475 | 3 | 0.929 | 2.3 | 1.6 | 0.930 | -0.1 | 0.0 | 0.0 | -0.1 |
| 18 | $3.20 \cdot 10^{-3}$ | 0.062 | 2421 | 5 | 0.851 | 2.4 | 1.8 | 0.851 | 0.0 | 0.0 | 0.0 | 0.0 |
| 18 | $5.00 \cdot 10^{-3}$ | 0.040 | 4363 | 1 | 0.739 | 1.7 | 1.7 | 0.739 | 0.0 | 0.0 | 0.0 | 0.0 |
| 22 | $5.00 \cdot 10^{-4}$ | 0.488 | 2430 | 184 | 1.357 | 2.5 | 4.0 | 1.425 | -4.8 | 0.1 | 0.0 | -4.8 |
| 22 | $8.00 \cdot 10^{-4}$ | 0.305 | 2884 | 51 | 1.267 | 2.2 | 1.9 | 1.287 | -1.6 | 0.0 | 0.0 | -1.6 |
| 22 | $1.30 \cdot 10^{-3}$ | 0.188 | 2830 | 15 | 1.065 | 2.3 | 2.3 | 1.071 | -0.6 | 0.0 | 0.0 | -0.5 |
| 22 | $2.10 \cdot 10^{-3}$ | 0.116 | 2033 | 2 | 1.001 | 2.7 | 2.0 | 1.003 | -0.2 | 0.0 | 0.0 | -0.2 |
| 22 | $3.20 \cdot 10^{-3}$ | 0.076 | 1986 | 5 | 0.905 | 2.6 | 1.7 | 0.905 | -0.1 | 0.0 | 0.0 | -0.1 |
| 22 | $5.00 \cdot 10^{-3}$ | 0.049 | 2268 | 1 | 0.771 | 2.4 | 1.7 | 0.771 | 0.0 | 0.0 | 0.0 | 0.0 |
| 22 | $8.00 \cdot 10^{-3}$ | 0.030 | 2161 | 0 | 0.656 | 2.4 | 1.7 | 0.656 | 0.0 | 0.0 | 0.0 | 0.0 |
| 22 | $1.30 \cdot 10^{-2}$ | 0.019 | 3398 | 0 | 0.581 | 2.0 | 5.6 | 0.581 | 0.0 | 0.0 | 0.0 | 0.0 |
| 22 | $3.20 \cdot 10^{-2}$ | 0.008 | 4541 | 0 | 0.512 | 1.8 | 6.9 | 0.512 | 0.0 | 0.0 | 0.0 | 0.0 |
| 27 | $5.00 \cdot 10^{-4}$ | 0.599 | 1492 | 99 | 1.497 | 3.3 | 4.0 | 1.620 | -7.6 | 0.0 | 0.0 | -7.6 |
| 27 | $8.00 \cdot 10^{-4}$ | 0.374 | 2460 | 58 | 1.294 | 2.5 | 2.2 | 1.325 | -2.4 | 0.0 | 0.0 | -2.4 |
| 27 | $1.30 \cdot 10^{-3}$ | 0.230 | 2639 | 16 | 1.181 | 2.3 | 2.7 | 1.190 | -0.8 | 0.0 | 0.0 | -0.8 |
| 27 | $2.10 \cdot 10^{-3}$ | 0.143 | 1910 | 4 | 1.095 | 2.8 | 1.7 | 1.098 | -0.3 | 0.0 | 0.0 | -0.3 |
| 27 | $3.20 \cdot 10^{-3}$ | 0.094 | 1699 | 2 | 0.901 | 2.7 | 2.1 | 0.902 | -0.1 | 0.0 | 0.0 | -0.1 |
| 27 | $5.00 \cdot 10^{-3}$ | 0.060 | 2016 | 0 | 0.804 | 2.5 | 2.5 | 0.804 | 0.0 | 0.0 | 0.0 | 0.0 |
| 27 | $8.00 \cdot 10^{-3}$ | 0.037 | 1997 | 0 | 0.731 | 2.6 | 2.7 | 0.731 | 0.0 | 0.0 | 0.0 | 0.0 |
| 27 | $1.30 \cdot 10^{-2}$ | 0.023 | 3210 | 0 | 0.634 | 2.0 | 2.9 | 0.634 | 0.0 | 0.0 | 0.0 | 0.0 |
| 35 | $8.00 \cdot 10^{-4}$ | 0.485 | 23836 | 950 | 1.404 | 1.5 | 2.9 | 1.467 | -4.3 | 0.1 | 0.0 | -4.3 |
| 35 | $1.30 \cdot 10^{-3}$ | 0.298 | 25024 | 204 | 1.225 | 1.4 | 1.8 | 1.241 | -1.3 | 0.0 | 0.0 | -1.3 |
| 35 | $2.10 \cdot 10^{-3}$ | 0.185 | 17347 | 15 | 1.108 | 1.6 | 1.6 | 1.113 | -0.5 | 0.0 | 0.0 | -0.4 |
| 35 | $3.20 \cdot 10^{-3}$ | 0.121 | 16475 | 5 | 0.961 | 1.7 | 1.6 | 0.963 | -0.2 | 0.0 | 0.0 | -0.2 |
| 35 | $5.00 \cdot 10^{-3}$ | 0.078 | 19271 | 13 | 0.864 | 1.6 | 1.5 | 0.865 | 0.0 | 0.0 | 0.0 | -0.1 |
| 35 | $8.00 \cdot 10^{-3}$ | 0.049 | 18246 | 0 | 0.727 | 1.6 | 1.8 | 0.727 | 0.0 | 0.0 | 0.0 | 0.0 |
| 35 | $1.30 \cdot 10^{-2}$ | 0.030 | 13171 | 0 | 0.646 | 1.9 | 2.2 | 0.646 | 0.0 | 0.0 | 0.0 | 0.0 |
| 35 | $2.10 \cdot 10^{-2}$ | 0.018 | 14507 | 0 | 0.555 | 1.8 | 2.8 | 0.555 | 0.0 | 0.0 | 0.0 | 0.0 |
| 35 | $3.20 \cdot 10^{-2}$ | 0.012 | 32311 | 0 | 0.496 | 1.1 | 7.6 | 0.496 | 0.0 | 0.0 | 0.0 | 0.0 |
| 45 | $8.00 \cdot 10^{-4}$ | 0.624 | 13446 | 697 | 1.482 | 1.8 | 3.6 | 1.604 | -7.6 | 0.0 | 0.0 | -7.5 |
| 45 | $1.30 \cdot 10^{-3}$ | 0.384 | 20011 | 245 | 1.297 | 1.3 | 2.1 | 1.327 | -2.2 | 0.1 | 0.0 | -2.3 |
| 45 | $2.10 \cdot 10^{-3}$ | 0.238 | 14091 | 56 | 1.131 | 1.7 | 1.7 | 1.139 | -0.7 | 0.0 | 0.0 | -0.7 |
| 45 | $3.20 \cdot 10^{-3}$ | 0.156 | 13613 | 5 | 0.998 | 1.5 | 1.5 | 1.001 | -0.3 | 0.0 | 0.0 | -0.3 |
| 45 | $5.00 \cdot 10^{-3}$ | 0.100 | 15984 | 2 | 0.909 | 1.4 | 1.3 | 0.910 | -0.1 | 0.0 | 0.0 | -0.1 |
| 45 | $8.00 \cdot 10^{-3}$ | 0.062 | 15340 | 0 | 0.777 | 1.4 | 1.5 | 0.777 | 0.0 | 0.0 | 0.0 | 0.0 |
| 45 | $1.30 \cdot 10^{-2}$ | 0.038 | 10876 | 0 | 0.661 | 1.7 | 1.7 | 0.661 | 0.0 | 0.0 | 0.0 | 0.0 |
| 45 | $2.10 \cdot 10^{-2}$ | 0.024 | 11920 | 0 | 0.587 | 1.7 | 2.1 | 0.587 | 0.0 | 0.0 | 0.0 | 0.0 |
| 60 | $1.30 \cdot 10^{-3}$ | 0.512 | 18040 | 373 | 1.387 | 1.3 | 2.2 | 1.450 | -4.3 | 0.0 | 0.0 | -4.3 |
| 60 | $2.10 \cdot 10^{-3}$ | 0.317 | 13355 | 143 | 1.213 | 1.5 | 1.8 | 1.230 | -1.3 | 0.0 | 0.0 | -1.4 |
| 60 | $3.20 \cdot 10^{-3}$ | 0.208 | 13038 | 31 | 1.074 | 1.5 | 1.5 | 1.079 | -0.5 | 0.1 | 0.0 | -0.5 |
| 60 | $5.00 \cdot 10^{-3}$ | 0.133 | 15083 | 6 | 0.900 | 1.4 | 1.5 | 0.902 | -0.2 | 0.0 | 0.0 | -0.2 |
| 60 | $8.00 \cdot 10^{-3}$ | 0.083 | 14371 | 0 | 0.809 | 1.4 | 1.5 | 0.809 | 0.0 | 0.0 | 0.0 | -0.1 |
| 60 | $1.30 \cdot 10^{-2}$ | 0.051 | 10381 | 0 | 0.689 | 1.7 | 1.9 | 0.689 | 0.0 | 0.0 | 0.0 | 0.0 |
| 60 | $2.10 \cdot 10^{-2}$ | 0.032 | 11570 | 0 | 0.598 | 1.6 | 2.4 | 0.598 | 0.0 | 0.1 | 0.0 | 0.0 |
| 60 | $3.20 \cdot 10^{-2}$ | 0.021 | 8000 | 0 | 0.537 | 2.0 | 3.3 | 0.536 | 0.0 | 0.0 | 0.0 | 0.0 |
| 60 | $5.00 \cdot 10^{-2}$ | 0.013 | 14942 | 0 | 0.512 | 1.5 | 6.8 | 0.512 | 0.0 | 0.0 | 0.0 | 0.0 |
| 70 | $1.30 \cdot 10^{-3}$ | 0.597 | 9308 | 144 | 1.438 | 1.8 | 2.2 | 1.531 | -6.1 | 0.1 | 0.0 | -6.1 |
| 70 | $2.10 \cdot 10^{-3}$ | 0.370 | 10323 | 142 | 1.238 | 1.7 | 1.9 | 1.262 | -1.9 | 0.0 | 0.0 | -1.9 |
| 70 | $3.20 \cdot 10^{-3}$ | 0.243 | 9986 | 17 | 1.087 | 1.7 | 1.6 | 1.094 | -0.6 | 0.1 | 0.0 | -0.7 |

Table 1. (continued)

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \\ \hline \end{array}$ | $x$ | $y$ | $N_{\text {data }}$ | $N_{p h p}$ | $\tilde{\sigma}^{e^{+} p}$ | $\begin{array}{r} \delta_{\text {stat }} \\ (\%) \end{array}$ | $\begin{gathered} \delta_{\text {sys }} \\ (\%) \end{gathered}$ | $F_{2}^{e m}$ | $\begin{array}{r} \Delta_{\text {all }} \\ (\%) \end{array}$ | $\begin{gathered} \Delta_{F_{2}} \\ (\%) \end{gathered}$ | $\begin{array}{r} \Delta_{x F_{3}} \\ (\%) \end{array}$ | $\begin{gathered} \Delta_{F_{L}} \\ (\%) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 70 | $5.00 \cdot 10^{-3}$ | 0.155 | 11939 | 2 | 0.984 | 1.5 | 1.5 | 0.986 | -0.2 | 0.0 | 0.0 | -0.2 |
| 70 | $8.00 \cdot 10^{-3}$ | 0.097 | 11402 | 0 | 0.815 | 1.6 | 1.4 | 0.815 | 0.0 | 0.0 | 0.0 | -0.1 |
| 70 | $1.30 \cdot 10^{-2}$ | 0.060 | 8257 | 0 | 0.708 | 1.9 | 1.7 | 0.708 | 0.0 | 0.0 | 0.0 | 0.0 |
| 70 | $2.10 \cdot 10^{-2}$ | 0.037 | 9567 | 0 | 0.620 | 1.8 | 1.6 | 0.620 | 0.0 | 0.0 | 0.0 | 0.0 |
| 70 | $3.20 \cdot 10^{-2}$ | 0.024 | 6805 | 0 | 0.557 | 2.1 | 2.8 | 0.557 | 0.0 | 0.0 | 0.0 | 0.0 |
| 70 | $5.00 \cdot 10^{-2}$ | 0.016 | 15253 | 0 | 0.500 | 1.6 | 6.2 | 0.500 | 0.0 | 0.0 | 0.0 | 0.0 |
| 90 | $2.10 \cdot 10^{-3}$ | 0.475 | 6476 | 83 | 1.236 | 2.0 | 2.2 | 1.277 | -3.2 | 0.1 | 0.0 | -3.3 |
| 90 | $3.20 \cdot 10^{-3}$ | 0.312 | 7362 | 10 | 1.168 | 1.9 | 1.7 | 1.182 | -1.1 | 0.0 | 0.0 | -1.2 |
| 90 | $5.00 \cdot 10^{-3}$ | 0.200 | 8940 | 0 | 1.029 | 1.7 | 1.7 | 1.032 | -0.4 | 0.1 | 0.0 | -0.4 |
| 90 | $8.00 \cdot 10^{-3}$ | 0.125 | 8642 | 2 | 0.896 | 1.8 | 1.5 | 0.897 | -0.1 | 0.1 | 0.0 | -0.1 |
| 90 | $1.30 \cdot 10^{-2}$ | 0.077 | 6311 | 0 | 0.749 | 2.1 | 1.6 | 0.749 | 0.0 | 0.1 | 0.0 | 0.0 |
| 90 | $2.10 \cdot 10^{-2}$ | 0.048 | 7083 | 0 | 0.622 | 1.9 | 1.9 | 0.621 | 0.0 | 0.1 | 0.0 | 0.0 |
| 90 | $3.20 \cdot 10^{-2}$ | 0.031 | 4891 | 1 | 0.535 | 2.3 | 2.9 | 0.535 | 0.0 | 0.1 | 0.0 | 0.0 |
| 90 | $5.00 \cdot 10^{-2}$ | 0.020 | 5982 | 0 | 0.484 | 2.1 | 6.3 | 0.484 | 0.0 | 0.1 | 0.0 | 0.0 |
| 90 | $8.00 \cdot 10^{-2}$ | 0.012 | 7223 | 0 | 0.435 | 2.0 | 2.8 | 0.435 | 0.0 | 0.0 | 0.0 | 0.0 |
| 90 | $1.80 \cdot 10^{-1}$ | 0.006 | 2981 | 0 | 0.340 | 3.1 | 5.3 | 0.339 | 0.1 | 0.1 | 0.0 | 0.0 |
| 120 | $2.10 \cdot 10^{-3}$ | 0.634 | 3273 | 2 | 1.317 | 2.2 | 2.5 | 1.404 | -6.2 | 0.1 | 0.0 | -6.2 |
| 120 | $3.20 \cdot 10^{-3}$ | 0.416 | 4812 | 13 | 1.173 | 1.9 | 2.0 | 1.198 | -2.1 | 0.1 | 0.0 | -2.2 |
| 120 | $5.00 \cdot 10^{-3}$ | 0.266 | 6066 | 13 | 1.038 | 1.7 | 1.8 | 1.045 | -0.7 | 0.1 | 0.0 | -0.7 |
| 120 | $8.00 \cdot 10^{-3}$ | 0.166 | 6203 | 0 | 0.908 | 1.8 | 1.4 | 0.910 | -0.2 | 0.1 | 0.0 | -0.2 |
| 120 | $1.30 \cdot 10^{-2}$ | 0.102 | 4477 | 0 | 0.760 | 2.0 | 1.8 | 0.760 | 0.0 | 0.1 | 0.0 | -0.1 |
| 120 | $2.10 \cdot 10^{-2}$ | 0.063 | 5225 | 0 | 0.664 | 1.9 | 1.8 | 0.664 | 0.0 | 0.1 | 0.0 | 0.0 |
| 120 | $3.20 \cdot 10^{-2}$ | 0.042 | 3712 | 0 | 0.579 | 2.3 | 2.2 | 0.579 | 0.1 | 0.1 | 0.0 | 0.0 |
| 120 | $5.00 \cdot 10^{-2}$ | 0.027 | 4133 | 0 | 0.472 | 2.1 | 5.1 | 0.472 | 0.1 | 0.1 | 0.0 | 0.0 |
| 120 | $8.00 \cdot 10^{-2}$ | 0.017 | 5409 | 0 | 0.437 | 1.9 | 4.7 | 0.436 | 0.0 | 0.1 | 0.0 | 0.0 |
| 120 | $1.80 \cdot 10^{-1}$ | 0.007 | 2653 | 0 | 0.328 | 2.7 | 8.8 | 0.328 | 0.1 | 0.1 | 0.0 | 0.0 |
| 150 | $3.20 \cdot 10^{-3}$ | 0.520 | 3655 | 7 | 1.194 | 2.1 | 2.3 | 1.238 | -3.6 | 0.1 | -0.1 | -3.6 |
| 150 | $5.00 \cdot 10^{-3}$ | 0.333 | 4977 | 0 | 1.037 | 1.9 | 1.8 | 1.049 | -1.1 | 0.1 | -0.1 | -1.2 |
| 150 | $8.00 \cdot 10^{-3}$ | 0.208 | 5111 | 0 | 0.887 | 1.8 | 1.6 | 0.890 | -0.3 | 0.1 | 0.0 | -0.4 |
| 150 | $1.30 \cdot 10^{-2}$ | 0.128 | 3912 | 0 | 0.805 | 2.1 | 2.0 | 0.805 | -0.1 | 0.1 | 0.0 | -0.1 |
| 150 | $2.10 \cdot 10^{-2}$ | 0.079 | 4382 | 0 | 0.661 | 2.0 | 1.9 | 0.660 | 0.0 | 0.1 | 0.0 | 0.0 |
| 150 | $3.20 \cdot 10^{-2}$ | 0.052 | 3144 | 0 | 0.574 | 2.4 | 1.7 | 0.573 | 0.1 | 0.1 | 0.0 | 0.0 |
| 150 | $5.00 \cdot 10^{-2}$ | 0.033 | 3622 | 0 | 0.515 | 2.2 | 2.3 | 0.515 | 0.1 | 0.1 | 0.0 | 0.0 |
| 150 | $8.00 \cdot 10^{-2}$ | 0.021 | 4716 | 0 | 0.431 | 2.0 | 5.0 | 0.431 | 0.1 | 0.1 | 0.0 | 0.0 |
| 150 | $1.80 \cdot 10^{-1}$ | 0.009 | 2580 | 0 | 0.326 | 2.7 | 6.4 | 0.326 | 0.1 | 0.1 | 0.0 | 0.0 |
| 200 | $3.20 \cdot 10^{-3}$ | 0.693 | 1197 | 2 | 1.162 | 3.8 | 3.2 | 1.248 | -6.9 | 0.2 | -0.1 | -6.9 |
| 200 | $5.00 \cdot 10^{-3}$ | 0.443 | 3151 | 0 | 1.065 | 2.3 | 2.1 | 1.087 | -2.1 | 0.2 | -0.1 | -2.2 |
| 200 | $8.00 \cdot 10^{-3}$ | 0.277 | 3478 | 0 | 0.899 | 2.2 | 1.8 | 0.905 | -0.6 | 0.2 | -0.1 | -0.7 |
| 200 | $1.30 \cdot 10^{-2}$ | 0.171 | 3451 | 0 | 0.794 | 2.2 | 1.5 | 0.795 | -0.1 | 0.1 | -0.1 | -0.2 |
| 200 | $2.10 \cdot 10^{-2}$ | 0.106 | 2248 | 0 | 0.659 | 2.7 | 1.8 | 0.659 | 0.0 | 0.1 | -0.1 | -0.1 |
| 200 | $3.20 \cdot 10^{-2}$ | 0.069 | 2142 | 0 | 0.570 | 2.8 | 1.9 | 0.570 | 0.1 | 0.1 | -0.1 | 0.0 |
| 200 | $5.00 \cdot 10^{-2}$ | 0.044 | 2393 | 0 | 0.508 | 2.8 | 2.4 | 0.508 | 0.1 | 0.1 | 0.0 | 0.0 |
| 200 | $8.00 \cdot 10^{-2}$ | 0.028 | 3124 | 0 | 0.434 | 2.4 | 2.7 | 0.433 | 0.1 | 0.1 | 0.0 | 0.0 |
| 200 | $1.80 \cdot 10^{-1}$ | 0.012 | 2161 | 0 | 0.340 | 2.9 | 3.7 | 0.339 | 0.1 | 0.1 | 0.0 | 0.0 |
| 250 | $5.00 \cdot 10^{-3}$ | 0.554 | 1971 | 0 | 1.066 | 3.0 | 2.9 | 1.105 | -3.6 | 0.3 | -0.2 | -3.7 |
| 250 | $8.00 \cdot 10^{-3}$ | 0.346 | 2446 | 0 | 0.927 | 2.6 | 1.7 | 0.937 | -1.1 | 0.2 | -0.1 | -1.1 |
| 250 | $1.30 \cdot 10^{-2}$ | 0.213 | 2634 | 0 | 0.821 | 2.7 | 1.8 | 0.824 | -0.3 | 0.2 | -0.1 | -0.3 |
| 250 | $2.10 \cdot 10^{-2}$ | 0.132 | 1644 | 0 | 0.644 | 3.5 | 1.8 | 0.644 | 0.0 | 0.2 | -0.1 | -0.1 |
| 250 | $3.20 \cdot 10^{-2}$ | 0.087 | 1646 | 0 | 0.612 | 3.4 | 2.3 | 0.612 | 0.1 | 0.2 | -0.1 | 0.0 |
| 250 | $5.00 \cdot 10^{-2}$ | 0.055 | 1838 | 0 | 0.517 | 3.2 | 1.8 | 0.516 | 0.1 | 0.2 | -0.1 | 0.0 |
| 250 | $8.00 \cdot 10^{-2}$ | 0.035 | 2365 | 0 | 0.445 | 2.8 | 5.1 | 0.445 | 0.1 | 0.2 | -0.1 | 0.0 |
| 250 | $1.80 \cdot 10^{-1}$ | 0.015 | 1729 | 0 | 0.341 | 3.2 | 5.6 | 0.340 | 0.1 | 0.2 | 0.0 | 0.0 |
| 350 | $5.00 \cdot 10^{-3}$ | 0.776 | 810 | 0 | 1.103 | 4.4 | 3.5 | 1.196 | -7.8 | 0.3 | -0.3 | -7.8 |
| 350 | $8.00 \cdot 10^{-3}$ | 0.485 | 1924 | 2 | 0.988 | 2.9 | 2.1 | 1.011 | -2.3 | 0.3 | -0.3 | -2.4 |
| 350 | $1.30 \cdot 10^{-2}$ | 0.298 | 2037 | 0 | 0.824 | 2.9 | 1.7 | 0.829 | -0.6 | 0.3 | -0.2 | -0.7 |

Table 1. (continued)

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \\ \hline \end{array}$ | $x$ | $y$ | $N_{\text {data }}$ | $N_{p h p}$ | $\tilde{\sigma}^{e^{+} p}$ | $\delta_{s t a t}$ <br> (\%) | $\begin{gathered} \delta_{s y s} \\ (\%) \\ \hline \end{gathered}$ | $F_{2}^{e m}$ | $\begin{gathered} \Delta_{\text {all }} \\ (\%) \end{gathered}$ | $\begin{gathered} \Delta_{F_{2}} \\ (\%) \end{gathered}$ | $\begin{array}{r} \Delta_{x F_{3}} \\ (\%) \\ \hline \end{array}$ | $\begin{gathered} \Delta_{F_{L}} \\ (\%) \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 350 | $2.10 \cdot 10^{-2}$ | 0.185 | 1355 | 0 | 0.679 | 3.6 | 1.9 | 0.680 | -0.1 | 0.3 | -0.2 | -0.2 |
| 350 | $3.20 \cdot 10^{-2}$ | 0.121 | 1329 | 0 | 0.639 | 3.6 | 2.0 | 0.639 | 0.0 | 0.3 | -0.2 | -0.1 |
| 350 | $5.00 \cdot 10^{-2}$ | 0.078 | 1475 | 0 | 0.526 | 3.4 | 1.9 | 0.526 | 0.1 | 0.3 | -0.1 | 0.0 |
| 350 | $8.00 \cdot 10^{-2}$ | 0.049 | 1783 | 0 | 0.442 | 3.2 | 2.6 | 0.441 | 0.1 | 0.3 | -0.1 | 0.0 |
| 350 | $1.80 \cdot 10^{-1}$ | 0.022 | 1465 | 0 | 0.328 | 3.5 | 1.6 | 0.328 | 0.2 | 0.2 | -0.1 | 0.0 |
| 450 | $8.00 \cdot 10^{-3}$ | 0.624 | 1163 | 9 | 0.954 | 3.5 | 2.4 | 0.995 | -4.2 | 0.4 | -0.4 | -4.2 |
| 450 | $1.30 \cdot 10^{-2}$ | 0.384 | 1331 | 0 | 0.854 | 3.0 | 1.7 | 0.864 | -1.2 | 0.4 | -0.4 | -1.2 |
| 450 | $2.10 \cdot 10^{-2}$ | 0.238 | 932 | 0 | 0.702 | 3.6 | 1.6 | 0.704 | -0.3 | 0.4 | -0.3 | -0.3 |
| 450 | $3.20 \cdot 10^{-2}$ | 0.156 | 854 | 0 | 0.579 | 4.3 | 1.8 | 0.579 | 0.0 | 0.4 | -0.3 | -0.1 |
| 450 | $5.00 \cdot 10^{-2}$ | 0.100 | 1009 | 0 | 0.511 | 3.4 | 1.5 | 0.511 | 0.1 | 0.4 | -0.2 | 0.0 |
| 450 | $8.00 \cdot 10^{-2}$ | 0.062 | 909 | 0 | 0.439 | 3.6 | 2.3 | 0.439 | 0.2 | 0.4 | -0.2 | 0.0 |
| 450 | $1.30 \cdot 10^{-1}$ | 0.038 | 830 | 0 | 0.365 | 3.8 | 2.3 | 0.365 | 0.2 | 0.4 | -0.2 | 0.0 |
| 450 | $2.50 \cdot 10^{-1}$ | 0.020 | 588 | 0 | 0.264 | 4.5 | 7.7 | 0.263 | 0.2 | 0.3 | -0.1 | 0.0 |
| 650 | $8.00 \cdot 10^{-3}$ | 0.901 | 529 | 0 | 0.926 | 4.6 | 3.2 | 1.023 | -9.4 | 0.7 | -0.8 | -9.4 |
| 650 | $1.30 \cdot 10^{-2}$ | 0.554 | 1056 | 2 | 0.827 | 3.3 | 1.9 | 0.850 | -2.8 | 0.7 | -0.8 | -2.7 |
| 650 | $2.10 \cdot 10^{-2}$ | 0.343 | 792 | 0 | 0.722 | 3.8 | 1.7 | 0.728 | -0.8 | 0.7 | -0.7 | -0.8 |
| 650 | $3.20 \cdot 10^{-2}$ | 0.225 | 712 | 0 | 0.603 | 4.0 | 1.9 | 0.604 | -0.2 | 0.7 | -0.6 | -0.3 |
| 650 | $5.00 \cdot 10^{-2}$ | 0.144 | 751 | 0 | 0.475 | 4.3 | 2.1 | 0.475 | 0.1 | 0.7 | -0.5 | -0.1 |
| 650 | $8.00 \cdot 10^{-2}$ | 0.090 | 767 | 0 | 0.456 | 3.9 | 1.5 | 0.455 | 0.2 | 0.6 | -0.4 | 0.0 |
| 650 | $1.30 \cdot 10^{-1}$ | 0.055 | 641 | 0 | 0.357 | 4.2 | 3.2 | 0.356 | 0.3 | 0.6 | -0.3 | 0.0 |
| 650 | $2.50 \cdot 10^{-1}$ | 0.029 | 495 | 0 | 0.250 | 4.8 | 3.4 | 0.249 | 0.4 | 0.6 | -0.2 | 0.0 |
| 650 | $4.00 \cdot 10^{-1}$ | 0.018 | 132 | 0 | 0.152 | 9.5 | 14.5 | 0.151 | 0.3 | 0.4 | -0.1 | 0.0 |
| 800 | $1.30 \cdot 10^{-2}$ | 0.682 | 653 | 0 | 0.847 | 4.5 | 2.4 | 0.887 | -4.6 | 0.9 | -1.2 | -4.3 |
| 800 | $2.10 \cdot 10^{-2}$ | 0.422 | 436 | 0 | 0.665 | 7.5 | 1.6 | 0.675 | -1.3 | 0.9 | -1.0 | -1.2 |
| 800 | $3.20 \cdot 10^{-2}$ | 0.277 | 470 | 0 | 0.647 | 4.9 | 1.4 | 0.650 | -0.4 | 0.9 | -0.9 | -0.4 |
| 800 | $5.00 \cdot 10^{-2}$ | 0.177 | 449 | 0 | 0.451 | 5.0 | 2.3 | 0.451 | 0.0 | 0.9 | -0.8 | -0.1 |
| 800 | $8.00 \cdot 10^{-2}$ | 0.111 | 473 | 0 | 0.420 | 5.8 | 2.6 | 0.419 | 0.2 | 0.8 | -0.6 | 0.0 |
| 800 | $1.30 \cdot 10^{-1}$ | 0.068 | 425 | 0 | 0.370 | 5.2 | 2.5 | 0.369 | 0.3 | 0.8 | -0.5 | 0.0 |
| 800 | $2.50 \cdot 10^{-1}$ | 0.035 | 337 | 0 | 0.260 | 5.9 | 1.8 | 0.259 | 0.4 | 0.7 | -0.3 | 0.0 |
| 800 | $4.00 \cdot 10^{-1}$ | 0.022 | 88 | 0 | 0.114 | 11.4 | 13.5 | 0.114 | 0.4 | 0.7 | -0.2 | 0.0 |
| 1200 | $1.40 \cdot 10^{-2}$ | 0.950 | 376 | 0 | 0.872 | 5.5 | 2.9 | 0.960 | -9.2 | 1.6 | -2.1 | -8.7 |
| 1200 | $2.10 \cdot 10^{-2}$ | 0.634 | 442 | 0 | 0.681 | 5.1 | 1.8 | 0.707 | -3.7 | 1.6 | -2.3 | -3.0 |
| 1200 | $3.20 \cdot 10^{-2}$ | 0.416 | 449 | 0 | 0.563 | 5.0 | 2.0 | 0.571 | -1.4 | 1.6 | -2.0 | -1.0 |
| 1200 | $5.00 \cdot 10^{-2}$ | 0.266 | 525 | 0 | 0.490 | 4.6 | 1.6 | 0.493 | -0.4 | 1.5 | -1.7 | -0.3 |
| 1200 | $8.00 \cdot 10^{-2}$ | 0.166 | 546 | 0 | 0.434 | 5.3 | 1.5 | 0.434 | 0.0 | 1.5 | -1.4 | -0.1 |
| 1200 | $1.30 \cdot 10^{-1}$ | 0.102 | 460 | 0 | 0.336 | 5.8 | 3.1 | 0.335 | 0.3 | 1.4 | -1.1 | 0.0 |
| 1200 | $2.50 \cdot 10^{-1}$ | 0.053 | 392 | 0 | 0.254 | 5.4 | 2.0 | 0.253 | 0.6 | 1.3 | -0.7 | 0.0 |
| 1200 | $4.00 \cdot 10^{-1}$ | 0.033 | 176 | 0 | 0.149 | 8.1 | 10.3 | 0.148 | 0.7 | 1.2 | -0.5 | 0.0 |
| 1200 | $6.50 \cdot 10^{-1}$ | 0.020 | 9 | 0 | 0.021 | 35.6 | 25.1 | 0.021 | 0.7 | 1.0 | -0.3 | 0.0 |
| 1500 | $2.10 \cdot 10^{-2}$ | 0.792 | 206 | 0 | 0.666 | 7.9 | 6.2 | 0.710 | -6.1 | 2.2 | -3.3 | -5.1 |
| 1500 | $3.20 \cdot 10^{-2}$ | 0.520 | 260 | 0 | 0.540 | 10.3 | 2.0 | 0.554 | -2.5 | 2.2 | -3.1 | -1.6 |
| 1500 | $5.00 \cdot 10^{-2}$ | 0.333 | 320 | 0 | 0.518 | 5.8 | 1.8 | 0.523 | -1.0 | 2.1 | -2.6 | -0.5 |
| 1500 | $8.00 \cdot 10^{-2}$ | 0.208 | 338 | 0 | 0.476 | 5.7 | 2.6 | 0.477 | -0.2 | 2.0 | -2.1 | -0.1 |
| 1500 | $1.30 \cdot 10^{-1}$ | 0.128 | 216 | 0 | 0.301 | 11.8 | 1.9 | 0.300 | 0.3 | 1.9 | -1.6 | 0.0 |
| 1500 | $1.80 \cdot 10^{-1}$ | 0.092 | 202 | 0 | 0.313 | 7.3 | 5.9 | 0.311 | 0.5 | 1.8 | -1.3 | 0.0 |
| 1500 | $2.50 \cdot 10^{-1}$ | 0.067 | 146 | 0 | 0.270 | 8.6 | 3.0 | 0.268 | 0.7 | 1.7 | -1.0 | 0.0 |
| 1500 | $4.00 \cdot 10^{-1}$ | 0.042 | 69 | 0 | 0.146 | 12.6 | 13.7 | 0.145 | 0.9 | 1.6 | -0.7 | 0.0 |
| 1500 | $6.50 \cdot 10^{-1}$ | 0.026 | 11 | 0 | 0.012 | 30.9 | 19.0 | 0.012 | 0.9 | 1.4 | -0.4 | 0.0 |
| 2000 | $3.20 \cdot 10^{-2}$ | 0.693 | 156 | 0 | 0.586 | 8.3 | 3.4 | 0.617 | -5.1 | 3.2 | -5.2 | -3.1 |
| 2000 | $5.00 \cdot 10^{-2}$ | 0.443 | 195 | 0 | 0.476 | 7.4 | 2.0 | 0.487 | -2.3 | 3.2 | -4.6 | -0.9 |
| 2000 | $8.00 \cdot 10^{-2}$ | 0.277 | 213 | 0 | 0.444 | 7.1 | 2.0 | 0.448 | -0.9 | 3.0 | -3.7 | -0.2 |
| 2000 | $1.30 \cdot 10^{-1}$ | 0.171 | 154 | 0 | 0.340 | 8.3 | 3.9 | 0.340 | 0.0 | 2.9 | -2.8 | -0.1 |
| 2000 | $1.80 \cdot 10^{-1}$ | 0.123 | 131 | 0 | 0.282 | 9.1 | 2.6 | 0.280 | 0.4 | 2.7 | -2.3 | 0.0 |
| 2000 | $2.50 \cdot 10^{-1}$ | 0.089 | 100 | 0 | 0.234 | 10.4 | 5.3 | 0.232 | 0.7 | 2.5 | -1.8 | 0.0 |
| 2000 | $4.00 \cdot 10^{-1}$ | 0.055 | 58 | 0 | 0.170 | 13.8 | 10.1 | 0.169 | 1.1 | 2.3 | -1.2 | 0.0 |

Table 1. (continued)

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{array}$ | $x$ | $y$ | $N_{\text {data }}$ | $N_{p h p}$ | $\tilde{\sigma}^{e^{+} p}$ | $\delta_{\text {stat }}$ <br> (\%) | $\delta_{s y s}$ <br> (\%) | $F_{2}^{e m}$ | $\begin{array}{r} \Delta_{\text {all }} \\ (\%) \\ \hline \end{array}$ | $\begin{gathered} \Delta_{F_{2}} \\ (\%) \\ \hline \end{gathered}$ | $\begin{array}{r} \Delta_{x F_{3}} \\ (\%) \\ \hline \end{array}$ | $\begin{gathered} \Delta_{F_{L}} \\ (\%) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3000 | $5.00 \cdot 10^{-2}$ | 0.665 | 140 | 0 | 0.542 | 8.7 | 2.1 | 0.578 | -6.2 | 5.5 | -9.4 | -2.3 |
| 3000 | $8.00 \cdot 10^{-2}$ | 0.416 | 144 | 0 | 0.438 | 8.5 | 2.2 | 0.452 | -3.2 | 5.2 | -7.9 | -0.6 |
| 3000 | $1.30 \cdot 10^{-1}$ | 0.256 | 110 | 0 | 0.325 | 13.9 | 4.4 | 0.329 | -1.1 | 4.9 | -5.9 | -0.1 |
| 3000 | $1.80 \cdot 10^{-1}$ | 0.185 | 76 | 0 | 0.243 | 11.7 | 3.8 | 0.243 | -0.2 | 4.6 | -4.8 | -0.1 |
| 3000 | $2.50 \cdot 10^{-1}$ | 0.133 | 62 | 0 | 0.193 | 17.7 | 4.2 | 0.192 | 0.6 | 4.4 | -3.7 | 0.0 |
| 3000 | $4.00 \cdot 10^{-1}$ | 0.083 | 22 | 0 | 0.089 | 21.7 | 9.1 | 0.088 | 1.4 | 3.9 | -2.5 | 0.0 |
| 3000 | $6.50 \cdot 10^{-1}$ | 0.051 | 9 | 0 | 0.016 | 33.9 | 15.3 | 0.016 | 2.0 | 3.5 | -1.5 | 0.0 |
| 5000 | $8.00 \cdot 10^{-2}$ | 0.693 | 142 | 2 | 0.376 | 9.0 | 2.7 | 0.422 | -10.9 | 9.8 | -18.8 | -1.9 |
| 5000 | $1.30 \cdot 10^{-1}$ | 0.426 | 76 | 0 | 0.284 | 11.7 | 2.0 | 0.302 | -6.1 | 9.2 | -14.8 | -0.4 |
| 5000 | $1.80 \cdot 10^{-1}$ | 0.308 | 81 | 0 | 0.278 | 11.3 | 5.2 | 0.288 | -3.3 | 8.7 | -11.9 | -0.2 |
| 5000 | $2.50 \cdot 10^{-1}$ | 0.222 | 60 | 0 | 0.223 | 13.2 | 3.5 | 0.226 | -1.2 | 8.1 | -9.2 | -0.1 |
| 5000 | $4.00 \cdot 10^{-1}$ | 0.139 | 31 | 0 | 0.126 | 18.3 | 5.4 | 0.125 | 1.2 | 7.2 | -6.0 | 0.0 |
| 8000 | $1.30 \cdot 10^{-1}$ | 0.682 | 48 | 0 | 0.241 | 14.5 | 4.2 | 0.293 | -17.8 | 15.1 | -31.5 | -1.3 |
| 8000 | $1.80 \cdot 10^{-1}$ | 0.493 | 43 | 0 | 0.272 | 15.4 | 5.6 | 0.311 | -12.4 | 14.2 | -26.1 | -0.5 |
| 8000 | $2.50 \cdot 10^{-1}$ | 0.355 | 31 | 0 | 0.199 | 18.1 | 10.8 | 0.214 | -7.2 | 13.2 | -20.2 | -0.2 |
| 8000 | $4.00 \cdot 10^{-1}$ | 0.222 | 18 | 0 | 0.119 | 23.8 | 6.8 | 0.121 | -1.4 | 11.7 | -13.1 | 0.0 |
| 8000 | $6.50 \cdot 10^{-1}$ | 0.136 | 11 | 0 | 0.036 | 30.6 | 14.2 | 0.035 | 2.8 | 10.6 | -7.9 | 0.0 |
| 12000 | $1.80 \cdot 10^{-1}$ | 0.739 | 25 | 0 | 0.238 | 20.1 | 3.7 | 0.330 | -27.9 | 20.1 | -46.7 | -1.2 |
| 12000 | $2.50 \cdot 10^{-1}$ | 0.532 | 15 | 0 | 0.172 | 25.9 | 4.1 | 0.214 | -19.8 | 18.6 | -38.0 | -0.4 |
| 12000 | $4.00 \cdot 10^{-1}$ | 0.333 | 7 | 0 | 0.084 | 38.0 | 3.3 | 0.092 | -8.2 | 16.6 | -24.7 | -0.1 |
| 20000 | $2.50 \cdot 10^{-1}$ | 0.887 | 8 | 0 | 0.161 | 35.4 | 5.2 | 0.293 | -44.9 | 26.1 | -69.5 | -1.4 |
| 20000 | $4.00 \cdot 10^{-1}$ | 0.554 | 4 | 0 | 0.080 | ${ }_{-48}^{+79}$ | 13.9 | 0.112 | -28.9 | 23.2 | -51.8 | -0.3 |
| 30000 | $4.00 \cdot 10^{-1}$ | 0.831 | 1 | 0 | 0.039 | ${ }_{-83}^{+230}$ | 6.8 | 0.086 | -53.9 | 28.2 | -81.4 | -0.7 |

more, good acceptance $(A>20 \%)$ and purity $(P>30 \%)$ were required for each bin. The acceptance was defined as the fraction of events generated in a bin that passed the event selection. The purity was defined as the fraction of events reconstructed in a bin that were generated in that bin. The lowest values of acceptance occur at the lowest $Q^{2}$ values where the box cut (see Sect.9.1) becomes effective. The bins with lowest purity occur at the lowest $y$ values. In the majority of bins, the acceptance is greater than $80 \%$ and the purity is greater than $50 \%$, as shown in Fig. 3.

### 9.3 Background estimate

The final sample contains a small number of background events from the following sources:

- non-ep background. The background not associated with $e p$ collisions was determined from the number of events observed in unpaired or empty bunches. This background was subtracted statistically, taking into account the appropriate ratios of bunch currents and numbers of bunches. It is less than $1 \%$ for most bins and reaches $3 \%$ for a few bins at low $y$;
- photoproduction background. The events from the photoproduction MC generator PYTHIA were analysed in the same way as the data and the number of events passing the selection cuts in each bin was determined. This number of events was subtracted. In
the region $\delta<40 \mathrm{GeV}$, the photoproduction contribution was measured directly through events tagged by the positron tagger, as seen in Fig. 4a. The MC photoproduction background sample was normalised to the measurement at $\delta<40 \mathrm{GeV}$ and subtracted bin by bin. The total photoproduction background contribution to each bin is given in Table 1. The maximum background fraction of about $20 \%$ occurs in bins at high $y$ and low $Q^{2}$.
- charged current background. The background from charged current interactions is negligible.


### 9.4 Event characteristics

Figure 5 shows the characteristics of both low- and high$Q^{2}$ samples of accepted events. To obtain these distributions, the MC events were reweighted to the structure function obtained from the NLO QCD fit to these data (see Sect. 10.4). The energy, angular and transverse momentum distributions are well reproduced by the MC simulation (also indicated in the figures).

Figure 6 shows the distributions in $y_{\mathrm{PT}}, x_{\mathrm{PT}}$ and $Q_{\mathrm{PT}}^{2}$ together with the MC distributions normalised to the integrated luminosity of the data. Good agreement for the high- $Q^{2}$ sample is seen.

The discrepancies seen at low $y$ and low $Q^{2}, Q^{2} \lesssim$ $15 \mathrm{GeV}^{2}$ and $y \lesssim 10^{-2}$, are located in three $\left(x, Q^{2}\right)$-bins, the highest $x$ bins at $Q^{2}=3.5,6.5$ and $10 \mathrm{GeV}^{2}$, from a total of 242 bins. The systematic uncertainty in this low-y

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Fig. 4. a Distribution of $\delta$, see (3), for the low$Q^{2}$ sample (filled circles), for that subset of the data tagged by the positron tagger (stars) and for these tagged data corrected for the acceptance of the positron tagger (open circles). The lines show MC distributions after reweighting as discussed in the text. b The photoproduction MC (PHP) and the scaled tagged data are shown on a linear scale. The shaded area shows the uncertainty assigned to the photoproduction background, see Sect. 10.2


Fig. 5. Distributions of measured quantities, as discussed in the text, are shown for the low- $Q^{2}$ sample and the high$Q^{2}$ sample. The MC histograms are based on the NLO QCD fit to the data and are normalised according to the luminosity. The diffractive component and the photoproduction background contributions are separately shown. The inserts show details of the corresponding distributions


Fig. 6. Distributions of kinematic variables are shown for the low- $Q^{2}$ sample and the high- $Q^{2}$ sample. The MC histograms are based on the NLO QCD fit to the data and are normalised according to the luminosity. The diffractive component and the photoproduction background contributions are shown separately
region (see Sect. 10.2) is $10 \%$. This discrepancy between data and MC simulation is approximately equal to the systematic uncertainty (not shown in the figure). Reweighting the MC simulation to get agreement in this region does not affect the extracted structure-function values.

## 10 Results

### 10.1 Extraction of the proton-structure function $F_{2}^{e m}$ and the reduced cross section

Monte Carlo events were generated according to (1) including QED radiative effects. The value of $\tilde{\sigma}^{e^{+} p}$, at fixed $\left(x, Q^{2}\right)$ within a bin, was obtained from the ratio of the number of observed events to the number of events predicted from the MC simulation in that bin, multiplied by the reduced cross section from the Monte Carlo generator, $\tilde{\sigma}_{\mathrm{MC}}^{e^{+} p}\left(x, Q^{2}\right)$. The acceptance correction and bin-centering correction were thus taken from the MC simulation. The MC events contain transverse and longitudinal photon and $Z^{0}$ contributions, which can be written as $F_{2}^{i n t}, F_{2}^{w k}, x F_{3}$ and $F_{L}$ or, equivalently, the relative corrections $\Delta_{F_{2}}$ (see (2)), $\Delta_{x F_{3}}$ and $\Delta_{F_{L}}$ according to:

$$
\begin{equation*}
\tilde{\sigma}^{e^{+} p}=F_{2}^{e m}\left(1+\Delta_{F_{2}}+\Delta_{x F_{3}}+\Delta_{F_{L}}\right)=F_{2}^{e m}\left(1+\Delta_{\text {all }}\right) . \tag{7}
\end{equation*}
$$

The result for $F_{2}^{e m}$ was obtained by correcting the reduced cross section, $\tilde{\sigma}^{e^{+} p}$, for these relative contributions, $\Delta_{\text {all }}$, from the CTEQ4D parton distribution functions [34] used in the MC generation. The reduced cross section and $F_{2}^{e m}$ results do not significantly change if the NLO QCD fit results, instead of CTEQ4D, are used in the unfolding procedure.

The statistical uncertainties on the reduced cross sections were calculated from the number of events measured in the bins, taking into account the background and the statistical uncertainty from the MC simulation. Poisson statistics was used for the two bins that contain fewer than 5 events.

The reduced cross sections and $F_{2}^{e m}$ values were corrected for higher-order QED radiative effects not included in HERACLES. These second-order leading-logarithmic corrections, as well as third- and higher-order terms coming from soft-photon exponentiation, were evaluated using the program HECTOR 1.00 [35]. They lie between $-0.2 \%$ and $-0.5 \%$ and vary smoothly with $Q^{2}$. A constant correction of $-0.35 \%$ was applied.

### 10.2 Systematic uncertainties

Several factors contribute to the systematic uncertainties of $F_{2}^{e m}$ and $\tilde{\sigma}^{e^{+} p}$. The relative uncertainties in $\tilde{\sigma}^{e^{+} p}$ are identical to those quoted for $F_{2}^{e m}$. Ten sources of systematic uncertainties were found to be correlated between the different $F_{2}^{e m}$ bins and are denoted by $\}$ in the text and in Table 2.

The ten correlated systematic uncertainties are:
\{1\} positron finding and efficiency.
The positron identification efficiency was varied within the uncertainty found in the QED Compton study, which is $\sim 2 \%$ for positron energies $E_{e}^{\prime} \sim 8$ GeV and $\sim 1 \%$ for energies above 15 GeV . The effect on $F_{2}^{e m}$ is $\sim 1 \%$, except in the lowest $x$ bins where it reaches about $2 \%$;
$\{2\}$ positron scattering angle - A.
The uncertainty in the alignment of the different detector elements results in a systematic uncertainty in the angle of the scattered positron. The difference between the extrapolated track position at the face of the CAL and the position found from the CAL, the HES or the SRTD shows that this uncertainty was about 2 mm . The largest systematic uncertainty was obtained if the separation between the $+X$ and $-X$ halves of the RCAL was changed by $\pm 2 \mathrm{~mm}$, resulting in an uncertainty of $\sim 1 \%$ for $Q^{2} \lesssim 200 \mathrm{GeV}^{2}$;
$\{3\}$ positron scattering angle - B.
For positron angles measured with the tracking detector, the scattering angle $\theta_{e}$ was changed by the estimated systematic uncertainty of $\pm 1 \mathrm{mrad}$ in the track angle. This change had a $1 \%$ effect on the measured $F_{2}^{e m}$ at high $Q^{2}$;
$\{4\} \quad$ positron energy scale.
The systematic uncertainties in $F_{2}^{e m}$ due to the uncertainties in the absolute calorimeter-energy calibration for scattered positrons was estimated by changing the energy scale in the MC simulation: the change is $2 \%$ at 8 GeV , linearly decreasing to $1 \%$ at 15 GeV (see Sect. 7.2). Systematic variations of $1 \%$ in $F_{2}^{e m}$ are observed;
\{5-7\} hadronic energy measurement.
The hadronic energy scales of the different calorimeter regions in the MC simulation were changed by $\pm 2 \%$ in turn for the FCAL $\{5\}$, BCAL $\{6\}$ and RCAL $\{7\}$, while leaving the positron energy scale unchanged. The value of $2 \%$ is based on detailed comparisons of the distributions of the quantity $P_{T, h} / P_{T, e}$ from data and MC simulation, see Sect. 8 . The effect on $F_{2}^{e m}$ was typically $2 \%$ in these bins at the highest or lowest $y$ and about $0.5 \%$ elsewhere;
$\{8\}$ hadronic energy flow - A.
The hadronic energy flow for diffractive events is different from that for non-diffractive events. To investigate the sensitivity of the PT reconstruction method to the size of the diffractive component, the diffractive scattering cross section in the MC simulation was reweighted such that it changed by $\pm 50 \%$ while leaving the correction function $\mathcal{C}$ (5) unchanged. Effects of at most $2 \%$ at low $Q^{2}$ and high $y$ in $F_{2}^{e m}$ were observed. The variations in $F_{2}^{e m}$ were negligible elsewhere. This is also a check on the effect of different simulations of the hadronic energy flow between the struck parton and the proton remnant;

Table 2. Relative uncertainties in the reduced cross section. The systematic uncertainties, denoted by the $\}$, are explained in Sect. 10.2. The overall normalisation uncertainty of $2 \%$ is not included, neither is the additional uncertainty of $1 \%$ at low $Q^{2}$

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{array}$ | $x$ | $\tilde{\sigma}^{e^{+} p}$ | $\begin{gathered} \delta_{\text {stat }} \\ (\%) \end{gathered}$ | $\begin{aligned} & \delta_{s y s} \\ & (\%) \end{aligned}$ | $\delta_{u n c}$ <br> (\%) | $\delta_{\{1\}}$ <br> (\%) | $\delta_{\{2\}}$ <br> (\%) | $\delta_{\{3\}}$ <br> (\%) | $\begin{gathered} \delta_{\{4\}} \\ (\%) \end{gathered}$ | $\delta_{\{5\}}$ <br> (\%) | $\delta_{\{6\}}$ <br> (\%) | $\delta_{\{7\}}$ <br> (\%) | $\delta_{\{8\}}$ <br> (\%) | $\delta_{\{9\}}$ <br> (\%) | $\begin{array}{r} \delta_{\{10\}} \\ (\%) \\ \hline \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.7 | $6.32 \cdot 10^{-5}$ | 0.832 | 1.7 | 4.7 | 2.0 | 1.9 | -0.1 | 0.0 | 0.2 | 0.1 | 0.2 | 1.4 | 1.9 | 2.9 | 0.3 |
| 2.7 | $1.61 \cdot 10^{-4}$ | 0.740 | 1.0 | 2.4 | 1.8 | 1.4 | 0.3 | 0.0 | -0.2 | 0.1 | 0.2 | -0.3 | 0.1 | 0.4 | -0.2 |
| 2.7 | $4.00 \cdot 10^{-4}$ | 0.649 | 1.1 | 3.9 | 3.6 | 1.2 | 0.3 | 0.0 | -0.3 | 0.1 | 0.4 | -0.4 | 0.7 | 0.1 | 0.0 |
| 2.7 | $6.32 \cdot 10^{-4}$ | 0.607 | 1.5 | 4.3 | 3.4 | 1.1 | 0.3 | 0.0 | -0.5 | 0.6 | -0.2 | 0.1 | 2.3 | 0.0 | -0.1 |
| 2.7 | $2.00 \cdot 10^{-3}$ | 0.477 | 0.9 | 4.2 | 2.7 | 1.1 | 0.1 | 0.0 | -0.2 | 0.5 | -0.1 | -0.1 | 0.8 | 0.0 | 2.8 |
| 3.5 | $6.32 \cdot 10^{-5}$ | 1.076 | 2.7 | 12.7 | 3.9 | 2.6 | 0.6 | 0.0 | 0.8 | -0.1 | 0.4 | 2.6 | 4.3 | 10.6 | 0.1 |
| 3.5 | $1.02 \cdot 10^{-4}$ | 0.951 | 2.0 | 3.6 | 2.0 | 1.7 | 0.3 | 0.0 | 0.7 | 0.1 | 0.1 | 0.8 | 0.8 | 1.9 | 0.1 |
| 3.5 | $1.61 \cdot 10^{-4}$ | 0.882 | 2.0 | 2.5 | 1.9 | 1.4 | 0.1 | 0.0 | 0.3 | -0.1 | 0.3 | 0.3 | 0.1 | 0.4 | 0.0 |
| 3.5 | $2.53 \cdot 10^{-4}$ | 0.829 | 1.9 | 2.4 | 1.7 | 1.3 | -0.4 | 0.0 | -0.5 | 0.1 | 0.4 | -0.6 | -0.2 | 0.2 | -0.2 |
| 3.5 | $4.00 \cdot 10^{-4}$ | 0.787 | 1.9 | 2.7 | 2.1 | 1.2 | -0.3 | 0.0 | -0.4 | -0.3 | 0.3 | -0.5 | 0.8 | 0.0 | 0.3 |
| 3.5 | $6.32 \cdot 10^{-4}$ | 0.700 | 1.9 | 3.0 | 2.5 | 1.1 | 0.2 | 0.0 | 0.4 | 0.3 | 0.3 | -0.1 | 1.0 | 0.0 | 0.1 |
| 3.5 | $2.00 \cdot 10^{-3}$ | 0.554 | 0.9 | 2.5 | 1.4 | 1.1 | -0.1 | 0.0 | -0.3 | 0.5 | -0.1 | 0.0 | 0.8 | 0.0 | 1.5 |
| 3.5 | $8.00 \cdot 10^{-3}$ | 0.472 | 1.0 | 6.3 | 1.6 | 1.1 | -0.2 | 0.0 | -0.5 | 0.7 | -0.1 | -0.1 | -0.2 | 0.0 | 5.9 |
| 4.5 | $1.02 \cdot 10^{-4}$ | 1.125 | 2.0 | 5.8 | 1.3 | 2.0 | -0.4 | 0.0 | 0.3 | -0.1 | 0.2 | 1.7 | 1.9 | 4.6 | -0.1 |
| 4.5 | $1.61 \cdot 10^{-4}$ | 0.994 | 2.0 | 2.2 | 0.7 | 1.6 | 0.6 | 0.0 | 0.3 | 0.1 | 0.2 | 0.3 | 0.7 | 0.7 | 0.0 |
| 4.5 | $2.53 \cdot 10^{-4}$ | 0.923 | 2.0 | 2.6 | 1.9 | 1.3 | 0.0 | 0.0 | -0.5 | 0.1 | 0.2 | -0.3 | 1.1 | 0.1 | 0.0 |
| 4.5 | $4.00 \cdot 10^{-4}$ | 0.882 | 2.2 | 2.4 | 1.9 | 1.2 | -0.4 | 0.0 | -0.4 | -0.1 | -0.2 | -0.6 | 0.3 | 0.0 | -0.3 |
| 4.5 | $6.32 \cdot 10^{-4}$ | 0.803 | 2.2 | 2.0 | 1.4 | 1.2 | -0.4 | 0.0 | -0.1 | 0.2 | 0.3 | -0.2 | 0.7 | 0.0 | 0.2 |
| 6.5 | $1.02 \cdot 10^{-4}$ | 1.210 | 2.4 | 10.2 | 1.0 | 2.5 | 0.3 | 0.0 | 0.3 | -0.2 | 0.5 | 2.5 | 2.4 | 9.2 | 0.2 |
| 6.5 | $1.61 \cdot 10^{-4}$ | 1.159 | 1.5 | 3.6 | 0.4 | 1.8 | 0.0 | 0.0 | 0.6 | 0.1 | 0.1 | 0.6 | 0.9 | 2.8 | 0.0 |
| 6.5 | $2.53 \cdot 10^{-4}$ | 1.078 | 1.5 | 1.9 | 0.5 | 1.5 | 0.4 | 0.0 | 0.5 | 0.1 | 0.3 | 0.3 | 0.5 | 0.6 | 0.0 |
| 6.5 | $4.00 \cdot 10^{-4}$ | 0.972 | 1.5 | 1.8 | 0.9 | 1.3 | -0.1 | 0.0 | -0.4 | -0.1 | 0.3 | -0.6 | -0.2 | 0.2 | 0.0 |
| 6.5 | $6.32 \cdot 10^{-4}$ | 0.898 | 1.6 | 3.1 | 2.7 | 1.2 | -0.2 | 0.0 | -0.2 | 0.2 | 0.3 | -0.4 | 0.4 | 0.0 | 0.0 |
| 6.5 | $1.02 \cdot 10^{-3}$ | 0.828 | 1.6 | 2.1 | 1.6 | 1.1 | 0.3 | 0.0 | -0.3 | 0.3 | 0.2 | 0.1 | 0.6 | 0.0 | -0.1 |
| 6.5 | $1.61 \cdot 10^{-3}$ | 0.740 | 1.8 | 2.5 | 2.0 | 1.1 | -0.7 | 0.0 | -0.4 | -0.5 | -0.3 | -0.1 | 0.4 | 0.0 | 0.6 |
| 6.5 | $2.53 \cdot 10^{-3}$ | 0.666 | 1.2 | 1.9 | 1.1 | 1.1 | 0.2 | 0.0 | 0.3 | 0.6 | 0.0 | 0.0 | 0.6 | 0.0 | 0.5 |
| 6.5 | $5.00 \cdot 10^{-3}$ | 0.576 | 0.9 | 4.1 | 0.9 | 1.1 | 0.2 | 0.0 | -0.3 | 0.4 | 0.0 | 0.1 | 0.0 | 0.0 | 3.9 |
| 6.5 | $2.10 \cdot 10^{-2}$ | 0.487 | 1.2 | 8.7 | 1.9 | 1.1 | -0.2 | 0.0 | -0.7 | 0.8 | -0.4 | 0.0 | -0.5 | 0.0 | 8.3 |
| 8.5 | $1.61 \cdot 10^{-4}$ | 1.212 | 2.5 | 9.9 | 1.7 | 2.4 | 0.6 | 0.0 | -1.0 | 0.2 | 0.4 | 1.8 | 1.8 | 9.1 | 0.0 |
| 8.5 | $2.53 \cdot 10^{-4}$ | 1.188 | 1.6 | 2.7 | 0.9 | 1.6 | 0.6 | 0.0 | 0.6 | -0.1 | 0.2 | 0.4 | 0.2 | 1.6 | 0.2 |
| 8.5 | $4.00 \cdot 10^{-4}$ | 1.067 | 1.6 | 2.0 | 0.9 | 1.4 | 0.6 | 0.0 | -0.4 | 0.1 | 0.2 | -0.4 | 0.1 | 0.6 | -0.2 |
| 8.5 | $6.32 \cdot 10^{-4}$ | 0.973 | 1.6 | 2.8 | 1.9 | 1.2 | 1.4 | 0.0 | 0.3 | -0.1 | 0.4 | -0.3 | -0.2 | 0.1 | 0.3 |
| 8.5 | $1.02 \cdot 10^{-3}$ | 0.875 | 1.6 | 1.6 | 0.8 | 1.2 | -0.4 | 0.0 | -0.4 | -0.1 | 0.2 | -0.3 | 0.0 | 0.1 | -0.2 |
| 8.5 | $1.61 \cdot 10^{-3}$ | 0.813 | 1.8 | 1.6 | 1.0 | 1.1 | 0.3 | 0.0 | -0.1 | 0.3 | 0.0 | -0.1 | 0.5 | 0.1 | -0.3 |
| 10.0 | $1.61 \cdot 10^{-4}$ | 1.230 | 4.2 | 12.8 | 1.4 | 2.7 | -0.7 | 0.0 | -0.6 | -0.1 | 0.5 | 3.1 | 1.3 | 11.9 | 0.1 |
| 10.0 | $2.53 \cdot 10^{-4}$ | 1.237 | 2.1 | 4.5 | 1.3 | 1.9 | 0.3 | 0.0 | -0.3 | -0.2 | -0.1 | 0.7 | -0.1 | 3.8 | 0.1 |
| 10.0 | $4.00 \cdot 10^{-4}$ | 1.099 | 1.9 | 2.3 | 1.1 | 1.5 | 0.8 | 0.0 | -0.2 | 0.1 | 0.3 | -0.5 | 0.2 | 1.0 | 0.0 |
| 10.0 | $6.32 \cdot 10^{-4}$ | 1.016 | 1.8 | 1.9 | 1.2 | 1.3 | 0.4 | 0.0 | -0.1 | -0.2 | 0.4 | -0.6 | -0.1 | 0.2 | -0.1 |
| 10.0 | $1.02 \cdot 10^{-3}$ | 0.934 | 1.8 | 1.9 | 1.3 | 1.2 | 0.2 | 0.0 | -0.4 | -0.2 | 0.2 | -0.4 | -0.2 | 0.0 | 0.0 |
| 10.0 | $1.61 \cdot 10^{-3}$ | 0.851 | 2.1 | 2.0 | 1.2 | 1.1 | 1.1 | 0.0 | 0.2 | -0.3 | 0.1 | 0.1 | 0.0 | 0.0 | -0.1 |
| 10.0 | $2.53 \cdot 10^{-3}$ | 0.761 | 2.0 | 2.4 | 1.8 | 1.1 | 0.9 | 0.0 | -0.2 | 0.4 | -0.2 | -0.1 | 0.4 | 0.0 | -0.4 |
| 10.0 | $5.00 \cdot 10^{-3}$ | 0.624 | 1.1 | 2.9 | 0.8 | 1.1 | 0.8 | 0.0 | -0.2 | 0.6 | -0.1 | 0.0 | 0.1 | 0.0 | 2.4 |
| 10.0 | $2.10 \cdot 10^{-2}$ | 0.503 | 1.1 | 6.8 | 1.3 | 1.0 | 0.7 | 0.0 | -0.7 | 0.9 | -0.2 | 0.1 | -0.2 | 0.0 | 6.5 |
| 12.0 | $2.53 \cdot 10^{-4}$ | 1.322 | 2.6 | 7.5 | 1.0 | 2.2 | 1.3 | 0.0 | -1.0 | 0.2 | 0.3 | 1.0 | 0.5 | 6.8 | 0.2 |
| 12.0 | $4.00 \cdot 10^{-4}$ | 1.191 | 2.3 | 2.7 | 0.9 | 1.6 | 0.6 | 0.0 | 0.2 | -0.2 | -0.1 | 0.1 | 0.3 | 1.9 | 0.0 |
| 12.0 | $6.32 \cdot 10^{-4}$ | 1.070 | 2.1 | 2.3 | 1.6 | 1.3 | 0.3 | 0.0 | -0.1 | -0.2 | 0.2 | -0.6 | 0.5 | 0.3 | 0.1 |
| 12.0 | $1.02 \cdot 10^{-3}$ | 0.983 | 2.0 | 1.9 | 1.1 | 1.2 | 0.8 | 0.0 | -0.2 | 0.2 | 0.4 | -0.5 | 0.3 | 0.0 | -0.1 |
| 12.0 | $1.61 \cdot 10^{-3}$ | 0.899 | 2.2 | 1.6 | 0.7 | 1.1 | 0.8 | 0.0 | -0.2 | 0.1 | 0.2 | -0.1 | 0.1 | 0.1 | 0.1 |
| 12.0 | $2.53 \cdot 10^{-3}$ | 0.793 | 2.1 | 1.9 | 1.0 | 1.1 | 0.6 | 0.0 | 0.4 | 0.3 | -0.1 | 0.0 | 0.1 | 0.0 | 0.9 |
| 15.0 | $2.53 \cdot 10^{-4}$ | 1.397 | 2.9 | 7.6 | 1.5 | 2.3 | 0.7 | 0.0 | 0.5 | 0.2 | 0.2 | 2.0 | 0.7 | 6.7 | -0.1 |
| 15.0 | $4.00 \cdot 10^{-4}$ | 1.258 | 2.3 | 3.7 | 1.5 | 1.8 | 0.4 | 0.0 | -0.3 | -0.2 | 0.3 | 0.1 | 0.8 | 2.7 | 0.0 |
| 15.0 | $6.32 \cdot 10^{-4}$ | 1.137 | 2.0 | 2.2 | 0.9 | 1.4 | 1.0 | 0.0 | -0.2 | -0.1 | 0.3 | -0.2 | 0.3 | 0.8 | -0.2 |
| 15.0 | $1.02 \cdot 10^{-3}$ | 1.055 | 1.9 | 1.9 | 1.1 | 1.2 | 0.3 | 0.0 | -0.2 | -0.3 | 0.3 | -0.6 | 0.2 | 0.1 | 0.1 |
| 15.0 | $1.61 \cdot 10^{-3}$ | 0.944 | 2.2 | 1.6 | 0.7 | 1.2 | 0.6 | 0.0 | -0.5 | 0.1 | 0.0 | -0.2 | 0.1 | 0.0 | -0.1 |

Table 2. (continued)

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{array}$ | $x$ | $\tilde{\sigma}^{e^{+} p}$ | $\begin{gathered} \delta_{\text {stat }} \\ (\%) \\ \hline \end{gathered}$ | $\begin{gathered} \delta_{\text {sys }} \\ (\%) \end{gathered}$ | $\delta_{u n c}$ <br> (\%) | $\delta_{\{1\}}$ <br> (\%) | $\delta_{\{2\}}$ <br> (\%) | $\delta_{\{3\}}$ <br> (\%) | $\begin{gathered} \delta_{\{4\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{5\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{6\}} \\ (\%) \end{gathered}$ | $\delta_{\{7\}}$ <br> (\%) | $\delta_{\{8\}}$ <br> (\%) | $\delta_{\{9\}}$ <br> (\%) | $\begin{array}{r} \delta_{\{10\}} \\ (\%) \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 15.0 | $2.53 \cdot 10^{-3}$ | 0.836 | 2.1 | 1.6 | 0.9 | 1.1 | 0.6 | 0.0 | -0.3 | 0.3 | -0.1 | -0.1 | 0.1 | 0.0 | -0.2 |
| 15.0 | $5.00 \cdot 10^{-3}$ | 0.706 | 1.5 | 1.5 | 0.8 | 1.1 | 0.3 | 0.0 | -0.1 | 0.5 | -0.1 | 0.0 | 0.2 | 0.0 | 0.1 |
| 15.0 | $2.10 \cdot 10^{-2}$ | 0.519 | 1.0 | 6.9 | 1.0 | 1.0 | 0.7 | 0.0 | -0.6 | 0.5 | -0.2 | 0.1 | -0.2 | 0.0 | 6.6 |
| 18.0 | $5.00 \cdot 10^{-4}$ | 1.284 | 1.9 | 4.1 | 0.7 | 1.9 | 0.4 | 0.0 | -0.5 | 0.1 | 0.3 | 0.4 | 0.2 | 3.5 | 0.1 |
| 18.0 | $8.00 \cdot 10^{-4}$ | 1.157 | 2.2 | 2.4 | 0.9 | 1.4 | 1.4 | 0.0 | -0.1 | -0.1 | 0.3 | -0.6 | 0.1 | 0.5 | 0.3 |
| 18.0 | $1.30 \cdot 10^{-3}$ | 1.038 | 2.0 | 2.2 | 1.5 | 1.2 | 0.8 | 0.0 | -0.3 | -0.1 | 0.4 | -0.4 | 0.3 | 0.1 | 0.1 |
| 18.0 | $2.10 \cdot 10^{-3}$ | 0.929 | 2.3 | 1.6 | 0.9 | 1.1 | 0.2 | 0.0 | 0.0 | -0.2 | -0.2 | -0.2 | 0.4 | 0.0 | -0.5 |
| 18.0 | $3.20 \cdot 10^{-3}$ | 0.851 | 2.4 | 1.8 | 1.0 | 1.1 | 0.9 | 0.0 | -0.5 | 0.2 | 0.2 | 0.1 | 0.2 | 0.1 | 0.2 |
| 18.0 | $5.00 \cdot 10^{-3}$ | 0.739 | 1.7 | 1.7 | 0.6 | 1.1 | 0.9 | 0.0 | -0.2 | 0.4 | -0.2 | 0.0 | -0.1 | 0.0 | 0.6 |
| 22.0 | $5.00 \cdot 10^{-4}$ | 1.357 | 2.5 | 4.0 | 1.2 | 2.0 | 0.5 | 0.0 | -1.0 | 0.1 | 0.2 | 0.6 | 0.7 | 2.8 | 0.0 |
| 22.0 | $8.00 \cdot 10^{-4}$ | 1.267 | 2.2 | 1.9 | 0.6 | 1.5 | -0.1 | 0.0 | 0.3 | -0.1 | 0.3 | -0.1 | 0.3 | 0.6 | -0.5 |
| 22.0 | $1.30 \cdot 10^{-3}$ | 1.065 | 2.3 | 2.3 | 1.4 | 1.3 | 1.0 | 0.0 | -0.1 | -0.2 | 0.4 | -0.5 | 0.0 | 0.2 | 0.1 |
| 22.0 | $2.10 \cdot 10^{-3}$ | 1.001 | 2.7 | 2.0 | 0.8 | 1.2 | 0.8 | 0.0 | -1.0 | 0.2 | -0.3 | -0.2 | 0.3 | 0.0 | -0.1 |
| 22.0 | $3.20 \cdot 10^{-3}$ | 0.905 | 2.6 | 1.7 | 1.2 | 1.1 | 0.3 | 0.0 | 0.1 | -0.4 | 0.1 | -0.1 | 0.3 | 0.1 | 0.1 |
| 22.0 | $5.00 \cdot 10^{-3}$ | 0.771 | 2.4 | 1.7 | 0.9 | 1.1 | 0.4 | 0.0 | -0.3 | 0.3 | -0.2 | -0.1 | 0.2 | 0.0 | -0.7 |
| 22.0 | $8.00 \cdot 10^{-3}$ | 0.656 | 2.4 | 1.7 | 0.9 | 1.1 | 0.4 | 0.0 | -0.7 | 0.5 | -0.3 | 0.0 | 0.3 | 0.0 | 0.1 |
| 22.0 | $1.30 \cdot 10^{-2}$ | 0.581 | 2.0 | 5.6 | 1.8 | 1.0 | 0.9 | 0.0 | -0.3 | 0.4 | 0.1 | -0.1 | 0.0 | 0.0 | 5.1 |
| 22.0 | $3.20 \cdot 10^{-2}$ | 0.512 | 1.8 | 6.9 | 1.6 | 1.0 | 0.9 | 0.0 | -0.7 | 1.4 | -0.3 | 0.1 | -0.3 | 0.0 | 6.4 |
| 27.0 | $5.00 \cdot 10^{-4}$ | 1.497 | 3.3 | 4.0 | 1.5 | 2.1 | 0.7 | 0.0 | 0.1 | 0.2 | 0.2 | 1.3 | 0.9 | 2.5 | 0.3 |
| 27.0 | $8.00 \cdot 10^{-4}$ | 1.294 | 2.5 | 2.2 | 0.8 | 1.7 | 0.4 | 0.0 | 0.1 | 0.1 | 0.3 | 0.2 | 0.4 | 0.9 | 0.0 |
| 27.0 | $1.30 \cdot 10^{-3}$ | 1.181 | 2.3 | 2.7 | 1.8 | 1.4 | 1.1 | 0.0 | -0.2 | 0.1 | 0.2 | -0.6 | 0.1 | 0.2 | 0.0 |
| 27.0 | $2.10 \cdot 10^{-3}$ | 1.095 | 2.8 | 1.7 | 1.0 | 1.2 | 0.5 | 0.0 | 0.2 | -0.1 | 0.3 | -0.2 | 0.0 | 0.1 | 0.0 |
| 27.0 | $3.20 \cdot 10^{-3}$ | 0.901 | 2.7 | 2.1 | 1.5 | 1.2 | 0.8 | 0.0 | 0.1 | 0.2 | -0.2 | 0.0 | 0.5 | 0.1 | 0.0 |
| 27.0 | $5.00 \cdot 10^{-3}$ | 0.804 | 2.5 | 2.5 | 1.5 | 1.1 | 1.0 | 0.0 | -1.1 | 0.2 | -0.5 | -0.1 | 0.3 | 0.0 | -0.2 |
| 27.0 | $8.00 \cdot 10^{-3}$ | 0.731 | 2.6 | 2.7 | 1.3 | 1.1 | 1.2 | 0.0 | -0.4 | 0.7 | -0.2 | 0.1 | 0.1 | 0.0 | 1.4 |
| 27.0 | $1.30 \cdot 10^{-2}$ | 0.634 | 2.0 | 2.9 | 1.4 | 1.0 | 0.6 | 0.0 | -0.4 | 0.3 | 0.0 | 0.1 | -0.1 | 0.0 | 2.2 |
| 35.0 | $8.00 \cdot 10^{-4}$ | 1.404 | 1.5 | 2.9 | 0.7 | 1.9 | 0.4 | 0.0 | -1.0 | 0.0 | 0.0 | 0.3 | 1.1 | 1.4 | 0.0 |
| 35.0 | $1.30 \cdot 10^{-3}$ | 1.225 | 1.4 | 1.8 | 0.3 | 1.5 | 0.5 | 0.0 | -0.3 | -0.1 | 0.4 | -0.5 | 0.3 | 0.3 | 0.1 |
| 35.0 | $2.10 \cdot 10^{-3}$ | 1.108 | 1.6 | 1.6 | 0.4 | 1.3 | 0.6 | 0.0 | -0.5 | -0.1 | 0.3 | -0.4 | 0.1 | 0.0 | -0.1 |
| 35.0 | $3.20 \cdot 10^{-3}$ | 0.961 | 1.7 | 1.6 | 0.7 | 1.2 | 0.6 | 0.0 | 0.3 | 0.1 | 0.1 | -0.1 | -0.4 | 0.0 | -0.2 |
| 35.0 | $5.00 \cdot 10^{-3}$ | 0.864 | 1.6 | 1.5 | 0.4 | 1.1 | 0.6 | 0.0 | -0.6 | 0.2 | 0.1 | 0.1 | -0.1 | 0.0 | -0.1 |
| 35.0 | $8.00 \cdot 10^{-3}$ | 0.727 | 1.6 | 1.8 | 0.7 | 1.1 | 0.7 | 0.0 | -0.4 | 0.5 | -0.4 | 0.0 | 0.1 | 0.0 | -0.6 |
| 35.0 | $1.30 \cdot 10^{-2}$ | 0.646 | 1.9 | 2.2 | 0.9 | 1.1 | 0.5 | 0.0 | -0.3 | 0.9 | -0.1 | -0.1 | 0.0 | 0.0 | 1.4 |
| 35.0 | $2.10 \cdot 10^{-2}$ | 0.555 | 1.8 | 2.8 | 1.1 | 1.0 | 1.0 | 0.0 | -0.5 | -0.3 | -0.2 | 0.2 | -0.2 | 0.0 | 2.0 |
| 35.0 | $3.20 \cdot 10^{-2}$ | 0.496 | 1.1 | 7.6 | 1.0 | 1.0 | 0.9 | 0.0 | -1.2 | 0.9 | -0.2 | 0.1 | -0.2 | 0.0 | 7.2 |
| 45.0 | $8.00 \cdot 10^{-4}$ | 1.482 | 1.8 | 3.6 | 1.4 | 2.0 | 0.4 | 0.0 | 0.3 | -0.1 | 0.3 | 1.6 | 0.1 | 1.9 | 0.0 |
| 45.0 | $1.30 \cdot 10^{-3}$ | 1.297 | 1.3 | 2.1 | 0.3 | 1.6 | 0.5 | 0.0 | -0.7 | 0.0 | 0.3 | -0.8 | 0.2 | 0.4 | 0.0 |
| 45.0 | $2.10 \cdot 10^{-3}$ | 1.131 | 1.7 | 1.7 | 0.4 | 1.4 | 0.5 | 0.0 | -0.2 | 0.1 | 0.6 | -0.3 | -0.1 | 0.1 | 0.1 |
| 45.0 | $3.20 \cdot 10^{-3}$ | 0.998 | 1.5 | 1.5 | 0.5 | 1.2 | 0.6 | 0.0 | -0.2 | 0.2 | 0.1 | -0.2 | 0.0 | 0.0 | -0.2 |
| 45.0 | $5.00 \cdot 10^{-3}$ | 0.909 | 1.4 | 1.3 | 0.4 | 1.2 | 0.2 | 0.0 | -0.2 | 0.2 | 0.1 | -0.2 | -0.2 | 0.0 | 0.1 |
| 45.0 | $8.00 \cdot 10^{-3}$ | 0.777 | 1.4 | 1.5 | 0.4 | 1.1 | 0.5 | 0.0 | -0.5 | -0.4 | -0.3 | -0.1 | 0.3 | 0.0 | 0.1 |
| 45.0 | $1.30 \cdot 10^{-2}$ | 0.661 | 1.7 | 1.7 | 0.8 | 1.1 | 0.2 | 0.0 | -0.8 | 0.6 | -0.2 | -0.1 | -0.1 | 0.0 | -0.1 |
| 45.0 | $2.10 \cdot 10^{-2}$ | 0.587 | 1.7 | 2.1 | 0.3 | 1.0 | 0.4 | 0.0 | -0.6 | 0.3 | 0.0 | 0.1 | -0.3 | 0.0 | 1.6 |
| 60.0 | $1.30 \cdot 10^{-3}$ | 1.387 | 1.3 | 2.2 | 0.6 | 1.8 | 0.5 | -0.1 | -0.4 | 0.1 | 0.1 | -0.1 | 0.4 | 0.7 | 0.0 |
| 60.0 | $2.10 \cdot 10^{-3}$ | 1.213 | 1.5 | 1.8 | 0.5 | 1.5 | 0.4 | 0.0 | -0.2 | -0.2 | 0.4 | -0.4 | 0.4 | 0.4 | -0.1 |
| 60.0 | $3.20 \cdot 10^{-3}$ | 1.074 | 1.5 | 1.5 | 0.4 | 1.3 | 0.4 | 0.0 | 0.2 | -0.1 | 0.4 | -0.3 | -0.2 | 0.1 | -0.1 |
| 60.0 | $5.00 \cdot 10^{-3}$ | 0.900 | 1.4 | 1.5 | 0.5 | 1.2 | 0.4 | 0.0 | 0.4 | 0.2 | 0.3 | -0.1 | 0.2 | 0.0 | 0.1 |
| 60.0 | $8.00 \cdot 10^{-3}$ | 0.809 | 1.4 | 1.5 | 0.7 | 1.1 | 0.5 | 0.0 | -0.6 | 0.2 | -0.3 | 0.1 | 0.2 | 0.0 | 0.1 |
| 60.0 | $1.30 \cdot 10^{-2}$ | 0.689 | 1.7 | 1.9 | 0.8 | 1.1 | 0.7 | 0.0 | -0.6 | 0.7 | -0.3 | 0.1 | 0.2 | 0.0 | -0.5 |
| 60.0 | $2.10 \cdot 10^{-2}$ | 0.598 | 1.6 | 2.4 | 1.1 | 1.1 | 0.6 | 0.0 | -0.7 | 0.2 | -0.1 | 0.1 | 0.0 | 0.0 | 1.5 |
| 60.0 | $3.20 \cdot 10^{-2}$ | 0.537 | 2.0 | 3.3 | 1.3 | 1.0 | 0.5 | 0.0 | -0.6 | 0.7 | 0.0 | 0.2 | -0.2 | 0.0 | 2.7 |
| 60.0 | $5.00 \cdot 10^{-2}$ | 0.512 | 1.5 | 6.8 | 1.9 | 1.0 | 0.0 | 0.0 | -0.8 | 0.9 | -0.3 | -0.1 | -0.1 | 0.0 | 6.3 |

Table 2. (continued)

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{array}$ | $x$ | $\tilde{\sigma}^{e^{+} p}$ | $\begin{gathered} \delta_{\text {stat }} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\text {sys }} \\ (\%) \end{gathered}$ | $\delta_{u n c}$ <br> (\%) | $\begin{gathered} \delta_{\{1\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{2\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{3\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{4\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{5\}} \\ (\%) \end{gathered}$ | $\delta_{\{6\}}$ <br> (\%) | $\begin{gathered} \delta_{\{7\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{8\}} \\ (\%) \end{gathered}$ | $\delta_{\{9\}}$ <br> (\%) | $\begin{array}{r} \delta_{\{10\}} \\ (\%) \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 70.0 | $1.30 \cdot 10^{-3}$ | 1.438 | 1.8 | 2.2 | 0.6 | 1.9 | 0.3 | -0.3 | -0.5 | 0.1 | 0.1 | 0.3 | -0.3 | 0.5 | 0.0 |
| 70.0 | $2.10 \cdot 10^{-3}$ | 1.238 | 1.7 | 1.9 | 0.3 | 1.6 | 0.4 | -0.1 | 0.3 | 0.1 | 0.4 | -0.3 | 0.4 | 0.5 | 0.1 |
| 70.0 | $3.20 \cdot 10^{-3}$ | 1.087 | 1.7 | 1.6 | 0.5 | 1.4 | 0.5 | 0.0 | -0.2 | -0.1 | 0.3 | -0.3 | 0.0 | 0.1 | -0.1 |
| 70.0 | $5.00 \cdot 10^{-3}$ | 0.984 | 1.5 | 1.5 | 0.4 | 1.2 | 0.5 | 0.0 | -0.3 | 0.2 | 0.2 | -0.1 | -0.2 | 0.0 | 0.2 |
| 70.0 | $8.00 \cdot 10^{-3}$ | 0.815 | 1.6 | 1.4 | 0.5 | 1.1 | 0.2 | 0.0 | -0.4 | 0.2 | 0.1 | -0.1 | -0.2 | 0.0 | -0.3 |
| 70.0 | $1.30 \cdot 10^{-2}$ | 0.708 | 1.9 | 1.7 | 0.5 | 1.1 | 0.3 | 0.0 | -0.6 | 0.8 | -0.3 | -0.1 | 0.2 | 0.0 | -0.4 |
| 70.0 | $2.10 \cdot 10^{-2}$ | 0.620 | 1.8 | 1.6 | 0.7 | 1.1 | 0.4 | 0.0 | -0.6 | 0.6 | -0.2 | 0.1 | 0.0 | 0.0 | 0.5 |
| 70.0 | $3.20 \cdot 10^{-2}$ | 0.557 | 2.1 | 2.8 | 0.9 | 1.0 | 0.3 | 0.0 | -0.3 | -0.2 | 0.3 | -0.1 | -0.2 | 0.0 | 2.3 |
| 70.0 | $5.00 \cdot 10^{-2}$ | 0.500 | 1.6 | 6.2 | 0.9 | 1.0 | 0.3 | 0.0 | -0.7 | 0.5 | -0.2 | 0.2 | -0.1 | 0.0 | 6.0 |
| 90.0 | $2.10 \cdot 10^{-3}$ | 1.236 | 2.0 | 2.2 | 0.4 | 1.8 | 0.2 | -0.4 | -0.8 | 0.0 | 0.4 | -0.2 | 0.2 | 0.5 | 0.0 |
| 90.0 | $3.20 \cdot 10^{-3}$ | 1.168 | 1.9 | 1.7 | 0.2 | 1.5 | 0.4 | -0.3 | 0.3 | 0.1 | 0.4 | -0.2 | -0.1 | 0.0 | 0.1 |
| 90.0 | $5.00 \cdot 10^{-3}$ | 1.029 | 1.7 | 1.7 | 0.6 | 1.3 | 0.2 | -0.1 | -0.7 | -0.1 | 0.1 | -0.2 | -0.3 | 0.0 | -0.3 |
| 90.0 | $8.00 \cdot 10^{-3}$ | 0.896 | 1.8 | 1.5 | 0.2 | 1.2 | 0.7 | 0.0 | -0.4 | -0.2 | -0.1 | -0.2 | 0.0 | 0.0 | 0.1 |
| 90.0 | $1.30 \cdot 10^{-2}$ | 0.749 | 2.1 | 1.6 | 1.0 | 1.1 | 0.4 | 0.0 | -0.2 | 0.3 | -0.3 | -0.1 | 0.2 | 0.0 | 0.1 |
| 90.0 | $2.10 \cdot 10^{-2}$ | 0.622 | 1.9 | 1.9 | 0.6 | 1.1 | 0.4 | 0.0 | -0.3 | 0.9 | -0.3 | 0.2 | 0.1 | 0.0 | -1.0 |
| 90.0 | $3.20 \cdot 10^{-2}$ | 0.535 | 2.3 | 2.9 | 1.2 | 1.0 | 0.6 | 0.0 | -0.3 | 0.5 | 0.2 | 0.0 | -0.1 | 0.0 | 2.2 |
| 90.0 | $5.00 \cdot 10^{-2}$ | 0.484 | 2.1 | 6.3 | 0.9 | 1.0 | 0.2 | 0.0 | 0.2 | 0.1 | 0.2 | 0.1 | -0.2 | 0.0 | 6.1 |
| 90.0 | $8.00 \cdot 10^{-2}$ | 0.435 | 2.0 | 2.8 | 0.9 | 1.0 | 0.2 | 0.0 | -0.6 | 0.7 | -0.1 | 0.1 | 0.0 | 0.0 | 2.3 |
| 90.0 | $1.80 \cdot 10^{-1}$ | 0.340 | 3.1 | 5.3 | 3.0 | 1.0 | 0.7 | 0.0 | -3.1 | 1.6 | -1.4 | -0.7 | 0.0 | 0.0 | -1.8 |
| 120.0 | $2.10 \cdot 10^{-3}$ | 1.317 | 2.2 | 2.5 | 0.8 | 2.0 | 0.0 | 0.6 | -0.9 | 0.1 | 0.1 | 0.3 | -0.6 | 0.0 | -0.1 |
| 120.0 | $3.20 \cdot 10^{-3}$ | 1.173 | 1.9 | 2.0 | 0.7 | 1.7 | 0.3 | 0.1 | -0.7 | 0.0 | 0.3 | -0.4 | 0.0 | 0.1 | 0.0 |
| 120.0 | $5.00 \cdot 10^{-3}$ | 1.038 | 1.7 | 1.8 | 0.5 | 1.4 | 0.5 | -0.7 | 0.1 | -0.2 | 0.4 | -0.2 | -0.2 | 0.1 | 0.0 |
| 120.0 | $8.00 \cdot 10^{-3}$ | 0.908 | 1.8 | 1.4 | 0.4 | 1.2 | 0.4 | -0.3 | -0.1 | 0.1 | 0.0 | -0.1 | -0.1 | 0.0 | 0.0 |
| 120.0 | $1.30 \cdot 10^{-2}$ | 0.760 | 2.0 | 1.8 | 0.5 | 1.1 | 0.3 | 0.0 | -1.1 | 0.2 | -0.2 | -0.1 | -0.1 | 0.0 | 0.4 |
| 120.0 | $2.10 \cdot 10^{-2}$ | 0.664 | 1.9 | 1.8 | 0.7 | 1.1 | 0.4 | -0.1 | -0.2 | 1.0 | -0.4 | 0.1 | -0.2 | 0.0 | -0.2 |
| 120.0 | $3.20 \cdot 10^{-2}$ | 0.579 | 2.3 | 2.2 | 0.9 | 1.0 | 0.4 | 0.0 | -0.5 | 0.1 | 0.1 | 0.4 | -0.1 | 0.0 | -1.5 |
| 120.0 | $5.00 \cdot 10^{-2}$ | 0.472 | 2.1 | 5.1 | 0.8 | 1.0 | 0.7 | 0.0 | 0.1 | -0.3 | -0.3 | 0.1 | -0.1 | 0.0 | 4.9 |
| 120.0 | $8.00 \cdot 10^{-2}$ | 0.437 | 1.9 | 4.7 | 0.7 | 1.0 | 0.4 | 0.0 | -0.9 | 0.6 | -0.2 | 0.0 | -0.1 | 0.0 | 4.4 |
| 120.0 | $1.80 \cdot 10^{-1}$ | 0.328 | 2.7 | 8.8 | 3.2 | 1.0 | 0.6 | 0.0 | -2.2 | -1.3 | -0.9 | -0.6 | 0.0 | 0.0 | -7.6 |
| 150.0 | $3.20 \cdot 10^{-3}$ | 1.194 | 2.1 | 2.3 | 0.6 | 1.9 | 0.0 | 0.4 | -0.9 | -0.1 | 0.1 | 0.3 | -0.5 | 0.1 | 0.0 |
| 150.0 | $5.00 \cdot 10^{-3}$ | 1.037 | 1.9 | 1.8 | 0.5 | 1.5 | 0.1 | 0.4 | -0.2 | 0.1 | 0.5 | -0.3 | -0.3 | 0.0 | -0.1 |
| 150.0 | $8.00 \cdot 10^{-3}$ | 0.887 | 1.8 | 1.6 | 0.7 | 1.3 | 0.3 | -0.3 | -0.4 | 0.1 | -0.2 | 0.1 | -0.2 | 0.0 | 0.0 |
| 150.0 | $1.30 \cdot 10^{-2}$ | 0.805 | 2.1 | 2.0 | 0.5 | 1.2 | 0.4 | -1.0 | -1.0 | 0.2 | -0.2 | -0.1 | -0.3 | 0.0 | -0.3 |
| 150.0 | $2.10 \cdot 10^{-2}$ | 0.661 | 2.0 | 1.9 | 0.6 | 1.1 | 0.5 | -0.7 | -0.4 | 0.7 | -0.4 | 0.1 | 0.0 | 0.0 | 0.7 |
| 150.0 | $3.20 \cdot 10^{-2}$ | 0.574 | 2.4 | 1.7 | 0.7 | 1.0 | 0.3 | -0.7 | 0.0 | 0.7 | -0.2 | -0.1 | -0.3 | 0.0 | 0.3 |
| 150.0 | $5.00 \cdot 10^{-2}$ | 0.515 | 2.2 | 2.3 | 1.3 | 1.0 | 0.6 | -0.5 | -0.5 | -0.4 | 0.2 | 0.2 | -0.1 | 0.0 | 1.3 |
| 150.0 | $8.00 \cdot 10^{-2}$ | 0.431 | 2.0 | 5.0 | 1.0 | 1.0 | 0.5 | -0.5 | -0.4 | 0.6 | 0.1 | 0.1 | -0.1 | 0.0 | 4.7 |
| 150.0 | $1.80 \cdot 10^{-1}$ | 0.326 | 2.7 | 6.4 | 2.3 | 1.0 | 0.5 | -0.5 | -2.5 | -1.3 | -1.0 | -0.4 | 0.0 | 0.0 | -5.0 |
| 200.0 | $3.20 \cdot 10^{-3}$ | 1.162 | 3.8 | 3.2 | 2.0 | 2.0 | 0.0 | 0.8 | -1.2 | 0.2 | 0.3 | 0.4 | -0.4 | 0.1 | 0.0 |
| 200.0 | $5.00 \cdot 10^{-3}$ | 1.065 | 2.3 | 2.1 | 0.4 | 1.7 | 0.0 | 0.6 | -0.8 | -0.1 | 0.4 | -0.3 | 0.0 | 0.0 | 0.2 |
| 200.0 | $8.00 \cdot 10^{-3}$ | 0.899 | 2.2 | 1.8 | 0.6 | 1.4 | 0.1 | 0.5 | 0.6 | 0.1 | 0.4 | -0.2 | 0.1 | 0.0 | 0.0 |
| 200.0 | $1.30 \cdot 10^{-2}$ | 0.794 | 2.2 | 1.5 | 0.2 | 1.2 | 0.1 | 0.3 | -0.5 | 0.2 | -0.5 | 0.0 | -0.1 | 0.0 | -0.2 |
| 200.0 | $2.10 \cdot 10^{-2}$ | 0.659 | 2.7 | 1.8 | 0.7 | 1.1 | -0.1 | 0.7 | 0.5 | 0.4 | -0.7 | -0.1 | -0.1 | 0.0 | 0.1 |
| 200.0 | $3.20 \cdot 10^{-2}$ | 0.570 | 2.8 | 1.9 | 0.5 | 1.1 | 0.0 | 0.3 | -0.8 | 0.7 | -0.5 | 0.2 | 0.0 | 0.0 | 0.9 |
| 200.0 | $5.00 \cdot 10^{-2}$ | 0.508 | 2.8 | 2.4 | 0.5 | 1.0 | 0.3 | -0.3 | -0.6 | -0.4 | -0.2 | -0.2 | -0.1 | 0.0 | 2.0 |
| 200.0 | $8.00 \cdot 10^{-2}$ | 0.434 | 2.4 | 2.7 | 0.5 | 1.0 | -0.3 | -0.7 | 0.1 | 0.4 | 0.4 | 0.3 | -0.1 | 0.0 | 2.3 |
| 200.0 | $1.80 \cdot 10^{-1}$ | 0.340 | 2.9 | 3.7 | 0.6 | 1.0 | 0.3 | -0.1 | -2.4 | -1.2 | -0.9 | -0.3 | 0.0 | 0.0 | -1.9 |
| 250.0 | $5.00 \cdot 10^{-3}$ | 1.066 | 3.0 | 2.9 | 1.3 | 1.9 | 0.0 | 0.6 | -1.6 | 0.1 | 0.4 | -0.2 | 0.1 | 0.0 | 0.0 |
| 250.0 | $8.00 \cdot 10^{-3}$ | 0.927 | 2.6 | 1.7 | 0.5 | 1.5 | 0.0 | 0.5 | -0.2 | -0.1 | 0.4 | -0.2 | -0.1 | 0.0 | 0.0 |
| 250.0 | $1.30 \cdot 10^{-2}$ | 0.821 | 2.7 | 1.8 | 0.9 | 1.3 | 0.0 | 0.4 | 0.5 | 0.3 | 0.5 | 0.1 | -0.1 | 0.0 | 0.4 |
| 250.0 | $2.10 \cdot 10^{-2}$ | 0.644 | 3.5 | 1.8 | 0.6 | 1.1 | 0.0 | -0.4 | -0.8 | -0.3 | -0.3 | -0.2 | 0.0 | 0.0 | -0.7 |
| 250.0 | $3.20 \cdot 10^{-2}$ | 0.612 | 3.4 | 2.3 | 1.0 | 1.1 | 0.1 | 0.9 | -0.7 | 0.9 | -1.0 | 0.0 | 0.1 | 0.0 | -0.3 |

Table 2. (continued)

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{array}$ | $x$ | $\tilde{\sigma}^{e^{+} p}$ | $\delta_{\text {stat }}$ <br> (\%) | $\begin{gathered} \delta_{\text {sys }} \\ (\%) \end{gathered}$ | $\delta_{u n c}$ <br> (\%) | $\begin{gathered} \delta_{\{1\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{2\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{3\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{4\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{5\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{6\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{7\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{8\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{9\}} \\ (\%) \end{gathered}$ | $\begin{array}{r} \delta_{\{10\}} \\ (\%) \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 250.0 | $5.00 \cdot 10^{-2}$ | 0.517 | 3.2 | 1.8 | 1.1 | 1.0 | 0.1 | 0.5 | 0.3 | 0.4 | -0.1 | 0.1 | -0.2 | 0.0 | -0.3 |
| 250.0 | $8.00 \cdot 10^{-2}$ | 0.445 | 2.8 | 5.1 | 0.3 | 1.0 | 0.1 | 1.0 | -0.6 | -0.2 | 0.2 | -0.3 | -0.1 | 0.0 | 4.8 |
| 250.0 | $1.80 \cdot 10^{-1}$ | 0.341 | 3.2 | 5.6 | 1.1 | 1.0 | 0.1 | 0.5 | -1.0 | 1.3 | -0.2 | 0.3 | 0.0 | 0.0 | -5.1 |
| 350.0 | $5.00 \cdot 10^{-3}$ | 1.103 | 4.4 | 3.5 | 2.7 | 2.0 | 0.0 | -0.2 | -0.6 | -0.2 | 0.3 | 0.4 | 0.0 | 0.0 | -0.2 |
| 350.0 | $8.00 \cdot 10^{-3}$ | 0.988 | 2.9 | 2.1 | 0.4 | 1.7 | 0.0 | 0.2 | -0.8 | -0.1 | 0.8 | 0.1 | -0.4 | 0.0 | -0.2 |
| 350.0 | $1.30 \cdot 10^{-2}$ | 0.824 | 2.9 | 1.7 | 0.5 | 1.3 | 0.0 | 0.7 | -0.3 | -0.1 | 0.1 | -0.1 | -0.3 | 0.0 | 0.0 |
| 350.0 | $2.10 \cdot 10^{-2}$ | 0.679 | 3.6 | 1.9 | 1.1 | 1.2 | 0.0 | 0.8 | -0.4 | 0.2 | -0.4 | -0.1 | 0.0 | 0.0 | 0.0 |
| 350.0 | $3.20 \cdot 10^{-2}$ | 0.639 | 3.6 | 2.0 | 0.8 | 1.1 | 0.0 | 0.6 | 0.5 | 0.8 | -0.8 | 0.1 | -0.2 | 0.0 | -0.5 |
| 350.0 | $5.00 \cdot 10^{-2}$ | 0.526 | 3.4 | 1.9 | 0.6 | 1.0 | 0.0 | 0.8 | -0.8 | 0.3 | -0.5 | -0.1 | -0.1 | 0.0 | 0.8 |
| 350.0 | $8.00 \cdot 10^{-2}$ | 0.442 | 3.2 | 2.6 | 0.6 | 1.0 | 0.0 | 0.5 | -0.7 | -0.5 | -0.2 | 0.0 | -0.1 | 0.0 | 2.1 |
| 350.0 | $1.80 \cdot 10^{-1}$ | 0.328 | 3.5 | 1.6 | 0.4 | 1.0 | 0.0 | 0.2 | -0.8 | 0.6 | -0.3 | 0.3 | 0.0 | 0.0 | -0.3 |
| 450.0 | $8.00 \cdot 10^{-3}$ | 0.954 | 3.5 | 2.4 | 1.4 | 1.8 | 0.0 | -0.2 | -0.1 | -0.1 | 0.6 | -0.1 | -0.1 | 0.3 | -0.1 |
| 450.0 | $1.30 \cdot 10^{-2}$ | 0.854 | 3.0 | 1.7 | 0.6 | 1.4 | 0.0 | 0.4 | -0.4 | 0.1 | 0.3 | -0.1 | -0.1 | 0.0 | 0.1 |
| 450.0 | $2.10 \cdot 10^{-2}$ | 0.702 | 3.6 | 1.6 | 0.8 | 1.2 | 0.0 | 0.2 | 0.5 | 0.1 | 0.2 | 0.1 | 0.1 | 0.0 | 0.2 |
| 450.0 | $3.20 \cdot 10^{-2}$ | 0.579 | 4.3 | 1.8 | 0.3 | 1.1 | 0.0 | 0.7 | -0.7 | 0.7 | -0.6 | -0.1 | 0.0 | 0.0 | -0.5 |
| 450.0 | $5.00 \cdot 10^{-2}$ | 0.511 | 3.4 | 1.5 | 0.5 | 1.0 | 0.0 | 0.3 | -0.4 | 0.6 | -0.5 | 0.1 | -0.3 | 0.0 | 0.0 |
| 450.0 | $8.00 \cdot 10^{-2}$ | 0.439 | 3.6 | 2.3 | 1.1 | 1.0 | 0.0 | 0.5 | -0.5 | -0.7 | -0.1 | 0.0 | -0.1 | 0.0 | 1.3 |
| 450.0 | $1.30 \cdot 10^{-1}$ | 0.365 | 3.8 | 2.3 | 1.0 | 1.0 | 0.0 | 0.9 | -0.2 | 0.2 | 0.2 | 0.1 | 0.0 | 0.0 | 1.6 |
| 450.0 | $2.50 \cdot 10^{-1}$ | 0.264 | 4.5 | 7.7 | 1.7 | 1.0 | 0.0 | 0.9 | -2.4 | -0.8 | -0.7 | -0.3 | 0.0 | 0.0 | -6.9 |
| 650.0 | $8.00 \cdot 10^{-3}$ | 0.926 | 4.6 | 3.2 | 1.8 | 1.9 | 0.0 | 0.2 | -1.5 | -0.1 | 0.0 | 0.5 | -0.7 | 0.0 | 0.0 |
| 650.0 | $1.30 \cdot 10^{-2}$ | 0.827 | 3.3 | 1.9 | 0.7 | 1.5 | 0.0 | 0.4 | -0.3 | -0.1 | 0.6 | -0.2 | -0.2 | 0.1 | 0.1 |
| 650.0 | $2.10 \cdot 10^{-2}$ | 0.722 | 3.8 | 1.7 | 0.9 | 1.2 | 0.0 | 0.4 | -0.5 | 0.2 | 0.3 | 0.0 | 0.1 | 0.0 | -0.1 |
| 650.0 | $3.20 \cdot 10^{-2}$ | 0.603 | 4.0 | 1.9 | 0.9 | 1.1 | 0.0 | 0.3 | -0.4 | 0.2 | -0.1 | 0.1 | 0.1 | 0.0 | 1.1 |
| 650.0 | $5.00 \cdot 10^{-2}$ | 0.475 | 4.3 | 2.1 | 1.0 | 1.0 | 0.0 | 0.4 | -0.4 | 1.1 | -0.9 | -0.1 | 0.0 | 0.0 | -0.2 |
| 650.0 | $8.00 \cdot 10^{-2}$ | 0.456 | 3.9 | 1.5 | 0.8 | 1.0 | 0.0 | -0.3 | -0.4 | -0.3 | 0.1 | 0.2 | -0.1 | 0.0 | 0.6 |
| 650.0 | $1.30 \cdot 10^{-1}$ | 0.357 | 4.2 | 3.2 | 1.4 | 1.0 | 0.0 | 0.3 | -0.8 | 0.3 | 0.2 | 0.1 | 0.0 | 0.0 | 2.5 |
| 650.0 | $2.50 \cdot 10^{-1}$ | 0.250 | 4.8 | 3.4 | 1.7 | 1.0 | 0.0 | 0.7 | -1.8 | -0.4 | -0.6 | -0.1 | 0.0 | 0.0 | 1.8 |
| 650.0 | $4.00 \cdot 10^{-1}$ | 0.152 | 9.5 | 14.5 | 0.8 | 1.0 | 0.0 | 1.1 | -5.6 | -3.9 | -1.3 | -1.3 | 0.0 | 0.0 | -12.5 |
| 800.0 | $1.30 \cdot 10^{-2}$ | 0.847 | 4.5 | 2.4 | 1.0 | 1.7 | 0.0 | 0.3 | -1.0 | 0.0 | 0.4 | -0.2 | -0.4 | 0.0 | -0.4 |
| 800.0 | $2.10 \cdot 10^{-2}$ | 0.665 | 7.5 | 1.6 | 0.6 | 1.3 | 0.0 | 0.1 | 0.6 | -0.1 | 0.4 | -0.2 | 0.1 | 0.0 | -0.4 |
| 800.0 | $3.20 \cdot 10^{-2}$ | 0.647 | 4.9 | 1.4 | 0.6 | 1.1 | 0.0 | 0.5 | -0.4 | -0.1 | -0.3 | 0.0 | -0.1 | 0.0 | -0.2 |
| 800.0 | $5.00 \cdot 10^{-2}$ | 0.451 | 5.0 | 2.3 | 1.3 | 1.0 | 0.0 | 0.3 | 0.3 | 1.1 | -0.8 | 0.0 | 0.0 | 0.0 | 0.6 |
| 800.0 | $8.00 \cdot 10^{-2}$ | 0.420 | 5.8 | 2.6 | 1.0 | 1.0 | 0.0 | 0.9 | -1.4 | 0.6 | -0.4 | -0.1 | 0.0 | 0.0 | 1.2 |
| 800.0 | $1.30 \cdot 10^{-1}$ | 0.370 | 5.2 | 2.5 | 0.8 | 1.0 | 0.0 | 0.6 | -0.7 | -0.5 | 0.0 | -0.1 | 0.0 | 0.0 | -1.9 |
| 800.0 | $2.50 \cdot 10^{-1}$ | 0.260 | 5.9 | 1.8 | 0.4 | 1.0 | 0.0 | -0.1 | -0.5 | 0.3 | -0.2 | 0.2 | 0.0 | 0.0 | 1.3 |
| 800.0 | $4.00 \cdot 10^{-1}$ | 0.114 | 11.4 | 13.5 | 2.2 | 1.0 | 0.0 | 0.8 | -5.3 | -3.1 | -0.8 | -0.8 | 0.0 | 0.0 | -11.7 |
| 1200.0 | $1.40 \cdot 10^{-2}$ | 0.872 | 5.5 | 2.9 | 1.1 | 1.7 | 0.0 | 0.2 | -2.0 | 0.1 | 0.5 | 0.3 | -0.3 | 0.0 | 0.0 |
| 1200.0 | $2.10 \cdot 10^{-2}$ | 0.681 | 5.1 | 1.8 | 0.8 | 1.3 | 0.0 | 0.2 | -0.6 | -0.1 | 0.5 | -0.1 | 0.3 | 0.0 | 0.0 |
| 1200.0 | $3.20 \cdot 10^{-2}$ | 0.563 | 5.0 | 2.0 | 1.5 | 1.1 | 0.0 | 0.3 | 0.5 | 0.2 | 0.1 | 0.1 | 0.2 | 0.0 | -0.1 |
| 1200.0 | $5.00 \cdot 10^{-2}$ | 0.490 | 4.6 | 1.6 | 1.0 | 1.0 | 0.0 | 0.4 | -0.2 | 0.4 | -0.2 | -0.2 | -0.1 | 0.0 | -0.2 |
| 1200.0 | $8.00 \cdot 10^{-2}$ | 0.434 | 5.3 | 1.5 | 0.4 | 1.0 | 0.0 | -0.1 | -0.3 | 0.7 | -0.6 | 0.2 | -0.2 | 0.0 | -0.1 |
| 1200.0 | $1.30 \cdot 10^{-1}$ | 0.336 | 5.8 | 3.1 | 0.6 | 1.0 | 0.0 | 0.3 | -0.6 | 0.5 | 0.1 | -0.1 | 0.0 | 0.0 | 2.8 |
| 1200.0 | $2.50 \cdot 10^{-1}$ | 0.254 | 5.4 | 2.0 | 1.0 | 1.0 | 0.0 | 0.5 | -1.3 | -0.3 | -0.1 | 0.1 | 0.0 | 0.0 | 0.1 |
| 1200.0 | $4.00 \cdot 10^{-1}$ | 0.149 | 8.1 | 10.3 | 2.9 | 1.0 | 0.0 | 0.3 | -3.1 | -2.4 | -0.7 | -0.2 | 0.0 | 0.0 | -9.0 |
| 1200.0 | $6.50 \cdot 10^{-1}$ | 0.021 | 35.6 | 25.1 | 17.5 | 1.0 | 0.0 | -0.9 | -13.3 | -5.3 | -1.5 | -2.2 | 0.0 | 0.0 | -10.5 |
| 1500.0 | $2.10 \cdot 10^{-2}$ | 0.666 | 7.9 | 6.2 | 5.8 | 1.5 | 0.0 | 0.3 | 1.0 | 0.0 | 0.3 | -0.4 | 0.2 | 0.0 | -0.5 |
| 1500.0 | $3.20 \cdot 10^{-2}$ | 0.540 | 10.3 | 2.0 | 1.3 | 1.1 | 0.0 | 0.2 | -0.2 | 0.2 | 0.5 | 0.0 | 0.3 | 0.0 | -0.8 |
| 1500.0 | $5.00 \cdot 10^{-2}$ | 0.518 | 5.8 | 1.8 | 1.1 | 1.0 | 0.0 | 0.3 | -0.4 | -0.2 | -0.2 | -0.1 | -0.4 | 0.0 | 0.5 |
| 1500.0 | $8.00 \cdot 10^{-2}$ | 0.476 | 5.7 | 2.6 | 1.0 | 1.0 | 0.0 | 0.5 | -0.6 | 0.8 | -1.0 | -0.1 | 0.1 | 0.0 | -1.6 |
| 1500.0 | $1.30 \cdot 10^{-1}$ | 0.301 | 11.8 | 1.9 | 1.2 | 1.0 | 0.0 | 0.2 | 0.1 | -0.1 | -0.3 | 0.2 | 0.0 | 0.0 | 1.0 |
| 1500.0 | $1.80 \cdot 10^{-1}$ | 0.313 | 7.3 | 5.9 | 1.2 | 1.0 | 0.0 | 0.2 | -0.9 | 0.6 | 0.3 | -0.1 | 0.0 | 0.0 | 5.6 |
| 1500.0 | $2.50 \cdot 10^{-1}$ | 0.270 | 8.6 | 3.0 | 2.2 | 1.0 | 0.0 | 0.1 | -1.7 | -0.5 | -0.1 | 0.1 | 0.0 | 0.0 | 0.0 |

Table 2. (continued)

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{array}$ | $x$ | $\tilde{\sigma}^{e^{+} p}$ | $\delta_{\text {stat }}$ <br> (\%) | $\delta_{s y s}$ <br> (\%) | $\delta_{u n c}$ <br> (\%) | $\delta_{\{1\}}$ <br> (\%) | $\begin{gathered} \delta_{\{2\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{3\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{4\}} \\ (\%) \end{gathered}$ | $\delta_{\{5\}}$ <br> (\%) | $\delta_{\{6\}}$ <br> (\%) | $\begin{gathered} \delta_{\{7\}} \\ (\%) \end{gathered}$ | $\begin{gathered} \delta_{\{8\}} \\ (\%) \\ \hline \end{gathered}$ | $\begin{gathered} \delta_{\{9\}} \\ (\%) \\ \hline \end{gathered}$ | $\begin{array}{r} \delta_{\{10\}} \\ (\%) \\ \hline \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1500.0 | $4.00 \cdot 10^{-1}$ | 0.146 | 12.6 | 13.7 | 1.9 | 1.0 | 0.0 | 0.4 | -3.9 | -1.7 | -0.9 | -0.2 | 0.0 | 0.0 | -12.8 |
| 1500.0 | $6.50 \cdot 10^{-1}$ | 0.012 | 30.9 | 19.0 | 8.8 | 1.0 | 0.0 | 0.8 | -7.6 | -6.0 | -1.0 | -0.4 | 0.0 | 0.0 | 13.6 |
| 2000.0 | $3.20 \cdot 10^{-2}$ | 0.586 | 8.3 | 3.4 | 3.1 | 1.1 | 0.0 | 0.4 | -0.4 | -0.1 | 0.5 | -0.2 | -0.4 | 0.0 | 0.6 |
| 2000.0 | $5.00 \cdot 10^{-2}$ | 0.476 | 7.4 | 2.0 | 1.4 | 1.0 | 0.0 | 0.3 | -0.8 | 0.1 | 0.0 | 0.1 | -0.2 | 0.0 | -0.4 |
| 2000.0 | $8.00 \cdot 10^{-2}$ | 0.444 | 7.1 | 2.0 | 1.1 | 1.0 | 0.0 | 0.3 | -0.3 | 0.8 | -0.6 | 0.1 | -0.1 | 0.0 | 0.8 |
| 2000.0 | $1.30 \cdot 10^{-1}$ | 0.340 | 8.3 | 3.9 | 1.9 | 1.0 | 0.0 | 0.3 | -0.6 | 0.4 | -0.4 | -0.1 | 0.0 | 0.0 | -3.1 |
| 2000.0 | $1.80 \cdot 10^{-1}$ | 0.282 | 9.1 | 2.6 | 1.8 | 1.0 | 0.0 | 0.4 | -1.2 | -0.7 | 0.1 | -0.1 | 0.0 | 0.0 | -0.5 |
| 2000.0 | $2.50 \cdot 10^{-1}$ | 0.234 | 10.4 | 5.3 | 2.3 | 1.0 | 0.0 | 0.4 | -0.9 | -0.4 | -0.2 | 0.1 | 0.0 | 0.0 | 4.5 |
| 2000.0 | $4.00 \cdot 10^{-1}$ | 0.170 | 13.8 | 10.1 | 2.0 | 1.0 | 0.0 | -0.2 | -4.0 | -2.5 | -0.5 | -0.2 | 0.0 | 0.0 | -8.6 |
| 3000.0 | $5.00 \cdot 10^{-2}$ | 0.542 | 8.7 | 2.1 | 1.6 | 1.0 | 0.0 | 0.1 | 0.9 | -0.1 | 0.4 | -0.2 | 0.2 | 0.0 | -0.1 |
| 3000.0 | $8.00 \cdot 10^{-2}$ | 0.438 | 8.5 | 2.2 | 0.8 | 1.0 | 0.0 | 0.3 | -0.4 | -0.1 | -0.3 | 0.0 | -0.2 | 0.0 | -1.6 |
| 3000.0 | $1.30 \cdot 10^{-1}$ | 0.325 | 13.9 | 4.4 | 1.4 | 1.0 | 0.0 | 0.3 | 0.6 | 1.3 | -1.0 | -0.1 | 0.0 | 0.0 | 3.7 |
| 3000.0 | $1.80 \cdot 10^{-1}$ | 0.243 | 11.7 | 3.8 | 1.4 | 1.0 | 0.0 | 0.3 | -1.0 | -0.5 | -0.1 | 0.1 | 0.0 | 0.0 | -3.1 |
| 3000.0 | $2.50 \cdot 10^{-1}$ | 0.193 | 17.7 | 4.2 | 2.2 | 1.0 | 0.0 | -0.2 | -1.3 | -0.5 | -0.1 | -0.2 | 0.7 | 0.0 | -3.1 |
| 3000.0 | $4.00 \cdot 10^{-1}$ | 0.089 | 21.7 | 9.1 | 6.7 | 1.0 | 0.0 | 0.5 | -4.3 | -1.3 | -0.6 | 0.4 | 0.0 | 0.0 | 4.1 |
| 3000.0 | $6.50 \cdot 10^{-1}$ | 0.016 | 33.9 | 15.3 | 8.1 | 1.0 | 0.0 | -0.1 | -10.8 | -7.0 | -0.7 | -0.2 | 0.0 | 0.0 | 1.1 |
| 5000.0 | $8.00 \cdot 10^{-2}$ | 0.376 | 9.0 | 2.7 | 2.2 | 1.0 | 0.0 | 0.2 | -0.9 | 0.0 | 0.5 | 0.0 | 0.1 | 0.6 | 0.0 |
| 5000.0 | $1.30 \cdot 10^{-1}$ | 0.284 | 11.7 | 2.0 | 0.8 | 1.0 | 0.0 | 0.4 | 1.3 | 0.6 | -0.6 | 0.0 | 0.0 | 0.0 | 0.3 |
| 5000.0 | $1.80 \cdot 10^{-1}$ | 0.278 | 11.3 | 5.2 | 2.4 | 1.0 | 0.0 | 0.1 | -0.7 | 0.3 | -0.5 | 0.1 | 0.0 | 0.0 | 4.4 |
| 5000.0 | $2.50 \cdot 10^{-1}$ | 0.223 | 13.2 | 3.5 | 2.6 | 1.0 | 0.0 | 0.2 | -2.1 | 0.5 | 0.1 | -0.2 | 0.0 | 0.0 | 0.1 |
| 5000.0 | $4.00 \cdot 10^{-1}$ | 0.126 | 18.3 | 5.4 | 1.6 | 1.0 | 0.0 | -0.3 | -2.9 | -1.6 | -0.3 | 0.3 | 0.0 | 0.0 | -3.8 |
| 8000.0 | $1.30 \cdot 10^{-1}$ | 0.241 | 14.5 | 4.2 | 4.0 | 1.0 | 0.0 | 0.3 | -0.7 | -0.1 | -0.1 | 0.0 | 0.0 | 0.0 | 0.0 |
| 8000.0 | $1.80 \cdot 10^{-1}$ | 0.272 | 15.4 | 5.6 | 3.6 | 1.0 | 0.0 | 0.3 | -0.5 | 1.0 | -0.8 | -0.1 | 0.0 | 0.0 | 4.0 |
| 8000.0 | $2.50 \cdot 10^{-1}$ | 0.199 | 18.1 | 10.8 | 0.6 | 1.0 | 0.0 | 0.5 | -1.6 | -0.9 | -0.1 | 0.1 | 0.0 | 0.0 | -10.6 |
| 8000.0 | $4.00 \cdot 10^{-1}$ | 0.119 | 23.8 | 6.8 | 0.8 | 1.0 | 0.0 | -0.2 | -4.1 | -1.6 | -0.4 | -0.2 | 0.0 | 0.0 | 5.0 |
| 8000.0 | $6.50 \cdot 10^{-1}$ | 0.036 | 30.6 | 14.2 | 5.7 | 1.0 | 0.0 | -0.6 | -8.1 | -4.1 | -0.2 | 0.1 | 0.0 | 0.0 | -9.1 |
| 12000.0 | $1.80 \cdot 10^{-1}$ | 0.238 | 20.1 | 3.7 | 3.1 | 1.0 | 0.0 | 0.2 | -1.7 | 0.2 | 0.3 | 0.1 | 0.0 | 0.0 | 0.1 |
| 12000.0 | $2.50 \cdot 10^{-1}$ | 0.172 | 25.9 | 4.1 | 3.7 | 1.0 | 0.0 | 0.1 | -0.6 | 0.4 | -0.3 | -0.1 | 0.0 | 0.0 | 1.2 |
| 12000.0 | $4.00 \cdot 10^{-1}$ | 0.084 | 38.0 | 3.3 | 0.8 | 1.0 | 0.0 | 0.5 | -2.7 | -1.2 | 0.0 | -0.1 | 0.0 | 0.0 | 0.5 |
| 20000.0 | $2.50 \cdot 10^{-1}$ | 0.161 | 35.4 | 5.2 | 4.8 | 1.0 | 0.0 | 0.7 | -1.5 | 0.0 | 0.1 | 0.0 | 0.0 | 0.0 | -0.1 |
| 20000.0 | $4.00 \cdot 10^{-1}$ | 0.080 | +79 +48 | 13.9 | 12.6 | 1.0 | 0.0 | -0.5 | -4.7 | -1.4 | -0.5 | -0.1 | 0.0 | 0.0 | -3.0 |
| 30000.0 | $4.00 \cdot 10^{-1}$ | 0.039 | ${ }_{-83}^{+230}$ | 6.8 | 4.6 | 1.0 | 0.0 | 0.5 | -4.9 | -0.4 | 0.2 | 0.0 | 0.0 | 0.0 | -0.6 |

\{9\} background subtractions.
As described above, the photoproduction background estimated using the PYTHIA MC simulation agreed with the photoproduction contribution extracted from events tagged by the positron tagger. Uncertainties in the positron-tagger efficiency and in the fake-positron background in the PYTHIA MC simulation led to an overall uncertainty in the photoproduction background of $\pm 35 \%$, as is indicated in Fig.4b. This resulted in an uncertainty in $F_{2}^{e m}$ which is at most $11 \%$ for $y \gtrsim 0.4$ and negligible elsewhere;
$\{10\}$ hadronic energy flow - B.
The hadronic angle $\gamma_{\mathrm{PT}}$ was calculated using the energy and angle of hadronic clusters (see Sect.5). The hadronic angle can lie inside the forward beamhole for low- $y$ events, since the energy deposits around the beamhole are merged into one cluster.

To estimate the uncertainty in the cluster algorithm and the simulation of the hadronic final state in the forward direction, a summation over energy deposits in calorimeter cells, instead of EFOs, was used in the hadronic reconstruction. Systematic variations in $F_{2}^{e m}$ up to $10 \%$ were observed in the lowest $W$ bins, leaving $F_{2}^{e m}$ at medium and high $y$ unchanged.

The four uncorrelated systematic uncertainties are:

1) positron finding and efficiency.

Varying the isolation criterion requirement of 5 GeV by $\pm 1 \mathrm{GeV}$ around an $\eta-\phi$ cone of radius 0.8 centred on the positron had a negligible effect on $F_{2}^{e m}$;
2) positron scattering angle.

Enlarging the box cut, discussed in Sect. 9.1, by 1 cm in both data and MC simulation had an effect of $\pm 1-3 \%$ on $F_{2}^{e m}$ for low values of $Q^{2} \lesssim 10 \mathrm{GeV}^{2}$ and a negligible effect elsewhere;

## ZEUS



Fig. 7. a-d The relative uncorrelated systematic, total systematic, statistical, and total systematic uncertainty. Three regions in $Q^{2}$ are denoted by different symbols. e-h Four typical sources of the correlated systematic uncertainty, where the number in $\}$ corresponds to the number in Sect.10.2. The dotted lines are at $\pm 2 \%$
3) hadronic energy flow - A.

The value of $\gamma_{\max }$ (see Sect. 8) was varied by $\pm 10^{\circ}$, hence varying the amount of hadronic energy removed from the event. Only small variations in the bias of $\delta_{h}$ were seen in the MC simulation. This reflects the uncertainty in the MC simulation of low-energy deposits in the CAL and the albedo effect. Variations of up to $2 \%$ in $F_{2}^{e m}$ were observed in the bins at $y \lesssim 0.02$; elsewhere the variations were within $1 \%$. In the low statistics bins at $x=0.65$ and $Q^{2}>1200 \mathrm{GeV}^{2}$, this effect increased to $\sim 10 \%$;
4) hadronic energy flow - B.

The fraction of events removed by the $P_{T, h} / P_{T, e}$ cut is sensitive to the amount of $P_{T, h}$ lost in the forward beampipe and thus to the details of the jet and proton-
remnant fragmentation. The $P_{T, h} / P_{T, e}$ cut was varied from 0.3 by $\pm 0.1$, which resulted in changes of $1 \%$ at low $y$.

The systematic uncertainties that did not exhibit a clear correlation were added in quadrature and are listed in Table 2 as $\delta_{u n c}$.

The satellite bunches were varied in the MC simulation by $\pm 30 \%$ to estimate the uncertainty in the luminosity corresponding to the events in the satellite bunch. Effects on the measured $F_{2}^{e m}$ were typically $0.5-1 \%$ and were included in the normalisation uncertainty.

Both the uncorrelated and correlated contributions to the systematic uncertainty were symmetrised by averaging the positive and negative deviations. For the asymmetric

## ZEUS



Fig. 8. a The statistical uncertainty on $F_{2}^{e m}$. b The systematic uncertainty. Over a large region of phase space both the statistical and systematic uncertainties are below $2 \%$. At the edges of the measurable region the systematic uncertainty increases to $\sim 10 \%$. For $y \gtrsim 0.5$, the uncertainty increases due to the uncertainty on the photoproduction background, whereas for $y \lesssim 0.01$ the uncertainty increases mainly because of uncertainties on the hadronic energy flow
systematic checks, such as the boxcut and the check on the clustering, where only a variation in one direction was applied, the variation of the $F_{2}^{e m}$ measurement was symmetrised by mirroring the variation.

Figure 7 shows the statistical and systematic uncertainties on $F_{2}^{e m}$ for each bin as a function of $y$. Four contributions to the correlated systematic uncertainty are also shown. The two largest contributions to the correlated systematic uncertainty come from $\{9\}$ and $\{10\}$. Most sources of the systematic uncertainty show a correlation between the points as a function of $y$. The variation of the gap between the two RCAL halves shows a correlation in $Q^{2}$ Figure 8 shows the statistical and total systematic uncertainty in each individual bin in the $\left(x, Q^{2}\right)$-plane.

The systematic uncertainties presented above do not include the uncertainty in the measurement of the integrated luminosity ( $\pm 1.5 \%$ ), the overall trigger efficiency
( $\pm 1.0 \%$ ), the higher-order electroweak radiative corrections $( \pm 0.5 \%)$ or the determination of the vertex distribution ( $\pm 1.0 \%$ ). These effects lead to a combined uncertainty of $\pm 2 \%$ on the overall normalisation of $F_{2}^{e m}$. An additional $\pm 1 \%$ relative normalisation uncertainty was assigned to the low- $Q^{2}$ sample to account for the additional uncertainty on the shape of the vertex distribution in the MC simulation.

### 10.3 Reduced cross section and $F_{2}^{e m}$

The values of the reduced cross section, $\tilde{\sigma}^{e^{+} p}$, and the electromagnetic structure function, $F_{2}^{e m}$, are given in Table 1 together with their statistical $\left(\delta_{\text {stat }}\right)$ and systematic ( $\delta_{\text {sys }}$ ) uncertainties, the corrections due to the $Z^{0}$ contribution to $F_{2}, \Delta_{F_{2}}$, to the parity-violating component of

ZEUS


Fig. 9. The results for $F_{2}^{e m}$ (points) are shown versus $x$ for fixed $Q^{2}$. The full curves show results from the ZEUS NLO QCD fit and the dashed curves show predictions from CTEQ4D. The inner error bars (delimited by the horizontal lines) show the statistical uncertainties; the outer ones show the statistical and systematic uncertainties added in quadrature. The overall normalisation uncertainties, see Sect. 10.2, are not included
the $Z^{0}$ exchange, $\Delta_{x F_{3}}$, and to the longitudinal structure function, $\Delta_{F_{L}}$, see $(7)^{3}$. The contribution of $F_{L}$ to $\tilde{\sigma}^{++} p$ becomes large towards large values of $y$; the contribution of $Z^{0}$ exchange is substantial at large $Q^{2}$.

The $F_{2}^{e m}$ values are displayed versus $x$ for fixed values of $Q^{2}$ in Figs. 9-12. The structure function is measured with much improved precision with respect to the previous ZEUS results [1]. Also shown is the result of the ZEUS NLO QCD fit described below.

Figures 13 and 14 show the $F_{2}^{e m}$ values as a function of $Q^{2}$ for fixed $x$. Scaling violations are observed, which decrease as $x$ increases. Where the data from the measurements presented here at high $x$ overlap the $x$ range covered by fixed-target experiments [36-38], the agreement

[^4]
## ZEUS



Fig. 10. The results for $F_{2}^{e m}$ (points) are shown versus $x$ for fixed $Q^{2}$. For details, see caption of Fig. 9
is good, as shown in Fig. 14. The results also agree well with the recently published data of the H1 collaboration $[2,3]$ and with the recent parameterisations from CTEQ [39] and MRST [40].

The $Q^{2}$-range has increased substantially with respect to previous ZEUS measurements. The ZEUS data in combination with fixed-target data now span almost four orders of magnitude in $Q^{2}$ at $x \sim 0.4$.

### 10.4 NLO QCD fit

A next-to-leading-order (NLO) QCD fit to the reduced cross section from this analysis has been performed. Apart from the present data, proton-target data from NMC [36], BCDMS [37], and E665 [38] were used. Deuteron-target data were used from NMC [36] and E665 [38], as well as data on the ratio $F_{2}^{n} / F_{2}^{p}$ from NMC [41]. The only neutrino results included in the fit are $x F_{3}$ data from CCFR [42].

To remain in the kinematic region where pQCD should be applicable, only data with $Q^{2}>2.5 \mathrm{GeV}^{2}$ were included in the fit. The resonance region and the region

ZEUS


Fig. 11. The results for $F_{2}^{e m}$ (points) are shown versus $x$ for fixed $Q^{2}$. For details, see caption of Fig. 9
affected by higher-twist contributions and target-mass effects ( $x \gtrsim 0.1, Q^{2} \lesssim 20 \mathrm{GeV}^{2}$ ) were avoided by requiring $W^{2}>20 \mathrm{GeV}^{2}$.

The parameterisations of the parton density functions (PDFs) were assumed to have the following functional form, adapted from MRST [40]:

$$
x f\left(x, Q_{0}^{2}\right)=A x^{\delta}(1-x)^{\eta}(1+\gamma x)
$$

and were evaluated at $Q_{0}^{2}=7 \mathrm{GeV}^{2}$. The $x^{\delta}$ term parameterises the steep rise at low $x$, whereas $(1-x)^{\eta}$ describes the shape of the valence quarks at high $x$.

The fit was done in the $\overline{\mathrm{MS}}$ scheme with a variable-flavour-number ansatz (RT-VFN) [43]. A charm mass of 1.35 GeV was assumed. The strange-quark distribution, $x(s(x)+\bar{s}(x))=2 x \bar{s}(x)$, was fixed to the world-average value 0.20 of the total sea, according to measurements of CCFR [44]. The value of $\alpha_{S}$ was fixed to the world average value, $\alpha_{S}\left(M_{Z}^{2}\right)=0.118$.

The QCD NLO fit reproduces the data over the full kinematic range, indicating that NLO DGLAP evolution can give a consistent description. The overall $\chi^{2} /$ ndf of the fit is 0.95 for 1263 data points, where the $\chi^{2}$ is calculated using the statistical and total systematic uncertainty added in quadrature. The $\chi^{2} / \mathrm{ndf}$ for the present data is 0.84 for the 242 ZEUS data points.

## ZEUS



Fig. 12. The results for $F_{2}^{e m}$ (points) are shown versus $x$ for fixed $Q^{2}$. For details, see caption of Fig. 9

## ZEUS



Fig. 13. The results for $F_{2}^{e m}$ (points) versus $Q^{2}$ are shown for fixed $x$. The fixed-target results from NMC, BCDMS and E665 (triangles) and the ZEUS NLO QCD fit (curve) are also shown

## ZEUS



Fig. 14. The results for $F_{2}^{e m}$ versus $Q^{2}$, for six bins at fixed $x$, are compared with results from NMC, BCDMS, E665 (triangles) and the recently published H1 results (open symbols). The inner error bars (delimited by the horizontal lines) show the statistical uncertainties; the outer ones show the statistical and systematic uncertainties added in quadrature, both for ZEUS and H1

## 11 Conclusions

Measurements of the reduced cross section and the proton structure function, $F_{2}^{e m}$, have been presented from an analysis of inelastic positron-proton neutral current scattering data. The data were taken with the ZEUS detector at HERA during 1996 and 1997. The structure function $F_{2}^{e m}$ has now been measured over a substantially larger phase space than covered by previous ZEUS measurements and with statistical and systematic uncertainties below $2 \%$ in most of the ( $x, Q^{2}$ ) region covered. The data cover $Q^{2}$ values between 2.7 and $30000 \mathrm{GeV}^{2}$ and $x$ values between $6 \cdot 10^{-5}$ and 0.65 .

At large $x$ values and $Q^{2}$ values up to $70 \mathrm{GeV}^{2}$, where the new data reach the $x$-range covered by fixed-target experiments, good agreement with these experiments is found. There is also good agreement with the data of the H1 collaboration. Strong scaling violations are observed
for $x<0.02$. The measured $x-Q^{2}$ behaviour of $F_{2}^{e m}$ can be described by QCD using NLO DGLAP evolution over the full kinematic range.

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[^2]:    ${ }^{1}$ Data on $F_{2}^{e m}$ based on similar statistics and in a similar kinematic range have recently been presented by the H 1 collaboration $[2,3]$

[^3]:    ${ }^{2}$ The ZEUS coordinate system is a right-handed Cartesian system, with the $Z$ axis pointing in the proton beam direction, referred to as the "forward direction", and the $X$ axis pointing left towards the centre of HERA. The coordinate origin is at the nominal interaction point

[^4]:    ${ }^{3}$ The table with the results and the uncertainities can be obtained from: http://www-zeus.desy.de/publications.php3

