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Measurement of the Nucleon Nucleon Scattering Length with the ESC04 Interaction

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ABSTRACT

We have determined a value for the $^1S_0$ neutron-neutron scattering length (a_{nn}). The scattering length result is presented for the extended-soft-core (ESC04) interaction. The value obtained in the present work is a_{nn} = −18.6249 fm. The method of solution of the radial Schrödinger equation with nonlocal potential for nucleon-nucleon pairs is described and the result is consistent with previous determinations of $a_{nn} = -18.63 \pm 0.10$ (statistical) ± 0.44 (systematic) ± 0.30 (theoretical) fm. The nonlocal potentials are of the central, spin-spin, spin-orbital, and tensor type. The analysis from the ESC04 interaction is done at energies $0 \leq T_{lab} \leq 350$ MeV. We compare the present result with experimental S-wave phase shifts analysis and agreement is found.

KEYWORDS

Nucleon-Induced Reactions; S-Matrix Theory; Scattering Theory

1. Introduction

In nuclear physics, important information can be obtained from the scattering length associated with low-energy nucleon-nucleon scattering. At these energies, the nucleon-nucleon interaction can be treated non-relativistically and the scattering was studied by means of a single particle Schrödinger equation which involves a non-local effective potential, derived from [1-4] using an extended soft-core model (ESC interaction). In the present manuscript, we consider a potential that involves a central part, a spin-spin interaction, a spin-orbital interaction and a tensor part and perform a numerical study of the associated Schrödinger equation. Also, we determine a numerical value for proton-proton and neutron-proton scattering lengths.

The present work is realized by considering energies in the range of $0 \leq T_{lab} \leq 350$ MeV. For nucleon-nucleon scattering, it has been demonstrated that the interaction from the ESC model gives a description that is in good agreement with the nucleon-nucleon data. The extended soft-core model, also known as ESC, is used for nucleon-nucleon (NN), hyperon-nucleon (YN), and hyperon-hyperon (YY) scatterings. The particular version of the model ESC, called ESC04 [T. A. Rijken, Phys. Rev. C 73, 04007 (2006)], describes NN and YN interaction in an unified way using broken SU (3) symmetry.

A good fit with the experimental data is obtained by using the ESC04 model. The manuscript is organized as follows: in Section II, we give a theoretical review of the model; in Section III, we present our numerical results and in Section IV, we draw our conclusions.

2. Theory

2.1. The Schroedinger Equation with Non-Local Potential

The model we are going to study numerically involves a radial Schrödinger equation with ESC04 potential; namely

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V(r)\right]\Psi(\vec{r}) = E\Psi(\vec{r}), \qquad (1)$$

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where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is the reduced mass of the nucleons whose individual masses are m_1 and m_2, and have spins $\overrightarrow{\sigma_1}$ and $\overrightarrow{\sigma_2}$; r is the distance between the nucleons. The potential is parameterized as

$$V(r) = V_c(r) + V_{SS}(r)\vec{S}\bullet\vec{S} + V_{LS}(r)\vec{L}\bullet\vec{S} + S_{12}V_T(r)$$

where $S_{12} = 3\left(\overrightarrow{\sigma_1}\bullet\vec{r}\right)\left(\overrightarrow{\sigma_2}\bullet\vec{r}\right) - \left(\overrightarrow{\sigma_1}\bullet\overrightarrow{\sigma_2}\right)$ is a second rank tensor operator.

For an S-state we introduce $u(r)$, where

$$\Psi(\vec{r}) = \Psi(r) = \frac{u(r)}{r}.$$

For a given value of the quantum number J,

$$\Psi(\vec{r}) = \sum_L \frac{u_L(r)}{r}\Phi_{JML}, \quad (2)$$

where we introduce

$$\Phi_{JML} = \sum_{M_L=-L}^{L} \left(L1J \mid M_L M_S M\right) Y_{LM_L}\chi_{M_S}, \quad (3)$$

where the symbol $\left(L1J \mid M_L M_S M\right)$ denotes a Clebsch-Gordan coefficient, and Y_{LML} are the spherical harmonics, and

$$\chi_{+1} = \alpha_1\alpha_2;$$

$$\chi_0 = \frac{1}{\sqrt{2}}\left(\alpha_1\beta_2 + \alpha_2\beta_1\right);$$

$$\chi_{-1} = \beta_1\beta_2.$$

The subscript on χ refers to the magnetic projection quantum number M_S of the spin-1 state, while α and β represent spin up and spin down for the particular spin-½ nucleon indicated by the subscript.

The Equation (2) forms an orthonormal set spanning the space of spin-1 functions and functions of the direction ***r***. The normalization of $\Psi(r)$ requires that the radial functions satisfy,

$$\sum_L \int_0^\infty u_L^2(r)\,dr = 1. \quad (4)$$

The Schrödinger equation [Equation (1)] is processed by the method of separation of variables, we obtain as its radial component,

$$\frac{d^2}{dr^2}R(r) + \frac{2}{r}\frac{d}{dr}R(r) + \left[\frac{2\mu}{\hbar^2}\left\{E - V(r)\right\} - \frac{L(L+1)}{r^2}\right]R(r) = 0. \quad (5)$$

We use the parametrized potential

$$V(r) = V_c(r) + V_{SS}(r)\vec{S}\bullet\vec{S} + V_{LS}(r)\vec{L}\bullet\vec{S} + S_{12}V_T(r)$$

and

$$R(r) = \frac{u(r)}{r}$$

for an *S-state* to obtain,

$$\begin{aligned}&\frac{d^2}{dr^2}u_L(r) + \frac{2\mu}{\hbar^2}\left\{E - \frac{\hbar^2}{2\mu r^2}L(L+1) - V_c(r) - S(S+1)V_{SS}(r) - \frac{1}{2}V_{LS}(r)\left[J(J+1) - L(L+1) - S(S+1)\right]\right\}u_L \\ &-\frac{2\mu}{\hbar^2}V_T(r)\sum_{L'} S_{JLL'}u_{L'}(r) = 0\end{aligned}, \quad (6)$$

where $S_{JLL'} = \int\left(\Phi_{JML'}, S_{12}\Phi_{JML'}\right)d\vec{r}$ [5], and S_{12} may be written as an operator of the form

$$\sum_{qq'}\left(j_1 j_2 \lambda \middle| qq'M\right)\overrightarrow{\sigma_{1q}}\,\overrightarrow{\sigma_{2q'}}$$

with λ = 2 and $j_1 = j_2 = 1$. Here $\left(j_1 j_2 \lambda \middle| qq'M\right)$ is the Clebsch-Gordan coefficient.

Using Racha algebra (see appendix A of [6]) we can show that

$$S_{JLL'} = \left(2\sqrt{6}\right)LL'\delta_{JJ'}(-1)^{1+J}\left(LL'2\middle|000\right)W\left(LL'11;2J\right) = 2\delta_{JJ'}\left[\delta_{LL'} - 3\left(J1L\middle|000\right)\left(J1L'\middle|000\right)\right]. \quad (7)$$

2.2. Numerical Solution of the Schrödinger Equation

Considering the single state for the $^1S_0$ wave, Equation (6) for the neutron-neutron system has the form ($S = J = L = 0$, $L' = -1, 0, 1$),

$$\frac{d^2}{dr^2}u_0(r)+\frac{2\mu}{\hbar^2}\{E-V_c(r)\}u_0(r)$$
$$-\frac{2\mu}{\hbar^2}V_T(r)\left[S_{00-1}u_{-1}(r)+S_{000}u_0(r)+S_{001}u_1(r)\right]=0 \tag{8}$$

where $S_{00\text{-}1} = S_{001} = 0$, $S_{000} = 2$ are calculated from Equation (7).

For the proton-proton system we add the Coulomb effect to Equation (8), $\left[E-V_c(r)\right]\to\left[E-V_c(r)+V_{coul}(r)\right]$.

The numerical techniques necessary to solve equation (8) with this ESC04 potential are explained in chapter 3, Equation (3.28) of [7]. The solutions of u_0 from Equation (8) are introduced in the S matrix (Equation (10.58) of [7], which is,

$$S_l=\frac{U_l(r_{n-1})r_n h_l^-(kr_n)-U_l(r_n)r_{n-1}h_l^-(kr_{n-1})}{U_l(r_n)r_{n-1}h_l^+(kr_{n-1})-U_l(r_{n-1})r_n h_l^+(kr_n)}, \tag{9}$$

where the S matrix is evaluated in the last two points on a mesh of size ε ($r=0,\varepsilon,2\varepsilon,\cdots N\varepsilon$). U_l are the solutions to Equation (8) with the ESC04 potential previously calculated and h_l are the spherical Hankel functions defined in Equation (10.52) of [7].

We insert the numerical solution of the S matrix in the solution of the S matrix for a real potential

$$S_l=e^{2i\delta_l}, \tag{10}$$

where δ_l is real and is known as the phase shift.

Once the δ_0 phase shift is found the a_{nn} scattering length and the effective range r_{nn} are calculated. For $l = 0$ the expression for $k\cot(\delta_0)$ can be parameterized in the following form,

$$k\cot(\delta_0)=-\frac{1}{a}+\frac{1}{2}r_0k^2+\cdots. \tag{11}$$

The quantity a is called the scattering length and r_0 is known as the effective range.

In the limit of low energies the scattering length is given in terms of the *s-wave* phase shift (see appendix B of [8]),

$$a=\lim_{k\to0}\Re\left\{-\frac{1}{k}e^{i\delta_0}\sin(\delta_0)\right\}, \tag{12}$$

where $k^2=2\mu E/\hbar^2$ is the center-of-mass momentum (the wave number) and $\Re$ indicates the real part.

2.3. Extended Soft-Core Potential (ESC04)

An Extended Soft-core potential is calculated consisting of a central, spin-spin, spin-orbital, and a tensor part. The potential of the ESC04 model is generated by one-boson-exchange (OBE), two-meson-exchange (TME) and meson-pair-exchange (MPE); this potential is calculated and explained in [1-4]. In **Figure 1** the total ESC04 potential is plotted as a function of the r distance. In **Figure 2** we show the central, spin-spin, spin-orbital, and tensor part of this total potential.

The algoritms for the *YN* potential are found in [9].

3. Results

The a_{nn} Scattering Length

The a_{nn} scattering length is calculated obtaining a numerical value $a_{nn} = -18.62497$ fm and an effective range of $r_{nn} = 2.746615$ fm. We use an ESC04 potential below 350 MeV. In **Figures 3** and **4** the phase shift $\delta(^1S_0)$ is plotted for the proton-proton and neutron-proton case.

Table 1 shows the results for the low-energy parameters from the scattering lengths and the effective ranges for neutron-proton, proton-proton and neutron-neutron system using the ESC04 interaction.

4. Conclusions

In the present work, we have numerically solved the Schrödinger equation with an ESC04 potential and obtained the nucleon-nucleon scattering lengths. Summarizing our main conclusions:

1) Recent calculations using the ESC04 interaction for nucleon-nucleon dispersion have been realized [4], and reproduced with the Schrödinger equation.

2) The numerical solution of the radial Schrödinger equation has been realized and has been demonstrated to give a good fit to the nucleon-nucleon data.

3) The scattering lengths a_{pp}, a_{np} and a_{nn} have been calculated and are consistent with the experimental re-

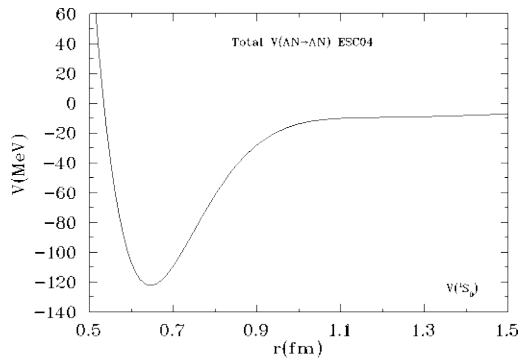

Figure 1. Total potential in the partial wave $^1S_0$, for $I = ½$.

Table 1. ESC04 low-energy parameters: *S*-wave scattering lengths and effective ranges.

	Experimental data	ESC04
a_{pp} ($^1S_0$)	−7.823 ± 0.010	−7.98
r_{pp} ($^1S_0$)	2.794 ± 0.015	2.762
a_{np} ($^1S_0$)	−23.715 ± 0.015	−23.801
r_{np} ($^1S_0$)	2.760 ± 0.030	2.773
a_{nn} ($^1S_0$)	−18.70 ± 0.60	−18.625
r_{nn} ($^1S_0$)	2.750 ± 0.11	2.747

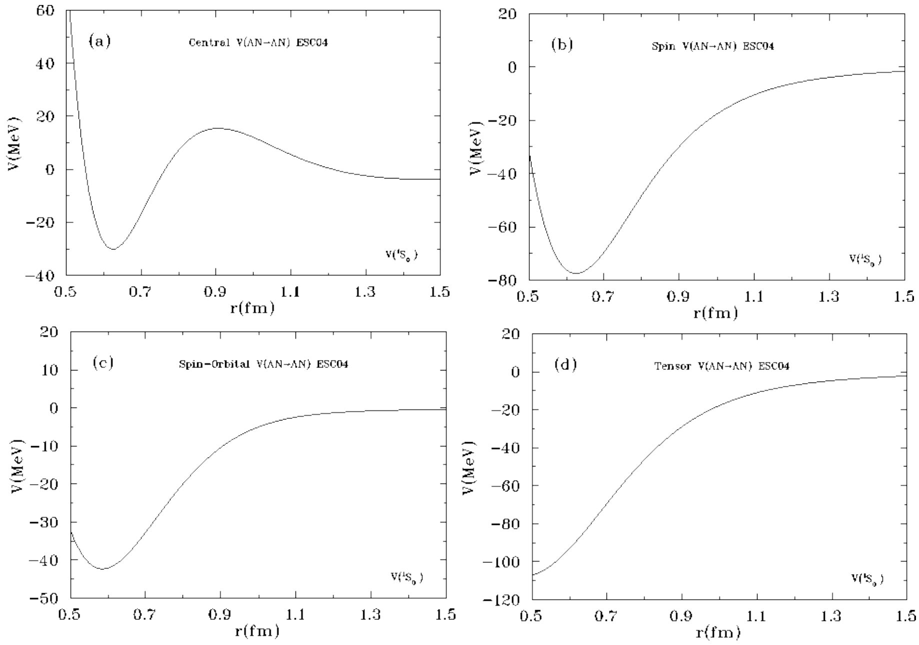

Figure 2. **Central (a), spin-spin (b), spin-orbital (c), and tensor (d) part of the YN potential.**

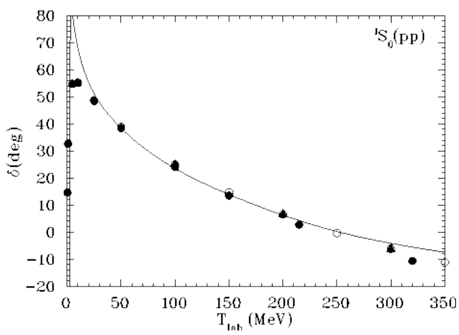

Figure 3. **Solid curve, proton-proton *I* = 1 phase shifts (degrees), as a function of T_{lab} (MeV), numerical solution for the ESC04 model. Dots, phases of the Rijken analysis [4]. Circles, s.e. phases of the Nijmegen93 PW analysis. Triangles, the m.e. phases of the Nijmegen93 PW analysis [10].**

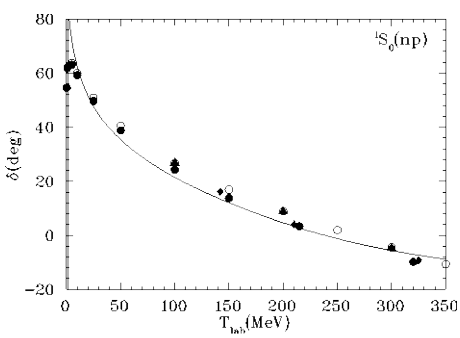

Figure 4. **Solid curve, neutron-proton *I* = 0 phase shifts (degrees), as a function of T_{lab} (MeV), numerical solution for the ESC04 model. Dots, phases of the Rijken analysis [4]. Circles, s.e. phases of the Nijmegen93 PW analysis. Triangles, the m.e. phases of the Nijmegen93 PW analysis [10]. Diamonds, Bugg s.e. [11].**

sults. The final value for a_{nn} from this study is $a_{nn} = -18.625$ fm. Results from previous studies are

$$\begin{aligned} a_{nn} &= -18.60 \pm 0.34 \pm 0.26 \pm 0.30 \text{ fm} \\ &= -18.60 \pm 0.52 \text{ fm} \end{aligned} \quad [12],$$

$$\begin{aligned} a_{nn} &= -18.70 \pm 0.42 \pm 0.39 \pm 0.30 \text{ fm} \\ &= -18.70 \pm 0.65 \text{ fm} \end{aligned} \quad [13],$$

and

$$\begin{aligned} a_{nn} &= -18.63 \pm 0.10 \pm 0.44 \pm 0.30 \text{ fm} \\ &= -18.63 \pm 0.48 \text{ fm} \end{aligned} \quad [14],$$

The presented ESC model is thus successful in describing the NN data.

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