Measurement of the Spatial Coherence Function of Undulator Radiation using a Phase Mask

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A measurement of the horizontal coherence function of 7.9 keV radiation from an undulator beam line at the Advanced Photon Source is reported. X-ray diffraction from a phase-shifting mask was used, and the coherence function was measured as a function of the width of beam-conditioning slits in the beam line. The coherence distribution is found to be best described by a Lorentzian function.

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A principal motivation for the construction of third-generation synchrotrons is to make available highly coherent x rays for high resolution and high coherence experiments. The importance of coherent x rays is evidenced by the construction of third-generation synchrotrons, by the ongoing development of soft x-ray lasers [1], and by the work towards fourth generation x-ray free electron laser sources.

The coherence properties of x rays are frequently encapsulated into a single number such as the spatial coherence length, or the coherent fraction of the light. However, in order to fully understand the sources of changes in diffracted intensity, fringe visibility and speckle structure for experiments involving diffraction or speckle interferometry [2], more detailed information concerning the coherence properties of the light is required.

A full measurement of the coherence using a series of Young's experiments, as described in elementary textbooks, is a time-consuming activity that is rarely performed. However, partial measurements have been reported using the Young's slit approach for extreme ultraviolet (EUV) radiation from a laser [3,4] as well as for EUV radiation [5] and x-radiation [6] from a synchrotron. The spatial coherence length for hard x rays has been estimated using diffraction [7] and measured [8,9] using the intensity correlation method due to Hanbury Brown and Twiss [10].

Much synchrotron research uses x rays with moderate to high energies and so, because of the high aspect ratio required, fabrication of high contrast absorption masks represents a considerable manufacturing challenge. However, it is now well established that, after the x rays have propagated a short distance, phase structures generate high contrast intensity modulations [11]. In this Letter we report a measurement of the spatial coherence function using the intensity distribution created by diffraction from a phase-shifting mask.

Wolf [12] has developed a coherence theory formalism known as the coherent mode model. In this theory, the mutual optical intensity is written in the form $J(\vec{r}_1, \vec{r}_2) = \sum_{n=-\infty}^{\infty} \alpha_n \psi_n(\vec{r}_1)\psi_n^*(\vec{r}_2)$, where the α_n are real, positive constants, and $\int_{-\infty}^{\infty} \psi_m(\vec{r})\psi_n^*(\vec{r})d\vec{r} = \delta_{mn}$, where $\delta_{mn} = 1$ when $m = n$, and 0 otherwise. The component wave functions, $\psi_n(\vec{r})$, are mutually incoherent.

In the case of a sufficiently long beam line of length z the phase deviations from spherical waves are negligible (the Fresnel approximation) and so a component wave function has the form $\psi_m(\vec{r}) \approx \frac{1}{\lambda z} e^{i\pi(\vec{r}-\vec{r}_m)^2/\lambda z}$, where $\vec{r}_m$ is the effective source point. Note that for the remainder of the paper position vectors denote a two-dimensional position in the plane perpendicular to the optic axis z. Under the Fresnel approximation, the complex degree of coherence [13] for the radiation can be written

$$j(\vec{r}_1, \vec{r}_2) = e^{i\pi(r_1^2 - r_2^2)/\lambda z} g(\vec{r}_1 - \vec{r}_2), \tag{1}$$

where

$$g(\vec{x}) \equiv \sum_{n=-\infty}^{\infty} c_n \exp[i\vec{k}_n \cdot \vec{x}], \tag{2}$$

and $\vec{k}_m \equiv 2\pi\vec{r}_m/\lambda z$, $\vec{x} \equiv \vec{r}_1 - \vec{r}_2$ and $c_m \equiv \alpha_m / \sum_n \alpha_n$. Under this approximation the values of c_m, and therefore the function $g(\vec{x})$, contain all the coherence information. We refer to $g(\vec{x})$ as the complex coherence factor (CCF).

Define the Fourier transform of the CCF as

$$G(\vec{u}) \equiv \int g(\vec{x}) \exp[2\pi i \vec{x} \cdot \vec{u}/\lambda] d\vec{r} = \sum_{n=-\infty}^{\infty} c_n \delta\left(\frac{2\pi}{\lambda}\vec{u} + \vec{k}_n\right), \tag{3}$$

where we have used Eq. (2). Under the Fresnel approximation, the diffraction pattern of an aperture illuminated by partially coherent radiation is given by [14] $I(\vec{r}) = \int G(\vec{r}'/z_d)P(\vec{r} - \vec{r}')d\vec{r}'$ where P is the Fresnel diffraction pattern produced with a coherent spherical wave, and z_d is the distance between the diffracting aperture and the detector. Thus the total diffraction pattern is the convolution of the coherent Fresnel diffraction pattern with the

Fourier transform of the CCF. Provided the aperture form is well known and its diffraction pattern contains the complete range of spatial frequencies, it is possible to recover the complex coherence factor using simple Fourier deconvolution. In the case that the diffraction pattern is missing some spatial frequencies, the corresponding parts of the CCF will not be able to be recovered. As a limiting case, note that a Young's experiment produces a harmonic intensity pattern containing only a single nonzero spatial frequency—this form of experiment is therefore able to measure only one c_m at a time.

A coherence function measurement method based on a mask known as a uniformly redundant array (URA) was proposed [15] and used for soft x-ray lasers [16]. A URA is a complex one- or two-dimensional mask with multiple apertures such that an infinite URA contains all aperture separations a precisely equal number of times. Diffraction by a URA is therefore a superposition of multiple Young's experiments with a uniform distribution of aperture separations. In practice, a finite mask is used that can only approximate the ideal properties of an infinite array. The theory above provides a basis for interpreting the diffraction pattern from a URA mask.

We calculated a 1D URA pattern for this work using the algorithm described by Fenimore and Cannon [17], based on the prime numbers 137 and 139, and manufactured it using standard contact optical lithography and gold electroforming onto a silicon nitride (Si_3N_4) support membrane. The overall size of the URA was 700 μm $\times$ 700 μm. The gold was 1.6 μm thick and the minimum slit size was 5.0 μm. A micrograph of the structure is shown in Fig. 1. The metallic parts of the aperture will transmit 51% of, and impart a π phase shift to, 7.9 keV x rays. The URA was measured and its coherent diffraction properties calculated using conventional Fresnel diffraction theory and the known composition and thickness of the URA (Fig. 2).

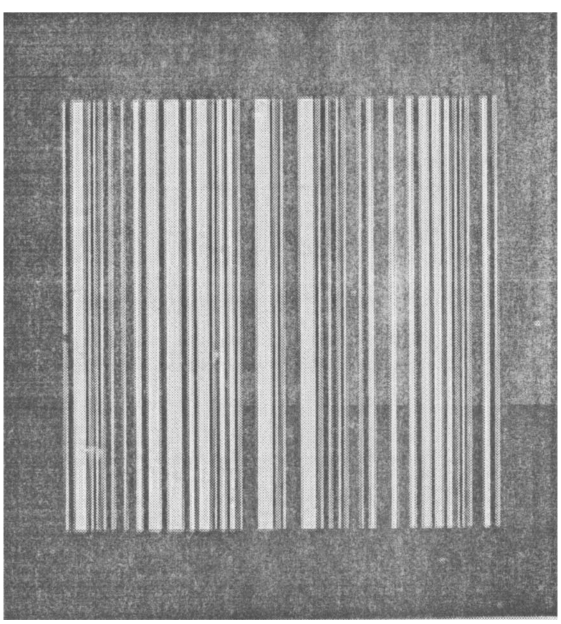

FIG. 1. Optical microscope image of the URA. The overall width is 700 μm.

The coherence experiments were conducted on the 2-ID-D undulator beam line of the Advanced Photon Source, Argonne National Laboratory near Chicago, Illinois. The beam line contains an array of optical components, including mirrors, a double crystal monochromator and beam-conditioning slits. The URA was a distance of 70.4 m from the undulator and a distance of 43.4 m from the beam-conditioning slits. The diffracted x rays passed through an evacuated tube and were detected using a CCD camera a further 5.4 m beyond the URA. This geometry implies that deviations from spherical waves are negligible, validating the use of Eq. (1).

The camera system consists of a 5 μm yttrium aluminum garnet (YAG:Ce) crystal, 10$\times$ microscope objective, and a Princeton Instruments MicroMax x-ray camera using a Kodak model KAF-1400 CCD chip. The nominal spatial resolution of the camera is ≈ 2 μm. The modulation transfer function (MTF) of the CCD camera system was not sufficient to fully resolve fringes produced by a 700 μm slit separation, but the observed fringe contrast was found to be negligible for fringes produced by much smaller slit separations such that the MTF has a negligible effect on the data reported here. Its effect was therefore not incorporated into the analysis.

The coherence of the radiation in the horizontal direction was measured as a function of the beam-conditioning slit size. The properties of the undulator are well known and, treating it as a distant incoherent x-ray source, it is possible to calculate the coherence at the URA using the van Cittert–Zernike theorem [18]. This value should set a lower bound on the coherence. The beam-conditioning slits in the beam line will select out a subset of the phase space of the radiation and so the coherence delivered to the experiment will be higher. Measurements were taken with nominal slit widths 10, 50, 90, and 170 μm.

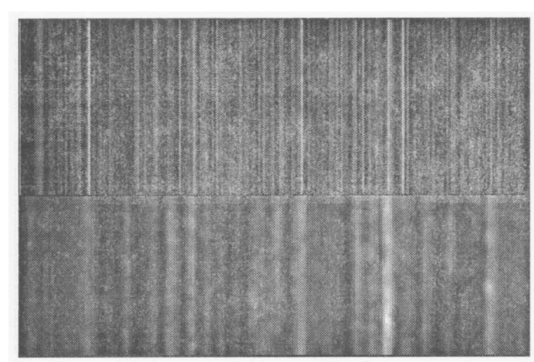

FIG. 2. The upper image shows a calculated fully coherent diffraction pattern. The lower image shows the pattern obtained with a nominal beam-conditioning slit width of 170 μm.

The measured diffraction pattern for the 170 μm slit setting is also shown in Fig. 2. Examination indicates that the patterns are consistent, confirming the calculated pattern used in the analysis. Some simple deviations from a nonideal URA profile were modeled and no significant effects on the results were found. The calculated URA coherent diffraction pattern was used to deconvolve the data using standard Fourier transform techniques for each slit width.

Figure 3 shows the absolute value of the CCF for the range of different slit settings. The uncertainties indicated on this plot were estimated using a Monte Carlo approach whereby a set of synthetic data sets obeying the measured noise statistics were generated and the resulting variation in the computed CCFs was used as the error estimate. The CCF is essentially a plot of fringe visibility as a function of spatial frequency so that an uncertainty in a particular data pixel will contribute a component of error at each location in the CCF. The uncertainties for each point are therefore not independent of each other. Conversely, where a spatial frequency is poorly measured, the value of the CCF at the point corresponding to that frequency will also have a large error. As the URA diffraction provides a relatively uniform distribution of spatial frequencies, this source of uncertainty is minimized in the experimental approach reported here.

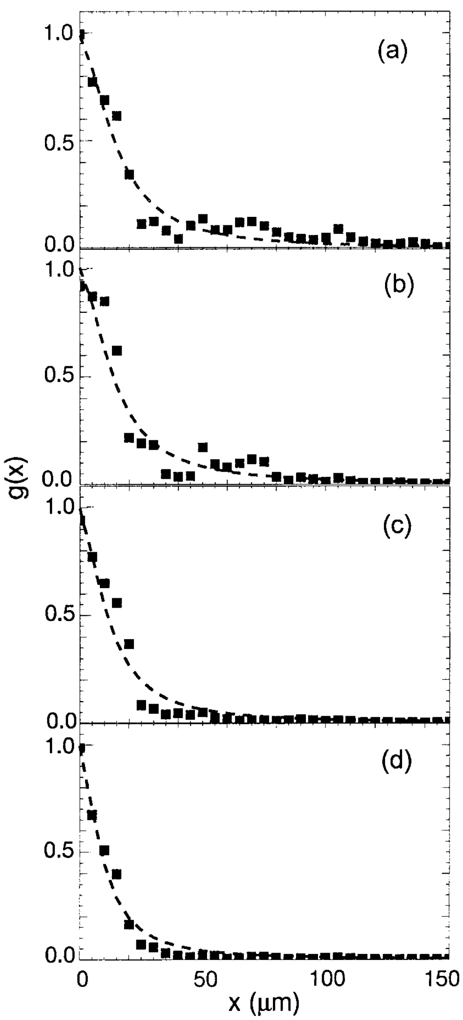

FIG. 3. Measured complex coherence factor and Lorentzian fit for a beam conditioning slit width of (a) 10 μm, (b) 50 μm, (c) 90 μm, and (d) 170 μm. The uncertainty in each data point is indicated by the height of the symbol.

The effective source within the undulator is Gaussian and so, from the van Cittert–Zernike theorem, the CCF should itself be Gaussian. However, satisfactory Gaussian fits could not be obtained, and a Lorentzian was found to be in better agreement with the measurement. Figure 3 shows the Lorentzian fits to the data.

The reasons for the non-Gaussian form are not known; however, recent work has predicted that random refractive effects due to imperfections in optical elements such as a beryllium window can result in a Lorentzian distribution [19]. Complex phase space selection may occur as the beam travels through the beam line optics, and this will also have an effect on the correlation function.

A broad understanding of the effect of the beam-conditioning slits can be obtained by fitting to the CCF and describing its half width at half maximum as the "coherence length" for the radiation [20]. It was found, as expected, that the coherence did increase as the slit width reduced. As a typical example, the coherence length with the beam-conditioning slit set at a nominal width of 160 μm was found to be (11 ± 1) μm. A naive model [21] incorporating the effect of the beam-conditioning slit predicts a value of (9 ± 1) μm, where the errors in the model arise from uncertainties in the beam-conditioning slit width. The coherence lengths obtained were all consistent with this model.

In summary, the method described in this Letter opens the way for a complete measurement of the coherence of radiation emerging from a synchrotron as a standard diagnostic. The use of the refractive properties of the array also implies that this method can be applied to highly energetic x rays without the need for very high aspect ratio apertures.

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[21] The coherence length is estimated by calculating the expected coherence length of the source, calculating the coherence length that would be produced by an incoherent source just behind the beam-conditioning slit, and adding the two in quadrature.