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Measurement of the τ-lepton Lifetime at Belle

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The lifetime of the τ lepton is measured using the process $e^+e^- \to \tau^+\tau^-$, where both τ leptons decay to $3\pi\nu_\tau$. The result for the mean lifetime, based on 711 fb^{-1} of data collected with the Belle detector at the $\Upsilon(4S)$ resonance and 60 MeV below, is $\tau = (290.17 \pm 0.53(\text{stat}) \pm 0.33(\text{syst})) \times 10^{-15}$ s. The first measurement of the lifetime difference between τ^+ and τ^- is performed. The upper limit on the relative lifetime difference between positive and negative τ leptons is $|\Delta\tau|/\tau < 7.0 \times 10^{-3}$ at 90% C.L.

High precision measurements of the mass, lifetime, and leptonic branching fractions of the τ lepton can be used to test lepton universality [1], which is assumed in the standard model. Among the recent experimental results that may manifest the violation of the lepton universality in the case of the τ lepton, the combined measurement of the ratio of the branching fraction of W-boson decay to $\tau\nu_\tau$ to the mean branching fraction of W-boson decay to $\mu\nu_\mu$ and $e\nu_e$ by the four LEP experiments stands out: $2\mathcal{B}(W \to \tau\nu_\tau)/(\mathcal{B}(W \to \mu\nu_\mu) + \mathcal{B}(W \to e\nu_e)) = 1.066 \pm 0.025$ [2], which differs from unity by 2.6 standard deviations. The present Particle Data Group (PDG) value of the τ-lepton lifetime $(290.6 \pm 1.0) \times 10^{-15}$ s [3] is dominated by the results obtained in the LEP experiments [4].

A high-statistics data sample collected at Belle allows us to select $\tau^+\tau^-$ events where both τ leptons decay to three charged pions and a neutrino. As explained later, for these events the directions of the τ leptons can be determined with an accuracy better than that given by the thrust axis of the event. At an asymmetric-energy collider, the laboratory frame angle between the produced τ leptons is not equal to 180 deg, so their production point can be determined from the intersection of two trajectories defined by the τ-lepton decay vertices and their momentum directions. The direction of each τ lepton in the laboratory system can be determined with twofold ambiguity. These special features of the asymmetric-energy B-factory experiments allow a high precision measurement of the τ-lepton lifetime with

systematic uncertainties that differ from those of the LEP experiments. Belle has a possibility to measure separately the τ^+ and τ^- lifetimes, which allows us to test CPT symmetry in τ-lepton decays.

In the following, we use symbols with and without an asterisk for quantities in the e^+e^- center-of-mass (c.m.) and laboratory frame, respectively. In the c.m. frame, τ^+ and τ^- leptons emerge back to back with the energy E_τ^* equal to the beam energy $E_{\rm beam}^*$ if we neglect the initial- (ISR) and final-state radiation (FSR). We determine the direction of the τ-lepton momentum in the c.m. frame as follows. If the neutrino mass is assumed to be zero for the hadronic decay $\tau \to X\nu_\tau$ (X representing the hadronic system with mass m_X and energy E_X^*), the angle θ^* between the momentum $\vec{P}_X^*$ of the hadronic system and that of the τ lepton is given by

$$\cos\theta^* = \frac{2E_\tau^* E_X^* - m_\tau^2 - m_X^2}{2P_X^*\sqrt{(E_\tau^*)^2 - m_\tau^2}}. \quad (1)$$

The requirement that the τ leptons be back to back in the c.m. frame can be written as a system of three equations: two linear and one quadratic. For the components x^*, y^*, z^* of the unit vector $\hat{n}_+^*$ representing the direction of the positive τ lepton, we write

$$x^* \times P_{1x}^* + y^* \times P_{1y}^* + z^* \times P_{1z}^* = |P_1^*| \cos\theta_1^*,$$
$$x^* \times P_{2x}^* + y^* \times P_{2y}^* + z^* \times P_{2z}^* = -|P_2^*| \cos\theta_2^*,$$
$$(x^*)^2 + (y^*)^2 + (z^*)^2 = 1, \quad (2)$$

where $\vec{P}_1^*$ and $\vec{P}_2^*$ are the momenta of the hadronic systems in the c.m. frame and $\cos\theta_i^*$ ($i=1,2$) are given by Eq. (1). Index 1 (2) is used for the positive (negative) τ lepton.

There are two solutions for Eq. (2), so the direction $\hat{n}_+^*$ is determined with twofold ambiguity. In the present analysis, we take the mean vector of the two solutions of Eq. (2) as the direction of the τ lepton in c.m. frame. The analysis of Monte Carlo (MC) simulated events shows that there is no bias due to this choice.

Each direction $\hat{n}_\pm^*$ is converted to a four-momentum using the $e^\pm$ beam energy and the τ mass. Both four-momenta are then boosted into the laboratory frame, each passing through the corresponding τ decay vertex $\vec{V}_i$ that is determined by the three pion-daughter tracks (see Fig. 1). We approximate the trajectory of τ leptons in the magnetic field of the Belle detector with a straight line. Due to the finite detector resolution, these straight lines do not intersect at the $\tau^+\tau^-$ production point. The three-dimensional separation between these lines is characterized by the distance dl between the two points ($\vec{V}_{01}$ and $\vec{V}_{02}$) of closest approach. The typical size of dl is ~0.01 cm. For the production point of each τ lepton, we take the points $\vec{V}_{01}$ and $\vec{V}_{02}$. The flight distance l_1 (l_2) of the τ^+ (τ^-) in the laboratory frame is defined as the distance between the points $\vec{V}_{01}$ and $\vec{V}_1$ ($\vec{V}_{02}$ and $\vec{V}_2$). The proper time t (the product of the speed of light and the
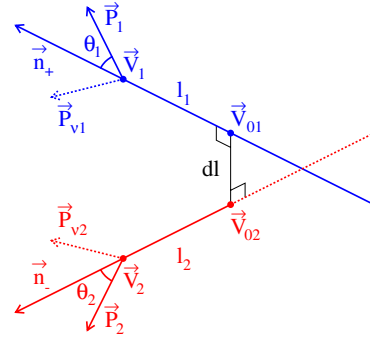

FIG. 1 (color online). The schematic view of the $\tau^+\tau^-$ event in the laboratory frame.

decay time of the τ lepton) for the positive τ lepton is equal to the distance l_1 divided by its relativistic kinematic factor $\beta\gamma$ in the laboratory frame: $t_1 = l_1/(\beta\gamma)_1$. The corresponding parameter for the negative τ lepton is $t_2 = l_2/(\beta\gamma)_2$.

The analysis presented here is based on the data collected with the Belle detector [5] at the KEKB asymmetric-energy e^+e^- (3.5 on 8 GeV) collider [6] operating at the $\Upsilon(4S)$ resonance and 60 MeV below. The total integrated luminosity of the data sample used in the analysis is 711 fb^{-1}. Two inner detector configurations were used. A 2.0 cm beampipe and a three-layer silicon vertex detector (SVD1) were used for the first sample of 157 fb^{-1}, while a 1.5 cm beampipe, a four-layer silicon detector (SVD2), and a small-cell inner drift chamber were used to record the remaining 554 fb^{-1} [7]. The integrated luminosity of the data sample at the energy below the $\Upsilon(4S)$ resonance is about 10% of the total data sample. All analyzed distributions for the on- and off-resonance data coincide within the statistical uncertainties with each other; this justifies our combination of the on- and off-resonance t distributions in the present analysis.

The following requirements are applied for the exclusive selection of the $\tau^+\tau^-$ events where both τ leptons decay into three charged pions and a neutrino: there are exactly six charged pions with zero net charge and there are no other charged tracks; the thrust value of the event in the c.m. frame is greater than 0.9; the square of the transverse momentum of the 6π system is required to be greater than 0.25 (GeV/c)^2 to suppress the $e^+e^- \to e^+e^-6\pi$ two-photon events; the mass $M(6\pi)$ of the 6π system should fulfill the requirement 4 GeV/$c^2 < M(6\pi) < 10.25$ GeV/c^2 to suppress other background events; there should be three pions (triplet) with net charge equal to ±1 in each hemisphere (separated by the plane perpendicular to the thrust axis in the c.m.), the pseudomass (see the definition in Ref. [8]) of each triplet of pions must be less than 1.8 GeV/c^2 and each pion-triplet vertex-fit quality must satisfy $\chi^2 < 20$; the discriminant D of Eq. (2) should satisfy $D > -0.05$ (with slightly negative values arising from experimental uncertainties; if this happens, we use $D = 0$ when solving the equation); the distance of closest approach must satisfy $dl < 0.03$ cm to

reject events with large uncertainties in the reconstructed momenta and vertex positions. All of these selection criteria are based on a study of the signal and background MC simulated events.

For the signal MC samples, we use $\tau^+\tau^-$ events produced by the KKMC generator [9] with the mean lifetimes $\langle\tau\rangle = 87.11$ (present PDG value), 84, and 90 μm, which are about 10σ below and above the PDG value. In the first sample all τ decay channels are switched on, while in the last two samples both τ leptons are forced to decay into three charged pions and a neutrino. For the background estimation, we use the MC samples of events generated by the EVTGEN program [10], which correspond to the one-photon annihilation diagram $e^+e^- \to q\bar{q}$, where $q\bar{q}$ are $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ (*uds* events), $c\bar{c}$ (charm events), and $e^+e^- \to \Upsilon(4S) \to B^+B^-, B^0\bar{B}^0$ (beauty events). For the estimation of the background from the process $\gamma\gamma \to$ *hadrons* ($\gamma\gamma$ events), we use events generated by PYTHIA [11]. The statistics in these MC samples are equivalent to the integrated luminosity of the data; i.e., the number of events of a given category is equal to the product of the integrated luminosity of the data and the expected cross section from theory. All these MC events are fed to the full detector simulation based on GEANT 3 [12] and are passed through the same reconstruction procedures as for the data.

In the measured proper time distribution, the exponential behavior is smeared by the experimental resolution. This resolution has been studied with MC simulation (see the Supplemental Material [13] for details) and is found to be described well by the expression

$$R(\Delta t) = (1 - A\Delta t)e^{-(\Delta t - t_0)^2/2\sigma^2}, \qquad (3)$$

where $\Delta t = t_{\text{reconstructed}} - t_{\text{true}}$ and $\sigma = a + b|\Delta t - t_0|^{1/2} + c|\Delta t - t_0| + d|\Delta t - t_0|^{3/2}$. The parameters t_0, a, b, c, and d are allowed to vary freely in the fit, while the asymmetry $A = 2.5$ cm^{-1} is fixed because of its strong correlation with the lifetime parameter τ.

After applying all the selection criteria, the contamination of the background in the data is about 2%. The dominant background arises from *uds* events. For these events, all six pions emerge (typically) from one primary vertex and these *uds* events are similar to the $\tau^+\tau^-$ events with zero lifetime. Using the MC simulation, we check that the decay time distributions of *uds* events that pass the selection criteria are well described by the resolution function of Eq. (3). The same behavior is found for $\gamma\gamma$ events, whose fraction in all the selected events is about 1.4×10^{-4}. Other sources of background contribute to the selected data sample at the per thousand level.

The measured proper time distribution is parametrized by

$$F(t) = N \int e^{-t'/\tau}R(t - t')dt' + A_{uds}R(t) + B_{cb}(t), \quad (4)$$

where the resolution function $R(t)$ is given by Eq. (3), A_{uds} is the normalization of the combined *uds* and $\gamma\gamma$ background, and $B_{cb}(t)$ is the background distribution due to charm and beauty events. The shapes and yields of the backgrounds ($B_{cb}(t)$, A_{uds}) are fixed from the MC simulation; the free parameters of the fit are the normalization N, the τ-lepton lifetime τ, and the five parameters of the resolution function t_0, a, b, c, and d.

The result of the fit of the experimental data to Eq. (4) is shown in Fig. 2, together with the contributions from the sum of *uds* and $\gamma\gamma$ events and the sum of charm and beauty events. The curves on these contributions are the result of the fit with Eq. (4), the $A_{uds}R(t)$ function (with the fixed value of A_{uds}) for *uds* plus $\gamma\gamma$ events, and the fixed sum of two Gaussians for charm plus beauty events.

The relation of the parameter τ in Eq. (4) to the generated value of the τ-lepton mean lifetime is analyzed using three MC $\tau^+\tau^-$ samples with the mean lifetime values of 84, 87.11, and 90 μm. The dependence of parameter τ on the input mean lifetime value $\langle\tau\rangle$ is found to be linear: $(\tau - 87) = (0.97 \pm 0.03)(\langle\tau\rangle - 87) + (0.001 \pm 0.07)$ [in units of μm] with a χ^2 value of 0.2.
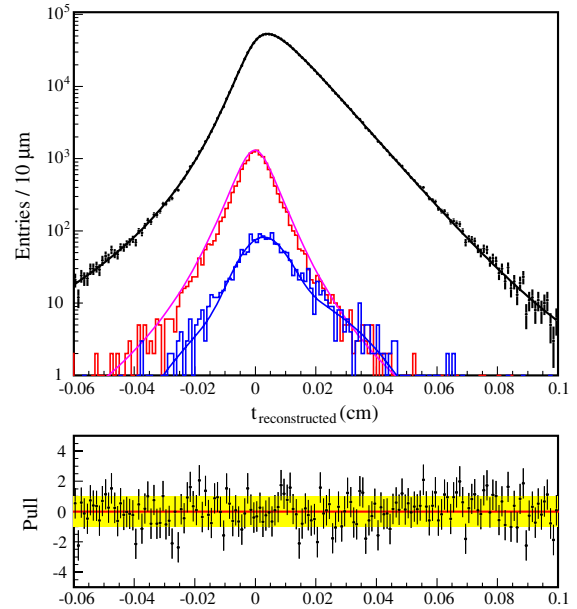

FIG. 2 (color online). The measured proper time t distribution for the data (filled circles with errors). The black line (passing through these circles) is the result of the fit by Eq. (4). The first histogram below the data circles (red histogram) is the MC prediction for the sum of the *uds* and $\gamma\gamma$ background contributions. The magenta line near this histogram is the contribution of the *uds* + $\gamma\gamma$ obtained in the fit by Eq. (4). The lowest (blue) histogram is the MC prediction for the sum of the charm and beauty background contributions. The blue line (passing through this histogram) is the smoothed distribution of the charm and beauty contributions that is used in the fit. The distribution of residuals $[(data - fit)/error]$ for the fit is shown in the bottom panel.

A check is performed for any bias in the fitted lifetime arising from the selection criteria for the signal MC sample before and after applying cuts, and no bias is found for all the selection criteria listed above. To check that the fitting procedure gives the correct estimation of the input lifetime value for different resolution functions, we perform the fits of the decay time distributions for MC samples with a lifetime of 87.11 μm for the sum of the SVD1 and SVD2 samples, for SVD1 and SVD2 samples separately, and for samples with lifetimes equal to 84 and 90 μm. In all cases, the value of the parameter τ is equal to the slope of the exponential distribution of the selected events at the generation level within the statistical error of the parameter τ.

The value of the parameter τ obtained from the fit to the real data is 86.99 ± 0.16 μm. The conversion of this parameter to the value of the τ-lepton mean lifetime using the straight-line parameters of the fit described above gives the same value: 86.99 ± 0.16 μm. The error here is statistical.

The demonstration of the stability of the obtained result to the choice of the cut on dl, which gives the maximal bias to the slope parameter obtained in the fit, is described in the Supplemental Material [13].

The following sources of systematic uncertainties are considered and summarized in Table I.

A study of the influence of the SVD misalignment on the systematic shift in the τ-lepton lifetime measurement is performed in the following way. We use 4.8×10^6 generated $\tau^+\tau^-$ events that decay with the $3\pi\nu_\tau - 3\pi\nu_\tau$ topology and standard Belle SVD alignment. After all selection cuts, about 1.2×10^6 events remain (compared with 1.1×10^6 events in the data). We shift the sensitive elements of SVD along the $X/Y/Z$ axes by sampling from a Gaussian function with $\sigma = 10$ μm and rotation around these axes by sampling from a Gaussian function with $\sigma = 0.1$ mrad. The values of 10 μm and 0.1 mrad are obtained from the dedicated studies of SVD alignment [7]. We prepare the following decay time MC distributions: with default alignment (4.8×10^6 generated events), one sample with misalignment according to the aforementioned shifts and rotations (4.8×10^6 generated events), several samples with misalignments according to these shifts and rotations with fewer generated events; all these samples have the same events at the generator level. The maximal

TABLE I. Summary of systematic uncertainties.

Source	$\Delta\langle\tau\rangle$ (μm)
SVD alignment	0.090
Asymmetry fixing	0.030
Beam energy, ISR and FSR description	0.024
Fit range	0.020
Background contribution	0.010
τ-lepton mass	0.009
Total	0.101

difference of the parameter τ obtained in these fits is 0.07 μm. This is due to the possible effect of misalignment and limited MC statistics. We also perform global SVD shifts and rotations with respect to the central drift chamber by 20 μm and 1 mrad, respectively. The values of 20 μm and 1 mrad are conservative estimates from the SVD alignment study. The variation of the τ parameter is within 0.06 μm for these shifts. We take the value $\sqrt{0.07^2 + 0.06^2} = 0.09$ μm for the systematics due to the SVD misalignment. For an additional check of the alignment of the tracking detectors, we divide our data sample into two nonintersecting samples by the azimuthal angle (ϕ) of the momentum direction of the positive τ lepton. In the first sample (vertical), the direction of the positive τ lepton should have ϕ between 45 and 135 deg or between 225 and 315 deg. The second sample (horizontal) contains all the remaining events. The obtained τ parameters are the same within statistical errors, so we do not assign additional systematics due to the azimuthal dependence of the tracking system alignment.

The systematic uncertainty due to fixing the parameter $A = 2.5$ cm^{-1} is estimated by removing the asymmetry term $(1 - A\Delta t)$ in the resolution function in Eq. (3). The difference in the obtained lifetime, which is equal to 0.03 μm, is taken as a systematic uncertainty.

For the estimation of the accuracy of the initial and final state radiation description by the KKMC generator, we analyze the distributions of $M(\mu^+\mu^-)c^2 - 2E^*_{\text{beam}}$ for $e^+e^- \to \mu^+\mu^-$ events for the data and KKMC events passed through the full Belle simulation and reconstruction procedure. Due to the ISR and FSR, these distributions are asymmetric and their maxima are shifted from zero to the left. If the KKMC description of the ISR and FSR energy spectrum is harder or softer than for the data, we would observe the MC peak position shifted from the one in the data. The result of our comparison of the data and MC simulation gives the difference of peak positions between the data and MC simulation of (3 ± 2) MeV. We take the relative error 3 MeV/10.58 GeV $= 2.8 \times 10^{-4}$ as a combined uncertainty from the ISR and FSR description, the beam energy calibration, and the calibration of the tracking system.

The variation of the fit range within about 30% of that shown in Fig. 2 contributes an uncertainty on τ of ± 0.02 μm.

During the fit of the real data, the level of the background contribution (parameter A_{uds}) is fixed to the nominal value predicted by the MC simulation in a "nominal" fit. The contribution to the systematic error of the $\langle\tau\rangle$ value due to the uncertainty of the background level is tested by changing the background level in the range of the uncertainty of the $q\bar{q}$ continuum and other backgrounds, from -50% to $+150\%$. This range is estimated conservatively from the control sample with looser selection

criteria. The maximal variation of the τ parameter is 0.01 μm.

The relative uncertainty due to the accuracy of the τ-lepton mass [3] is $(0.16\ \text{MeV}/c^2)/(1776.82\ \text{MeV}/c^2) = 9.0 \times 10^{-5}$.

To check the stability of the result for the different periods of Belle operation and vertex detector geometries, we repeat the analysis for three subsamples of the data. The obtained results are consistent within statistical errors.

The present PDG listings provide only the average lifetime of the positive and negative τ leptons. Our measurement determines the lifetimes for positive and negative τ leptons separately. The difference of $\langle\tau\rangle$ for positive and negative τ leptons obtained in the corresponding fits is $(0.07 \pm 0.33)\ \mu$m. Most of the sources of systematic uncertainties affect the result for positive and negative τ leptons in the same way, so their contributions to the lifetime difference cancel. The upper limit on the relative lifetime difference is calculated according to Ref. [14] as

$$|\langle\tau_{\tau^+}\rangle - \langle\tau_{\tau^-}\rangle|/\langle\tau_\tau\rangle < 7.0 \times 10^{-3} \quad \text{at 90\% C.L.} \quad (5)$$

The systematic uncertainty of the lifetime difference is at least 1 order of magnitude smaller than the statistical one, and is neglected.

In summary, the τ-lepton lifetime has been measured using the technique of the direct decay time measurement in fully kinetically reconstructed $e^+e^- \to \tau^+\tau^- \to 3\pi\nu_\tau 3\pi\nu_\tau$ events. The obtained result for the product of the mean lifetime and speed of light is

$$\langle\tau_\tau\rangle = [86.99 \pm 0.16(\text{stat}) \pm 0.10(\text{syst})]\ \mu\text{m}, \quad (6)$$

or in units of seconds

$$(290.17 \pm 0.53(\text{stat}) \pm 0.33(\text{syst})) \times 10^{-15}\ \text{s}.$$

The first measurement of the lifetime difference between τ^+ and τ^- is performed. The obtained upper limit on the relative lifetime difference between positive and negative τ leptons is $|\langle\tau_{\tau^+}\rangle - \langle\tau_{\tau^-}\rangle|/\langle\tau_\tau\rangle < 7.0 \times 10^{-3}$ at 90% C.L.

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