CHAPTER 120

MEASUREMENT OF DENSITY CURRENTS IN AN IDEALIZED MODEL

By Timm STUCKRATH [1)]

1 INTRODUCTION

The mixing of salt and fresh water in estuaries of tidal rivers is related to two major phenomenons turbulent diffusion and density currents

The turbulent diffusion can be read off from the horizontal and vertical salinity distribution, the density current can be determined from the velocity distribution on the vertical axes

The physical description of tidal mixing in a mathematically closed system is not possible, because turbulent diffusion and gravitational convection are varied by such different influences as tidal action, fresh water flow, river bed form and roughnes, and gravitational and CORIOLIS forces

Most research in this field has been done on turbulent diffusion, especially to predict the mean horizontal (longitudinal) salinity distribution IPPEN, HARLEMAN (ref 11, 12), and others have found out by various model tests that FICK's law of diffusion used in one-dimensional form is a good physical description for the longitudinal salinity distribution and that the tidal energy dissipation, the cross section, and the fresh water flow are a good means of classifying tidal estuaries from the viewpoint of stratification

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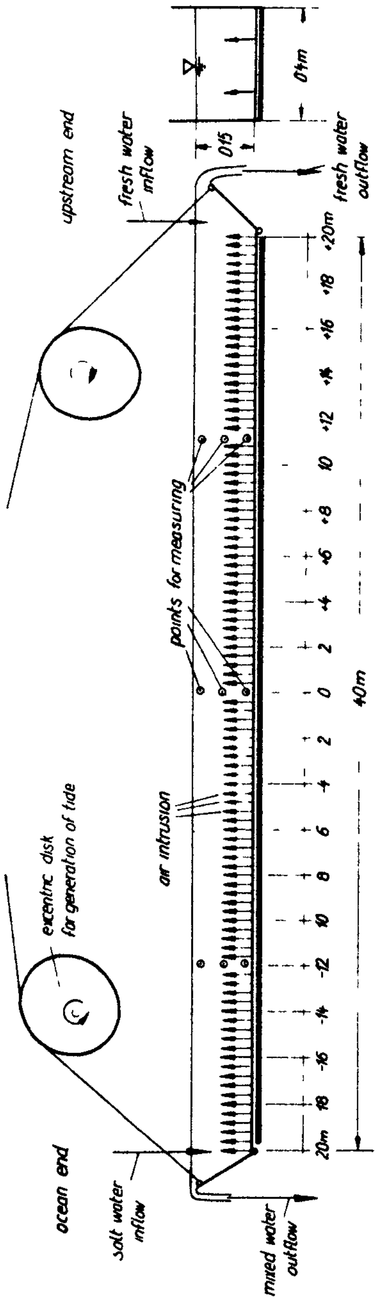


FIG 1 TIDAL FLUME OF THE FRANZIUS-INSTITUT

Much research is still needed to predict the density currents in tidal estuaries Most theoretical and practical research on density currents is still limited to rivers or model tests without tide and with high stratification In these channels salt and fresh water are seperated into layers The model tests which were carried out with sponsorship of the DEUTSCHE FORSCHUNGSGEMEINSCHAFT in the FRANZIUS-INSTITUT in Hannover, were assigned to obtain informations about density currents in mixed estuaries

2 EXPERIMENTAL EQUIPMENT

The flume that was used, is shown in fig 1 It was 40 m long, 0 4 m wide and 0 3 m deep The flume was supplied with fresh water from the upstream end and with salt water from the ocean end The tide was generated on both sides with tidal weirs which were moved by excentric disks The flume should represent the zone of brackish water in an estuary Due to the water supply the upstream end of the brackish zone during the whole tidal circle was within the flume, while the ocean end of the brackish zone could move downstream past the ocean end of the flume (fig 2)

As the intensity of turbulence in the model was nearly zero, artificial turbulence was generated by an air supply from the bottom of the flume (fig 1) Constant supply with air gives constant energy dissipation per unit of mass, even if there are great differences in water depth due to the tidal amplitude or changes in bed level An artificial air supply is therefore a good means of achieving a definite turbulent energy dissipation in tidal models

About 70 model tests were carried out with variation of

fresh water velocity from 0 20 to 1 35 cm/sec (4 series),
ocean salinity from 0 to 35 ^{o}/oo,
air supply from 4 to 85 cm^3/sec m^2

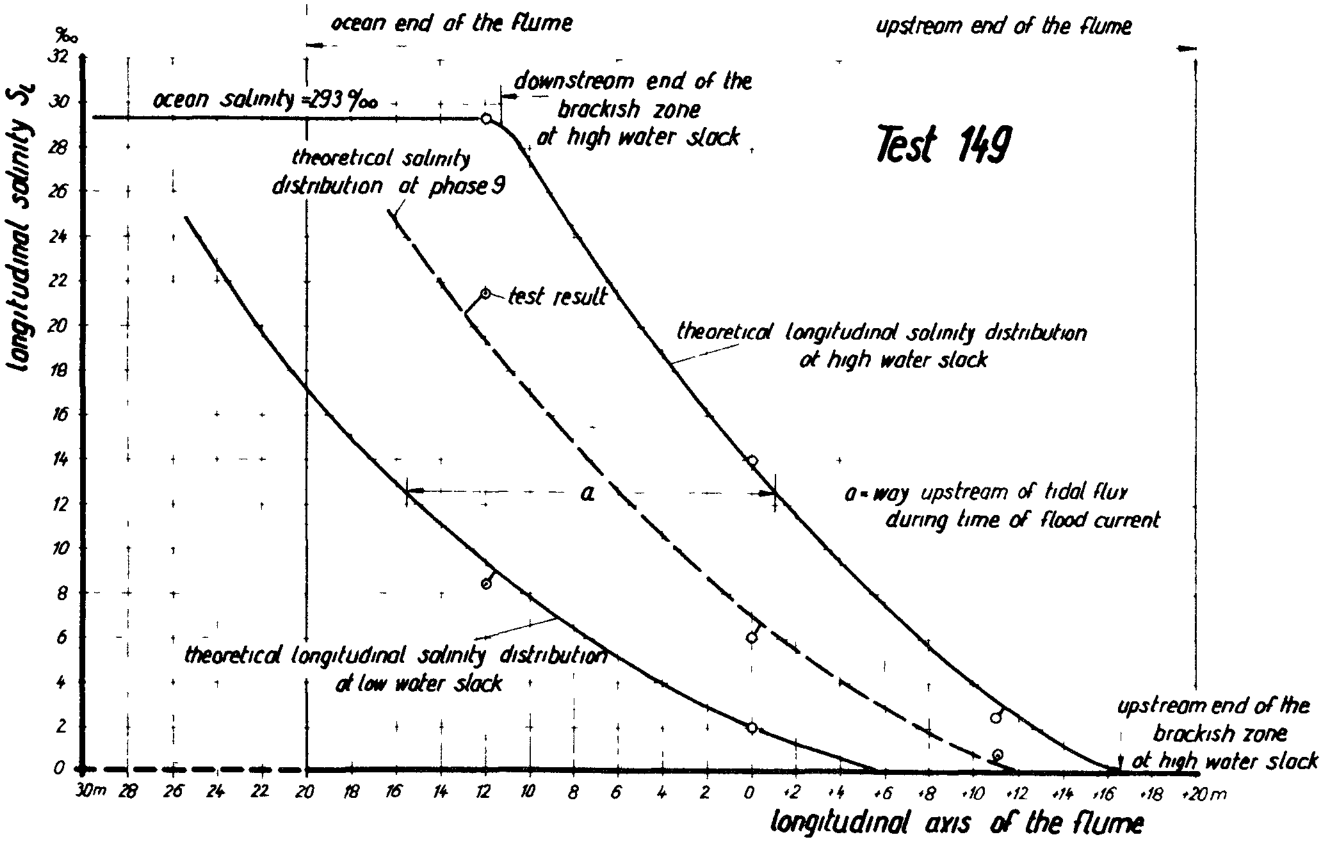


FIG. 2 LONGITUDINAL SALINITY DISTRIBUTION IN THE FLUME

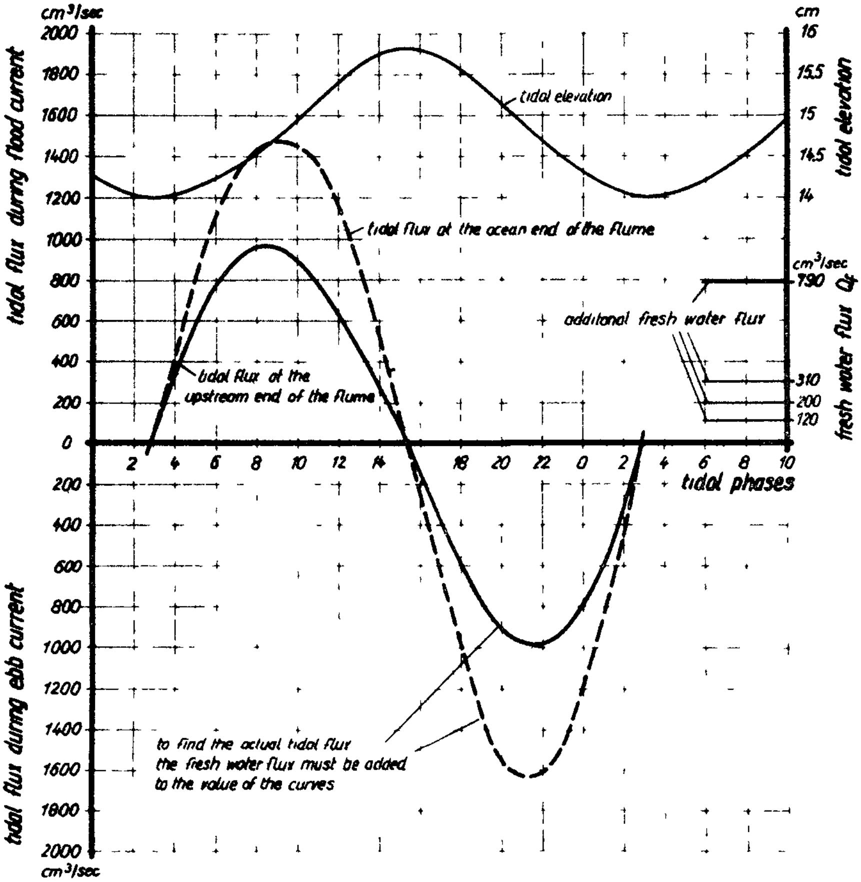


FIG 3 TIDAL AND FRESH WATER FLUX

Since the tide in the model had nearly no influence on turbulence, the experiments were carried out with a single tide which changed the depth from 14 cm (low water) to 15 8 cm (high water) The time for one tidal circle was 30 minutes The tidal flux on both ends of the flume during one tidal circle (24 tidal phases) and the fresh water flux of the 4 series conducted, can be read from fig 3

The salinity was measured by the electrical conductivity of salt water at 9 fixed points (3 vertical axes with 3 levels) (fig 1) The low spead currents were measured in the vecimity of these points with elastic pendulums as shown on fig 4

The elastic flexion of the pendulum is restricted to a short spring plate and recorded by strain gauges which were attached to both sides of the spring Currents of 0 2 cm/sec up to 10 cm/sec could be measured with this technique with good accuracy

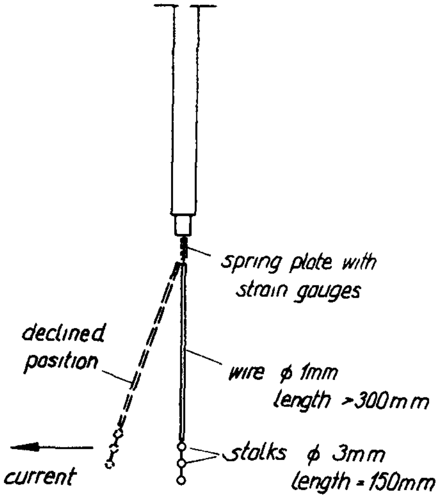


FIG 4 INSTRUMENT FOR MEASURING LOW SPEED CURRENTS

3 SALINITY DISTRIBUTION

Tidal research has shown that FICK's law of diffusion can be applied to rivers of various conditions of mixing (ref 5, 8, 9, 14, 15) It was therefore also important to verify the model tests of the FRANZIUS-INSTITUT with this theory Since the applied theoretical developments and their agreements with the test results cannot be shortened appreciably without harming their understanding, they shall not be explained here It was worthwile to know that all test results could be approximated with this theory closely (fig. 2) and that the longitudinal and vertical salinity distribution in the model could be expressed by physical parameters

Instead of the tidal energy dissipation - which is the major cause for mixing in prototype estuaries - the energy dissipation of the air introduced was used, similar to the experiments which were carried out at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY with occillating grids (ref 6, 7, 10)

For the analysis of density currents it is not neccessary to have a theoretical prediction of the salinity distribution It is however presumed that the salinity at any point and at any time is known, because its distribution and changing is the cause for density currents

4 DENSITY CURRENTS

One classical problem of density current was to determine the initial velocity v_1 which occurs, when a vertical seperation between two liquids of different density (ρ_1 and ρ_2) is suddenly removed (fig 5)

The equation which describes this velocity is

$$v_1 = K \sqrt{\frac{\rho_1 - \rho_2}{\rho} \; g \; h}$$

K = constant

ρ_1 = density of the water with greater density

ρ_2 = density of the water with lesser density

$\rho = \frac{\rho_1 + \rho_2}{2}$

g = acceleration of gravity

h = water depth

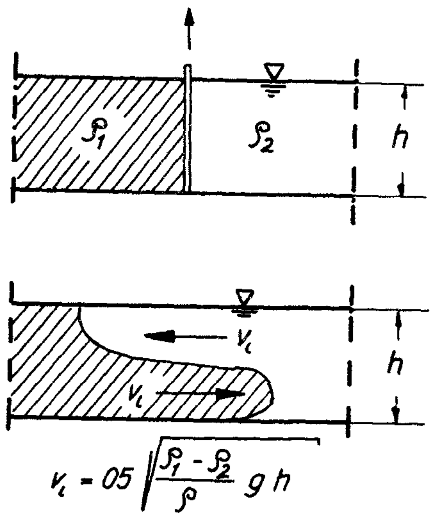


FIG 5 THE INITIAL VELOCITY v_1

Many model experiments have been carried out to determine the initial velocity. O'BRIEN and CHERNO (ref. 13), KEULEGAN (ref 4), YIH (ref. 3), BARR (ref 4), and others have proved that the constant before the root, which can also be computed by theoretical considerations as ABRAHAM and v d. BURGH (ref. 2) have shown, has nearly the value of

$$K = 0{,}5.$$

ALLEN and PRICE (ref. 3) ascertained this value at ship locks.

After the vertical seperation is removed (fig 5) the density current is

$$\Delta v = 2v_1 .$$

So the equation of the initial velocity can be rewritten

$$\Delta v = 2K \sqrt{\frac{\rho_1 - \rho_2}{\rho} \cdot g \cdot h}$$

$$F_d = 2K = \frac{\Delta v}{\sqrt{\frac{\rho_1 - \rho_2}{\rho} \cdot g \cdot h}}$$

F_d is called the densimetric FROUDE number For the initial velocity (fig 5) is $F_d = 1$.

In a river mouth the density current can be determined as in fig 6 as the difference between the measured velocity and the tidal velocity without density.

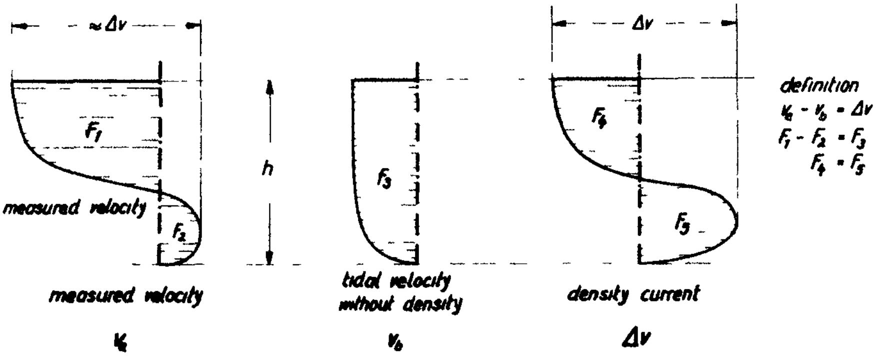


FIG. 6 DEFINITION OF DENSITY CURRENT

The experiments in the flume of the FRANZIUS-INSTITUT have shown that the densimetric FROUDE number is a good parameter for approximating the density currents in mixed channels also. In one vertical axis ρ_1 is the density on the bottom and ρ_2 is the density on the surface of the flume

Fig 7 compares the average density differences (which are linear to the salinity differences) over the whole tidal circle in the three vertical axes of fig 1 with the average density current Fig 7 shows that all experiments give an average densimetric FROUDE number smaller than $F_d = 1$

The compensating straight line gives

$$F_d = 0\ 6$$

The computed densimetric FROUDE numbers for every test result of fig. 7 are shown on fig. 8 with the average hori-

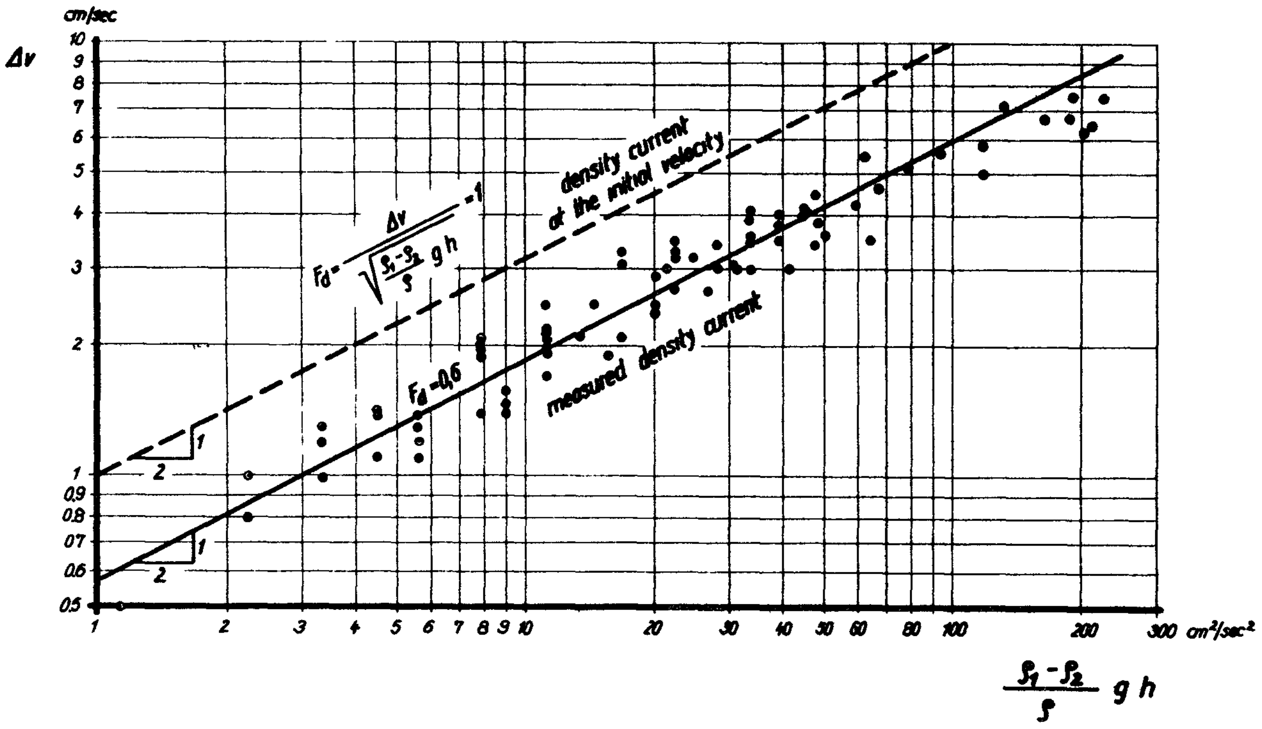
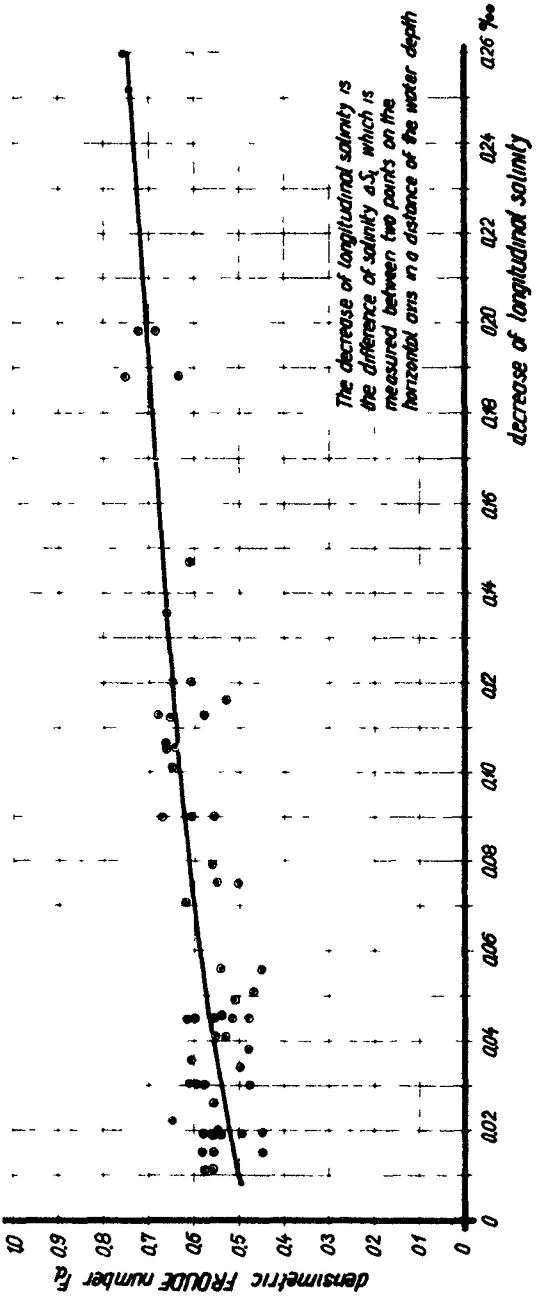


FIG. 7 DENSITY CURRENT VERSUS VERTICAL DENSITY DIFFERENCE



The decrease of longitudinal salinity is the difference of salinity ΔS_L which is measured between two points on the horizontal axis in a distance of the water depth

FIG. 8 DENSIMETRIC FROUDE NUMBER AND AVERAGE DECREASE OF LONGITUDINAL SALINITY zontal salinity gradient of the same experiment The average horizontal salinity gradient here is explained as the decrease of longitudinal salinity in a horizontal distance of the same length as the water depth

From fig 8 can be derived what ABBOTT (ref 1) has proposed in a different form from prototype observation If two estuaries have the same vertical salinity distribution and the same depth but unequal lengths of the salinity intrusion, the estuary with the shorter intrusion length has higher density currents than the longer one

Fig 8 cannot show the dependence of density currents on the change of the salinity distribution, because it is derived from time-averaged measurements This dependence can be read from fig 9 which shows the maximum density currents In the flume the maximum density currents occured shortly before the highest density on the bottom was reached On fig 9 the densimetric FROUDE number is compared with the change of density difference during the time of two tidal phases (2/24 of one tidal circle)

$\Delta\rho_n$ denotes the vertical density difference at the tidal phase of maximum density current, $\Delta\rho_{n-2}$ denotes the vertical density difference measured 2 tidal phases before If the change of density difference is zero, the value on the abszissa is

$$\frac{\Delta\rho_n}{\Delta\rho_{n-2}} = \frac{\Delta S_n}{\Delta S_{n-2}} = 1$$

At this point fig 9 gives F_d = about 0,5 So the maximum density current in an estuary where the salinity differences do not change quickly, can be computed as the average density current in fig 8

If

$$\frac{\Delta\rho_n}{\Delta\rho_{n-2}} = \frac{\Delta S_n}{\Delta S_{n-2}} \quad \text{tends to } \infty, \ F_d \text{ tends to } 1$$

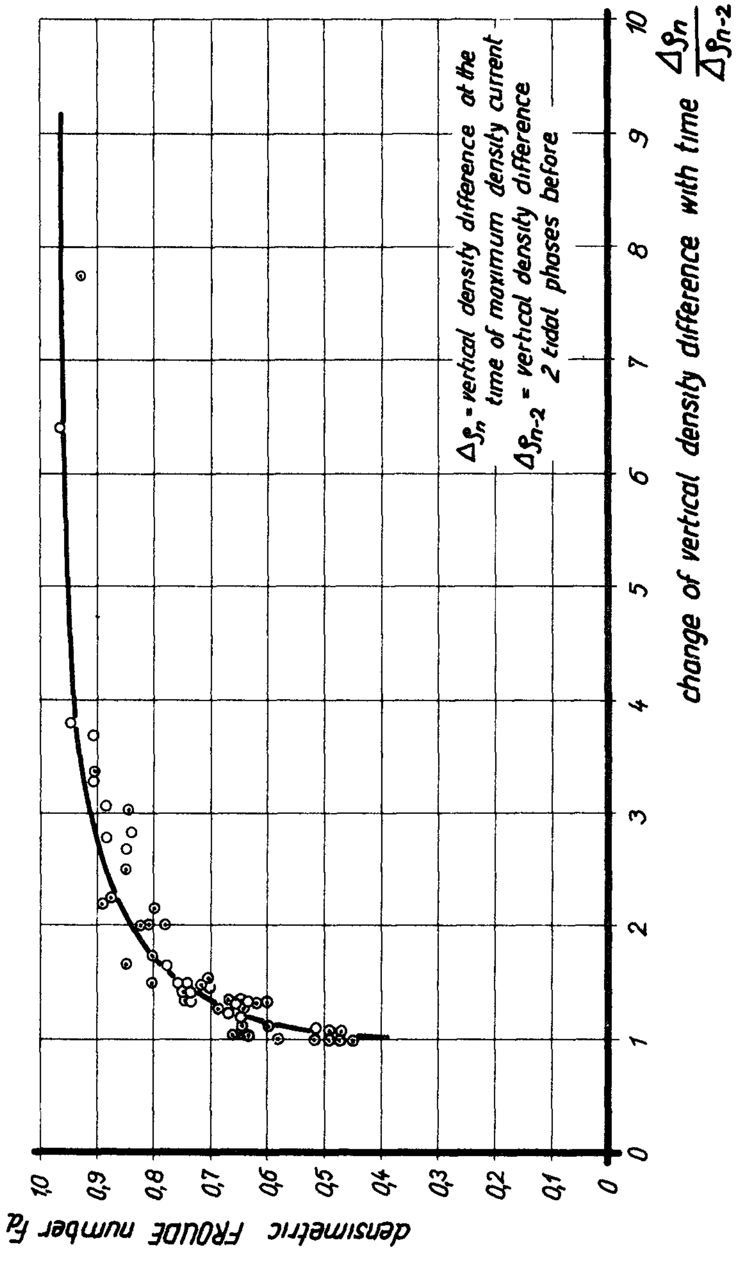


FIG 9 DENSIMETRIC FROUDE NUMBER AND CHANGE OF DENSITY DIFFERENCE WITH TIME

This is evident, because estuaries in which density differences change abruptly, approach the state of the initial velocity as in the experiment of fig 5

5 CONCLUSIONS

The model experiments of the FRANZIUS-INSTITUT represented the salinity distribution in an idealized estuary of various mixing stages The density currents show a dependence on the vertical salinity distribution (densimetric FROUDE number), on the longitudinal salinity distribution, and on the change of salinity with time

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