Measurement Procedures for Electromagnetic Compatibility Assessment of Electroexplosive Devices

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Abstract—**Electroexplosive devices (EED's) are electrically fired explosive initiators used in a wide variety of applications. The nature of most of these applications requires that the devices function with near certainty when required and otherwise remain inactive. Recent concern with pulsed electromagnetic interference (EMI) and the nuclear electromagnetic pulse (EMP) made apparent the lack of methodology for assessing EED vulnerability. A new and rigorous approach for characterizing EED firing levels is developed in the context of statistical linear models and is demonstrated in this paper. We combine statistical theory and methodology with thermodynamic modeling to determine the probability that an EED of a particular type fires when excited by a pulse of a given width and amplitude. The results can be applied to any type of EED for which the hot wire explosive binder does not melt below the firing temperature of the primary explosive. Methods for assessing model validity and for obtaining probability plots, called firing likelihood plots (FLP's), are included. These statistical methods are both more general and more efficient than previous methods for EED assessment. The results provide information that is crucial for evaluating the effects of currents induced by impulsive electromagnetic fields of short duration relative to the thermal time constant of an EED.**

Methods of measuring the thermal time constant of an EED and the energy needed to fire an EED with a single current impulse are given. These parameters are necessary not only to determine suitable ranges in the design of the statistical experiment, but also to assess the effect of pulses on EED's in EMC analyses.

Key Words—electroexplosive device (EED), EED response to pulsed currents, electromagnetic compatability (EMC), firing likelihood plots (FLP), thermal time constant of EED.

I. Background and Introduction

A HOT-WIRE electroexplosive device (EED) is an initiator that sets off a small charge of primary explosive by joule heating due to electrical current flowing in its bridgewire. When the primary explosive reaches its critical temperature due to this heating, it explodes and detonates a secondary explosive that serves as an actuator. A typical EED is shown in Fig. 1. The parameters commonly used for describing EED performance are all-fire current and no-fire current. Additional parameters are needed for electromagnetic compatability (EMC) analysis.

It is necessary to quantify both electrical and heat flow characteristics of the EED. The heating power (p) is a function of current (i) and electrical resistance (R_e). Additional parameters that must be measured are the critical temperature (θ_c) of the explosive, the thermal resistance (R) of the EED, and the thermal capacity (C) of the EED. The thermal time constant (τ) of the EED may be calculated from the RC product. Another parameter that must be measured is the energy (U) required to fire an EED with a single pulse of such short duration that practically no heat energy flows out over the duration of the pulse. By using a number of isolated rectangular pulses, we were able to generate a family of curves that relate pulse width and peak power to the likelihood of firing. We call these curves "firing likelihood plots" (FLP's).

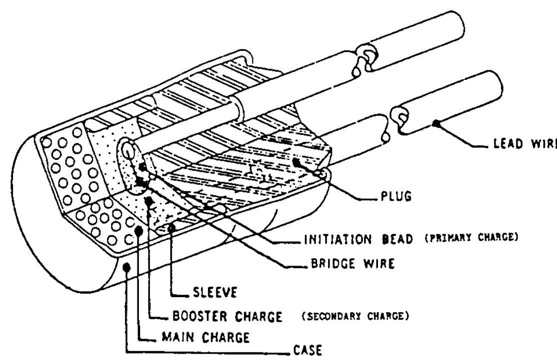

Fig. 1. Structural diagram of a typical EED.

Two distinct measurement procedures are needed to obtain these parameters and curves. Not all of these parameters can be measured on any one EED since each measurement destroys the EED. Subsequent measurements must be made on other EED's. For example, the firing current and thermal resistance may be measured by using a slowly increasing current ramp. The thermal capacity and energy to fire may be measured with another procedure that uses an impulse of current. These data may then be used to calculate other parameters and to design the statistical experiment. The statistical experiment requires many repetitions of the second measurement procedure, where the width and height of the impulses are varied over a carefully chosen range of values.

The electromagnetic environment that may induce stray currents in the wire of an EED is usually poorly known. The theory of how energy may be transferred from this environment by unintended antennas (any electrical conductor) is also poorly understood. These two very relevant topics are not within the scope of this paper, but motivate the work reported. They must be considered in any comprehensive EMC analysis.

There are several widely used standards or guidelines for

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evaluating and handling EED's in the presence of EMI [1]–[4]. The Bruceton up-down procedure [5] has been used for years to measure the firing current of EED's but it provides little information on extreme firing levels. In practice, since the extremes of the firing current distribution are of interest, e.g., the minimum all-fire current from an operational standpoint, and the maximum no-fire current from a safety standpoint, alternate procedures are used. In one such procedure [3], 50 EED's randomly selected from a lot are tested for 5 min at an arbitrarily set no-fire current level. These same EED's are then fired at an arbitrarily set all-fire level. If any of the samples fire at the no-fire level, or if any do not fire at the all-fire level, the lot fails.

Some of these direct measurement methods are not more widely used due to a practical problem with the binder used to hold the primary explosive around the wire of some of the older designs of EED's. The binder softens at a temperature below the critical temperature of the primary explosive. This allows flow of the explosive-binder mixture, which changes the thermal characteristics of the EED and often causes dudding. Later designs do not have this problem, and direct measurement of the firing current gives a better measure of the average and standard deviation for a given sample size than can be obtained with the Bruceton method.

An omission in existing EMC analyses on EED's is the effect of impulsive EM fields, either from periodic pulses such as may be generated by radar, or aperiodic pulses such as may be generated by lightning, nuclear EMP, or arcing of dc machinery. Whether these transient problems are of consequence is not clear. The methodology presented in this paper may help to answer these questions.

We propose a new way of characterizing the response of EED's to impulsive fields. The probability of firing is determined statistically as a function of width and power of a rectangular input pulse. We present general statistical procedures that can be used for characterizing EED's and describe the proper experimental methodology.

II. Sensitivity Testing—Related Methods

Sensitivity testing is the name that has been used for the general methodology associated with EED testing. The class of experiments is characterized by a binary response, fire or no-fire in this case, and a continuous stimulus. The stimulus is adjusted to a predetermined set of levels and the proportion of "fire" responses at each level is determined. Many test specifications spell out such procedures [2].

A procedure called the Bruceton method or the up-down method has been used for such tests [5]–[7]. The stimulus in general may represent very different attributes such as input voltage, height of drop, temperature, etc. The experiment consists of selecting an equispaced lattice of stimulus levels: $\cdots, s_{-2}, s_{-1}, s_0, s_1, s_2, \cdots$ centered at a nominal 50-percent firing level. Begin by applying the stimulus to a randomly selected EED at level s_0. The remaining settings are determined by the previous outcomes. If the first EED fires, the second is tested at level s_{-1}. If the first EED does not fire, the second is tested at level s_1. Each subsequent EED is tested according to this procedure: one level up if no response and one level down if a response. The advantage of this test is that it concentrates the test levels near the mean and improves accuracy of that estimate. Fewer EED's are therefore required on the average for a given accuracy. The disadvantages of the up-down method are that it provides relatively poor estimates of the dispersion, requires one-at-a-time sequential testing, and deals with only one-dimensional stimuli. Procedures for computing the estimates of the mean firing level and standard deviation are given in the references.

Another possible approach to characterizing such devices is to adapt methodology used in statistical bioassay known as quantal-response models. In these methods the probability of response is expressed as a linear function of the levels of the stimuli. Multiple stimuli are possible with this approach. Considerable methodology is available on this topic because of its many years of use and development in biological experiments. Two models have envolved as the most commonly used. The Probit model is based on a Gaussian probability distribution and the Logit model on a logistic distribution. They are very similar in their results and assumptions, but the Logit model is computationally simpler. See [8], [9] for details. They have been considered for EED applications [3], but it is not clear why they have not been used.

Both procedures require information on the proportion of (in this case) EED's in independent trials that fired at preset levels of the stimuli. To our knowledge Probit and Logit methods have not been used in EED characterizations, but they are the natural choice for certain EED experiments. They differ from the Bruceton method in several ways. They are not sequential in nature and the statistical design is predetermined by preliminary experimentation or *a priori* information. Stimulus levels and the number of EED's tested at each level are fixed. The model is more specific than the Bruceton method and must be validated with data for the results to be defensible. Good diagnostic tools are available for these methods, however. All three methods are suited to experiments that are somewhat wasteful of information since exact firing levels are not observed. EED's not fired are wasted unless they have not been affected by attempts at firing and can possibly be reused. We originally considered the Logit method for the EED characterization problem addressed here, but since it was possible with our measurement system and the particular EED's tested to obtain more information than is necessary for the Logit model, we developed the more efficient procedure described in this paper. Details of the statistical basis for the methodology are given in [10].

III. Heat Flow Equations

The relevant thermodynamic concepts are important for an understanding of the statistical model and its limitations, for obtaining prior information on the region of model validity, and for identifying thermodynamic parameters relevant for EED characterizations. The methodology introduced in this section is based on physical principles rather than statistical modeling. Only a small number of the available EED's were allocated to these measurements. Parameter estimates are obtained directly from the equations, not from statistical principles. These procedures are adequate for the intended purposes. They follow basic heat flow equations similar to those used in determining temperature distribution around a barretter wire [11]. In situations of temperatures less than red glow of the wire, heat flow by radiation is minimal. Also, for small volumes, the Grashof number is low, indicating minimal heat flow due to convection [12]. These effects have been simulated for a similar structure, a barretter wire in air, with an iterative mathematical procedure [11], and found to cause relatively minor variations in absolute temperature. Thus the simple models based on conductive heat flow should be adequate. They provide approximate "estimates" of the thermodynamic parameters and *a priori* information on design boundaries for the statistical model. The final statistical results demonstrate no inconsistencies with these initial thermodynamic measurements and assumptions.

The units for temperature are degrees Celsius throughout this paper.

The heat flow equation that applies during joule heating is [12]

$$\theta(t)=\theta_a+p(t)R(1-e^{-t/\tau}), \quad t>0 \tag{1}$$

where

- $\theta(t)$ is the temperature as a function of time (degrees),
- θ_a is the ambient temperature (degrees),
- t is the time after application of current (seconds),
- $p(t)$ is the power due to joule heating, $p(t) = i(t)^2R_e$ (watts), where $i(t)$ is current (amperes) as a function of time, R_e is the electrical resistance of the EED wire (ohms). The current in this case is constant over the given interval of time.
- R is the thermal resistance of the heat-leak path out of the EED (degrees per watt),
- τ is the thermal time constant (seconds). τ is the product of R and C, where C is the thermal capacity of the EED wire and explosive-binder mixture (joules per degree).

Differentiating (1) gives the rate of rise of temperature

$$d\theta(t)/dt=R\left[\frac{p(t)e^{-t/\tau}}{\tau}+\frac{dp(t)}{dt}(1-e^{-t/\tau})\right]. \tag{2}$$

If $p(t)$ is independent of time, as is the case while power is applied with a rectangular pulse, (2) reduces to

$$d\theta(t)/dt=(p(t)/C)e^{-t/\tau}, \quad \theta_0>\theta_a. \tag{3}$$

During cooling, the temperature decreases from initial temperature (θ_0) according to

$$\theta(t)=\theta_a+(\theta_0-\theta_a)e^{-t/\tau}. \tag{4}$$

The temperature rise and fall due to a single impulse function of current may be calculated using (1) and (4). For times much less than the thermal time constant of the EED, the limit of (3) may be used as given by

$$d\theta(t)/dt=p(0)/C, \quad t=0. \tag{5}$$

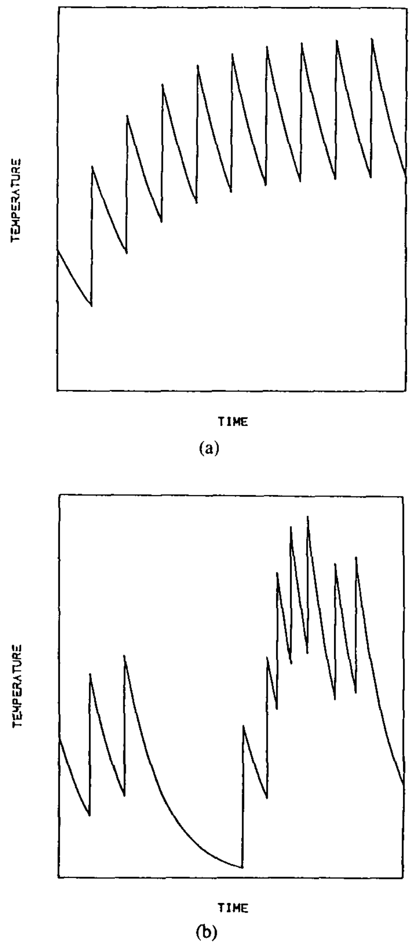

Fig. 2. Illustration of the cumulative heating (stacking) of an EED due to a train of periodic (a) or aperiodic (b) pulses.

This form is useful to determine the power and hence the current needed to fire an EED within a specified time.

For excitation caused by a sequence of either periodic or aperiodic pulses, where the cooling time between pulses is of the same order as the thermal time constant, a combination of (1) and (4) may be used to determine cumulative temperature rise, commonly known as stacking. This is illustrated in Fig. 2.

A. Measurement Procedures

The results of our experiments are presented to illustrate the methodology but not for the purpose of characterizing the type of EED used in this study. Squibs were used instead of EED's for these measurements. A squib is an EED of low explosive charge, usually without any secondary explosive. The particular squibs used simulate a common form of EED, an electrically fired commercial blasting cap. Due to the age and unsealed design of these squibs, the data obtained are not considered typical of data that would be obtained from normal EED's. The squibs used, while all of the same type, were not suitably controlled prior to testing. They were readily available, but the lot number was not recorded and the storage time and conditions were not fully known. They were used to develop and to validate the method. Variability from these unknown factors, if any, was confounded with our random errors.

Two procedures were used to measure the parameters of this type of EED. Since each measurement procedure destroyed the EED, only a subset of the relevant parameters were measured on any one EED. The first procedure used a slowly increasing current ramp. The second procedure used a rectangular impulse of current.

The first measurement procedure was as follows. Ambient temperature was recorded. The critical temperature of the primary explosive may be obtained either from the manufacturer or it may be measured with a special oven with five solid walls and a sixth weak wall made of some easily replaceable material that could serve as a pressure-release blast wall. The electrical resistance R_e of each EED was measured with a special multimeter that applied much less current than the no-fire current. A slow current ramp was used to heat the EED to detonation. The value of current at which the EED fires was recorded as I_f. The thermal resistance was obtained by

$$R = (\theta_c - \theta_a)/P_f \tag{6}$$

where

- θ_c is the critical temperature of the primary explosive (degrees)
- θ_a is the ambient temperature at time of EED firing (degrees),
- P_f is the power applied at the time of firing, i.e., $P_f = I_f^2 R_e$. Values of three independent parameters were measured, R_e, I_f, and θ_a, and

two values were calculated, R and P_f.

Data from the first measurement procedure are summarized in Table I. The average and standard deviation were calculated for the electrical resistance, the firing current, the firing power, and the thermal resistance. The values of θ_a were not included in the table since their only use was in (6). These are based on data from the firing of 10 EED's.

The second procedure also required knowledge of the critical temperature of the primary explosive and measurement of the ambient temperature and electrical resistance of each EED. An approximate value of the thermal time constant, unknown at this point, was needed to make estimates of pulse width and spacing to be used. A few EED's were fired in order to obtain this estimate. We started with a pulse width of 1 ms and a pulse spacing of 500 ms. The values meet the minimum heat leak and cooling time requirements for an EED with thermal time constant in the range of 10–100 ms. The amplitude of the pulse was then increased slowly until the EED under test fired. The amplitude at which the EED fires was recorded as I_p. The energy needed to fire an EED with a single rectangular pulse was calculated as follows:

$$U_f = Pw \tag{7}$$

where

- U_f is the energy (joules) to fire with a single impulse,
- P is the peak power (watts), $P = I_p^2 R_e$,
- w is the pulse width (seconds).

TABLE I
AVERAGE AND STANDARD DEVIATION OF FOUR PARAMETERS OBTAINED USING THE FIRST MEASUREMENT PROCEDURE

Parameter	Average	SD	Units
Electrical Resistance	1.42	0.123	ohms
DC firing Current	0.39	0.015	amperes
Thermal Resistance	1365.76	188.4	degrees per watt
Power to Fire	0.22	0.027	watts

The thermal capacity C of the EED (joules per degree) was calculated as follows:

$$C = U_f/(\theta_c - \theta_a) \tag{8}$$

where

U_f is defined in (7),

θ_c and θ_a are critical and ambient temperatures, respectively.

Using the assumed value of pulse width, values of three independent parameters R_e, θ_a, and I_p were measured. U_f and C were calculated from these three, plus knowledge of θ_c.

Data from the second measurement procedure are summarized in Table II, based on data from firing 11 EED's. Values of I_p and w were not included, since their values were used in (7) to calculate U_f.

The product of the average thermal resistance from the first procedure and the average thermal capacity from the second procedure gave a calculated value of thermal, time constant τ of 53.4 ms. If this had fallen outside the 10–100 ms range suitable for the trial pulse width and spacing that we used, a second measurement iteration would have been needed using suitably adjusted pulse width and spacing to accommodate this calculated time constant.

The statistical experiment is discussed in Section IV. Information obtained from the first two procedures was used to determine the appropriate ranges of parameters for the statistical experiment.

B. Instrumentation

The instrumentation to make these measurements consisted of four pieces of equipment plus a restricted area for setting off the EED's. See Fig. 3 for a block diagram. The pulse generator, storage oscilloscope, and digital multimeter are commercially available items. The pulse amplifier is a special design; it should be able to reach peak pulse currents of 20 A or more with a rise time of about 10 ms. It required a high-current capacity ancillary power supply. See Fig. 4 for one such design. The multimeter must be able to measure resistance in the 1-Ω range to three significant figures. Most good pulse generators and oscilloscopes meet the rise time requirements. A dummy 1-Ω, 50-W load was substituted for the EED to make adjustments on the range scales of the pulse generator and oscilloscope.

TABLE II
AVERAGE AND STANDARD DEVIATION OF THREE PARAMETERS OBTAINED USING THE SECOND MEASUREMENT PROCEDURE

Parameter	Average	SD	Units
Electrical resistance	1.45	0.09	ohms
Energy to Fire	11.8	[illegible]	millijoules
Thermal Capacity	39.1	[illegible]	microjoules per degree

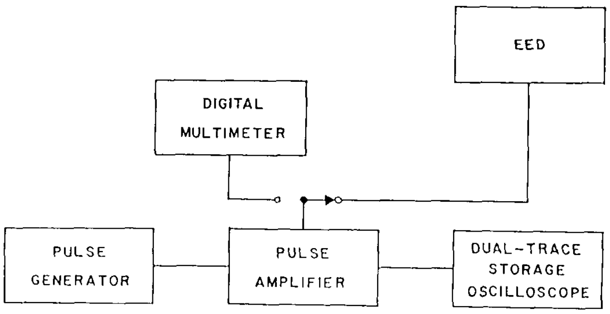

Fig. 3. Block diagram of instrumentation.

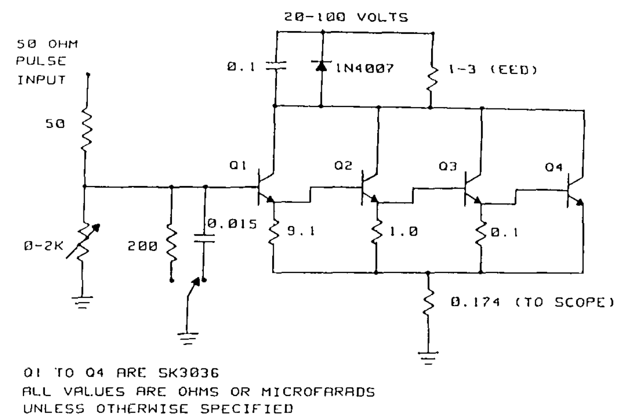

Fig. 4. Pulse amplifier schematic.

The requirements for the area used for setting off the EED's were based mainly on safety and convenience. A 200-l (55 gal) drum with about 30 cm of sand in the bottom served as an absorber and director for blast energy and shrapnel particles. Firing current was fed through a coaxial cable and a connector on the side of the drum. This allowed final activation of the circuit by connecting the cable to the outside of the drum, so that the person making the connection was never exposed to an armed EED. Electrical resistance measurements were made through this same cable, and were calculated by subtracting short-circuit resistance of the cable from the total measured resistance. This also assured a reliable electrical connection which prevented apparent dudding.

IV. The Statistical Experiment

In this section we describe the statistical experiment and its design. This is the direct application of the statistical methodology developed in [10] combined with the thermodynamic concepts in the previous section. Some statistical notation is now introduced.

A general linear model is given by

$$\underline{Y} = \underline{X}\underline{\beta} + \underline{\epsilon} \tag{9}$$

where

- $\underline{Y}$ is an $n \times 1$ vector of response variables.
- $\underline{X}$ is an $n \times r$ matrix of regressor variables. Its elements are fixed, not random variables. The dimension r ($r < n$) equals the number of unknown parameters in the model. The structure of this matrix fully specifies the form of the linear model.
- $\underline{\beta}$ is an $r \times 1$ vector of unknown parameters.
- $\underline{\epsilon}$ is an $n \times 1$ vector of random errors. Statistical assumptions are zero mean ($E(\underline{\epsilon}) = \underline{0}$) and, in the most general case, that its variances and covariances are specified by a positive definite symmetric $n \times n$ matrix $\underline{\Sigma}$. The exact form of $\underline{\Sigma}$ is usually unknown.

The elements ϵ_i of ϵ are the errors in the i^{th} observation. When the ϵ_i are uncorrelated, the off-diagonal elements of $\underline{\Sigma}$ will be zero. If the ϵ_i also have constant variance σ^2, then $\underline{\Sigma} = \sigma^2\underline{I}$ where $\underline{I}$ is the $n \times n$ identity matrix. The latter assumption is not made in this paper. When it can also be assumed that the ϵ_i have a Gaussian distribution, then uncorrelated ϵ_i are also statistically independent. If the errors are Gaussian then the statistical properties of the model and its parameter estimates are well known [13].

Equation (9) is the basis for the statistical estimation procedures used. Since the linear least squares model represented by (9) is used in a nonstandard way, it was necessary to introduce the additional notation. In particular, the X and Y vectors each represent the same attributes, pulse width and pulse amplitude, but are interchanged during the experiment.

Consider (10a) and (10b), which are relevant to the specific problem at hand.

$$u = \beta v^{-1} + \epsilon v^{-1} \tag{10a}$$

$$v = \beta u^{-1} + \epsilon u^{-1}. \tag{10b}$$

These equations (10) suggest an interpretation of bilinearity when written in the equivalent hyperbolic form of (11) and each is a special case of (9) for $r = 1$.

$$uv = \beta + \epsilon. \tag{11}$$

While mathematically equivalent to (10), the form of (11) is not an adequate basis for a statistical analysis where u and v are being varied as separate factors, and the region of model validity must be determined. We will show that once the model has been proven valid for a particular type of EED, (11) can be used for simple computations of $\hat{\beta}$, the estimate of β.

The two attributes of concern are pulse width and pulse amplitude. Two experiments are now defined according to which of these attributes is fixed and which is measured.

Experiment 1: The pulse amplitude is fixed within a set of predetermined design levels and the pulse width at which each EED fires is measured.

Experiment 2: The pulse width is fixed within a set of predetermined design levels and the pulse amplitude at which each EED fires is measured.

Equations (10) correspond to the two experiments. The pulse width, whether fixed or measured, is in units of time (milliseconds). The choice of units for pulse amplitude is not as simple. Possible parameters are voltage, current, or power. Each requires a different experiment and statistical model. The choice will also affect interpretation of the conclusions and uncertainty of the final estimates.

The experiments were conducted in a region where the instrumentation supplies a rectangular pulse. If the pulse width is short relative to the thermal time constant of the EED, there will be negligible heat loss during the pulse. Therefore, the temperature rise will be proportional to energy, the product of pulse power and pulse width.

If current or voltage had been used as a measure of amplitude, two things would have been considered. First, the mathematical model would not have an appealing physical interpretation as discussed in the previous paragraph, i.e., the temperature-energy relation would be absent. The mathematical model would have a square-law component in the amplitude and would not have the symmetry suggested by (10) or the resulting simplified analysis possible with (11). Second, the error structure of the experiment would be different. If only current or voltage were measured, the electrical resistance of the EED, which varies for each EED, would contribute to the uncertainty of the parameter estimate. Since it is possible to measure the electrical resistance safely and accurately, pulse power was selected as the amplitude parameter.

This choice of pulse power as one of the parameters has several advantages. The parameter and variables have physical meaning and the EED electrical resistance variations are eliminated from the estimation procedures. The statistical model, though nonstandard, can be analyzed with standard, least-squares estimation procedures.

The binder material of the EED tested was not affected below the firing temperature. This is not true of all EED's, as some binders soften at temperatures below the critical temperature of the primary explosive. Such EED's must be tested by a different procedure.

A. Statistical and Thermodynamic Design of the Experiment

The design of the experiment is done in two stages. The first stage is based largely on physical theory. Its objective is to determine a range of pulse amplitudes and widths within which it is possible to perform the experiment and be assured that crucial assumptions hold. The second stage is the statistical design of the experiment within this feasible region. R from the first measurement procedure and C from the second procedure give an estimate of τ necessary in determining this feasible region. The second procedure is then used to measure pulse energy, as is now described.

Limits of the instrumentation used in the experiment, thermodynamic properties of the EED, and mathematical-statistical assumptions underlying the analysis must be considered for the first stage of the design. A feasible region will then be established within which pulse width and pulse amplitude may be set. The minimum interpulse spacing is also determined at this time. This variable does not arise explicitly in the experiment but is crucial for model validity. The details for determining the feasible regions of pulse width, pulse amplitude, and interpulse spacing are now given.

There is a limitation on both the maximum pulse width and the minimum pulse width. The minimum pulse width is limited by instrumentation capabilities. The pulse amplifier rise time will distort the leading edge of a pulse. For long pulse widths this rise time distortion is negligible and the pulses will appear to be rectangular. For very short pulses the imperfect leading edge will dominate its shape and the pulse can no longer be considered rectangular. Rise time limitations of the pulse amplifier can be determined from the amplifier design specifications and should also be measured in the test system with a suitably fast oscilloscope. We determined that pulses of about 0.1 ms minimum duration were rectangular. In the final statistical design the shortest pulse width assigned for experimental purposes was 0.2 ms or twice the minimum pulse width. The requirement of rectangular pulses enables use of the model described in (9) and its special case (10). This is true because the pulse can then be characterized by its width-power product, or equivalently, its energy. The EED temperature is related to power in (1), but in order to simplify this relationship so that (5) may be used, a limit must be placed on the maximum pulse width.

The maximum feasible width is not determined by instrument limitations but by thermodynamic properties of the EED and the desire to minimize the complexity of the model and the analysis. The rectangular pulse requirement of the previous paragraph enables simple computation of energy. Statistically this would enable the use of a simple model except that the binary response (fire or no-fire) is in general not linear in energy but in temperature. In order for the EED input energy and EED bridgewire temperature to be approximately linearly related, there must be minimal heat loss from the EED during the pulse. Using (3) and (5) we can determine the percent error between the linear and exponential models for various pulse widths. This error is 0.5 percent for $\tau/100$, 2.5 percent for $\tau/20$, 5.1 percent for $\tau/10$, 8.2 percent for $\tau/6$, and 13 percent for $\tau/4$. To enable as wide a range of pulse widths as possible, $\tau/20$ was chosen as the maximum preset pulse width for the experiment. The value of EED thermal time constant from Section III-A was 53.4 ms. We used a maximum pulse width of 2.7 ms, which keeps this error less than 2.5 percent.

The maximum limits on pulse amplitude are due to the instrumentation. The pulse amplifier exhibits some distortion at high powers and begins to saturate above 50 W. It was possible to generate pulses over 50 W, but for levels much greater, some distortion in the rectangular shape was visible. Therefore the upper power limit was set at 50 W. There is no lower limit on pulse amplitude per se, but it is limited implicitly by the pulse width maximum. A longer pulse width is required to fire an EED as pulse power decreases. The pulse amplitude will, for this reason, have a lower limit.

The minimal spacing between pulses must also be predetermined. While not explicitly required for the statistical design, it is necessary to space the pulses sufficiently far apart to allow all, or almost all, of the heat from previous pulses to dissipate. The response of each pulse can therefore be treated independently of previous pulses. Equation (4) describes how the hot wire temperature returns to ambient after a pulse is turned off. Approximately 95 percent of the heat energy from any pulse will be dissipated in three time constants, 98 percent in four time constants, and over 99 percent in five time constants. This is true for any ambient temperature and any pulse width or power below the firing level. An interpulse spacing of 500 ms was chosen. This is greater than five time constants (267 ms).

The numerical values for these bounds are relevant only for the particular type of EED and instrumentation used. The methods described here may be used for obtaining the corresponding limits for any other EED for which the binder material does not melt before firing. The experimental limits could have been obtained by other methods, statistical, for example. The methods given, however, are simple to implement, have useful physical interpretations, and can be obtained with minimal data (using few EED's). The bounds that have been determined are used for the present values of pulse width or pulse amplitude only. If measured values exceed these bounds, the effects will have to be evaluated statistically.

Given this preliminary information, we chose to allocate the approximately 100 EED's that remained in a 2 × 5 experiment. The two level factor is the experiment type. The two experiments described in Section IV are characterized by which variable is fixed and which is observed. The second factor is the level at which the fixed variable is set. Five equally spaced levels were chosen within the predetermined experimental region for each experiment. The chosen levels are given in Table III.

Ideally the available EED's should be allocated equally among all 10 design points. This could not be done because a small proportion of the EED's were expected to be defective, i.e., duds. We used unsealed squibs that are not as reliable as sealed EED's. Since each EED is destroyed upon functioning, the defective ones could be identified only when tested.

After the levels of the experiment have been determined, it is desirable to randomize the order in which the experiment is performed. We decided to combine and randomize across the ten levels of the two factors. To randomly assign each EED would have been impractical. Setting the fixed variable once and testing a set of EED's insured better repeatability and required considerably less time. While there is no reason to believe that there is a time effect in this measurement system, the randomization is done to insure against inadvertent time trends. Table A36 from Natrella [5] was used and the ordering in Table IV was obtained. The procedure is simply to assign an integer to each level in Table III and then move sequentially, in any direction, through Table A36 from a randomly chosen starting point. The levels are determined by the sequence of random integers, ignoring repeats.

TABLE III
FACTOR LEVELS CHOSEN FOR PULSE WIDTH AND PULSE POWER

Width:	0.2	0.8	1.4	2.0	2.6	milliseconds
Power:	2.0	14.0	26.0	38.0	50.0	watts

TABLE IV
RANDOMIZED DESIGN SEQUENCE FOR MEASUREMENT OF EED DESIGN LEVELS

Test sequence	Fixed variable	Level
1	Width	0.8 milliseconds
2	Width	2.0 milliseconds
3	Power	38.0 watts
4	Width	0.2 milliseconds
5	Power	2.0 watts
6	Width	1.4 milliseconds
7	Power	50.0 watts
8	Power	26.0 watts
9	Width	2.6 milliseconds
10	Power	14.0 watts

V. EXPERIMENTAL RESULTS

The experimental data are given in Tables V and VI, along with some relevant transformations of the data. Table V contains the data from Experiment 1 (pulse power fixed) and Table VI contains the data from Experiment 2 (pulse width fixed). All levels of Experiment 1 have 7 or 8 observations. This is reasonably balanced considering the random nature of the EED reliability discussed previously. In Experiment 2 all levels have 7 or 8 observations except one, the 0.2 ms width level, which contains 16 observations. This was due to an equipment failure that occurred in the system which generates and measures the pulses. After the equipment was repaired and recalibrated, the level that was completed just prior to the failure was repeated. When the new measurements exhibited no significant change, the new data were combined with the old and the experiment continued. Partly as a result of this failure, there were 46 EED's tested in Experiment 2 as compared to 36 in Experiment 1.

Two EED's that are outliers in firing energy are also outliers in electrical resistance. In this case the 15th EED is a moderate outlier with lower resistance than all the others and the 67th EED is a high-resistance outlier. Both, however, required a higher energy to fire than was typical of the other EED's. Since the EED's that required excessive firing energies also exhibited nontypical resistances, the possibility of screening EED's by measuring their electrical resistance is suggested. While this is possible, it was not done here. The present study was not directed toward studying outliers and there is insufficient information in these data to say anything definite about them.

A. Statistical Analysis

The linear model given in (9) is a general model that enables a wide choice of specific models. The exact model is specified by the choice of the design matrix. It is good practice to fit several likely models to the data and choose the best one. "Best" can be interpreted as the model that provides a good fit determined by statistical methods, yet contains only necessary parameters.

Several models were fit to the data beginning with the

TABLE V
LISTING OF DATA AND RELEVANT TRANSFORMATIONS OF THE DATA FROM EXPERIMENT 1

EXPERIMENT 1 (1-POWER FIXED, 2-WIDTH FIXED)

I	EX	NO	LV	WIDTH	POWER	CURRENT	RESISTANCE	ENERGY	LOGWIDTH	LOGPOWER	LOGENERGY	FIXEDVAR	MEAS VAR	FIXLOGVAR	MSDLOGVAR
1	1	1	4	.305E+00	.380E+02	.521E+01	.140E+01	.115E+02	-.119E+01	.364E+01	.245E+01	.380E+02	.305E+00	.364E+01	-.119E+01
2	1	2	4	.285E+00	.380E+02	.479E+01	.166E+01	.108E+02	-.126E+01	.364E+01	.238E+01	.380E+02	.285E+00	.364E+01	-.126E+01
3	1	3	4	.300E+00	.380E+02	.489E+01	.159E+01	.114E+02	-.120E+01	.364E+01	.243E+01	.380E+02	.300E+00	.364E+01	-.120E+01
4	1	4	4	.350E+00	.380E+02	.495E+01	.155E+01	.133E+02	-.105E+01	.364E+01	.259E+01	.380E+02	.350E+00	.364E+01	-.105E+01
5	1	5	4	.270E+00	.379E+02	.511E+01	.145E+01	.102E+02	-.131E+01	.363E+01	.232E+01	.379E+02	.270E+00	.363E+01	-.131E+01
6	1	6	4	.250E+00	.380E+02	.524E+01	.138E+01	.950E+01	-.139E+01	.364E+01	.225E+01	.380E+02	.250E+00	.364E+01	-.139E+01
7	1	7	4	.325E+00	.380E+02	.537E+01	.132E+01	.123E+02	-.112E+01	.364E+01	.251E+01	.380E+02	.325E+00	.364E+01	-.112E+01
8	1	8	4	.330E+00	.380E+02	.529E+01	.136E+01	.125E+02	-.111E+01	.364E+01	.253E+01	.380E+02	.330E+00	.364E+01	-.111E+01
9	1	9	1	.570E+01	.202E+01	.115E+01	.153E+01	.115E+02	.174E+01	.703E+00	.245E+01	.202E+01	.570E+01	.703E+00	.174E+01
10	1	10	1	.590E+01	.200E+01	.114E+01	.155E+01	.118E+02	.177E+01	.693E+00	.247E+01	.200E+01	.590E+01	.693E+00	.177E+01
11	1	11	1	.630E+01	.200E+01	.121E+01	.138E+01	.126E+02	.184E+01	.693E+00	.253E+01	.200E+01	.630E+01	.693E+00	.184E+01
12	1	12	1	.690E+01	.200E+01	.116E+01	.148E+01	.138E+02	.193E+01	.692E+00	.262E+01	.200E+01	.690E+01	.692E+00	.193E+01
13	1	13	1	.580E+01	.200E+01	.115E+01	.152E+01	.116E+02	.176E+01	.693E+00	.245E+01	.200E+01	.580E+01	.693E+00	.176E+01
14	1	14	1	.560E+01	.200E+01	.113E+01	.157E+01	.112E+02	.172E+01	.692E+00	.241E+01	.200E+01	.560E+01	.692E+00	.172E+01
15	1	15	1	.870E+01	.200E+01	.130E+01	.119E+01	.174E+02	.216E+01	.693E+00	.286E+01	.200E+01	.870E+01	.693E+00	.216E+01
16	1	16	5	.235E+00	.500E+02	.608E+01	.136E+01	.118E+02	-.145E+01	.391E+01	.246E+01	.500E+02	.235E+00	.391E+01	-.145E+01
17	1	17	5	.200E+00	.500E+02	.605E+01	.137E+01	.100E+02	-.161E+01	.391E+01	.230E+01	.500E+02	.200E+00	.391E+01	-.161E+01
18	1	18	5	.250E+00	.500E+02	.612E+01	.134E+01	.125E+02	-.139E+01	.391E+01	.253E+01	.500E+02	.250E+00	.391E+01	-.139E+01
19	1	19	5	.205E+00	.500E+02	.624E+01	.129E+01	.103E+02	-.158E+01	.391E+01	.233E+01	.500E+02	.205E+00	.391E+01	-.158E+01
20	1	20	5	.240E+00	.500E+02	.582E+01	.148E+01	.120E+02	-.143E+01	.391E+01	.248E+01	.500E+02	.240E+00	.391E+01	-.143E+01
21	1	21	5	.240E+00	.500E+02	.599E+01	.140E+01	.120E+02	-.143E+01	.391E+01	.248E+01	.500E+02	.240E+00	.391E+01	-.143E+01
22	1	22	5	.196E+00	.500E+02	.601E+01	.139E+01	.980E+01	-.163E+01	.391E+01	.228E+01	.500E+02	.196E+00	.391E+01	-.163E+01
23	1	23	3	.520E+00	.260E+02	.418E+01	.149E+01	.135E+02	-.654E+00	.326E+01	.260E+01	.260E+02	.520E+00	.326E+01	-.654E+00
24	1	24	3	.420E+00	.260E+02	.408E+01	.157E+01	.109E+02	-.868E+00	.326E+01	.239E+01	.260E+02	.420E+00	.326E+01	-.868E+00
25	1	25	3	.450E+00	.260E+02	.429E+01	.142E+01	.117E+02	-.799E+00	.326E+01	.246E+01	.260E+02	.450E+00	.326E+01	-.799E+00
26	1	26	3	.520E+00	.260E+02	.433E+01	.139E+01	.135E+02	-.654E+00	.326E+01	.260E+01	.260E+02	.520E+00	.326E+01	-.654E+00
27	1	27	3	.370E+00	.260E+02	.427E+01	.143E+01	.962E+01	-.994E+00	.326E+01	.226E+01	.260E+02	.370E+00	.326E+01	-.994E+00
28	1	28	3	.510E+00	.260E+02	.443E+01	.132E+01	.133E+02	-.673E+00	.326E+01	.259E+01	.260E+02	.510E+00	.326E+01	-.673E+00
29	1	29	3	.410E+00	.260E+02	.441E+01	.134E+01	.107E+02	-.892E+00	.326E+01	.237E+01	.260E+02	.410E+00	.326E+01	-.892E+00
30	1	30	2	.840E+00	.140E+02	.328E+01	.130E+01	.118E+02	-.174E+00	.264E+01	.246E+01	.140E+02	.840E+00	.264E+01	-.174E+00
31	1	31	2	.730E+00	.140E+02	.317E+01	.139E+01	.102E+02	-.315E+00	.264E+01	.232E+01	.140E+02	.730E+00	.264E+01	-.315E+00
32	1	32	2	.770E+00	.140E+02	.315E+01	.141E+01	.108E+02	-.261E+00	.264E+01	.238E+01	.140E+02	.770E+00	.264E+01	-.261E+00
33	1	33	2	.870E+00	.140E+02	.331E+01	.128E+01	.122E+02	-.139E+00	.264E+01	.250E+01	.140E+02	.870E+00	.264E+01	-.139E+00
34	1	34	2	.920E+00	.140E+02	.316E+01	.140E+01	.129E+02	-.834E-01	.264E+01	.256E+01	.140E+02	.920E+00	.264E+01	-.834E-01
35	1	35	2	.780E+00	.140E+02	.319E+01	.138E+01	.109E+02	-.248E+00	.264E+01	.239E+01	.140E+02	.780E+00	.264E+01	-.248E+00
36	1	36	2	.880E+00	.140E+02	.320E+01	.137E+01	.123E+02	-.128E+00	.264E+01	.251E+01	.140E+02	.880E+00	.264E+01	-.128E+00

TABLE VI
LISTING OF DATA AND RELEVANT TRANSFORMATIONS OF THE DATA FROM EXPERIMENT 2

EXPERIMENT 2 (1-POWER FIXED, 2-WIDTH FIXED)

I	EX	NO	LV	WIDTH	POWER	CURRENT	RESISTANCE	ENERGY	LOGWIDTH	LOGPOWER	LOGENERGY	FIXEDVAR	MEAS VAR	FIXLOGVAR	MSDLOGVAR
37	2	1	2	.800E+00	.150E+02	.304E+01	.163E+01	.120E+02	-.223E+00	.271E+01	.248E+01	.800E+00	.150E+02	-.223E+00	.271E+01
38	2	2	2	.800E+00	.135E+02	.327E+01	.126E+01	.108E+02	-.223E+00	.260E+01	.238E+01	.800E+00	.135E+02	-.223E+00	.260E+01
39	2	3	2	.800E+00	.133E+02	.310E+01	.139E+01	.106E+02	-.223E+00	.259E+01	.237E+01	.800E+00	.133E+02	-.223E+00	.259E+01
40	2	4	2	.800E+00	.132E+02	.292E+01	.155E+01	.106E+02	-.223E+00	.258E+01	.236E+01	.800E+00	.132E+02	-.223E+00	.258E+01
41	2	5	2	.800E+00	.144E+02	.321E+01	.139E+01	.115E+02	-.223E+00	.266E+01	.244E+01	.800E+00	.144E+02	-.223E+00	.266E+01
42	2	6	2	.800E+00	.164E+02	.345E+01	.138E+01	.131E+02	-.223E+00	.280E+01	.258E+01	.800E+00	.164E+02	-.223E+00	.280E+01
43	2	7	2	.800E+00	.149E+02	.333E+01	.134E+01	.119E+02	-.223E+00	.270E+01	.248E+01	.800E+00	.149E+02	-.223E+00	.270E+01
44	2	8	2	.800E+00	.167E+02	.345E+01	.141E+01	.134E+02	-.223E+00	.282E+01	.259E+01	.800E+00	.167E+02	-.223E+00	.282E+01
45	2	9	4	.200E+01	.679E+01	.226E+01	.133E+01	.136E+02	.693E+00	.192E+01	.261E+01	.200E+01	.679E+01	.693E+00	.192E+01
46	2	10	4	.200E+01	.605E+01	.214E+01	.132E+01	.121E+02	.693E+00	.180E+01	.249E+01	.200E+01	.605E+01	.693E+00	.180E+01
47	2	11	4	.200E+01	.584E+01	.208E+01	.135E+01	.117E+02	.693E+00	.176E+01	.246E+01	.200E+01	.584E+01	.693E+00	.176E+01
48	2	12	4	.200E+01	.555E+01	.202E+01	.136E+01	.111E+02	.693E+00	.171E+01	.241E+01	.200E+01	.555E+01	.693E+00	.171E+01
49	2	13	4	.200E+01	.500E+01	.185E+01	.147E+01	.100E+02	.693E+00	.161E+01	.230E+01	.200E+01	.500E+01	.693E+00	.161E+01
50	2	14	4	.200E+01	.501E+01	.182E+01	.152E+01	.100E+02	.693E+00	.161E+01	.230E+01	.200E+01	.501E+01	.693E+00	.161E+01
51	2	15	4	.200E+01	.637E+01	.214E+01	.139E+01	.127E+02	.693E+00	.185E+01	.254E+01	.200E+01	.637E+01	.693E+00	.185E+01
52	2	16	4	.200E+01	.510E+01	.191E+01	.141E+01	.102E+02	.693E+00	.163E+01	.232E+01	.200E+01	.510E+01	.693E+00	.163E+01
53	2	17	1	.200E+00	.531E+02	.619E+01	.139E+01	.106E+02	-.161E+01	.397E+01	.236E+01	.200E+00	.531E+02	-.161E+01	.397E+01
54	2	18	1	.200E+00	.512E+02	.619E+01	.134E+01	.102E+02	-.161E+01	.393E+01	.233E+01	.200E+00	.512E+02	-.161E+01	.393E+01
55	2	19	1	.200E+00	.553E+02	.631E+01	.139E+01	.111E+02	-.161E+01	.401E+01	.240E+01	.200E+00	.553E+02	-.161E+01	.401E+01
56	2	20	1	.200E+00	.440E+02	.571E+01	.135E+01	.880E+01	-.161E+01	.378E+01	.218E+01	.200E+00	.440E+02	-.161E+01	.378E+01
57	2	21	1	.200E+00	.487E+02	.619E+01	.127E+01	.973E+01	-.161E+01	.388E+01	.228E+01	.200E+00	.487E+02	-.161E+01	.388E+01
58	2	22	1	.200E+00	.525E+02	.643E+01	.127E+01	.105E+02	-.161E+01	.396E+01	.235E+01	.200E+00	.525E+02	-.161E+01	.396E+01
59	2	23	1	.200E+00	.562E+02	.643E+01	.136E+01	.112E+02	-.161E+01	.403E+01	.242E+01	.200E+00	.562E+02	-.161E+01	.403E+01
60	2	24	1	.200E+00	.524E+02	.631E+01	.132E+01	.105E+02	-.161E+01	.396E+01	.235E+01	.200E+00	.524E+02	-.161E+01	.396E+01
61	2	25	1	.200E+00	.481E+02	.619E+01	.126E+01	.962E+01	-.161E+01	.387E+01	.226E+01	.200E+00	.481E+02	-.161E+01	.387E+01
62	2	26	1	.200E+00	.457E+02	.536E+01	.159E+01	.914E+01	-.161E+01	.382E+01	.221E+01	.200E+00	.457E+02	-.161E+01	.382E+01
63	2	27	1	.200E+00	.581E+02	.655E+01	.136E+01	.116E+02	-.161E+01	.406E+01	.245E+01	.200E+00	.581E+02	-.161E+01	.406E+01
64	2	28	1	.200E+00	.517E+02	.583E+01	.152E+01	.103E+02	-.161E+01	.394E+01	.234E+01	.200E+00	.517E+02	-.161E+01	.394E+01
65	2	29	1	.200E+00	.539E+02	.619E+01	.141E+01	.108E+02	-.161E+01	.399E+01	.238E+01	.200E+00	.539E+02	-.161E+01	.399E+01
66	2	30	1	.200E+00	.634E+02	.679E+01	.138E+01	.127E+02	-.161E+01	.415E+01	.254E+01	.200E+00	.634E+02	-.161E+01	.415E+01
67	2	31	1	.200E+00	.880E+02	.607E+01	.239E+01	.176E+02	-.161E+01	.448E+01	.287E+01	.200E+00	.880E+02	-.161E+01	.448E+01
68	2	32	1	.200E+00	.585E+02	.630E+01	.148E+01	.117E+02	-.161E+01	.407E+01	.246E+01	.200E+00	.585E+02	-.161E+01	.407E+01
69	2	33	3	.140E+01	.978E+01	.232E+01	.182E+01	.137E+02	.336E+00	.228E+01	.262E+01	.140E+01	.978E+01	.336E+00	.228E+01
70	2	34	3	.140E+01	.966E+01	.250E+01	.155E+01	.135E+02	.336E+00	.227E+01	.260E+01	.140E+01	.966E+01	.336E+00	.227E+01
71	2	35	3	.140E+01	.957E+01	.262E+01	.140E+01	.134E+02	.336E+00	.226E+01	.260E+01	.140E+01	.957E+01	.336E+00	.226E+01
72	2	36	3	.140E+01	.866E+01	.244E+01	.146E+01	.121E+02	.336E+00	.216E+01	.250E+01	.140E+01	.866E+01	.336E+00	.216E+01
73	2	37	3	.140E+01	.897E+01	.250E+01	.144E+01	.126E+02	.336E+00	.219E+01	.253E+01	.140E+01	.897E+01	.336E+00	.219E+01
74	2	38	3	.140E+01	.719E+01	.232E+01	.134E+01	.101E+02	.336E+00	.197E+01	.231E+01	.140E+01	.719E+01	.336E+00	.197E+01
75	2	39	3	.140E+01	.853E+01	.250E+01	.137E+01	.119E+02	.336E+00	.214E+01	.248E+01	.140E+01	.853E+01	.336E+00	.214E+01
76	2	40	5	.260E+01	.461E+01	.185E+01	.134E+01	.120E+02	.956E+00	.153E+01	.248E+01	.260E+01	.461E+01	.956E+00	.153E+01
77	2	41	5	.260E+01	.458E+01	.179E+01	.144E+01	.119E+02	.956E+00	.152E+01	.248E+01	.260E+01	.458E+01	.956E+00	.152E+01
78	2	42	5	.260E+01	.423E+01	.179E+01	.133E+01	.110E+02	.956E+00	.144E+01	.240E+01	.260E+01	.423E+01	.956E+00	.144E+01
79	2	43	5	.260E+01	.495E+01	.185E+01	.146E+01	.129E+02	.956E+00	.160E+01	.256E+01	.260E+01	.495E+01	.956E+00	.160E+01
80	2	44	5	.260E+01	.445E+01	.173E+01	.150E+01	.116E+02	.956E+00	.149E+01	.245E+01	.260E+01	.445E+01	.956E+00	.149E+01
81	2	45	5	.260E+01	.363E+01	.167E+01	.131E+01	.943E+01	.956E+00	.129E+01	.224E+01	.260E+01	.363E+01	.956E+00	.129E+01
82	2	46	5	.260E+01	.376E+01	.173E+01	.126E+01	.977E+01	.956E+00	.132E+01	.228E+01	.260E+01	.376E+01	.956E+00	.132E+01

general design matrix; it was then determined that the firing level behavior of the EED's can be adequately described by the relatively simple (10) and (11).

The chosen design matrix has dimensions 82 × 1 (82 EED's tested), and its components are the inverse values of the fixed variable (width or power). The residual variances were nonhomogeneous; therefore weighted least squares were necessary. The form of the nonhomogeneity is specified in (10), and was suggested by statistical theory. The weights are the inverse of the error variance under this model and proved effective at stabilizing the residual variances on the given data. The parameter vector β reduces, in this case, to a scalar β.

The weighted least squares estimate of β is 11.60 and the standard deviation of the estimate is 0.17 based on the data from 82 EED's. The units of β are watt-milliseconds (millijoules). Therefore corresponding relationships

$$P = 11.60 w^{-1} \tag{12a}$$

$$w = 11.60 P^{-1} \tag{12b}$$

are the best predictors of peak power P and width w, respectively within the range of experimentation. The values must be in the correct units, milliseconds for w and watts for P. Equations (12) exhibit the symmetry of (10).

The estimate of β is simple the arithmetic mean of the firing energies. This fact suggests a simpler analysis. It is not recommended, however, except as an exploratory tool or perhaps for testing a batch of EED's for which this model has already been proven correct and its valid experimental region already determined. The full linear model approach that we describe makes possible a thorough assessment of the appropriateness of the model for the data. Foremost, it allows the effects of the two factors, width and power, to be separated, whereas they would be confounded in a simple energy model. Since the utility of the method suggested here, and of the resulting graphics, is to determine the likelihood of firing for a given pulse configuration (width and amplitude), these factors must remain explicit in the experimentation and analysis. Another advantage, since computation is done almost exclusively on computers, is that good statistical software for linear models is readily available. Such software usually provides excellent diagnostic tools for use in the model validation, for graphical display of data and for exploratory analysis.

Data from the experiments are given in Tables V and VI. Detailed analysis given in [10] shows that the data fit the model.

VI. Implementation of the Firing Probability Plots

A most useful tool for assessing EED behavior that is made possible by this research is the firing likelihood plot (FLP). It graphically summarizes relevant information from the analysis in a format that is useful and easily interpreted. In this section we describe how to implement these plots from the data analysis that was described in Section V. It is first necessary to discuss two distinct topics related to the FLP's. This is done in Sections VI-A and VI-B.

A. Tolerance Intervals

A commonly used estimate of statistical interval is the confidence interval. It is an interval which, given validity of the assumptions, has a known probability of containing the unknown parameter of interest. For example, a 99-percent confidence interval could be derived for parameter β. It would provide information on where the mean firing energy is for the type of EED. An interval for the mean firing energy would be of little use, however. Concern in EED applications is for the probability that a given type of EED fires (or does not fire) given a particular input pulse or train of pulses.

Tolerance intervals address this issue more directly. They are intervals within which, with a given probability γ, a chosen proportion P_r of the population will lie. In the case of EED's, a two-sided, γ percent tolerance interval provides bounds on the range of firing energies, expressed in terms of pulse width and pulse amplitude within which 99 percent of the EED's would fire. The coefficient γ is the probability that the resulting interval is correct. It is analogous to the confidence coefficient in a confidence interval. There are also one-sided tolerance intervals that provide either lower or upper bounds on the percentile of the distribution. Different tables are required depending on whether one-sided or two-sided intervals are being used. A good reference for both tables and instructions for implementation of tolerance intervals is Natrella [5]. The need for all-fire and no-fire levels for EED applications dictates the use of upper and lower one-sided tolerance intervals. This is not a conventional usage.

The mean firing energy estimate $\hat{\beta}$ and the standard deviation s are central in adapting tolerance intervals for firing likelihood plots. A proportion P_r of the underlying population, Gaussian in this case, must be chosen. Then the desired probability γ that the interval is correct must be chosen. Table A7 of [5], a look-up table of values of K for calculating one-sided tolerance limits, is then used to obtain the specific value of K for the desired γ, P_r, and df. By symmetry of the Gaussian distribution, the upper and lower bounds for the tolerance interval are given by the limits

$$\hat{\beta} + Ks \tag{13a}$$

or

$$\hat{\beta} - Ks \tag{13b}$$

whichever is appropriate. Each probability contour on the firing likelihood plot corresponds to one of these bounds. The ϵ of (10) directly affects the bounds of (13), increasing the interval as the variance of ϵ increases. Plot implementation will be discussed next to show how we can use these intervals.

B. Plot Implementation

A problem might arise in the algorithm for generating plots. This will apply to most grahics software. The issue is the hyperbolic nature of the function being plotted. Choosing equally spaced points along either axis will generate plots that have a dense grid on the chosen axis but are very sparse on the orthogonal axis. Even logarithmic scaling will not alleviate this problem.

The approach we took was to derive a parametric equation, develop an equispaced index for the parameter, and then generate the plotted points from these values. The procedure is as follows:
The basic contours are described by the equation $Pw = \theta^2$ where P is the pulse power value and w is the pulse width value. We choose the following pair of parametric equations:

$$w = \theta^{1-t} \tag{14a}$$

$$P = \theta^{1+t}. \tag{14b}$$

To determine bounds on t, we observe that

$$w_{\max} = \theta^{1-t_{\min}}$$

$$P_{\max} = \theta^{1+t_{\max}}.$$

These equations are equivalent to

$$t_{min} = 1 - \frac{\log w_{\max}}{\log \theta}$$

$$t_{\max} = \frac{\log P_{\max}}{\log \theta} - 1.$$

Given the maximum w and P chosen for the plots, we find $t_{\min}$ and t_{max}, and then choose n, the number of points to be plotted. Compute

$$t_i = t_{\min} + \frac{(t_{\max} - t_{\min})}{n-1}(i-1);\ i = 1, \cdots, n$$

and the corresponding pairs of points

$$w_i = \theta^{1-t_i} \tag{15a}$$

$$p_i = \theta^{1+t_i};\ \text{where } i = 1, \cdots, n. \tag{15b}$$

The parametrization corresponds to $Pw = \theta^2$, not θ. Therefore, if $\hat{\beta}$ is the estimate in the linear model, the proper relationship for plotting the mean firing level would be $\theta = (\hat{\beta})^{1/2}$. Details for relating this parameterization to plotting other contours will be given in the following section.

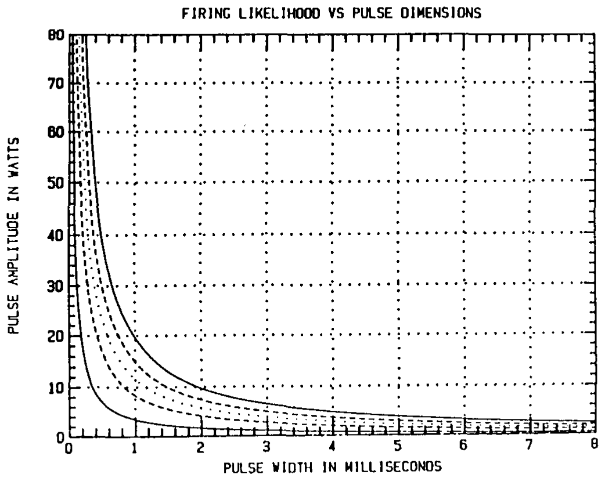

Fig. 5. Firing likelihood Plots for EED type tested. Dotted line: mean-firing level, dashed lines: upper and lower one-sided 95-percent tolerance intervals, solid lines: upper and lower one-sided 99.999-percent tolerance intervals.

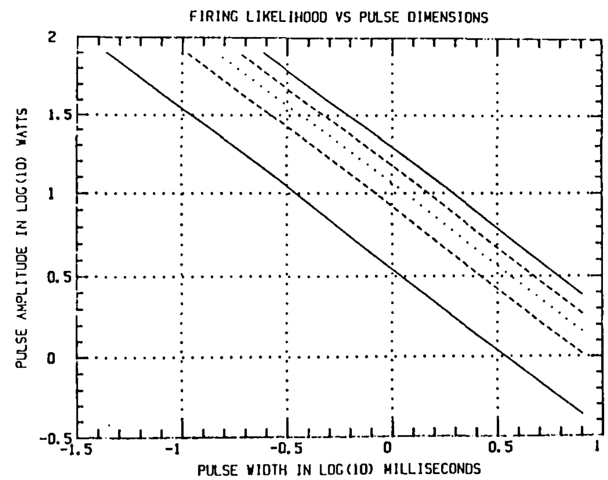

Fig. 6. Logarithmic firing likelihood plots for EED type tested. Dotted line: mean firing level, dashed lines: upper and lower one-sided 95-percent tolerance intervals, solid lines: upper and lower one-sided 99.999-percent tolerance intervals.

C. Generating the Graphs

A pair of contours for the firing likelihood plots are now simply implemented. For a given P_r and γ, obtain the limits given by (13a) and (13b). Then let $\theta_1 = (\hat{\beta} + Ks)^{1/2}$ and $\theta_2 = (\hat{\beta} - Ks)^{1/2}$, where θ_1 and θ_2 are each substituted for θ in (15a) and (15b). Generate a set of points to be plotted, as explained in Section VI-B, for both θ_1 and θ_2. These families of points will generate the 100 P_r percent firing likelihood curves. This process is repeated for each P_r for which a curve is desired. Examples are given in Figs. 5 and 6 for the EED's tested.

D. Interpreting the Graphs

Firing likelihood plots provide a convenient representation of an operating characteristic of a class of EED's. We can graphically assess the probability of firing for an EED from this class when it is subjected to a rectangular input pulse of a given width and amplitude. Once the experiment is performed on a representative sample of EED's and the resulting statistical analysis is completed, any relevant probability level (P_r) may be chosen for the plot contours. Each contour defines a bound on pulse dimensions that has a probabiltiy γ to include a proportion P_r of the EED's. In most cases the extreme values of P_r will be of interest, near 0 and near 1, corresponding to maximum no-fire and minimum all-fire levels, respectively. In the example given the contours may be thought of as distinct no-fire/all-fire limits.

It may be possible to relate the curves directly to electromagnetic field intensity for a given physical configuration. Since pulse power is proportional to E^2, electric field strength squared, the power axis of the plots may also be expressed in terms of the peak field strength squared of an impulsive field. A value of E^2 that corresponds to a specific power level is required for determining the position of the E^2 scale. This may be obtained by measuring steady-state values of E and power for a particular EED support structure. Usually the coupling of electromagnetic energy from a field to the structure and hence to the EED is poorly known.

Radiated pulses are not likely to be rectangular, and even if they are, most antennas will cause ringing due to phase distortion. Probabilities that are based on an ideal (rectangular) test situation are therefore likely to be conservative because of the decrease in actual energy coupled into the EED by an irregularly shaped pulse whose maximum values of amplitude and duration are equal to those of a rectangular pulse. Such a representation can extend the usefulness of the firing likelihood plots to cover most irregular pulse shapes.

There is no physical reason that the left end (high amplitude, low width) of the firing likelihood plots cannot be extended beyond the experimental region. Figure 6, a log-log plot from the same data used in Fig. 5, facilitates this extension. The limitation was imposed due to the inability of the test system to generate rectangular pulses of higher power and lesser pulse width. Extension beyond this limit could not be experimentally verified and statistical prediction was not considered. Such an extension of the plots should therefore be used with discretion as it is possible that unanticipated factors may affect the model in this region. Extensions to the right of the experimental region (low amplitude, long width) are not possible with our model for physical reasons previously stated.

The interpretation assumes adequate cooling time between pulses. If pulse repetition occurs in less than approximately 5 thermal time constants then (1) and (4) will have to be used to estimate the cumulative heating effect. The consequent cumulative heating will cause the EED to fire at a lower pulse energy level. Since temperature increases proportionally with power, it is possible to calculate a correction factor for closely spaced periodic pulses. It is also possible to relate the firing likelihood plots to aperiodic pulse trains but computations are more complex and require consideration of specific aperiodic sequences.

VII. Summary

A new method which integrates both statistical and engineering concepts has been proposed for characterizing EED's. The method provides a useful description of performance for a class of EED's based on rigorous and efficient statistical procedures. The methodology and measurement techniques have been proven and demonstrated in an actual experiment and are applicable to a wide class of EED's. The resulting firing likelihood plots provide information which is relevant and not previously available.

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