


Measurements as a roadblock to near-term practical quantum advantage in chemistry: Resource analysis

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Recent advances in quantum computing devices have brought attention to hybrid quantum-classical algorithms like the variational quantum eigensolver (VQE) as a potential route to practical quantum advantage in chemistry. However, it is not yet clear whether such algorithms, even in the absence of device error, could actually achieve quantum advantage for systems of practical interest. We have performed an exhaustive analysis to estimate the number of qubits and number of measurements required to compute the combustion energies of small organic molecules and related systems to within chemical accuracy of experimental values using the VQE. We consider several key modern improvements to the VQE, including low-rank factorizations of the Hamiltonian. Our results indicate that, although these techniques are useful, they will not be sufficient to achieve practical quantum computational advantage for our molecular set, or for similar molecules. This suggests that novel approaches to operator estimation leveraging quantum coherence, such as enhanced likelihood functions, may be required.

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I. INTRODUCTION

In the last decade, quantum computers have evolved from laboratory prototypes of a few qubits to machines with tens of qubits that are commercially available for researchers and businesses to use [1,2]. In 2019, Google announced the realization of a quantum supremacy milestone: their 53-qubit chip accomplished a specific task that would be extremely difficult to simulate with a classical supercomputer [3], though recent work improved considerably on initial classical runtime estimates [4]. This task was specifically designed to be well suited to the quantum processor and challenging for classical computers, and does not solve a practical problem. The next milestone, and arguably the most pressing one [5], is finding a *practical quantum advantage* with noisy intermediate-scale quantum (NISQ) devices [6], that is, running an algorithm on a NISQ device that provides an improved solution for a commercially relevant task. This improvement can manifest in different ways, either as a reduction in the time to solution or an increase in the quality of the solution. Accomplishing this goal requires first a steady improvement in the quality of quantum computing hardware. Fortunately, we are witnessing a rapid growth in the number of qubits and fidelity of these machines as indicated by the recent trends in metrics such as quantum volume [7]: in the past couple of years this went from

32 in January 2020 [8], to 128 in September 2020 [9], then 2048 in December 2021 [10] and 4096 in April 2022 [11]. The second requirement towards quantum advantage is identifying commercially relevant tasks for which a near-term quantum algorithm can provide a measurable improvement compared to classical alternatives.

Quantum chemistry has been identified as a likely candidate [12–14] for quantum advantage for multiple reasons. First, electronic structure calculations are used extensively in the development of many technologies, for example in the chemicals industry [15], drug development [16], and battery materials research [17]. Second, electronic structure calculations rely on the Schrödinger equation, for which a general exact solution has exponential cost on a classical computer with all known classical methods. Third, quantum computers can store exponentially scaling representations of the wave function on a linear number of qubits and also provide means to implement Hamiltonian evolution efficiently, giving rise to quantum algorithms to estimate ground state energies of some molecular Hamiltonians using polynomially scaling resources.

Quantum approaches to electronic structure calculations can be divided into two categories: (1) algorithms based on the quantum phase estimation subroutine and related techniques and (2) quantum heuristic algorithms [13], such as the variational quantum eigensolver (VQE) [18] and related methods based on different versions of the time-dependent variational principle [14]. Under certain assumptions, approaches in the first category can provide an advantage in computational scaling compared to exact classical algorithms, however they require a fault-tolerant implementation [12,19–23], and therefore are not applicable in the near term. In contrast, quantum heuristics such as the VQE can be implemented on NISQ

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devices thanks to the flexibility in their construction, but do not provide any proven asymptotic advantage compared to classical algorithms. Demonstrating a quantum advantage in this context requires a comparison of the computational cost and performance of the quantum heuristic against the state-of-the-art classical approximations for specific problem instances. Performance metrics might include aspects such as total runtime and accuracy of the solution. In addition, this comparison must go hand in hand with an optimization of the algorithmic choices of the heuristic to maximize performance. From this perspective, a necessary step towards achieving quantum advantage in the near term is establishing protocols for evaluating quantum computational resources for specific sets of problem instances and target accuracy of the solution, a procedure we will refer to as resource and performance assessment (RPA).

While many studies have estimated resource requirements for quantum chemistry using fault-tolerant algorithms such as quantum phase estimation [19–24], only a small number have assessed the resource requirements for NISQ approaches. McClean *et al.* have analyzed the asymptotic measurement requirements of the VQE [25] while Kühn *et al.* numerically examined qubit requirements and required circuit depth for UCC-derived *Ansätze* [26]. Numerical studies have explored the VQE measurement costs of diatomic molecules and hydrogen chains or rings when applying fermionic marginal constraints and a low-rank factorization of the Hamiltonian [27,28]. Elfving *et al.* estimated the number of spin orbitals required for industrially relevant calculations and concluded that the required VQE execution time was prohibitively large, although the method used to estimate the execution time was not specified [24].

While these studies provide valuable insight into the performance of the VQE and its variants, several questions about the feasibility of these techniques for systems of practical interest remain unanswered. For example, although VQE measurement requirements have been assessed for small basis sets, such analysis has not been carried out for the basis sets required to achieve a useful accuracy with respect to the infinite basis set limit. Another key question is how these measurement requirements compare to the runtime of state-of-the-art classical quantum-chemistry techniques. Furthermore, previous studies have estimated measurements by employing canonical orbitals, and have not considered frozen natural orbitals (FNOs) [29–31], which are known to significantly reduce both the computational cost of classical wave-function-based quantum chemistry methods and the requirements on the number of qubits for quantum computing applications [32,33].

To address these questions, we have performed an RPA to estimate the number of qubits, number of measurements, and total runtime required for calculating combustion energies for small organic molecules to within chemical accuracy (defined as 4.2 kJ/mol or 1.6 mHa [34]) with a single VQE energy evaluation, leaving the problem of VQE parameter optimization to future work. These estimates consider frozen natural orbitals as well as measurement reduction techniques such as Hamiltonian grouping of commuting [35–39] or anticommuting [40,41] terms, the application of fermionic marginal

constraints [27], and low-rank factorization of the Hamiltonian [28,42].

Our results indicate that between 120 and 260 qubits are required for chemical accuracy for our benchmark systems. Under optimistic assumptions about the *Ansatz* requirements and the sampling rate of the device, we show that a single energy evaluation could take several days to weeks, rendering the calculations impractical and inferior to classical methods, in particular when considering the large number of such evaluations required for the optimization loop of VQE. Our results also show that although certain grouping techniques greatly reduce the number of measurements, they are not enough to guarantee practical runtimes in the regime where quantum advantage is expected. This suggests that making VQE practical in the near term requires the use of new approaches to measurement that leverage quantum coherence to reduce estimation runtimes, such as the recently proposed Bayesian likelihood function techniques [43,44].

The rest of the paper is organized as follows: in Sec. II, we describe our methods for each step of the RPA in detail. In Sec. III, we present the numerical results for our RPA. There, we establish accurate classical quantum chemistry reference values by comparison with experimental reaction energies. We then truncate the active space to establish the minimal number of qubits necessary to preserve chemical accuracy. The last step of our estimation evaluates the number of necessary measurements to reach chemical accuracy on a quantum computer, including measurement reduction techniques. This evaluation is made for increasing active space sizes to establish asymptotic scaling relationships. Finally, we introduce an empirical extrapolation formula to establish runtime and resource requirements for more general systems than the ones specifically studied in this paper. In Sec. IV, we discuss our results, their implications, and further research avenues.

II. METHODS

In this section we describe our methodology for resource estimation, starting with an outline of the RPA concept as applied to VQE, followed by a detailed description of the methods employed for the estimation of classical and quantum computational resources. All calculations were deployed using Zapata Computing’s Orquestra workflow management platform.

A. Outline of the resource and performance assessment

The goal of an RPA is simple: we want to estimate as accurately as possible the resources, such as number of qubits, number of measurements, fidelity, among others, needed to achieve a given quality of solution for a specific choice of quantum algorithm and a set of problem instances. By fixing a target quality in the solution, it is possible to compare the cost with that of state-of-the-art classical approaches, establishing whether a quantum advantage is possible. The process can be divided in five stages.

(1) Define a set of problem instances and the quantum algorithm to be assessed.

(2) Set a target metric for performance. For example, choose a target quality of solution or time to solution.

(3) Select a classical approach for comparison, ideally the state-of-the-art method for the instances of interest, and estimate the amount of classical resources required to achieve the target performance.

(4) Estimate the amount of quantum computational resources required to achieve the target performance using the quantum algorithm.

(5) Compare the performances and computational cost of the quantum and classical approaches. Establish whether a practical quantum advantage is attainable.

In this paper, the problem under study is the calculation of the combustion energy for a set of small organic molecules. We selected the gold standard for quantum chemistry—coupled cluster with singles, doubles, and perturbative triples [CCSD(T)]—as our classical algorithm, and evaluated the cost of estimating combustion reaction energies to chemical accuracy. This resource estimate includes determining the basis set and number of spin orbitals needed. The performance of the classical approach provides the reference to be outperformed by the quantum algorithm, setting chemical accuracy as the target metric for quantum advantage. In principle, it is also possible to set experimental data as the reference, provided reliable enthalpic corrections to the electronic energies are available. In our case, we instead ensured that the classical approach was reproducing experimental data, and then used the classical electronic energy results as reference for the quantum algorithms.

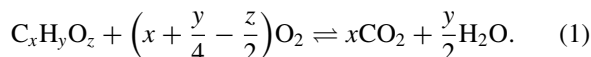
With the target accuracy fixed, we proceeded to estimate the number of qubits and total number of measurements needed to achieve such accuracy assuming access to a sufficiently expressive *Ansatz* and high enough gate fidelity. By incorporating assumptions about the characteristics of the variational circuit and the quantum hardware, we established realistic runtime estimates for achieving chemical accuracy. Crucially, our analysis takes into account the system and size dependence of different resource requirements. For this reason, we focus on techniques that can be scaled up to larger molecules or clusters, which is why considerations of spatial symmetry for example are not included.

While the most accurate RPA would require executing the algorithm, this might be too computationally costly. We can instead take advantage of our knowledge of the algorithm and the problem to investigate empirical scaling of the resources with system size in order to establish relationships that allow extrapolation to larger instances. Some performance metrics, such as the number of measurements in the VQE, only depend on properties of the problem instance. More specifically, the number of measurements to reach a target accuracy can be predicted from the Hamiltonian of each molecule and classical estimations for variances (see Sec. II C 3). In the rest of this section we describe in detail the benchmark data set chosen for our RPA as well as the methodology for estimating classical and quantum computational resources.

B. Benchmark data set

In this paper, we aim to establish resource estimates for computing electronic ground state energies with the VQE. We wish to apply our resource estimation procedure to a benchmark set of molecules to facilitate extrapolation to larger

systems. Ideally, this set would be of practical relevance, contain small enough molecules to allow chemically accurate classical computations, and correspond to well-established, accurate experimental data. For these reasons, we chose to study combustion reactions for the following small hydrocarbons: methane, methanol, ethane, ethene, ethyne, ethanol, propane, propene, and propyne (depicted in Fig. S1 in [45]). For clarity, we explicitly write the general formula for the reaction's stoichiometry:



Experimental enthalpies of combustion for the hydrocarbons in our benchmark set can easily be calculated from available enthalpies of formation [46] (see Table S1 [45]). By combining electronic ground state energies with vibrational, rotational, and translational enthalpic contributions, we can obtain simulated combustion enthalpies that can be compared to the experimental values. Most of our paper focuses on getting accurate electronic energies, as harmonic vibrational corrections to enthalpies are obtained from the second derivatives of the electronic energies. Anharmonic effects are expected to be important for larger, flexible molecules but only play a very minor role in our benchmark reactions, as numerically verified in Table S4 [45].

Algorithms to compute anharmonic vibrational spectra on quantum computers exist [47–50], and have been argued to be better candidates than electronic structure for early quantum advantage [51]. This assessment was based on considerations of scaling of the number of terms and their locality in the respective Hamiltonians. In addition, the relative magnitude of the Hamiltonian coefficients also favored vibrational Hamiltonians for the systems considered. We hope that our method for resource estimation provides an accurate picture of the prospects of electronic structure algorithms on quantum computers for concrete examples, thus facilitating the comparison with prospects of quantum algorithms for vibrational structure.

Our chosen set of molecules is dominated by dynamical correlation. As pointed out by Elfving *et al.*, this means that a very large number of orbitals is needed for accurate treatment [24]. Hence, a very large number of qubits would be needed on a quantum computer to rival quantum chemistry capabilities on classical computers. In that sense, systems dominated by nondynamical correlations would be better candidates for demonstrations of near-term quantum advantage. However, we believe that most of our extrapolation and resource estimation results are valid for general molecular systems, whether dominated by dynamical or by nondynamical correlations. In particular, our results regarding the scaling of the number of measurements necessary to reach chemical accuracy with the size of the system should be transferable to most cases.

C. Methodology for resource estimation

1. Classical benchmarks

The first component of the RPA consists of establishing a reference classical quantum chemistry approach and evaluating the classical resources needed to achieve chemical accuracy for a benchmark set of molecules. The current gold

standard for ground state electronic structure calculations is the CCSD(T) method [52]. For closed-shell molecules, and with sufficiently large basis sets, CCSD(T) can reach chemical accuracy, i.e., an error of 4.2 kJ/mol compared to experimental data [34,53]. Unfortunately, its N^7 scaling limits its application to small systems. For completeness, we note that approximate CCSD(T) methods were developed [54–57] that take advantage of the spatial locality of electron correlation, which allowed computation of much larger systems [57,58] by reducing the overall asymptotic scaling. However, the accuracy of the local approximations in coupled-cluster methods was recently questioned [59,60].

We assessed the suitability of CCSD(T) for our benchmark set. We also explored which basis set was large enough to reach chemical accuracy for CCSD(T), by performing computations in various aug-cc-pVXZ basis sets (denoted AVXZ) with X = D, T, Q, or 5 [61] denoting increasing angular momentum of the basis functions, and in the def2-TZVPPD basis set [62,63]. The details of our computational methods and results for this assessment are reported in the Supplemental Material [45] for the interested reader.

2. Number of qubits

To estimate the minimal number of qubits that can be used while recovering CCSD(T)/AV5Z results with sufficient accuracy, we explored different choices of orbitals and active spaces. In practice, we evaluated the CCSD(T)/AV5Z energy with and without truncations and we assumed that the resulting errors are reflective of those that would be obtained with an accurate VQE *Ansatz*.

The simplest choice for orbital truncation is to eliminate Hartree-Fock canonical virtual (i.e., unoccupied) orbitals with the highest energy. However, a better choice is known in the quantum chemistry literature: the FNO [29–31] method. FNOs have been successfully applied to reduce the number of qubits needed in quantum chemistry simulations on quantum computers [32,33], however the corresponding number of measurements has to our knowledge not been estimated. As is usual, we apply a perturbation theory correction to partly compensate for the truncated energy, for both canonical and frozen natural orbitals. Further technical details are reported in the Supplemental Material [45].

We also note that there exist other methods to reduce the number of qubits necessary to encode a problem: improved basis sets [64], exploitation of symmetries [65,66], or partitioning methods [67,68]. We leave their study to future work.

3. Measurement analysis

General considerations. To obtain the energy in the VQE algorithm, it is necessary to estimate the expectation value of the Hamiltonian by performing many measurements and averaging their results. The total number of necessary measurements M can be estimated as follows:

$$M = \frac{K}{\epsilon^2}, \quad (2)$$

where ϵ is the desired precision on the estimation and K is a proportionality constant that depends on the Hamiltonian, the state being measured, and the measurement strategy employed

for the estimation [25,69], as described below. Note that in quantum chemistry, an accuracy of 1.6 mHa with respect to the exact ground state in the infinite basis set limit is typically desired, which means that the uncertainty due to sampling error ϵ must be less than this amount.

After transformation to the qubit representation, the molecular Hamiltonian takes the form

$$\hat{H} = \sum_i h_i \hat{P}_i, \quad (3)$$

where \hat{P}_i is a product of Pauli operators acting on one or more qubits and h_i is the associated coefficient, obtained from the one- and two-electron integrals calculated with Psi4 [70].

While the simplest approach to estimating the expectation value of \hat{H} would be to measure each \hat{P}_i independently, it is possible to measure two operators \hat{P}_i and \hat{P}_j at the same time if they commute, thereby reducing the total number of measurements needed. In general, measuring two commuting operators \hat{P}_i and \hat{P}_j implies multiqubit measurements [37] or appending a unitary transformation to the circuit [71]. Hence, we first consider grouping methods that do not increase the depth of the circuit and rely only on single-qubit measurements, i.e., qubitwise commuting (QWC) groups [36,37]. QWC implies that for both \hat{P}_i and \hat{P}_j , the Pauli operators acting on the same qubit individually commute. Finding the optimal QWC grouping is equivalent to solving the minimum clique cover graph problem and is NP-hard in the general case [37]. Here we use a heuristic greedy algorithm that goes through all operators and adds each one to the first group with which it is qubitwise commuting [35,72,73]. In addition, we sort the list of operators according to their coefficients h_i , so operators with the largest coefficients are grouped first [74].

We also consider the basis rotation approach to Hamiltonian decomposition [28,42,73]. In the variant applied in this paper, the Hamiltonian terms that only contain \hat{Z} operators are measured in the usual way, while an eigenvalue decomposition is used to obtain a low-rank factorization of the remaining two-body terms. The expectation values of this low-rank factorization and the remaining one-body terms can be obtained by applying a linear-depth basis rotation circuit after the *Ansatz*.

Other methods to obtain the expectation value of the Hamiltonian at reduced cost exist [38,39,75,76], for example it is possible to measure groups of fully commuting Pauli terms by appending the appropriate circuit to the *Ansatz* [71], or to use locally biased [77] or derandomized [78] classical shadows as an alternative to QWC grouping. In the current paper, we only assessed the performance of one additional method based on grouping mutually anticommuting Pauli terms [40,41]. Our results show that anticommuting grouping is less performant than QWC grouping (see Fig. S12 [45]), therefore these results are not included in the main text. Benchmarking of additional methods like those mentioned above will be the object of future work. A particularly promising approach makes use of overlapping Pauli grouping [75].

Measurement estimation. The grouped Hamiltonian can be rewritten

$$\hat{H} = \sum_C \sum_{\alpha \in C} h_\alpha \hat{P}_\alpha, \quad (4)$$

where C indexes groups and α labels terms in a group. Applying the Lagrangian approach of Rubin *et al.* [27] shows that the optimal allocation of measurements to groups gives the following expression for the proportionality constant K :

$$K = \left[\sum_C \sqrt{\sum_{\alpha, \beta \in C} h_\alpha h_\beta \text{Covar}(\hat{P}_\alpha, \hat{P}_\beta)} \right]^2. \quad (5)$$

This measurement allocation scheme assumes that the covariances between all operators \hat{P}_α and \hat{P}_β are already known, including the variances $\text{Var}(\hat{P}_\alpha) = \text{Covar}(\hat{P}_\alpha, \hat{P}_\alpha)$. In general, one does not know the values of these covariances and must estimate them.

Because K depends on the variances and covariances of the operators \hat{P}_i , K depends on the quantum state being measured, i.e., it changes through the VQE optimization. To avoid these complications, and allow estimation of K for up to 80 qubits, we employ some simplifying approximations. For the variances, we consider two approximations. In the first, we assume variances to be 1.0, the upper bound. In the second, we estimate variances from configuration interaction singles and doubles (CISD) density matrices computed with Psi4 [79]. The variances can be approximated from the expectation value $\langle \hat{P} \rangle$ of each operator since $\hat{P}^2 = 1$:

$$\text{Var}(\hat{P}) = \langle \hat{P}^2 \rangle - \langle \hat{P} \rangle^2 = 1 - \langle \hat{P} \rangle^2 \quad (6)$$

where we approximate the exact expectation value by its CISD counterpart.

We assume all of the covariances between different terms are zero. This does not correspond to a worst or best case scenario, but approximates the effect of a random distribution of covariances within bounds given by the Cauchy-Schwarz inequality:

$$\left| \sqrt{\text{Var}(\hat{P}_\alpha) \text{Var}(\hat{P}_\beta)} \right| \geq \text{Covar}(\hat{P}_\alpha, \hat{P}_\beta). \quad (7)$$

Indeed, covariances can be positive or negative, but their magnitude is bounded by Eq. (7). Hence a random distribution is expected to have an average around zero. That is exactly the case when the quantum state being measured is a Haar random distribution [38]. In practice, we observed that this approximation resulted in K being overestimated by a factor of ≈ 2 relative to estimates using covariances obtained from circuit simulations of optimized *Ansätze*. Note that when the upper bound is used for variances and covariances are set to zero, the estimated value of K is entirely determined by the coefficients of the Hamiltonian.

A precise assessment of the number of measurements required and its scaling with increasing system size for both QWC and basis rotation grouping is fundamental in predicting the runtime of energy estimation on NISQ devices. For this purpose, we computed K for all the molecules in our benchmark set with the exception of O_2 for technical reasons. For each molecule, we generated the Hamiltonians from one- and two-electron integrals obtained from Psi4 for different active space sizes, where we always used an integer number of qubits per active electron to facilitate extrapolation. K was computed for all these Hamiltonians with up to 80 qubits, and a power fit of K as a function of the number of qubits was used to

extrapolate the number of measurements necessary for the 100 to 200 qubits region. In addition, we performed these estimations for both the upper bound approximation to the variances and variances computed from CISD. Finally, for each case we also computed the Hamiltonian coefficients based on canonical orbitals (as is usual in most VQE publications) and based on FNOs (consistent with our active space size estimations), relying on the aug-cc-pVDZ (denoted AVDZ) Dunning basis set in all cases. This results in a total of eight K estimations for each molecule and active space, giving us unprecedented insight into the relative performance of the variants examined.

Variance reduction. Grouping the Hamiltonian terms is not the only possibility to reduce the total number of measurements needed. The Hamiltonian can also be transformed so that its overall variance is reduced. Here, we explored the reduced density matrix constraints (RDMC) method [27] proposed by Rubin *et al.* In brief, this method adds operators to the Hamiltonian that sum to zero and optimizes their coefficients to reduce the total variance. We implemented this method directly in the qubit picture, which was suggested by Rubin *et al.* to have better performance than the original implementation in the fermionic picture. We present a comparison of both implementations in Fig. S11 [45] for the interested reader. We apply RDMC to a small set of molecules, with up to 20 qubits included in the active space, and examine the reduction obtained in K for the case of no grouping, for QWC grouping, and for the basis rotation grouping. The Hamiltonians examined were computed with FNOs based on the aug-cc-pVTZ (denoted AVTZ) Dunning basis set and variances were estimated from CISD density matrices.

In conclusion, our resource and performance assessment includes a benchmark of classical methods, which can then be used as a reference to estimate the number of qubits needed to reach chemical accuracy in the general case. We establish empirical scaling relations for the number of measurements using state-of-the-art grouping and measurement reduction techniques, various approximations for the variances involved, and two different molecular orbital bases. These scaling relations and their prefactors allow us to estimate the number of measurements needed to reach chemical accuracy when the qubit active space for the molecules in our benchmark reaches 100 to 200 qubits. These relations are also useful as a general guide for the scaling of QWC and basis rotation methods, and for the performance of RDMC.

III. RESULTS

A. Benchmarking classical chemistry methods

The main purpose of this section is to establish whether classical quantum chemistry methods can reach chemical accuracy for combustion reaction of small, closed-shell hydrocarbons, and to quantify how much effort is necessary to reach chemical accuracy. The results of our assessment of classical resources will establish a reference for the next step of our resource evaluation which is concerned with the number of qubits required for chemical accuracy.

The first step in this assessment is to check whether affordable computational methods yield accurate enough results, in which case it would be unnecessary to consider more

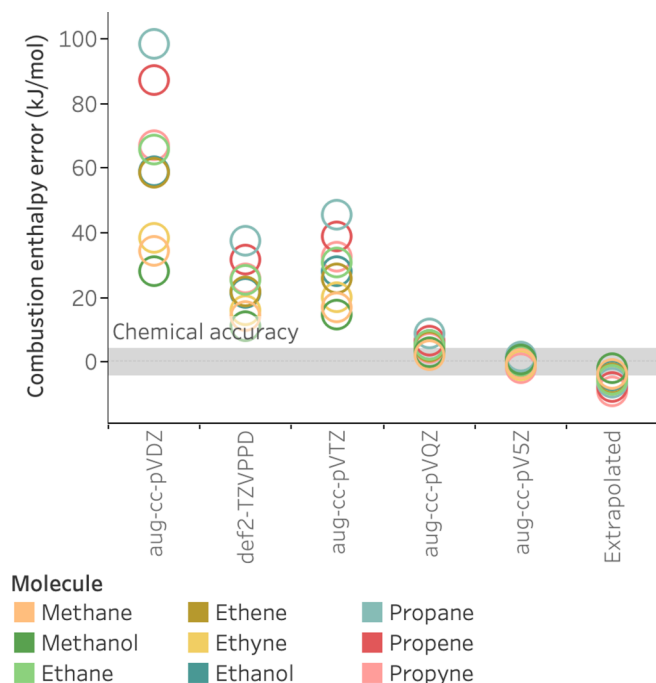


FIG. 1. CCSD(T) combustion enthalpy errors in kJ/mol in various basis sets and with AVQZ/AV5Z extrapolation. Harmonic enthalpy contributions are included.

computationally expensive methods. Initial tests showed that current density functional theory functionals and perturbation theory were not accurate enough for our purpose, while the inclusion of harmonic enthalpic contributions is necessary for comparison with experiments. For details, we refer the reader to the Supplemental Material [45]. Here, we directly investigate the “gold standard” of quantum chemistry, CCSD(T) complemented by the harmonic enthalpic contributions. In particular, we are looking to select an appropriate basis set for this method. In Fig. 1, we see that convergence of the error for CCSD(T) as a function of the basis set angular momentum is quite slow. At the AVTZ level, errors reach 100 kJ/mol, whereas chemical accuracy is reached for all reactions at the AV5Z level. We also plotted results with the def2-TZVPPD basis set, which performs slightly better than AVTZ in spite of being slightly smaller. Even at the AV5Z level, the agreement of the sum of CCSD(T) and harmonic enthalpy contributions with experimental values is somewhat fortuitous, and the energies are not completely converged yet. Indeed, a cubic extrapolation of the AVQZ and AV5Z correlation energies increases the error from experimental values, while providing results that should be closer to the complete basis set limit. The final extrapolated errors vary between -2 and -10 kJ/mol.

In conclusion, the combination of harmonic enthalpies and CCSD(T)/AV5Z electronic energies provides combustion enthalpies within chemical accuracy. These accurate results may not transfer to all systems, especially those where high-order correlation effects become important. Provided *Ansätze* on quantum computers can take into account high-order excitations at a sufficiently low polynomial cost, they could provide a better path to chemical accuracy. However, a significant

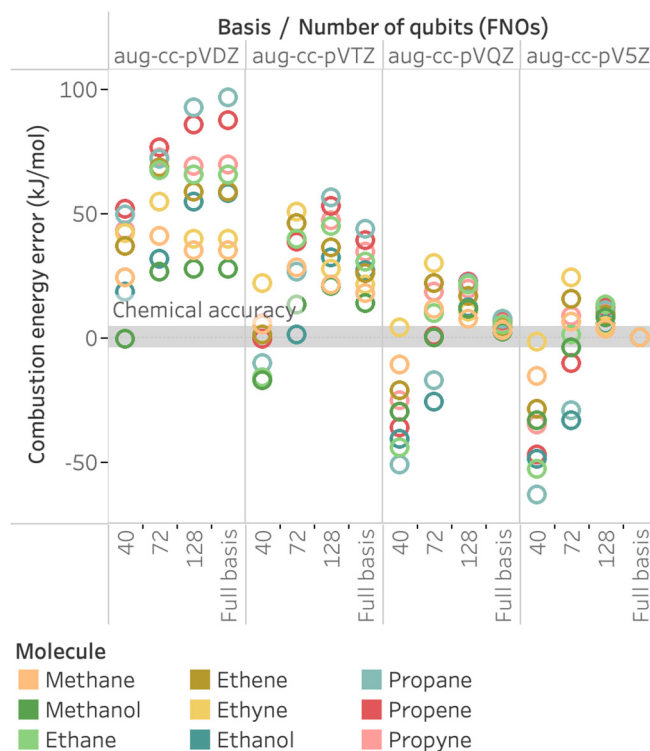


FIG. 2. Error relative to CCSD(T)/AV5Z for the FNO method with a fixed number of qubits and frozen core orbitals. Perturbation theory correction is included in the results (see Supplemental Material [45]).

number of qubits would be needed, as we demonstrate in the next section.

B. Number of qubits

In this section we explore how truncation of the orbital active space impacts the combustion energy errors. We take as reference the CCSD(T)/AV5Z electronic combustion energies unless indicated otherwise, and compute the energy difference with respect to the CCSD(T) combustion energy in various basis sets with truncated virtual spaces. In all cases a perturbation theory correction is included to compensate part of the truncation error (see Supplemental Material [45]). In our first experiment, we truncate the virtual space by keeping a fixed number of spin orbitals: 40, 72, or 128.

As was previously reported in the literature [30], canonical virtual orbitals are not an optimal basis for virtual space truncation. Indeed, we observe very large errors in that case even with 128 qubits and the AV5Z basis set, which are the largest active space and basis sets explored, respectively (see Fig. S6 [45]). The smallest errors range is between -50 and -200 kJ/mol. Moreover, the errors do not converge smoothly as the active space size is increased from 40 to 128 qubits.

A better truncation basis for correlated calculations is provided by FNOs. In Fig. 2, we plot the errors obtained for different basis sets and active space sizes. We first notice that combustion energy errors visibly converge towards the full basis set limit for each basis examined when going from 40 to 128 qubits. However, even in the largest active space chemical

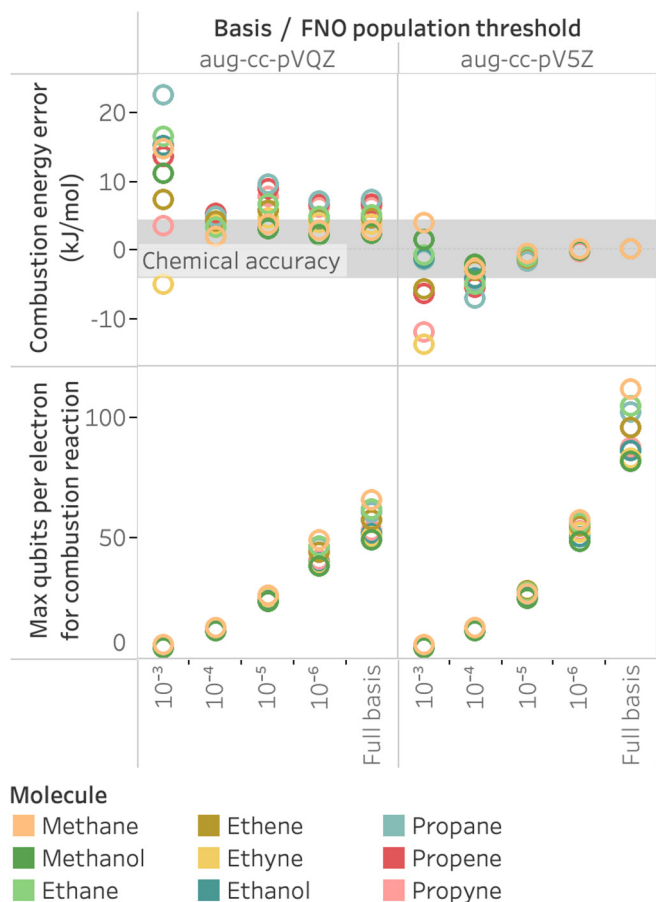


FIG. 3. Top: Error relative to CCSD(T)/AV5Z for the FNO method using the FNO threshold for truncation and frozen core orbitals. Bottom: Largest number of qubits per active electrons that would be needed to compute the combustion energy for each molecule in the given active space. Perturbation theory correction is included in the results (see Supplemental Material [45]).

accuracy cannot be reached and the final errors range from 3 to 13 kJ/mol for AV5Z in 128 qubits.

To facilitate the exploration of active space sizes, our second experiment switches to the FNO occupation threshold as a criterion to select active virtual orbitals. Focusing on AVQZ and AV5Z, we present results for thresholds of 10^{-3} , 10^{-4} , 10^{-5} , and 10^{-6} . The upper part of Fig. 3 shows that the combustion energy error is indistinguishable from the full basis value at a threshold of 10^{-6} . A threshold of 10^{-5} yields a maximum deviation of -1.8 kJ/mol from the full basis result, whereas a threshold of 10^{-4} results in a maximum error of -7.1 kJ/mol, larger than chemical accuracy.

To connect the FNO threshold to the size of the active space in a transferable way, we plot the maximum number of qubits per active electron for each combustion reaction in the lower part of Fig. 3. This number is obtained by dividing the number of active FNO spin orbitals by the number of active electrons for each target molecule, CO_2 , H_2O , and O_2 , separately. The maximum number of qubits per active electron is the largest result among the four molecules. A threshold of 10^{-4} corresponds to about 13 qubits per electron, which is the number we will use to estimate the size of the active space necessary

to reach chemical accuracy. This is an optimistic estimate: the errors we observe are slightly larger than chemical accuracy relative to the full basis limit for AV5Z, but this could be compensated for by including orbital optimization [80], or by using some of the qubit reduction techniques [65,66] mentioned above.

To conclude, our estimation for the number of qubits N_q necessary to obtain accurate dynamical correlation energies is at least

$$N_q \approx 13N_{\text{el}} \quad (8)$$

where N_{el} is the number of active electrons in the system.

C. Measurement estimation

In this section, our aim is to estimate the number of measurements needed for a single energy estimation step in the VQE procedure. We consider measurement reduction techniques based on QWC of Pauli terms [37] and orbital basis rotation [28,42], realistic variance estimation, and an efficient orbital basis so that our final estimates reflect conditions close to a large experiment. We aim to obtain empirical extrapolation formulas for the number of measurements for each molecule in our benchmark set. This will allow us to extrapolate the number of necessary measurements for the large qubit active spaces needed for chemical accuracy (see Sec. III B). We also provide empirical scaling relations for two grouping methods with the size of the system.

1. Hamiltonian decomposition methods

We evaluated the Hamiltonian estimator variance K for QWC and basis rotation grouping with two different orbital bases for the Hamiltonian (canonical orbitals or FNOs) and two different estimates for the variances (upper bounds or CISD, see Sec. II C 3), giving a total of four different variants for each grouping method. We ran computations for all molecules in our benchmark set (Fig. S1 [45]), and also included H_2O and CO_2 that are necessary for computing combustion energies. Due to technical limitations in our code at the time of computation, the open-shell O_2 was omitted. For each molecule, we computed different active spaces with an integer number of qubits per active electron up to a total of 80 qubits. This represents the most extensive investigation of the number of measurements in the VQE to our knowledge. We fit our results to a power law for each grouping method:

$$K = a(N_q)^b \quad (9)$$

where N_q is the number of qubits, and a and b are fitted parameters. The obtained scaling exponents b are reported next to the corresponding curves in Fig. 4.

The number of terms in the quantum chemistry Hamiltonian scales as N^4 , where N is the number of qubits. However, the QWC grouping method with optimal measurement allocation approximately scales between N^5 and N^6 . The optimal measurement allocation tends to attribute very little to no measurements to terms with very small Hamiltonian coefficients that can safely be neglected. Thus, the observed scaling for QWC grouping only constitutes a modest improvement over the estimated upper bound of N^6 for scaling without grouping [25].

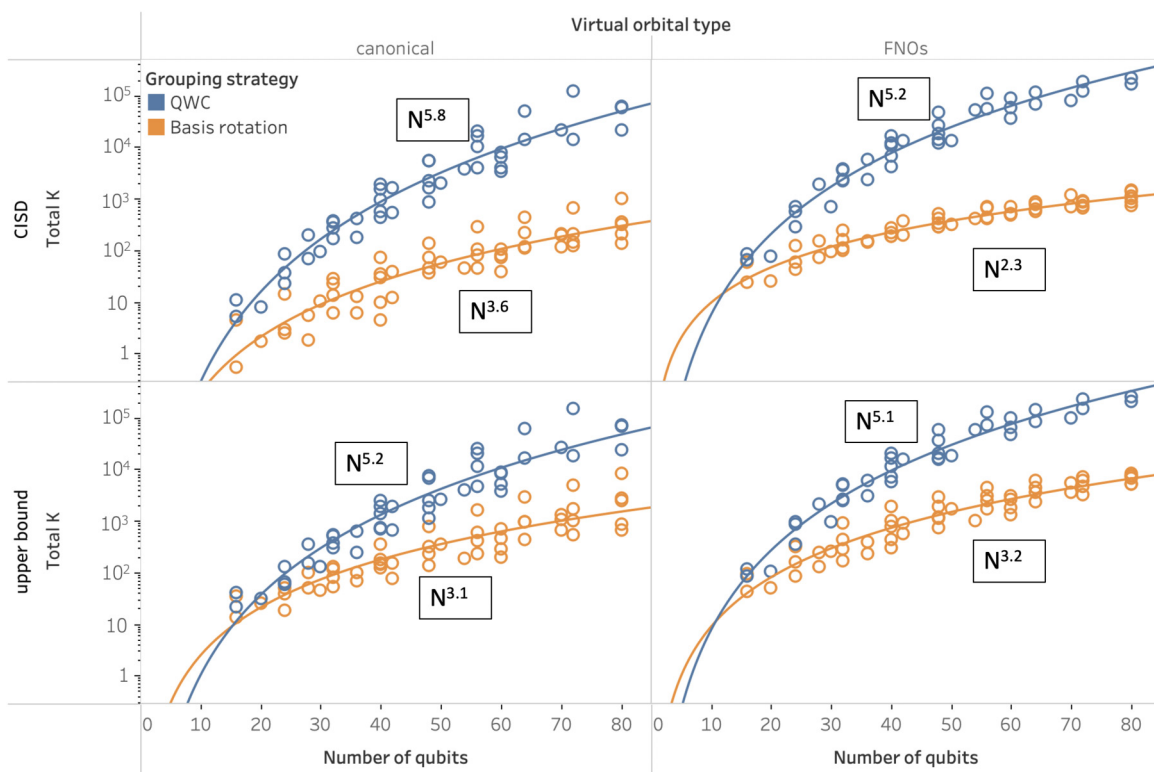


FIG. 4. Values of K computed for molecules in our benchmark set using QWC grouping (blue) and basis rotation grouping (orange). The top row approximates variances with CISC density matrices and the bottom row sets variances to their upper bounds. Covariances are set to zero in both cases. The left column represents the Hamiltonians in the canonical orbital basis and the right column in the FNO basis. A power law is fit through the data for each grouping method and the obtained exponent is reported next to the curve.

Basis rotation grouping offers better scaling, as hinted by the data presented by Huggins *et al.* for up to 32 qubits [28]. We observe that the scaling varies between $N^{2.3}$ and $N^{3.6}$, a very significant improvement compared to QWC grouping results. We note that the empirically obtained scaling exponents are very close to the optimal bound of N^2 derived for the measurement of fermionic two-particle density matrices [81]. In addition, the effect of this improved scaling is already beneficial at low number of qubits, so that QWC grouping never appears advantageous in our computed data. The power law fits indicate that there is a crossover point at which QWC grouping could be preferred, but it only happens below 15 or 20 qubits. Such an example appears in the next section, in Fig. 5 for 12 qubits. Basis rotation grouping practically always yields a lower number of measurements, however it necessitates the addition of a basis rotation circuit before measurements are performed. Although this circuit has a very shallow depth [82], in some situations the additional noise induced might become excessive.

To facilitate the comparison of K computed with upper bound and CISC variances, we plotted again the data in Fig. 4 so as to highlight the difference between the two variance estimation methods in Fig. S7 [45]. As expected, this clearly shows that CISC variances always yield a lower number of measurements, albeit by only 20 to 30% when combined with QWC grouping. With basis rotation grouping, the benefit is significantly larger and reaches a factor 5 to 10. This shows that variance approximation is an important aspect to consider

when estimating measurements: errors of an order of magnitude can occur when using upper bounds.

The effect of changing the orbital basis of the Hamiltonian from canonical orbitals to FNOs is visualized in Fig. S8 [45], which contains the same data as Fig. 4 but highlighting the difference of interest in color. When computing the number of qubits needed (see Sec. III B), we showed that FNOs yield significantly more correlation energy than canonical orbitals for the same number of qubits, which allows chemically accurate results in smaller active spaces. However, this increased accuracy comes at a price since the value of K is systematically higher for FNOs, by a factor of up to 10 in some cases. This is slightly compensated by a lower scaling exponent (compare left and right column of Fig. 4), that reflects the fact that K saturates faster for FNOs. Indeed, when all virtual orbitals are included, the canonical and FNO spaces are the same and they must have the same K .

To obtain extrapolations of the value of K for each molecule, we must choose one of the eight variants investigated. For variance estimation, the CISC approximation is closest to what would be experimentally observed. In spite of the increased number of measurements, we believe the FNO basis is more advantageous since it yields more compact active spaces. Finally, we consider that the circuit fidelity is high enough to afford the orbital rotation circuit from basis rotation grouping. Hence, we fit Eq. (9) for each molecule using K computed with basis rotation grouping, FNO Hamiltonians, and CISC variances. We report our results in Table I,

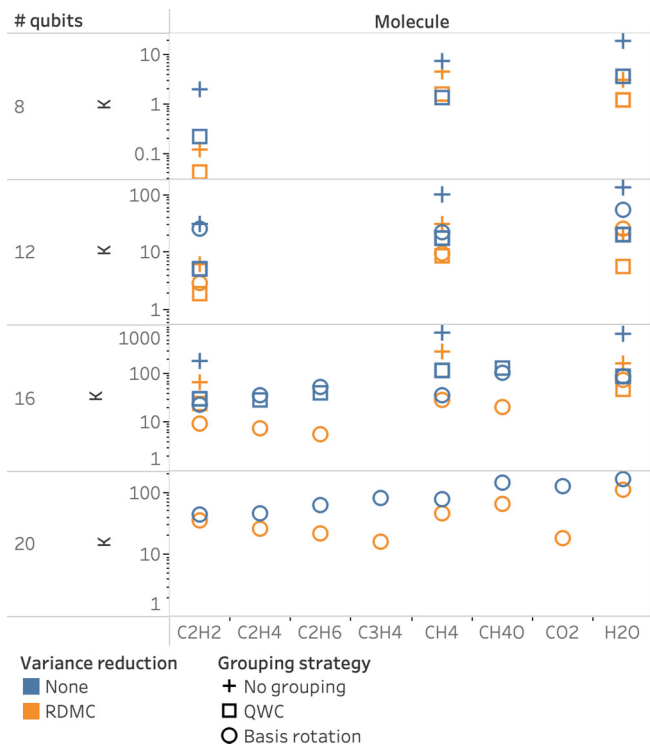


FIG. 5. Values of K computed for various molecules with no grouping (crosses), QWC grouping (squares), and basis rotation grouping (circles), both with (orange) and without (blue) RDMC. Eight qubit data freeze six electrons for CH_4 and H_2O ; others only freeze core electrons. Hamiltonians were represented with FNOs in the AVTZ basis set and variances estimated from CISD.

and we also plot the fits in Fig. S9 [45]. The fit to all molecular data presented in Fig. 4 yielded an exponent of 2.3, while for individual molecules b varies between 1.8 and 2.7. Most prefactors a have the same order of magnitude, except for H_2O where the prefactor is five to ten times larger than for other molecules. However, H_2O also has the lowest exponent.

The fits presented above represent the scaling of K when increasing the number of qubits for a fixed molecule, which

TABLE I. Fitting coefficients for K for each molecule with $K = a(N_q)^b$ [Eq. (9)] when using basis rotation grouping, CISD variances and FNOs. a is multiplied by 100 in the table for clarity.

Molecule	b	$a \times 10^2$
H_2O	1.8	45
CO_2	2.4	4.4
Methane	2.2	5.8
Methanol	2.2	9.0
Ethane	2.5	1.9
Ethene	2.6	1.3
Ethyne	2.5	1.6
Ethanol	2.4	3.8
Propane	2.4	2.5
Propene	2.6	1.6
Propyne	2.7	1.0

is convenient to extrapolate K to the very large active spaces needed for chemical accuracy. This “virtual scaling” is not the same as the “size scaling,” where both the number of active electrons and the number of qubits increase. To investigate size scaling, we fitted Eq. (9) through our data for increasing numbers of active electrons while keeping the number of qubits per active electron fixed. We only have enough data to obtain meaningful fits for up to five qubits per electrons. Beyond that, the extrapolation gives incoherent results where larger active spaces would need fewer measurements, whereas for two, three, four, and five qubits per electrons the obtained scaling is consistent. Overall, size scalings are slightly more favorable than virtual scalings (see Fig. S10 [45]). The QWC grouping method scales around N^4 to $N^{5.5}$ in most cases, whereas the basis rotation method scales between N^2 and $N^{2.5}$. Thus, these data suggest that the basis rotation method provides a considerable asymptotic improvement in the number of measurements compared to QWC and related approaches.

2. Variance reduction

We now turn to a method that transforms the Hamiltonian to reduce the number of required measurements: the application of fermionic marginal constraints introduced by Rubin *et al.* [27], that we abbreviate as RDMC. RDMC as formulated in the original publication scales in principle as N^4 where N is the number of orbitals. The implementation we are using is based on OPENFERMION [73] and formulates RDMC as a linear program, which takes significant classical resources. Therefore, we restrict our paper to a few molecules and active spaces. We note that we expect an optimized implementation of RDMC to be applicable to much larger systems. Our goal is to obtain an empirical estimation of the improvement in K that RDMC yields. Our results are presented in Fig. 5, where we compare the performance of various grouping methods combined with or without RDMC. We see that in all cases, RDMC yields reductions in the values of K . The reduction factor obtained is about 3 to 5 when no grouping method is used. In the case of QWC grouping, the reduction provided by RDMC decreases a bit to a factor of 2 to 3. Basis rotation grouping usually yields the lowest K and has the best scaling with molecular size or number of qubits. Even in this case, RDMC is able to yield an additional improvement to K , of approximately a factor of 2. We note that the observed performance of RDMC seems to vary significantly among tested cases, and a factor of 2 is a somewhat conservative estimate. In general, the smaller reduction factors are obtained for larger number of qubits.

For low number of qubits, there are some irregularities in the patterns usually observed. For example, in active spaces of 12 qubits, the QWC grouping method generally performs better than the basis rotation grouping. This also happens for H_2O in 16 qubits with RDMC. At this low number of qubits, very few virtual orbitals are included for each active electron, less than one for 12 qubits. This makes it difficult to extrapolate the behavior of the methods examined to large qubit numbers, and highlights the importance of running systematic benchmarks on large enough systems.

As highlighted in Sec. II C 3, our RDMC implementation performs the Hamiltonian transformation in the qubit picture, as was suggested in the original work [27]. In Fig. S11 [45], we compare our results to the original implementation in the fermionic picture, and confirm that the qubit picture implementation systematically yields equivalent or better results. In general, RDMC shows a reduction in measurement count in all cases tested and therefore it could provide practical improvements for the implementation of VQE in the near term. However, a more extensive analysis of the classical computational cost of this technique and the magnitude of its improvement when scaled to larger systems would improve the current assessment.

D. Overall qubit and runtime requirements

In this section, we summarize and gather the previous results to obtain estimates for the number of qubits, number of measurements, and runtimes required to reach chemically accurate results for the set of investigated combustion reactions. The number of qubits N_q is estimated simply from Eq. (8) and the number of valence electrons in each molecule. The number of measurements is computed as

$$M = \frac{K}{2\epsilon^2} \quad (10)$$

where K is extrapolated for each molecule separately from Eq. (9) with a and b taken from Table I. The extrapolation takes into account basis rotation grouping and approximated variances from CISD and assumes the Hamiltonian is expressed in the FNO basis. The extra factor of 1/2 in Eq. (10) approximately accounts for the additional measurement reduction provided by RDMC on top of the basis rotation grouping. We fix $\epsilon = 0.5$ mHa instead of the usual chemical accuracy of 1.6 mHa. Indeed, we allow 1.1 mHa for additional errors arising from truncation of the active virtual space and from device noise effects. Note that reducing the effect of device noise to below chemical accuracy in general is still a subject of research, and the low error we are assuming can only be achieved on the smallest circuits with the best devices currently.

To convert the number of measurements to actual runtimes, several additional assumptions are necessary. The first and perhaps most speculative regards the *Ansatz*. Although the UCCSD *Ansatz* generally yields good results in spite of deficiencies for strong correlation [83], the corresponding quantum circuit is extremely deep and not appropriate for NISQ devices. Alternatives have been designed [84,85], however we will assume here that we can use a shallower, hardware-efficient *Ansatz*. Such an *Ansatz* makes use of parametrized entangling gates that are taken to be hardware native or easily compiled to hardware native gates without significant overhead. We are assuming a linear connectivity of the qubit array, in which case a single layer of a hardware-efficient *Ansatz* is defined as the circuit of depth 2 that entangles every neighboring pair of qubits. We further assume that the number of layers needed to reach the ground state energy scales linearly with the number of qubits, and for the purposes of our estimation, we choose the prefactor in the scaling to be 2. It is likely that this depth constitutes a lower bound for the

TABLE II. Estimated runtimes t in days for a single energy evaluation using the number of measurements M from extrapolated values of K [Eq. (9) and Table I], with $\epsilon = 0.5$ mHa and the effect of RDM constraints included by a factor of 1/2 [see Eq. (10)]. The number of qubits N_q is computed from the number of active electrons N_{el} and our empirical estimations of active space size [Eq. (8)].

Molecule	N_{el}	N_q	$K \times 10^{-3}$	$M \times 10^{-9}$	t (days)
H ₂ O	8	104	1.9	3.9	2.3
CO ₂	16	208	16	32	39
Methane	8	104	1.6	3.2	1.9
Methanol	14	182	8.4	17	18
Ethane	14	182	8.5	17	18
Ethene	12	156	6.6	13	12
Ethyne	10	130	3.1	6.2	4.6
Ethanol	20	260	24	48	71
Propane	20	260	16	31	47
Propene	18	234	23	46	62
Propyne	16	208	18	36	44

Ansatz depth that would be necessary in practice. Since our extrapolation for K assumes the basis rotation grouping, we also need to add the depth of the circuit for basis rotations, which is $N_q - 3$ on a linear array of qubits if α and β spins can be transformed independently [82]. The final depth of the circuit would then be $5N_q - 3$ in terms of two-qubit gates. Our final assumption is that runtime is dominated by execution times of two-qubit gates, which is assumed to be 100 ns, a value on the faster side of current superconducting gate times (see Table I in the review by Kjaergaard *et al.* [86]). The final formula we use to obtain runtimes t in seconds from the values of M and N_q reads

$$t = 10^{-7}M(5N_q - 3). \quad (11)$$

We report the results of our runtime estimates in Table II. We also plot our estimated runtimes from the computed K values of Fig. 4 and their extrapolation in Fig. S13 [45]. The picture painted by these runtimes is very pessimistic for VQE. The shortest runtime for energy estimation, for CH₄, is 1.9 days. This is in spite of using rather optimistic estimates for the *Ansatz* depth and the number of qubits needed and neglecting the time for qubit reset, cloud latency times, or measurement overheads for error mitigation. Moreover, we highlight again that this is the time necessary for a single energy evaluation. Running the full VQE algorithm involves optimizing the circuit parameters, which requires at least a few dozen to hundreds of iterations even with excellent optimizers. Hence, the total VQE runtime would be about a month for the smallest molecules in our test set. Larger molecules like ethanol already have a runtime of 71 days for a single energy evaluation.

These runtimes originate essentially in the considerable number of measurements necessary to obtain chemically accurate energies for molecules. Even on devices where the error rate would be small enough to warrant reliable VQE execution, the runtime to solution would be prohibitive for molecules in our benchmark set. Parallelization of measurements over several quantum devices is a potential solution,

provided all of these quantum devices are sufficiently similar, and the distribution of measurements designed to achieve chemical accuracy. However, parallelization could only bring a constant factor improvement and will not change the scaling of the runtimes with molecular size. In the case of systems dominated by nondynamical correlation, a smaller active space might be sufficient to demonstrate quantum advantage over classical computing power. A recent paper [24] proposes the chromium dimer with a (24, 24) active space as a potential candidate. At 48 qubits, our extrapolation indicates a runtime of a few hours (see Fig. S13 [45]), which could allow for a full VQE optimization with considerable effort. However, Hamiltonian coefficients for heavier, strongly correlated atoms like Cr might be larger, which would result in larger values of K . Moreover, even if such a computation becomes possible, the transition to practically relevant advantage could require active spaces beyond 100 qubits [24].

Focusing on the scaling b and omitting the prefactor a , our results for the basis rotation grouping technique suggest that VQE has the potential to scale better with system size than methods such as coupled cluster. To transform this difference in scaling into an actual practical advantage, research should focus on two directions: (1) developing linear scaling *Ansätze* that provide sufficient accuracy on NISQ devices and (2) improving the measurement techniques, in particular to reduce the dependency of the number of measurements on the required precision. Regarding the first direction, having sufficiently accurate *Ansätze* for the VQE with a circuit depth scaling only linearly implies an empirical runtime scaling of N^3 to N^4 , which would be advantageous over the scaling of approaches such as CCSD(T). A number of *Ansätze* with linear scaling have been proposed [82,87], but more studies should be devoted to investigating their representational power for chemical systems of interest, their trainability, and the impact of noise on their accuracy. Along this line, the development and benchmarking of error mitigation techniques is crucial towards achieving sufficient accuracy on NISQ devices. Regarding the second direction, methods that can reduce the dependency of the number of measurements with respect to the required accuracy should be prioritized to make the VQE competitive. One such method has been recently proposed by Wang *et al.* [43] and Koh *et al.* [44] which trade circuit fidelity for a reduction in the number of measurements.

IV. DISCUSSION

The VQE is a heuristic algorithm, which does not have yet a demonstrated quantum speedup over classical algorithms for quantum chemistry. Hence, it is of utmost importance to adequately benchmark the VQE to evaluate its performance and prospects for quantum advantage. One significant step has recently been made in this direction [24] by identifying what molecules are the most likely candidates for quantum advantage, and in particular for practically relevant quantum advantage.

Here, we outlined a general procedure to assess quantum advantage with a quantum heuristic by carrying out a RPA. We performed a specific RPA for computing a set of combustion energies with the VQE, but our general method

is also applicable to other variational algorithms. First, it is essential to assess the performance of state-of-the-art classical algorithms to check whether they can solve the problem at hand and estimate the compute resources required. Then, the number of qubits necessary to obtain a solution that is accurate enough should be established. Finally, a rigorous estimation of the number of measurements needed to evaluate expectation values with sufficient accuracy is performed. Measurement requirements are crucial to obtain approximate runtimes, which are ultimately decisive for the practicality of the quantum algorithm.

Our classical benchmarks show that CCSD(T)/AV5Z complemented with harmonic enthalpic corrections is sufficient to reproduce experimental combustion enthalpies to within chemical accuracy. CCSD(T)/AV5Z is taken as our reference energy to estimate the minimal size of the active space that still yields chemical accuracy. Using the well-known FNO method for virtual space truncation, we observe that at least 13 qubits per active electrons must be included to obtain dynamical correlation energies within chemical accuracy. In practice, early quantum advantage will probably be obtained by treating only a small active space on the quantum computer and computing the remaining dynamical correlation energy classically. We believe our results regarding the number of measurements needed are transferable to that case as well.

Our RPA results further show that the number of measurements necessary for QWC grouping scales as N^5 to N^6 , whereas the basis rotation grouping only needs about N^2 measurements, at the cost of a small addition to the overall circuit depth. The application of reduced density matrix constraints on the Hamiltonian in addition to grouping warrants another reduction in the number of measurements by a factor of 2. Unfortunately, the ϵ^{-2} precision dependence of measurement requirements introduces a very large multiplicative factor. With optimistic assumptions regarding the total circuit depth and the execution time of quantum circuits, estimating a single energy for molecules in our benchmark set to chemical accuracy would take between a few days and a couple of months. Combined with the necessity for a large number of energy evaluations to optimize VQE parameters, this indicates that a VQE with sample averaging is not currently practical even for molecules with only a few heavy atoms.

There are several possible ways to resolve this issue. One is to work on better Hamiltonian decomposition methods, and hopefully achieve reduction in the prefactor or the scaling of the number of measurements needed as a function of the system size. Another would be to work on improving Hamiltonian transformations to reduce the Hamiltonian variance further. Some of these directions have been explored by the authors without significant success. However, a more concrete improvement tackles the ϵ^{-2} dependence of the number of measurements. Recently, the use of Bayesian techniques combined with engineered likelihood functions [43,44] offered a way to exploit better device fidelity to reduce the number of measurements, bridging the VQE and quantum phase estimation in a practical way. Engineered likelihood functions may then be combined with grouping and variance reduction techniques to further reduce measurement requirements and runtime. Any proposed solution to the measurement

bottleneck for the application of the VQE should be benchmarked on various molecules and active spaces to assess its robustness and scaling with system size.

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