Measurements of direct CP violation

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1 Introduction

The experimental fact that not all natural phenomena are insensitive to the mirror-reversal of their spatial arrangement and the simultaneous replacement of all the particles with the corresponding antiparticles, *i.e.* the violation of CP symmetry (or CP violation in short), is a feature of the fundamental physical laws which is not yet satisfactorily explained.

Since its discovery [1] 39 years ago, CP violation has been a very active field of research at the heart of particle physics. The reason is that CP violation is deeply related to such fundamental and diverse issues [2] as the microscopic time-reversibility of physical laws and the origin of the baryonic asymmetry of the universe; it is the only known phenomenon which allows an absolute distinction between particles and anti-particles [3]. The very successful Standard Model of particle physics (often indicated as SM in what follows) can accommodate CP violation within its framework, but it does not shade any real light on its origin.

Since CP violation was discovered in the decays of flavoured neutral mesons, which remain to date the only systems in which it has been observed, the term "*direct* CP violation" was coined to indicate effects appearing in the amplitudes describing the physical decays of such particles, as opposed to effects in the peculiar meson-antimeson oscillations which such systems exhibit, dubbed as manifestations of "*indirect* CP violation". For quite some time, all measured CP violation effects could be related to a single phenomenological parameter describing an asymmetry in the mixing of the neutral kaon and its antiparticle (indirect CP violation), despite many experiments devoted to searches for effects in other systems, and in particular differences in the decay properties of particles and anti-particles.

The 1990's marked important progress for the understanding of CP violation, with the first experimental evidence of such phenomenon outside the kaon system coming just a few years after the proof of the existence of direct CP violation in neutral kaon decays was obtained.

The unequivocal evidence of the occurrence of *direct* CP violation in Nature, recently obtained after a search several decades long, clears the way to the interpretation of the lack of symmetry under CP as a truly universal property of weak interactions. As such it is expected to be present, albeit to different degree, in several aspects of Nature governed by these interactions.

To date, the original observation of direct CP violation in neutral kaon decays remains the only detected manifestation of this kind of effect. A review of the current situation, and of the long road which led to it, seems therefore appropriate at this time, when important milestones have been achieved, and several new experimental studies are underway or in preparation.

The literature on CP violation is extremely vast, and even a compilation of the most significant papers would be a daunting task; introductory discussions can be found in any particle physics textbook, and many review articles exist. The literature can be traced starting from recent [4], [5] or by now classic [6] books on the subject. First-hand accounts of the early years of CP violation can be read in [2], [7], [8]. Previous review articles specifically devoted to the search for direct CP violation include [9] and particularly [10], while a comprehensive review of CP violation effects in kaon decays can be found in [11].

The emphasis of this article is on the description of the experimental progress in the field, from its origins to the present experiments. The approach is a phenomenological one, and any attempt to make justice to the huge theoretical literature would be outside the scope of this work.

The plan of the paper is as follows: in section 2 we review the basic concept of direct CP violation, focusing on light and heavy meson decays. Section 3 - the longest one - is devoted to the search and experimental evidence for direct CP violation in the neutral kaon system. Section 4 describes searches in charged kaon decays, while section 5 accounts for the relevant experimental activities in heavier flavoured meson systems (D and B). In section 6 some searches for direct CP violation effects in other processes are described, while section 7 briefly touches on the related issues of CPT and T violation. Some conclusions and perspectives are presented in section 8.

2 Direct CP violation

CP violation is a lack of symmetry of a physical process when all the spatial coordinates are inverted (parity operation, P) and particles are replaced by their anti-particles (charge conjugation, C). As such it has connections with the odd number of (ordinary) spatial dimensions, and with the roots of the current quantum field theoretical paradigm for explaining Nature at the microscopic level. Both the P and C symmetries have been found experimentally to be valid with good accuracy in all processes driven by the strong and electromagnetic interactions, and badly broken by the weak interactions, which however approximately respect their combination CP.

A general property of any Lorentz invariant quantum field theory based on a hermitian, local Lagrangian, is that the combined operation CPT, in which T represents the time reversal operation, is a valid symmetry [12]. No violation of this law has ever been detected. Throughout this paper we will assume the validity of CPT symmetry, except where explicitly noted; experimental tests of its validity will be briefly discussed in section 7.

The first manifestation of CP violation in Nature, to be detected experimentally, was the decay of the long-lived neutral K meson (K_L) in both two- and three- pion states [1], in 1964. The number of proposed “exotic” explanations for this so-called “Princeton effect” (see *e.g.* [13] for an excellent review written few years after the discovery) clearly indicates the physicists’ uneasiness[1] in definitely dropping the concept of Nature’s left-right symmetry, shaken at the roots but not yet entirely invalidated after the discovery of parity violation [14] in 1956, since it could be apparently restored by complementing space inversion with charge conjugation.

No evidence for CP violation has ever been found in processes induced by the strong or electromagnetic interactions, and so it is usually assumed that such an effect arises uniquely in weak interactions.

[1] “What shook all concerned now was that with CP gone there was nothing elegant to replace it with.”, A. Pais in [7].

Within the approach of quantum field theory, the appearance of CP violation is linked to the presence of ineliminable complex terms in the Lagrangian density. Given the real nature of observables, CP violation can be experimentally detected only when measurable quantities are affected by the quantum-mechanical interference of two or more terms which have relative phases different from 0 and π; this somewhat indirect link between the effect and its mathematical description is one of the main reasons for the complexity of the formalism, which sometimes tends to obscure the relevant physical facts.

The simplest manifestation of CP violation is the spontaneous transformation of a system which is in a well-defined CP eigenstate into another state which is either a CP eigenstate with a different eigenvalue or not a CP eigenstate at all. Although the term has received slightly different meaning during the years, this is indeed what is called *direct* CP violation. Since its discovery, CP violation has been experimentally accessible only in the weak decays of neutral mesons with flavour, in which case it can manifest itself also in a different and subtler, but sometimes dominant way, *i.e.* through virtual meson-antimeson transitions [15], the so-called *indirect* CP violation. For flavoured neutral mesons M, distinguished from their anti-particles $\overline{M}$ by the flavour eigenvalue $F = \pm 1$, indirect CP violation appears in $\Delta F = 2$ virtual processes, as an asymmetry between the $M \to \overline{M}$ and $\overline{M} \to M$ transition rates, while direct CP violation can be defined [16] as that occurring in $\Delta F = 1$ physical decays[2].

It should be stressed that indirect CP violation is a feature of neutral mesons only, since conserved quantum numbers make particle-antiparticle mixing impossible for charged particles, baryons and leptons[3], while most of the possible CP violation effects which can be considered in microscopic phenomena are of the direct type[4]. It happens however that the most straightforward manifestation of CP violation (namely direct CP violation) is somewhat hidden and masked by a larger effect (indirect CP violation) in the system in which the phenomenon was actually discovered.

The importance of distinguishing among CP violation in the $\Delta F = 2$ and $\Delta F = 1$ transitions lies in the fact that the former only appears in the mixing of flavoured mesons and anti-mesons, which is observable as an intrinsic property of the physical meson states, induced by their mutual virtual interactions (which in the Standard Model are among the few measurable manifestation of electroweak interactions at the second order), while the latter is a general property of the flavour-changing weak interactions at first order.

The Standard Model provides us with some justification for the smallness of observed CP violation effects, and it also leads us to expect direct CP violation to be an ubiquitous feature in all phenomena controlled by weak interactions[5]; the measurable effects are expected to be larger in heavy meson systems, which

[2] CP violation in $\Delta F = 0$ transitions, such as $K^* \to K\pi$ decays, would also be referred to as direct CP violation.

[3] As long as the relevant conservation laws are valid, which actually appears not to be exactly the case for baryons (and possibly for neutrinos as well), even within the Standard Model.

[4] Intrinsic electric dipole moments of elementary particles would be flavour-conserving CP-violating static properties.

[5] It is interesting to note that the existence of direct CP violation can also test the validity of the fundamental concepts of quantum mechanics against some "hidden variable" alternatives [17].

are unfortunately more difficult to study from an experimental point of view.

The description of CP violation in the Standard Model is that it originates in both forms from a single irreducible complex term in the mixing matrix relating "physical" quarks (mass eigenstates) to the states which participate in the charged-current weak interactions (flavour eigenstates). It is well known that such a complex term can have physical significance only when at least three (non-degenerate) quark generations are present [18]; this implies in turn that the magnitude of CP violation effects in hadrons is necessarily linked to the amount of mixing among the quark generations themselves [19]. Such mixing happens to be small, therefore suppressing most CP violation effects within the Standard Model. The above picture also implies that, for such effects to occur at all in a given process, their amplitudes must receive contributions from all three quark generations.

The subtle and elusive nature of CP violation is such that its dominant manifestation for light quark systems requires a quantum-mechanical interplay of particle-antiparticle mixing. This is the reason why CP violation was actually discovered in neutral kaons, being the lightest system with enough complexity to allow for such phenomena. Remarkably, the neutral kaon system actually exhibits a very rich phenomenology of CP violation, with all possible kinds of CP violation contributing, depending on the circumstances under investigation. There is no *a priori* reason for direct CP violation to be smaller than its indirect counterpart, but this turns out to be the case for neutral kaons.

In heavier flavoured meson systems (D, B mesons) some kinds of CP violation are believed to be irrelevant in the framework of the Standard Model, while the searches for asymmetries in exclusive decay channels, with tiny branching ratios, and the difficulties in relating the measurements to the fundamental parameters of the underlying theory, make the study of CP violation in these systems a very challenging, although promising, enterprise.

The smallness of CP violation effects contributes in making theoretical predictions difficult and in many cases quite uncertain, the main reason being the present limited capability to deal with hadronic states in the non-perturbative QCD regime. To overcome as much as possible these problems, a massive development of theoretical and numerical techniques took place; the interested reader is referred to recent reviews such as [20] or [21].

2.1 Meson decays

The general features and experimental signatures of CP violation are discussed in any review paper on the subject; we just recall the main points in the following.

Due to the almost complete matter-antimatter asymmetry of the known universe, it is clear that detection of CP-violating effects can only be achieved by studying microscopic phenomena, such as the static properties of "elementary" particles and their simplest transformations, namely spontaneous decays (the study of asymmetries in multi-particle interactions is also possible in some cases, but is experimentally more challenging).

The most straightforward indication of CP violation would be a rate asymmetry between two CP-conjugate decays; it is well known that CPT symmetry by itself forces the equality of the fundamental basic properties of particles and anti-particles (charge conjugation partners, or CP-partners since parity does not play a role for space-integrated observables), such as the total decay rate. This CPT-enforced equality also extends to partial decay rates for classes of final states which are not distinguished by the CP-conserving interactions (assumed to be the strong and electromagnetic ones). This means that, assuming CPT symmetry as we do in most of the following, any CP-violating partial rate asymmetry must show up in at least two related decay channels, in such a way that they compensate each other when summed together: if $f_1, f_2, \ldots, f_n$ make up a complete set of allowed final states for the decay of a particle A, which are not distinguished by the strong and electromagnetic interactions, one must have for the partial rates Γ:

$$\Gamma(A \to f_1) - \Gamma(\overline{A} \to \overline{f}_1) = -\sum_{i=2}^{n} \left[\Gamma(A \to f_i) - \Gamma(\overline{A} \to \overline{f}_i)\right] \qquad (1)$$

In several cases, conservation laws other than the one related to CP symmetry (*i.e.* baryon number, lepton number) hinder the possibility of having CP-violating decays, and therefore flavoured (not self-CP-conjugate) mesons are the natural candidates for searches of CP violation.

Neutral mesons M can be coincident with their anti-particles (self-conjugate), as is the case for the π^0 or, when they are characterised by some non-zero additive quantum number (such as flavour), they can have distinct anti-particles $\overline{M}$, and in this case they exhibit a richer phenomenology. If one defines

$$CP|M\rangle = |\overline{M}\rangle \qquad\qquad CP|\overline{M}\rangle = |M\rangle \qquad (2)$$

the CP eigenstates are

$$|M_1\rangle \equiv \tfrac{1}{\sqrt{2}}\left(|M\rangle + |\overline{M}\rangle\right) \quad (CP = +1) \qquad (3)$$

$$|M_2\rangle \equiv \tfrac{1}{\sqrt{2}}\left(|M\rangle - |\overline{M}\rangle\right) \quad (CP = -1) \qquad (4)$$

If CP symmetry is valid, such states coincide with the physical eigenstates. The mutual coupling of M and $\overline{M}$ through weak interactions gives origin to a mass shift and a mass splitting Δm, so that in the time evolution of free mesons the phenomenon of flavour oscillations takes place, with a characteristic period $\hbar/\Delta m c^2$.

Considering a single decay channel f for a non-self-conjugate (*e.g.* charged) meson M (and its antiparticle $\overline{M}$ decay to $\overline{f}$), when the decay can proceed through two different elementary amplitudes A_1, A_2 one can write, for the total decay amplitude:

$$A(M \to f) = |A_1| e^{i\phi_1} e^{i\delta_1} + |A_2| e^{i\phi_2} e^{i\delta_2} \qquad (5)$$

$$A(\overline{M} \to \overline{f}) = |A_1| e^{-i\phi_1} e^{i\delta_1} + |A_2| e^{-i\phi_2} e^{i\delta_2} \qquad (6)$$

in which a part of the amplitude phase (ϕ_i) changes sign for the CP-conjugate decay (the actual CP-violating phase, often called "weak" phase) while another part (δ_i) does not (often called the "final state interaction", "strong" or "rescattering" phase). One can easily show that the asymmetry of the partial decay rates is given by

$$A_{CP}^{(f)} \equiv \tfrac{\Gamma(\overline{M}\to\overline{f})-\Gamma(M\to f)}{\Gamma(\overline{M}\to f)+\Gamma(M\to\overline{f})} = \tfrac{2|A_1||A_2|\sin(\delta_1-\delta_2)\sin(\phi_1-\phi_2)}{|A_1|^2+|A_2|^2+2|A_1||A_2|\cos(\delta_1-\delta_2)\cos(\phi_1-\phi_2)} \qquad (7)$$

It should be clear that any single phase in an amplitude is irrelevant, since it can be always redefined away, and only phase *differences* are significant, so that the presence of at least two interfering amplitudes is required to have an effect. Moreover, expression (7) for the asymmetry clearly displays that in order to have an observable effect in the rates (squared modulus amplitudes in quantum mechanics) induced by the difference of the CP-violating phases ϕ_i, these two amplitudes must also have different final-state interaction phases. The latter actually provide the “reference” phases against which the sign change of the weak phases can be detected, and are usually induced by the (CP-symmetric) strong or electromagnetic interactions of the final-state particles, as indicated above. These strong phases are often related to the experimentally measurable phases, which describe the scattering of the final state particles, via the Fermi-Watson theorem [22]; they are difficult to compute from first principles, and this leads to the difficulty of predicting the partial rate asymmetries or extracting the weak phases from the measured values of such asymmetries.

From the above expression it is also clear that, in order to have a large asymmetry, the two interfering amplitudes must be of comparable magnitude: the best decay modes in which to look for direct CP violation are therefore the ones in which the dominant, lowest-order (*tree* level) graphs are suppressed because of the small inter-generation mixing (“Cabibbo suppressed”), or by some other selection rule, so that one is left with higher-order graphs (*penguin*) of comparable magnitude.

As an example, CP violation in $K^0 \to \pi\pi$ ($\Delta S = 1$, where S is strangeness) decays can arise in the Standard Model because of different weak phases in the tree amplitude and the (electro-weak or gluonic) penguin one, shown diagrammatically in fig. 1. Since the first one contributes to the isospin $I = 0, 2$ $\pi\pi$ final states, while the second one only to the $I = 0$ state, different strong phases are expected.

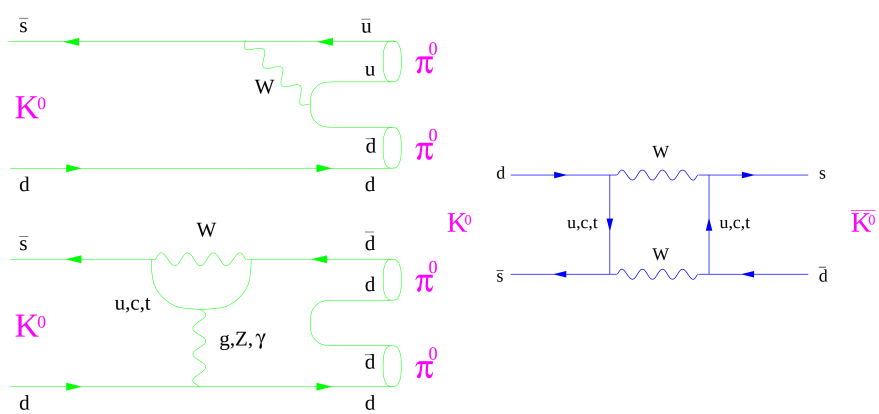

Figure 1: Left: the “tree” and “penguin” diagrams originating $K^0 \to \pi^+\pi^-$ decays in the Standard Model. Right: The “box” diagram originating $K^0 -- \overline{K}^0$ transitions in the Standard Model.

The above requirements for the observability of a CP-violating effect are not specific to partial decay rate asymmetries, but apply to several other particle-antiparticle asymmetries. The two interfering amplitudes can be due to competing elementary processes in the same decay, possibly inducing so-called CP violation "in the decay". Decays of self-conjugate neutral mesons into CP eigenstates with different CP eigenvalues, such as $\eta \to \pi\pi$, would also be indications of this kind of CP violation.

In neutral flavoured meson systems one can have also CP violation purely "in the mixing", actually the dominant effect in the neutral kaon system. This is an asymmetry in the virtual transitions $M \to \overline{M}$ with respect to $\overline{M} \to M$, originating physical states which do not contain the same amount of M and $\overline{M}$ components, *i.e.* states which are not CP eigenstates. This kind of CP asymmetry is therefore an intrinsic property of the meson system itself, driven by its (virtual) interactions with other states. If CP symmetry is approximately valid, the physical states will be largely composed of a single CP eigenstate, with a small "impurity" component belonging to the opposite CP eigenvalue. In the Standard Model, $\Delta F = 2$ transitions occur at second order in the weak interactions, and are described by the so-called *box* diagrams (fig. 1, for the neutral kaon system).

CP violation in the mixing can be singled out in the decays of the physical $M - -\overline{M}$ coherent mixtures to *flavour-specific* final states (*i.e.* states to which either M or $\overline{M}$ can decay but not both, due to some selection rule) for which only a single elementary amplitude can contribute. In the Standard Model, semi-leptonic decays of neutral mesons ($M \to X^{\mp} l^{\pm} \nu(\overline{\nu})$, where X is an hadronic state and $l^{\pm}$ a charged lepton) are of this kind, and as such they can be exploited for the study of CP violation in the mixing. As long as the $\Delta F = \Delta Q$ rule, linking the change in flavour eigenvalue and in electric charge for the hadrons involved in the semi-leptonic decay (this rule being justified by the tree level quark diagrams in the Standard Model) is valid, these decays are *flavour-specific*. As an example only $K^0 \to \pi^- l^+ \nu$ and $\overline{K}^0 \to \pi^+ l^- \overline{\nu}$ decays are allowed, but not the ones in which the roles of K^0 and $\overline{K}^0$ are exchanged. Lacking in each case a second amplitude which can interfere, CPT symmetry forces the two amplitudes to be equal; by observing the decays of the physical flavour mixtures, any rate difference among the decay to states with positive or negative leptons reflects the asymmetry in the composition of the mesons, *i.e.* measures CP violation in the mixing.

In the decays of neutral flavoured mesons, the amplitudes for reaching a final state f (common to M and $\overline{M}$, *i.e.* not flavour-specific) without and with mixing ($M \to f$ and $M \to \overline{M} \to f$) can interfere; in this case the "reference" phase, which allows the effect of a non-zero phase in the decay amplitude to be observed, is the one of the mixing amplitude, and no final-state interactions are required to have an asymmetry. This is the so-called CP violation "in the interference of decay with and without mixing" (or "in the interference of mixing and decay", or simply "mixing-induced").

While CP violation in the decay is clearly a manifestation of direct CP violation, and CP violation in the mixing is clearly a manifestation of indirect CP violation, in the mixing-induced case one cannot ascribe in a physically meaningful way the CP violation to either the $M - -\overline{M}$ mixing or the $M, \overline{M} \to f$ decay, so that one is dealing with an undefined mixture of direct and indirect CP violation. However, it should be clear that, being the mixing an intrinsic property of the decaying meson system, any difference between the amount of mixing-induced CP violation in decays to different final states must be due to direct CP violation; this allows to identify this component without ambiguities by measuring several channels to which mixing-induced CP violation can contribute. Figure 2 summarises in a graphic way the different kinds of CP violation effects in mesons, which will be discussed in more detail in section 5.

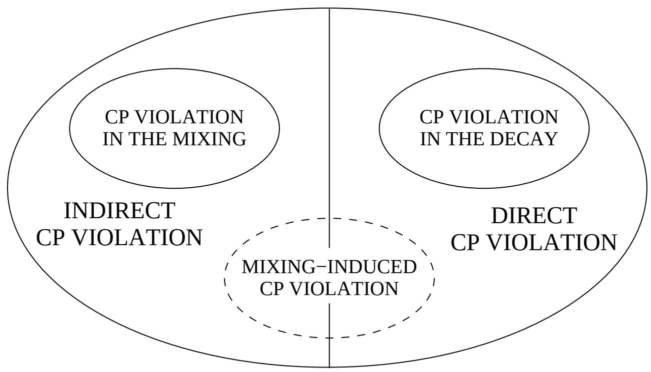

Figure 2: Graphical schematization of different kinds of CP violation in meson processes.

The quantitative understanding of meson decays within a theoretical model is clearly easier when approximate selection rules or dynamical effects are such that these processes are driven by a single elementary amplitude, so that the measurements of the decay parameters (including the branching ratio itself) can be related in a straightforward way to the fundamental parameters of the model. For this reason, the theoretical and experimental studies of CP violation in heavy meson decays has been focused initially on the mixing-induced one (CP violation in the mixing being small in these systems). The recent measurements of CP violation in neutral B_d meson decays [23] [24] [25], providing the first evidence of CP violation outside the neutral kaon system, are actually an example of a class of mixing-induced CP violation phenomena which can be related to the underlying parameters of the Standard Model in a reliable way.

From an experimental point of view, the mixing properties of neutral kaons can be studied rather easily, since such mesons appear in Nature as two flavour superpositions with very different lifetimes; this (accidental) property is not at all shared by heavier neutral meson systems, for which the physical states have very similar lifetimes. CP violation in heavy flavoured neutral mesons is therefore better studied in the "orthogonal reference frame" of *tagged* flavour eigenstates, *i.e.* states whose flavour eigenvalue at production time is known, by exploiting either associate production or the production of coherent meson-antimeson pairs; such approach has been used also for kaons.

While charged mesons are the most straightforward testing ground for direct CP violation, a possibility of extracting direct CP violation signals in neutral meson decays lies in the accurate measurement of the time-dependent decay asymmetries for non flavour-specific final states, which can be fed by both M and $\overline{M}$, as will be discussed in detail when addressing heavy meson decays. It should be clear that, if the flavour oscillations are not too fast, in the limit of short proper decay times any CP violation effect due to mixing would be relatively less important than the ones due to decay, which manifest themselves in a time independent way.

For channels with only two (pseudo-)scalar mesons or a (pseudo-)scalar and a vector meson in the final state, a CP asymmetry can only become apparent as a difference of the partial decay rates, while for states with more complex spin content or larger number of particles in the final state, asymmetries in the distributions of kinematic variables can also be used to search for CP violation.

Summarising, evidence for direct CP violation in meson decays could come from either:

- Any difference between the properties of distinct CP-conjugate decays.
- Any difference between the decay properties of CP-conjugate states to a common final state, in cases in which particle-antiparticle mixing cannot occur (*i.e.* any CP asymmetry for charged particles).
- Any difference between the amount of CP violation in decays of a given physical meson to different final states, *e.g.* differences in the mixing-induced neutral meson decay asymmetries to different final states.
- The study of the time-dependent decay asymmetries in neutral meson decays.
- Any non-zero CP-violating decay asymmetries of *flavour-tagged* mesons to "right-sign" *flavour-specific* final states.

The above signatures will be discussed in the following.

2.2 T-odd correlations

Experiments on transverse muon polarization in $K_{\mu 3}$ decays, to be discussed in the following, are an example of a class of measurements searching for CP violation as non-zero values of T-odd observables (assuming CPT symmetry); such searches were performed on several systems (see *e.g.* references in [26]), so far with null results.

When event (decay) rates are measured for an isolated system, Lorentz invariance only allows them to depend on scalar (or pseudoscalar) quantities; just as the rate dependence of beta decay on the pseudoscalar quantity $\mathbf{S}\cdot\mathbf{p}$ ($\mathbf{S}$ being a polarization vector and $\mathbf{p}$ a three-momentum) gave evidence for parity violation, any rate dependence on a T-odd quantity could be expected to indicate T violation. Lorentz symmetry requires that the simplest of such T-odd quantities involve three vectors as a triple product; at least three particles are required to get a non-trivial value for such a quantity, *e.g.* $\mathbf{S}\cdot\mathbf{p}_1\times\mathbf{p}_2$ in a three-body decay, in which a polarization vector and two three-momenta are involved. In order to avoid using spin information, at least four particles are required to form a non-null T-odd correlation, such as $\mathbf{p}_1\cdot\mathbf{p}_2\times\mathbf{p}_3$, because the vectors have to be independent; moreover, no identical particles should appear in the final state, otherwise the quantity vanishes trivially.

An important feature of T-odd correlations is that final-state interactions can themselves generate a non-null value for such quantities, even in absence of T violation, so that the measurement of a non-zero expectation value is not, by itself, evidence for violation of T symmetry[6]. In order to detect a true signal of T violation, one should therefore either subtract the effect due to final-state interactions, if known, or study processes in which such interactions are known to have negligible effects. Conversely, it should be noted that T-odd correlations provide an example of how direct CP violation (assuming CPT) can appear even in absence of any final-state interactions, contrary to the case of partial rate or angular distribution asymmetries, in which "strong" phases are an essential requirement to make CP violation observable. When such phases are indeed present but small in magnitude, triple product correlations may happen to be the observables most sensitive to CP violation.

This class of measurements (assuming CPT symmetry) is actually complementary to that of partial decay rate asymmetries, for the decay modes in which no final-state interactions appear, when any measurement of a non-zero value for a T-odd correlation directly indicates CP violation, are actually the ones in which the comparison of the decay rates for particles and anti-particles give no information on the validity of CP symmetry, their equality being enforced by CPT.

It is important to note that even when, possibly uncontrollable, final-state interactions are present, the study of T-odd correlations is relevant to CP symmetry because their value for the CP-conjugate process is just the opposite of that for the original one, if CP symmetry is valid. One can therefore detect an unambiguous signal of CP violation by comparing non-zero values of T-odd correlations for CP-conjugate processes, even in presence of unknown final-state interaction effects. Clearly, such comparisons have to be performed in absence of instrumental CP-asymmetric effects: the study of states produced in particle-antiparticle collisions (the initial state being a CP eigenstate) is therefore a good environment for such kind of experiments.

3 Neutral kaon decays

The system of neutral kaons, K^0 and $\overline{K}^0$, is an extraordinary physics laboratory, being the lightest system of two coupled particles (as "elementary" as isolated observation allows) which are distinguished only by a quantum number (flavour, strangeness S in this case) which is conserved by the strong and electromagnetic interactions but *not* by the weak interactions responsible for their decay. As such, neutral kaons allow the direct observation of characteristic quantum-mechanical interference effects of a small magnitude.

The central role of kaons in the development of particle physics cannot be overstated: as the lightest particles with a flavour quantum number not to be found in ordinary matter but readily produced at GeV energies, they stimulated the study of such diverse issues as flavour mixing, parity non-conservation, the existence of the *charm* quark, before providing the first (and, for a long time, unique) evidence of CP non-conservation. For the neutral kaon system the peculiar features of particle-antiparticle state mixing can be easily observed, and a large variety of experimental measurements can be performed, since both the

[6]This happens because the inversion of spins and momenta (sometimes called "naive" time reversal) is not the only effect of time reversal, and therefore such a non-zero T-odd correlation can actually be invariant under T, as indeed the part due to strong or electromagnetic final-state interactions is.

flavour eigenstates $K^0, \overline{K}^0$ and the physical mass eigenstates K_S, K_L are separately accessible for experimentation: the system offers the unique possibility of observing in principle arbitrary superpositions of flavour eigenstates. This occurs due to the relatively small ratio of the $K^0, \overline{K}^0$ mass to the pion masses, which together with the approximate validity of CP symmetry leads to a very large lifetime difference for the physical states: $\tau_S = 8.9 \cdot 10^{-11}$ s for K_S, for which the dominant hadronic decay channel $\pi\pi$ is allowed, and $\tau_L = 5.2 \cdot 10^{-8}$ s for K_L, for which the $\pi\pi$ decay channel is largely suppressed by the (approximate) CP symmetry. This lifetime difference makes easy the experimentation with pure K_L, obtained by considering the neutral kaons surviving undecayed after a long ($\gg \beta\gamma c\tau_S$) decay region. K_S decays can be statistically studied in a neutral kaon beam close to the production point, which initially contains the same amount of K_S and K_L components (as long as the CPT symmetry is valid). To experiment with pure K_S, the coherent production of neutral kaon pairs (*e.g.* from ϕ decays) can be exploited.

However, since within the Standard Model CP violation requires the involvement of all three quark families, which in the kaon system only occurs through virtual processes, and not in first order, the magnitude of the observable effects is generally small.

3.1 Phenomenology

The discussion of the phenomenology of the neutral kaon system and CP violation can be found in standard particle physics textbooks and in several reviews (see *e.g.* [11] [27] [28]). Unfortunately, while the discussion of the formalism has its common roots in the Weisskopf-Wigner approach to coupled unstable systems [29] and in the classic paper of Wu and Yang [30], the variety of different conventions and approximations can be the source of some confusion: as an example, at least 5 different definitions of the direct CP violation parameter ϵ' in $K \to \pi\pi$ decays can be found in the literature, and they coincide only when some approximations are made, which are often justified only in some class of phase conventions for the meson states. CP violation is expressed in the formalism by the presence of complex quantities, but since the mapping of quantum-mechanical states to physical states is to within a phase, it is crucial to separate the irrelevant (convention-dependent) phases from the "physical" ones which characterise CP violation[7] (see *e.g.* [32] and references therein).

We now summarise the consistent definitions that we use in the following discussion. Instead of using more specific but uncommon formalism (see *e.g.* [33] [34]) we adopt the usual one, trying to state clearly the approximations and the phase conventions relevant to deduce a given relation.

The relative phase of K^0 and $\overline{K}^0$ states is not measurable and can be fixed by convention (even independently from the quark field phases). Given the phase convention

$$CP|K^0\rangle = e^{i\xi_{CP}}|\overline{K}^0\rangle \tag{8}$$

$$CP|\overline{K}^0\rangle = e^{-i\xi_{CP}}|K^0\rangle \tag{9}$$

[7] "Anyone who has played with these invariances knows that it is an orgy of relative phases", A. Pais in [31].

(with real ξ_{CP}) the CP eigenstates are defined to be

$$|K_1\rangle \equiv \frac{1}{\sqrt{2}}(e^{-i\xi_{CP}/2}|K^0\rangle + e^{i\xi_{CP}/2}|\overline{K}^0\rangle) \quad (CP = +1) \tag{10}$$

$$|K_2\rangle \equiv \frac{1}{\sqrt{2}}(e^{-i\xi_{CP}/2}|K^0\rangle - e^{i\xi_{CP}/2}|\overline{K}^0\rangle) \quad (CP = -1) \tag{11}$$

The ξ_{CP} phase can be seen as the parameter of a transformation whose generator is the strangeness operator: since $K^0, \overline{K}^0$ states are defined by the strangeness-conserving strong interactions, no observable can depend on the value of such quantity. The simplest phase convention, adopted in the following (except where explicitly stated) is $\xi_{CP} = 0$, *i.e.*

$$CP|K^0\rangle = |\overline{K}^0\rangle \qquad\qquad CP|\overline{K}^0\rangle = |K^0\rangle \tag{12}$$

In this case the "physical"[8] states K_S, K_L (short- and long-lived, respectively) can be written in terms of the strangeness eigenstates $K^0, \overline{K}^0$ as

$$\begin{aligned} |K_S\rangle &= \tfrac{1}{\sqrt{1+|\epsilon_S|^2}}(|K_1\rangle + \epsilon_S|K_2\rangle) = \\ &\tfrac{1}{\sqrt{2(1+|\epsilon_S|^2)}}\left[(1+\epsilon_S)|K^0\rangle + (1-\epsilon_S)|\overline{K}^0\rangle\right] \end{aligned} \tag{13}$$

$$\begin{aligned} |K_L\rangle &= \tfrac{1}{\sqrt{1+|\epsilon_L|^2}}(|K_2\rangle + \epsilon_L|K_1\rangle) = \\ &\tfrac{1}{\sqrt{2(1+|\epsilon_L|^2)}}\left[(1+\epsilon_L)|K^0\rangle - (1-\epsilon_L)|\overline{K}^0\rangle\right] \end{aligned} \tag{14}$$

CPT symmetry requires $\epsilon_S = \epsilon_L$, leaving a single (complex) CP impurity parameter to parameterise the composition of physical states:

$$\overline{\epsilon} \equiv (\epsilon_S + \epsilon_L)/2 \tag{15}$$

This parameter would be zero if CP symmetry were valid in $K^0 - -\overline{K}^0$ state mixing. The impurity parameter $\overline{\epsilon}$ is not physically measurable and depends on the choice of phase convention; a phase convention independent term which parameterises CP violation in the mixing of states is the measure of the non-orthogonality of the physical states:

$$\langle K_S|K_L\rangle = \frac{2\mathrm{Re}(\overline{\epsilon})}{1+|\overline{\epsilon}|^2} \tag{16}$$

The statement that the real part of $\overline{\epsilon}$ is a measurable quantity parameterising the amount of indirect CP violation is seen to be approximately valid in every so called "physical" phase convention in which $|\overline{\epsilon}| \ll 1$; that such a class of phase conventions is allowed at all clearly follows from the fact that CP violation is experimentally known to be small with respect to the dominant weak interaction effects, so that a formalism in which all the parameters related to CP violation are small must exist, although a phase convention in which $|\overline{\epsilon}| \gg 1$ could also be adopted at will (see *e.g.* [35]). In a different formalism, widely used for heavier meson systems, one defines

$$\frac{q}{p} \equiv \frac{1-\overline{\epsilon}}{1+\overline{\epsilon}} \tag{17}$$

[8]Although still non-observable in principle.

Due to their coupling through the strangeness-violating weak interactions, the time evolution of K^0 and $\overline{K}^0$ has to be described together, and the appropriate formalism to treat such coupled systems is that developed by Weisskopf and Wigner (see *e.g.* [13]). Considering the projection onto the two-dimensional subspace of K^0 and $\overline{K}^0$ components, the time evolution of a generic state Ψ is described by a non-hermitian quasi-Hamiltonian $\mathcal{H}$:

$$i\hbar\frac{\partial}{\partial t}\Psi = \mathcal{H}\Psi \qquad (18)$$

The 2×2 matrix $\mathcal{H}$ can be written in full generality as

$$\mathcal{H} \equiv \mathbf{M} - \frac{i}{2}\mathbf{\Gamma} \qquad (19)$$

where $\mathbf{M}$ and $\mathbf{\Gamma}$ (the "mass matrix" and the "decay matrix") are both hermitian matrices, whose components, at lowest order in the small (weak) Hamiltonian H_I responsible for $K^0, \overline{K}^0$ not being mass eigenstates, are[9]

$$M_{11} = m_K + \langle K^0|H_I|K^0\rangle + O(H_I^2) \qquad (20)$$

$$M_{22} = m_K + \langle \overline{K}^0|H_I|\overline{K}^0\rangle + O(H_I^2) \qquad (21)$$

$$M_{12} = M_{21}^* = \langle K^0|H_I|\overline{K}^0\rangle + O(H_I^2) \qquad (22)$$

$$\Gamma_{11} = 2\pi \textstyle\sum_f |\langle f|H_I|K^0\rangle|^2 \delta(m_K - E_f) + O(H_I^4) \qquad (23)$$

$$\Gamma_{22} = 2\pi \textstyle\sum_f |\langle f|H_I|\overline{K}^0\rangle|^2 \delta(m_K - E_f) + O(H_I^4) \qquad (24)$$

$$\Gamma_{12} = \Gamma_{21}^* = 2\pi \textstyle\sum_f \langle K^0|H_I|f\rangle\langle f|H_I|\overline{K}^0\rangle \delta(m_K - E_f) + O(H_I^4) \qquad (25)$$

where m_K is the (common) mass of K^0 and $\overline{K}^0$, as defined by the CP-conserving strong interactions, and f is any final state (of energy E_f) accessible to K^0 or $\overline{K}^0$ in physical decays.

CPT symmetry requires the diagonal elements of the quasi-Hamiltonian to be equal: $M_{11} = M_{22}$, $\Gamma_{11} = \Gamma_{22}$. The off-diagonal elements M_{12} and Γ_{12} are responsible for strangeness oscillations, and T symmetry requires them to be real. CP symmetry imposes the same constraints on these quantities, namely the equality of the diagonal matrix elements and the reality of the off-diagonal ones. It is interesting to note that a Hamiltonian H_I which changes strangeness by one unit ($|\Delta S| = 1$), such as the one describing weak interactions in the Standard Model, can only contribute to $\mathbf{\Gamma}$ and not to $\mathbf{M}$ in lowest order.

The states K_S, K_L of definite mass and lifetime are the eigenstates of $\mathcal{H}$:

$$\mathcal{H}|K_{S,L}\rangle = \lambda_{S,L}|K_{S,L}\rangle \qquad (26)$$

where the eigenvalues are $\lambda_{S,L} \equiv m_{S,L} - i\Gamma_{S,L}/2$ ($m_{S,L}, \Gamma_{S,L}$ real), which (assuming CPT symmetry) are given by

$$\lambda_{S,L} = \mathcal{H}_{11} \pm \sqrt{\mathcal{H}_{12}\mathcal{H}_{21}} \qquad (27)$$

As mentioned, the decay widths are very different, with $\Gamma_L/\Gamma_S \approx 600$. The physical states K_L, K_S have also a tiny mass difference, $m_L - m_S \simeq 3.5 \cdot 10^{-12}$

[9] In the following index 1 refers to K^0 and 2 to $\overline{K}^0$ and $\hbar = c = 1$ unless explicitely noted.

MeV/c^2, induced in the Standard Model by second-order weak interactions, according to the Glashow-Iliopoulos-Maiani mechanism, and a correspondingly long flavour oscillation length in the laboratory.

Of the eight real parameters defining the matrix $\mathcal{H}$, one is an irrelevant global phase, four can be chosen as the masses and lifetimes of the physical states, two vanish if CPT symmetry is valid, and the last one expresses CP violation: it can be chosen to be the phase convention independent part of $\overline{\epsilon}$. One has

$$\frac{1-\overline{\epsilon}}{1+\overline{\epsilon}} \equiv \frac{q}{p} = \sqrt{\frac{\mathcal{H}_{21}}{\mathcal{H}_{12}}} = \frac{\Delta\lambda}{2\mathcal{H}_{12}} \tag{28}$$

(where $\Delta\lambda \equiv \lambda_S - \lambda_L$), and in the limit $|\overline{\epsilon}| \ll 1$ (possible for small CP violation) one has

$$\overline{\epsilon} \simeq \frac{\mathcal{H}_{12} - \mathcal{H}_{21}}{2\Delta\lambda} = \frac{\mathrm{Im}(M_{12}) - i\mathrm{Im}(\Gamma_{12})/2}{i\Delta m - \Delta\Gamma/2} \tag{29}$$

where, as is conventional in the case of the $K^0 - -\overline{K}^0$ system, Δm and $\Delta\Gamma$ are defined to be positive and, in the limit of small CP violation:

$$\Delta m \equiv m_L - m_S \simeq -2\mathrm{Re}(M_{12}) \tag{30}$$

$$\Delta\Gamma \equiv \Gamma_S - \Gamma_L \simeq 2\mathrm{Re}(\Gamma_{12}) \tag{31}$$

In the neutral kaon system the following empirical relation holds

$$\Delta m \simeq \Gamma_S/2 \simeq \Delta\Gamma/2 \tag{32}$$

and therefore one obtains in the end the expression

$$\overline{\epsilon} \simeq -\frac{e^{i\pi/4}}{\Delta m\sqrt{2}}\left[\mathrm{Im}(M_{12}) - i\mathrm{Im}(\Gamma_{12})/2\right] \simeq \frac{e^{i\pi/4}}{2\sqrt{2}}\left[\frac{\mathrm{Im}(M_{12})}{\mathrm{Re}(M_{12})} + i\frac{\mathrm{Im}(\Gamma_{12})}{\mathrm{Re}(\Gamma_{12})}\right] \tag{33}$$

so that indeed the CP impurity parameter $\overline{\epsilon}$ is linked to the off-diagonal elements of $\mathbf{M}$ and $\mathbf{\Gamma}$ being complex. It can be easily seen that what actually determines the presence of CP violation in the quasi-Hamiltonian $\mathcal{H}$ describing meson-antimeson mixing is a non-zero *relative* phase between M_{12} and Γ_{12}: CP violation is present in the quasi-Hamiltonian if and only if $|\mathcal{H}_{12}| \neq |\mathcal{H}_{21}|$, and

$$\langle K_S|K_L\rangle \propto \mathrm{Im}(M_{12}^*\Gamma_{12}) \tag{34}$$

CP symmetry would imply that this relative phase is zero, and the coefficients of the two flavour eigenstates in the physical states differ at most by a pure phase:

$$\left|\frac{1-\overline{\epsilon}}{1+\overline{\epsilon}}\right| \equiv \left|\frac{q}{p}\right| = 1 \tag{35}$$

In presence of small CP violation one has instead

$$\left|\frac{1-\overline{\epsilon}}{1+\overline{\epsilon}}\right| \equiv \left|\frac{q}{p}\right| \simeq 1 - \frac{1}{2}\sin\left[\arg\left(\frac{\Gamma_{12}}{M_{12}}\right)\right] \tag{36}$$

The charge asymmetry for semi-leptonic K_L or K_S decays is a pure measurement of CP violation in the mixing, as long as CPT symmetry and the

$\Delta S = \Delta Q$ rule[10] are valid:

$$\delta^{(l)}_{L,S} \equiv \frac{\Gamma(K_L, K_S \to \pi^- l^+ \nu) - \Gamma(K_L, K_S \to \pi^+ l^- \overline{\nu})}{\Gamma(K_L, K_S \to \pi^- l^+ \nu) + \Gamma(K_L, K_S \to \pi^+ l^- \overline{\nu})} = \langle K_S | K_L \rangle \qquad (37)$$

Semi-leptonic decays of K_L are easily accessible from an experimental point of view, the branching ratios being $\simeq 39\%$ for $\pi^\pm e^\mp \nu$ (K_{e3}) and $\simeq 27\%$ for $\pi^\pm \mu^\mp \nu$ ($K_{\mu 3}$); the measured charge asymmetries are [36] [37] [38]:

$$\delta^{(e)}_L = (0.3322 \pm 0.0055)\% \qquad (38)$$

$$\delta^{(\mu)}_L = (0.304 \pm 0.025)\% \qquad (39)$$

So far the only measurement of the charge asymmetry in semi-leptonic K_S decays has been carried out by the KLOE experiment. A preliminary result based on 170 pb^{-1} (out of the ~ 500 collected until 2002) gave [39]:

$$\delta^{(e)}_S = (1.9 \pm 1.8)\% \qquad (40)$$

If CPT symmetry and the $\Delta S = \Delta Q$ rule are valid, one expects $\delta^{(l)}_L = \delta^{(l)}_S = 2\mathrm{Re}(\overline{\epsilon})/(1 + |\overline{\epsilon}|^2)$, from which $\mathrm{Re}(\overline{\epsilon}) = (1.655 \pm 0.003) \cdot 10^{-3}$ is obtained.

Within a few years of the discovery of CP violation, the crucial question became that of whether the single parameter $\overline{\epsilon}$ describing the asymmetry of $K^0 - -\overline{K}^0$ mixing in the effective quasi-Hamiltonian $\mathcal{H}$ could account for all the CP non-conservation effects in Nature. Already in 1964, L. Wolfenstein pointed out [40] that a hypothetical "super-weak" interaction, capable of driving $K^0 - -\overline{K}^0$ transitions in first order (*i.e.* satisfying a $\Delta S = 2$ selection rule) would induce $K_L \to \pi\pi$ decays through the CP impurity of the physical states, and that the smallness of the coupling required to give the measured amount of CP violation through this mechanism would effectively confine all the measurable effects of such a new interaction to the very sensitive neutral kaon system. The introduction of a small complex, non-diagonal term in the effective quasi-Hamiltonian, induced by a new Hamiltonian H_{SW}

$$M^{(SW)}_{12} = \langle K^0 | H_{SW} | \overline{K}^0 \rangle = i\Delta_{SW} \qquad (41)$$

would give by diagonalization

$$\overline{\epsilon} \simeq \frac{-\Delta_{SW}}{\Delta m} \frac{1+i}{2} \qquad (42)$$

and inserting the value of the $K_L - K_S$ mass difference one obtains (in an appropriate phase convention) $|\Delta_{SW}| \simeq \sqrt{2}|\overline{\epsilon}|\Delta m = 1.1 \cdot 10^{-8}$ eV. Writing $|\Delta_{SW}| \sim G_{SW}\, m_K^3$, so to have a super-weak coupling constant G_{SW} dimensionally comparable to G_F, one has $G_{SW} \sim 10^{-11} G_F$. Δ_{SW} is suppressed by a factor $|\overline{\epsilon}|$ with respect to Δm, which is a second-order weak interaction effect: the tiny mass difference between K_L and K_S effectively boosts the effects of an extremely weak interaction.

[10] Experimental information on the validity of this rule come from the limits on $BR(\Sigma^+ \to n e^+ \nu)$ and the detailed study of semi-leptonic K^0 decays by CPLEAR which give [36] $A(\Delta S = -\Delta Q)/A(\Delta S = \Delta Q) = (-0.002 \pm 0.006)\ + i(0.0012 \pm 0.0021)$.

Although the super-weak scenario followed the familiar pattern of having weaker fundamental interactions respecting all but a few of the symmetries which are valid for the stronger ones (a fact which has to do with the logical structure of our physical theories and tells little about Nature), it actually stood more as a paradigm for classifying theories of CP violation rather than a realistic model. In recent times the concept of "super-weak CP violation" acquired a more blurred meaning: its extension beyond the neutral kaon sector can be done in different ways, *i.e.* implying either no CP violation in other meson systems, or CP violation being limited to $\Delta F = 2$ flavour-changing interactions in every system. Various models actually implement the super-weak idea in different ways, leading to different estimates of CP violation outside the kaon system. Moreover, when considering CP violation effects in neutral meson systems induced by the interference of decays with and without mixing, it is a matter of convention whether the CP violation phases are assumed to be in the $\Delta F = 1$ (direct) or in the $\Delta F = 2$ (indirect) sector for a *single*, given decay mode: only by comparing different decays an unambiguous signature for direct CP violation can be found, and super-weak CP violation indicates a framework in which CP violation in *any* decay mode can be ascribed to the $\Delta F = 2$ mixing.

The Standard Model does not belong to the super-weak class, CP violation arising in an ubiquitous phase appearing in most weak decays driven by the charged current of quarks.

Always assuming CPT symmetry, considering a CP eigenstate f with eigenvalue $CP = +1$ (such as $\pi\pi$), so that the K_2 state would not decay to it in absence of CP violation, the measure of CP violation is usually expressed in terms of

$$\eta_f = |\eta_f| e^{i\phi_f} \equiv \frac{A(K_L \to f)}{A(K_S \to f)} \quad (CP|f\rangle = +|f\rangle) \tag{43}$$

Both the modulus and the phase of η_f can be measured[11]: the first from a branching ratio measurement, and the second by studying the $K_L - K_S$ interference term in the decay rate as a function of proper time t, in the region where the contribution by both physical states are comparable (around $t \approx 12\tau_S$): this requires a good knowledge of the admixture of K^0 and $\overline{K}^0$ in the beam used to perform the measurement, such as is obtained by using pure K_L or K_S beams.

To first order in the (small) parameter $|\overline{\epsilon}|$ (*i.e.* in an appropriate phase convention) one has

$$\eta_f \simeq \overline{\epsilon} + \epsilon_f \tag{44}$$

where

$$\epsilon_f \equiv \frac{A(K_2 \to f)}{A(K_1 \to f)} \quad (CP|f\rangle = +|f\rangle) \tag{45}$$

The above decomposition of η_f depends on the choice of phase convention; ϵ_f represents the direct CP violation part of the amplitude ratio, while $\overline{\epsilon}$ represents the part proceeding through the CP-conserving decay of the small component with opposite CP eigenvalue (K_1 in this case).

[11] The phase convention independent expression for the physical parameter η_f should be properly written as $\eta_f = \frac{A(K_L \to f)}{A(K_S \to f)} \frac{\langle K^0 | K_S \rangle}{\langle K^0 | K_L \rangle}$, where the second factor is 1 with the standard phase convention.

For the experimentally less accessible case of final states which are CP eigenstates with $CP = -1$, the measure of CP violation due to K_S decays is similar:

$$\eta_f = |\eta_f| e^{i\phi_f} \equiv \frac{A(K_S \to f)}{A(K_L \to f)} \quad (CP|f\rangle = -|f\rangle) \tag{46}$$

$$\epsilon_f \equiv \frac{A(K_1 \to f)}{A(K_2 \to f)} \quad (CP|f\rangle = -|f\rangle) \tag{47}$$

While in principle the measurement of a single η_f could give a measurement of ϵ_f by comparison with the value of $\overline{\epsilon}$ obtained from the K_L semi-leptonic charge asymmetry, ϵ_f turns to be so small that such an approach cannot reach the required precision, being affected by systematic errors which cannot be controlled at the level required. Moreover, as already stated, for a single final state f the amount of CP violation ascribed to ϵ_f is phase-convention dependent, and only a measurement of two different decay channels allows a definite answer to be given on the existence of CP violation.

The K_L decay channels with largest branching ratio which can support direct CP violation are $\pi^+\pi^-$ and $\pi^0\pi^0$, for which respectively [36] [41]:

$$|\eta_{+-}| = (2.287 \pm 0.017) \cdot 10^{-3} \qquad \phi_{+-} = (43.4 \pm 0.7)^\circ \tag{48}$$

$$|\eta_{00}| = (2.23 \pm 0.011) \cdot 10^{-3} \qquad \phi_{00} = (43.2 \pm 1.0)^\circ \tag{49}$$

When considering neutral kaon decays to $\pi\pi$, which are the dominant final states common to both K_S and K_L (through CP violation), one has to take into account the fact that the decay channels which are independent from the point of view of the CP-conserving strong interactions are actually the states of definite isospin $I = 0, 2$ ($I = 1$ being forbidden by Bose symmetry, since kaons have zero spin). One can define the (unmeasurable) amplitude ratios to a $\pi\pi$ final state of definite isospin:

$$\eta_I \equiv \frac{A(K_L \to (\pi\pi)_I)}{A(K_S \to (\pi\pi)_I)} \tag{50}$$

which for the dominant amplitude, the one for the isospin 0 state indicated as $(\pi\pi)_{I=0}$, is usually named ϵ:

$$\epsilon \equiv \eta_0 \tag{51}$$

In the limit of isospin symmetry, the Fermi-Watson theorem (based on CPT and unitarity) allows to factor out from the decay amplitudes the strong phases δ_I, corresponding to $\pi\pi$ scattering in the isospin I eigenstate at energy equal to the K^0 mass:

$$A_I \equiv A(K^0 \to (\pi\pi)_I) = a_I e^{i\delta_I} \tag{52}$$

$$\overline{A}_I \equiv A(\overline{K}^0 \to (\pi\pi)_I) = \overline{a}_I e^{i\delta_I} \tag{53}$$

which is useful because, due to CPT symmetry one has $\overline{a}_I = a_I^*$, implying that CP violation is just described by the imaginary part of a_I.

The experimental fact that the $I = 0$ isospin state of $\pi\pi$ dominates over the $I = 2$ one in K decays (as can readily be seen from the ratio of $\pi^+\pi^-$ and $\pi^0\pi^0$ branching ratios) constitutes the so-called $\Delta I = 1/2$ "rule", for which no convincing theoretical explanation has been found so far. The violation of this "rule" is parameterised by the quantity

$$\omega \equiv \frac{A(K_S \to (\pi\pi)_{I=2})}{A(K_S \to (\pi\pi)_{I=0})} \tag{54}$$

and experimentally (neglecting possible amplitudes with $|\Delta I| > 3/2$, as allowed by the data) $|\omega| \simeq 1/22.2$ (although including isospin breaking effects one could get $|\omega| \simeq 1/29.5$ [42]).

Given this strong dominance, a practical choice of phase convention is the one in which the corresponding decay amplitude is real (Wu-Yang phase convention); only in this case the $K^0 - -\overline{K}^0$ mixing parameter $\overline{\epsilon}$ coincides with ϵ, while in general (dropping terms of order $\overline{\epsilon}\,\mathrm{Re}(a_2)/Rea(a_0), \overline{\epsilon}\,\mathrm{Im}(a_2)/\mathrm{Re}(a_0)$ and in the limit $|\overline{\epsilon}| \ll 1$) one has:

$$\epsilon = \overline{\epsilon} + i\frac{\mathrm{Im}(a_0)}{\mathrm{Re}(a_0)} \tag{55}$$

The parameter ϵ is independent from the choice of phase convention; its real part $\mathrm{Re}(\epsilon) \simeq \mathrm{Re}(\overline{\epsilon})$ describes CP violation in the mixing, while its imaginary part expresses CP violation in the interference of mixing and decay for the dominant isospin 0 $\pi\pi$ channel, which as such is not a definite signal of direct CP violation. The phase of ϵ is determined by CPT symmetry to be close to the so-called "super-weak phase" [36] [41]:

$$\phi_{SW} \equiv \arctan\left[\frac{2\Delta m}{\Delta\Gamma}\right] \simeq (43.46 \pm 0.05)^\circ \tag{56}$$

the two phases being equal in the limit in which no direct CP violation occurs in $\Delta S = 1$ decay amplitudes of neutral kaons, except for the dominant one to the $(\pi\pi)_{I=0}$ state (and small overall CP violation), since

$$\epsilon \simeq \frac{\mathrm{Im}(\Gamma_{12}^* M_{12})}{(\Delta m)^2\sqrt{2}} e^{i\phi_{SW}} + i\left[\frac{\mathrm{Im}(\Gamma_{12})}{\Delta\Gamma} + \frac{\mathrm{Im}(a_0)}{\mathrm{Re}(a_0)}\right] \tag{57}$$

and the second term cancels in the above limit.

Direct CP violation in $\pi\pi$ decays of neutral kaons can arise as a difference between the amount of CP violation in different isospin channels, and is parameterised by

$$\epsilon' \equiv \frac{\omega}{\sqrt{2}}(\eta_2 - \eta_0) = \frac{\eta_0}{\sqrt{2}}\left[\frac{A(K_L \to (\pi\pi)_{I=2})}{A(K_L \to (\pi\pi)_{I=0})} - \frac{A(K_S \to (\pi\pi)_{I=2})}{A(K_S \to (\pi\pi)_{I=0})}\right] \tag{58}$$

Both isospin states contribute (differently) to the two measurable channels $\pi^+\pi^-$ and $\pi^0\pi^0$: from the isospin decomposition (neglecting possible $\Delta I > 3/2$ amplitudes, see [42])

$$A(K^0 \to \pi^+\pi^-) = \tfrac{1}{\sqrt{3}}(\sqrt{2}A_0 + A_2) \tag{59}$$

$$A(K^0 \to \pi^0\pi^0) = \tfrac{-1}{\sqrt{3}}(A_0 - \sqrt{2}A_2) \tag{60}$$

and one obtains

$$\eta_{+-} = \epsilon + \tfrac{\epsilon'}{1+\omega/\sqrt{2}} \tag{61}$$

$$\eta_{00} = \epsilon - \tfrac{2\epsilon'}{1-\omega\sqrt{2}} \tag{62}$$

without any approximation. All the above parameters are independent from the choice of phase convention.

The experimental similarity of η_{+-} and η_{00}, both in modulus and phase, indicate that $|\epsilon'| \ll |\epsilon|$, *i.e.* direct CP violation is relatively small in kaon decays.

The above formulæ can be simplified by introducing some approximations. Since $|\omega| \ll 1$ one has

$$\eta_{+-} \simeq \epsilon + \epsilon' \qquad\qquad \eta_{00} \simeq \epsilon - 2\epsilon' \tag{63}$$

which in the literature are sometimes promoted (inconsistently) to exact relations defining (suitably different) ϵ and ϵ' parameters.

The smallness of CP violation ($|\epsilon|, |\epsilon'| \ll 1$) allows (together with CPT symmetry) to approximate

$$\omega \simeq e^{i(\delta_2-\delta_0)} \mathrm{Re}\left(\tfrac{a_2}{a_0}\right) \tag{64}$$

$$\epsilon' \simeq \tfrac{i}{\sqrt{2}} e^{i(\delta_2-\delta_0)} \tfrac{\mathrm{Re}(a_2)}{\mathrm{Re}(a_0)} \left[\tfrac{\mathrm{Im}(a_2)}{\mathrm{Re}(a_2)} - \tfrac{\mathrm{Im}(a_0)}{\mathrm{Re}(a_0)}\right] \simeq \tfrac{i}{\sqrt{2}} e^{i(\delta_2-\delta_0)} \mathrm{Im}\left(\tfrac{a_2}{a_0}\right) \tag{65}$$

which are valid in the limit in which all the quantities $|\overline{\epsilon}|$, $|\overline{\epsilon}\,\mathrm{Re}(a_2)/\mathrm{Re}(a_0)|$, $|\overline{\epsilon}\,\mathrm{Im}(a_2)/\mathrm{Re}(a_0)|$, $|\overline{\epsilon}\,\mathrm{Im}(a_0)/\mathrm{Re}(a_0)|$ are much lower than 1.

In this approximation the phase of ϵ' is determined by the strong phases δ_I; from the analysis of $\pi\pi$ interaction data, one obtains [43]: $\delta_2 - \delta_0 = (-47.7^\circ \pm 1.5^\circ)$ as the best estimate of this phase difference[12].

The reader could notice that the parameter ϵ' does not vanish even if the strong scattering phases δ_I for $I = 0, 2$ are equal to each other. This is so because ϵ' is not a measure of CP violation in the decay into *a single* channel, but actually refers to decay modes receiving contributions from two distinct amplitudes, corresponding to definite isospin states $I = 0, 2$. CP violation in the decay is singled out in the real part of ϵ' which, in order to be non-zero, requires the strong phases for the two channels to be different ($\delta_2 - \delta_0 \neq 0, \pi$). The imaginary part of ϵ' instead, does not disentangle CP violation in the decay from the mixing-induced one, but since it involves the difference of CP violation in two decay modes, it is an unambiguous indicator of direct CP violation nevertheless.

For a final state f which is a CP eigenstate one has in general

$$\frac{\Gamma(K^0 \to f) - \Gamma(\overline{K}^0 \to f)}{\Gamma(K^0 \to f) + \Gamma(\overline{K}^0 \to f)} \simeq 2\mathrm{Re}(\epsilon_f) \tag{66}$$

(valid in the limit $|\eta_f| \ll 1$ if $|\overline{\epsilon}| \ll 1$) and for $\pi^+\pi^-$ or $\pi^0\pi^0$ states the ϵ_f parameters are $\epsilon_{+-} \simeq \epsilon'$, $\epsilon_{00} \simeq -2\epsilon'$, respectively.

Since ϵ' arises from the interference of two amplitudes, one of which is empirically found to be suppressed by the rather small factor ω, one can see that this measure of direct CP violation is suppressed with respect to the "natural"

[12] The long-standing problem of the discrepancy of such measurements with the values obtained by the isospin analysis of the neutral and charged kaons decay amplitudes to two pions, which are affected by larger theoretical uncertainties (see *e.g.* [44] [42] and references therein), might be clarified by better measurements of the neutral kaon $\pi\pi$ branching ratios, *i.e.* $\delta_2 - \delta_0 \simeq (-47.8 \pm 2.8)^\circ$ from KLOE [45], [46].

magnitude which could be expected from the size of the complex phases in the elementary amplitudes.

It could also be noticed that the (conventional) pseudoscalar nature of kaons and pions implies that the state which decays dominantly into $\pi\pi$ is the C-odd one, so that indirect CP violation described by ϵ is C-odd and P-even, while direct CP violation described by ϵ' is C-even and P-odd.

A great deal of theoretical effort was devoted to the computation of ϵ' in the Standard Model (see fig. 3), which turns out to be a daunting task (see [47] for a recent review): no reliable approach exists yet to compute all required matrix elements, in which strong interaction effects play a relevant role. The two dominant terms which contribute to ϵ' turn out to have similar phases, so that an additional cancellation makes the final result both smaller and more sensitive to theoretical uncertainties.

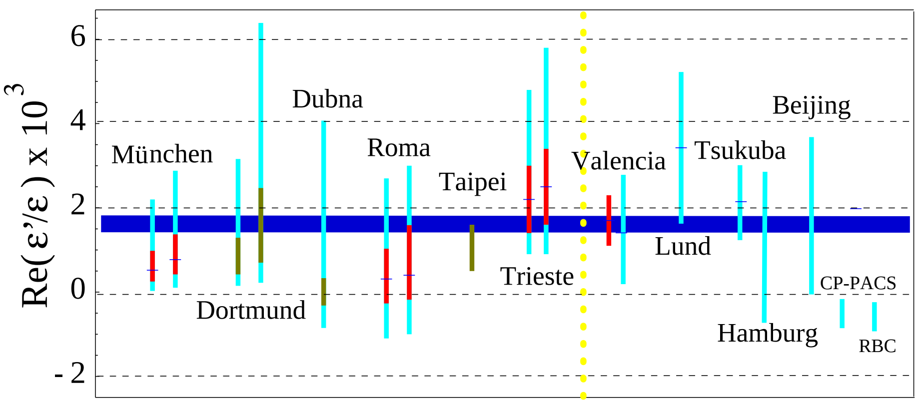

Figure 3: Recent Standard Model predictions for $\mathrm{Re}(\epsilon'/\epsilon)$ by different groups along the years. The vertical dotted line indicates the time at which the results from the latest generation of experiments become available; the horizontal band is the experimental world average. Where darker error bars are shown, they refer to independent "Gaussian" variation of the input parameters, while the larger ones correspond to the full "scanning" of the parameter space.

It should be clear, therefore, that the smallness of ϵ'/ϵ is to a large extent accidental, and as such it does not preclude the fact that direct CP violating effects can be larger, and even dominant, in other kinds of processes.

It should be noticed that the phase of ϵ' is quite close to that of ϵ ($\simeq \phi_{SW}$). This accidental similarity of phases (which would not be valid if the short-lived neutral kaon were heavier than the long-lived one, instead of the opposite) implies that for small direct CP violation ($|\epsilon'| \ll |\epsilon|$), the CPT-enforced equality of the phases ϕ_{00} and ϕ_{+-} with ϕ_{SW} is already well constrained. From the above discussion it should be clear that, in first approximation, the measurement of the single real number $\mathrm{Re}(\epsilon'/\epsilon)$ is sufficient to get information on direct CP violation, while a significant non-zero value of $\mathrm{Im}(\epsilon'/\epsilon)$ would be a signal of CPT violation.

Small violations of Bose statistics, not experimentally excluded otherwise, have been discussed [48] as possible contributions to the difference among η_{+-}

and η_{00}: by allowing a small component of the isospin state $I = 1$ in $\pi^+\pi^-$ with zero relative angular momentum, one can have a CP-conserving contribution to $K_L \to \pi^+\pi^-$. Therefore, the measured difference of CP violation in the two $\pi\pi$ decay channels, usually ascribed to direct CP violation ($\epsilon' \neq 0$) can be used to give an upper bound to the probability β_B^2 of wrong-statistics admixture, in the limit in which no direct CP violation is present, namely

$$\beta_B^2 \leq 2.7 \cdot 10^{-6} \tag{67}$$

3.2 Experiments on $K \to \pi\pi$ decays

If the only mechanism for CP violation would be the super-weak one, inducing an asymmetry in $K^0 - -\overline{K}^0$ mixing, all $\pi\pi$ decays of neutral kaons would be CP-conserving decays of the K_1 component, both for K_S and K_L: in this case all their properties, such as the ratio of $\pi^+\pi^-$ ("charged") to $\pi^0\pi^0$ ("neutral") decays, should be the same for both physical states. In other words, if CP violation only manifests itself as a "constituent" property of the decaying meson itself, it should appear to be the same through any kind of decay process; considering the $\pi\pi$ decays in which CP violation was originally found, the CP-violating amplitude ratios η_{+-} and η_{00} should be exactly equal in this case. The ratio of these two CP-violating amplitude ratios is thus a quantity sensitive to the presence of (direct) CP violation in the decay amplitudes.

The squared modulus of the ratio of the above mentioned CP-violating parameters is an experimentally accessible quantity, being the double ratio R of partial decay amplitudes

$$R \equiv \frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)} \frac{\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^+\pi^-)} = \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 \tag{68}$$

Experiments measuring R are basically counting experiments, which must accurately measure the number of $\pi\pi$ events from K_L and K_S, within regions of same volume in phase space, which for such 2-body decays means kaon momentum, decay position, and centre of mass angular coordinates.

Given the approximate validity of the $\Delta I = 1/2$ "rule" ($|\omega| \ll 1$), and the smallness of direct CP violation ($|\epsilon'| \ll 1$), the following approximate relation holds

$$\mathrm{Re}(\epsilon'/\epsilon) \simeq \frac{1}{6}(1 - R) \tag{69}$$

The extraction of ϵ'/ϵ from the comparison of η_{+-} or η_{00} with the mixing parameter ϵ, as obtained from the charge asymmetry δ_L in semi-leptonic K_L decays, is not precise enough to draw any conclusion on direct CP violation: neglecting $|\omega|$ one has

$$\mathrm{Re}(\epsilon'/\epsilon) \simeq \left|\tfrac{\epsilon'}{\epsilon}\right| \simeq \tfrac{2|\eta_{+-}|\cos(\phi_{+-})}{\delta_L} - 1 = (17 \pm 39) \cdot 10^{-3} \tag{70}$$

$$\mathrm{Re}(\epsilon'/\epsilon) \simeq \left|\tfrac{\epsilon'}{\epsilon}\right| \simeq \tfrac{1}{2} - \tfrac{|\eta_{00}|\cos(\phi_{00})}{\delta_L} = (11 \pm 36) \cdot 10^{-3} \tag{71}$$

while assuming CPT symmetry (as well as $\phi(\epsilon) = \phi_{SW}$) and $|\epsilon'|^2 \ll 1$ one has also

$$\mathrm{Re}(\epsilon'/\epsilon) \simeq \tfrac{1}{2}\left[\tfrac{4|\eta_{+-}|^2}{\delta_L^2(1+\tan^2\phi_{SW})} - 1\right] = (15 \pm 39) \cdot 10^{-3} \tag{72}$$

$$\mathrm{Re}(\epsilon'/\epsilon) \simeq \tfrac{1}{4}\left[1 - \tfrac{4|\eta_{00}|^2}{\delta_L^2(1+\tan^2\phi_{SW})}\right] = (5 \pm 30) \cdot 10^{-3} \tag{73}$$

From an experimental point of view, initially the main difficulty in a measurement of the double ratio R was the determination of $\Gamma(K_L \to \pi^0\pi^0)$ with sufficient precision. The difficulties of measuring accurately the energy and direction of the final-state photons, and the presence of the severe background due to the CP-conserving $K_L \to 3\pi^0$ decays, 200 times more frequent, made such measurements quite challenging.

In experiments in which the K_S flux is not a limiting factor, the statistical error on the double ratio is usually limited by the number N_{00}^L of collected $K_L \to \pi^0\pi^0$ decays, and is roughly given by $1.2/\sqrt{N_{00}^L}$ if the experimental acceptances for $\pi^+\pi^-$ are comparable to the the ones for $\pi^0\pi^0$. Intense kaon beams are therefore a primary requirement for any experiment which aims at measuring R with high precision.

K_L decays are readily obtained from a neutral beam produced at a sufficiently large distance from the region of observation: such beams usually contain also large amounts of neutrons and photons from π^0 decays.

One way of producing K_S decays is that of exploiting the phenomenon of kaon regeneration in matter (see *e.g.* [28]). Early experiments used regenerators placed on neutral (K_L) beams to produce K_S. The proper decay time t distribution into a final state f with $CP = +1$ can be described in terms of the magnitude and phase of the CP-violating amplitude ratio η_f and of the regeneration amplitude ρ:

$$I_f(t) \propto |\rho|^2 e^{-\Gamma_S t} + 2|\rho||\eta_f| \cos(\Delta m\, t - \phi_f + \phi_\rho) e^{-(\Gamma_S+\Gamma_L)t/2} + |\eta_f|^2 e^{-\Gamma_L t} \quad (74)$$

where $f = \pi^+\pi^-$ or $\pi^0\pi^0$, and ϕ_ρ is the phase of the regeneration amplitude ρ (which can be measured in dedicated experiments); the expression exhibits the oscillating term due to the interference of K_S and K_L decay amplitudes.

It is evident from the above expression that even with regeneration amplitudes typically below 10%, most of the $K \to \pi\pi$ decay rate downstream of the regenerator is due to the K_S component. A major concern with this technique is due to the incoherently regenerated K_S events which, being produced at a finite angle, can be measured in the detector with different acceptance relative to K_L; the fraction of such events has to be carefully estimated, or measured, and then subtracted.

Without making use of a regenerator, a neutral beam containing both K_S and K_L is readily available close to the production target; thanks to CPT symmetry, K_S are produced in the same amount as K_L, and the proper-time distribution of decays to a given $\pi\pi$ state f is

$$I_f(t) \propto e^{-\Gamma_S t} + 2D(p_K)|\eta_f| \cos(\Delta m t - \phi_f) e^{-(\Gamma_S+\Gamma_L)t/2} + |\eta_f|^2 e^{-\Gamma_L t} \quad (75)$$

In the above formula the "dilution factor" $D(p_K)$ is defined as the ratio of (incoherent) K^0 and $\overline{K}^0$ production at the target, for a given value of the kaon momentum p_K

$$D(p_K) \equiv \frac{N_{K^0}(p_K) - N_{\overline{K}^0}(p_K)}{N_{K^0}(p_K) + N_{\overline{K}^0}(p_K)} \quad (76)$$

which depends on the details of the interaction (targeting angle, etc.) and can also be fitted from the $I_f(t)$ distribution for either decay mode.

Due to the large K_L/K_S lifetime ratio and $|\eta_f| \ll 1$, the $\pi\pi$ decays occurring in a region close to the target ($t \lesssim$ a few τ_S) are largely dominated by the K_S

component. A practical concern in this case, in which one has to collect decays close to the production target, is the required shielding of the detector from the high rate of particles generated in the collision. On the other hand, not using a regenerator frees from the need of knowing (or fitting) the regeneration amplitude in order to perform a measurement of the decay parameters.

Pure K_S beams can only be obtained by exploiting the correlated production of $K_S K_L$ pairs, as is done in e^+e^- collider experiments working at the energy of the ϕ resonance, using a detected K_L decay in the opposite hemisphere to "tag" a K_S, as discussed in section 3.5.

Instead of working with kaon mass eigenstates, associate production of neutral kaons such as the study of the proton-antiproton annihilation reactions $p\overline{p} \to \pi^+ K^- K^0$ and $p\overline{p} \to \pi^- K^+ \overline{K}^0$ allows to compare asymmetries as a function of time in decays of neutral kaons tagged to be initially in a strangeness eigenstate, as discussed in section 3.6.

The early results suggested that CP violation in $\pi^0\pi^0$ decays was about twice as large than in $\pi^+\pi^-$ decays, and therefore ϵ' was a large number. Improved experiments however showed that $|\eta_{00}|$ was close to $|\eta_{+-}|$, and when ϕ_{00} was measured in interference experiments and shown to be comparable to ϕ_{+-}, the smallness of ϵ' was proved.

The results of the first two experiments specifically designed to detect direct CP violation by comparing $\pi\pi$ decays in a pure K_L beam with those in a beam where the great majority of such decays are due to the K_S component were reported in 1972. Despite exploiting the best experimental techniques then available, both could collect only a few dozen $K_L \to \pi^0\pi^0$ decays. Given this basic statistical limitation, only an upper limit for ϵ'/ϵ of historical interest could be inferred, but the methods employed were relevant for the subsequent developments.

With no hint to the size of the effect, no measurements of direct CP violation were performed for some time. With the advent of the Cabibbo-Kobayashi-Maskawa model for weak interactions, definite predictions became available ($\epsilon'/\epsilon \sim 1/450$ [49] or even $10^{-1} \div 10^{-2}$ [50]) and an intense experimental activity started again in the '80s.

3.3 First experiments on direct CP violation in $K \to \pi\pi$ decays

BNL-AGS experiment (1972)

The measurement of the Princeton group at the Brookhaven AGS [51] used a 20 cm thick movable uranium regenerator to measure the $K_S \to \pi\pi$ decay rates; the regeneration amplitude cancels in forming the double ratio R. The experimental setup was switched between detection of charged and neutral decay modes, so that all four decay rates entering the double ratio were eventually measured, for kaon momenta in the 3-10 GeV/c range. For neutral events, at least one of the four photons from $\pi^0\pi^0$ decays was required to convert to an e^+e^- pair and be measured in a spark chamber magnetic spectrometer; the other 3 photons were detected, and their impact positions were measured, in a large lead plate chamber. The (subtracted) background fraction in the neutral mode, due to $K_L \to 3\pi^0$ decays was at the level of 2.5%. For charged decays a Cerenkov counter and a muon veto were also used, in order to reject semi-leptonic K_L decays.

The optimal thickness of the photon converter (~ 0.1 radiation lengths) necessary for triggering and decay vertex determination, coupled with the limited solid angle through the spectrometer magnet, limited in an important way the acceptance for $\pi^0\pi^0$ decays. Regenerator (K_S) events were weighted as a function of the regenerator position along the beam, in order to get a decay-point distribution similar to the one due to K_L. Incoherently regenerated K_S were statistically subtracted; their fraction was directly measured by the transverse momentum distribution for charged decays, and used for neutral events after estimating the neutral/charged acceptance ratio by Monte Carlo simulation. K_L flux normalisation was performed using beam target activity monitors.

With a sample of 124 $K_L \to \pi^0\pi^0$ decays, the result was $\mathrm{Re}(\epsilon'/\epsilon) = (-10 \pm 24) \cdot 10^{-3}$.

CERN-PS experiment (1972)

The experiment of the Aachen-CERN-Torino group at the CERN PS [52] also alternated movable-regenerator runs and "vacuum" runs (no regenerator on the K_L beam) to measure the most challenging $K_{S,L} \to \pi^0\pi^0$ decays with a non-magnetic detector based on a lead-glass calorimeter, with kaons in the 2-6 GeV/c momentum range. As for the BNL experiment, data was collected with the regenerator placed at several different longitudinal positions, in order to match the decay vertex distribution of K_L along the 4 m decay volume. By requiring at least two photons to convert in thin lead foils before entering the detector and be measured in spark chambers, the directions of all four photons hitting the calorimeter could be known. In this way the resolution on the reconstructed $\gamma\gamma$ invariant mass was about 10 MeV/c^2, and the subtracted $K_L \to 3\pi^0$ background was kept below 2%.

With this method, the diffractively regenerated K_S component, limited by the destructive interference with the coherently regenerated one due to the choice of regenerator thickness, could be directly measured from the angular distribution in the data, and was found to be around 11%. Flux normalisation was performed by using $K_L \to 3\pi^0$ decays, mostly unaffected by regeneration, from which however a large ($\simeq 33\%$) diffracted component had to be subtracted.

With 167 $K_L \to \pi^0\pi^0$ decays, and using independent measurements of $|\eta_{+-}/\rho|^2$ obtained with a different detector placed on the same beam, the result was $\mathrm{Re}(\epsilon'/\epsilon) = (0 \pm 20) \cdot 10^{-3}$. The largest systematic error was actually due to the error on the knowledge of the regeneration amplitude, other important contributions being due to the knowledge of the regenerator position and the K_L scattering subtraction.

Second BNL-AGS experiment (1979)

A second experiment at the AGS by a New York group [53] exploited the detection of both charged and neutral $\pi\pi$ decays from a beam of neutral kaons of 8-18 GeV/c momentum, using a detector arrangement which could be moved along the beam direction, placed at short distance (≈ 7 m) from the production target.

The good proper decay time resolution of $\simeq 0.4\tau_S$ achieved also for neutral decays, allowed the measurement of the $K_S - -K_L$ interference in both the $\pi^0\pi^0$ and $\pi^+\pi^-$ modes, in the proper decay time intervals 4-11 τ_S (for neutral) and 4-18 τ_S (for charged). From the fit of the $\pi\pi$ decay distributions, using the known value of Δm as input, both the moduli and phases of η_{+-} and η_{00} were measured (as well as the dilution factor).

Charged and neutral decays were collected with alternate detector settings: a magnetic spectrometer based on proportional wire chambers providing a 9 MeV/c^2 kaon mass resolution for the former, and a lead-glass calorimeter providing a 21 MeV/c^2 π^0 mass resolution for the latter. The detection of $\pi^0\pi^0$ decays required at least two of the decay photons to be converted to e^+e^- in a set of three lead sheets, for triggering and measurement purposes.

Backgrounds from 3-body K_L decays were important, of the order of 10% for the neutral mode and even larger for the charged one; backgrounds from coherent regeneration and diffractive scattering in the photon converter and collimators also had to be taken into account.

The comparison of the amplitude ratios for the two $\pi\pi$ decay modes from the same experiment was free from most systematic errors at the level of accuracy allowed by the statistical sample. Measurements performed at two longitudinal positions of the detector, shifted by 1 m, were combined after correcting for the acceptance dependence; their consistency was used as a systematic check.

The result on direct CP violation was $\mathrm{Re}(\epsilon'/\epsilon) = (0 \pm 30) \cdot 10^{-3}$, based on 85000 $K_{L,S} \to \pi^0\pi^0$ decays.

BNL E749 (1985)

In 1979 an interesting proposal for the measurement of direct CP violation at the AGS with a different experimental approach was submitted to BNL by a Yale-Brookhaven group [54]. Although the experiment was not actually performed with the approach initially discussed, it is interesting to consider its proposed scheme.

The idea is to search for a difference in the charge ratio ($\pi^0\pi^0$ to $\pi^+\pi^-$) for $\pi\pi$ decays of pure K_L and K_S, induced by the direct CP violating $K_2 \to \pi\pi$ decay. The dominance of CP violation induced by the $K^0 - -\overline{K}^0$ mixing implies that most of the $\pi\pi$ decays, both from K_L and K_S are due to their K_1 components, while a possible small fraction of decays of the K_2 component could have different properties (such as the charge ratio).

Considering decays after a regenerator placed on a K_L beam, the $\pi\pi$ decay amplitude from coherently regenerated K_S at any given point is coherent with the ones due to K_L (through K_1 or K_2), their phase difference varying linearly with the distance of the decay point from the regeneration point due to the K_L-K_S mass difference (see *e.g.* [13]); the ratio of regenerated K_S to transmitted K_L amplitudes at the exit face of the regenerator of thickness L is:

$$\frac{\rho}{A_L} = \frac{2\pi n}{m_K(\lambda_S - \lambda_L)} \frac{f(0) - \overline{f}(0)}{2} [1 - e^{-i(\lambda_S - \lambda_L)L/\beta\gamma}] \qquad (77)$$

where n is the density of scattering centres, $f(0)$ and $\overline{f}(0)$ are the forward scattering amplitudes for K^0 and $\overline{K}^0$ respectively, and β, γ the relativistic factors for the kaon. The corresponding $\pi^+\pi^-$ decay intensity in vacuum as a function of proper time t (with $t = 0$ at the exit face of the regenerator) is therefore

$$R(\pi^+\pi^-;t) \propto \left[\left|\frac{\rho}{A_L}\right|^2 e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} + 2\left|\frac{\rho}{A_L}\right| |\eta_{+-}| \cos(\Delta m t - \phi_{+-} + \phi_\rho) e^{-(\Gamma_S+\Gamma_L)t/2}\right] \quad (78)$$

where the regeneration phase, containing a term due to the regenerator thickness

$$\phi_\rho = \arg[i(f(0) - \overline{f}(0))] + \arg[(1 - e^{-i\Delta\lambda L/\beta\gamma})/i\Delta\lambda] \quad (79)$$

can be experimentally determined.

In presence of a direct CP violating amplitude ($K_2 \to \pi\pi$), the charge ratio of $\pi\pi$ decays could be expected to vary along the beam. With an appropriate choice of regenerator, in some proper decay time interval there could be significant destructive interference between the regenerated K_S ($\approx K_1$) and (indirect CP-violating) K_1 amplitudes from K_L; in this region any effect on the charge ratio due to a direct CP-violating $K_2 \to \pi\pi$ decay would be strongly put in evidence. Given the fact that for K_1 decays the $\pi^0\pi^0/\pi^+\pi^-$ ratio is $\simeq 1/2$ (due to the dominance of the isospin $I = 0$ component in the final $\pi\pi$ state), while for K_2 it would be expected to be $\simeq 2$ ($\pi\pi$ in the $I = 2$ isospin state being required), and that any change in the charge ratio could only be due to direct CP violation, this proposal represented an appealing strategy for its detection. Unfortunately, knowing that $K_L \to \pi\pi$ decays proceed dominantly through indirect CP violation, the same decay time interval under consideration is also the one in which the number of expected $\pi\pi$ events will be smallest.

The plan was to make $\pi\pi$ yield measurements with different settings of a (thick) fixed and a (thin) movable regenerator at a distance ~ 6 m from the production target, in order to measure the charge ratio for pure K_L, almost pure K_S, and a coherent mixture in which the indirect CP-violating component would be made small by the above discussed interference. The goal was to measure the charge ratios with an accuracy better than 1%, potentially reaching a sensitivity of $\simeq 1.6 \cdot 10^{-3}$ on $|\epsilon'/\epsilon|$.

The experiment was performed at BNL [55], with a magnetic detector based on proportional wire chambers and a lead-glass calorimeter which could measure simultaneously both charged and neutral decays, requiring a converted photon for the latter. Anti-coincidence counters and muon veto counters helped in reducing the 3-body background from K_L decays. The detector was placed at ~ 10 m from the target. An 80 cm thick graphite regenerator was placed on the neutral beam at short regular intervals, to alternate the collection of K_L and K_S decays in the 1.2 m long fiducial region, for a 7-14 GeV/c kaon momentum range.

Residual backgrounds for the neutral mode were at the level of 1.5% from incoherent K_S regeneration, and 17.5% from K_L 3-body background, and much smaller for charged decays. The $K_L - K_S$ acceptance difference cancellation was achieved by analyzing the data in bins of kaon energy and longitudinal decay position, which was determined with good accuracy also for neutral decays by extrapolating the converted photon trajectory to its intersection with the narrow neutral beam.

With 1122 $K_L \to \pi^0\pi^0$ decays, a result of $\mathrm{Re}(\epsilon'/\epsilon) = (1.7 \pm 7.2 \pm 4.3) \cdot 10^{-3}$ was obtained with the double ratio method, the first error quoted being statistical and the second systematic, dominated by the uncertainty in the K_L background subtraction.

FNAL E617 experiment (1985)

About at the same time of the BNL E749 experiment, the Chicago-Saclay experiment E617 at FNAL was performed [56], using a double-beam technique to reduce the systematic uncertainties linked to the separate data-collection of K_L and K_S decays. In this approach, two identical neutral beams, produced by 800 GeV/c protons impinging on a single target, entered the 13 m long evacuated decay volume side-by-side. A thick regenerator at fixed longitudinal position was set on one of them, alternating from one beam to the other at each accelerator pulse, in order to provide K_S decays while cancelling the effect of any left-right asymmetry of either the beam line or the detection apparatus. K_L and K_S decays were distinguished on the basis of their reconstructed transverse coordinates at the regenerator plane.

The detector setup was switched between charged and neutral mode by adding or removing a lead photon converter: for $\pi^0\pi^0$ decays the direction of at least one converted photon was required to be measured in the drift-chamber magnetic spectrometer, thus allowing the measurement of the kaon transverse momentum with respect to the beam axis, so that inelastically regenerated events could be identified and removed. A lead-glass calorimeter was used for the measurements of the photon energies and impact point positions, leading to a 6.5 MeV/c^2 kaon mass resolution; the longitudinal decay point position was determined as the weighted average of the two (consistent) ones obtained by imposing the π^0 mass constraint to $\gamma\gamma$ pairs.

Muon veto counters were used to suppress $K_{\mu 3}$ decays. The subtracted backgrounds due to inelastically regenerated K_S were at the level of 13% for neutral and 2% for charged decays, while backgrounds from K_L 3-body decays were at the 8% and 3% level respectively; the uncertainties on these background subtractions were the largest sources of systematic errors.

The irreducible difference in the detector acceptance for K_L and K_S decays (of order 10%) arising from the different lifetimes was corrected by Monte Carlo simulation. The value of ϵ' was extracted from a fit of the ratios of regenerated to vacuum events in kaon momentum bins for each of the four modes, from which the magnitude of the coherent regeneration amplitude, assumed to follow a power-law behaviour as a function of kaon momentum, was also determined; the phase of the regeneration amplitude was used as an input to the fit.

With 3152 $K_L \to \pi^0\pi^0$ events the result was $\mathrm{Re}(\epsilon'/\epsilon) = (-4.6 \pm 5.3 \pm 2.4) \cdot 10^{-3}$, the first error being statistical and the second systematic.

3.4 Recent experiments on $\mathrm{Re}(\epsilon'/\epsilon)$

E731 at FNAL

The E731 experiment, performed at FNAL at the end of the '80s, adopted the double-beam technique as its predecessor E617, with a different detector.

After a short test run in 1985 (E731A), from which $\mathrm{Re}(\epsilon'/\epsilon) = (3.2 \pm 2.8 \pm 1.2) \cdot 10^{-3}$ was obtained [57], the experiment was extensively upgraded. The original setup required at least one of the photons from the $\pi^0\pi^0$ decay to convert in a thin lead sheet, which was added in the middle of the 40 m long evacuated decay region during neutral mode running, in order to obtain the decay vertex position by tracking the e^+e^- pair. The upgraded detector allowed running without this requirement, therefore collecting a much higher statistics for the neutral mode. Most of the data-taking period, in 1987 and 1988, alternated collection of $\pi^+\pi^-$ and $\pi^0\pi^0$ decays, with different proton beam intensities; in the last 20% of the data-taking period all four decay modes were collected simultaneously, further reducing the sensitivity to differential biases.

The regenerator and an upstream absorber were placed alternately on one of the two neutral beams at every accelerator pulse. A lead-glass calorimeter was used to detect $\pi^0\pi^0$ decays, while rejecting the $K_L \to 3\pi^0$ background down to 1.8% and, together with the magnetic spectrometer used for $\pi^+\pi^-$ measurement, to suppress K_{e3} background down to the 0.3% level in the vacuum beam. The $K_{\mu 3}$ background was rejected by muon veto counters.

Events were assigned to the vacuum or the regenerator beam according to the extrapolated kaon direction (for $\pi^+\pi^-$ decays) or using the energy-weighted average photon impact position (for $\pi^0\pi^0$ decays). This procedure is affected by background due to incoherently regenerated K_S, which are produced at finite angle: veto counters in the regenerator helped in suppressing events resulting from inelastic reactions. Such background was at the 0.2% level for $\pi^+\pi^-$ decays from the regenerator beam, measured by extrapolating the kaon squared transverse momentum (p_T^2) distribution. For neutral decays this background was at the 2-3% level in the regenerator as well as in the vacuum beam (due to large-angle K_S scattering), and was subtracted by modelling the distributions in Monte Carlo using the measured p_T^2 spectra from charged decays as input.

To maximise the $K_L \to \pi^0\pi^0$ statistics, the fiducial region for the neutral decay mode was longer than the one for charged decays. In order to better control the large acceptance correction induced by the different lifetimes, all the limiting geometrical apertures were defined by active veto elements; the correction was based on accurate Monte Carlo simulation of the apparatus, checked against large data samples of $K_L \to 3\pi^0$ and K_{e3} decays.

The direct CP violation parameter was extracted by a global fit (performed in kaon momentum bins in the 40-160 GeV/c range) of the ratio of $\pi\pi$ events in the two beams, corrected for acceptance; the longitudinal vertex range used was 110-137 m from the target for the charged mode and 110-152 m from the target for the neutral mode. In the fit, the regeneration amplitude at a reference kaon momentum, and the exponent of its assumed power-law dependence from such momentum were left as free parameters, as well as a parameter describing the energy dependence of the kaon absorption in the absorber and regenerator. CPT symmetry was assumed in the fit, the phase of ϵ' was set to the value given by the $\pi\pi$ phase shift analysis, while Δm and τ_S were set at the values measured by the same experiment. Averaging the results of each bin, the result was much less sensitive to the energy dependence of the absorption cross section.

The result on direct CP violation [58] with the total sample of 410000 $K_L \to \pi^0\pi^0$ and 329000 $K_L \to \pi^+\pi^-$ decays was $\mathrm{Re}(\epsilon'/\epsilon) = (0.74 \pm 0.52 \pm 0.29) \cdot 10^{-3}$ (the first error being statistical and the second systematic), consistent with no direct CP violation within the errors.

The dominant systematic errors were due to the imperfect knowledge of the electromagnetic calorimeter response as a function of energy, accidental activity effects in the detector and the knowledge of the acceptance correction; a thorough discussion of the E731 experiment can be found in [59].

NA31 at CERN

The NA31 experiment, performed at CERN at the same time as the E731 experiment, detected simultaneously $\pi^0\pi^0$ and $\pi^+\pi^-$ decays originating from two alternating neutral beams produced by 450 GeV/c protons impinging onto targets located at different longitudinal positions. The K_L beam was produced by an intense proton beam on a target placed at ≈ 240 m from the detector, while the K_S beam was produced by an attenuated (by a factor $\sim 3 \cdot 10^{-4}$) proton beam impinging on a similar target assembly, mounted on a movable support, which could be positioned at a distance $\approx$ 80-130 m from the detector, inside the evacuated decay region. The two neutral beams were collinear and contained within an evacuated beam pipe passing through a central hole in the detectors, in order to reduce K_L elastic scattering, K_S regeneration and neutron interactions to negligible levels.

The non-magnetic detector was based on two large four-plane drift chambers and a liquid-argon/lead sandwich calorimeter, read-out in sets of transverse (x and y) strips. This was followed by an iron-plastic scintillator hadronic calorimeter. Plastic scintillator hodoscopes were used for triggering. Veto and muon counters helped reducing background from K_L semi-leptonic decays.

Data were collected alternating the K_L and K_S beams. At the average kaon momentum of 100 GeV/c the K_L decay distribution is essentially flat ($\beta\gamma c\tau_L \simeq$ 3 km); by moving the K_S target along the 50 m long decay region, the effective longitudinal decay vertex distribution for K_S ($\beta\gamma c\tau_S \simeq 5$ m) was made similar to that of K_L, therefore making the relative acceptance correction, performed in bins of kaon energy and decay position, very small in the analysis.

The subtracted background, only significant for K_L, was due to three-body K^0 decays, mainly in the neutral channel where it was below 3%. With no magnetic analysis, a significant fraction (about 40%) of good $\pi^+\pi^-$ events were eliminated by the cuts used to reject the main K_{e3} background, based on the energy deposition in the calorimeters; in the later data-taking runs a transition radiation detector was added to the setup, strongly reducing the K_{e3} background with a large gain in efficiency for the $\pi^+\pi^-$ mode. Event losses due to activity in the detector uncorrelated with the kaon decay ("accidentals"), at the level of a few percent, were made similar for the two decay modes.

The experiment took data in 1987, 1988 and 1989, and its result on direct CP violation [60] with the total sample of 428000 $K_L \to \pi^0\pi^0$ decays was $\mathrm{Re}(\epsilon'/\epsilon) = (2.30 \pm 0.65) \cdot 10^{-3}$, indicating the existence of direct CP violation. The dominant systematic errors were due to the understanding of biases related to accidental activity in the detector, K_L background subtraction in the neutral mode, and the knowledge of the absolute energy scale and its stability for neutral decays.

KTeV at FNAL

The final results of E731 and NA31 were not in good agreement, their probability to be consistent being 7.7%, and left the fundamental question of the existence of direct CP violation not confirmed: while NA31 had a 3.5 standard deviation signal, the world average $\mathrm{Re}(\epsilon'/\epsilon) = (14.5 \pm 7.8) \cdot 10^{-3}$ was only 1.9 standard deviations from zero, after inflating the error to take the disagreement ($\chi^2 = 3.2$) into account according to the PDG recipe [36]. This situation stimulated both the FNAL and the CERN groups to design new and higher precision experiments, in order to clarify the matter, aiming at a reduction of the experimental errors by a factor ≈ 3.

The KTeV program at FNAL comprises the E832 experiment for the search of direct CP violation in neutral K decays, and the E799 experiment for studies of CP violation in rare K_L decays, sharing most of the experimental set-up. The E832 experiment uses the double-beam technique of its predecessors, with several important improvements. All four decay modes are collected at the same time, therefore allowing a better control of several systematic effects. The uncertainties related to inelastic scattering and regeneration are reduced by using an active lead-scintillator regenerator, which allows the vetoing of any event with an energy release detected in the device.

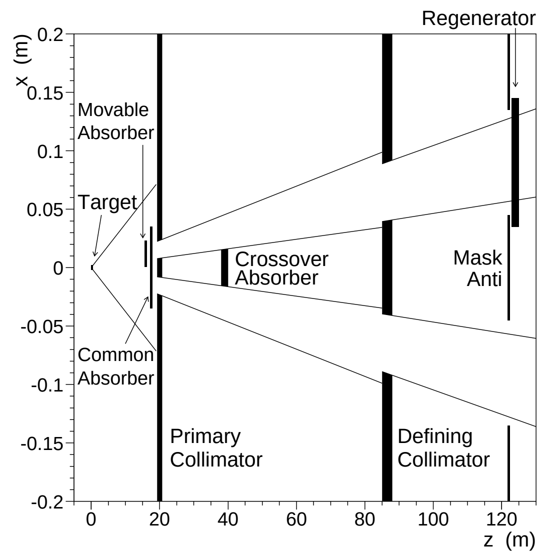

Figure 4: Scheme of the KTeV double beam arrangement.

As in previous experiments, both the 2 hadronic interaction lengths thick regenerator and an upstream absorber are moved alternately on either of the twin beams at each accelerator pulse, therefore resulting in an effective cancellation of any bias related to spatial detector asymmetries (see fig. 4). The twin neutral beams are produced by $3 \cdot 10^{12}$ 800 GeV/c protons in a 20 s spill every minute, providing a 0.9 MHz kaon flux in front of the regenerator. The average magnitude of the regeneration amplitude is 3%, and in the regenerator beam the K_L-related component accounts for 20% of the decay rate; in the "vacuum" beam K_L decays dominate, less than 1% being due to the residual K_S component.

The detector, placed at $\sim$ 160 m from the target (see fig. 5), has as main components a magnetic spectrometer with a 410 MeV/c kick based on proportional wire chambers and a high-performance pure CsI crystal electromagnetic calorimeter with excellent energy resolution and small non-Gaussian resolution tails, which allows a very accurate measurement of $\pi^0\pi^0$ decays.

Decays of diffractively regenerated K_S ($\sim 1\%$ of the coherently regenerated ones) are reduced in the data by kinematic cuts in the analysis, while the dominant inelastically regenerated component (a factor 100 larger than the coherent one) is reduced by vetoing on activity in the active regenerator.

Backgrounds in the vacuum beam include misidentified 3-body K decays, at the level of 0.09% in the $\pi^+\pi^-$ mode and 0.11% in the $\pi^0\pi^0$ mode. Scattering in the regenerator (and in the collimators, one order of magnitude less) are the major sources of background in the regenerator beam, at the level of 0.08% in the $\pi^+\pi^-$ mode and 1.2% in the $\pi^0\pi^0$ mode, but are also the largest background source ($\sim 0.4\%$) in the vacuum beam for the neutral mode, where kaons from the regenerator beam scattered at large angles can be reconstructed in the vacuum beam, leading to a counting error in the double ratio.

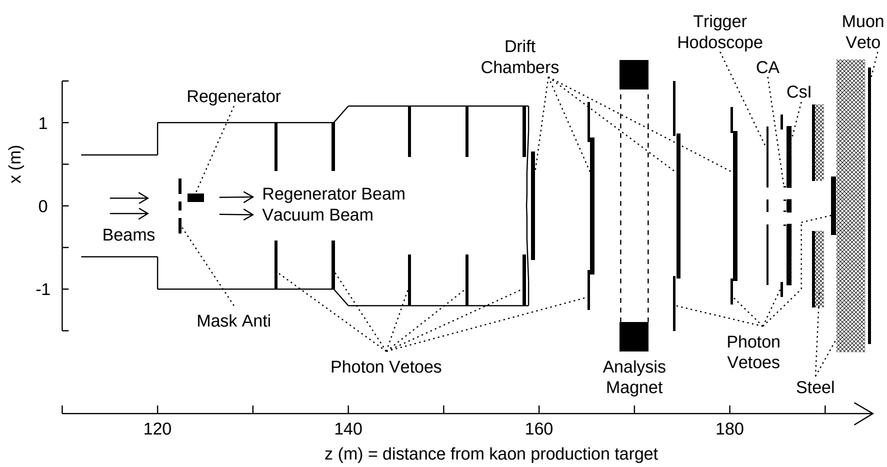

Figure 5: Scheme of the KTeV detector.

Kaons in the momentum range 40 to 160 GeV/c, decaying in an evacuated region between 110 and 158 m from the target, are used in the analysis, which is performed in 10 GeV/c wide kaon momentum bins, in order to reduce the sensitivity to detector acceptance effects and to the knowledge of the spectra, while also allowing to account for the momentum dependence of the regeneration amplitude. No binning is used in the longitudinal decay position.

The result is extracted from a fit of the event yields in the four modes entering the double ratio, using acceptance functions determined by Monte Carlo simulation; the acceptance correction induces a shift of $\sim 85 \cdot 10^{-4}$ on $\mathrm{Re}(\epsilon'/\epsilon)$, mostly (85%) due to geometry. Only 5% of the collected events are actually used for the measurement, most of them being instead checked to understand the detector acceptance and efficiency, and to model them in the simulation.

Other factors entering the fit are the momentum-dependent part of the vacuum-to-regenerator kaon flux ratio (measured from $K_L \to \pi^+\pi^-\pi^0$ decays), the kaon lifetimes and mass difference (fixed), the phases of the CP-violating amplitude ratios ϕ_{+-} and ϕ_{00} (also fixed to the super-weak value ϕ_{SW}, assuming CPT), and the regeneration amplitude, for which a power-law dependence on kaon momentum is assumed, from which the phase is determined using theoretical input.

E832 collected data both in 1996-97 (3.3 million $K_L \to \pi^0\pi^0$ decays) and in 1999 (a sample of similar size). Results from the analysis of the 1996-97 sample are available [61] [41]:

$$\mathrm{Re}(\epsilon'/\epsilon) = (2.071 \pm 0.148_{\,\mathrm{stat}} \pm 0.239_{\,\mathrm{syst}}) \cdot 10^{-3} = (2.07 \pm 0.28) \cdot 10^{-3} \quad (80)$$

from the global fit with $\chi^2 = 27.6$ with 21 degrees of freedom.

The data was also analyzed with a weighting technique similar to the one used by the NA48 experiment (see below), which does not depend on an accurate Monte Carlo acceptance correction; with this approach the statistics is effectively reduced by a factor 1.7, and the obtained result is fully consistent with the one quoted above.

This measurement of direct CP violation by KTeV is fully consistent with the NA31 measurement, and at 2 standard deviations from the E731 one. The error appears to be dominated by systematics, although part of it can be reduced by including the 1999 data sample; the largest systematic uncertainties are due to the reconstruction and the knowledge of the background for the $\pi^0\pi^0$ channel. Improvements on these systematics are being pursued, to better exploit the $1 \cdot 10^{-4}$ statistical error which can be reached with the total sample.

NA48 at CERN

The NA48 experiment at CERN adopted a double-target approach as NA31, but with several important differences with respect to its predecessor, the most important being that all four decay modes were detected simultaneously. The neutral beam for K_S decays is obtained by a selecting a small fraction ($\sim 2{\cdot}10^{-5}$) of the primary 450 GeV/c protons, used to produce the K_L beam on a target at ≈ 240 m from the detector, channelling them in a single bent crystal and steering them on a secondary target close to the beginning of the fiducial decay region, at ≈ 120 m from the detector. Two quasi-collinear neutral beams, converging at a 0.6 mrad angle at the centre of the detector, are therefore simultaneously available, and after emerging from the evacuated decay region they cross the experimental apparatus always remaining within a thin vacuum pipe, in order to avoid interactions of the large neutron and photon flux (see fig. 6). The proton targeting angles are chosen in such a way that the spectra of decaying K_S and K_L are similar, and any small residual difference is irrelevant for the analysis performed in small kaon momentum bins.

The experiment collects all four decay modes simultaneously, and the identification of K_S and K_L decays is performed by using a time coincidence technique: an array of scintillator counters, placed on the 20 MHz secondary proton beam directed to the K_S target, allows the precise measurement of the time of passage of each single proton. By comparing such times with the event time provided by high-resolution detectors, events for which a proton is found in a 4 ns wide coincidence window are flagged as K_S. Such technique does not induce any asymmetry in first order, but only a small dilution effect (corrected for); small higher-order effects due to charged-neutral asymmetries in the time measurement tails or rate-induced effects were accurately measured.

The close target is not movable, thus allowing for a much improved collimation and shielding than in NA31; nevertheless, the acceptance correction due to

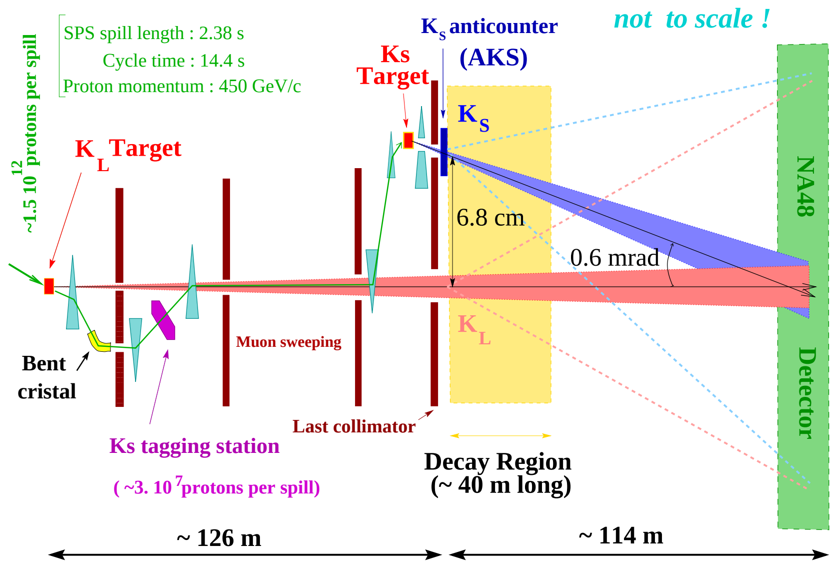

Figure 6: Scheme of the NA48 beam arrangement.

the different lifetime of K_S and K_L is made small also in NA48 by using only K_L decays in the region where also K_S decays are present, and by weighting offline the K_L events as a function of the decay proper time, to have very similar longitudinal decay distributions at the price of losing statistics in the smallest CP-violating sample of decays.

Another important difference with respect to NA31 is the presence of a magnetic spectrometer with four 4-view large drift chambers (see fig. 7), which allows a significant reduction in the backgrounds due to semi-leptonic K_L decays in the charged mode. $\pi^0\pi^0$ decays are measured by a quasi-homogeneous liquid Krypton electromagnetic calorimeter with projective tower structure, working as a ionization chamber, whose excellent energy, space and time resolutions are complemented by a very good stability, uniformity and ease of calibration.

Backgrounds are almost exclusively due to misidentified 3-body K_L decays, and are at the level of 0.06% for neutral and 0.2% for charged K_L decays, while they are negligible for K_S decays. A 0.1% contribution due to beam scattering is measured and subtracted from the $K_L \to \pi^0\pi^0$ sample.

The result is extracted by averaging the double ratio measured in 5 GeV/c wide kaon momentum bins, in the range 70-170 GeV/c, for decays in a longitudinal region corresponding to 3.5 τ_S in proper time.

NA48 collected data for the direct CP violation measurement in 1997 ($0.5 \cdot 10^6$ $K_L \to \pi^0\pi^0$ decays) [62], 1998, 1999 ($3 \cdot 10^6$) [63]. After the implosion of the vacuum pipe and the complete rebuilding of the damaged drift chambers, a new data-taking period in 2001 with a better duty cycle ($\times 1.8$) and reduced instantaneous intensity (-30%) was used to perform systematic cross-checks on rate-related effects and to complement the statistics ($1.5 \cdot 10^6$ $K_L \to \pi^0\pi^0$

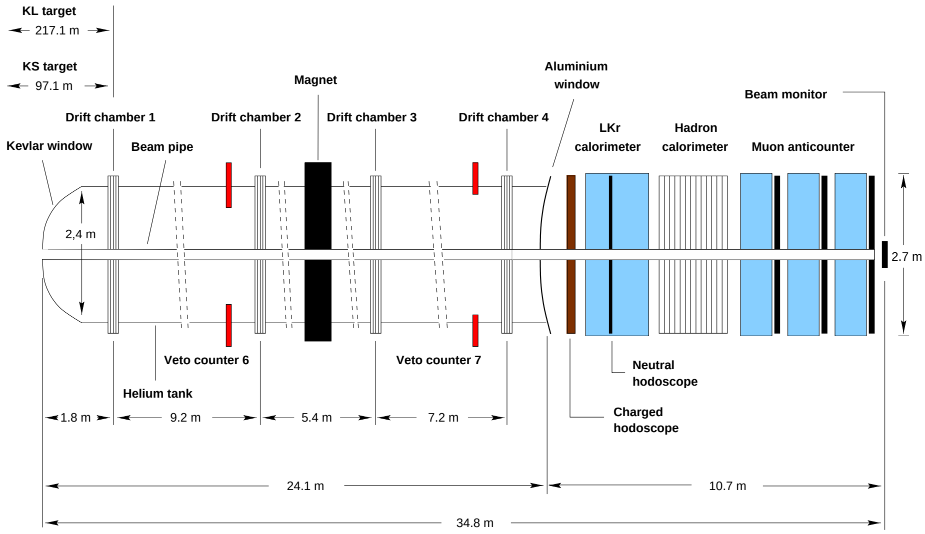

Figure 7: Scheme of the NA48 detector.

decays) [64]. The final combined result from all the collected data is [64]:

$$\mathrm{Re}(\epsilon'/\epsilon) = (1.47 \pm 0.14_{\mathrm{stat}} \pm 0.09_{\mathrm{stat/syst}} \pm 0.15_{\mathrm{syst}}) \cdot 10^{-3} = (1.47 \pm 0.22) \cdot 10^{-3} \tag{81}$$

where the first quoted error is purely statistical, the second is the one induced by the finite statistic of the control samples used to study systematic effects, and the third is purely systematic. The largest systematic uncertainties are due to the reconstruction of the $\pi^0\pi^0$ decays and the knowledge of the $\pi^+\pi^-$ trigger efficiency.

The measurement of direct CP violation by NA48 is well consistent with the NA31 and the KTeV results.

3.5 ϕ factories

A different technique for studying decays of K mesons, exploiting their production in correlated coherent pairs, was proposed a long time ago [65], and is actually being pursued in the KLOE experiment at the DAΦNE e^+e^- storage ring in Frascati and, with lower luminosity, at the VEPP-2M storage ring in Novosibirsk.

The ϕ meson, produced practically at rest, decays with $\sim 34\%$ probability in a $K_S K_L$ pair, providing an intense source of monochromatic back-to-back K_S and K_L beams (and K^+K^- pairs, with 49% branching ratio). By considering decays of neutral kaon pairs to $\pi\pi$, one can extract information on direct CP violation in a different way. Since the ϕ is a C-odd meson, C-conservation in its decay to pairs of neutral kaons, induced by strong interactions, implies that only $K_S K_L$ or $K^0\overline{K}^0$ states are allowed in its decay, independently from the validity of CP or CPT symmetries:

$$|\phi\rangle \to \frac{1}{\sqrt{2}}\left[|\overline{K}^0(\mathbf{p})K^0(-\mathbf{p})\rangle - |K^0(\mathbf{p})\overline{K}^0(-\mathbf{p})\rangle\right] =$$

$$\frac{1}{\sqrt{2}}\frac{1+|\epsilon|^2}{1-\epsilon^2}\left[|K_S(\mathbf{p})K_L(-\mathbf{p})\rangle - |K_L(\mathbf{p})K_S(-\mathbf{p})\rangle\right] \tag{82}$$

where $\mathbf{p}$ denotes the kaon momentum. This feature allows to tag either the mass eigenstate or the strangeness (at that time) of a decaying kaon, by detecting respectively a $\pi\pi$ decay close to the interaction point (at the level of validity of CP symmetry) or the lepton charge in a semi-leptonic decay (at the level of validity of the $\Delta S = \Delta Q$ rule) for its companion. Angular momentum conservation in the decay of the spin 1 ϕ forces the kaon pair to be in a p-wave orbital angular momentum state; Bose symmetry then implies that the two kaons cannot decay simultaneously to the same $\pi\pi$ final state, since two identical spinless bosonic systems cannot be in an antisymmetric state. Both the above features are independent from the validity of the CP and CPT symmetries.

Bose symmetry alone allows in principle to detect direct CP violation: if one of the two kaons decays to $\pi^+\pi^-$, the other one must be, at that time, that particular combination of K^0 and $\overline{K}^0$ which cannot decay to $\pi^+\pi^-$. In absence of direct CP violation this combination is just K_2, which cannot decay to $\pi^0\pi^0$ either. Therefore, an event in which one detects simultaneous decays to $\pi^+\pi^-$ and $\pi^0\pi^0$, while allowed by Bose symmetry, would be evidence for direct CP violation: the amplitude for simultaneous decay to $\pi^+\pi^-$ and $\pi^0\pi^0$ is actually proportional to $\eta_{+-} - \eta_{00}$. Experimental resolution will blur the definition of simultaneity, but the study of the decay distribution to the $\pi^+\pi^-\pi^0\pi^0$ state as a function of the decay time difference $\Delta t \equiv t_{+-} - t_{00}$, integrating on their sum $t_{+-} + t_{00}$, can provide information on direct CP violation [65] [66] [67]. Such distribution (neglecting terms of order $|\epsilon|^2$)

$$\begin{gathered} I(\pi^+\pi^-,\pi^0\pi^0;\Delta t) = \\ \int dt_{+-}\,dt_{00}\,\delta(t_{+-}-t_{00}-\Delta t)\left|A(\phi\to\pi^+\pi^-,\pi^0\pi^0;t_{+-},t_{00})\right|^2 \simeq \\ \frac{2\Gamma_S(\pi^+\pi^-)\Gamma_S(\pi^0\pi^0)}{\Gamma_S+\Gamma_L} e^{-(\Gamma_S+\Gamma_L)|\Delta t|/2}\left[|\eta_{+-}|^2 e^{\Delta\Gamma\Delta t/2} + \right. \\ \left. |\eta_{00}|^2 e^{-\Delta\Gamma\Delta t/2} - 2\mathrm{Re}\left(\eta^*_{+-}\eta_{00}e^{i\Delta m\Delta t}\right)\right] \end{gathered} \tag{83}$$

is roughly constant for Δt between about 10 and 100 τ_S, and is symmetric for $\Delta t \to -\Delta t$ if $\epsilon' = 0$. Indeed the intensity asymmetry, for small $|\epsilon'/\epsilon|$ and neglecting $|\omega|$, is given by

$$\begin{gathered} A_I(\pi^+\pi^-,\pi^0\pi^0;\Delta t) \equiv \frac{I(\pi^+\pi^-,\pi^0\pi^0;\Delta t)-I(\pi^+\pi^-,\pi^0\pi^0;-\Delta t)}{I(\pi^+\pi^-,\pi^0\pi^0;\Delta t)+I(\pi^+\pi^-,\pi^0\pi^0;-\Delta t)} \simeq \\ \frac{3\mathrm{Re}(\epsilon'/\epsilon)\sinh(\Delta\Gamma\Delta t/2)-3\mathrm{Im}(\epsilon'/\epsilon)\sin(\Delta m\Delta t)}{[1-\mathrm{Re}(\epsilon'/\epsilon)][\cosh(\Delta\Gamma\Delta t/2)-\cos(\Delta m\Delta t)]} \end{gathered} \tag{84}$$

so that one is sensitive to the imaginary part of ϵ'/ϵ by using a limited range of Δt, while the asymmetry for $\Delta t \to \infty$ is only sensitive to the real part of ϵ'/ϵ, and contains the same information as the double ratio of decay widths.

Partially integrated rate asymmetries can also be used

$$A_\Gamma(t) \equiv \frac{\Gamma(0<\Delta t<t)-\Gamma(-t<\Delta t<0)}{\Gamma(0<\Delta t<t)+\Gamma(-t<\Delta t<0)} \tag{85}$$

where $\Gamma(t_1 < \Delta t < t_2) \equiv \int_{t_1}^{t_2} d(\Delta t) I(\Delta t)$. The asymptotic value of such asymmetry for $\pi^+\pi^-,\pi^0\pi^0$ decays reduces to

$$A_\Gamma(\pi^+\pi^-,\pi^0\pi^0;\infty) \simeq 3[\mathrm{Re}(\epsilon'/\epsilon) - 2(\Gamma_L/\Gamma_S)\mathrm{Im}(\epsilon'/\epsilon)] \tag{86}$$

and also has only a limited sensitivity to the imaginary part of ϵ'/ϵ.

In principle, by fitting the shape of the $I(\Delta t)$ distribution directly, one could extract both the real and the imaginary parts of ϵ'/ϵ; it is perhaps only of academic interest to point out that such shape analysis, being also sensitive to the phase difference $\phi_{+-} - \phi_{00}$, could distinguish the cases in which, even if $\epsilon' \neq 0$, one has $|\eta_{00}| = |\eta_{+-}|$ and therefore the double ratio method would give a null result. On the other hand, this approach is very sensitive to experimental resolution effects on the measurement of the decay time (*i.e.* flight path) difference, particularly in the region around $\Delta t = 0$ where the distribution is rapidly changing, being also the region which is sensitive to $\mathrm{Im}(\epsilon'/\epsilon)$. Estimates of achievable accuracies with this approach at a ϕ factory [68] are $\sim 2 \cdot 10^{-4}$ and $\sim 3 \cdot 10^{-3}$ for the real and imaginary parts of ϵ'/ϵ respectively.

In experiments at a ϕ factory, in which K_L and K_S are produced at the same point, detector acceptance (and its uniformity) is an important issue, since the K_L lifetime corresponds to about 340 cm. Interferometric measurements require a very accurate and precise knowledge of the decay vertex position.

Radiative decays of the ϕ with soft photons generate a C-even background due to $\phi \to \gamma(K^0\overline{K}^0)_{C=+1}$, and therefore $|K_S K_S\rangle$ or $|K_L K_L\rangle$ final states (and $|K_S(\mathbf{p})K_L(-\mathbf{p})\rangle + |K_L(\mathbf{p})K_S(-\mathbf{p})\rangle$ if CPT symmetry is violated). Although such processes are expected to be suppressed by large factors $O(10^7)$ with respect to the non-radiative ϕ decay to C-odd $K^0\overline{K}^0$ pairs, they can dilute, and in some cases fake, the asymmetries discussed above, and their effect has to be taken into account [11].

Apart from the interferometric technique discussed above, also at ϕ factories direct CP violation can be measured by the double ratio method, as in fixed target experiments. The statistical error on $\mathrm{Re}(\epsilon'/\epsilon)$ from the measurement of A_Γ is $\sim 1/(3\sqrt{N})$ (N being the number of $\phi \to \pi^+\pi^-, \pi^0\pi^0$ decays). In order to reach the 10^{-4} level on $\mathrm{Re}(\epsilon'/\epsilon)$, integrated luminosities of the order of ~ 10 fb^{-1} are required, with both techniques, corresponding to about 2 years running at $5 \cdot 10^{32}$ cm^{-2} s^{-1} luminosity.

The VEPP-2M collider at Novosibirsk, with $5 \cdot 10^{30}$ cm^{-2} s^{-1} luminosity, collected 33 pb^{-1} at the ϕ centre of mass energy in 1999-2000. The two installed experiments are CMD-2 [69], with cylindrical drift chambers and calorimeters in a 1.5 T super-conducting magnet, and the non-magnetic SND [70], with smaller drift chambers and a spherical NaI(Tl) crystal calorimeter ($0.9 \cdot 4\pi$ acceptance); these experiments collected $2 \cdot 10^6$ $K_S K_L$ decays and $1 \cdot 10^6$ K^+K^- decays in the above period [71]. An upgraded machine VEPP-2000, which should reach peak luminosity $1 \cdot 10^{32}$ cm^{-2} s^{-1}, is in preparation at Novosibirsk.

The DAΦNE collider at Frascati reached in 2002 a luminosity $\simeq 8 \cdot 10^{31}$ cm^{-2} s^{-1}, steadily increasing toward the design value of $5 \cdot 10^{32}$ cm^{-2} s^{-1}, and the KLOE experiment [72] collected $\simeq 500$ pb^{-1} so far. The KLOE detector (see fig. 8) consists of a large volume drift chamber surrounded by a lead-scintillating fibre electromagnetic calorimeter, both enclosed in a super-conducting solenoid providing a 2 T $\cdot$ m field integral. The tracking detector, with stereo views, is rather light and works with a helium-based gas mixture to minimize regeneration and scattering. The calorimeter is sensitive to low photon energies and has excellent time resolution, which is crucial since particle identification uses velocity measurements. No results on $\mathrm{Re}(\epsilon'/\epsilon)$ are yet available; a ten-fold increase in data is required to reach a statistical error in the 10^{-4} range.

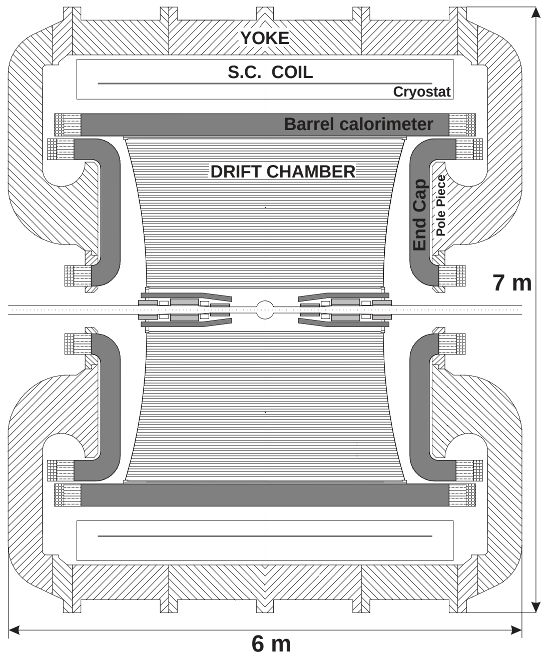

Figure 8: Schematic drawing of the main elements of the KLOE detector.

3.6 The CPLEAR approach

The CPLEAR experiment [73] at the CERN antiproton ring performed an extensive set of measurements in the neutral kaon system with a different technique, analogous to one used today in heavy-meson experiments at hadron machines: by exploiting the strangeness-conserving associate production reactions (driven by strong interactions)

$$\overline{p}p\,(\text{at rest}) \rightarrow \left\{ \begin{array}{l} K^+\pi^- K^0 \\ K^-\pi^+\overline{K}^0 \end{array} \right. \tag{87}$$

which amount to 0.4% of the total $\overline{p}p$ cross-section at rest, the initial strangeness of the produced neutral kaons could be known, and their time evolution and flavour-dependent symmetries could be studied. Also, by exploiting an initial state of precisely known energy and momentum, the neutral kaon direction and momentum could be determined by the production kinematics.

The partial decay rates to a state f accessible to both K^0 and $\overline{K}^0$, for states being initially K^0 or $\overline{K}^0$, indicated as $\Gamma_f(t)$ and $\overline{\Gamma}_f(t)$ respectively, do evolve in time in such a way that their time-dependent asymmetry is

$$A^{(f)}_{CP}(t) \equiv \frac{\Gamma_f(t)-\overline{\Gamma}_f(t)}{\Gamma_f(t)+\overline{\Gamma}_f(t)} = \frac{[-2\mathrm{Re}(\overline{\epsilon})/(1+|\overline{\epsilon}|^2)](e^{-\Gamma_S t}+|\eta_f|^2 e^{-\Gamma_L t})+2|\eta_f|e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta m t-\phi_f)}{(e^{-\Gamma_S t}+|\eta_f|^2 e^{-\Gamma_L t})-[4\mathrm{Re}(\overline{\epsilon})/(1+|\overline{\epsilon}|^2)]|\eta_f|e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta m t-\phi_f)} \tag{88}$$

Any asymmetry between the instantaneous partial decay rates for states being initially K^0 or $\overline{K}^0$ to decay to a given $\pi\pi$ state (*i.e.* either $\pi^+\pi^-$ or $\pi^0\pi^0$)

could be used to extract a measurement of direct CP violation [74]: the initial value (at $t=0$) of this asymmetry is indeed

$$A_{CP}^{(\pi\pi)}(0) \simeq 2\mathrm{Re}(\eta_{\pi\pi}) - 2\mathrm{Re}(\epsilon) \simeq \begin{cases} +2\mathrm{Re}(\epsilon') & (\text{for } \pi^+\pi^-) \\ -4\mathrm{Re}(\epsilon') & (\text{for } \pi^0\pi^0) \end{cases} \tag{89}$$

Due to the finite experimental resolution on the proper decay time, in order to be able to measure an asymmetry with a statistically significant sample of events one has to resort to time-integrated measurements. The sensitivity to direct CP violation effects is largest for short integration times: this is readily understood since any effect of CP violation due to $K^0 - -\overline{K}^0$ mixing is relatively less important for $t \ll \tau_S$, before strangeness oscillations can take place, while direct CP violation effects are independent of time.

Neglecting terms of order $|\epsilon|^3$, the time-integrated asymmetry between time 0 and T is given for short times ($T \ll \tau_S$) by [75]

$$I^{(\pi\pi)}(T) \equiv \int_0^T A_{CP}^{(\pi\pi)}(t)\,dt \simeq -2\mathrm{Re}(\epsilon) + 2\mathrm{Re}(\eta_{\pi\pi})\frac{\Gamma_S T}{1-e^{-\Gamma_S T}} \quad (T \ll \tau_S) \tag{90}$$

in which, as discussed above, the term independent from T only depends on ϵ', while mixing only contributes at finite times. However, in practice, the information which can be obtained on $\mathrm{Re}(\epsilon')$ is drastically limited by statistics, and the uncertainty on the knowledge of $\mathrm{Re}(\epsilon)$.

Another way of extracting the information on direct CP violation is by considering the difference of the two integrated asymmetries for $\pi^+\pi^-$ and $\pi^0\pi^0$ states which, in the same approximation, is given by

$$I^{(\pi^+\pi^-)}(T) - I^{(\pi^0\pi^0)}(T) \simeq \frac{6\Gamma_S T}{1-e^{-\Gamma_S T}}\mathrm{Re}(\epsilon') \quad (T \ll \tau_S) \tag{91}$$

For intermediate integration times ($\tau_S \ll T \ll \tau_L$), the approximate expression for the time-integrated asymmetry (in the limit $\mathrm{Im}(\epsilon'/\epsilon) \simeq 0$ and $\phi(\epsilon) \simeq \pi/4$) is

$$I^{(\pi\pi)}(T) \simeq \begin{cases} 2\mathrm{Re}(\epsilon)[1+2\mathrm{Re}(\epsilon'/\epsilon)] & (\text{for } \pi^+\pi^-) \\ 2\mathrm{Re}(\epsilon)[1-4\mathrm{Re}(\epsilon'/\epsilon)] & (\text{for } \pi^0\pi^0) \end{cases} \tag{92}$$

and consequently from the ratio of the two asymmetries one obtains, in the case $\mathrm{Re}(\epsilon'/\epsilon) \ll 1$

$$\mathrm{Re}(\epsilon'/\epsilon) \simeq \frac{1}{6}\left(1 - \frac{I^{(00)}(T)}{I^{(+-)}(T)}\right) \tag{93}$$

This allows to extract $\mathrm{Re}(\epsilon'/\epsilon)$ without the need to measure $\mathrm{Re}(\epsilon)$ to the (unattainable) level of precision which would be necessary using only one of the decay modes, *e.g.* from $I^{(\pi^+\pi^-)}$. Given the fact that $I^{(\pi^+\pi^-)} \simeq 5 \cdot 10^{-3}$ the number of events needed to reach the same statistical accuracy as with the double ratio method used in the comparison of K_S and K_L decays, is $\sim 4 \cdot 10^4$ times larger, in practice again an insurmountable handicap.

For $\pi^+\pi^-$ decays, in which the decay vertex position can be measured with good accuracy, the modulus $|\eta_{+-}|$ and phase ϕ_{+-} can be extracted from the fit of the $K^0, \overline{K}^0$ proper decay time distributions. Without information on photon directions, the same cannot be done for $\pi^0\pi^0$ decays to any reasonable accuracy, and the integrated decay rates between 0 and 20 τ_S can be used only for those, to obtain

$$|\epsilon'/\epsilon| \simeq \frac{1}{10}\left[2 - \frac{I^{(\pi^0\pi^0)}(T \approx 20\tau_S)}{\mathrm{Re}(\eta_{+-})}\right] \tag{94}$$

Clearly, the determination of $|\eta_{+-}|$ and $|\eta_{00}|$ by the separate measurement of $\pi^+\pi^-$ and $\pi^0\pi^0$ decays could also allow a measurement of direct CP violation through the usual double ratio of decay rates. The above amplitude ratios could be measured either ignoring the initial strangeness of the kaon, in which case the decay rate reduces to the sum of two exponentials, from which $|\eta_f|^2$ can be extracted, or by fitting the interference term in the rate asymmetry in the proper decay time region around $\approx 14\tau_S$. The latter approach is more sensitive due to the small value of $|\eta_f|$, but the systematic uncertainties are very different in the two cases.

Intense fluxes of flavour-tagged K^0 and $\overline{K}^0$ were produced by stopping low-energy anti-protons (200 MeV/c, $10^6\,\overline{p}/s$) from the CERN LEAR ring in a low-density gaseous hydrogen target. The detector consisted in a cylindrical tracking chamber, a threshold Cerenkov counter to identify charged kaons, and an electromagnetic calorimeter made of lead plates inter-spaced with streamer tubes, all enclosed in a solenoidal magnet providing a constant 0.44 T field. The experimental acceptance extended to 20 τ_S on average (60 cm).

The differences in the interactions of $K^+\pi^-$ and $K^-\pi^+$ with the detector required all the observables to be independently normalised for K^0 and $\overline{K}^0$, to avoid systematic asymmetries; these can only be important close to the production target, before strangeness oscillations have taken place. Inefficiencies are less dangerous since they can only "dilute" the measurement of an existing asymmetry but cannot induce a fake one by themselves.

The final results of CPLEAR on the amplitude ratios are [76]

$$|\eta_{+-}| = (2.264 \pm 0.035) \cdot 10^{-3} \qquad |\eta_{00}| = (2.47 \pm 0.39) \cdot 10^{-3} \tag{95}$$

The systematic errors for the η_{+-} measurement arise primarily from the imperfect knowledge of the regeneration amplitude due to detector material, (affecting the $K^0, \overline{K}^0$ rates through the interference with the K_S amplitude), from the subtraction of the background due to semi-leptonic K_L decays, and the imperfect knowledge of the experimental time resolution function; the latter, and its perturbation by photons not related to the neutral kaon decay process, were the main sources of systematic errors for the η_{00} measurement.

From the above ratios one would get naively (without taking into account correlated uncertainties) $\mathrm{Re}(\epsilon'/\epsilon) = (-31.7 \pm 62.9) \cdot 10^{-3}$.

Strangeness-tagged kaon decays also allow the measurement of several other kinds of asymmetries, which could be useful to disentangle CP violation in the decay amplitudes from the contribution due to the strangeness mixing [75].

3.7 Comments on strategies of experimental approach

The combination at face value of the world averages of the independent measurements of $|\eta_{+-}|$ and $|\eta_{00}|$ [36] gives

$$\mathrm{Re}(\epsilon'/\epsilon) = (8 \pm 16) \cdot 10^{-3} \tag{96}$$

To go significantly below this level of error, dedicated experiments are required, and only the ones based on the double ratio method provided precise results, so that in the following only these will be discussed.

The experimental techniques to measure ϵ' evolved steadily in time, with improvements being dictated by the systematic limitations found in the previous generation of experiments, in a continuous effort to increase the level of accuracy; clear trends can be identified, with each collaboration attacking the most critical sources of systematic errors and eliminating by clever design the elements which lead to larger uncertainties in previous results.

Table 3.7 presents a schematic comparison of several parameters for some experiments measuring the double ratio: in this table are reported the (average) momentum and (instantaneous) intensities of the primary proton beam and of the kaons, the amount of neutrons in the neutral beams, some indicative (average) detector resolution parameters, the size and background fractions of the collected samples.

In the measurements of direct CP violation in neutral kaon decays through the double ratio method, several possible sources of systematic errors arise:

1. Imperfect knowledge of relative K_L and K_S fluxes.
2. Differences in the triggering, detection or reconstruction efficiencies for $\pi^+\pi^-$ and $\pi^0\pi^0$ decays.
3. Detector acceptance differences for K_L and K_S decays.
4. Differential resolution effects.
5. Changes in detector properties during the data taking period.
6. Rate and accidental-induced differential effects.
7. Contamination by K_L scattered in a regenerator and K_S regenerated inelastically or by neutrons in the beam.
8. Imperfect background subtractions.
9. Imperfect knowledge of the fiducial regions in the kaon phase-space.

We will discuss the above issues and the techniques used to address them (not necessarily in the above order, since many of them are tightly interlinked), focusing on the recent experiments at hadronic machines, which provided results on direct CP violation.

Apart from coherent production of K_L K_S pairs, no hadronic interaction can provide a pure K_S beam, but only a coherent mixture of K_S and K_L in equal proportions (thanks to CPT symmetry, independently from CP conservation in strong interactions, and independently from the $K^0/\overline{K}^0$ production cross section ratio). While beams with any required K_L purity are easily available at a suitable distance from the production target, "K_S beams" are invariably 50%-50% K_S $-K_L$ mixtures; nevertheless, when considering $\pi\pi$ decays, the large ratio of partial decay widths (due to the approximate validity of CP symmetry) results in a large effective suppression of $K_L \to \pi\pi$ decays with respect to K_S in the region close to the production target (by a factor $|\eta|^2 \simeq 5 \cdot 10^{-6}$ at the target). The above considerations make clear that the intrinsic impurity of the

	BNL E749	FNAL E617	FNAL E731†	CERN NA31	FNAL KTeV††	CERN NA48
c)	28	400	800	450 (K_L) 360 (K_S)	800	450
y (s^{-1})	$3 \cdot 10^{11}$	$7 \cdot 10^{12}$	$0.15 - 1 \cdot 10^{11}$	$0.4 \cdot 10^{11}$ (K_L) $1.2 \cdot 10^{7}$ (K_S)	$1.5 \cdot 10^{11}$	$6.3 \cdot 10^{11}$ (K_L) $1.2 \cdot 10^{7}$ (K_S)
s^{-1})	$\sim 10^2$	~ 20	$0.9 - 6 \cdot 10^2$	$0.7 \cdot 10^3$	$0.3 \cdot 10^2$	$1.1 \cdot 10^2$
s^{-1})	$3 \cdot 10^6$	$7 \cdot 10^5$	$1.3 - 8.3 \cdot 10^6$	$0.4 \cdot 10^6$	$1.8 \cdot 10^5$	$8.4 \cdot 10^6$
eam	≈ 30	10	≈ 1	8-9	0.8	≈ 5
on (m[τ_S])	1.2 [2.1]	13 [3]	42 [11] ($\pi^0\pi^0$) 17 [4.5] ($\pi^+\pi^-$)	46.8	48 [13]	13-32 [3.5]
/c)	7-14	50-140 ($\pi^0\pi^0$) 30-120 ($\pi^+\pi^-$)	40-160	60-180	40-160	70-170
(m)	0.12 ($\pi^0\pi^0$) ≈ 0.5 ($\pi^+\pi^-$)	1.7 ($\pi^0\pi^0$)	≈ 1 ($\pi^0\pi^0$) ≈ 0.15 ($\pi^+\pi^-$)	≈ 1.2 ($\pi^0\pi^0$) ≈ 0.8 ($\pi^+\pi^-$)	0.25 ($\pi^0\pi^0$) ≈ 0.2 ($\pi^+\pi^-$)	0.5 ($\pi^0\pi^0$) ≈ 0.5 ($\pi^+\pi^-$)
γ	8%	3%	1.0%	1.4%	0.6%	0.8%
) (MeV/c^2)	15	4.5	3.5	≈ 20	1.6	2.5
$\pi^0\pi^0$) (10^3)	1.1	3.2	410.3	428	3348	4837
n-$\pi\pi$)/tot	17.5%	7.6%	1.8%	2.67%	0.11%	0.06%
)/tot	–	2.6%	3.4%	–	0.38%	0.1%
$\pi^+\pi^-$) (10^3)	8.1	10.6	329.0	1142	11126	21554
n-$\pi\pi$)/tot	$\approx 7\%$	3.0%	0.34%	0.54%	0.1%	0.16%
)/tot	–	0.4 %	–	0.09%	0.01%	–
$\pi^0\pi^0$) (10^3)	3.2	5.7	800.0	2254	5556	7370
n-$\pi\pi$)/tot	1.2%	0.5%	0.05%	0.07%	–	–
)/tot	1.5%	12.7%	2.6%	–	1.22%	–
$\pi^+\pi^-$) (10^3)	19.9	25.8	1060.7	5541	19291	31830
n-$\pi\pi$)/tot	≈ 0	0.2%	–	–	–	–
)/tot	0.2%	1.7%	0.15%	0.03%	0.08%	–
t.	background	background	γ energy	accidental	$\pi^0\pi^0$ rec.	$\pi^0\pi^0$ rec.
ing			1985-1988	1986-1989	1996-1999	1997-2001
ate	1985	1985	1990-1993	1988-1993	1999-	1999-2002
($\times 10^4$)	$17 \pm 72 \pm 43$	$-46 \pm 53 \pm 24$	$7.4 \pm 5.2 \pm 2.9$	23.0 ± 6.5	$20.71 \pm 1.48 \pm 2.39$	$14.7 \pm 1.4 \pm 1.7$

beams has to be taken into account in the analysis, this being however a trivial task. Intense neutral beams have usually rather wide momentum spectra, and at the higher end of the spectrum K_S could still account for a sizable fraction of $\pi\pi$ decays, even at rather large distances from the production target. On the other hand a fraction of K_L contributes $\pi\pi$ decays in a "K_S beam", depending on the distance from their production point at which the fiducial region starts and its size. The term describing $K_S - K_L$ interference in the expression for the yield of $\pi\pi$ decays must also to be taken into account: the coefficient of this term contains the already mentioned "dilution factor", which has to be measured experimentally.

All but the earliest experiments are independent from the knowledge of the absolute flux normalisation, by detecting at the same time at least two of the four decay channels which enter the double ratio (*i.e.* having simultaneous K_L and K_S beams and/or detecting at the same time $\pi^+\pi^-$ and $\pi^0\pi^0$ decays). Note that this is true for the double ratio measurement, while in order to measure the "single ratios" (*i.e.* $|\eta_{+-}|^2$ and $|\eta_{00}|^2$) the knowledge of flux normalisation is required.

From an experimental point of view, the detection of charged and neutral decays involves different parts of the apparatus, and is therefore subject to rather different instrumental effects. On the other hand, clearly there cannot be any instrumental difference in the detection of K_S or K_L decays to the same final state, provided they occur at the same place and time, which however is not usually the case in real experiments.

When K_L and K_S decays to a given mode are collected at the same time, but separately for charged and neutral decays, the K_L/K_S flux ratio is irrelevant for the measurement of R, provided such ratio is exactly the same during the detection periods dedicated to the two modes. In this case R is computed by dividing the "single ratios", known up to a (common) multiplicative constant related to the flux ratio.

By using a double-beam technique with a regenerator or the production of coherent kaon pairs, K_L/K_S flux normalisation is (approximately or exactly) not an issue. In the case of the FNAL twin beam experiments (E617, E731, E832-KTeV), the K_L/K_S beam intensity ratio is constant, provided the geometry of the beam line elements and the regenerator properties do not change in time; the same is of course true for K_LK_S pairs produced at a ϕ-factory, so that, for what concerns primary flux cancellation, simultaneous charged and neutral mode data collection is not an unavoidable requirement: any instrumental charged-neutral detection difference has no effect on the double ratio in this case.

When measuring simultaneously the charged and neutral decays, with K_L and K_S collected at different times, flux normalisation is irrelevant as long as the relative detection efficiency for the two decay modes is stable in time and independent from the (different) environmental conditions (*e.g.* rates and accidental activity) when running with the different beams. In this case the double ratio is computed as the ratio of the $\pi\pi$ charge ratios ($\pi^+\pi^-/\pi^0\pi^0$), known up to a (common) multiplicative constant related to the detection efficiencies; this was the case of the NA31 experiment at CERN.

In the CERN NA48 double-beam approach, although the secondary beam used to produce K_S is derived from the primary one used to produce K_L, some limited residual K_L/K_S beam intensity variations (at the level of $\approx 10\%$) were present, due to different transmission and focusing of the beams; these variations could bias the measurement if coupled to time variations in the relative charged-neutral detection efficiencies on the same time scale; the possible effects had therefore to be measured and controlled, and eventually cancelled by intensity-ratio weighting.

The latest generation of experiments (KTeV and NA48, but also E731 in its last phase) were simultaneously sensitive to all four decay modes, therefore making flux cancellation more effective; the following discussion will focus mostly on these experiments, in which most of the corrections to be applied to the measured double ratio are zero to first order; some interesting discussion can be found in [77].

Even for experiments with simultaneous K_S and K_L sources, detector acceptance differences for their decays are in general present. If the geometrical parameters of the two beams (angular divergence and emittance distribution) are equal, and their incidence on the detector is the same, either intrinsically (because they coincide spatially at the detector plane, as in the CERN experiments) or statistically (because they alternate positions, as in the FNAL experiments), the acceptance for a given final state only depends on the kaon momentum and on its longitudinal decay position.

The angular parameters of the beams can be made similar by collimation, and are identical in case of coherent regeneration. The momentum spectra of the two beams are usually rather similar, either due to the choice of suitable proton targeting angles (as in NA48) or because of the fact that the squared regeneration amplitude has a $1/p_K^\alpha$ dependence (p_K being the incident K_L momentum, and $\alpha \simeq 1.2$) quite matching the $1/p_K$ kinematic factor which appears in the spectrum of decaying K_S (as in regenerator-based experiments). For this reason the acceptance dependence on the kaon momentum can be safely taken into account by performing the analysis in kaon energy bins, with resolution effects being kept under control by the choice of adequate bin sizes.

On the other hand, longitudinal decay point distributions are intrinsically very different for K_S and K_L of the same momentum, whatever the production point; binning in such a variable could give rise to biases induced by bin to bin migration effects caused by the finite (and different for charged and neutral modes) experimental resolution, coupled with the very different distributions. For the above reason, acceptance cancellation can only be achieved by artificially changing one (or both) the distributions to make them similar; this was performed in NA31 and some previous experiments by collecting data with the K_S target positioned at different longitudinal positions and combining the measurements, and in NA48 by weighting the K_L events at the analysis stage and trading off statistical power in the procedure, also not using K_L decays occurring at longitudinal positions in which no K_S decays are available. The FNAL experiments chose instead not to have any intrinsic acceptance cancellation, gaining in statistics at the price of having to deal with a large acceptance correction, and therefore a very accurate Monte Carlo simulation of the apparatus, checked against large samples of K_{e3} and $3\pi^0$ decays.

The general experimental layout for these high-energy experiments is rather similar, consisting of a long, evacuated decay region followed by a detector region filled with helium to reduce multiple scattering, kaon regeneration and photon conversion and, in the case of KTeV, also the interactions of the neutral beam particles. The magnetic spectrometer, as light as possible in order to limit the impact on the electromagnetic calorimeter performance, requires a large-gap magnet providing a uniform transverse momentum kick. Excellent electromagnetic calorimetry, in terms of resolution, uniformity and calibration, is an essential requirement in order to reduce backgrounds and to control the energy and position scales for neutral events. Veto elements around the active detector region are used to reduce trigger rates and backgrounds due to incompletely contained events, while others behind absorber walls are used for muon background suppression. Among the advantages of using high-energy (~ 100 GeV) kaon beams, besides the possibility of exploiting better resolutions for photon detection, is the fact that the small solid angle acceptance allows planar detectors, which can be kept accurately under control for what concerns their geometry and positions.

Triggering and reconstruction efficiencies are in general different for charged and neutral decays: the knowledge of their ratio is however not needed as long as the K_L and K_S decays are collected simultaneously. In particular, if the short- and long-lived kaon decays originate from beams which have different properties, care should be taken to check any possible effect due to local rate differences: different detector illuminations or different time structures, which could lead to a systematically different time distribution of charged and neutral decays, can potentially result in a bias when coupled to detector or trigger efficiency variations on the same scale.

Trigger efficiencies have to be continuously monitored, by collecting large enough downscaled fractions of “minimum-bias” triggers. Triggering and detection efficiencies have in general a non-null correlation (partially different for charged and neutral modes) with instantaneous rates in the detectors, so that they should be checked for variations on all the time scales on which K_L/K_S time variations can be expected.

The time variation of detector performances could induce biases when coupled with variations of the K_L/K_S flux ratio on the same time scale. Local inefficiencies due to malfunctioning wire chamber wires or calorimeter cells have to be continuously monitored; they were avoided in KTeV by data rejection and immediate repair, and tracked in time by NA48, to be taken into account in analysis and detector simulation. The periodic reversal of the spectrometer magnetic field helps reducing any residual left-right asymmetry which could couple to any spatial asymmetry of the two beams.

Differential effects induced by accidental activity, due to kaon decays or other particles in the beams, can also cause the same kind of effects, resulting in a non-linear dependence of the number of collected events on the beam intensity. The accidental activity, while largely affecting both beams at the same level, is clearly higher in a regenerator experiment such as KTeV. Both experiments study these effects by using the software overlay of events collected randomly in time at a rate proportional to the instantaneous beam intensity. The production of K_S by using a close target is a more efficient technique than the use of a regenerator, requiring a lower proton intensity to have the same kaon yield, and therefore resulting in a lower level of accidental activity. However, with this technique the region of the close target must be appropriately shielded to avoid unwanted accidental particles hitting the detector; in particular, possible effects induced by the simultaneous presence of other particles, produced by the same proton interaction from which the decaying K_S emerged, and therefore “in-time” with respect to the decay, should be checked as potential sources of bias. On the other hand, a regenerator is also a source of large unwanted backgrounds.

The experimental requirement of making simultaneous K_L and K_S beams as similar as possible to avoid any bias, is of course at odds with the necessity of being able to distinguish the K_L or K_S nature of the decay to actually form the double ratio[13]. The origin of a single $\pi\pi$ decay has to be attributed to a K_S or K_L on an event by event basis, in order to measure the double ratio: this clearly requires two distinct beams (in time or space).

In the KTeV twin beams arrangement, the two quasi-parallel beams (1.6 mrad divergence) are well separated in the transverse plane along all the decay region and also at the detector; the transverse kaon position can therefore be used to identify the beam from which the event originated. For $\pi^+\pi^-$ decays, the transverse position of the extrapolated kaon trajectory at the regenerator plane, measured with good accuracy, is used; for $\pi^0\pi^0$ decays, measured in a destructive detector, the kaon trajectory is not known and the energy-weighted impact position of the four photons, corresponding to the intercept of the kaon trajectory with the calorimeter plane[14], must be used: even with a good resolution in this variable, much smaller than the beam separation thanks to the high-performance calorimeter (~ 1 mm vs. ≈ 300 mm separation of the 9×9 cm^2 beams), this approach cannot discriminate against kaons which are scattered at large angle in the regenerator or in collimators, which give the largest contribution to the background, to be measured and subtracted. As already mentioned, any instrumental asymmetry due to the separation of the beams at the detector plane is statistically cancelled by alternating the regenerator on the two beams at every accelerator pulse.

In the NA48 approach with very close K_L and K_S beams converging at the centre of the detector ($\simeq 60$ mm separation at the beginning of the fiducial region, 0.6 mrad angle), illumination differences are intrinsically very small, depending only on the different beam divergences, and the assignment of an event to either beam is performed with a time-of-flight technique on tagged protons. This approach requires precise event time information from the detectors, and the only asymmetries which can be induced in the double ratio are the ones due to different time measurement tails for charged and neutral decays, leading to asymmetric mis-tagging of a K_S decay. The above systematic effect, as well as the small ones due to effective rate differences for charged and neutral decays, induced by rate-dependent trigger efficiencies, had to be checked and corrected for using an alternative tagging method, possible for $\pi^+\pi^-$, based on the measured kaon trajectory at the collimator plane, which clearly identifies the beam from which the event originates.

Background due to 3-body kaon decays is an intrinsically asymmetric component entering the double ratio, being decay mode dependent and different for K_L and K_S. Using redundant information in the reconstruction of the final state, this background can be subtracted, but the accuracy of the procedure is intrinsically limited by the statistical fluctuations of the subtracted number of events, and also systematically limited by the fact that the distribution of the

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[13] The ultimate arrangement of measuring kaon decays from a *single* neutral beam in the region around 12 τ_S from the production target, where the yields of $\pi\pi$ decays from K_L and K_S are comparable, is also the one in which mass eigenstates are intrinsically indistinguishable!

[14] Actually with a plane at a position corresponding to the energy-weighted average of the longitudinal shower development in the detector, which depends on the decay vertex position for a non-projective calorimeter such as the one of KTeV, which is however rather compact (50 cm thick), thus reducing the dependence on such an effect.

background in the signal region for the relevant variables must be extrapolated using some model (possibly extracted from the data). High-resolution detectors are therefore used in order to be able to impose tighter cuts on the signal, thereby reducing the background fraction before the subtraction.

All experiments are affected by background due to the dominant 3-body decay modes of the intense K_L beams: $\pi^+\pi^-\pi^0$, $\pi^\pm e^\mp \nu$ (K_{e3}) and $\pi^\pm\mu^\mp\nu$ ($K_{\mu 3}$) for the charged mode, $3\pi^0$ for the neutral mode.

For the charged mode, while the $\pi^+\pi^-\pi^0$ background can be subtracted with relative ease by magnetic analysis, due to its good signature in the detector, semi-leptonic modes with an undetected neutrino usually contaminate the signal region for the relevant kinematic variables. Particle identification is used both to reject such semi-leptonic backgrounds and to positively identify the residual contamination, in order to subtract it: transition radiation detectors or calorimetric information combined with momentum measurement are used for $e^\pm$ identification, and muon veto counters to identify $\mu^\pm$.

A fraction of the $\pi^+\pi^-\gamma$ decays is unavoidably included in the $\pi^+\pi^-$ sample: the irreducible inner bremsstrahlung contribution, dominant for small photon energies, however appears in the same K_L/K_S ratio as the non-radiative decay from which it derives, and as such cannot change the measured double ratio at all. The direct emission component, which is important for higher photon energies ($\sim 1.5\%$ of the $\pi^+\pi^-$ mode for $E_\gamma > 20$ MeV [78]), being only present for K_L could create a bias on the double ratio; however a good $\pi^+\pi^-$ invariant mass resolution allows this component to be rejected from the sample.

In experiments with a close target, one also gets hyperon decays from the neutral beam produced there, since hyperon lifetimes are comparable to that of K_S : the contamination of the $\pi^+\pi^-$ sample by misidentified $\Lambda \to p\pi^-$ decays can be made very small by rejecting the kinematic region corresponding to large momentum asymmetry of the two charged particles in the laboratory system.

The background in the neutral mode due to $K_L \to 3\pi^0$ decays is partly suppressed by photon veto detectors surrounding the decay volume; the remaining background fraction with missing or superimposed photons in the detector is reduced and subtracted by using the kinematic constraints of the $K^0 \to 2\pi^0 \to 4\gamma$ decay chain. Assuming zero transverse momentum of the decaying kaon with respect to the beam axis (a good approximation for collimated beams in the $\sim$ 100 GeV/c momentum range), one of the three mass constraints is used to compute the longitudinal decay vertex position, leaving two more for background rejection and control: defining E_i as the photon energies and r_{ij} as the distance between two photon clusters ($i, j = 1, 4$), the longitudinal distance d of the decay vertex from the calorimeter is computed in KTeV as the weighted average of the two values

$$d_{ij} = \frac{\sqrt{E_i E_j r_{ij}^2}}{m_{\pi^0}} \tag{97}$$

corresponding to the photon pairing which gives the best vertex agreement, and in NA48 as

$$d = \sum_{i>j} \frac{\sqrt{E_i E_j r_{ij}^2}}{m_K} \tag{98}$$

Since the reconstructed decay vertex position in the neutral mode is obtained by constraining the invariant masses of sets of photons, background events due to $3\pi^0$ decays with missing energy in the detector are reconstructed downstream of their true decay position (smaller d values): the neutral background fraction then generally increases in the region closer to the detector. For this reason, depending on the size and the location of the fiducial region, the amount of $3\pi^0$ background in the neutral channel can be quite different.

In this context, $K_L \to \pi^0\pi^0\gamma$ is a negligible background ($< 6 \cdot 10^{-3}$ of the $\pi^0\pi^0$ mode).

In regenerator-based experiments, a different additional class of backgrounds arises from scattering in the regenerator itself, and can possibly affect both the K_L and K_S samples. Apart from the coherently regenerated K_S component, a diffractively regenerated K_S component is produced at non-zero angle with respect to the incident K_L beam, and inelastic K_L interactions can contribute large backgrounds as well. The latter are usually eliminated by veto counters in the regenerator detecting recoil charged particles, while the former have to be subtracted by analyzing the kaon transverse momentum distribution. Kaon scattering is induced by the presence of matter along the kaon path: regenerators and collimators (in a ratio ≈ 10 to 1 for KTeV). Scattered kaons, besides requiring a correction when the extrapolated kaon impact position on the detector is used to tag the nature of the kaon, have to be subtracted because of their different regeneration properties and acceptance, compared to unscattered ones. Any effect induced by scattering, however, is intrinsically the same for both decay modes of a given kaon type (K_S or K_L), and therefore cancels in the double ratio to the extent at which the analysis cuts can be made equal for $\pi^+\pi^-$ and $\pi^0\pi^0$; the residual correction to be applied is due to the unavoidable differences in the selection of the two channels, mostly performed using different detectors. Moreover, the scattering background can be studied and measured in the charged mode, where more information on the event is usually available due to tracking, and with the same caveats mentioned above, this knowledge can be applied to the neutral mode.

One more potential source of background is the interaction of the intense flux of photons and neutrons with material along the beam line. In the case of KTeV, photons are partially absorbed by a 14 radiation lengths thick lead slab after the target; the neutron flux, reduced to below the kaon one by more absorbers, is largely symmetric among the different modes. In NA48, with a higher neutron and photon flux ratio with respect to kaons, the use of an evacuated beam pipe crossing all the detectors avoids that the neutral beams traverse any significant amount of material.

The presence of holes in the detectors is also a difference among KTeV and NA48: while in both experiments the neutral beams themselves do not touch the active part of the calorimeters, they do cross an active part of the drift chambers in KTeV (although, since calorimeter information is necessary also for charged decays, a fraction of this area is excluded from the acceptance), and not in NA48. Since most of the particle rate (both due to the neutral particles in the beams and to kaon decay products) is sharply peaked close to the beam(s) in the transverse plane, one has either to worry about detector efficiencies as a function of the distance to the beam axis, or to cope with acceptance effects due to slight differences in beam illumination close to the hole itself (the bulk of the acceptance correction in NA48); in both cases the effects have to be modelled by Monte Carlo simulation which includes the hole in acceptance due in particular to the requirement of identifying electrons by their showering in the calorimeters, in order to reject background from K_{e3} decays.

One important systematic issue is that of the knowledge of the relative accuracy of the measurements for kinematic variables on which cuts are performed, and for which K_S and K_L have different distributions. This is the case for the kaon momentum and particularly for its longitudinal decay vertex position, so that the energy and longitudinal (but also transverse) scales of the charged and neutral detectors have to be known accurately. The spectrometer scales are directly determined by its well known geometry, and checked with the value of the reconstructed kaon mass for $\pi^+\pi^-$ decays (or the Λ mass in $p\pi^-$ decays), the adjustment of which fixes the overall momentum scale, linked to the absolute value of the spectrometer magnetic field. For NA48, in which the K_S and K_L kaon spectra differences can be larger, the kaon momentum is reconstructed using the ratio of pion momenta and their opening angle at the decay vertex, in a way which is independent from the knowledge of the absolute value of the magnetic field.

For neutral decays, the decay vertex position is determined by imposing kinematic constraints for the decay process, and is therefore linked to the absolute energy scale of the calorimeters as discussed above. This scale can be measured to a limited level of accuracy by calibration with $e^{\pm}$ beams, but ultimately has to be fitted from $K_S \to \pi\pi$ data. This is done (for charged decays as well) by adjusting the reconstructed positions of well defined detector edges for decays in which such a detector did not fire. The accuracy and the stability of this adjustment procedure (at the 10^{-4} level) as a function of time and of kinematic variables determines the systematic error, and was one of the main reasons for building high performance calorimeters. When a Monte Carlo is not used for the largest part of the acceptance correction as in NA48, the sensitivity of the result to the knowledge of the above scales, which directly affects both the position and the size of the fiducial region, with a potentially large effect on the steep K_S vertex distributions, is greatly reduced by fixing the most sensitive upstream edge of such region with a hardware cut (defined by a veto counter); event weights in NA48 are defined in terms of the measured kaon proper lifetime, instead of position, which is seen to be independent of the absolute energy scale of the calorimeter. In both experiments the neutral energy scales are also checked, at different points, by exploiting the prompt $3\pi^0$ decays of η mesons produced on the vacuum window (KTeV) or on thin polyethylene targets during special π^- beam runs (NA48): the reconstruction of the known decay vertex position for such events allows further checks on the energy scale and non-linearities.

The world average of ϵ'/ϵ measurement is

$$\mathrm{Re}(\epsilon'/\epsilon) = (16.7 \pm 2.3) \cdot 10^{-4} \tag{99}$$

where the error has been inflated by a factor 1.44 according to the procedure adopted by the PDG [36], due to the poor χ^2 value of 6.2 (with 3 degrees of freedom). The probability of the four most precise measurements to be consistent is 10%, and it varies between 7% and 20% when a single measurement is ignored. A graphical depiction of the present data is shown in figure 3.7.

Despite the somewhat unsatisfactory consistence, such averaged result is still more than 7 standard deviations from zero, therefore proving at last the existence of direct CP violation in neutral kaon decays, after a long history of measurements, graphically summarised in figure 3.7.

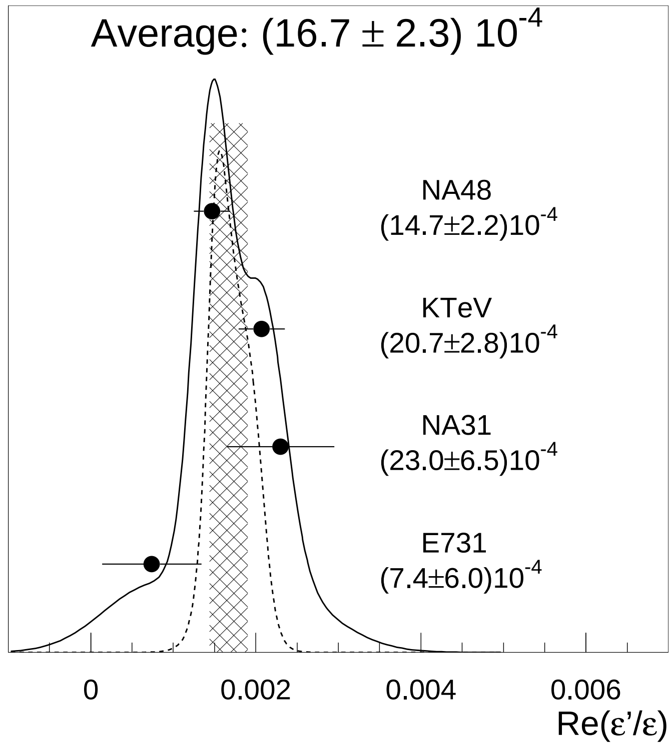

Figure 9: Ideogram of recent published $\mathrm{Re}(\epsilon'/\epsilon)$ measurements. The curves show (unnormalised) probability distributions according to the PDG procedure [36] (solid line) or a Bayesian "skeptical" approach [79] (dashed line).

The value of $\mathrm{Re}(\epsilon'/\epsilon)$ usually quoted by experiments is derived from the double ratio of partial decay widths according to the simple relation (69), which neglects terms of order $|\epsilon'|^2$ and also $|\omega|$; the proper comparison with theoretical predictions should be done retaining the small correction due to the latter, so that the actual formula becomes

$$\mathrm{Re}(\epsilon'/\epsilon) = \frac{1}{6}\left(1 - \frac{|\omega|}{\sqrt{2}}\cos(\delta_2 - \delta_0)\right)\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right) \tag{100}$$

This represents a $\sim 2\%$ correction, completely negligible compared to the present uncertainties in the theoretical computation, and also to the present experimental accuracy, but not to the size of the corrections which the experiments do consider to quote the central value. The experimental value of $\mathrm{Re}(\epsilon'/\epsilon)$ to be compared with theoretical predictions is therefore

$$\mathrm{Re}(\epsilon'/\epsilon) = (16.3 \pm 2.3) \cdot 10^{-4} \tag{101}$$

and it should be noted that the uncertainty on the value of $|\omega|$ (see [42] and references therein) hardly affects any comparison with theory in itself, since

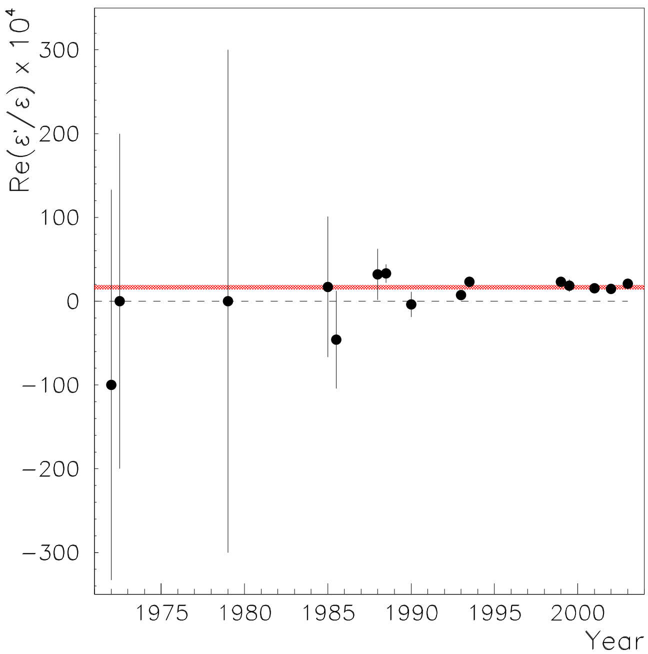

Figure 10: Time evolution of $\mathrm{Re}(\epsilon'/\epsilon)$ measurements. The horizontal band represents the current world average.

the empirical value of such parameter is used both in the computation and in extracting the value of $\mathrm{Re}(\epsilon'/\epsilon)$ from the experiments.

With no constraint on the relative phase of ϵ' and ϵ is imposed, the relation at the same level of approximation is

$$1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = 6\,\mathrm{Re}(\epsilon'/\epsilon)\left(1 + \frac{|\omega|}{\sqrt{2}}\cos(\delta_2 - \delta_0)\right) - \mathrm{Im}(\epsilon'/\epsilon)\left(3\sqrt{2}|\omega|\sin(\delta_2 - \delta_0)\right) \tag{102}$$

but the above phase difference is $\sim \delta_2 - \delta_0 + \pi/2 - \phi_{SW} = (-1.2 \pm 1.5)^\circ$ and therefore the last term can be safely neglected.

While the order of magnitude of the measured value of $\mathrm{Re}(\epsilon'/\epsilon)$ is not at odds with what can be expected in the Standard Model, it does not pose any strong constraint on its underlying picture of CP violation, since the computation of the hadronic part of the decay process is not yet completely under theoretical control. The common expectation is that improvements in the accuracy of lattice QCD computations will ultimately allow precise quantitative comparisons to be performed[15].

Being evidence of direct CP violation, the measured non-zero value of $\mathrm{Re}(\epsilon'/\epsilon)$

[15] It is interesting to note, however, that the two most recent lattice QCD computations of ϵ'/ϵ in the SM are in gross disagreement with the experimental measurement [80].

	$K_L \to \pi^+\pi^-\pi^0$	$K_S \to \pi^+\pi^-\pi^0$	$K_L \to \pi^0\pi^0\pi^0$	$K_S \to \pi^0\pi^0\pi^0$
BR	$(12.58 \pm 0.19)\%$	$(3.2^{+1.2}_{-1.0}) \cdot 10^{-7}$	$(21.08 \pm 0.27)\%$	$< 3 \cdot 10^{-7}$
		$(2 \pm 10) \cdot 10^{-9}(\dagger)$		(90% CL)[82]
g	(0.678 ± 0.008)		–	–
h	(0.076 ± 0.006)		$(-5.0 \pm 1.4) \cdot 10^{-3}$	
j	$(1.1 \pm 0.8) \cdot 10^{-3}$		–	–
k	$(9.9 \pm 1.5) \cdot 10^{-3}$		see h	see h
f	$(4.5 \pm 6.4) \cdot 10^{-3}$		–	–

Table 2: Experimental values of $K^0 \to 3\pi$ branching ratios and decay parameters, from [36] except where otherwise noted. (†): CP-violating component.

translates to an asymmetry between CP-conjugate processes

$$\frac{\Gamma(K^0 \to \pi^+\pi^-) - \Gamma(\overline{K}^0 \to \pi^+\pi^-)}{\Gamma(K^0 \to \pi^+\pi^-) + \Gamma(\overline{K}^0 \to \pi^+\pi^-)} = (5.56 \pm 0.84) \cdot 10^{-6} \tag{103}$$

$$\frac{\Gamma(K^0 \to \pi^0\pi^0) - \Gamma(\overline{K}^0 \to \pi^0\pi^0)}{\Gamma(K^0 \to \pi^0\pi^0) + \Gamma(\overline{K}^0 \to \pi^0\pi^0)} = (-11.1 \pm 1.7) \cdot 10^{-6} \tag{104}$$

which makes the effect of direct CP violation more self-evident.

3.8 Other neutral K decays

$K_{S,L} \to 3\pi$

Experimentally, $K \to 3\pi$ decays [81] are analyzed in terms of two adimensional, Lorentz-invariant, independent variables u, v (sometimes called Y, X):

$$u \equiv \frac{s_3 - s_0}{m_\pi^2} \qquad v \equiv \frac{s_1 - s_2}{m_\pi^2} \tag{105}$$

having defined

$$s_i \equiv (p_K - p_i)^2 \tag{106}$$

$$s_0 \equiv (s_1 + s_2 + s_3)/3 = m_K^2/3 + \textstyle\sum_i m_{\pi_i}^2/3 \tag{107}$$

where p_K and p_i are the kaon and pion four-momenta, with the convention that the index $i = 3$ refers to the "odd" pion (the neutral one in $\pi^+\pi^-\pi^0$). Decay distributions are expressed as a power series in u, v, usually stopping at second order due to the limited phase space (Q-values being in the 75-93 MeV range):

$$|A(K \to 3\pi)|^2 \propto 1 + gu + hu^2 + jv + kv^2 + fuv \tag{108}$$

where the parameters g, h, j, k, f are usually referred to as "Dalitz plot slopes". Note that in case of identical pions in the final state the odd terms in u, v are intrinsically undefined and other terms may coincide (*e.g.* for $3\pi^0$ decays there are no g, j, f terms and a single quadratic slope term). Any non-zero linear slope parameter for v ($j \neq 0$ or $f \neq 0$) would be evidence of CP violation.

The possible 3π final states for neutral kaon decays are $\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$; the measured branching ratios and slopes are reported in Table 2.

Bose symmetry only allows isospin states I=1,3 for $3\pi^0$, so that this is a pure $CP = -1$ eigenstate. Since the K_L readily decays to $3\pi^0$, the (so far unobserved) transition $K_S \to 3\pi^0$ requires CP violation, and the decay $K_1 \to 3\pi^0$ would imply direct CP violation. If direct CP violation is not dominant in such decay, the expected branching ratio is

$$BR(K_S \to 3\pi^0) = BR(K_L \to 3\pi^0)\frac{\tau_S}{\tau_L}|\epsilon|^2 \simeq 1.9 \cdot 10^{-9} \tag{109}$$

The CP eigenvalue of the $\pi^+\pi^-\pi^0$ state is $(-1)^{l+1}$, where l is the eigenvalue of the orbital angular momentum between the two charged pions; such a state can have therefore both $CP = +1$ (for I=0,2) and $CP = -1$ (for I=1,3), so that both kaon CP eigenstates can decay into it without violating CP; an angular momentum analysis is required to measure CP violation in such decays. The I=0,2 states must have a non-zero orbital angular momentum between any pair of pions, so that kaon decays into them are strongly suppressed by the centrifugal barrier, due to the limited phase space available. This is the case for the CP-conserving $K_S \to (\pi^+\pi^-\pi^0)_{I=0,2}$ transition, which being odd under the exchange of the $\pi^+\pi^-$ momenta ($v \to -v$), gives a vanishing contribution to the decay amplitude when integrated over the whole Dalitz plot (or at its centre): a measurement of $K_S \to \pi^+\pi^-\pi^0$ in such a region would be an indication of CP violation.

The following CP-violating quantities are usually defined:

$$\eta_{+-0} \equiv \left.\frac{A(K_S \to \pi^+\pi^-\pi^0)}{A(K_L \to \pi^+\pi^-\pi^0)}\right|_{u=v=0} \tag{110}$$

$$\eta_{000} \equiv \left.\frac{A(K_S \to 3\pi^0)}{A(K_L \to 3\pi^0)}\right|_{u=v=0} \tag{111}$$

$$\eta^v_{+-0} \equiv \left.\frac{\partial A(K_L \to \pi^+\pi^-\pi^0)/\partial v}{\partial A(K_S \to \pi^+\pi^-\pi^0)/\partial v}\right|_{u=v=0} \tag{112}$$

Since the CP-conserving $K_S \to \pi^+\pi^-\pi^0$ amplitude vanishes at the centre of the Dalitz plot due to Bose symmetry, η_{+-0} can also be defined as the ratio of the K_L to K_S transition amplitudes to the $CP = -1$ eigenstate.

Since $\pi^+\pi^-\pi^0$ is not a CP eigenstate, $K_S - -K_L$ interference in the partial decay rate to this final state is an indication for CP violation only when it is present after integration over the internal angle variables. The CP-conserving decay amplitude for $K_L \to \pi^+\pi^-\pi^0$ interferes with both the CP-conserving and the CP-violating K_S decay amplitudes to the same final state, but the interference terms are respectively odd and even in v, and this feature can be used to separately measure them: as mentioned, the CP-conserving term vanishes upon integration over the whole phase space, allowing the extraction of the CP-violating interference term

$$\eta_{+-0} = \frac{\int dudv\, A^*(K_L \to \pi^+\pi^-\pi^0)A(K_S \to \pi^+\pi^-\pi^0; CP = -1)}{\int dudv\, |A(K_L \to \pi^+\pi^-\pi^0)|^2} \tag{113}$$

In analogy to the case of $\pi\pi$ decays, one often writes

$$\eta_{+-0} = \epsilon + \epsilon'_{+-0} \qquad\qquad \eta_{000} = \epsilon + \epsilon'_{000} \tag{114}$$

in order to have parameters independent from the phase convention.

When the final state is an isospin eigenstate with definite permutation symmetry (which would be the case for $3\pi^0$ in absence of $\Delta I > 1/2$ transitions), or in absence of final-state interactions, one has (assuming CPT symmetry)

$$\epsilon'_f = i \left[\frac{\mathrm{Im}[A(K^0 \to f)]}{\mathrm{Re}[A(K^0 \to f)]} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \right] \tag{115}$$

where $A_0 \equiv A(K^0 \to (\pi\pi)_{I=0})$, for any $CP = +1$ eigenstate f. One can see that in this case a non-null imaginary part of η_f requires either CP violation in a decay amplitude or in the interference of mixing and decay. In the general case, the presence of either of these kinds of CP violation can originate a difference of η_f for different states f. The measurement of such a difference with high precision presents an experimental challenge, since cancellations of systematic effects cannot be usually achieved; while such a measurement cannot disentangle the two above types of CP violation, it would be an indication of direct CP violation.

The experimental study of CP violation in 3π decays of the neutral kaons requires intense sources of K_S mesons: such studies can be performed by analyzing decays from intense K^0 beams in the first few lifetimes after production (E621 at FNAL [83], NA48 at CERN [82]), by comparing K^0 and $\overline{K}^0$ decays of strangeness-tagged neutral kaons (CPLEAR at CERN [73]), or with tagged K_S at a ϕ factory (KLOE at DAΦNE [68]).

The very small expected branching ratios $O(10^{-9})$ make the detection of CP violation in these channels very challenging; present experiments were only able to put upper limits on the $\eta_{3\pi}$ parameters for K_S (η_{+-0}, η_{000}), without detecting CP violation even at the level of ϵ.

By using strangeness-tagged neutral kaon decays, the event yield asymmetry for opposite strangeness of the kaon at production time can be studied as a function of proper time t:

$$A_{CP}^{(3\pi)}(t) \equiv \frac{\Gamma(K^0 \to 3\pi) - \Gamma(\overline{K}^0 \to 3\pi)}{\Gamma(K^0 \to 3\pi) + \Gamma(\overline{K}^0 \to 3\pi)} = \\ -2\mathrm{Re}(\epsilon_S) + 2|\eta_{3\pi}| \cos(\Delta m t + \phi_{3\pi}) e^{-\Delta\Gamma t/2} \tag{116}$$

where $\phi_{3\pi}$ is the phases of $\eta_{3\pi}$). The value of $\eta_{3\pi}$ can be extracted by fitting the $K_S - -K_L$ interference term, while a non-zero asymmetry at $t = 0$ is an indication of direct CP violation.

While direct CP violation in $\pi\pi$ decays of neutral kaons is suppressed by the $\Delta I = 1/2$ rule, this need not be the case for other decay modes, but it turns out to be true also for the 3π mode, in first approximation. In the Standard Model, the parameters ϵ'_{+-0} and ϵ'_{000}, can be substantially enhanced over their lowest-order chiral perturbation theory estimate (the Li-Wolfenstein relation $|\epsilon'_{+-0}| \simeq |\epsilon'_{000}| \simeq 2|\epsilon'|$, valid at the centre of the Dalitz plot for $\pi^+\pi^-\pi^0$) due to the presence of higher-order contributions not suppressed by the $\Delta I = 1/2$ rule, but they still remain small with respect to ϵ (see [85] and references therein).

The measurements of η_{+-0} are dominated by the CPLEAR results obtained with this technique [86]:

$$\mathrm{Re}(\eta_{+-0}) = (-2 \pm 7^{+4}_{-1}) \cdot 10^{-3} \tag{117}$$

$$\mathrm{Im}(\eta_{+-0}) = (-2 \pm 9^{+2}_{-1}) \cdot 10^{-3} \tag{118}$$

corresponding to a limit $|\eta_{+-0}| < 17 \cdot 10^{-3}$ at 90% CL, which translates to $BR_{CPV}(K_S \to \pi^+\pi^-\pi^0) < 6.3 \cdot 10^{-8}$ for the CP-violating part. For η_{000} a preliminary NA48 result [82] gives

$$\text{Re}(\eta_{000}) = (-26 \pm 10 \pm 5) \cdot 10^{-3} \tag{119}$$
$$\text{Im}(\eta_{000}) = (-34 \pm 10 \pm 11) \cdot 10^{-3} \tag{120}$$

and when $\text{Re}(\eta_{000})$ is fixed to $\text{Re}(\epsilon)$ one has a limit $|\eta_{000}| < 29 \cdot 10^{-3}$ at 90% CL, corresponding to $BR(K_S \to 3\pi^0) < 3.1 \cdot 10^{-7}$.

CP violation (even of the indirect type) is not yet seen in these channels, and higher statistic experiments would be required for a search of direct CP violation.

In experiments at ϕ factories one could study the interference terms of the relative time intensity distributions when one kaon decays to 3π and the other semi-leptonically [68]; this approach is statistically more powerful since the interference term can be proportional to η instead of $|\eta|^2$. Using the same notation as for the discussion of $\pi\pi$ decays, the time-dependent asymmetry

$$A_{3\pi}(\Delta t) = \frac{I(3\pi, l^+\pi^-\nu;\Delta t) - I(3\pi, l^-\pi^+\overline{\nu};\Delta t)}{I(3\pi, l^+\pi^-\nu;\Delta t) + I(3\pi, l^-\pi^+\overline{\nu};\Delta t)} = \frac{2\text{Re}(\epsilon)e^{\Delta\Gamma\Delta t/2} - 2\text{Re}(\eta_{3\pi}e^{i\Delta m\Delta t})}{e^{\Delta\Gamma\Delta t/2} + |\Gamma_S(3\pi)/\Gamma_L(3\pi)|^2 e^{-\Delta\Gamma\Delta t/2}} \tag{121}$$

reduces to $2\text{Re}(\epsilon)$ for large positive Δt but has a larger sensitivity to $\epsilon'_{3\pi}$ for $\Delta t < 0$.

$K_{S,L} \to \pi\pi\gamma$

Among the final states accessible to the neutral kaons, the radiative $\pi^+\pi^-\gamma$ mode, with

$$BR(K_S \to \pi^+\pi^-\gamma) \simeq 1.8 \cdot 10^{-3} \tag{122}$$
$$BR(K_L \to \pi^+\pi^-\gamma) \simeq 4.4 \cdot 10^{-5} \tag{123}$$

has also been considered for searches of direct CP violation. Since $\pi\pi\gamma$ is not a pure CP eigenstate, CP violation cannot be detected as a simple violation of a selection rule, although as for all non-leptonic decay modes, the measurement of an interference effect between K_S and K_L is evidence [87] for CP violation[16].

As for all radiative decays, a (dominant) fraction of its rate, corresponding to low-energy photons, is unavoidably included in the measurements of the corresponding non-radiative mode, due to the finite energy threshold for photon detection [46].

The decay of neutral kaons to the $\pi^+\pi^-\gamma$ final state can occur through a so called *inner-bremsstrahlung* (IB) process, *i.e.* an electric odd-multipole photon emission from one of the pions in the $\pi^+\pi^-$ state. This electromagnetic process is CP-conserving, and therefore the (approximate) CP symmetry strongly hinders the $K_L \to \pi^+\pi^-\gamma$ decay to proceed in this way (the CP eigenvalue of $\pi\pi\gamma$ states is $(-1)^{l+1}$ for electric multipoles El and $(-1)^l$ for magnetic ones Ml, *i.e.* $CP = +1$ for E1, M2, ..., and $CP = -1$ for M1, E2, ...). Other contributions to $\pi^+\pi^-\gamma$ decays of neutral kaons due to radiation from the decay vertex can

[16]This is true when the final state is summed over internal angle variables and polarizations.

be present, and are usually called *direct emission* (DE) terms; these are generally smaller than IB, and lowest multipolarity (dipole, $l = 1$) terms strongly dominate. CP symmetry strongly suppresses any M1(E1) DE term in $K_S(K_L)$ decays to $\pi^+\pi^-\gamma$.

The photon energy spectra for the two types of emission (IB and DE) are different, therefore allowing the measurement of their relative importance in K_L decays. Experimental results [36] show that the DE term in K_L decays is mainly of M1 nature.

Any possible CP-violating asymmetry between π^+ and π^- in this decay would require the interference of $\pi\pi$ states of opposite parity, and therefore both odd and even multipoles; a DE E2 term is therefore the necessary ingredient to have such kind of effects, which are therefore expected to be strongly suppressed by the smallness of the multipole expansion parameter. Charge asymmetries of this kind larger than 2.4% are excluded at 90% CL by KTeV [78]. In the case in which the photon polarization is observed, a sign asymmetry in the angle between the direction of such polarization and the $\pi^+\pi^-$ plane also requires higher order multipoles. Ignoring the above asymmetries, only the lowest-order E1 and M1 DE terms can be considered, as will be done in the following.

While the $K_S \to \pi^+\pi^-\gamma$ decay is dominated by the unsuppressed IB process, $K_L \to \pi^+\pi^-\gamma$ receives two competing contributions: the IB, induced through the K_1 component (suppressed by the approximate CP symmetry), and the (intrinsically smaller, mainly M1) DE from the dominant K_2 component, which does not interfere with the previous one in the total rate.

A CP-violating amplitude ratio can be defined for a CP eigenstate such as

$$\eta_{+-\gamma} \equiv \frac{A(K_L \to \pi^+\pi^-\gamma; E1)}{A(K_S \to \pi^+\pi^-\gamma; E1)} \tag{124}$$

where only the lowest order multipole has been considered.

The proper decay time distribution of $\pi^+\pi^-\gamma$ decays from a mono-energetic neutral kaon beam is described by

$$\begin{aligned} I(\pi^+\pi^-\gamma; t) \propto C_S\, e^{-\Gamma_S t} + [C_L^{(IB)}|\eta_{+-\gamma}|^2 + C_L^{(DE)}]e^{-\Gamma_L t} + \\ C_{\rm int} D(p_K)|\eta_{+-\gamma}| \cos(\Delta m t - \phi_{+-\gamma})\, e^{-(\Gamma_S+\Gamma_L)t/2} \end{aligned} \tag{125}$$

where $\phi_{+-\gamma}$ is the phase of $\eta_{+-\gamma}$. The C_S term describes K_S (IB-dominated, E1) decays, and the $C_L^{(IB)}$ and $C_L^{(DE)}$ terms K_L decays due to IB (E1, indirect CP-violating) and DE (mostly CP-conserving M1, but possibly also direct CP-violating E1) respectively. The coefficient $C_{\rm int}$ of the interference term is multiplied by the "dilution factor" $D(p_K)$ describing the incoherent mixture of K^0 and $\overline{K}^0$ present in the beam at the production point, defined earlier.

In absence of direct CP violation, the dominant DE contribution to $K_L \to \pi^+\pi^-\gamma$ decay is of M1 multipolarity (CP conserving $K_2 \to \pi^+\pi^-\gamma$ decay) and does not interfere with the IB one (CP conserving E1 $K_1 \to \pi^+\pi^-\gamma$ decay), so that the $K_S - -K_L$ interference term $C_{\rm int}$ (due to E1 amplitudes) is unaffected by CP violation, which only modifies the coefficient of the $e^{-\Gamma_L t}$ term. In this case the CP-violating amplitude ratio $\eta_{+-\gamma}$ is equal to $\eta_{+-} \simeq \epsilon$, and the ratio of the E1 $\pi^+\pi^-\gamma$ branching ratio to the $\pi^+\pi^-$ branching ratio is clearly the same for K_S and K_L.

An E1 direct emission contribution from the decay of the dominant K_2 component of the K_L would be a direct CP-violating term, which would affect the interference term and shift the measured value of $\eta_{+-\gamma}$ away from ϵ.

Since the inner bremsstrahlung contribution is determined by the amplitude for the corresponding non-radiative decay $K_L \to \pi^+\pi^-$, one has, at first order in the ratio of DE/IB K_S decay amplitudes [11]:

$$\eta_{+-\gamma} = \eta_{+-} + \epsilon'_{+-\gamma} \simeq$$
$$\eta_{+-} + \frac{(\overline{\epsilon}-\eta_{+-})A(K_1\to\pi^+\pi^-\gamma;E1)+A(K_2\to\pi^+\pi^-\gamma;E1)}{A(K_S\to\pi^+\pi^-\gamma;E1(IB))} \tag{126}$$

Even if the direct CP violating term $\epsilon'_{+-\gamma}$ is not suppressed by the $\Delta I = 1/2$ rule as it happens for ϵ', it contains the small factor given by the ratio of the direct emission and the inner bremsstrahlung amplitudes; since also in this case the two interfering amplitudes have widely different magnitude, theoretical predictions for $\epsilon'_{+-\gamma}$ are in the 10^{-5} range [11].

Evidence for direct CP violation in $\pi^+\pi^-\gamma$ decays of neutral kaons could therefore be obtained by a non-zero difference in the measured fractions of radiative (IB) to non-radiative $\pi^+\pi^-$ decays for K_S and K_L: such a measurement would require a precise subtraction of the DE component in K_L decays and either a pure K_S beam or a precise subtraction of the K_L component in a mixed beam, both difficult at the level of accuracy required. A better approach is that of analyzing $\pi^+\pi^-\gamma$ decays at short proper times (close to production target or regenerator), where the interference term dominates on the K_L decay terms, and fit their proper decay time distributions to extract the $\eta_{+-\gamma}$ parameter, searching for differences from ϵ. In the limit in which K_S decays are given by the IB term only, the $\eta_{+-\gamma}$ parameter which is measured is actually the ratio of the CP-violating part of the K_L decay amplitude over the (CP-conserving) K_S decay amplitude. This kind of measurement was actually performed by the FNAL experiments E731 [88] and E773 [89] for decays downstream of a regenerator. The amount of CP violation measured is consistent with what is expected from indirect CP violation only [36]:

$$|\eta_{+-\gamma}| = (2.35 \pm 0.07) \cdot 10^{-3} \tag{127}$$
$$\phi_{+-\gamma} = (44 \pm 4)^\circ \tag{128}$$

and assuming any difference between $\eta_{+-\gamma}$ and η_{+-} to be due to direct CP violation, E731 [88] quoted

$$|\epsilon'_{+-\gamma}|/|\epsilon| < 0.3 \quad (90\%\, CL) \tag{129}$$

By measuring the polarization of the photon in the final state, more information could be obtained: in presence of M1 DE terms, the decay amplitudes depend on the photon polarization; however, assuming CPT symmetry and neglecting higher order multipoles, CP violation could induce a net photon polarization in pure $\pi^+\pi^-\gamma$ decays only if there are differences in the final-state interactions of the interfering two-pion states (with different isospin): approximate CP symmetry and IB dominance strongly suppress any such effect in K_S decays, while they could be present in K_L decays.

In the time evolution of a generic $K_S - -K_L$ mixture, an oscillating net photon polarization is expected, independently from CP violation effects. Downstream of a regenerator it would be possible to reach, for some given value

	$K_S \to \pi^+\pi^-e^+e^-$	$K_L \to \pi^+\pi^-e^+e^-$
BR	$(4.7 \pm 0.3) \cdot 10^{-5}$	$(3.38 \pm 0.13) \cdot 10^{-7}$
Asymmetry	$(0.5 \pm 4.3)\%$	$(13.5 \pm 1.5)\%$

Table 3: Experimental data on $K^0 \to \pi^+\pi^-e^+e^-$ decays, from [36] [95] [93].

of proper time, complete photon polarization; in presence of direct CP violation this value of proper time could be different for the two polarization states. The experimental difficulties of photon polarization measurements however make the above measurements less appealing (but see the following section on $\pi^+\pi^-e^+e^-$).

The $\pi^+\pi^-\gamma$ decay could provide new information on CP violation, although any direct CP-violating contribution is predicted to be rather small in the SM (*e.g.* the authors of [90] quote $|\epsilon'_{+-\gamma}/\epsilon| < 0.02$ at best).

In experiments with correlated kaon pairs at a ϕ factory one could exploit the time difference distribution for radiative $\pi^+\pi^-$ and semi-leptonic decays to extract information on $\eta_{+-\gamma}$ [68]:

$$I(\pi^\pm l^\mp\nu, \pi^+\pi^-\gamma; \Delta t < 0) \simeq \frac{\Gamma_L(\pi^\pm l^\mp\nu)\Gamma_S(\pi^+\pi^-\gamma)}{\Gamma_S+\Gamma_L} = \left[\frac{\Gamma(K_L\to\pi^+\pi^-\gamma)}{\Gamma(K_S\to\pi^+\pi^-\gamma)} e^{-\Gamma_L|\Delta t|} + e^{-\Gamma_S|\Delta t|} \pm 2e^{-(\Gamma_S+\Gamma_L)|\Delta t|/2}|\mathrm{Re}(\eta_{+-\gamma})|\cos(\Delta m|\Delta t| - \phi_{+-\gamma})\right] \quad (130)$$

For $K_{S,L} \to \pi^0\pi^0\gamma$ decays no IB contribution is present, and Bose symmetry forbids odd multipole contributions. The rates being highly suppressed (predicted BR $< 10^{-8}$ [11]), these decays (never observed so far) are not very promising for CP violation studies.

$K_{S,L} \to \pi^+\pi^-e^+e^-$

The decays $K_{S,L} \to \pi^+\pi^-e^+e^-$ are expected to proceed through an intermediate state $\pi^+\pi^-\gamma^*$, followed by internal conversion, and are therefore related to the $\pi^+\pi^-\gamma$ decays discussed above. Although suppressed by two orders of magnitude with respect to those, the decays discussed here have the advantage of giving easier experimental access to the polarization of the (virtual) photon, which can induce asymmetries in the orientation of the e^+e^- decay plane with respect to the $\pi^+\pi^-$ one.

The angular distribution in the angle ϕ between the normals to the $\pi^+\pi^-$ and e^+e^- planes can be parameterized as

$$\frac{d\Gamma}{d\phi} = I_1\cos^2\phi + I_2\sin^2\phi + I_3\sin\phi\cos\phi \quad (131)$$

where the $\sin\phi\cos\phi$ term, which changes sign for $\phi \to -\phi$, contains the interference between the two dominant contributions to the decay, the indirect CP-violating inner bremsstrahlung (E1) and the CP-conserving direct emission (M1). Other small contributions could come from a (direct CP-violating) E1 DE term and through a CP-conserving "charge-radius" $K_L \to K_S\gamma$ transition.

Table 3 summarizes the available experimental information: a large asymmetry ($\approx 14\%$) in the above-mentioned angle ϕ was predicted [91] and actually observed [92] in K_L decays, and not in K_S decays [93] as expected.

Introducing the angle θ_e between the three-momentum of the e^+ and that of the di-pion system, as measured in the e^+e^- rest frame, the differential decay rate can be written (dropping terms proportional to m_e^2) as [94]

$$\begin{aligned}\frac{d\Gamma}{d\cos\theta_e d\phi} = A_1 + A_2\cos 2\theta_e + A_3\sin^2\theta_e\cos 2\phi + A_4\sin 2\theta_e\cos\phi \quad + \\ A_5\sin\theta_e\cos\phi + A_6\cos\theta_e + \\ A_7\sin\theta_e\sin\phi + A_8\sin 2\theta_e\sin\phi + A_9\sin^2\theta\sin 2\phi \end{aligned} \tag{132}$$

where the terms with coefficients A_4, A_7 and A_9 are CP-violating. The only numerically relevant coefficient in the SM is A_9, which generates the decay plane asymmetry discussed above. While A_4 and A_9 are mainly induced by indirect CP violation, A_7 only contains direct CP violation contributions, which would induce an asymmetry in the distribution of decays in $\sin\theta_e\sin\phi$. Unfortunately the ratio of direct to indirect CP violation for this decay is predicted to be at most of order 10^{-3} [94] in the Standard Model. A measurement of direct CP violation from the angular distribution in the $K_L \to \pi^+\pi^-e^+e^-$ decay would require a much larger statistics than the current world data sample ($\sim 6\cdot 10^3$ events), besides a very accurate knowledge of the detection angular acceptance, and looks currently beyond reach.

CP violation effects can also show up in the rare decays of neutral kaons to four leptons, with [36] [96] [97]

$$BR(K_L \to e^+e^-e^+e^-) = (4.06 \pm 0.22)\cdot 10^{-8} \tag{133}$$

$$BR(K_L \to \mu^+\mu^-e^+e^-) = (2.7 \pm 0.3)\cdot 10^{-9} \tag{134}$$

in the distribution of the angle between the planes of the two l^+l^- pairs in the kaon rest frame. Such distributions have been actually measured with limited precision by KTeV [98] [96] [97], but the statistics does not allow to identify the presence of CP violation in these decays.

$K_L \to \pi^0 l\bar{l}$

An intense theoretical and experimental activity has been (and is being) devoted to the study of the decays $K_L \to \pi^0 l\bar{l}$ (where l is a lepton, e, μ or ν), since such loop-dominated decays (so far unobserved) are expected to be mostly CP-violating, with a direct-indirect CP violation hierarchy rather different from the one of $\pi\pi$ decays. Moreover, in some cases their properties can be related with reasonable confidence to the fundamental parameters of the theory, thus allowing sensitive searches for new physics. Contrary to the case of $\pi\pi$ decays, the presence of a single hadron in the final state reduces the theoretical difficulties in linking the elementary amplitudes, expressed in terms of quarks, to the measured ones for physical mesons: the hadronic part of the amplitude can be extracted from the experimental knowledge of the $K^+ \to \pi^0 e^+\nu$ decay process via isospin symmetry.

Within the Standard Model, the amplitudes for these decays receive three different kinds of contributions (see *e.g.* [11]). The first contribution, only present for final states with charged leptons, is the one induced by $K \to \pi\gamma^*\gamma^*$

transitions, the only one which can be CP-conserving. Theoretical predictions of such contribution are unreliable at present, but the experimental measurements of the $K_L \to \pi^0\gamma\gamma$ decay allow to bound it: two recent measurements of such decay mode [99] [100] are poorly consistent among them in this respect, and can lead to predictions for the contribution to $K_L \to \pi^0 l^+ l^-$ which can differ by an order of magnitude (see [101] and references therein). The CP conserving contribution is predicted to be similar for the electron and the muon modes.

The second and most interesting contribution is a short-distance "direct" CP violating one[17]; in the SM this component, dominated by diagrams with top-quark loops, can be predicted with good precision as a function of the CKM mixing matrix parameters, and is smaller by a factor $\simeq 5$ for the muon mode than for the electron one.

The third contribution is a CP-violating one, mostly induced by $K \to \pi\gamma^*(Z^*)$ transitions (l^+l^- in the $J^{PC} = 1^{--}$ state), which is largely a manifestation of indirect CP violation due to the K_1 component of K_L. This component is negligible for the final state with two neutrinos [103], since the diagram with a virtual Z is heavily suppressed ($\sim 10^{-7}$) with respect to that with a virtual photon. The latter cannot be predicted in a reliable way, but can be determined by a measurement of $K_S \to \pi^0 l\bar{l}$:

$$BR_{\mathrm{ind}}(K_L \to \pi^0 l^+ l^-) = |\epsilon|^2 \frac{\tau_L}{\tau_S} BR(K_S \to \pi^0 l^+ l^-) \tag{135}$$

For the electron mode, the K_S decay was measured by the NA48/1 experiment, which presented a preliminary result [104]

$$BR(K_S \to \pi^0 e^+ e^-) = (5.8^{+2.8}_{-2.3} \pm 0.3 \pm 0.8_{\mathrm{theo}}) \cdot 10^{-9} \tag{136}$$

where the first error is statistical, the second systematic, and the third one is due to the theoretical uncertainty in the extrapolation to the full phase space[18] This corresponds to

$$BR_{\mathrm{ind}}(K_L \to \pi^0 l^+ l^-) = 17.7^{+8.6}_{-6.9} \cdot 10^{-12} \tag{137}$$

for the indirect CP-violating contribution to the corresponding K_L decay. The indirect CP-violating amplitude can interfere with the direct one, so that a two-fold ambiguity remains in the relation between the branching ratio and the "direct" CP-violating contribution, depending on the relative sign of the two CP-violating components; the expectation for the total $K_L \to \pi^0 e^+ e^-$ branching ratio is now $1 \div 4 \cdot 10^{-11}$, but in case of destructive interference with the large indirect CP-violating component the branching ratio could have almost no sensitivity to the "direct" CP-violating part.

Standard Model predictions for the branching ratios of $K_L \to \pi^0 l\bar{l}$ decays are in the $10^{-11} - 10^{-12}$ range; those for the related $K_S \to \pi^0 l^+ l^-$ decay are in the $10^{-8} - 10^{-10}$ range; such expectations and the current experimental limits are summarized in table 4.

From the experimental point of view, the search for $K_L \to \pi^0 e^+ e^-$ requires high rate experiments with very good electromagnetic calorimetry and photon

[17] Since there is a single hadron in the final state, and negligible phases from final-state interactions, this is actually CP violation in the interference of mixing and decay, containing both direct and indirect CP violation [102].

[18] A form factor derived from [105] was used.

	$K_L \to \pi^0 e^+e^-$	$K_L \to \pi^0 \mu^+\mu^-$	$K_L \to \pi^0 \nu\overline{\nu}$
$BR_{\rm CPC}$	$0.3 \div 7 \cdot 10^{-12}$	$(0.3 \div 7) \cdot 10^{-12}$	0
$BR_{\rm CPV\ dir}$	$4 \cdot 10^{-12}$	$8 \cdot 10^{-13}$	$3 \cdot 10^{-11}$
$BR_{\rm CPV\ ind}$	$(0.15 \div 15) \cdot 10^{-11}$	$(0.3 \div 30) \cdot 10^{-12}$	$6 \cdot 10^{-15}$
$BR_{\rm tot}$	$(3 \div 10) \cdot 10^{-12}$	$(4 \div 10) \cdot 10^{-12}$	$3 \cdot 10^{-11}$
$BR_{\rm exp}$ (90% CL)	$< 2.8 \cdot 10^{-10}$	$< 3.8 \cdot 10^{-10}$ (†)	$< 5.9 \cdot 10^{-7}$ (†)
$BR(l^+l^-\gamma\gamma)$	$6 \cdot 10^{-7}$	$1 \cdot 10^{-8}$	–

Table 4: Indicative ranges of the Standard Model theoretical expectations ([106] and references therein), experimental limits from KTeV [36] [96] [107] [108] [109] for $K_L \to \pi^0 l\bar{l}$ branching ratios, and branching ratios of the $l\bar{l}\gamma\gamma$ background. (†): 1997 sample only.

detection. The largest backgrounds are due to the accidental superposition of $2\pi^0$ and $3\pi^0$ decays with single or double Dalitz decay of a π^0, and to the radiative Dalitz decay $K_L \to e^+e^-\gamma\gamma$ ("Greenlee" background [110], with $BR = (6.0 \pm 0.3) \cdot 10^{-7}$) which can only be reduced by the π^0 mass constraint on the $\gamma\gamma$ pair; the measurement of $K_L \to \pi^0 e^+e^-$ will ultimately require a reliable background subtraction and therefore an even higher sensitivity than dictated by the branching ratio.

For the $K_L \to \pi^0\mu^+\mu^-$ decays the background due to the muonic radiative Dalitz decay is a factor 60 smaller than for the e^+e^- mode, $BR(K_L \to \mu^+\mu^-\gamma\gamma) = (1.0 \pm 0.7) \cdot 10^{-8}$), but the kinematic cuts used to reduce it are also less effective, and the prediction for the "direct" CP-violating components is a factor ≈ 5 smaller.

As far as direct CP violation is concerned, the detection of $K_L \to \pi^0 l^+l^-$ decays needs to be complemented by accurate measurements of other kaon decays, since only the short-distance term can be predicted reliably. Additional experimental input, such as measurements of e^+e^- energy distribution asymmetries or the $K_S - -K_L$ interference term in the time dependence of the rate, would disentangle the different contributions.

The decay $K_L \to \pi^0\nu\overline{\nu}$ is the most interesting one: neglecting neutrino masses and assuming lepton flavour conservation, the final state is a pure $CP = +1$ eigenstate ($\nu\overline{\nu}$ pair produced in a state with angular momentum 1), so that there is no CP-conserving contribution, while the indirect CP-violating one is expected to be strongly suppressed in the Standard Model [103] and can be bound by the measurement of $K^+ \to \pi^+\nu\overline{\nu}$. The decay is therefore dominantly due to mixing-induced CP violation, and as such its decay rate can be predicted reliably. The process occurs in the SM through second-order weak interactions only: weak penguin diagrams and W-exchange box diagrams in which only the top-quark loops are relevant; the branching ratio can be predicted with very good ($\sim 10^{-2}$) accuracy, depending only on the top-quark mass, the strong coupling constant α_S and CKM matrix elements, so that its measurement could put strong constraints on the flavour mixing structure. With the present knowledge of the mixing matrix the prediction is $BR(K_L \to \pi^0\nu\overline{\nu})$=$(2.6 \pm 1.2) \cdot 10^{-11}$. An indirect limit can be obtained in a model-independent way from the measurement of the related $K^+ \to \pi^+\nu\overline{\nu}$ [111]: $BR(K_L \to \pi^0\nu\overline{\nu})$ <$1.7 \cdot 10^{-9}$.

Clearly, this channel is also the most challenging from an experimental point of view: its signature is a single π^0 in the detector, and to suppress the $K_L \to \pi^0\pi^0$ background with two missing photons, which has a branching ratio 10^8 times larger[19], a hermetic and highly efficient ($\sim 1\cdot 10^{-4}$) photon detector is required, but photo-nuclear interactions pose an intrinsic limit to the efficiency for low energy photons. The best branching ratio limit to date was obtained by the KTeV experiment [109], requiring the Dalitz decay of the π^0 in order to be able to reconstruct the decay vertex from the e^+e^- pair and cut on the $\gamma\gamma$ invariant mass. While the above requirement reduces the sensitivity by two orders of magnitude, without imposing it the same experiment obtained [112], in a 1-day special run with a "pencil" beam, a worse limit (by a factor 2.7), background-dominated by beam neutron interactions with the material in front of the detector. Extrapolations show that the use of the $\pi^0 \to \gamma\gamma$ decay mode and improvements in background suppression are required for future searches (see *e.g.* [113] for a recent review).

Two major projects are underway to search for this decay ([114] and [115], see also [116]) with an expected sensitivity of a few tens of events.

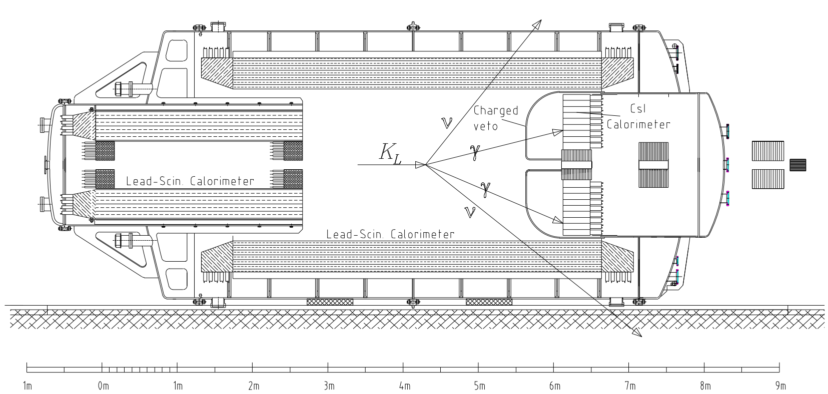

Figure 11: Schematic drawing of the E391a experimental apparatus.

One experiment is planned to run at the new J-PARC 50 GeV proton synchrotron at Tokai (beam expected in 2008); its approach is going to be tested by the E391a pilot project [114] running at the 12 GeV KEK PS in 2004. The experimental technique is based on a high transverse momentum ($p_T > 120$ MeV/c) π^0 selection, making use of a well collimated, low energy (2 GeV/c) "pencil" beam, free from hyperon background, entering a hermetic, highly evacuated double decay region, used to suppress the beam halo and to reject decays occurring upstream of the fiducial volume (see fig. 11). The advantage of this approach is the relatively high acceptance for the signal ($\sim 8\%$). Photon detection is based on a CsI calorimeter efficient down to energies of 1 MeV, and

[19] "Yesterday's signal is today's calibration and tomorrow's background".

the efficiency requirements on the lead-scintillator veto counters for low energy photons are reduced to the 10^{-4} level by suppressing the fully neutral $\pi^0\pi^0$ and $\gamma\gamma$ backgrounds with the p_T cut. The single event sensitivity of E391a is estimated to be above the level of the Standard Model predictions ($\sim 2 \cdot 10^{-10}$ with less than 1 background event), while 1000 events would be expected (for $BR = 3 \cdot 10^{-11}$) in the full-scale experiment (with $\sim 16\%$ acceptance).

The KOPIO (E926) experiment [115] at the 24 GeV BNL AGS, planned for 2006, aims at a full kinematic reconstruction of the event by using a 2 radiation length thick pre-radiator to measure photon directions, and an intense 800 MeV/c RF-microbunched neutral beam (200 ps wide bunches every 40 ns) to obtain the K_L momentum by time-of-flight measurement from the electromagnetic "shashlyk" calorimeter (see fig. 12). The soft spectrum for the neutral beam is such that K_L and neutrons are well below the π^0 hadro-production threshold, although the beam region will be in high vacuum. The beam will be very small in one of the two transverse dimensions, thus providing an additional constraint for the decay vertex reconstruction without limiting its acceptance. While also this approach reduces the requirements on the vetoing efficiency to manageable levels, a complete charged particle and photon veto system, including a forward "beam catcher" with aerogel Cerenkov counters, will complement the detector. With a $\sim 1.6\%$ acceptance, 50 signal events are expected in 3 years of running, with a signal to background ratio of 2.

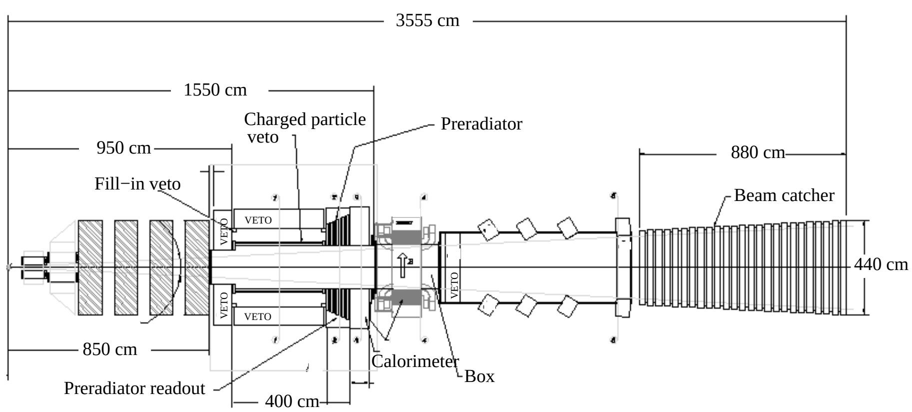

Figure 12: Schematic drawing of the KOPIO experimental apparatus.

Both collaborations have already shown encouraging results concerning the required beam and detector performances: E391a had a successful engineering run in 2002 on a neutral beam, during which the calorimeter was calibrated; KOPIO had results close and sometimes better than required on the beam micro-bunching and the pre-shower angular resolution.

For the $K_L \to \pi^0\mu^+\mu^-$ decay, the use of additional information obtained from the measurement of muon polarizations through their decay asymmetries can be conceived; this has been proposed [117] as a superior way to disentangle the different contributions and extract the CP violation in the decay amplitudes. Indeed, the P-odd longitudinal polarization of the μ^+ in the $\mu^+\mu^-$ rest frame is found for this decay to be proportional to the CP-violating phase of the

$K_2 \to \pi^0\gamma\gamma$ amplitudes[20]. Numerical estimates of the expected polarizations confirm that they can be large, as expected from the prediction of comparable sizes for the terms in the decay amplitude.

$K_{S,L} \to \gamma\gamma$ and $K_{S,L} \to l^+l^-$

The $\gamma\gamma$ final state from K^0 decay is not a CP-eigenstate, but a superposition of two CP-eigenstates which are distinguished by photon polarizations being parallel or orthogonal:

$$|(\gamma\gamma)_{CP=+1}\rangle \equiv |(\gamma\gamma)_{\parallel}\rangle = \frac{1}{\sqrt{2}}[|LL\rangle + |RR\rangle] \quad (138)$$

$$|(\gamma\gamma)_{CP=-1}\rangle \equiv |(\gamma\gamma)_{\perp}\rangle = \frac{1}{\sqrt{2}}[|LL\rangle - |RR\rangle] \quad (139)$$

where R, L indicate right or left photon helicity. One can define two CP-violating amplitude ratios:

$$\eta_+ = \frac{A(K_L \to (\gamma\gamma)_{CP=+1})}{A(K_S \to (\gamma\gamma)_{CP=+1})} = \epsilon + \epsilon'_+ \quad (140)$$

$$\eta_- = \frac{A(K_S \to (\gamma\gamma)_{CP=-1})}{A(K_L \to (\gamma\gamma)_{CP=-1})} = \epsilon + \epsilon'_- \quad (141)$$

Direct CP violation is expected to be larger by more than an order of magnitude than in $\pi\pi$ decays (see [11] and references therein), particularly for the $CP = -1$ final state.

The decay rates for K_S and K_L decays to this states are small and similar, since [36] [118] :

$$BR(K_S \to \gamma\gamma) = (2.77 \pm 0.07) \cdot 10^{-6} \quad (142)$$

$$BR(K_L \to \gamma\gamma) = (5.93 \pm 0.08) \cdot 10^{-4} \quad (143)$$

and therefore CP violation experiments are rather difficult.

One possibility for measuring CP violation in these decays would be the study of the time dependence of the decay rate in a CP eigenstate for strangeness-tagged beams: the $K_S - -K_L$ interference term changes sign for initially pure K^0 or $\overline{K}^0$. Unfortunately, the measurement of photon polarizations to study CP-eigenstates induces large suppression factors, making such experiments very difficult, given the small branching ratios. If photon polarizations are not measured, the total rates can be studied, in which CP-violating asymmetries between K^0 and $\overline{K}^0$ are also expected due to $K_S - -K_L$ interference: large amounts of tagged decays ($> 10^6$) would be required to measure any direct CP-violating effect, making also these measurements very challenging. Very intense kaon beams would allow such studies, by analyzing the decays in the first few K_S lifetimes from the production target.

The $K^0 \to \gamma\gamma$ transitions are also the main contribution to the $K^0 \to l^+l^-$ decay amplitudes, in proportion to the lepton mass. Only K_L decays to these channels have been observed so far (see table 5). CP-eigenstates can be defined as for the $\gamma\gamma$ final state. In presence of final-state interactions, CP-conserving and CP-violating amplitudes can interfere and lead to CP violation which can

[20] By comparison, the muon polarization transverse to the decay plane is (barring FSI effects) T-violating by itself, but does not allow to disentangle the indirect CP-violating contribution.

Decay	Branching ratio	Notes
$K_L \to e^+e^-$	$9^{+6}_{-4} \cdot 10^{-12}$	4 evts. (BNL E871 1998)
$K_L \to \mu^+\mu^-$	$(7.25 \pm 0.16) \cdot 10^{-9}$	$6.2 \cdot 10^3$ evts. (BNL E871 2000)
$K_S \to e^+e^-$	$< 1.4 \cdot 10^{-7}$ (90% CL)	(CPLEAR 1997)
$K_S \to \mu^+\mu^-$	$< 3.2 \cdot 10^{-7}$ (90% CL)	(CERN 1973)

Table 5: Experimental data [36] for the $K^0 \to l\bar{l}$ branching ratios.

be observed by measuring a non-zero asymmetry corresponding to a net longitudinal polarization of either lepton:

$$\langle P_l \rangle = \frac{N(l^-;R) - N(l^-;L)}{N(l^-;R) + N(l^-;L)} \tag{144}$$

where $N(l^-;R,L)$ are the numbers of l^- emitted with positive or negative helicity. In practice only μ^+ polarization can be measured; predictions of this asymmetry for $K_L \to \mu^+\mu^-$ decays are in the 10^{-3} range within the SM [11]; the direct CP-violating contribution is however estimated to be negligible with respect to the indirect one, expected at the level of $\sim 2 \cdot 10^{-3}$ (see [119] and references therein).

4 Charged K decays

For charged kaons, electric charge conservation forbids mixing: any difference between K^+ and K^- decay parameters would be a signal of (direct) CP violation. As usual, at least two interfering decay amplitudes with different strong and weak phases are required in order to generate an observable asymmetry. This rules out the dominant decay channels accounting for 93% of the decay width: leptonic and semi-leptonic decays, as well as $\pi\pi$ decays for which (neglecting small electromagnetic isospin-breaking effects [120] or possible violations of Bose statistics [48]) the final state can only be in the single $I = 2$ isospin state. CP violation effects can only be detected in other decay channels, *e.g.* in the 3π channels, which account for most of the remaining decay width; in this case however, the limited phase space available reduces the possible size of the strong phase shifts, and therefore of the expected asymmetries.

4.1 Partial rate asymmetries

The 3π states are the ones with largest branching ratios in which CP violation effects can be expected to be present.

The phenomenological description of these decays was discussed for the case of neutral kaons; the experimental values for branching ratios and Dalitz plot slopes are summarised in table 6.

The rate and slope asymmetries are defined as

$$A_\Gamma^{(f)} \equiv \frac{\Gamma(K^+ \to f) - \Gamma(K^- \to \overline{f})}{\Gamma(K^+ \to f) + \Gamma(K^- \to \overline{f})} \tag{145}$$

$$A_g^{(f)} \equiv \frac{g(K^+ \to f) - g(K^- \to \overline{f})}{g(K^+ \to f) + g(K^- \to \overline{f})} \tag{146}$$

	$K^+ \to \pi^+\pi^+\pi^-$	$K^- \to \pi^-\pi^-\pi^+$
BR	$(5.576 \pm 0.031)\%$	
g	(-0.2154 ± 0.0035)	(-0.217 ± 0.007)
h	(0.012 ± 0.08)	(0.010 ± 0.06)
k	(-0.0101 ± 0.0034)	(-0.0084 ± 0.0019)
	$K^+ \to \pi^0\pi^0\pi^+$	$K^- \to \pi^0\pi^0\pi^-$
BR	$(1.73 \pm 0.04)\%$	
g	(0.685 ± 0.033)	(0.642 ± 0.057)
h	(0.066 ± 0.024)	(0.064 ± 0.040)
k	(0.0197 ± 0.0054)	(0.006 ± 0.004)

Table 6: Experimental values of $K^\pm \to 3\pi$ branching ratios and decay parameters [36] [121]; see also [122].

$K^\pm \to \pi^\pm\pi^0\gamma$		
A_Γ	$(0.9 \pm 1.3)\%$	Direct, $\sim 4 \cdot 10^4$ evts. (Argonne 1969)
$K^\pm \to \pi^\pm\pi^\pm\pi^\mp$		
A_Γ	$(0.07 \pm 0.12)\%$	Direct, $3.2 \cdot 10^6$ evts. (BNL 1970)
A_g	$(-7.0 \pm 5.3) \cdot 10^{-3}$	Direct, $3.2 \cdot 10^6$ evts. (1970) (see also [122])
A_h	(0.1 ± 4.4)	
A_k	(0.09 ± 0.20)	
$K^\pm \to \pi^0\pi^0\pi^\pm$		
A_Γ	$(0.0 \pm 0.6)\%$	Direct, $\sim 1.6 \cdot 10^4$ evts. (Argonne 1969)
A_g	(0.032 ± 0.050)	(see also [123])
A_h	(0.02 ± 0.36)	
A_k	(0.53 ± 0.26)	
$K^\pm \to \mu^\pm\nu(\overline{\nu})$		
A_Γ	$(-0.54 \pm 0.41)\%$	Direct measurement, (BNL 1967)
$K^\pm \to \pi^\pm\pi^0$		
A_Γ	$(0.8 \pm 1.2)\%$	Direct measurement, $4 \cdot 10^3$ evts. (BNL 1973)
$K^\pm \to \pi^\pm\mu^+\mu^-$		
A_Γ	(-0.02 ± 0.12)	Direct measurement, $\sim 10^2$ evts. [124]

Table 7: Experimental data on direct CP-violating asymmetries in $K^\pm$ decays, from [36] unless otherwise noted.

and similarly for the other slope parameters. Due to the limited phase space available for the decays, quadratic slopes are generally smaller than the linear ones; as a consequence partial rate asymmetries, which do not get any contribution from the integral of the linear terms over the whole Dalitz plot, tend to be suppressed with respect to slope asymmetries. Standard Model predictions for such asymmetries are generally at the 10^{-4} level or below, while experimental limits are at least an order of magnitude higher as shown in table 7.

Several experiments in the '70s measured partial decay rate asymmetries for charged kaons, using either absolute $K^{\pm}$ flux normalisation obtained with differential Cerenkov counters, or exploiting other sets of inclusive decays as normalisation.

More recently, the HyperCP (E871) experiment at FNAL, dedicated to the study of CP violation asymmetries in hyperon decays, collected in 1997 and 1999 a large sample of $\pi^{\pm}\pi^{+}\pi^{-}$ decays of charged kaons ($\approx 3.9 \cdot 10^{8}$ K^{+} and $\approx 1.6 \cdot 10^{8}$ K^{-}). A small fraction ($\sim 10\%$) of these decays has been analyzed to measure the A_g slope asymmetry, resulting in a preliminary value [122]

$$A_g^{(\pi^{\pm}\pi^{+}\pi^{-})} = (2.2 \pm 1.5 \pm 3.7) \cdot 10^{-3} \tag{147}$$

the first error being statistical and the second systematic (in the denominator the value $2g$ from [36] was used). For this preliminary study the dominant systematic effects were induced by the knowledge of the magnetic fields, the efficiency differences of parts of the detector for charge-conjugate states, and secondary beam effects. The difference of K^{+} and K^{-} momentum spectra, as well as the different interactions of π^{+} and π^{-} in the spectrometer, were also found to be sources of spurious asymmetries.

The ISTRA+ experiment at Protvino collected both $K^{+} \to \pi^{+}\pi^{0}\pi^{0}$ and $K^{-} \to \pi^{-}\pi^{0}\pi^{0}$ decays, which are being analyzed. From a sample of $\sim 5 \cdot 10^{5}$ $K^{\pm}$ decays (50% of the available data sample) a preliminary result was reported [123]:

$$A_g^{(\pi^{\pm}\pi^{0}\pi^{0})} = (-0.3 \pm 2.5) \cdot 10^{-3} \tag{148}$$

where the quoted error is only the statistical one.

Two other experiments have as a main goal the measurement of slope asymmetries in charged kaon decays. The NA48/2 experiment [125] uses essentially the NA48 detector on a new beam line, with two simultaneous and collinear unseparated charged meson beams with narrow momentum spectrum (60 GeV/c $\pm$ 5%); a 400 GeV/c primary intensity of $1 \cdot 10^{12}$ protons per pulse provides $\sim 5 \cdot 10^{6}$ simultaneous $K^{\pm}$ entering the fiducial decay volume every 16.8 s. Even as the fluxes and compositions of the two beams are different, the concurrent detection of $\pi^{\pm}\pi^{+}\pi^{-}$ decays in the same detector, coupled with the periodic reversal of the spectrometer magnetic field, will allow a good cancellations of systematic effects in the measurement. A sample in excess of 10^{9} $\pi^{\pm}\pi^{+}\pi^{-}$ decays is expected in a 120 day run, leading to a statistical sensitivity of the order $2 \cdot 10^{-4}$ on A_g, with systematic effects kept under control at the same level. The experiment starts taking data in 2003, and will also collect significant samples of $\pi^{\pm}\pi^{0}\pi^{0}$ decays, for which the slope asymmetry can also be measured with similar sensitivity and rather different systematics.

The OKA experiment [126] will exploit a new Protvino U-70 PS RF-separated beam (based on CERN-Karlsruhe separators used at CERN in the 70's). A 70

GeV/c primary protons intensity of $1 \cdot 10^{13}$ per pulse will provide either $4 \cdot 10^6$ K^+ or $1.3 \cdot 10^6$ K^- of 12.5 or 18 GeV/c momentum entering the fiducial region every 9 s. An electromagnetic calorimeter based on lead glass and PWO crystals, and a beam spectrometer made of proportional chambers and drift tubes in a large aperture magnet, providing a 3 T·m field integral, will be the main elements of the detector, derived from the SPHINX, ISTRA+ and GAMS setup. Although no simultaneous $K^\pm$ beams will be available, the periodic change of polarities of all the beam-line elements will help in controlling many systematics. The beam line is under construction and the first beam is foreseen for 2004; $\sim 4 \cdot 10^{11}$ charged K decays are expected in 3 months of data taking. The estimated statistical error on the measurement of the CP-violating slope asymmetry in $K^\pm \to \pi^+\pi^+\pi^-$ decays is $\sim 1 \cdot 10^{-4}$ in a 3 months run.

In the radiative decay $K^\pm \to \pi^\pm\pi^0\gamma$ ($BR \simeq 2.8 \cdot 10^{-4}$), the inner bremsstrahlung contribution is suppressed by the $\Delta I = 1/2$ rule, and the interference with an electric dipole direct emission term could give rise to a rate asymmetry, or to asymmetries in the Dalitz plot or in the photon spectrum, indications of direct CP violation. Predictions for such asymmetries are however at most 10^{-4} in the SM [11], and the current experimental limit on the rate asymmetry is two orders of magnitude higher.

Similar estimates hold for the rate asymmetry of $K^\pm \to \pi^\pm\gamma\gamma$, which has an even smaller branching ratio ($BR \simeq 1.1 \cdot 10^{-6}$) and is experimentally obscured by a $\pi^+\pi^0$ background with a rate $2 \cdot 10^4$ times larger.

Rate asymmetries in the $K^\pm \to \pi^\pm e^+e^-$ decays (with $BR \simeq 3 \cdot 10^{-7}$) are also predicted at the $O(10^{-5})$ level in the Standard Model [127], and $K^\pm$ asymmetries in the e^+e^- invariant mass distributions in these decays, which could reach $\sim 10^{-4}$, are also out of reach of forthcoming experiments. For the $K^\pm \to \pi^\pm\mu^+\mu^-$ decay the measurement of asymmetries involving muon polarization could also be conceived (see following section), but the tiny branching ratio makes this approach unattractive. None of the above measurements looks therefore experimentally accessible at present.

4.2 T-odd correlation experiments

In semi-leptonic $K \to \pi l \nu$ (K_{l3}) decays, with large branching ratios, the V-A structure of the weak current only allows two form factors[21], which are relatively real if T symmetry holds. K_{e3} experiments are not sensitive to one of the form factors, due to the smallness of the electron mass with respect to the kaon mass, so that measurable CP violation effects can only be expected in $K_{\mu3}$ decays.

The lepton polarization in the direction transverse to the (π, l) decay plane is

$$P_T(l) \propto \langle \mathbf{S}_l \cdot \mathbf{p}_l \times \mathbf{p}_\pi \rangle \tag{149}$$

where $\mathbf{S}_l$ is the lepton polarization vector and $\mathbf{p}_l, \mathbf{p}_\pi$ the lepton and pion three-momenta respectively. The above quantity is odd under the so-called "naive" time reversal (inversion of spin and momenta), and violates time reversal invariance if final-state interactions are neglected, being proportional to the imaginary part of the form factors ratio $\text{Im}(\xi)$. In $K^\pm \to \pi^0 l^\pm \nu$ decays, such final-state interactions are expected to be very small in the Standard Model ($P_T < 10^{-5}$), and a significant non-zero transverse polarization measurement in this decay

[21] No experimental evidence of scalar or tensor form factors was detected so far.

would be both a true signal of time reversal violation (and therefore of direct CP violation if CPT symmetry is valid), and indication for new physics. The transverse polarization induced by Standard Model final-state interactions in $K_L \to \pi^{\mp} l^{\pm} \nu(\overline{\nu})$ decays is larger $O(10^{-3})$, at the level of accuracy reached by experiments in the '80s.

In practice only μ^+ polarization can be measured reliably, so that experiments have been performed both in $K_L \to \pi^- \mu^+ \nu$ and $K^+ \to \pi^0 \mu^+ \nu$ decays.

The more accurate experiments use detectors with cylindrical symmetry, in which kaons decay in flight [128] or (for K^+) after stopping in a target [129]. In both cases events in which the detector (and beam) axis lies in the $(\mathbf{p}_\mu, \mathbf{p}_\pi)$ decay plane are selected. Toroidal magnetic fields guide decay muons to stop into polarimeters, without affecting the transverse component of their polarization; the muon polarization is correlated to the positron emission direction in its decay, and any component transverse to the decay plane induces an asymmetry in the measurement of the left and right (counterclockwise and clockwise with respect to the beam direction) positron counters. In decay in-flight experiments, muons were made to precess in a magnetic field parallel to the beam line before their decay was detected, thus eliminating systematic differences in detector efficiencies at the price of a lower analyzing power; left-right asymmetries in the polarimeters could be controlled by the accurate reversal of the axial precession magnetic field, which determined one of the dominant systematic uncertainties. Also, events with different orientation of the decay plane are expected to exhibit opposite asymmetries, thus providing a powerful systematic check.

The most recent experiment, E246 at KEK [129], analyzed $\sim 8.3 \cdot 10^6$ $\pi^0 \mu^+ \nu$ decays of 660 MeV/c K^+ stopped in a scintillating fibre target, extracting the asymmetry from a double ratio of clockwise and anticlockwise event rates for events with opposite decay plane orientations, in order to reduce the sensitivity to systematic errors which can potentially induce spurious asymmetries. Only experimental effects introducing a net screw asymmetry around the beam direction can mimic T violation: the largest systematic errors in this experiment arise from the asymmetry in the magnetic field shapes and the misalignment of the polarimeter.

Experiments have given null polarization results so far [130] [131]

$$P_T(\mu) = (1.7 \pm 5.6) \cdot 10^{-3} \qquad (K_L \to \pi^- \mu^+ \nu) \tag{150}$$

$$P_T(\mu) = (-1.12 \pm 2.34) \cdot 10^{-3} \qquad (K^+ \to \pi^0 \mu^+ \nu) \tag{151}$$

corresponding to [36] [131]

$$\mathrm{Im}(\xi)_{K_L} = -0.007 \pm 0.026 \tag{152}$$

$$\mathrm{Im}(\xi)_{K^+} = -0.0028 \pm 0.0075 \tag{153}$$

Improved experiments have been proposed, which could reach $\sim 1 \cdot 10^{-4}$ error on the transverse polarization, a factor ~ 20 better than achieved by present experiments.

Measurements of transverse muon polarization in other decays have been considered in the literature (see [119] for a thorough discussion). $K^+ \to \mu^+ \nu \gamma$ decays ($BR \simeq 5.5 \cdot 10^{-3}$) can be studied in the experiments designed to collect $K^+ \to \pi^0 \mu^+ \nu$, with minor changes to the setup; transverse polarizations at the 10^{-4} level are expected in the Standard Model due to final-state interactions,

Decay	BR	Notes
$K_L \to \pi^\pm e^\mp \nu\gamma$	$(3.53 \pm 0.06) \cdot 10^{-3}$	$15 \cdot 10^3$ evts. (KTeV 2001)
$K_L \to \pi^\pm \mu^\mp \nu\gamma$	$(5.7 \pm 0.7) \cdot 10^{-4}$	$2.5 \cdot 10^2$ evts. (NA48 1998)
$K^\pm \to \pi^0 e^\pm \nu\gamma$	$(2.65 \pm 0.20) \cdot 10^{-4}$	192 evts. (ISTRA 1986)
	$< 5.3 \cdot 10^{-5}$ (DE)	
$K^\pm \to \pi^0 \mu^\pm \nu\gamma$	$< 6.1 \cdot 10^{-5}$ (90% CL)	(Argonne 1973)

Table 8: Experimental data [36] for the radiative semi-leptonic decays of kaons.

and the KEK E246 experiment recently reported the first measurement of such quantity [132]:

$$P_T(\mu) = (-0.64 \pm 1.85) \cdot 10^{-2} \quad (K^+ \to \mu^+ \nu\gamma) \tag{154}$$

For the $K^+ \to \pi^+\mu^+\mu^-$ decay ($BR \sim 8 \cdot 10^{-8}$) the contribution to the transverse muon polarization due to Standard Model final state interactions can be $\sim 10^{-3}$, while spin-correlations involving both muons' polarizations are cleaner signals of T violation but prohibitive from an experimental point of view, because of the tiny branching ratio and the large background from $K^+ \to \pi^+\pi^+\pi^-$.

T-odd correlations (in the sense explained above) involving only momenta can be studied in 4-body decays with distinguishable particles in the final state: an example are the radiative semi-leptonic decays $K \to \pi l \nu\gamma$, which have branching ratios in the $10^{-3} \div 10^{-4}$ range for K_L decays and in the $10^{-4} \div 10^{-5}$ range for $K^\pm$ decays. For K_L, electromagnetic final-state interactions could be expected to induce a fake signal, which should be largely absent for $K^\pm$ decays. Unfortunately any CP violating effect in such radiative decays is suppressed by the dominance of the inner bremsstrahlung process; the necessary direct emission component, not yet observed in these decays, is expected at the level of a few percent of the former, and generally larger for the muonic decays. A measurement of the T-odd asymmetry in the $\mathbf{p}_\pi \cdot \mathbf{p}_e \times \mathbf{p}_\gamma$ distribution for $K^- \to \pi^0 e^- \nu\gamma$ decays by the ISTRA experiment [133] gave a null result: 0.03 ± 0.08 with 192 events. The OKA experiment at Protvino [126] plans to measure the T-odd asymmetry in the $\mathbf{p}_\pi \cdot \mathbf{p}_\mu \times \mathbf{p}_\gamma$ distribution for the (so far unobserved) $K^+ \to \pi^0 \mu^+ \nu\gamma$ decays (the predicted branching ratio is $\sim 2 \cdot 10^{-5}$ in the SM [134]), on an expected sample of $\sim 10^5$ events.

Table 8 summarizes the experimental information available on these decays.

CP violation measurements in other decays of charged kaons have been considered in the literature, such as the rare process $K^\pm \to \mu^\pm \nu e^+ e^-$ ($BR \simeq 1.3 \cdot 10^{-7}$), in which T-odd correlations related to muon polarization could be probed. Such measurements are however very difficult in practice, due either to large backgrounds or tiny branching ratios, which make polarization experiments prohibitive[22]

As already mentioned, independently from final-state interaction effects, an unambiguous direct CP violation signal could be obtained by measuring a difference in the magnitude of any T-odd correlation for K^+ and K^-.

[22]The BNL E865 experiment [135] recently collected $2.2 \cdot 10^3$ $K^\pm \to \mu^\pm \nu e^+ e^-$ decays, a 150-fold increase on the previous world sample, which could allow some CP violation studies to be performed.

5 Other meson systems

5.1 Phenomenology

Heavy flavoured meson systems, *i.e.* not self-conjugate mesons containing quarks heavier than the strange one, are the subject of very active investigations. In particular B meson decays, in which the effects of the three quark generations can be present at tree level, are a promising arena for the search of CP violation effects, since the asymmetries are less suppressed by the smallness of quark family mixing. The focus of the studies are the mixing-induced CP-violating decay rate asymmetries for neutral mesons, which in some cases can be expressed in a reliable way in terms of the elementary phases in the theory. Unfortunately, the measurements of such asymmetries involve specific final states, whose branching ratios for such high mass systems are usually very small, requiring large statistics to be experimentally detected, this being the reason for the building of the so-called "B factories". Also in heavy meson systems direct CP violation effects are difficult to predict theoretically from first principles, due to the non-perturbative physics of hadronization and final-state interactions.

The quantity $x \equiv \Delta m/\overline{\Gamma}$ (where $\overline{\Gamma}$ is the average decay width of the two mass eigenstates) determines whether the flavour oscillations are observable; it can have rather different values for the various neutral meson systems [36]:

$$x_K = 0.948 \tag{155}$$

$$x_D < 2.9 \cdot 10^{-2} \quad (95\% \, CL) \tag{156}$$

$$x_{B_d} = 0.755 \tag{157}$$

$$x_{B_s} > 19 \quad (95\% \, CL) \tag{158}$$

When compared to kaons, neutral B mesons have a much larger set of decay modes available, due to their larger mass, but they allow a more limited variety of experimental approaches for their study: the reason is that for them

$$\left|\frac{\Delta\Gamma}{\Delta m}\right| \sim \left|\frac{\Gamma_{12}}{M_{12}}\right| \ll 1 \tag{159}$$

This can be understood in a simple way as due to the fact that the width difference originates from the physically accessible decay modes common to M and $\overline{M}$, which are only a very small fraction of the total available to the heavy meson.

While for lighter mesons $\Delta F = 2$ transitions cannot be computed in a reliable way, for heavy ones such as the neutral B, with masses distant from the region of hadronic resonances, the mass and lifetime differences, linked to the off-diagonal elements of the effective Hamiltonian, are more tractable from a theoretical point of view, and the relation (159) can be shown to be valid in a rather model-independent way.

Such relation implies that $|y| \equiv |\Delta\Gamma/\overline{\Gamma}| \sim |x\Gamma_{12}/M_{12}|$ is also very small for B_d mesons, and could reach 1-10% for B_s mesons depending on the value of x_s. Experimentally [36], for heavy mesons:

$$(|\Delta\Gamma|/\overline{\Gamma})_D = (0.003 \pm 0.022) \tag{160}$$

$$(|\Delta\Gamma|/\overline{\Gamma})_{B_d} = (0.008 \pm 0.041) \tag{161}$$

$$(|\Delta\Gamma|/\overline{\Gamma})_{B_s} < 0.3 \tag{162}$$

The two physical states have therefore very similar decay widths and it is not possible to perform experiments with just one of them. For this reason the formalism used for heavy mesons does not use the amplitudes for physical states and their ratios η_f, but only the amplitudes for flavour eigenstates.

The relation among $|\Gamma_{12}|$ and $|M_{12}|$ also implies that the measure of CP violation in the mixing

$$1 - \left|\frac{1-\overline{\epsilon}}{1+\overline{\epsilon}}\right| = 1 - \left|\frac{q}{p}\right| \simeq \frac{1}{2}\left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\left[\arg\left(\frac{\Gamma_{12}}{M_{12}}\right)\right] \tag{163}$$

has a sensitivity to the CP-violating phase difference of the off-diagonal terms M_{12} and Γ_{12} which is suppressed by the smallness of $|\Gamma_{12}/M_{12}| \sim |\Delta\Gamma/\Delta m| \ll 1$.

CP-violating decay rate asymmetries are defined in general as[23]

$$A_{CP}^{(f)}(t) \equiv \frac{\Gamma(\overline{M} \to \overline{f}; t) - \Gamma(M \to f; t)}{\Gamma(\overline{M} \to \overline{f}; t) + \Gamma(M \to f; t)} \tag{164}$$

To measure CP violation in $M - -\overline{M}$ mixing, one can measure the meson flavour at a given time and exploit a decay mode (or set of modes) which cannot support CP violation in the decay amplitudes.

The time-dependent asymmetry of total (inclusive) decay rates for flavour-tagged mesons gives such a measure of CP violation in the mixing:

$$A_{CP}^{(\mathrm{incl})}(t) = \frac{\Gamma(\overline{M}(t) \to \mathrm{all}) - \Gamma(M(t) \to \mathrm{all})}{\Gamma(\overline{M}(t) \to \mathrm{all}) + \Gamma(M(t) \to \mathrm{all})} =$$
$$\frac{2\mathrm{Re}(\overline{\epsilon})}{1+|\overline{\epsilon}|^2}\left[\frac{-\cosh(\Delta\Gamma t/2) + y\sinh(\Delta\Gamma t/2) + \cos(\Delta m t) + x\sin(\Delta m t)}{\cosh(\Delta\Gamma t/2) - y\sinh(\Delta\Gamma t/2)}\right] \tag{165}$$

where $M(t), \overline{M}(t)$ indicate states which were flavour-tagged as $M, \overline{M}$ at $t = 0$; expression (165) is valid at first order in $2\mathrm{Re}(\overline{\epsilon})/(1 + |\overline{\epsilon}|^2)$. The time-integrated version vanishes by CPT symmetry, and in the limit $y \simeq 0$ (which is expected to be a good approximation for B mesons):

$$A_{CP}^{(\mathrm{incl})}(t) \simeq \frac{2\mathrm{Re}(\overline{\epsilon})}{1+|\overline{\epsilon}|^2}[x\sin(\Delta m t) - 2\sin^2(\Delta m t/2)] \tag{166}$$

The partial decay rate asymmetry for "wrong" (mixed) flavour-specific decays of flavour-tagged mesons, *i.e.*

$$A_{CP}^{(M)}(t) = \frac{\Gamma(\overline{M}(t) \to f) - \Gamma(M(t) \to \overline{f})}{\Gamma(\overline{M}(t) \to f) + \Gamma(M(t) \to \overline{f})} = \frac{|p/q|^2|A_f|^2 - |q/p|^2|\overline{A}_{\overline{f}}|^2}{|p/q|^2|A_f|^2 + |q/p|^2|\overline{A}_{\overline{f}}|^2} \tag{167}$$

where $M, \overline{M}$ cannot decay to $f, \overline{f}$ respectively, is seen to be independent of time. While this asymmetry cannot separate CP violation in the mixing and in the decay, it can happen that, due to the absence of final-state interactions, CPT symmetry itself imposes $|A_f| = |\overline{A}_{\overline{f}}|$ so that the latter type of CP violation is not possible: this is the case for semi-leptonic decays in the Standard Model.

[23] Note the conventional sign difference with respect to previous definitions, originating from the fact that the flavour of the decaying meson is usually determined by the *opposite* flavour of an associated particle produced in the same reaction.

The above asymmetry (167) is then a pure measurement of CP violation in the mixing:

$$A_{CP}^{(M)} = \frac{1-|q/p|^4}{1+|q/p|^4} = \frac{4\mathrm{Re}(\overline{\epsilon})}{1+|\overline{\epsilon}|^2} \qquad (|A_f| = |\overline{A}_{\overline{f}}|) \tag{168}$$

The asymmetry for "right" (unmixed) flavour-specific decays gives instead

$$A_{CP}^{(U)}(t) = \frac{\Gamma(\overline{M}(t)\to\overline{f}) - \Gamma(M(t)\to f)}{\Gamma(\overline{M}(t)\to\overline{f}) + \Gamma(M(t)\to f)} = \frac{|\overline{A}_{\overline{f}}|^2 - |A_f|^2}{|\overline{A}_{\overline{f}}|^2 + |A_f|^2} \tag{169}$$

which however in many cases is forced to be null by CPT symmetry.

In the case of neutral mesons, apart from time-integrated asymmetries, the analysis of mixing-induced CP violation can also give information on direct CP violation. The time-dependent asymmetry for decays to a common CP eigenstate f, for mesons tagged as $\overline{M}$ or M (at time $t=0$), can have a non-zero value when any kind of CP violation is present. Neglecting terms of second order in the CP-violating parameters, the full expression for such asymmetry is

$$A_{CP}^{(f)}(t) \simeq \frac{\mathcal{A}_{CP}^{(\mathrm{mix})}\cosh(\Delta\Gamma t) + \mathcal{A}_{CP}^{(\mathrm{m/d})}\cos(\Delta m t) + \mathcal{A}_{CP}^{(\mathrm{int})}\sin(\Delta m t)}{\cosh(\Delta\Gamma t/2) - \mathcal{A}_{\Delta\Gamma}\sinh(\Delta\Gamma t/2)} \tag{170}$$

where $\mathcal{A}_{CP}^{(\mathrm{mix})}$ expresses CP violation in the mixing, $\mathcal{A}_{CP}^{(\mathrm{m/d})}$ CP violation in either the mixing or in the decay amplitudes, $\mathcal{A}_{CP}^{(\mathrm{int})}$ CP violation in the interference of mixing and decay, and $\mathcal{A}_{\Delta\Gamma}$ is induced by the decay width difference.

The expression for the pure mixing term is

$$\mathcal{A}_{CP}^{(\mathrm{mix})} = \frac{2\mathrm{Re}(\overline{\epsilon})}{1+|\overline{\epsilon}|^2} = \frac{1-|q/p|^2}{1+|q/p|^2} \tag{171}$$

If CP violation in the mixing is small ($|q/p| \simeq 1$), at least when compared to the mixing-induced one, this term can be neglected; this is proved to be a good approximation for B_d mesons [136], so that when considering this system one often puts $\mathcal{A}_{CP}^{(\mathrm{mix})} = 0$.

The expressions for the other terms appearing in the asymmetry are

$$\mathcal{A}_{CP}^{(\mathrm{m/d})} = \frac{|\lambda_f|^2-1}{|\lambda_f|^2+1} \tag{172}$$

$$\mathcal{A}_{CP}^{(\mathrm{int})} = \frac{2\mathrm{Im}(\lambda_f)}{|\lambda_f|^2+1} \tag{173}$$

$$\mathcal{A}_{\Delta\Gamma} = \frac{2\mathrm{Re}(\lambda_f)}{|\lambda_f|^2+1} \tag{174}$$

For a final state f it is convenient to introduce the complex parameter

$$\lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f} = \left(\frac{1-\overline{\epsilon}}{1+\overline{\epsilon}}\right)\frac{\overline{A}_f}{A_f} = \eta_{CP}(f)\frac{q}{p}\frac{\overline{A}_{\overline{f}}}{A_f} \tag{175}$$

(independent from the choice of phase convention for the $M, \overline{M}$ states) where $\overline{\epsilon}$ is the CP impurity parameter appearing in the expression of physical states in terms of flavour eigenstates, and $A_f \equiv A(M \to f)$, $\overline{A}_f \equiv A(\overline{M} \to f)$. The last equality refers to a CP eigenstate with eigenvalue $\eta_{CP}(f)$, and in this case λ_f is a measure of CP violation; one can have a non-zero CP-violating decay asymmetry if the phase of λ_f is different from 0 or π, or if its modulus is different from 1. The first case can arise if the "mixing" phase

$$\phi_M \equiv \arg\left(\frac{1-\overline{\epsilon}}{1+\overline{\epsilon}}\right) \tag{176}$$

is different from the "decay" phase

$$\phi_D^{(f)} \equiv \arg\left(\frac{\overline{A}_f}{A_f}\right) \tag{177}$$

so that $\mathcal{A}_{CP}^{(\mathrm{int})} \neq 0$. This is mixing-induced CP violation (which can be both direct and indirect), as can be seen from the fact that the corresponding term in the decay asymmetry only builds up in time starting from zero at $t=0$, as $M--\overline{M}$ flavour mixing gets in effect.

The second case can arise either because $|(1-\overline{\epsilon})/(1+\overline{\epsilon})| \neq 1$, *i.e.* (indirect) CP violation in the mixing due to the CP impurity of the mass eigenstates (independent of the decay channel f), or because $|\overline{A}_f/A_f| \neq 1$, *i.e.* (direct) CP violation in the decay due to the interference of competing amplitudes with different phases (possibly specific to the decay mode f); in both cases $\mathcal{A}_{CP}^{(\mathrm{m/d})} \neq 0$. Clearly, both effects can also be present at the same time (while the two ratios entering the definition of λ_f have unphysical phases which depend on the convention chosen for the $M, \overline{M}$ states, their moduli have a physical meaning), and they both give rise to a decay asymmetry which is maximal at $t=0$, and gets washed out with time, due to flavour mixing.

If the width difference is neglected ($y \simeq 0$, see also [136]) one can set $\mathcal{A}_{\Delta\Gamma}=0$, and this assumption will be made in what follows. The expression for the asymmetry in absence of CP violation in the mixing then reduces to a sum of two terms, one proportional to the cosine and one to the sine of $\Delta m\, t$, whose coefficients are bounded to lie within the $\mathcal{A}_{CP}^{(\mathrm{m/d})2} + \mathcal{A}_{CP}^{(\mathrm{int})2} \leq 1$ region. When integrating over a time which is long on the time scale of flavour mixing ($t \gg 1/\Delta m$), both oscillating terms get averaged to zero; this is experimentally the case when the flavour mixing period is not much larger than the lifetime of the meson.

If CP violation in the mixing is small (as is the case for B^0, D^0 mesons in the Standard Model) one can write

$$\frac{q}{p} = \frac{1-\overline{\epsilon}}{1+\overline{\epsilon}} = e^{i\phi_M} \tag{178}$$

as a pure phase: $\overline{\epsilon}$ is then purely imaginary, and as such it can be redefined to zero by a flavour rotation.

In presence of a single elementary decay amplitude (or of several amplitudes with the same phase), CP violation in the decay cannot be present and the ratio of $\overline{M}, M$ amplitudes is also a pure phase

$$\frac{\overline{A}_f}{A_f} = \eta_{CP}(f) e^{i\phi_D^{(f)}} \tag{179}$$

When these two conditions are satisfied the cosine term in $A_{CP}^{(f)}(t)$ is absent, and the decay asymmetry directly measures the (sine of the) sum of the phases of the $M \leftrightarrow \overline{M}$ mixing process and the $M \to f$ decay process:

$$A_{CP}^{(f)}(t) = \eta_{CP}^{(f)} \sin[\phi_M + \phi_D^{(f)}] \sin(\Delta m\, t) \tag{180}$$

When considering decays to a CP self-conjugate set of quarks, the final state is in general a superposition of states with opposite CP eigenvalues, contributing opposite mixing-induced asymmetries which partially cancel; this is only possible when there is more than one particle with non-zero spin in the final state, or more than two particles, otherwise angular momentum conservation forces the CP-parity of the final state to a single value.

It is seen that, in the limit in which CP violation in the mixing is negligible, the appearance of a non-zero cosine term in the decay asymmetry is an indication of direct CP violation, due to interfering amplitudes with different phases. In this case the quantity λ_f contains the strong phases of the two elementary amplitudes, and therefore it is much harder to relate to the dynamical parameters of the underlying decay; at the same time the coefficient of the sine term is no longer simply related to the phases of the weak amplitudes (a product of elements of the quark mixing matrix in the SM). The largest direct CP violation effects are expected for decays to which two amplitudes of comparable magnitude contribute, *i.e.* when the lowest order contribution is suppressed.

Time-integrated decay asymmetries are measures of direct CP violation for neutral or charged mesons:

$$A_{CP}^{(f)} = \int_0^\infty dt\, A_{CP}^{(f)}(t) = \frac{1-y^2}{1+x^2} \frac{|\lambda_f|^2 - 1 + 2x\,\mathrm{Im}(\lambda_f)}{|\lambda_f|^2 + 1 - 2y\,\mathrm{Re}(\lambda_f)} \tag{181}$$

which are however suppressed if x is much larger than 1 or y^2 is close to 1.

CP-violating decay asymmetries can also be formed with untagged decays (see *e.g.* [4]), but their measurement cannot disentangle CP violation in the mixing and in the decay amplitudes. The untagged time-integrated asymmetry is

$$A_{CP}^{(M)} = -\frac{\mathcal{A}_{CP}^{(\mathrm{mix})}(x^2+y^2) - \mathcal{A}_{CP}^{(\mathrm{dir})}[1 + x^2 - \mathcal{A}_{CP}^{(\mathrm{mix})2}(1-y^2)]}{1 + x^2 - \mathcal{A}_{CP}^{(\mathrm{mix})2}(1-y^2) + \mathcal{A}_{CP}^{(\mathrm{dir})}\mathcal{A}_{CP}^{(\mathrm{mix})}(x^2+y^2)} \tag{182}$$

where

$$\mathcal{A}_{CP}^{(\mathrm{dir})} = \frac{|A_f|^2 - |\overline{A}_f|^2}{|A_f|^2 + |\overline{A}_f|^2} \tag{183}$$

Direct CP violation in heavy meson decays could also be studied without using any flavour tag, and therefore with a large statistical advantage, by analysing their decays to self-conjugate final states in which distinct, CP-conjugate resonances can be identified [137]: an example is $B^0 \to \pi^+\pi^-\pi^0$ in which the presence of both $\rho^+\pi^-$ and $\rho^-\pi^+$ intermediate states can allow the manifestation of direct CP violation, as Dalitz plot asymmetries, even in absence of any strong phases.

The study of decay asymmetries for inclusive modes has been proposed [138], not only because of the larger event yields, but also because by summing over all the states corresponding to a set of quantum numbers which are conserved by the strong interactions, the knowledge of the final-state interaction phases is not required to compute the asymmetries. For the same reason, though, such asymmetries do not probe direct CP violation.

Considering decays of coherent $M\overline{M}$ meson pairs, the asymmetries are expressed as a function of Δt, the time difference between the decay of the meson and that of its companion used to determine its flavour (at that time); the formulæare the same as above, with t replaced by Δt, which can also be negative. Mixing-induced CP violation can be observed as an asymmetry of the A_{CP} distributions with respect to $\Delta t = 0$ for decays of mesons of either flavour. When integrating such distributions symmetrically over Δt to get the total rate, mixing-induced CP violation, proportional to sin($\Delta m \Delta t$, cannot give an observable effect, while the other two kinds of CP violation can.

In experiments using $M\overline{M}$ pairs, CP violation in the mixing is measured by comparing the yields of positive and negative same-sign di-lepton events originating from (flavour-specific) semi-leptonic decays (for which no direct CP violation is present):

$$A_{CP}^{(l)} = \frac{N(l^+l^+) - N(l^-l^-)}{N(l^+l^+) + N(l^-l^-)} \tag{184}$$

For antisymmetric $M\overline{M}$ pairs, such as the ones produced by the decay of a vector quarkonium resonance ($J^{PC} = 1^{--}$), in the limit in which CP violation in the mixing is negligible, the measurement of a decay of the pair to two CP eigenstates with the same eigenvalue would be an indication of direct CP violation, since in such case the physical meson states can be chosen as CP eigenstates. Unfortunately, for heavy mesons the branching ratios to CP eigenstates are usually very small, so that the requirement of having both mesons decaying to such modes represents a heavy penalty which makes this approach unappealing.

The general (CPT-symmetric) expression [66] for the decay rate to two final states f_1, f_2 as a function of their time difference $\Delta t = t_1 - t_2$, reduces, in the limit $\Delta\Gamma \simeq 0$ to

$$I(f_1, f_2; \Delta t) \propto \left(1 - e^{-|\Delta t|\overline{\Gamma}}\right) \tag{185}$$

$$\left[\mathcal{A}_C \cos^2(\Delta m \Delta t/2) + \mathcal{A}_S \sin^2(\Delta m \Delta t/2) + \mathcal{A}_I \sin(\Delta m \Delta t)\right] \tag{186}$$

where

$$\mathcal{A}_C = 4\left|\frac{\lambda_1 - \lambda_2}{(1+\lambda_1)(1+\lambda_2)}\right|^2 \tag{187}$$

$$\mathcal{A}_S = 4\left|\frac{1 - \lambda_1\lambda_2}{(1+\lambda_1)(1+\lambda_2)}\right|^2 \tag{188}$$

$$\mathcal{A}_I = 2\mathrm{Im}\left(\frac{(1-\lambda_1)(1-\lambda_2^*)}{(1+\lambda_1)(1+\lambda_2^*)}\right) = 4\frac{(1-|\lambda_1|^2)\mathrm{Im}(\lambda_2) - (1-|\lambda_2|^2)\mathrm{Im}(\lambda_1)}{|1+\lambda_1|^2|1+\lambda_2|^2} \tag{189}$$

If $f_1 = f_2$, or if CP violation is only present in the mixing ($\lambda_1 = \lambda_2$), one has $\mathcal{A}_C = 0$ and $\mathcal{A}_I = 0$; if there is no CP violation in the mixing nor in the decay ($|\lambda_{1,2}| = 1$) then $\mathcal{A}_I = 0$. In both cases the expression for $I(f_1, f_2; \Delta t)$ is symmetric with respect to $\Delta t = 0$, so that any Δt asymmetry in such distribution

$$A_I(f_1, f_2; \Delta t) \equiv \frac{I(f_1, f_2; \Delta t) - I(f_1, f_2; -\Delta t)}{I(f_1, f_2; \Delta t) + I(f_1, f_2; -\Delta t)} \simeq \frac{\mathcal{A}_I \sin(\Delta m \Delta t)}{\mathcal{A}_+ + \mathcal{A}_- \cos(\Delta m \Delta t)} \tag{190}$$

where

$$\mathcal{A}_+ = 2\left|\frac{1+\lambda_2}{1-\lambda_2}\right|^2 \left[1 + |\lambda_1|^2 + |\lambda_2|^2 + |\lambda_1|^2|\lambda_2|^2 - 4\mathrm{Re}(\lambda_1)\mathrm{Re}(\lambda_2)\right] \tag{191}$$

$$\mathcal{A}_- = 2\left|\frac{1+\lambda_2}{1-\lambda_2}\right|^2 \left[-1 + |\lambda_1|^2 + |\lambda_2|^2 - |\lambda_1|^2|\lambda_2|^2 - 4\mathrm{Im}(\lambda_1)\mathrm{Im}(\lambda_2)\right] \tag{192}$$

is non-zero only if both mixing-induced and one other kind of CP violation are simultaneously present: for heavy meson systems, in which CP violation in the mixing is negligible, this requires the presence of (direct) CP violation in the decay.

Comparisons between the values of the parameters extracted from the study of time-dependent asymmetries in neutral meson decays to different CP eigenstates can also provide evidence for direct CP violation. Mixing-induced CP asymmetries arise because of a difference between the decay and the mixing phases and, as mentioned, the choice of phase convention always allows the former to be shifted to zero, at the expense of the latter. Since in general decay phases depends on the particular mode, when they are non-zero one cannot find any phase convention in which *all* the CP-violating phases only appear in the mixing amplitudes, thus giving evidence for direct CP violation. By comparing the decay of flavour-tagged mesons to two different CP eigenstates f_1 and f_2, if CP violation would only be due to state mixing one would get, for the coefficients of the sine term in the time-dependent decay asymmetries:

$$\mathcal{A}_{CP}^{(\mathrm{int})}(f_1) = \eta_{CP}(f_1)\eta_{CP}(f_2)\mathcal{A}_{CP}^{(\mathrm{int})}(f_2) \tag{193}$$

where $\eta_{CP}(f_{1,2})$ are the CP eigenvalues of the two states. Any deviation from the above relation is a signal of direct CP violation, which can be expressed as

$$\eta_{CP}(f_1)\lambda_{f_1} \neq \eta_{CP}(f_2)\lambda_{f_2} \tag{194}$$

This is (as for ϵ' in the neutral K system) a measure of direct CP violation which can be non-zero also in presence of a single elementary decay amplitude for each process, and in absence of final-state interactions (*i.e.* when $|\overline{A}_{f_1}/A_{f_1}| = 1 = |\overline{A}_{f_2}/A_{f_2}|$), since one is comparing two different decays: the difference $\mathrm{Im}(\lambda_{f_1}) \neq \mathrm{Im}(\lambda_{f_2})$ can only be induced by a difference in the weak phases of the decay amplitudes $\phi_D^{(f)}$. A super-weak scenario is defined as one in which a choice of phase convention exists in which all such phases can be made simultaneously real.

When the final state is an incoherent mixture of CP-even and CP-odd states, asymmetries are diluted by a factor $|1-2r|$, r being the fraction of events with a given CP eigenvalue; this is the case for decays to two-body final states in which at least one of the particles has spin, so that states with different orbital angular momentum can have different CP eigenvalues. The measurements of the asymmetries for such mixed states require an angular momentum analysis in order to either estimate the fractions of the two CP eigenstates on a statistical basis (*i.e.* measure an effective η_{CP} parameter), or to weight the events in the asymmetry fit according to the decay configuration.

When final states are considered which are not CP eigenstates, but still can be reached by both M and $\overline{M}$ mesons, the expressions for the asymmetries and the extraction of weak phases from them become more complicated [139]; CP violation can still be probed by any difference between the time-dependent decay distributions for $M \to f$ and $\overline{M} \to \overline{f}$, or by considering the inclusive state $f + \overline{f}$, but the need to consider two different final states makes the experimental investigation more demanding. By comparing the M and $\overline{M}$ decay distributions to a final state which is not a CP eigenstate, as a function of $|\Delta t|$ for mesons produced in correlated pairs, direct CP violation can be probed either by the shape of the distributions or by their normalization difference.

T-odd correlations in heavy meson decays have also been studied [140], such as $\langle \mathbf{S}_\tau \cdot \mathbf{p}_\tau \times \mathbf{p}_c \rangle$ in the inclusive semi-leptonic decays $b \to c\tau\overline{\nu}$, which requires measuring both momentum and polarization of the emitted τ lepton in events with a quark jet. Other T-odd correlations could be measured, such as the ones involving c quark polarization in the b decays, which require the measurement of exclusive channels in which the charmed quark hadronizes to a D^*, decaying to $D\pi$ and leading to a 4-body final state, in which triple correlations can be built just from the momenta. Exclusive decays of heavy mesons to vector particles, such as $B^0 \to K^* l^+ l^-$, also allow T-odd correlations to be studied, although the tiny branching ratios make experimental measurements very challenging.

In general, in the SM these correlations are estimated to give larger signals for B meson decays than for K, but no CP violation effect has been measured in this way so far.

5.2 Experimental considerations

As mentioned, searches for direct CP violation can be performed by measuring the time-integrated partial rate asymmetries $A_{CP}^{(f)}$ in any decay[24] of charged mesons or (if CP violation in the mixing is negligible) in decays of neutral mesons to *flavour-specific* final states. When using meson pairs (correlated or not), the relative normalization is not an issue, and if the decay is *flavour-specific*, or *self-tagging* ($M \nrightarrow \overline{f}$, $\overline{M} \nrightarrow f$, which is always the case for charged meson decays) such asymmetries are simply given by the asymmetry of measured (untagged) $\overline{f}$ and f events:

$$A_{CP}^{(U)} = \frac{N(\overline{f}) - N(f)}{N(\overline{f}) + N(f)} \tag{195}$$

We remark again that for neutral mesons this asymmetry would be non-zero also in case of CP violation in the mixing.

When considering instead states f which can be reached by both M and $\overline{M}$, flavour tagging information on the decaying meson is necessary; in this case the time-integrated asymmetry still contains diluted information on CP violation:

$$A_{CP} = \frac{N(\overline{M} \to f) - N(M \to f)}{N(\overline{M} \to f) + N(M \to f)} \tag{196}$$

where M, $\overline{M}$ refer to the meson flavour at the tagging time (different from the decay time) so that this asymmetry becomes

$$A_{CP} = \frac{\mathcal{A}_{CP}^{(\mathrm{m/d})} + x\mathcal{A}_{CP}^{(\mathrm{int})}}{1 + x^2} \tag{197}$$

(where $x \equiv \Delta m/\Gamma$), ignoring again any CP-violating effect in the mixing.

Since the statistical error of the measurement is $\sigma(A_{CP}) \sim 1/\sqrt{N}$, ($N$ being the measured number of events), the relevant quantity defining the observability of an asymmetry for a given decay mode is the product $BR \cdot A_{CP}^2$ (BR being the branching ratio).

[24] We omit here and after the superscript (f).

Channel	BR limit (90% CL)	Notes
$\eta \to \pi^+\pi^-$	$3.3 \cdot 10^{-4}$	CMD-2 1999
$\eta \to \pi^0\pi^0$	$4.3 \cdot 10^{-4}$	CMD-2 1999
$\eta \to 4\pi^0$	$6.9 \cdot 10^{-4}$	Crystal Ball 2000
$\eta' \to \pi^+\pi^-$	$2 \cdot 10^{-2}$	Bubble ch. 1969
$\eta' \to \pi^0\pi^0$	$9.4 \cdot 10^{-4}$	GAMS-2000 1987

Table 9: Limits on CP-violating decays of η, η' mesons [36].

The experimental measurement of asymmetries is also diluted in presence of background (assuming no CP asymmetry in the background), so that the measured asymmetry A_{CP}^{meas} is related to the one of the signal by

$$A_{CP}^{\text{meas}} = A_{CP} \frac{N_S}{N_S + N_B} \tag{198}$$

where N_S and N_B are the numbers of signal and background events respectively. Taking also into account the detection efficiency $\varepsilon(f)$, the number N of mesons required to measure an asymmetry at n standard deviations from zero in the decay to the final state f is

$$N > \frac{n^2}{\varepsilon(f) A_{CP}(f)^2 BR(f)} (1 + N_B/N_S) \tag{199}$$

from which one can argue that the study of rare decay modes for which however relatively large asymmetries are expected is favoured.

If the meson decay time is measured, the study of the shape of time-dependent asymmetries allows the search of CP violation as discussed in detail in the previous section; this kind of measurement only became possible in recent times thanks to the development of precise vertex detectors with μm resolutions.

5.3 Light unflavoured mesons

No mixing effects are present for the (CP self-conjugate) unflavoured light mesons, which are discussed here for completeness. The possibility of CP violation in the $\pi \to \mu \to e$ weak decay chain has been considered; final-state interactions due to weak interactions induce very small phases, leading to very tiny effects in the Standard Model. Direct comparison of π^+ and π^- require pions to decay in vacuum to overcome the large asymmetric effects due to pion interactions with matter (as compared to antimatter).

By comparing the μ^+ and the μ^- polarizations, as measured by the oscillation amplitudes $A_\pm$ of the $e^\pm$ counting rate in a magnetic field, in the muon $g-2$ experiments [141], a (direct) CP asymmetry limit was obtained [142]:

$$-0.01 < \frac{A_+ - A_-}{A_+ + A_-} < 0.02 \tag{200}$$

CP-forbidden decays of the $\eta(548)$ and $\eta'(958)$ self-conjugate mesons have also been searched for: since such mesons decay (strongly) to an odd number of pions, the observation of their decay to an even number of pions would be evidence of direct CP violation. Table 9 summarizes the current experimental limits for these decays.

5.4 D mesons

In the system of neutral D mesons, all CP violation effects are expected to be very small in the Standard Model, $O(0.01)$: $D^0 - -\overline{D}^0$ mixing is known to be very small and can be described rather well by physics of the first two quark generations only; the top-quark loops, which in the Standard Model induce the largest CP violating effects in the strange and beauty mesons, are here absent. The c quark can decay without suppression due to the small inter-generation mixing, and the lifetime of charged mesons is relatively short when compared to that of B mesons (scaled with their masses). In Cabibbo-allowed (such as $D^+, D^0 \to K^- X$) and doubly Cabibbo-suppressed (such as $D^0 \to K^+\pi^-$) decays of charmed mesons, there are usually no competing amplitudes in the Standard Model capable of inducing sizable direct CP violating asymmetries in the decay rates. The known indirect CP violation contribution in K^0 decays can induce a rate difference in the decays $D^\pm \to K_S\pi^\pm$ equal to $-2\mathrm{Re}\epsilon_K$, and any difference with respect to this value would be a signal of (direct) CP violation in the D decay process.

The search for direct CP violation in singly Cabibbo-suppressed decays of charmed mesons (such as $D^0 \to K^+K^-, \pi^+\pi^-$, or $D^+ \to K_S K^+$) is also very challenging, as far as Standard Model predictions are concerned (at the $\sim 10^{-3}$ level); searches for CP violation in D meson decays are therefore considered sensitive probes for new physics.

Partial rate asymmetries are defined as

$$A_{CP}^{(f)} = \frac{N(D^+, D^0 \to f) - N(D^-, \overline{D}^0 \to \overline{f})}{N(D^+, D^0 \to f) + N(D^-, \overline{D}^0 \to \overline{f})} \tag{201}$$

In photo-production experiments, such partial rate asymmetries are measured by normalizing to Cabibbo-allowed decays, since the production rates of D^0 and $\overline{D}^0$ are different (although in this way CP asymmetries from new physics in the normalization mode could mask a signal in the Cabibbo-suppressed mode). Searches for direct CP violation in Cabibbo-allowed decay modes (not expected in the Standard Model) actually look for differences in asymmetries among different decay channels.

By considering both "right sign" $D^0 \to \overline{f}$, $\overline{D}^0 \to f$ decays and "wrong sign" ones $D^0 \to f$, $\overline{D}^0 \to \overline{f}$, where f and $\overline{f}$ are in general not CP eigenstates ($|\overline{f}\rangle = CP|f\rangle$), one can study the "wrong sign" time-dependent decay rates for flavour-tagged neutral D mesons:

$$r(\tau) \equiv \frac{\Gamma(D^0(\tau)\to f)}{|\overline{A}_f|^2} = \frac{1}{2}\left|\frac{q}{p}\right|^2 \big[e^{-\tau}(1/|\lambda_f|^2+1)\cosh(y\tau) - 2\mathrm{Re}(1/\lambda_f)\sinh(y\tau) + (1/|\lambda_f|^2-1)\cos(x\tau) - 2\mathrm{Im}(1/\lambda_f)\sin(x\tau)\big] \tag{202}$$

$$\overline{r}(\tau) \equiv \frac{\Gamma(\overline{D}^0(\tau)\to \overline{f})}{|A_{\overline{f}}|^2} = \frac{1}{2}\left|\frac{p}{q}\right|^2 \Big[e^{-\tau}(|\lambda_{\overline{f}}|^2+1)\cosh(y\tau) - 2\mathrm{Re}(\lambda_{\overline{f}})\sinh(y\tau) + (|\lambda_{\overline{f}}|^2-1)\cos(x\tau) - 2\mathrm{Im}(\lambda_{\overline{f}})\sin(x\tau)\Big] \tag{203}$$

where the decay time is measured in units of the D^0 mean life: $\tau \equiv \overline{\Gamma}t$ (here $\Delta m = m_+ - m_-$, $\Delta\Gamma = \Gamma_+ - \Gamma_-$, where the subscripts indicate the CP eigenvalue). For flavour-specific (*e.g.* semi-leptonic) decays, and for small CP violation one has $1/|\lambda_f| \ll 1$ and $|\lambda_{\overline{f}}| \ll 1$; in the limit of small mixing ($x \ll 1$,

$y \ll 1$) the above expressions reduce to

$$r(\tau) \simeq \tfrac{e^{-\tau}}{4} \left|\tfrac{q}{p}\right|^2 (x^2+y^2)\tau^2 \tag{204}$$

$$\overline{r}(\tau) \simeq \tfrac{e^{-\tau}}{4} \left|\tfrac{p}{q}\right|^2 (x^2+y^2)\tau^2 \tag{205}$$

so that if there is no CP violation in the mixing one has in this case $r(\tau) = \overline{r}(\tau)$.

The phenomenology for hadronic decays of neutral D mesons is complicated by the possibility of doubly Cabibbo-suppressed decays ("wrong sign") leading to the same final states. To leading order in the CP-violating parameters:

$$r(\tau) = e^{-\tau}\left[\tfrac{R_D}{(1+A_D)^2}+ \sqrt{R_D}\tfrac{1+A_M}{1+A_D}\mathrm{Re}\left[(y+ix)e^{i(\delta+\phi_D+\phi_M)}\right]\tau + \tfrac{1}{2}R_M(1+A_M)^2\tau^2\right] \tag{206}$$

$$\overline{r}(\tau) = e^{-\tau}\left[R_D(1+A_D)^2+ \sqrt{R_D}\tfrac{1+A_D}{1+A_M}\mathrm{Re}\left[(y+ix)e^{i(\delta-\phi_D-\phi_M)}\right]\tau + \tfrac{1}{2}\tfrac{R_M}{(1+A_M)^2}\tau^2\right] \tag{207}$$

where R_D is a measure of the double Cabibbo suppression in the amplitude

$$\frac{A_f}{A_{\overline{f}}} = -\sqrt{R_D}e^{-i\delta} \tag{208}$$

(with δ a strong phase difference between the two amplitudes), R_M is a measure of the double Cabibbo suppressed mixing rate

$$\int_0^\infty dt\,\Gamma(D^0(t)\to f) = R_M|\overline{A}_f|^2 \tag{209}$$

and the real parameters A_M, A_D and $\phi_M+\phi_D$ characterize CP violation in the mixing, in the decay and in the interference of the two processes respectively:

$$\tfrac{q}{p} = (1+A_M)e^{i\phi_M} \tag{210}$$

$$\tfrac{\overline{A}_f}{A_{\overline{f}}} = (1+A_D)e^{i\phi_D} \tag{211}$$

In the limit of CP conservation A_M, A_D and $\phi_M+\phi_D$ are all zero and

$$r(\tau) = \overline{r}(\tau) = e^{-\tau}\left[R_D + \sqrt{R_D}\mathrm{Re}[(y+ix)e^{i\delta}]\tau + \frac{1}{2}R_M\tau^2\right] \tag{212}$$

For CP eigenstates, f and $\overline{f}$ coincide, and one has (for $CP|f\rangle = +|f\rangle$):

$$r(\tau) \propto \left|e^{-i(m_+-i\Gamma_+/2)\tau/\overline{\Gamma}} + \eta_f e^{-i(m_--i\Gamma_-/2)\tau/\overline{\Gamma}}\right|^2 \tag{213}$$

$$\overline{r}(\tau) \propto \left|e^{-i(m_+-i\Gamma_+/2)\tau/\overline{\Gamma}} - \eta_f e^{-i(m_--i\Gamma_-/2)\tau/\overline{\Gamma}}\right|^2 \tag{214}$$

where $\eta_f = (1-\lambda_f)/(1+\lambda_f)$ describes CP violation of all three types; for the opposite CP eigenvalue the above formula holds with $1 \leftrightarrow 2$ and $\eta_f \to 1/\eta_f$.

By analyzing decays of D^0 mesons produced from the (strong) decay chain $D^{*+} \to D^0\pi^+$ and $D^{*-} \to \overline{D}^0\pi^-$, the charge of the (slow) pion accompanying

Channel	BR	A_{CP}	Notes
$D^+ \to K_S \pi^+$	1.4%	-0.016 ± 0.017	FOCUS
$D^+ \to K_S K^+$	$2.9 \cdot 10^{-3}$	0.07 ± 0.06	FOCUS
$D^+ \to K^+ K^- \pi^+$	$8.8 \cdot 10^{-3}$	0.002 ± 0.011	E687, E791, FOCUS
$D^+ \to K^+ \overline{K}^{0*}$	$4.2 \cdot 10^{-3}$	-0.02 ± 0.05	E687, E791
$D^+ \to \phi \pi^+$	$6.1 \cdot 10^{-3}$	-0.014 ± 0.033	E687, E791
$D^+ \to \pi^+ \pi^+ \pi^-$	$3.1 \cdot 10^{-3}$	-0.02 ± 0.04	E791
$D^0 \to \pi^+ \pi^-$	$1.4 \cdot 10^{-3}$	0.021 ± 0.026	CLEO, E791, FOCUS
$D^0 \to \pi^0 \pi^0$	$8.4 \cdot 10^{-4}$	0.001 ± 0.048	CLEO
$D^0 \to K^\pm \pi^\mp$	$1.5 \cdot 10^{-4}$	$0.02^{+0.19}_{-0.20}$	CLEO
$D^0 \to K_S \pi^0$	1.14%	0.001 ± 0.013	CLEO
$D^0 \to K^+ K^-$	$4.1 \cdot 10^{-3}$	0.005 ± 0.016	CLEO, E687, E791, FOCUS
$D^0 \to \phi K_S$	$4.7 \cdot 10^{-3}$	-0.03 ± 0.09	CLEO [144]
$D^0 \to K_S K_S$	$3.6 \cdot 10^{-4}$	-0.23 ± 0.19	CLEO [144]
$D^0 \to K^\mp \pi^\pm \pi^0$	13%	-0.03 ± 0.09	CLEO
$D^0 \to K^\pm \pi^\mp \pi^0$	$5.6 \cdot 10^{-4}$	$0.09^{+0.25}_{-0.22}$	CLEO

Table 10: Measurements of direct-CP violating asymmetries in D meson decays, from [36] except where indicated otherwise.

the neutral D meson can be used to identify the initial flavour; this flavour-tagging technique has no significant bias for experiments performed at colliders in which quarks and anti-quarks are produced in pairs [143], and mis-tagging rates are usually $O(10^{-3})$. The decay of charmonium states to open charm, such as $J/\Psi(3770) \to D^0 \overline{D}^0$, can also be exploited to tag the flavour of neutral D mesons.

Systematic effects in the measurements can arise from asymmetries in the fitting of the signal and background components, and from flavour-tagging asymmetries due to charge-dependent biases in the detector acceptance and efficiency.

While no CP violation effects have been detected in the D meson system so far, table 10 and fig. 13 summarize the experimental results of searches for direct CP violation in their decays.

5.5 B mesons

In the system of neutral B mesons (see *e.g.* [145] for a recent review), CP violation effects in the mixing are generally expected to be small, $\lesssim O(10^{-2})$. The reason is that the total decay width difference $\Delta\Gamma$ of the two physical states is very small, being induced by the final states common to B and $\overline{B}$ decays, which have small $O(10^{-3})$ branching ratios for such a system with many open decay channels: $\Delta\Gamma/\Gamma \ll 1$, combined with the empirical fact that $|\Delta m| \sim \Gamma$ results in $|\Gamma_{12}| \ll |M_{12}|$ and therefore for the asymmetries due to mixing

$$A_{CP}^{(\mathrm{mix})} \propto 1 - \left|\frac{q}{p}\right|^2 \propto \mathrm{Im}(\Gamma_{12}/M_{12}) \lesssim \Delta\Gamma/\Delta m \lesssim 10^{-3} \tag{215}$$

In other terms, in the Standard Model, when final-state interaction effects are small, M_{12} and Γ_{12} acquire their phases from the same combination of CKM ma-

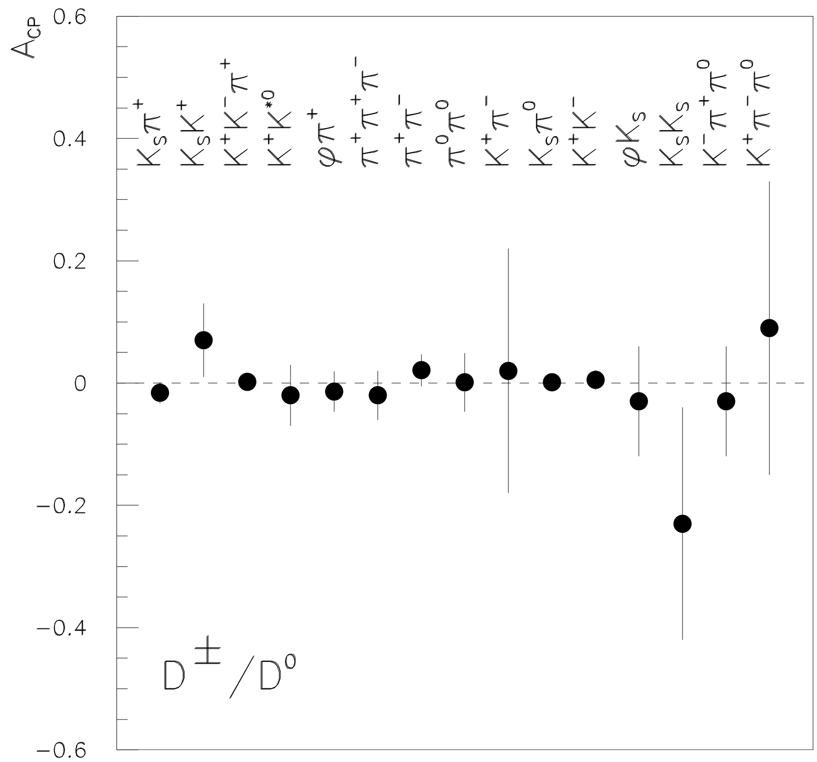

Figure 13: Graphical representation of time-integrated CP asymmetries for $D^\pm$ and $D^0(\overline{D}^0)$ decays.

trix elements, leading to a small relative phase and small asymmetries. Precise theoretical predictions are however rather difficult.

CP violation in mixing has not been experimentally detected yet for neutral B mesons. The CP-violating impurity in the physical B_d meson states is measured to be [36]

$$\frac{\mathrm{Re}(\bar{\epsilon}_{B_d})}{1+|\bar{\epsilon}_{B_d}|^2} = (0 \pm 4)\cdot 10^{-3} \tag{216}$$

from the (time-integrated or time-dependent) charge asymmetry in like-charge di-lepton events from semi-leptonic decays, and from the analysis of the time-dependent asymmetry of inclusive decays, using samples where the initial flavour state is tagged (see also [136]). For B_s mesons flavour oscillations have not been observed yet.

Mixing-induced CP violation is instead expected to give large $O(1)$ effects in several neutral B meson decay asymmetries. For channels in which the decay is dominated by a single elementary amplitude (and therefore CP violation effects in the decay are negligible), such asymmetries can be related in a direct way to the basic parameters of the underlying theory, with small theoretical uncertainties. This is among the main reasons for the great interest in the measurement of CP violation in B decays.

In the Standard Model, direct CP violation in B meson decays is expected to occur in charmless hadronic decays, in which *tree* and *penguin* $b \to u$ decay amplitudes of comparable magnitudes can interfere. Studied channels include $B_d^0 \to K^+\pi^-$, $B^+ \to K^+\pi^0$, $B^+ \to K_S\pi^+$ (together with their CP-conjugates, of course). Direct CP violation is instead expected to be negligible for modes to which only either *tree* or *penguin* graphs contribute: some examples are respectively $B^0 \to J/\Psi K^0$ and $B^0 \to \phi K^0$; searches for direct CP violation in these channels are therefore sensitive to physics beyond the Standard Model.

CP violation in the B meson system has been mostly studied in e^+e^- collider experiments (*e.g.* the LEP experiments, CLEO at CESR, BABAR at SLAC, BELLE at KEK), but significant results have been obtained in experiments at hadronic machines (*e.g.* CDF at the TeVatron). The e^+e^- environment, when compared to the hadronic one, provides a high signal to background ratio, cleaner events and a low interaction rate; the absolute production cross sections being however much smaller, very high luminosities are required.

B mesons can be studied at colliders by exploiting their non-resonant inclusive associate production ($b\overline{b}X$ final states): this is the case at hadron colliders and high-energy e^+e^- colliders.

At e^+e^- "B-factories", pairs of $B_d - \overline{B}_d$ mesons (both neutral and charged) are produced by decays of the $\Upsilon(4S)$ bottomonium state (just above the open beauty threshold) at its formation energy of 10.6 GeV: the branching ratio of this resonance to $B\overline{B}$ pairs is above 96% and the high instantaneous luminosities ($5-8\cdot10^{33}$ cm^{-2} s^{-1}) provide copious sources of B mesons. The time-dependent CP-violating asymmetries depend only on the time difference Δt of the two meson decays: the short lifetime of the B mesons, $c\tau_B \sim 500$ μm, corresponding to only 23 μm when produced from the decay of a $\Upsilon(4S)$ at rest, requires very precise vertex detectors for its measurement. The measurement of the distance between the two decay vertexes at a symmetric collider (usually the production point cannot be determined with sufficient accuracy) only gives the sum of the decay times. By using asymmetric beam energies, the Lorentz boost of the final state ($\langle\beta\gamma\rangle = 0.56$ at SLAC with 3.1+9 GeV e^+e^- beam energies, 0.43 at KEK with 3.5+8 GeV) allows the experimental measurement of time-dependent decay asymmetries, since the decay time difference is then given by $\Delta z \simeq \beta\gamma c\Delta t$, with an average separation between the two vertexes of 260 or 200 μm respectively, while the typical resolution achieved by the vertex detectors is about 180 μm.

While the study of time-dependent $B^0 - -\overline{B}^0$ meson asymmetries was performed at LEP, when running at the $\Upsilon(4S)$ resonance one can profit from the tight kinematic constraints which help in reducing the background; moreover, the combinatorial component is less important due to the absence of fragmentation products. The first evidence for (mixing-induced) CP violation outside the neutral kaon system was actually provided by the measurements of such asymmetries in $B_d^0, \overline{B}_d^0 \to J/\Psi K_S$ decays [24] [25].

While the flavour eigenvalue of $B^\pm$ mesons is determined by their charge, as measured from the decay products, the flavour eigenvalue for neutral B mesons, apart from flavour-specific decay modes, has to be determined by exploiting their associate production. In experiments using correlated pairs of B mesons, the flavour tagging of a decaying B meson provides flavour information for the opposite B at the same time. In experiments where B mesons are produced incoherently, their flavour (at a given time for neutral mesons) can be determined by that of the companion b hadron; the flavour tagging information obtained in such a way by a companion neutral meson is degraded by the flavour mixing, in contrast with what happens with correlated pairs where mixing does not affect the tagging. The B meson flavour can also be determined by exploiting its production from decays of excited B^{**} states through the decay chain $B^{**} \to B\pi$: in reconstructed B^{**} meson decays one obtains the flavour of the B^0 from the charge of the accompanying charged pion ($B^{**+} \to B_d\pi^+$, $B^{**-} \to \overline{B}_d\pi^-$).

Experiments at B-factories, discussed in the following, use large acceptance asymmetric detectors, which rely on excellent vertex tracking to identify secondary vertexes, discriminate against light quark vertexes, and measure decay time differences. Good particle identification over a wide kinematic range is also an important requirement: flavour tagging relies on the identification of electrons and muons from semi-leptonic decays, and kaons as well; moreover, the selection of exclusive decay channels needs good K/π discrimination. Both the BABAR [146] and the Belle [147] detectors have silicon vertex detectors, central drift chambers with helium-based gas to minimize multiple scattering, CsI(Tl) crystal electromagnetic calorimeters, and muon detection systems outside super-conducting solenoids providing 1.5 T magnetic field. Both experiments use Cerenkov detectors for hadron identification, based on silica quartz bars for BABAR, and on silica aerogel counters for Belle. The analysis is based on the full reconstruction of one B meson to a given final state, while the opposite one is partially reconstructed in order to determine its flavour through the identification of decay leptons, kaons or soft charged pions from D^* decays. The interesting decays are identified kinematically by constraining their centre of mass energies and momenta, and exploiting particle identification. Maximum likelihood fits which include the tagging efficiency parameters, background fractions and shapes in several discriminating variables, are used to extract event yields and asymmetries.

Direct CP violation has been searched for, both by measuring time-integrated decay asymmetries and by fitting the cosine term in time-dependent asymmetries.

The first approach is simpler: after event reconstruction, background rejection and signal fitting, the measured event asymmetry is formed. For non-self-tagging modes, such as $B^0(\overline{B}^0) \to K^0\pi^0$, B flavour tagging information is needed, and flavour mixing dilutes the measurement by a factor $1/(1+x^2)$ as mentioned.

The flavour of the B meson can be distinguished either by its charge or by the electric charge of one of its decay products (secondaries for states containing only neutral particles such as $B^0 \to \phi K^{0*} \to \phi K^\pm\pi^\mp$), by exploiting some selection rule. For neutral B mesons, semi-leptonic decays are mostly used, based on the $\Delta B = \Delta Q$ rule (valid to high accuracy in the Standard Model): the charge of the energetic lepton identifies the flavour of the meson at the time of its decay ($B^0 \to l^+X$, $\overline{B}^0 \to l^-X$), although the $b \to c \to l$ decay results in opposite lepton charges; alternatively, the charge of reconstructed charm mesons ($B_d^0 \to D^{*-}$, $B_s^0 \to D_s^-$), detected from the charge of the soft pions from $D^{*\pm} \to D^0(\overline{D}^0)\pi^\pm$, or that of kaons from $b \to c \to s$ decays, can be used, being all correlated with the flavour of the decaying B, to different extent.

Particle identification, achieved with multiple redundant measurements, is crucial for these measurements, as well as its charge symmetry together with that of track reconstruction. The measured CP-violating asymmetries are diluted by the errors due to the mis-tagging of the B meson flavour, *i.e.* by the (single or double) mis-identification of the final state particles, and have to be corrected accordingly:

$$A_{CP}^{\rm meas} = (1 - 2w)A_{CP} \tag{217}$$

where the wrong-tag fraction w (ranging from a few percent for the best signatures to several tens of percent for poor tagging information). The mis-tag probabilities for different samples can be extracted from the fit of the time-dependent asymmetries, or extracted from the time-integrated analysis of flavour-specific final states: assuming that the flavour of the fully reconstructed meson is correctly identified, the measured time-integrated fraction of mixed events $\chi^{\rm meas}$ is given by

$$\chi^{\rm meas} = \chi + (1 - 2\chi)w \tag{218}$$

where $\chi = x^2/[2(1 + x^2)]$. Fake asymmetries can be induced in case the mis-tagging probabilities are different for B^0 and $\overline{B}^0$.

Large backgrounds affect most of the channels which have very small branching ratios, and they are usually the dominant source of systematic error; in order to have small statistical errors on the signal samples, precise measurements of the backgrounds are also required, so that a fraction of data taking has to be spent off the resonance. Important systematic cross-checks are given by the measurements of the asymmetries for the background events, which in most cases have to be consistent with zero.

Results on CP-violating asymmetries for several *flavour-specific* final states have been reported by CLEO (with $\approx 1 \cdot 10^7$ $B\overline{B}$ events), BABAR and BELLE (more than $8 \cdot 10^7$ $B\overline{B}$ events each, so far); no clear evidence of direct CP violation has been found so far; table 11 and figures 14, 15 summarise the available experimental information on the time-independent CP asymmetries, while table 12 and figure 16 refer to the direct CP-violating coefficient of the time-dependent ones.

Due to the limited size of the statistical samples, in some cases in which no significant direct CP violation is expected in the Standard Model, the time-dependent asymmetry is analysed by assuming $|\lambda_f| = 1$ to get the most precise result on the underlying parameters of the theory, related to the mixing-induced CP-violating term.

A hint of direct CP violation in the B system comes from the BELLE fit of the $B_d^0 \to \pi^+\pi^-$ time-dependent decay distribution [164], in which a non-zero cosine term is found:

$$\mathcal{A}_{CP}^{(\rm m/d)} = +0.77 \pm 0.27 \pm 0.08 \tag{219}$$

$$\mathcal{A}_{CP}^{(\rm int)} = -1.23 \pm 0.41^{+0.08}_{-0.07} \tag{220}$$

(the first error being statistical and the second systematic). Such result is not confirmed by the BABAR analysis [163] of the same channel with similar

Channel	BR	A_{CP}	Notes
$B^+ \to \pi^+\pi^0$	$5.4 \cdot 10^{-6}$	-0.07 ± 0.15	BABAR [148], BELLE [149]
$B^+ \to K^+\pi^0$	$1.3 \cdot 10^{-5}$	0.01 ± 0.12	BABAR [148], BELLE [149], CLEO
$B^+ \to K_S\pi^+$	$1.1 \cdot 10^{-5}$	-0.02 ± 0.09	BABAR [150], BELLE [151], CLEO
$B^+ \to K^+\eta'$	$7.7 \cdot 10^{-5}$	0.02 ± 0.04	BABAR [152], BELLE [153], CLEO
$B^+ \to K^{*+}\eta$	$2.2 \cdot 10^{-5}$	$-0.05^{+0.25}_{-0.30}$	BELLE [149]
$B^+ \to \omega\pi^+$	$5.4 \cdot 10^{-6}$	-0.20 ± 0.19	BABAR, CLEO
$B^+ \to \omega K^+$	$5.3 \cdot 10^{-6}$	-0.21 ± 0.28	BELLE [154]
$B^+ \to J/\Psi\pi^+$	$4.2 \cdot 10^{-5}$	-0.01 ± 0.13	BABAR, BELLE [155]
$B^+ \to J/\Psi K^+$	$1.0 \cdot 10^{-3}$	-0.007 ± 0.018	BABAR, BELLE [155], CLEO
$B^+ \to \Psi' K^+$	$6.6 \cdot 10^{-4}$	-0.10 ± 0.07	BELLE [155], CLEO
$B^+ \to \phi K^+$	$1.2 \cdot 10^{-5}$	0.04 ± 0.09	BABAR [156]
$B^+ \to \phi K^{*+}$	$1.2 \cdot 10^{-5}$	0.16 ± 0.17	BABAR [157]
$B^+ \to K^{*+}\gamma$	$3.8 \cdot 10^{-5}$	0.05 ± 0.09	BELLE [158]
$B^+ \to D_1 K^+$		0.10 ± 0.15	BABAR [159], BELLE [160]
$B^+ \to D_2 K^+$		-0.19 ± 0.18	BELLE [160]
$B^+ \to D_{CP} K^+$		0.06 ± 0.18	BABAR [161]
$B^+ \to \pi^+\pi^+\pi^-$	$1.1 \cdot 10^{-5}$	-0.39 ± 0.35	BABAR [162]
$B^+ \to K^+\pi^+\pi^-$	$5.7 \cdot 10^{-5}$	0.01 ± 0.08	BABAR [162]
$B^+ \to K^+K^+K^-$	$3.4 \cdot 10^{-5}$	0.02 ± 0.08	BABAR [162]
$B_d^0 \to K^+\pi^-$	$1.8 \cdot 10^{-5}$	-0.09 ± 0.04	BABAR [163], BELLE [149], CLEO
$B_d^0 \to K_S\pi^0$	$5.4 \cdot 10^{-6}$	0.04 ± 0.12	BABAR [157]
$B_d^0 \to \eta K^{*0}$	$2.1 \cdot 10^{-5}$	$0.17^{+0.28}_{-0.25}$	BELLE [149]
$B_d^0 \to \phi K^{*0}$	$1.1 \cdot 10^{-5}$	0.04 ± 0.12	BABAR [157]
$B_d^0 \to \rho^+\pi^-$	$2.5 \cdot 10^{-5}$	-0.18 ± 0.09	BABAR [161]
$B_d^0 \to \rho^+K^-$	$7.3 \cdot 10^{-6}$	0.28 ± 0.19	BABAR [161]
$B_d^0 \to D^{*+}D^-$	$8.8 \cdot 10^{-4}$	-0.03 ± 0.12	BABAR [165]
$B_d^0 \to K^{*0}\gamma$		-0.06 ± 0.07	BELLE [158], CLEO
$B \to K^*\gamma$	$4.2 \cdot 10^{-5}$	-0.01 ± 0.07	BABAR, CLEO
$b \to s\gamma$	$3.3 \cdot 10^{-4}$	-0.08 ± 0.11	CLEO

Table 11: Measurements of time-integrated direct CP-violating asymmetries in B meson decays (from [36] unless otherwise indicated). Here $CP(D_1) = +1$, $CP(D_2) = -1$, $D_{CP} = D_1, D_2$.

Channel	BR	Value	Notes
$B_d^0 \to \pi^+\pi^-$	$4.5 \cdot 10^{-6}$	0.51 ± 0.23	BABAR [163], BELLE [164]
$B_d^0 \to \eta' K_S$	$3.1 \cdot 10^{-5}$	0.08 ± 0.18	BELLE [166]
$B_d^0 \to \rho^+\pi^-$	$2.5 \cdot 10^{-5}$	-0.36 ± 0.18	BABAR [161]
$B_d^0 \to \phi K_S$	$5.4 \cdot 10^{-6}$	0.17 ± 0.68	BABAR [167], BELLE [166]
$B_d^0 \to K^+K^-K_S$	$1.5 \cdot 10^{-5}$	-0.40 ± 0.43	BELLE [166]
$B_d^0 \to J/\Psi\pi^0$	$2.1 \cdot 10^{-5}$	-0.31 ± 0.28	BABAR [168], BELLE [169]
$B_d^0 \to D^{*+}D^{*-}$		-0.02 ± 0.28	BABAR [161]
$B_d^0 \to c\bar{c}K_S$	$---$	-0.052 ± 0.042	BABAR [170], BELLE [171]

Table 12: Measurements of the direct CP violation coefficient $\mathcal{A}_{CP}^{(\mathrm{m/d})}$ in time-dependent asymmetries of neutral B meson decays (from [36] unless otherwise indicated, see also [172]). Errors are scaled according to the PDG recipe.

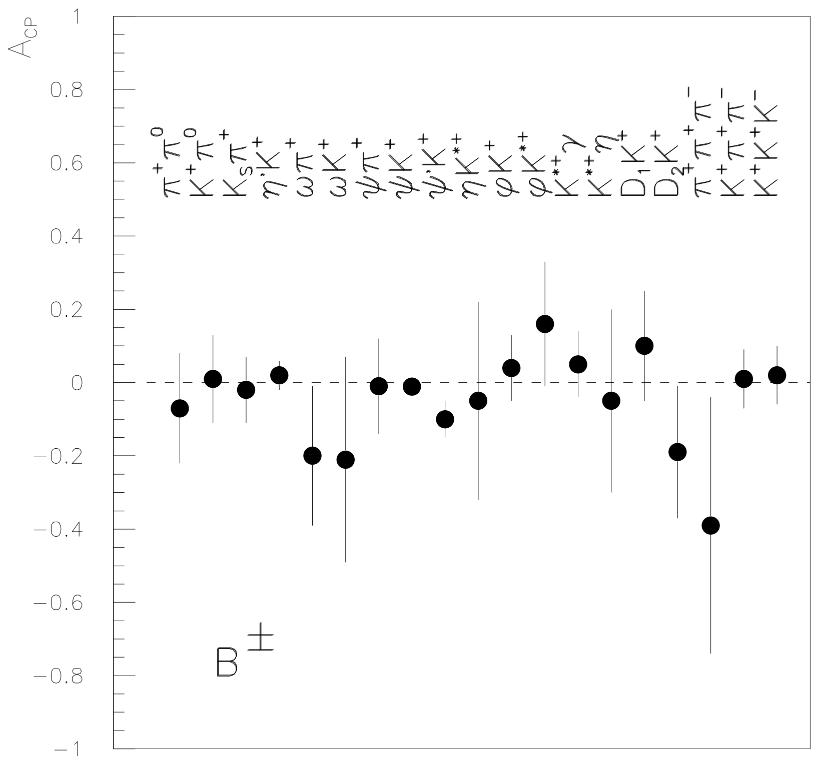

Figure 14: Graphical representation of time-integrated CP asymmetry measurements for $B^{\pm}$ decays.

statistics (~ 160 signal events):

$$\mathcal{A}_{CP}^{(\mathrm{m/d})} = +0.30 \pm 0.25 \pm 0.04 \tag{221}$$

$$\mathcal{A}_{CP}^{(\mathrm{int})} = +0.02 \pm 0.34 \pm 0.05 \tag{222}$$

The BELLE result for $\mathcal{A}_{CP}^{(\mathrm{m/d})}$ is 2.7 standard deviations from zero, and corresponds to a 2.2 standard deviation indication for direct CP violation, irrespective of the value of mixing-induced CP violation. This result however lies outside the physically allowed region $\mathcal{A}_{CP}^{(\mathrm{m/d})2} + \mathcal{A}_{CP}^{(\mathrm{int})2} \leq 1$; imposing the above constraint the result is $\mathcal{A}_{CP}^{(\mathrm{m/d})} = +0.57$. The BABAR experiment quotes a 90% confidence interval on the coefficient of the cosine term as $[-0.12, +0.72]$, and the naive average of the two results is $\mathcal{A}_{CP}^{(\mathrm{m/d})} = 0.51 \pm 0.23$ after scaling the error to account for their poor consistency. Clearly one has to wait for more data in order for the situation to be clarified.

As previously mentioned, precise measurements of mixing-induced asymme-

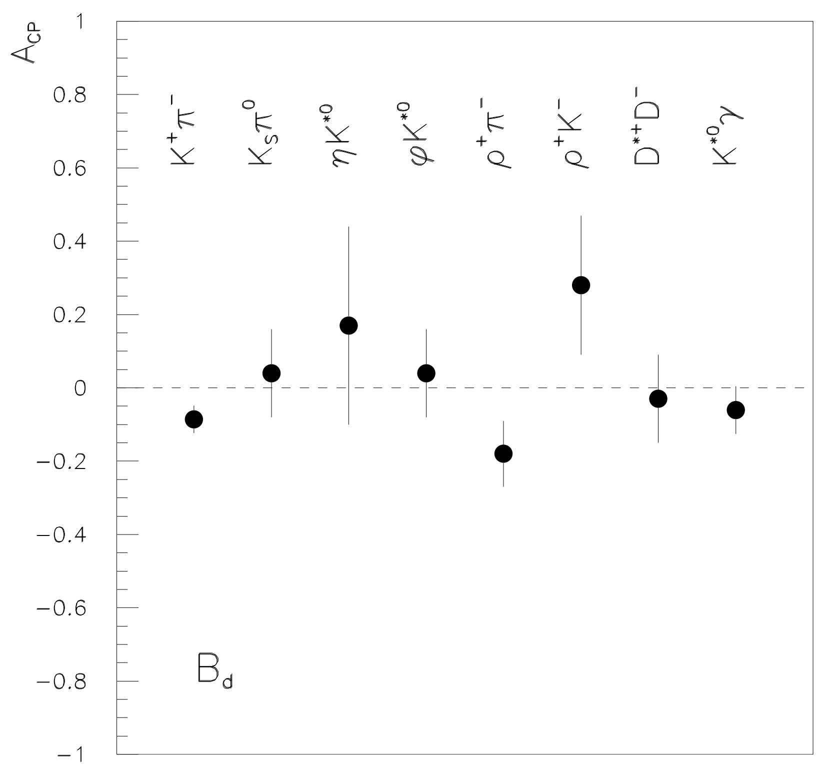

Figure 15: Graphical representation of time-integrated CP asymmetry measurements for $B_d^0(\overline{B}_d^0)$ decays.

tries in decays to different CP eigenstates, even if some of such asymmetries turn out to be zero (*i.e.* no CP violation), could give an indication of direct CP violation, by the comparison with the precisely measured asymmetry in the charmonium-K_S final state [173]. Table 13 and figure 17 summarise the available measurements: the probability of consistence is about 6%, no significant evidence of direct CP violation is seen yet. While the future increase in the statistical accuracy of the measurements is likely to reveal at some time differences among the above asymmetries, it is difficult to predict their pattern, since direct CP violation at present eludes the theoretical efforts of a precise estimation.

The CLEO experiment also studied the inclusive process $B \to X_s\gamma$, where X_s is any final state containing a strange quark [174], by considering events with high energy photons ($2.2 \div 2.7$ GeV): in this region of phase space there is little background from other B decay processes, although a significant continuum contribution has to be subtracted, by using several event shape variables. The measured (null) asymmetry also contains a small contribution (estimated to be

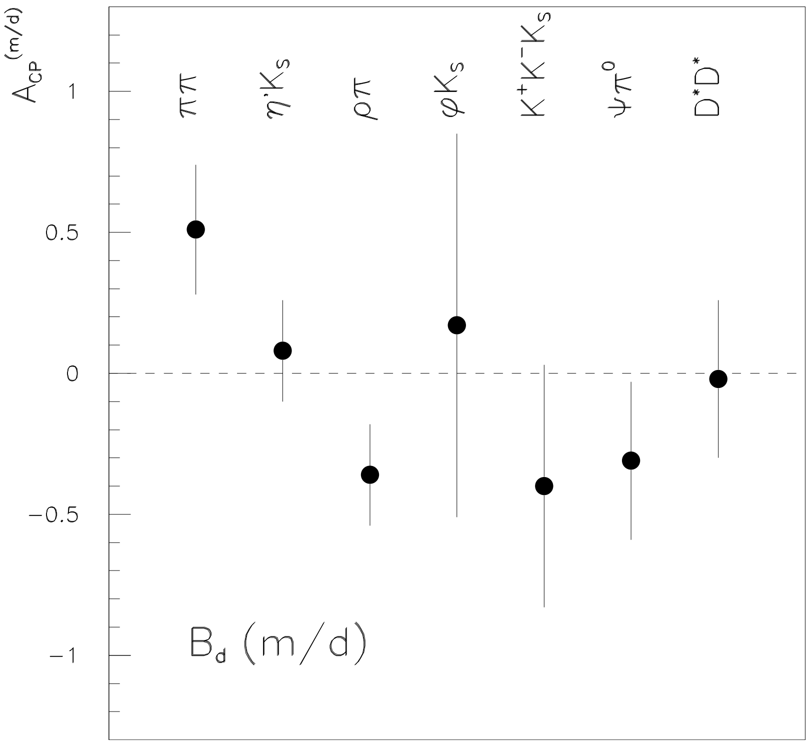

Figure 16: Graphical representation of the coefficient of the cosine term in time-dependent CP asymmetries for $B_d^0(\overline{B}_d^0)$ decays.

$\sim 2\%$) from a possible $b \to d\gamma$ asymmetry.

The experimental study of the heavier B_s^0 and $B_c^\pm$ mesons requires centre of mass energies above the $\Upsilon(4S)$. B_s^0 mesons have been studied both at e^+e^- (LEP, SLD) and $p\overline{p}$ (CDF) collider experiments; no signal of flavour oscillations has been identified yet, and no CP violation effects have been reported. These systems and their CP violation effects will be studied extensively at future hadron collider experiments, LHC-b [175] (but also ATLAS, CMS) at the CERN LHC and B-TeV [176] at the Fermilab TeVatron.

High-energy hadron colliders have the advantage of large $b\overline{b}$ production cross sections (100-500 μb), but the number of unrelated particles produced in each collision will be very large, making the analysis rather challenging. The measured decay-rate asymmetries have to be corrected for the detection efficiency asymmetries, and at pp colliders also for the different production rates of B^0 and $\overline{B}^0$ mesons.

The trigger systems, based on transverse momentum and secondary vertexes,

Channel	η_{CP}	Value	Notes
$B_d^0 \to \pi^+\pi^-$	+1	-0.47 ± 0.61	BABAR [163], BELLE [164]
$B_d^0 \to \eta' K_S$	–1	0.34 ± 0.34	BELLE [166]
$B_d^0 \to \phi K_S$	–1	-0.38 ± 0.41	BABAR [167], BELLE [166]
$B_d^0 \to K^+K^-K_S$	+1(†)	0.49 ± 0.55	BELLE [166]
$B_d^0 \to J/\Psi\pi^0$	+1	-0.43 ± 0.49	BABAR [168], BELLE [169]
$B_d^0 \to J/\Psi K_S$	–1	0.76 ± 0.06	BABAR [170], BELLE [171]
$B_d^0 \to J/\Psi K_L$	+1	-0.75 ± 0.12	BABAR [170], BELLE [171]
$B_d^0 \to \Psi' K_S$	–1	0.69 ± 0.24	BABAR [170]
$B_d^0 \to \Psi K^{*0}$	+1(†)	-0.15 ± 0.40	BABAR [170]
		-0.19 ± 0.56	(Effective for CP=+1)
$B_d^0 \to D^{*+}D^{*-}$	+1(†)	0.32 ± 0.47	BABAR [161]
		0.37 ± 0.55	(Effective for CP=+1)
$B_d^0 \to c\bar{c}K_S$	–1	0.75 ± 0.06	BABAR [170], BELLE [171]

Table 13: Measurements of the mixing-induced CP violation coefficient $\mathcal{A}_{CP}^{(\mathrm{int})}$ in time-dependent asymmetries of neutral B meson decays to CP eigenstates (from [36] unless otherwise indicated, see also [172]). Errors are scaled according to the PDG recipe. (†): dominant CP eigenvalue.

will be key elements for the physics performance; B-TeV plans to use the vertex detector already at the first trigger level. Both the LHC-b and the B-TeV detectors are single-arm spectrometers, with vertex detectors based on silicon strips for the former and silicon pixels for the latter; both experiments have aerogel RICH's for particle ID and are expected to run at $\sim 2 \cdot 10^{32}$ cm^{-2} s^{-1} luminosity, starting in 2007.

6 Other systems

6.1 Hyperon decays

Soon after the discovery of parity violation, it was realized [177] that weak hyperon decays could be an interesting laboratory for studying discrete symmetry violations. Since baryon number conservation effectively forbids mixing, any CP violation effect in baryon decays would be a signal of direct CP violation.

As for kaons, transitions described by gluonic penguin diagrams are thought to give rise to ineliminable phase differences between the decay amplitudes of hyperons and anti-hyperons. In the decays of Ξ and Λ, such differences can be measured through the interference between amplitudes of different final-state angular momentum (S and P wave), with different final-state interaction phases. Parity violation induces observable asymmetries in the angular decay distribution of final states from polarized hyperons, and in the polarization of the final state particles themselves. A single amplitude strongly dominates hyperon semi-leptonic decays in the SM, so that no significant CP violation effects are expected there.

In the non-leptonic decay $Y \to B\pi$ (where Y is a hyperon and B a baryon), if the parent Y is polarized, the angular distribution of B in the rest frame of

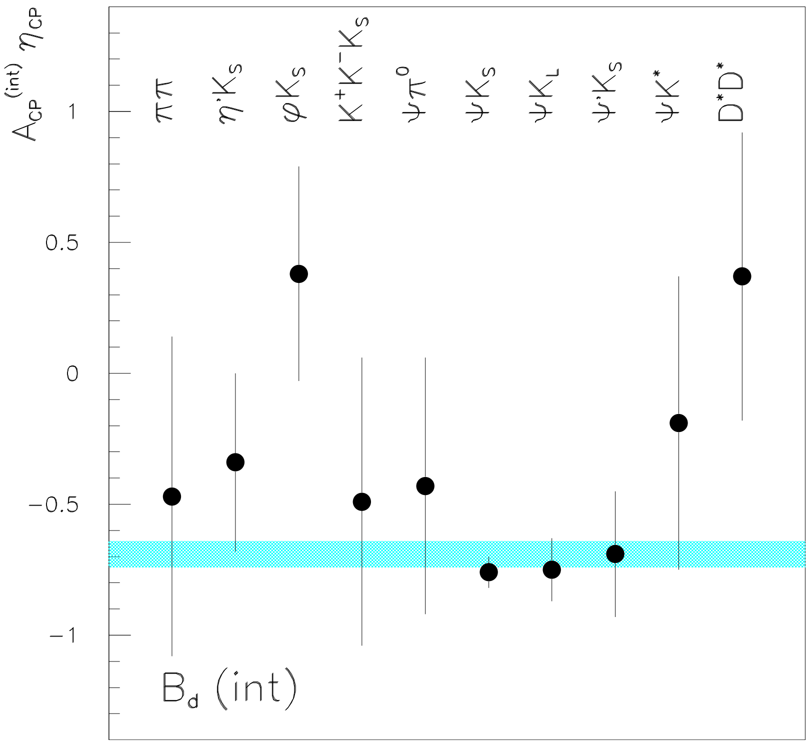

Figure 17: Graphical representation of the measured coefficient of the sine term (multiplied by the CP-parity η_{CP} of the final state) in time-dependent CP asymmetries for $B_d^0(\overline{B}_d^0)$ decays to CP eigenstates. The horizontal band represent the experimental average for all modes.

the parent (averaging over the daughter polarization states) is non-isotropic:

$$\frac{dN}{d\Omega} = \frac{1}{4\pi}(1 + \alpha_Y \mathbf{P}_Y \cdot \hat{\mathbf{p}}_B) \tag{223}$$

and its polarization vector is

$$\mathbf{P}_B = \frac{(\alpha_Y + \hat{\mathbf{p}}_B \cdot \mathbf{P}_Y)\hat{\mathbf{p}}_B - \beta_Y(\hat{\mathbf{p}}_B \times \mathbf{P}_Y) - \gamma_Y \hat{\mathbf{p}}_B \times (\hat{\mathbf{p}}_B \times \mathbf{P}_Y)}{1 + \alpha_Y + \hat{\mathbf{p}}_B \cdot \mathbf{P}_Y} \tag{224}$$

where $\mathbf{P}_{Y,B}$ are the polarization vectors of the hyperon and secondary baryon, and $\hat{\mathbf{p}}_B$ is the unit vector of the secondary baryon momentum in the parent hyperon rest frame. If the parent hyperon Y is unpolarized, the polarization of B reduces to $\mathbf{P}_B = \alpha_Y \hat{\mathbf{p}}_B$, and its angular distribution is isotropic in the rest frame of the parent. $\alpha_Y, \beta_Y, \gamma_Y$ are the decay asymmetry parameters ($\alpha_Y^2 + \beta_Y^2 + \gamma_Y^2 =$

1), related to the interference of the S and P wave decay amplitudes A_S and A_P:

$$\alpha_Y = \frac{2\mathrm{Re}(A_S^* A_P)}{|A_S|^2+|A_P|^2} \tag{225}$$

$$\beta_Y = \frac{2\mathrm{Im}(A_S^* A_P)}{|A_S|^2+|A_P|^2} \tag{226}$$

$$\gamma_Y = \frac{|A_S|^2-|A_P|^2}{|A_S|^2+|A_P|^2} \tag{227}$$

Since α_Y and β_Y change sign under a CP transformation, when comparing the decays of a hyperon and its antiparticle one can form three CP-violating observables:

$$\Delta \equiv \frac{\Gamma_Y - \Gamma_{\overline{Y}}}{\Gamma_Y + \Gamma_{\overline{Y}}} \qquad A \equiv \frac{\alpha_Y + \alpha_{\overline{Y}}}{\alpha_Y - \alpha_{\overline{Y}}} \qquad B \equiv \frac{\beta_Y + \beta_{\overline{Y}}}{\beta_Y - \beta_{\overline{Y}}} \tag{228}$$

where $\Gamma_Y \propto |A_S|^2 + |A_P|^2$ is the partial decay rate, and $\overline{Y}$ refers to the anti-hyperon decay. The A, B observables measure direct CP violation as the difference in the amount of parity violation in the decays of particles and anti-particles.

Also for hyperon decays, theoretical predictions are difficult because of hadronic uncertainties. The A_Ξ and A_Λ asymmetry parameters range from 10^{-4} to 10^{-5} in the Standard Model, and they can be larger in some super-symmetric models, even when direct CP violation in kaon decays is still predicted to be small.

Experimentally, A is the more accessible observable, requiring the measurement of the daughter polarization in the decay of unpolarized hyperons, or of the angular asymmetry parameters in the decay of polarized hyperons. B is generally predicted to be larger than A, and $B' \equiv (\beta_Y - \beta_{\overline{Y}})/(\alpha_Y - \alpha_{\overline{Y}})$ is also independent of the final-state interaction phases, while Δ is predicted to be much smaller.

Early approaches to the measurement of CP violation in hyperon decays focused on the measurement of the A asymmetry in $\Lambda, \overline{\Lambda}$ decays. Three experiments were performed, and the statistically-limited null results have a precision of 1-2% (see table 14).

If the secondary baryon is itself a hyperon, one can exploit the analyzing power of its parity-violating weak decay to get information on its polarization without performing any spin measurement. Considering the decay of unpolarized $\Xi^- \to \Lambda\pi^-$ followed by $\Lambda \to p\pi^-$, the angular decay distribution of the proton (averaging over its unmeasured polarization) actually measures the Λ polarization, induced by the α_Ξ parameter:

$$\frac{dN}{d\cos\theta_\Lambda} \propto 1 + \alpha_\Xi \alpha\Lambda \cos\theta_\Lambda \tag{229}$$

where θ_Λ is the angle between the proton direction in the Λ rest frame and the Λ direction in the Ξ rest frame. In this way the slope of the angular anisotropy of the protons in the Λ rest frame actually allows a measurement of the asymmetry

$$A_{\Xi\Lambda} = \frac{\alpha_\Xi\alpha_\Lambda - \alpha_{\overline{\Xi}}\alpha_{\overline{\Lambda}}}{\alpha_\Xi\alpha_\Lambda + \alpha_{\overline{\Xi}}\alpha_{\overline{\Lambda}}} \simeq A_\Xi + A_\Lambda \tag{230}$$

Each of the A asymmetries requires strong phase differences for the interfering decay amplitudes: while the $p\pi^-$ phase shift difference is measured to be

$\delta_P - \delta_S = (7.1 \pm 1.5)^\circ$, the $\Lambda\pi$ one is not measured and is predicted to be small, so that $A_{\Xi\Lambda}$ should be dominated by A_Λ.

The decays of Ξ^- and $\overline{\Xi}^+$ discussed above have been studied with the CLEO II detector at the CESR e^+e^- storage ring, running at a centre of mass energy just below the $\Upsilon(4S)$ resonance. Inclusively-produced Ξ^- (and $\overline{\Xi}^+$) were reconstructed in the $\Lambda\pi^-$ decay mode[25], with $\Lambda \to p\pi^-$; particle identification was provided by a combination of ionization energy loss and time-of-flight information. The number of reconstructed Ξ events after subtracting the $\approx 7\%$ combinatorial background was $8.4 \cdot 10^3$. An unbinned maximum likelihood fit to the $\cos\theta_\Lambda$ distribution was then performed using a fit function determined by a large Monte Carlo sample, generated with fixed values of the asymmetry parameters and suitably weighted. Several systematic effects in the measurement do cancel in forming the asymmetry, which is also insensitive to any Ξ production polarization. The result [178] is

$$A_{\Xi\Lambda} = (-5.7 \pm 6.4 \pm 3.9) \cdot 10^{-2} \qquad (231)$$

the first error being statistical and the second systematic.

The HyperCP (E781) experiment at Fermilab is dedicated to the measurement of the combined CP asymmetry parameter $\alpha_{\Xi\Lambda}$ using the same decay chain. Ξ^- (and $\overline{\Xi}^+$) are produced at 0° by $7.5 \cdot 10^9$ protons/s (of 800 GeV/c momentum) on a fixed target, and charged secondaries of 150 GeV/$c \pm 25\%$ are magnetically selected. Parity conservation in the strong interaction induced production process enforces the Ξ polarization to be zero when they are produced at zero angle. After a 13 m evacuated decay volume, the decay products are detected in a MWPC-based magnetic asymmetric spectrometer. The polarities of the collimator and spectrometer magnet were periodically reversed to switch from Ξ^- to $\overline{\Xi}^+$ running. The data collected in 1997 and 1999 contains about $2 \cdot 10^9$ Ξ^- and $0.510^{-9}\overline{\Xi}^+$ reconstructed decays, corresponding to a statistical sensitivity of $\sim 2 \cdot 10^{-4}$ on $A_{\Xi\Lambda}$.

The fact that the Λ helicity frame used to define the asymmetry changes from event to event reduces any sensitivity to local acceptance asymmetries of the detector. Two different analysis techniques are being pursued: one which uses Monte Carlo samples to correct for the detector acceptance, and one in which no acceptance correction is applied and the events are weighted to reduce any difference between the Ξ^- and $\overline{\Xi}^+$ samples.

The preliminary analysis of a sample corresponding to $\approx 2\%$ of the total has been presented [179], with the result

$$A_{\Xi\Lambda} = (-0.7 \pm 1.2 \pm 0.6) \cdot 10^{-3} \qquad (232)$$

where the first error is statistical and the second systematic, being presently limited by statistics.

The largest systematic effects affecting this preliminary result are the imperfect reversal of the magnetic fields in the apparatus and the effect of the earth's one, the imperfect symmetry of the detector efficiencies, and effects related to rates or the different interaction properties of π^+ vs. π^- and p vs. $\overline{p}$. Backgrounds, at the level of 0.3% are also different for the two CP conjugate states.

[25] Here and after the two CP-conjugate modes are implied.

Reaction	Asymmetry	Notes
$p\overline{p} \to \Lambda X$	$A_\Lambda = -0.02 \pm 0.14$	CERN R608 (1985)
$e^+e^- \to J/\Psi \to \Lambda\overline{\Lambda}$	$A_\Lambda = 0.01 \pm 0.10$	Orsay DM2 (1988)
$p\overline{p} \to \Lambda\overline{\Lambda}$	$A_\Lambda = 0.013 \pm 0.022$	CERN PS185 (1996)
$p\text{Be} \to \Xi^- X \to \Lambda\pi^- X$	$A_{\Xi\Lambda} = 0.012 \pm 0.014$	FNAL E756 (2000)
$e^+e^- \to \Xi^- X \to \Lambda\pi^- X$	$A_{\Xi\Lambda} = -0.057 \pm 0.075$	CLEO prel. (2000) [178]
$p\text{Be} \to \Xi^- X \to \Lambda\pi^- X$	$A_{\Xi\Lambda} = (-0.7 \pm 1.4) \cdot 10^{-3}$	HyperCP prel. (2002) [179]

Table 14: Experimental results on CP violating asymmetries in hyperon decays (from [36] unless otherwise indicated).

The understanding of several of the above systematic effects is limited by the size of the analyzed sample and is therefore expected to improve.

When its full data sample will be completely analyzed, the HyperCP experiment will provide by far the best test of (direct) CP asymmetries in hyperon decays. However most probably it will not reach the sensitivity corresponding to the Standard Model predictions; hence (although theoretical difficulties linked to hadronic physics makes precise predictions difficult) any positive signal would be a signal of new physics.

Table 14 summaries the experimental information available on (direct) CP violation asymmetries in hyperon decays; no signals of (direct) CP violation have been found so far.

As mentioned above, similar observables would be accessible by studying the decays of heavy baryons (Λ_b, Ξ_b), produced at hadron machines or future e^+e^- colliders running at the Z centre of mass energy.

T-odd triple product correlations in two-body decays, involving the hyperon polarization (in general non-zero in production) could have large values in the SM [180]: one example is $\langle \mathbf{p}_K \cdot \mathbf{S}_{\Lambda_b} \times \mathbf{S}_p \rangle$ for the decay $\Lambda_b \to pK^-$, which however requires the measurement of proton polarization; for similar decays such as $\Xi_b \to \Sigma^+ K^-$ one could exploit the self-analyzing decay of the secondary hyperon to build T-odd correlations using the final state momenta. Other kinds of T-odd correlations, not involving the polarization of the parent particle, are also possible for heavy baryon two-body decays, since their high mass allows vector mesons in the final states: asymmetries involving the polarization of both final state particles in $\Lambda_b \to pK^{*-}$ decays are an example, although the SM predicts tiny asymmetries in such channels. No significant CP violation is expected in the SM for T-odd correlations in (three-body) semi-leptonic decays, where a single amplitude strongly dominates.

A possible dependence of the angular decay distribution on the T-odd correlation $\langle \mathbf{S}_\Sigma \cdot (\mathbf{p}_e \times \mathbf{p}_\nu) \rangle$ in the semi-leptonic $\Sigma^- \to ne^-\overline{\nu}$ decay was also probed at the FNAL hyperon beam, with a null result for the relevant coefficient $C = (0.11 \pm 0.10)$.

6.2 Lepton decays

The recent increasing evidence for neutrino oscillations led to a re-examination of the possibility of CP violation in the leptonic sector. Such a phenomenon is not present in the Standard Model for charged leptons, but appears in several of its extensions; models predicting lepton flavour violation often imply also CP violation in lepton decays, usually as a consequence of the interference between transitions mediated by (some additional) scalar boson and the ones induced by the weak currents.

The analysis of muon decay parameters shows no signs of CP violation in that process, such as the e^+ polarization transverse to the plane defined by the μ^+ polarization and the e^+ momentum: $P_T(e^+) = 0.007 \pm 0.023$ [36]; scalar or tensor interactions would be required for such a polarization to appear.

CP violation in semi-hadronic τ decays has also been investigated recently (see *e.g.* [181]). By postulating a term in the decay amplitude mediated by scalar exchange, the relative phase of its coupling constant with respect to the Standard Model ($W^\pm$) term induces CP-odd terms in the decay distributions (which as usual require a difference in the strong phases for the vector and scalar exchange in order to be non-zero). Both spin-dependent and spin-averaged CP-odd terms can be present; the former can be measured by exploiting the spin correlation of τ leptons produced in $e^+e^- \to \tau^+\tau^-$. Several CP-odd observables can be considered, and their average values have been found to be consistent with zero, with errors which reach at best the 2% level in $\tau^\mp \to \pi^\mp\pi^0\nu_\tau(\overline{\nu}_\tau)$ and $\tau^\mp \to K^0\pi^\mp\nu_\tau(\overline{\nu}_\tau)$ decays.

6.3 Neutrino oscillations

The properties of neutrinos are experimentally difficult to determine, since they are only affected by the weak interaction. On the other hand, this same property makes them a very interesting system to extract information on the fundamental parameters of such interaction, without complications due to strong or electromagnetic effects; moreover the relationship with the theory is much more direct, since neutrinos are closer to being measurable free states than quarks, which are permanently confined within hadrons in a complicated and poorly known way.

Evidence for neutrino flavour oscillations (and therefore, indirectly, for non-degenerate neutrino masses) is now compelling, and therefore the issue of CP violation in the lepton sector is being actively considered. If neutrinos are non-degenerate in mass, a lepton mixing matrix which describes the relation of the physical (mass) eigenstates to the flavour eigenstates can be defined; as in the case of quarks, complex matrix elements could induce CP violation effects, and with three lepton flavours one complex phase remains which cannot be eliminated by field redefinitions. CP violation could be expected to give large effects in such a system, the intrinsic properties of which are completely determined by the CP-violating weak interactions. Moreover, results on neutrino oscillations indicate that, contrary to the case of quarks, one or more of the neutrino mixing angles could be large: it should be kept in mind that CP-violating effects require all three angles to be non-zero, while confirmed experimental indications of such a property only exist for two of them, as determined by solar and atmospheric neutrino experiments.

If neutrinos are Majorana particles (self-conjugate), two additional CP-violating phases appear in the mixing matrix (for 3 flavours); such phases do not produce any observable effect in oscillations, and only affect the rates for neutrinoless double-beta decay: the observation of such process, together with independent information on individual neutrino masses, could in principle allow to extract information on these phases, although this might turn out to be difficult in practice [182].

CP violation effects [183] can be present both in the time evolution of neutrinos in flavour space (oscillations) and in their interactions with matter (detection); the latter is clearly problematic to study, due to the macroscopic CP asymmetry of the detection system. CP violation in neutrino oscillations can be experimentally measured by comparing the probability of a neutrino, prepared in a given flavour state, to be detected after propagation in vacuum as a given different flavour, with the same probability for an antineutrino:

$$P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) \neq P(\nu_\alpha \to \nu_\beta) \tag{233}$$

indicates CP violation, if CPT symmetry is valid.

As usual, CPT symmetry imposes some constraints on the relations among the above probabilities: among these is the fact that CP violation effects in neutrino oscillations, implying differences in transition probabilities between different flavour states, can be studied only in appearance experiments, since the probability of a neutrino to remain in the same flavour eigenstate, measured in disappearance (reactor, solar, atmospheric) experiments, is equal to that for an antineutrino, if CPT symmetry is valid.

In order to have observable effects, it is also necessary that all three families do actively participate to the oscillations: one of the mixing angles is however known not to be large ($\sin(\vartheta_{13}) < 0.16$). Moreover, if the hierarchy of neutrino masses is such that, for a given experimental arrangement, one has a negligible effect from one of the mass differences ($\Delta m^2_{ij} L/2E \ll 1$, where Δm_{ij} ($i, j = 1, 2, 3$) is the mass difference between the two neutrino species, L the distance between the production and detection point, and E the average neutrino energy), the CP-violating asymmetry becomes negligible. It is usually assumed therefore that only long-baseline experiments, sensitive to all three neutrino masses, can aim at directly measuring CP violation effects with neutrinos [184].

The measurable CP violation effects in neutrino oscillations arise from the interference of two neutrino mixing amplitudes with different weak phases and different oscillation phases (due to the mass differences). The CP-asymmetric matter effects in long-baseline experiments add more complications, so that the simple CP-odd asymmetries between oscillation probabilities are no longer a sufficient signal for CP violation.

The best and less model-dependent approach to search for CP violation in neutrino oscillations seems to be the direct search for a CP asymmetry between $\nu_e \to \nu_\mu$ transitions and their CP-conjugates. This could be done exploiting future intense muon storage facilities ("neutrino factories"), presently under study (see *e.g.* [185]), providing very large fluxes of neutrinos, with only two different flavours and opposite helicity ($\nu_\mu, \overline{\nu}_e$ or $\overline{\nu}_\mu, \nu_e$), searching for the appearance of wrong sign muons and comparing the results for runs with opposite muon charges.

The sensitivity to CP-violating asymmetries when using intense conventional ν_μ neutrino beams would be limited by the intrinsic contamination of $\overline{\nu}_\mu$ ($\sim 1\%$) and ν_e ($\sim 0.5\%$), although this can be accurately predicted for a primary proton energy below the kaon production threshold.

The search of CP violation in neutrinos is clearly a very interesting field of research, since its detection would establish that such phenomenon is not a

e^-	$d_e = (6.9 \pm 7.4) \cdot 10^{-28}\, e$ cm
$\mu^\pm$	$d_\mu = (3.7 \pm 3.4) \cdot 10^{-19}\, e$ cm
$\tau^\pm$	$-2.2 \cdot 10^{-17} < \mathrm{Re}(d_\tau) < 4.5 \cdot 10^{-17}\, e$ cm (95% CL) [187]
	$-2.5 \cdot 10^{-17} < \mathrm{Im}(d_\tau) < 0.8 \cdot 10^{-17}\, e$ cm (95% CL) [187]
p	$d_p = (-3.7 \pm 6.3) \cdot 10^{-23}\, e$ cm
n	$d_n < 6.3 \cdot 10^{-26}\, e$ cm (90% CL)

Table 15: Experimental limits on electric dipole moments of elementary particles, from [36] unless otherwise indicated.

peculiarity of quarks. However, precise estimates of the size of expected effects require the knowledge of all the neutrino mixing angles, and it is clear that CP violation studies will require new facilities for sensitive studies.

6.4 Electric dipole moments

Studies of CP violation are not restricted to unstable elementary particles, but have been pursued, both in experiments and in theory, also for the constituents of ordinary matter. The most relevant example is the search for static, permanent electric dipole moments (EDM) of elementary, non-degenerate systems. Such a quantity is odd under both parity and time reversal transformations, and it indicates a violation of those symmetries if it has a non-zero value: in such systems the EDM vector can only be parallel to the only intrinsic vector, *i.e.* the particle spin, which however has opposite transformation properties with respect to P and T. The search for electric dipole moments of elementary particles has a long history dating back to the '50s; an exhaustive discussion of the subject can be found in [186].

Experimental approaches are based on detecting the electric "Zeeman effect" when the system is placed in an external electric field, and the key issue is the reduction of spurious effects in the measurement; a typical measurement is the change in the Larmor precession frequency of the system for parallel or anti-parallel magnetic and electric fields.

To avoid the experimental complications of working with a particle with non-zero electric monopole moment (charge), neutral particles were mostly studied: several experiments are devoted to the measurement of the neutron EDM with ultra-cold neutrons ($T \sim 2$ mK). In an atomic system the rearrangement of charges in an external electric field could shield its effects, but in heavy paramagnetic atoms relativistic effects lead instead to significant enhancements of the EDM induced by valence electrons, and very sensitive searches are performed to search for EDM in Cs and ^{205}Tl atoms. EDM in diamagnetic atoms would be driven instead by the nuclear spin direction, and searches in ^{129}Xe, ^{199}Hg and other atoms, and also heavy polar molecules (such as TlF), are an active target of study.

No positive signal of permanent EDM has been found so far; table 15 summaries the current experimental limits.

If CPT symmetry is valid, T violation is equivalent to CP violation, but in this case its classification as direct or indirect discussed above is not really appropriate: non-zero EDMs would be a signal of T violation in the static properties of a system and its interaction with the environment.

6.5 Production asymmetries

Electric dipole moments of unstable leptons have also been searched for by studying their coupling to photons in e^+e^- formation: the most general Lorentz invariant electromagnetic coupling is described by

$$F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2m_l}\sigma^{\mu\nu}q_\nu - F_3(q^2)\sigma^{\mu\nu}\gamma_5 q_\nu \tag{234}$$

where γ_μ, γ_5 e $\sigma^{\mu\nu}$ are Dirac matrices and their combinations, m_l is the lepton mass and q_ν the photon four-momentum. $F_1(0)$ is the electric charge, $F_2(0)$ the anomalous magnetic moment $(g-2)/2$ and $F_3(0)$ the ratio of electric dipole moment and electric charge, which is zero if P or T symmetry holds.

The study of the differential cross section for the reaction $e^+e^- \to \tau^+\tau^-$ (when dominated by virtual photon production) allows to put limits on the τ electric dipole moment. T violation would allow $\tau^+\tau^-$ production in a CP-odd state which, thanks to the relatively long lifetime of the τ lepton and the fact that its decay does not depend from the EDM value, would manifest as different momentum correlations, with respect to the dominant production mechanism in a CP-even state, for the final states from the decay of the τ lepton pair. Photon angular and energy distributions in the $e^+e^- \to \tau^+\tau^-\gamma$ state are also sensitive to the electric dipole moment.

When the production mechanism is through a virtual Z, several T-odd observables sensitive to anomalous couplings of the Z boson (weak dipole moment) can be built [188], by using the τ polarization vectors which can be determined by their self-analyzing decays; polarized beams would allow more T-odd correlations to be probed.

Anomalous couplings of the electroweak gauge bosons, $W^+W^-Z\gamma$, $Z\gamma\gamma^*$, $ZZ^*\gamma$, ZZZ^*, have been studied in e^+e^- and $p\overline{p}$ interactions with several final states, by detailed analysis of production cross sections and angular or energy distributions. Such anomalous couplings, which are zero at tree level in the Standard Model, could in principle introduce CP violation effects, for which limits have been set, mostly by LEP [36]. No CP-violating effects have been detected so far.

CP violation measurements in the $t\overline{t}$ production processes at present $p\overline{p}$ colliders ($10^4 \div 10^5$ $t\overline{t}$/year at the TeVatron) or future $pp, e^+e^-, \mu^+\mu^-, \gamma\gamma$ colliders ($10^7 \div 10^8$ $t\overline{t}$/year at LHC, $10^5 \div 10^6$ $t\overline{t}$/year at a future e^+e^- linear collider), have been discussed in the literature (see [189] for a comprehensive review). The study of top quark production presents some advantages: due to its large mass, this quark decays before hadronizing, so that its production and decay processes are not masked by non-perturbative physics; this also means that any CP violation in these processes is of the direct type. For the same reason, the spin of the top quark is an important observable (just as for leptons) which can be analyzed in its decay, particularly the semi-leptonic ones ($t \to bl^+\nu$). Moreover, while CP-violating effects in top-quark production are expected to be small in the Standard Model, the large mass of the t makes such processes highly sensitive to several other possible sources of CP violation from new physics.

If the $t \to bW^+$ decay dominates the other decay modes of the top quark, partial rate asymmetries are expected to be small, as they would vanish due to

CPT symmetry in the limit of a single decay mode; other decay modes would have larger asymmetries, but their rates are expected to be so small to make measurements difficult. Decay asymmetries which are not suppressed by the above argument are the ones obtained by integrating only over a part of the phase space; one example is the lepton average energy asymmetry in $t \to bl\nu$ decays

$$A_{CP}^{(E)} \equiv \frac{\langle E(l^+)\rangle - \langle E(l^-)\rangle}{\langle E(l^+)\rangle + \langle E(l^-)\rangle} \tag{235}$$

Such partially-integrated rate asymmetries in semi-leptonic decays turn out to be proportional to the lepton mass, so that the $l = \tau$ channel is usually considered. Asymmetries involving the lepton polarization in semi-leptonic decays are expected to be larger, in a model-independent way; examples are

$$A_y \equiv \frac{N(\tau^+;S_y>0)-N(\tau^+;S_y<0)+N(\tau^-;S_y>0)-N(\tau^-;S_y<0)}{N(\tau^+;S_y>0)+N(\tau^+;S_y<0)+N(\tau^+;S_y>0)-N(\tau^-;S_y<0)} \tag{236}$$

$$A_z \equiv \frac{N(\tau^+;S_z>0)-N(\tau^+;S_z<0)-N(\tau^-;S_z>0)+N(\tau^-;S_z<0)}{N(\tau^+;S_z>0)+N(\tau^+;S_z<0)+N(\tau^-;S_z>0)+N(\tau^-;S_z<0)} \tag{237}$$

where the τ spin components (S_x, S_y, S_z) refer to a τ rest frame in which the τ momentum is in the $-x$ direction and the y axis is in the decay plane along the b direction.

Other production asymmetries which could be studied at an e^+e^- collider are [189]

$$A_{CP}^{(C)} \equiv \frac{\int_{\theta_0}^{\pi-\theta_0} d\theta_l \left(\frac{d\sigma^+}{d\theta_l} - \frac{d\sigma^-}{d\theta_l}\right)}{\int_{\theta_0}^{\pi-\theta_0} d\theta_l \left(\frac{d\sigma^+}{d\theta_l} + \frac{d\sigma^-}{d\theta_l}\right)} \tag{238}$$

$$A_{CP}^{(FB)} \equiv \frac{\int_{\theta_0}^{\pi/2} d\theta_l \left(\frac{d\sigma^+}{d\theta_l} + \frac{d\sigma^-}{d\theta_l}\right) - \int_{\pi/2}^{\pi-\theta_0} d\theta_l \left(\frac{d\sigma^+}{d\theta_l} + \frac{d\sigma^-}{d\theta_l}\right)}{\int_{\theta_0}^{\pi-\theta_0} d\theta_l \left(\frac{d\sigma^+}{d\theta_l} + \frac{d\sigma^-}{d\theta_l}\right)} \tag{239}$$

$$A_{CP}^{(UD)} \equiv \frac{1}{2\sigma(\theta_0)} \int_{\theta_0}^{\pi-\theta_0} d\theta_l \left(\frac{d\sigma(\mathbf{p}_{ly} > 0)}{d\theta_l} - \frac{d\sigma(\mathbf{p}_{ly} < 0)}{d\theta_l}\right) \tag{240}$$

$$A_{CP}^{(LR)} \equiv \frac{1}{2\sigma(\theta_0)} \int_{\theta_0}^{\pi-\theta_0} d\theta_l \left(\frac{d\sigma(\mathbf{p}_{lx} > 0)}{d\theta_l} - \frac{d\sigma(\mathbf{p}_{lx} < 0)}{d\theta_l}\right) \tag{241}$$

where $\sigma^\pm$ are the inclusive cross section for producing positive or negative leptons ($\sigma = \sigma^+ + \sigma^-$), θ_l is the lepton polar angle with respect to the incident beams' axis, θ_0 a suitable cutoff angle, and $\mathbf{p}_{lx}, \mathbf{p}_{ly}$ are the components of the lepton momentum in the centre of mass system in which the z axis is defined by the incident particles, the x axis is within the $t\bar{t}$ production plane and the y axis is orthogonal to it.

Asymmetries involving top quark polarization have also been considered, such as

$$A_S \equiv \frac{\sigma_{LL} - \sigma_{RR}}{\sigma} \tag{242}$$

where the subscripts refer to the t and $\bar{t}$ helicities, which could be measured from the angular distributions of the decay products: considering *e.g.* semi-leptonic decays $t \to bl\nu$, the correlation of top-quark polarization and lepton direction is maximal.

The Standard Model does not predict measurable values for the above asymmetries, which could be therefore used as probes of new physics.

7 T, CPT violation and related issues

The CPT theorem [12] is based on very general principles of quantum field theory, and its validity makes CP violation equivalent to a violation of microscopic time reversibility in elementary physical processes. Despite its solid foundations, the validity of CPT symmetry has been questioned: the possible breakdown of locality at short distances, *e.g.* in string theories, of Lorentz invariance, as implemented in explicit models [190], or of quantum mechanics, would undermine the proof of its validity.

CPT symmetry has been tested experimentally by comparing masses, lifetimes, electric charges and other static properties of particles and anti-particles in several systems [36]. A complete discussion of CPT tests would be clearly beyond the scope of this work (see *e.g.* [5] and references therein), and we will just briefly comment on some of the methods used to check the validity of this symmetry, and on how its violation could affect previous considerations.

As for CP violation, in neutral meson systems one also distinguishes between indirect CPT violation, in the $\Delta F = 2$ transitions, and direct CPT violation in the $\Delta F = 1$ decay amplitudes. The former would lead to flavour components of different magnitudes in the physical states K_L and K_S, described by two mixing parameters

$$\overline{\epsilon}_S = \overline{\epsilon} + \Delta \qquad \overline{\epsilon}_L = \overline{\epsilon} - \Delta \tag{243}$$

so that $\Delta \neq 0$ indicates CPT violation in the effective Hamiltonian.

An accurate comparison of charge asymmetries in semi-leptonic decays of K_L and K_S could give information on the difference of such mixing parameters:

$$\delta^{(l)}_{L,S} = \frac{2\mathrm{Re}(\overline{\epsilon}_{L,S})}{1 + |\overline{\epsilon}_{L,S}|^2} - 2\mathrm{Re}(y) \pm 2\mathrm{Re}(x_-) \tag{244}$$

where y and x_- parameterise the violation of CPT in the decay amplitudes when the $\Delta S = \Delta Q$ rule is valid or not, respectively (the above expression is valid at first order in the small quantities $|\epsilon_{S,L}|$, $|y|$, $|x_-|$). Any difference among the two charge asymmetries above would indicate CPT violation, either "indirect" ($\mathrm{Re}(\Delta) \neq 0$) or "direct" ($y, x_- \neq 0$). With the inclusion of the statistics collected until 2002, the statistical error on the KLOE measurement of the charge asymmetry in semi-leptonic decays of K_S will be below 0.01, but still far from the level required for a significant quantitative test of CPT.

The comparison of the measured value of Δ with the value of $\mathrm{Re}(\epsilon)$ obtained from $\pi\pi$ decays [36] also allows to put bounds on CPT violation [37], [38]:

$$\mathrm{Re}(y + x_- - a_{CPT}) = \mathrm{Re}\left(\frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00}\right) - \frac{\delta_L}{2} = (5 \pm 32) \cdot 10^{-6} \tag{245}$$

where a_{CPT} parameterises direct CPT violation in $\pi\pi$ decays.

Information on $\overline{\epsilon}_S$ could also be extracted from the analysis of the flavour-tagged kaon decay asymmetry to the final state $\pi^+\pi^-\pi^0$ in a phase space region symmetric for π^+ and π^-, for large times (see eq. 76)

$$A_{CP}^{(\pi^+\pi^-\pi^0)}(t \to \infty) = 1 - \left|\frac{1 - \overline{\epsilon}_S}{1 + \overline{\epsilon}_S}\right|^2 \tag{246}$$

By using semi-leptonic decays of flavour-tagged kaons, CPLEAR searched for CPT-violating asymmetries [191]:

$$A_\Delta(t) = \frac{\overline{N}_+(t) - N_-(t)(1+4\mathrm{Re}(\overline{\epsilon}_L))}{\overline{N}_+(t) - N_-(t)(1+4\mathrm{Re}(\overline{\epsilon}_L))} + \frac{\overline{N}_-(t) - N_+(t)(1+4\mathrm{Re}(\overline{\epsilon}_L))}{\overline{N}_-(t) - N_+(t)(1+4\mathrm{Re}(\overline{\epsilon}_L))} \tag{247}$$

where $N_\pm$ ($\overline{N}_\pm$) is the number of K^0 ($\overline{K}^0$) decaying to $\pi^\pm e^\pm \nu(\overline{\nu})$. The value of $A_\Delta(t)$ for $t \gg \tau_S$ measures $\mathrm{Re}(\Delta)$, independently from the validity of the $\Delta S = \Delta Q$ rule and from direct CPT violation, while the study of its time dependence allows to extract information on $\mathrm{Im}(\Delta)$.

The phases of the CP violation parameters in the $\pi\pi$ decay modes allows checks of CPT violation, due to the fact that direct CP violation is very small. In the limit in which the $(\pi\pi)_{I=0}$ final state saturates the decay widths of neutral kaons[26], the phase of the ϵ parameter is ϕ_{SW} if CPT symmetry is valid, so that the phase difference $\phi_{+-} - \phi_{SW}$ checks this symmetry.

A recent test of CPT violation by KTeV [41] is the measurement of

$$\phi_{+-} - \phi_{SW} = (0.61 \pm 1.19)^\circ \tag{248}$$

obtained with a different fit to the longitudinal decay vertex distribution of the 1997 $K_{S,L} \to \pi^+\pi^-$ data used in the ϵ'/ϵ analysis. The error is dominated by the systematics due to the dependence on the geometrical acceptance cuts and on the model dependence of the nuclear screening corrections which modify the regeneration amplitude.

The same experiment also reported a test of direct CPT violation from the difference among the phases of η_{00} and η_{+-}:

$$\phi_{00} - \phi_{+-} = (0.39 \pm 0.50)^\circ \tag{249}$$

corresponding to

$$\mathrm{Im}(\epsilon'/\epsilon) = (-22.9 \pm 29.1) \cdot 10^{-4} \tag{250}$$

The real part of ϵ'/ϵ extracted from this KTeV analysis, when CPT symmetry is not imposed, is

$$\mathrm{Re}(\epsilon'/\epsilon) = (+22.5 \pm 1.9_{\mathrm{stat}}) \cdot 10^{-4} \tag{251}$$

where the error is only statistical and larger than in the standard analysis, due to the correlation with the imaginary part as shown in fig. 18.

In the same limit (and at first order in the deviation of the ϕ_{+-}, ϕ_{00} phases from ϕ_{SW}), the component of Δ orthogonal to the direction of ϕ_{SW} is given by

$$\Delta_\perp \simeq |\eta_{+-}| \left(\phi_{SW} - \frac{2}{3}\phi_{+-} - \frac{1}{3}\phi_{00} \right) - \frac{\mathrm{Re}(b_0)}{\mathrm{Re}(a_0)} \sin(\phi_{SW}) \tag{252}$$

where the b_I amplitudes measure direct CPT violation in the $K \to (\pi\pi)_I$ decay amplitudes:

$$A_I = (a_I + b_I)e^{i\delta_I} \tag{253}$$

$$\overline{A}_I = (a_I^* - b_I^*)e^{i\delta_I} \tag{254}$$

Neglecting CP violation, the $\Delta_\parallel$ component, along the direction of ϕ_{SW}, vanishes at the same level of approximation. $\Delta_\perp$ and $\Delta_\parallel$ are proportional to the

[26] Note that in this limit $\epsilon' = 0$, and CP violation in other decay modes is zero.

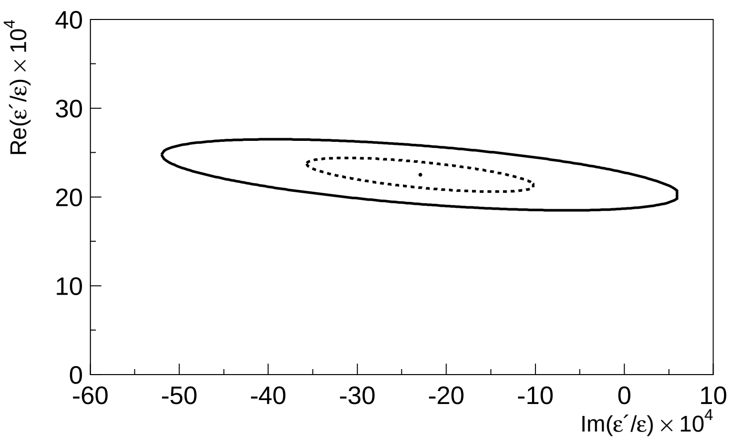

Figure 18: KTeV results for ϵ'/ϵ with no CPT assumptions.

mass and total decay width difference of K^0 and $\overline{K}^0$ respectively, so that the limit on $\Delta_\perp$, *assuming* no direct CPT violation, translates to the best limit on CPT symmetry, quoted as[82]

$$m(K^0) - m(\overline{K}^0) \simeq \frac{2\Delta m}{\sin(\phi_{SW})}|\eta_{+-}|\left(\phi_{SW} - \frac{2}{3}\phi_{+-} - \frac{1}{3}\phi_{00}\right) = (-1.7 \pm 4.2) \cdot 10^{-19}\ \ \mathrm{GeV}/c^2 \quad (255)$$

Analysis based on the constraint obtained from the Bell-Steinberger relation [192], which is a consequence of unitarity relating the amplitudes for all neutral kaon decay modes to the parameters describing CP violation in the effective Hamiltonian for the $K^0 - -\overline{K}^0$ system, allow the extraction of CP and CPT violation parameters in the effective Hamiltonian:

$$[1 + i\tan(\phi_{SW})][\mathrm{Re}(\overline{\epsilon}) - i\mathrm{Im}(\Delta)] = \sum_f \alpha_f \quad (256)$$

in which $\alpha_f = (1/\Gamma_S)A^*(K_S \to f)A(K_L \to f) \propto \eta_f BR(K_S \to f)$, and the sum runs over all decay channels f. The sum is dominated by the $\pi\pi$ channels, since the hadronic decay modes allowed for the K_L in absence of CP violation are suppressed by the small τ_S/τ_L ratio. With this approach $\mathrm{Im}(\Delta)$ can be obtained with a higher precision than that obtained by the $A_\Delta(t)$ measurement, without any dependence on the amount of direct CPT violation in the dominant decay to $(\pi\pi)_{I=0}$.

The real and imaginary parts of Δ in the neutral kaon system are measured to be zero with an accuracy of $\sim 10^{-4}$ for the real part and $\sim 10^{-3}$ for the imaginary part.

It should be noted that, if CPT violation arises through a violation of Lorentz invariance, such as can be the case in quantum field theory, the parameter Δ cannot be a constant but must depend on the four-momentum [193]; moreover, relations between the real and imaginary part of Δ are obtained in explicit models. A search for a dependence of the phase ϕ_{+-} in $K \to \pi^+\pi^-$ decays on the absolute direction of the beam in space, which would appear in a fit of such asymmetry as a function of sidereal time due to the Earth's rotation, was performed [194] with null results.

A thorough (although quite dated) analysis of CPT tests in the neutral kaon system can be found in [195]. It is important to note that in presence of CPT violation, new terms can arise in the expressions for the experimental observables which are used to study CP violation. As an example, the expressions for the ϵ, ϵ' parameters parameterising CP violation in $K \to \pi\pi$ decays become

$$\epsilon = \overline{\epsilon} + i\frac{\mathrm{Im}(a_0)}{\mathrm{Re}(a_0)} - \Delta + \frac{\mathrm{Re}(b_0)}{\mathrm{Re}(a_0)} \tag{257}$$

$$\epsilon' = i\frac{\omega}{\sqrt{2}}\left[\frac{\mathrm{Im}(a_2)}{\mathrm{Re}(a_2)} - \frac{\mathrm{Im}(a_0)}{\mathrm{Re}(a_0)} - i\left(\frac{\mathrm{Re}(b_2)}{\mathrm{Re}(a_2)} - \frac{\mathrm{Re}(b_0)}{\mathrm{Re}(a_0)}\right)\right] \tag{258}$$

so that in ϵ both direct and indirect CPT violation terms appear, and in ϵ' a term of direct CPT violation is present which is orthogonal to the one describing direct CP violation; after having factored out the strong phase shifts in ω, this term is real since it violates CPT and CP but not T.

The high-precision study of time-dependent or time-integrated asymmetries for pairs of correlated or uncorrelated neutral mesons can provide measurements of CPT violating parameters (see *e.g.* [67], [68]). The charge asymmetries of same-sign and opposite-sign di-lepton pairs from the decays of (C-odd) correlated mesons at ϕ or B factories are sensitive to CPT violation:

$$A_{CPT} = \frac{N_{-+} - N_{+-}}{N_{-+} + N_{+-}} \propto \frac{|A(M \to M)|^2 - |A(\overline{M} \to \overline{M})|^2}{|A(M \to M)|^2 + |A(\overline{M} \to \overline{M})|^2} \tag{259}$$

where N_{+-} (N_{-+}) are the number of di-lepton events in which the positive (negative) lepton is emitted before the oppositely charged one (the allowed decays being $M \to l^+X$ and $\overline{M} \to l^-X$). A_{CPT} is non-zero in presence of either CPT violation in the effective Hamiltonian ($\Delta \neq 0$) or violation of the $\Delta F = \Delta Q$ rule; when the latter is absent the asymmetry is

$$A_{CPT} = -4\mathrm{Re}(\Delta) - 8\mathrm{Im}(\Delta)\frac{\Delta m \Gamma_L}{\overline{\Gamma}^2 + (\Delta m)^2} \tag{260}$$

Limits on the validity of CPT symmetry in B mesons have been obtained from inclusive lepton decays of neutral B mesons produced at the Z centre of mass energy [196], and with the analysis of the time dependence of di-lepton yields for the decays of $B^0\overline{B}^0$ pairs produced by $\Upsilon(4S)$ ([197], [136] and references therein). These tests have not yet reached the level of precision available in the K system: the Δ parameter is known to be zero with a precision of $\sim 10^{-1}$ for the real part and $\sim 5 \cdot 10^{-2}$ for the imaginary part.

Many tests of the CPT-enforced equalities of particle and antiparticle properties have been performed, with relative precisions which range from a few per mille to $\sim 10^{-10}$ for the charge/mass ratio of proton and antiprotons; however, in absence of specific CPT violation models, the accuracy of the measurements cannot be translated to a level to which the symmetry itself is tested to be valid.

As far as CPT symmetry is valid, CP violation is equivalent to T violation. Some of the measurements discussed in the previous sections, however, are actual tests of T symmetry, such as those related to transverse polarization measurements. The only positive measurement of T violation to date is that obtained by the CPLEAR experiment [191] by measuring the asymmetry

$$A_T(t) = \frac{\overline{N}_+(t) - N_-(t)}{\overline{N}_+(t) + N_-(t)} \tag{261}$$

for flavour tagged neutral kaon decays in the semi-leptonic mode (with the same notation used in the definition of A_Δ, eq. (247)). The value of this asymmetry for $t \gg \tau_S$ measures time-reversal violation if CPT symmetry holds, independently from the validity of the $\Delta S = \Delta Q$ rule, but can also be non-zero if CPT (and CP) is violated and T holds.

The charge asymmetries of same-sign di-lepton pairs from correlated meson pairs can also test time reversal symmetry:

$$A_T = \frac{N_{++} - N_{--}}{N_{++} + N_{--}} \propto \frac{|A(\overline{M} \to M)|^2 - |A(M \to \overline{M})|^2}{|A(\overline{M} \to M)|^2 + |A(M \to \overline{M})|^2} \tag{262}$$

with the same notation used above in (259). This asymmetry is non-zero in presence of either T violation or CPT violation in the decay amplitudes.

A long-standing unresolved question, whose discussion is out of the scope of this article, is that of the so-called "strong CP problem" (see *e.g.* [198]): a free CP-violating parameter of the Standard Model is experimentally constrained to have an "unnatural" (and unstable) tiny value. Such parameter could induce large CP- (and P-) violating flavour-conserving effects, but the stringent limits on the EDMs of particles show that this is not the case.

Another of the outstanding problems of physics, which is directly related to the issue of direct CP violation, is that of explaining the baryon asymmetry of the universe. It is well known that the three necessary ingredients to generate such asymmetry are baryon number violation, C and CP violation, and departure from thermal equilibrium [199]. The Standard Model contains all such ingredients, but by now it has been made clear that it cannot account for the observed value of the baryon asymmetry: the mechanism of CP violation linked to the flavour Yukawa couplings is far too small to explain the presently observed ratio of matter to radiation in the framework of electroweak baryogenesis (see *e.g.* [200]). It is therefore clear that new, and so far unknown, features are essential to explain the universe as we observe it, so that also new mechanisms of CP violation are expected to exist in a fundamental theory, of which the Standard Model is thought to be the low-energy limit. Indeed many extensions of the known theory include several new ineliminable phases, often even too many, so that some fine tuning of the parameters is required for the theoretical predictions to be consistent with the observed CP violation.

The above considerations show that the phenomenon of CP violation is very likely to be of fundamental importance even at higher energy scales, in relation to the search for a more comprehensive theory of microscopic phenomena.

8 Conclusions and Perspectives

For 37 years the phenomenon of CP violation remained experimentally confined to the neutral kaon system, and for a similar period of time it could be described by a single parameter, related to a property of the decaying state, *i.e.* $K^0 - -\overline{K}^0$ mixing, or indirect CP violation. The lack of other measurements of

CP violation could raise the suspicion of it being a peculiar feature of the neutral kaon system, possibly through a new, unmeasurably small, fundamental interaction.

In this review, we tried to give an overview of the long path of experimental investigations which tried to shed some light on this fundamental issue, and which still continue today, with great dedication and intensity.

Today, with the experimental proof of the existence of direct CP violation in the decays of neutral kaons, and the first measurements of CP violation in the neutral B meson system, our knowledge has been remarkably improved, although this fundamental property of Nature still remains much of a mystery, only being detected via neutral meson decays and nowhere else.

The deep mystery shrouding the nature of CP violation can only push physicists to increased efforts, both in theory and experiment, aimed at reaching a better understanding, with the additional expectation that Nature will disclose to us more of her beautifully concealed secrets along the way.

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