CHAPTER FORTY NINE

MEASUREMENTS OF SURF BEAT AND SET-DOWN BENEATH WAVE GROUPS

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ABSTRACT

A laboratory study was conducted to measure the amplitudes of long waves in shallow water as induced by wave grouping. In a 55 m long wave channel with a plane beach at the end, two primary waves of nearly equal frequency were generated. Due to a sophisticated control of the wave paddle - including second order wave generation as well as active wave absorption at the paddle face - the wave action at the difference frequency was limited to an incident forced wave, propagating at the group velocity, and a reflected free wave generated in the surf zone. For the incident forced - or bound - wave, also known as set-down, the experimental results show good agreement with the existing theory. Furthermore, the experiments confirm qualitatively a theoretical model by Symonds et al. (1982) explaining two-dimensional surf beat as a result of the time-varying breakpoint of the incident primary waves.

1 INTRODUCTION

Long period sea-level fluctuations of several minutes in the nearshore zone were first observed by Munk (1949) and Tucker (1950). Their field records revealed a correlation between the amplitude of these fluctuations, since then termed surf beat, and the amplitude of the incoming swell. The mechanisms by which surf beat is generated have been discussed since then.

Several more recent field studies revealed that three-dimensional free edge waves can account significantly for nearshore long period energy. Gallagher (1971) was able to show theoretically that edge waves can be generated and resonantly excitated by nonlinear interaction of obliquely incident wave groups. However, since the first observations there has been some evidence that, apart from the resonant excitation of edge waves, two-dimensional surf beat due to normally incident waves also exists and contributes significantly to the long period energy.

Using the concept of radiation stress, Longuet-Higgins and Stewart (1962 and 1964) presented a theoretical explanation for the existence of a nonlinear forced - or bound - wave associated with grouped waves. This wave, propagating at the group velocity, results in a set-down beneath wave groups. For a constant water depth Bowers (1977) and Ottesen Hansen (1978) derived expressions for the velocity

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potential and the surface elevation of the bound long wave, using a Stokes expansion of the wave equations to the second order.

Referring to Tucker's observations, showing at some distance offshore a time lag of several minutes between the envelope of the incoming swell and tbe measured long wave, Longuet-Higgins and Stewart (1962) suggested that the incident bound wave is reflected from the surf zone and is radiated seaward as a free long wave, resulting in two-dimensional surf beat. However, no mechanism was proposed for this phenomenon. More recently, Symonds, Huntley and Bowen (1982) presented a theoretical model explaining the generation of free long waves, radiating seaward from a plane beach, as a result of the time-varying position of the breakpoint of normally incident grouped waves. Shoreward of the breakpoint standing wave solutions are found. This model has been extended by Symonds and Bowen (1984) to describe the long wave solutions generated over any offshore profile, including barred topographies.

Apart from coastal processes, surf beat is of special importance for the resonant excitation of harbour basins and moored structures. However, until recently a correct reproduction of these waves in physical models has been hampered for two reasons. Firstly, if a wave paddle is programmed to produce the primary waves only, the boundary conditions at the paddle are not fulfilled for the bound long waves. This results in the generation of spurious free long waves, as already pointed out by Bowers (1977). Secondly, disturbances occur when reflected long waves are re-reflected from the paddle. Flick et al. (1980) show that for critical frequencies this may even result in resonance, which makes model testing on surf beat phenomena virtually impossible. In the following sections methods to prevent both types of disturbances are discussed. Moreover, experimental results on both set-down beneath wave groups and free long waves generated in the surf zone and radiated seaward, are presented.

2 EXPERIMENTAL SET-UP

The experiments were carried out in a glass-wall wave channel of the Delft Hydraulics Laboratory. The cbannel is 55 m long, 1.0 m wide and 1.2 m high.

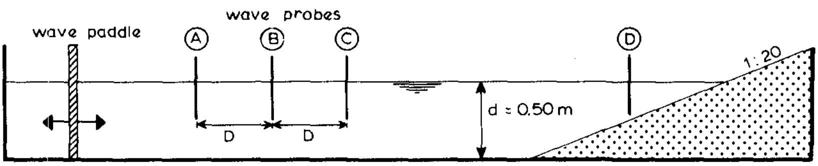


Fig. 1 Sketch of the wave channel

The facility is equiped with a hydraulically driven cradle-type wave generator which was operated in the piston mode only. For all tests a primary wave system made up of two frequencies was generated in a water depth of 0.50 m. These waves broke on a plane cemented beach of a 1:20 slope (see Fig. 1).

3 WAVE PADDLE CONTROL

Excluding reflection from the beach, the first-order water surface elevation in the wave channel is given by:

$$\eta = \hat{\eta}_1 \cos(\omega_1 t - k_1 x - \phi_1) + \hat{\eta}_2 \cos(\omega_2 t - k_2 x - \phi_2) \quad (1)$$

Application of the classical wave paddle control for the generation of these bichromatic waves will induce a secondary wave motion at the difference frequency, which may consist of the following four components:

- incident bound wave : $\hat{\zeta}_{ib} \cos(\Delta\omega t - \Delta k x - \phi_{ib})$
- incident free wave : $\hat{\zeta}_{if} \cos(\Delta\omega t - \kappa x - \phi_{if})$
- reflected bound wave : $\hat{\zeta}_{rb} \cos(\Delta\omega t + \Delta k x - \phi_{rb})$
- reflected free wave : $\hat{\zeta}_{rf} \cos(\Delta\omega t + \kappa x - \phi_{rf})$

where Δk, the wave number of the bound waves, equals $k_1 - k_2$, and the wave number of the free waves κ is coupled to the wave group frequency $\Delta\omega$ by the dispersion relation:

$$(\Delta\omega)^2 = (\omega_1 - \omega_2)^2 = g\kappa \tanh \kappa d \quad (2)$$

Ottesen Hansen (1978) shows that the amplitude of a bound wave is proportional to the product of the amplitudes of the corresponding primary waves. For the present study the 1:20 slope results in reflection coefficients for the primary waves of a few per cent. Therefore, the amplitudes of the reflected bound waves are negligible.

One of the objectives of the study was to measure the amplitude of the outgoing free long wave generated in the surf zone. To be able to do this, it is essential that all reflected low frequency energy is related to the outgoing free long wave only. However, without taking any precautions, and in spite of the mild slope of the beach, any incident free long wave will almost fully reflect due to its low steepness. Consequently, incident free long waves have to be avoided. In a wave channel there are two main reasons for the occurrence of these waves. Firstly, if a wavemaker is programmed to produce the primary wave system without the associated secondary bound wave, the boundary conditions at the paddle face are not fulfilled to second order. This results in the generation of spurious free long waves propagating onto the beach. Ottesen Hansen et al. (1980) give the theoretical expressions for these waves. Barthel et al. (1983) describe in detail how to correct the control signal for the wave paddle to second order, and verify this method, both for bichromatic wave systems and random waves. Secondly, free long waves reflected from the surf zone will re-reflect from the paddle face, resulting in another incident free wave. To prevent this, a recently developed device for active wave absorption has been applied. The special features of this electronic compensation are the capability of absorbing both primary and secondary waves and the independency of the mass of the paddle on the absorbing capacity.

The effect of second-order wave generation as well as active wave absorption on the resulting wave motion in the wave channel is illustrated in Figures 2a to 2d. These figures show respectively, from above to below, the control signal for the wave paddle, the wave signal recorded by the middle probe, and filtered low frequency signals at three increasing distances from the paddle (see Figure 1). Comparing these figures the effect of active wave absorption is obvious. The low frequency recordings obtained without absorption show instabilities - partly due to resonances in the channel - which almost completely vanish in the recordings obtained with absorption. An extra advantage of active wave absorption is the drastic reduction in time for a test series. Without absorption the low frequency water motion continues quite some time after the wavemaker is stopped, whereas with absorption one test can be started almost immediately after another. The effect of second-order wave generation can be seen best in the left-hand side of the figures, when the reflected waves are not yet interfering with the incident ones. If the wave generation is restricted to first order, the earliest long waves have different amplitudes at different positions. This is due to the interaction between the bound waves and the spurious free waves, which have different phase velocities. As shown in Figures 2c and 2d second-order wave generation results in equal amplitudes of the earliest waves at all positions, indicating that the spurious free waves are suppressed effectively.

4 MEASURING PROCEDURES

Water surface elevations were measured by means of resistance type wave gauges. Although incident bound waves and reflected free waves can be distinguished using two probes only, it was decided to use a set of three probes to check the effectiveness of the suppression of the incident free waves. These probes were positioned above the horizontal bed at equal distances D (see Figure 1).

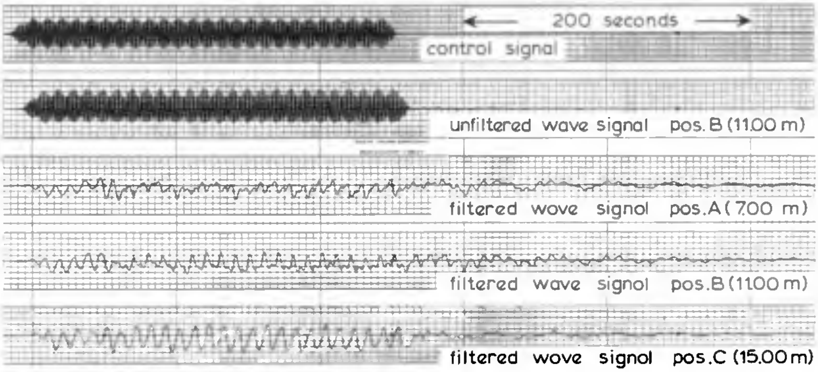
The analysis to obtain surf beat amplitudes is restricted to waves having the wave group frequency $\Delta\omega$. Thus, at a specific position $x = x_A$ in the channel, the low frequency surface elevation ζ_A can be written as:

$$\zeta_A = \hat{\zeta}_{ib} \cos(\Delta\omega t - \Delta k x_A - \phi_{ib}) + \hat{\zeta}_{if} \cos(\Delta\omega t - \kappa x_A - \phi_{if}) + \\ + \hat{\zeta}_{rf} \cos(\Delta\omega t + \kappa x_A - \phi_{rf}) \qquad (3)$$

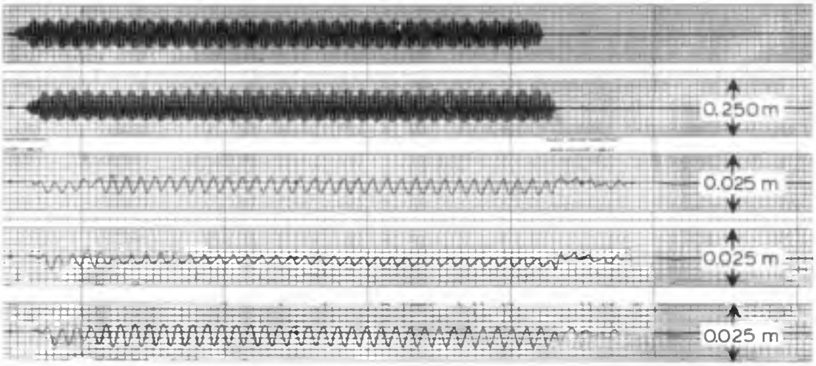
The recorded signals were low-pass filtered. To obtain the local amplitudes at the fundamental, the low frequency signals were decomposed by means of a Fourier analysis. The Fourier series of the signal recorded at $x = x_A$ can be written as:

$$\zeta_A = \frac{a_{o,A}}{2} + a_A \cos \Delta\omega t + b_A \sin \Delta\omega t + \cdots \text{ higher harmonics} \qquad (4)$$

For positions $x = x_B$ and $x = x_C$ similar equations can be formulated, resulting in a set of 6 equations. For the applied analysis it is essential that the Fourier series refer to the identical time interval of the simultaneous signals. Choosing the origin of the x-axis at the

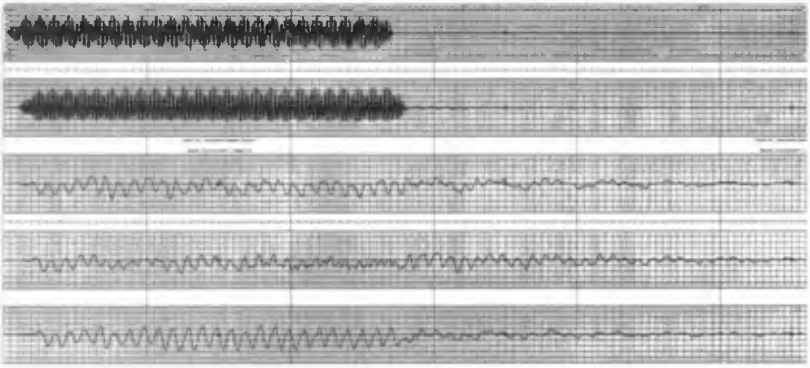


a. no active wave absorption; first order wave generation

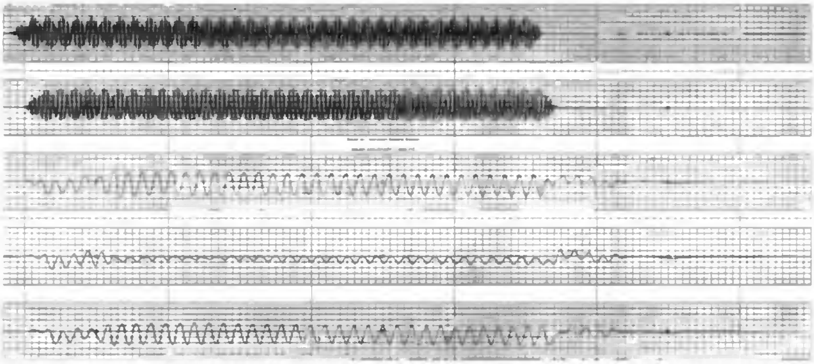


b. with active wave absorption; first order wave generation

Fig. 2 Typical recordings for different wave-paddle control techniques



c. no active wave absorption; second order wave generation



d. with active wave absorption; second order wave generation

Fig. 2 (contd.) Typical recordings for different wave-paddle control techniques middle probe, i.e. $x_B = 0$, after some trigonometry the following expressions can be obtained:

$$C_{ib} = \hat{\zeta}_{ib} \cos \phi_{ib} = \frac{(a_A+a_C)/2 - a_B \cos \kappa D}{\cos \Delta kD - \cos \kappa D} \tag{5}$$

$$S_{ib} = \hat{\zeta}_{ib} \sin \phi_{ib} = \frac{(b_A+b_C)/2 - b_B \cos \kappa D}{\cos \Delta kD - \cos \kappa D} \tag{6}$$

$$C_{if} = \hat{\zeta}_{if} \cos \phi_{if} = \frac{(-b_A+b_C)/2 + a_B \sin \kappa D - C_{ib}(\sin \Delta kD + \sin \kappa D)}{2 \sin \kappa D} \tag{7}$$

$$S_{if} = \hat{\zeta}_{if} \sin \phi_{if} = \frac{(a_A-a_C)/2 + b_B \sin \kappa D - S_{ib}(\sin \Delta kD + \sin \kappa D)}{2 \sin \kappa D} \tag{8}$$

$$C_{rf} = \hat{\zeta}_{rf} \cos \phi_{rf} = a_B - C_{ib} - C_{if} \tag{9}$$

$$S_{rf} = \hat{\zeta}_{rf} \sin \phi_{rf} = b_B - S_{ib} - S_{if} \tag{10}$$

The amplitudes of all long waves can be simply obtained from these expressions. The Fourier series were determined for 20 individual subsequent long waves. In this way, the average value as well as the standard deviation of $\hat{\zeta}_{ib}$, $\hat{\zeta}_{if}$, and $\hat{\zeta}_{rf}$ could be obtained. Subsequently, the test was repeated with an adjusted control signal for the wave paddle. An extra long wave, having the same amplitude as the free incident wave obtained in the former test but with a phase shift of 180°, was generated. The test with an optimum control of the wave paddle, in other words the test resulting in the lower computed amplitude of the incident free wave - in most cases some tithes of a millimeter -, was selected for the final analysis. Sometimes, the original control was even better than the second one, suggesting that at the optimum control, the computed amplitude of the incident free wave should be considered as an indicator for the accuracy of the applied analysis instead of being considered as a real wave. Therefore, for the final analysis, it was assumed that there is no incident free wave at all, which means that the second term at the right-hand side of equation (3) is omitted. Now, the recordings of two probes suffice to compute $\hat{\zeta}_{ib}$ and $\hat{\zeta}_{rf}$. Choosing the origin of the x-axis in between probes A and B yields the following set of equations:

$$C_{ib} = \hat{\zeta}_{ib} \cos \phi_{ib} = \frac{(a_A+a_B) \sin(\kappa D/2) - (b_A-b_B) \cos(\kappa D/2)}{2 \sin\{(\kappa+\Delta k)D/2\}} \tag{11}$$

$$S_{ib} = \hat{\zeta}_{ib} \sin \phi_{ib} = \frac{(a_A-a_B) \cos(\kappa D/2) + (b_A+b_B) \sin(\kappa D/2)}{2 \sin\{(\kappa+\Delta k)D/2\}} \tag{12}$$

$$C_{rf} = \hat{\zeta}_{rf} \cos \phi_{rf} = \frac{(a_A+a_B)/2 - C_{ib} \cos(\Delta kD/2)}{\cos(\kappa D/2)} \tag{13}$$

$$S_{rf} = \hat{\zeta}_{rf} \sin \phi_{rf} = \frac{(b_A + b_B)/2 - S_{ib} \cos(\Delta kD/2)}{\cos(\kappa D/2)} \qquad (14)$$

All presented results are obtained from the above set of equations.

A fourth probe was positioned above the slope just in front of the location where the highest waves break. In this way, it could be checked if the wave groups had the same modulation just before breaking as above the horizontal bed. No measuring devices were installed within the surf zone.

5 TEST PROGRAMME

The test conditions were selected to enable verification of the theoretical model presented by Symonds et al. (1982). They show that the zone between the minimum and the maximum positions of the breakpoint acts as a forcing region. Free long waves at the group period and harmonics are radiated away from this region both shoreward and seaward. As the waves which propagate shoreward are assumed to be fully reflected at the shoreline, standing waves are produced inshore of the forcing region. The reflected waves move through the forcing region and radiate seaward. Thus, the amplitudes of the resultant outgoing waves depend on the relative phase between the reflected waves and the waves radiated seaward directly from the forcing region. This is shown schematically in Figure 3 for the wave at the group frequency. The amplitude of this wave at the outer limit of the forcing

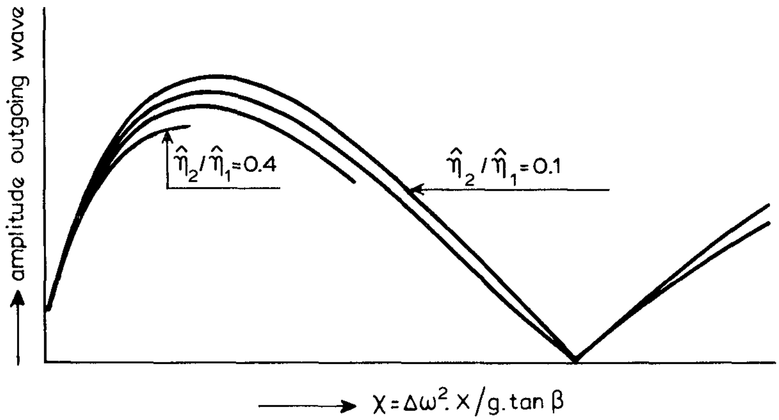


Fig. 3 Schematic representation of the results obtained by Symonds et al. (1982)

region, normalized by the variation about the mean of the steady state set-up, appears to be a function of the parameter χ and the modulation ratio of the incident primary waves $\hat{\eta}_1/\hat{\eta}_2$. The parameter χ depends on the mean distance X between the breakpoint and the shoreline, the group frequency $\Delta\omega$, and the beach slope β.

Table 1 presents the test programme, which was divided up into five test series. For the tabulated values of χ the mean position of the breakpoint had to be estimated. This quantity was obtained using the same equation as applied by Symonds et al.:

$$X = \frac{\hat{\eta}_1}{\gamma \tan \beta} \tag{15}$$

where the raio of wave amplitude to water depth at breaking has been assumed to be $\gamma = 0.4$ (see Bowen et al., 1968). The results of Symonds et al. imply among other things that the outgoing free wave does not depend on the individual primary wave frequencies ω_1 and ω_2, but on their difference $\Delta\omega$ only. To check this, series A and B, each repre-

test	χ	$\hat{\eta}_1$ (m)	$\hat{\eta}_2$ (m)	ω_1 (rad/s)	ω_2 (rad/s)	$\Delta\omega$ (rad/s)
A-1	4.72	0.055	0.011	3.062	2.145	0.917
A-2	3.29	0.055	0.011	3.062	2.296	0.766
A-3	2.08	0.055	0.011	3.065	2.456	0.609
A-4	1.18	0.055	0.011	3.077	2.618	0.459
A-5	0.53	0.055	0.011	3.063	2.755	0.308
B-1	4.78	0.055	0.011	4.295	3.372	0.923
B-2	3.34	0.055	0.011	4.065	3.293	0.772
B-3	2.12	0.055	0.011	4.070	3.455	0.615
B-4	1.19	0.055	0.011	4.071	3.609	0.462
B-5	0.53	0.055	0.011	4.070	3.762	0.308
C-1	4.86	0.080	0.016	4.294	3.522	0.772
C-2=B-2	3.34	0.055	0.011	4.065	3.293	0.772
C-3	2.13	0.035	0.007	4.295	3.523	0.772
D-1=A-3	2.08	0.055	0.011	3.065	2.456	0.609
D-2	1.32	0.035	0.007	3.065	2.456	0.609
D-3	1.13	0.030	0.006	3.065	2.456	0.609
E-1	3.04	0.035	0.028	4.295	3.372	0.923
E-2	2.13	0.035	0.028	4.294	3.522	0.772
E-3	1.35	0.035	0.028	4.296	3.681	0.615
E-4	0.76	0.035	0.028	4.294	3.833	0.461
E-5	0.34	0.035	0.028	4.296	3.988	0.308

Table 1 Test programme

senting one specific wave frequency ω_1, were designed to cover the same set of values for the parameter χ. In these series the variation of χ has been obtained by a change of ω_2. In series C and D the wave frequencies are fixed and, instead, the amplitudes of the primary waves are changed. In the model of Symonds et al. the breakpoint is taken to vary sinusoidally, implying a weak modulation of the primary wave system. Therefore, series A, B, C and D all have the same low ratio of modulation $(\hat{\eta}_2/\hat{\eta}_1 = 0.2)$. Of course, in nature far higher ratios are encountered. To obtain an idea of the effect of such modulations, series E was added to the programme.

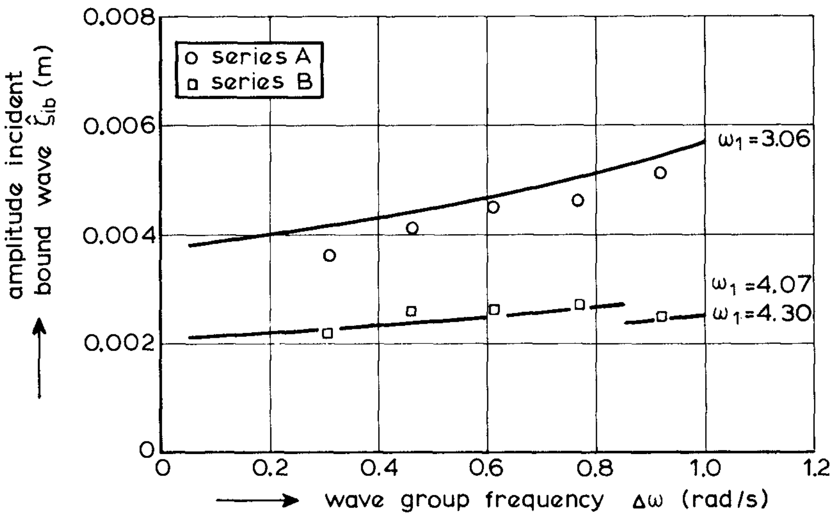


Fig. 4 Measured and theoretical bound long waves for weakly modulated primary waves, $\hat{\eta}_2/\hat{\eta}_1 = 0.2$

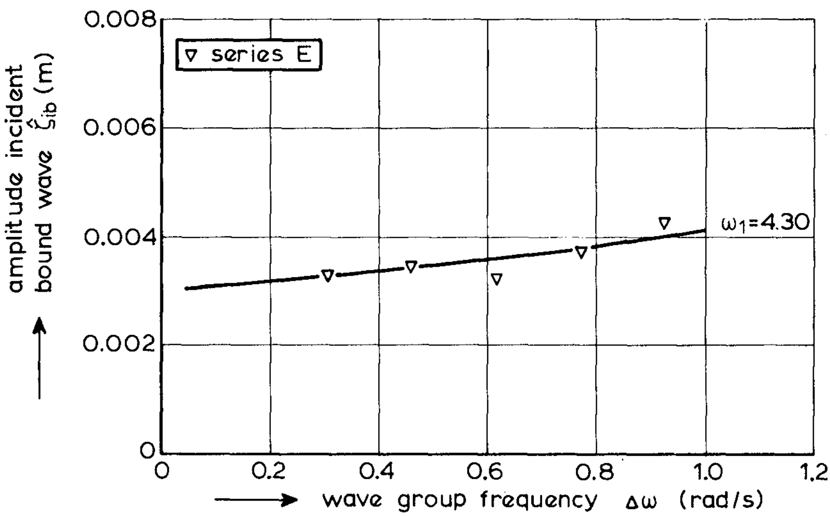


Fig. 5 Measured and theoretical bound long waves for almost fully modulated primary waves, $\hat{\eta}_2/\hat{\eta}_1 = 0.8$

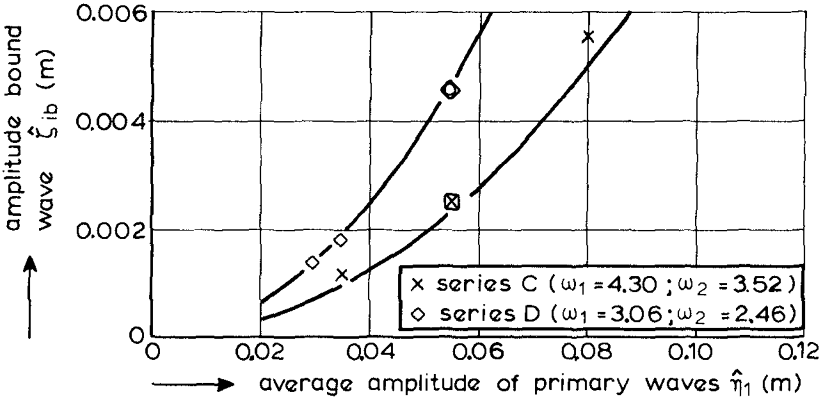


Fig. 6 The effect of the average amplitude of the primary waves on the bound long wave, $\hat{\eta}_2/\hat{\eta}_1 = 0.2$

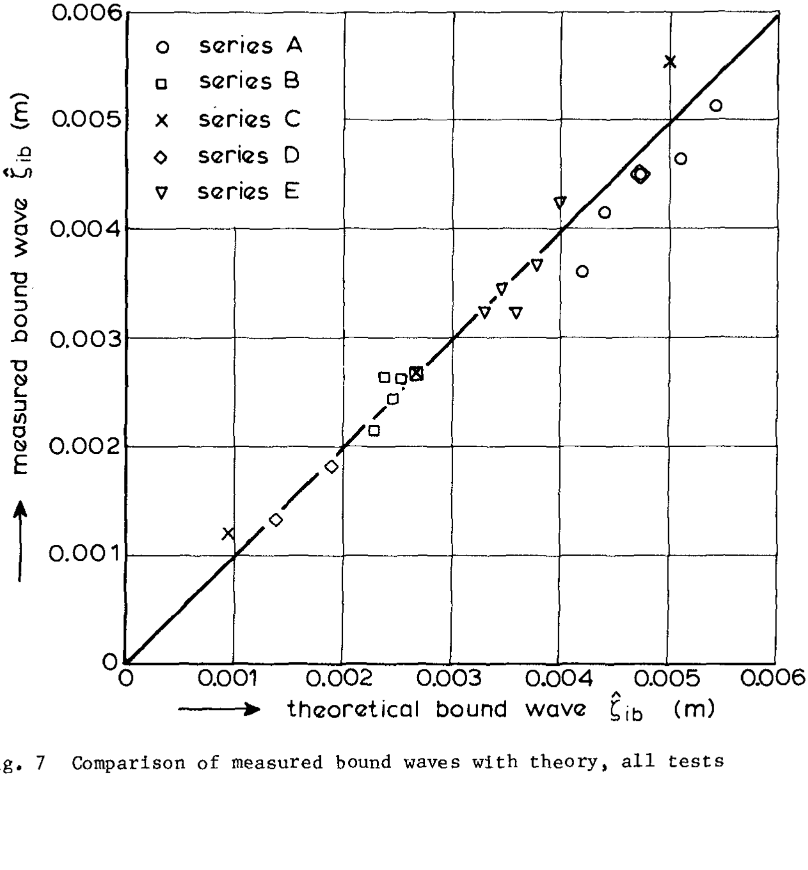


Fig. 7 Comparison of measured bound waves with theory, all tests

6 RESULTS

For all the test series, the measured amplitudes of the incident bound long waves are shown in Figures 4 to 6. The solid lines in these figures represent the theoretical values as given by Ottesen Hansen (1978). Both for weakly and almost fully modulated wave signals, the agreement between measured and theoretical values is quite satisfactory. According to the theory, the amplitude of the bound long wave should be proportional to the product of the primary wave amplitudes $\hat{\eta}_1$ and $\hat{\eta}_2$. As shown in Figure 6, this is confirmed by the measurements. In Figure 7 the measured values are plotted against the theoretical ones for all test conditions. The high correlation shown in this figure gives an indication of the accuracy of the measuring procedure applied.

Longuet-Higgins and Stewart (1962) have suggested that the incident bound wave is reflected from the surf zone and radiated seaward as a free long wave. Using this concept, one would expect some correlation between both waves. However, as shown in Figure 8, the correlation is very poor. Moreover, for some conditions the reflected wave is even higher than the incident one. Both results support the relevance of the model of Symonds et al.

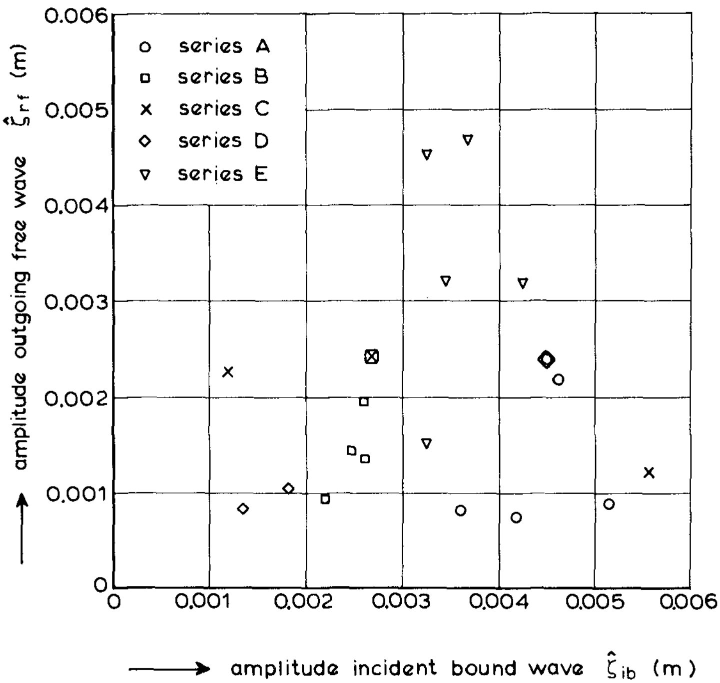


Fig. 8 Relation between incident bound long waves and outgoing free long waves

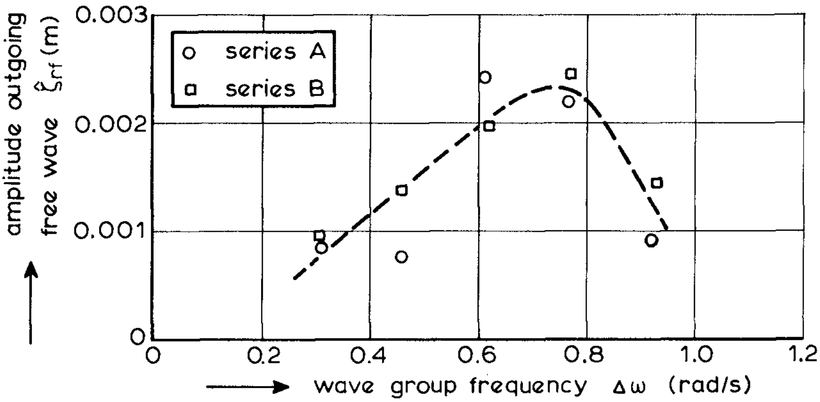


Fig. 9 Measured outgoing free long waves for weakly modulated primary waves, $\hat{\eta}_2/\hat{\eta}_1 = 0.2$

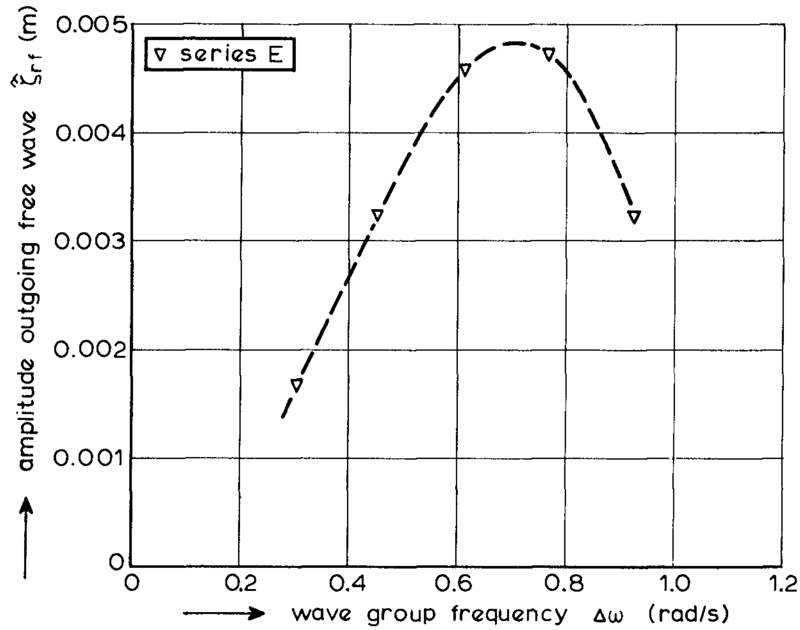


Fig. 10 Measured outgoing free long waves for almost fully modulated primary waves, $\hat{\eta}_2/\hat{\eta}_1 = 0.8$

The measured amplitudes of the outgoing free waves are shown for each test series separately in Figures 9 to 11. The barred lines on these figures merely connect the measured values; they don not refer to any theory. Figure 9 shows that the results for series A and B compare quite well with one another, which is again in accordance with the model of Symonds et al. This model is also qualitatively confirmed by the results shown in Figure 11. This figure shows clearly that increasing the primary wave amplitudes, or, in other words, increasing the mean position X of the breakpoint, does not necessarily result in an increase of the amplitude of the outgoing wave. The results of the tests with almost fully modulated primary waves show the same trend as that found for the weakly modulated waves (see Figure 10).

To be able to compare the experimental results with the model of Symonds et al. in a quantitative way, some assumptions have to be made. Firstly, the mean position X of the breakpoint was not measured. This quantity was obtained using equation (15). Secondly, the experimental results are obtained at a water depth of 0.50 m, whereas the theoretical results refer to the amplitude of the outgoing wave at the outer limit of the wave breaking region. This amplitude was estimated as follows:

$$\hat{\zeta}_{rf,b} = \left(\frac{0.50}{d_b}\right)^{\frac{1}{4}} \cdot \hat{\zeta}_{rf} = \left(\frac{0.50\ \gamma}{\hat{\eta}_1+\hat{\eta}_2}\right)^{\frac{1}{4}} \cdot \hat{\zeta}_{rf} \qquad (16)$$

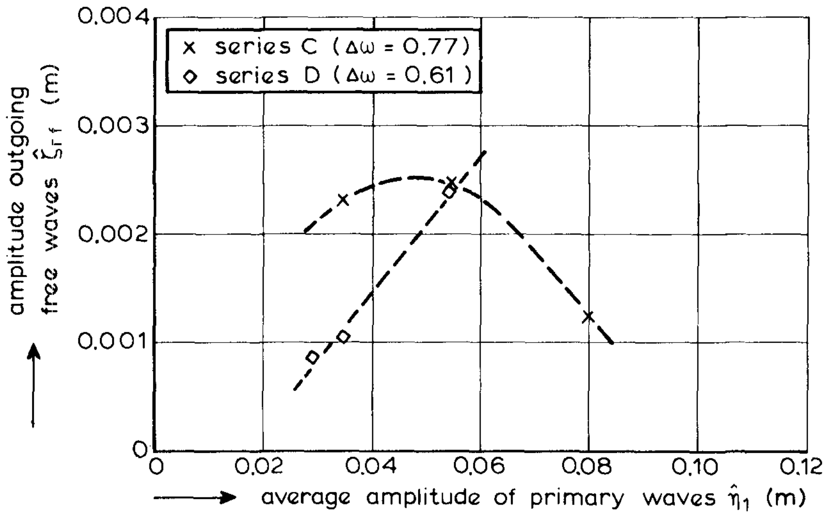


Fig. 11 The effect of the average amplitude of the primary waves on the outgoing free long wave, $\hat{\eta}_2/\hat{\eta}_1 = 0.2$

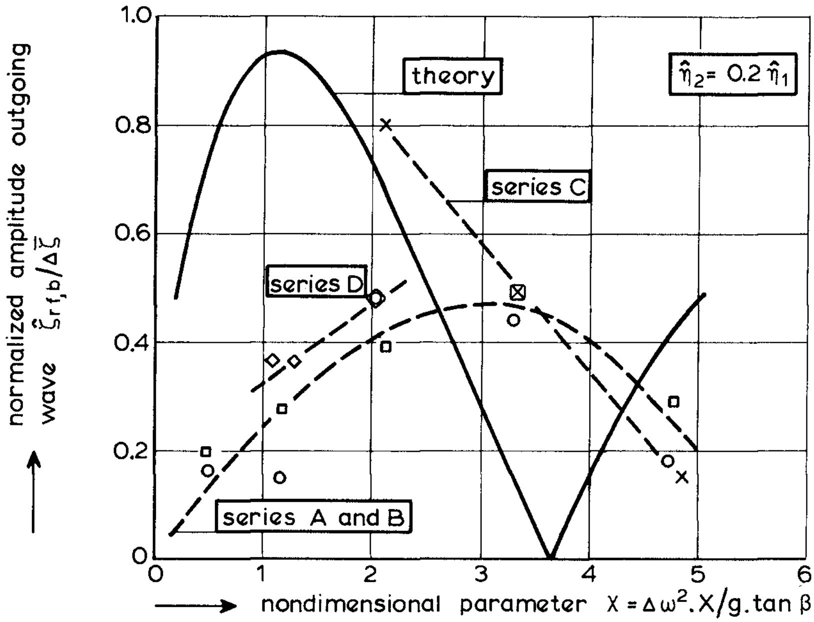


Fig. 12 Comparison of normalized outgoing long waves with the theory of Symonds et al., weakly modulated primary waves

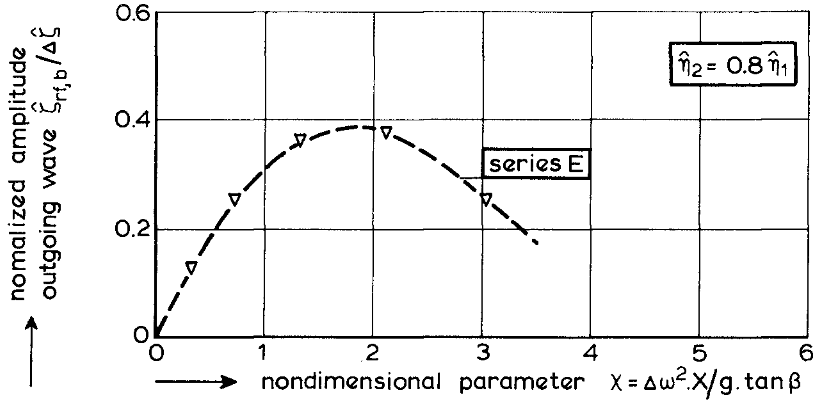


Fig. 13 Normalized outgoing long waves for almost fully modulated primary waves

Finally, the tbeoretical results are normalized by the variation about the mean of the set-up at the shoreline which would be obtained for monochromatic waves with the same amplitudes. Again in accordance with Symonds et al. this variation has been estimated to be:

$$\Delta\bar{\zeta} = \frac{3}{2}\gamma\,\hat{\eta}_2 \qquad (17)$$

Using equations (15) to (17), the experimental results can be plotted against the theory. Figure 12 shows, in spite of a qualitative agreement, that the deviations are significant. It is presumed that this is due to the assumptions underlying the conceptual model. For instance, the bound wave associated with the incoming wave groups, has not been included in the model. Moreover, the formulation precludes any modulation of the primary waves inside the surf zone. Another major assumption of the theoretical model is that the long waves propagating shoreward are supposed to be fully reflected at the shoreline.

Symonds et al. also assume a sinusoidal variation of the breakpoint, which restricts a qualitative comparison between the theory and experiments to those tests carried out with weakly modulated primary waves. Nonetheless, for comparison, the normalized results of series E are presented in Figure 13. Studying Figures 12 and 13 reveals that the effect of the ratio of wave modulation qualitatively agrees with the theoretical trend presented in Figure 3.

7 CONCLUSIONS

Two-dimensional surf beat induced by bichromatic waves has been investigated experimentally. The following conclusions can be drawn:

- The wave action at the wave group frequency can be reproduced correctly, applying a sophisticated control of the wave paddle, including second-order wave generation and active wave absorption at the paddle face.
- Two-dimensional surf beat induced by incident wave groups should be considered as the result of two wave systems, viz. incident bound waves, propagating at the group velocity, and outgoing free waves, which are generated in the surf zone.
- The experiments confirm the theoretical values for the amplitude of the incident bound wave, which is often termed as set-down, as obtained with a Stokes expansion of the wave equations to second order.
- There is no correlation between the incident bound waves and the outgoing free waves.
- The theoretical model of Symonds et al. and the experimental results for the free waves, which are generated in the surf zone and radiated seaward, agree qualitatively.

REFERENCES

Barthel, V., E.P.D. Mansard, S.E. Sand, and F.G. Vis, 1983: Group-bounded long waves in physical models. Ocean Engineering, Vol. 10, No.4, pp 261-294

Bowen, A.J., D.L. Inman, and V.P. Simmons, 1968: Wave set-down and set-up. J. Geophys. Res., Vol. 73, No. 8, pp. 2569-2577

Bowers, E.C., 1977: Harbour resonance due to set-down beneath wave groups. J. Fluid Mech., Vol. 79, Part 1, pp. 71-92

Flick, R.E., D.L. Inman, and R.T. Guza, 1980: Two-dimensional surf beat. Proc. 17th Int. Conf. Coastal Engng., Sydney, Vol. 1, pp. 624-638

Gallagher, B., 1971: Generation of surf beat by non-linear wave interactions. J. Fluid Mech., Vol. 49, Part 1, pp. 1-20

Longuet-Higgins, M.S., and R.W. Stewart, 1962: Radiation stress and mass transport in gravity waves, with application to "surf beats". J. Fluid Mech., Vol. 13, pp 481-504

Longuet-Higgins, M.S., and R.W. Stewart, 1964: Radiation stresses in water waves; a physical discussion, with applications. Deep Sea Res., Vol. 11, pp. 529-562

Munk, W.H., 1949: Surf beats. Trans. Am. Geoph. Union, Vol. 30, pp. 849-854

Ottesen Hansen, N.E., 1978: Long period waves in natural wave trains. Inst. Hydrodyn. and Hydraulic Engng., Techn. Univ. Denmark, Progress Rep. No. 46, pp. 13-24

Ottesen Hansen, N.E., S.E. Sand, H. Lundgren, T. Sorensen, and H. Gravesen, 1980: Correct reproduction of group-induced long waves. Proc. 17th Int. Conf. Coastal Engng., Sydney, Vol. 1, pp. 784-800

Symonds, G., D.A. Huntley, and A.J. Bowen, 1982: Two-dimensional surf beat: long wave generation by a time-varying breakpoint. J. Geoph. Res., Vol. 87, No. C1, pp. 492-498

Symonds, G., and A.J. Bowen, 1984: Interactions of nearshore bars with incoming wave groups. J. Geoph. Res., Vol. 89, No. C2, pp. 1953-1959

Tucker, M.J., 1950: Surf beats: sea waves of 1 to 5 minutes period. Proc. R. Soc., London, Ser. A, Vol. 202, pp. 565-573