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MEASURING AND EVALUATING EFFICIENCY AND EFFECTIVENESS USING GOAL PROGRAMMING AND DATA ENVELOPMENT ANALYSIS IN A FUZZY ENVIRONMENT

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Abstract: Generally, in most situations optimal achievement of multiple goals is rarely possible for crisp mathematical programming techniques. In such cases, a compromise achievement of goals that leads to a *satisfycing* solution rather than an *optimal* solution bears more relevance. The present research introduces a Fuzzy Goal Data Envelopment Analysis (Fuzzy GoDEA) framework to measure and evaluate the goals of efficiency and effectiveness in a fuzzy environment. Fuzzy GoDEA accommodates crisp input and output data but allows imprecise specification of the aspiration levels for the efficiency and effectiveness goals. A membership function is defined for each fuzzy constraint associated with the efficiency and effectiveness goals and represents the degree of achievement of that constraint. Further, the Fuzzy GoDEA framework is extended into several variations that (i) allow the assignment of relative importance to the goals of efficiency and effectiveness and (ii) model scenarios where one of the goals of efficiency and effectiveness is crisp and the other fuzzy. The Fuzzy GoDEA framework is implemented for a newspaper preprint insertion process (NPIP).

Keywords: Fuzzy goal programming, efficiency and effectiveness measurement, improvement of manufacturing organizations, Data Envelopment Analysis (DEA).

1. INTRODUCTION: MOTIVATION AND OBJECTIVES

Measurement and evaluation of efficiency has been an ongoing research issue in the management science literature (Fried, Lovell, and Schmidt (1993); Charnes, Cooper, Lewin, and Seiford (1994)). Further, there has been a growing interest in using information from the efficiency research literature for intelligent decision-making regarding the achievement of multiple organizational goals and resource allocation strategies (Thanassoulis and Dyson (1992); Athanassopoulos (1995); Hoopes, Triantis, and Partangel (2000)).

The data envelopment analysis (DEA) approach (Charnes, Cooper, and Rhodes (1978)) has been one of the important frameworks used for efficiency measurement. However, DEA usually requires deterministic data and crisp model constraints. That is, the data are required to be available in precise terms and the constraints of the model are required to be satisfied *precisely*. However, in reality, these conditions are not always met. Often data are imprecise or subject to incomplete knowledge (Triantis and Girod (1998)) and the model objective function and constraints are met to *some degree.*

In order to deal with the issue where the constraints and objective function are not satisfied exactly, Sengupta (1992) applied principles of fuzzy set theory (Zadeh (1965)) to DEA. He introduced fuzziness in the objective function and the constraints of the conventional DEA model but did not provide an application roadmap of his proposed framework.

In reality, when a decision-maker is faced with multiple goals the optimal achievement of all goals is rarely possible. More often than not the decision-maker is looking for *satisfycing* levels of goal achievement within some predefined acceptable limits rather than an optimal solution. Crisp mathematical programming approaches are limited in such cases as they provide only crisp representation of systems. Further, the decision-maker may want to assign the relative importance to the achievement of the goals but may be reluctant to assign quantitative preferences among goals. Also, the decision-maker may, in certain scenarios, desire crisp achievement of some goals while allowing imprecise achievement of other goals. The need to model such scenarios provides the fundamental motivation for this research.

Therefore, the primary objective of this research is to introduce a framework to measure and evaluate the goals of efficiency and effectiveness in a fuzzy environment. This framework is developed using Goal Programming, Data Envelopment Analysis and Fuzzy Set Theory. The proposed framework, called Fuzzy Goal Data Envelopment Analysis (Fuzzy GoDEA), uses Data Envelopment Analysis (DEA) type constraints to model the efficiency goal. The effectiveness goal is represented by the aggregate efficient contribution of the individual decision-making units toward achievement of the global organizational targets.

The fuzzy GoDEA framework is obtained through the following steps. First, the GoDEA model proposed by Athanassopoulos (1995) is reformulated. Then its fuzzy formulation is provided. Finally the crisp equivalent to the fuzzy formulation is developed as the base model in the Fuzzy GoDEA framework. This approach is outlined in detail in Sections 4 and 5 of this paper. The proposed Fuzzy GoDEA framework accommodates crisp input and output data but allows imprecise specification of the aspiration levels for the efficiency and effectiveness goals. The imprecision in goal achievement is allowed through the specification of an interval of acceptable achievement rather than a crisp value. A membership function is defined for each fuzzy constraint associated with the efficiency and effectiveness goals and represents the degree of achievement of that constraint. Further, the Fuzzy GoDEA framework is extended into several variations that (i) allow the assignment of the relative importance to the goals of efficiency and effectiveness and (ii) model scenarios where one of the goals of efficiency and effectiveness is crisp and the other fuzzy.

There are a number of salient features associated with the Fuzzy GoDEA framework. First, fuzzy peers are used to identify efficient and inefficient decision making units (DMUs), to help investigate root causes of inefficiency, and to allow comparisons with Banker, Charnes, and Cooper (1984) (BCC) *near-efficient* peers. Second, the activity variables facilitate the identification of dominant peers. Third, slack and deviation variables designate the amount of inefficiency and ineffectiveness across model variations. These characteristics amount to a considerable amount of information that can be subsequently used for performance improvement interventions. The features of the Fuzzy GoDEA framework are illustrated in Section 6 where it is applied to a newspaper preprint insertion process.

Therefore, the second objective of this research is to implement the Fuzzy GoDEA framework for the newspaper preprint insertion process (NPIP). This is consistent with Almond (1995) who states that fuzzy set theory approaches should provide implementation road maps otherwise they are of limited use. The data for this implementation are adopted from Girod (1996). Detailed analyses of the results are presented in this paper to describe the information available from the Fuzzy GoDEA methodology that can be used in conjunction with conventional DEA and fuzzy DEA (Girod and Triantis (1999)) analysis to assess and improve the efficiency and effectiveness performance of the NPIP process.

The decision-maker can be faced with several problems of interest in the context of meeting efficiency and effectiveness goals. The first problem is to measure and evaluate efficiency in terms of input consumption and output generation at the production process level. The second problem is to relate operational level efficiency to global organizational targets or effectiveness. The third problem is to provide higher-level decision-makers with a decision-making tool to evaluate current efficiency performance at the process level as well as the organization as a whole and make decisions regarding efficiency improvement, future resource allocation strategies and the achievement of global targets. The framework developed in this research can be used to address these issues.

The Fuzzy GoDEA methodology developed in this research employs attractive features of fuzzy set theory, goal programming and data envelopment analysis. The fuzzy element allows imprecise aspiration levels for the efficiency and effectiveness goals when the decision-maker chooses to attain a *satisfycing* rather than *optimizing* approach. With respect to the efficiency goal, the Fuzzy GoDEA formulation allows relaxation of the DEA structure and also enables inefficient units to be compared with units that are evaluated by conventional DEA analysis as not only 100% efficient but also less than 100%. In reality, such comparisons are valuable as it may be easier for inefficient units to achieve operating levels of units more efficient then them but not necessarily 100% efficient.

The rest of the paper is organized as follows. Section 2 provides a short description of the previous research work in the efficiency literature that deals with multiple objective performance evaluation and efficiency evaluation using fuzzy set theory. Fuzzy constraints are introduced in Section 3 where constraints related to inputs and outputs are represented as fuzzy numbers. Section 4 describes the reformulation of the Goal Programming Data Envelopment Analysis (GoDEA) introduced by Athanassopoulos (1995). The Fuzzy GoDEA formulation and its variations are presented in Section 5. Section 6 uses the proposed approach to provide an evaluation of the preprint insertion manufacturing process previously described by Girod and Triantis (1999). In Section 7 we conclude and make recommendations for future research.

2. BACKGROUND

As stated in the previous section, the primary objective of this research is to introduce a framework to measure and evaluate the goals of efficiency and effectiveness in a fuzzy environment. This builds on a number of developments in literature. These include the following four thrusts. a) How multi-level programming and more specifically goal programming approaches are used to address decision-making problems for multi-level organizations. b) The evaluation of multi-level organizations where efficiency performance is treated as a specific goal within the organization and which is modeled by a goal programming and data envelopment analysis approach. c) The use of fuzzy goal programming that captures the idea that goals are essentially and not precisely satisfied. d) Fuzzy decision-making when efficiency performance is the main performance criterion. We provide a brief overview of these four research thrusts.

Policy making in multi-level organizations is characterized by three main problems as described by Nijkamp and Rietveld (1981). These problems are (i) interdependencies between the subsystems; (ii) conflicts between the goals, priorities, and targets within each subsystem; and (iii) conflicts between the goals, priorities and targets between subsystems. For example, in the context of manufacturing environments analogies can be found for each of these three principal problems. Technological and administrative interdependencies exist among various production processes. Within a production process, conflicts exist with respect to short-term objectives; for example, quality assurance and throughput objectives are usually at odds, especially in the short term. Finally, conflicts with respect to manufacturing performance targets in terms of throughput, cost, efficiency, allocation of resources, etc. exist among the various manufacturing departments and processes.

Multi-level programming is an approach proposed in the literature to address these problems. Within the context of multi-level programming, goal programming is a modeling approach that has been extensively used. Athanassopoulos (1995) developed a model integrating Goal Programming and Data Envelopment Analysis (GoDEA) to incorporate target setting and resource allocation in multi-level planning problems. The GoDEA framework is proposed as a decision-making tool that combines conflicting objectives of efficiency, effectiveness and equity in resource allocation. Previously, Thanassoulis' and Dyson's (1992) formulation provided a method to estimate input/output targets for each individual decision making unit (DMU) in a system but failed to address planning and resource allocation issues at the global organizational level while considering all decision-making units (DMUs) simultaneously. More recently, Hoopes, Triantis, and Partangel (2000) augmented and implemented the GoDEA formulation to assess the performance of serial manufacturing technologies found in a two-level hierarchical manufacturing organization.

In a different research area, Bellman and Zadeh (1970) extended fuzzy set theory of Zadeh (1965) and developed a framework for decision-making in a fuzzy environment where the objectives and constraints can be treated as fuzzy sets in the decision space and a fuzzy decision then would be obtained as the intersection of these fuzzy sets. Narsimhan (1980) was the first to integrate the concepts of fuzzy set theory and goal programming.

Multi-criteria decision problems generally involve the resolution of multiple conflicting goals to achieve a *"satisfycing"* solution, rather than maximization of objectives, given a suitable aspiration level for each objective. The generalized goal programming approach seeks to minimize the negative (under achievement) and positive (over achievement) deviations from the goal targets. However, in most real life situations the aspiration levels for some or all objectives typically have an imprecise nature. In other words, the aspiration levels need a linguistic interpretation such as very good, good, and moderately good. To capture such scenarios it is appropriate to model the objective(s) and constraints with a certain specified tolerance limit. All the fuzzy goals and fuzzy constraints can be considered as fuzzy criteria. The central theme of fuzzy goal programming is that systems with ill-defined or imprecise characteristics are first modeled as fuzzy models. The fuzzy models are then formulated as crisp equivalent models that can be solved with existing decision-making methodologies.

Finally, fuzzy set theory has been used to evaluate the efficiency performance of organizations. There are many examples where DEA mathematical programming formulations were expanded using fuzzy set theory to incorporate the following: imprecision in the data as in Triantis and Girod (1998) and Girod and Triantis (1999), missing data and imprecise linguistic data as in Kao and Liu (2000), or fuzziness in the objective function and constraints as in Sengupta (1992) and Sheth (1999).

3. FUZZY SETS AND THE MEMBERSHIP FUNCTIONS FOR THE GOAL PROGRAMMING CONSTRAINTS

The fundamental assumption being made at this point is that the decision-maker can identify the most plausible production scenarios or occurrences for each input and output at any point in time when measurement occurs. This information is based on the decision-maker's knowledge of the technology and on his/her production experience (Girod and Triantis (1999)). One way to capture this information is to represent constraints as fuzzy numbers. In this case, we consider that all production occurrences for the constraints associated with each input and output to take values from a closed interval with a certain degree of plausibility.

Definition 1. *Let $\Re$ be the space of real numbers. A **fuzzy number** A is a set of ordered pairs $\{(z, \mu_A(z) | z \in \Re)\}$ where $\mu_A : \Re \to [0,1]$ and is upper semi-continuous. For each $z \in \Re$, $\mu_A(z)$ represents the degree of possibility (plausibility) that the quantity of A takes the value z.*

The membership function is constructed for both the input and output spaces based on the interpretation of the bounds or tolerance limits specified for the satisfaction of the fuzzy constraints. The assumptions for the membership functions are outlined next. Figure 1 shows the membership functions for the constraints associated with the input spaces. A similar representation exists for the membership functions for the constraints associated with the output spaces (Sheth (1999)).

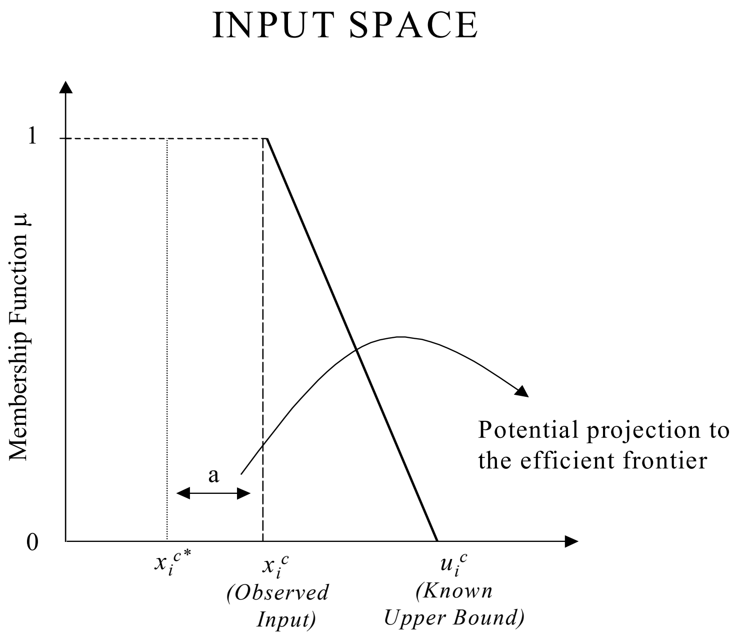

Figure 1: Membership Function μ for the Input Space

3.1. Assumptions

The following assumptions are made regarding the membership functions associated with the achievement of the fuzzy goals (or constraints). The membership functions are assumed to be linear and monotonically increasing or decreasing. The membership function associated with each constraint is evaluated as a linear expression (Zimmermann (1978)) when the constraint is satisfied *within* the specified tolerance limits *i.e., essentially satisfied*. The value of the membership function is equal to zero when the constraint is evaluated at or beyond the tolerance limits *i.e., completely dissatisfied* and is equal to one when the constraint is satisfied crisply. It should be noted that in general, the concept of a membership function does not have a unique semantic interpretation. In the context of some of the production processes studied (Girod (1996)); linearly increasing or decreasing membership functions seemed to be a reasonable assumption.

3.2 Membership Function Representations Associated with Constraints Associated with the Inputs and Outputs

Consider the membership function shown in Figure 1 for a constraint associated with the input space. The observed input i for production plan or DMU c is represented by x_i^c. Using the structural efficiency concept of DEA, the input efficiency of DMU c is assessed by comparing input x_i^c with the composite unit or convex combination in the Banker, Charnes, and Cooper (1984) (BCC) case of all the DMUs for input i in the system. The objective is to find a composite unit that is less than or equal to x_i^c *i.e.*, to find a composite unit that utilizes less than or as much of input i as DMU c.

This mathematical representation when fuzzified allows the inequality to be satisfied up to an upper bound u_i^c where $u_i^c \geq x_i^c$. That is, the decision-maker is satisfied to varying degrees when the inequality is satisfied within the interval (x_i^c, u_i^c). When the composite unit is greater than or equal to u_i^c then the constraint is dissatisfied completely. Therefore, an input realization greater than or equal to u_i^c is undesirable. Hence, the membership function takes the value zero at u_i^c and all values greater than u_i^c. The membership function increases monotonically from zero to one in the interval (x_i^c, u_i^c) as the input realization moves from u_i^c to x_i^c. This is consistent with crisp constraint satisfaction at x_i^c.

For the decision-maker, the membership function values for the efficiency constraints represent the degree of satisfaction of the DEA representation of the constraints. Therefore, when the membership functions for the DEA representation of an input is equal to one it implies crisp satisfaction of the DEA structure for that input and when the membership function value is less than one it implies a relaxation of the DEA structure. Accordingly, it follows that when the membership function for an input is equal to zero the DEA structure fails to hold for that input.

A priori there is no knowledge that x_i^c is an (in) efficient observation. Therefore, prior to the efficiency evaluation of the DMUs the input space membership function can be considered to have a *hypothetical* nature. In other words, the membership function in the input case is assumed to be one at all values equal to and less than the observed input realization x_i^c. Since DEA is based on "best observed practices" this assumption is justified in the sense that it may be possible to further reduce inputs as the best observed may not be the best possible. Therefore, if the observed input realization is evaluated as inefficient then the efficient frontier can be considered to lie at $x_i^{c^*}$ (efficient input usage) at a distance "a" units from x_i^c (observed input). When $a = 0$ then x_i^c (DMU c) is an efficient observation and lies on the efficient frontier $(x_i^c = x_i^{c^*})$. Alternately, "a" can be interpreted as the projection of DMU c on to the efficient frontier that would make DMU c an efficient observation.

4. THE REFORMULATED GODEA MODEL

Athanassopoulos' (1995) framework is modified to derive the reformulated GoDEA model for the current research as per the following justification. The central coordinating entity desires to maximize the attainment of pre-specified input/output global targets. To achieve this goal, the individual DMUs are expected to maximize their contribution toward achievement of the global organizational targets. In the current reformulation, the aim is to restrict global consumption of each input to less than or equal to the global target and to enable global production of output that is more than or equal to the global target. That is, the decision-maker desires to maximize the negative deviation from the input target and the positive deviation from the output target.

Global organizational targets are reflected in the objectives of efficiency (contribution of individual DMUs to individual targets), effectiveness (achievement of global organizational targets). However, for this research we will focus only on the objectives of efficiency and effectiveness and consequently we do not consider equity of resource allocation constraint. The reformulated model is developed for a two-level hierarchy where the global and individual DMU targets are known *a priori* (*e.g.,* from historical process knowledge). At a given decision-making level in the organizational hierarchy the decision-maker can prioritize the achievement of objectives according to their relative importance. Athanassopoulos' (1995) GoDEA model is reformulated as follows:

Model 1

$$\begin{aligned} &\min_{p_i, p_j, n_i, n_j} \sum_{k=1}^{N} \sum_{i \in I} \left(P_i^- \frac{n_i^k}{x_{ik}} + P_i^+ \frac{p_i^k}{x_{ik}} \right) + \sum_{k=1}^{N} \sum_{j \in J} \left(P_j^- \frac{n_j^k}{y_{jk}} + P_j^+ \frac{p_j^k}{y_{jk}} \right), \\ &\max_{d_i, d_j,} \left(\sum_{i \in I} P_i^g \frac{d_i^-}{TX_i} + \sum_{j \in J} P_j^g \frac{d_j^+}{TY_j} \right) \end{aligned} \tag{1}$$

Subject to:
DMU representation:

$$\sum_{k=1}^{N} \lambda_k^c y_{jk} + n_j^c - p_j^c = y_j^c, \qquad j \in J, \qquad \forall c \tag{2}$$

$$\sum_{k=1}^{N} \lambda_k^c x_{ik} + n_i^c - p_i^c = x_i^c, \qquad i \in I, \ \forall c \tag{3}$$

Effectiveness through Achievement of Global Targets:

$$\sum_{k=1}^{N} \lambda_k^1 x_{ik} + \ \dots \ + \sum_{k=1}^{N} \lambda_k^N x_{ik} + d_i^- = TX_i \ , \qquad \forall i \in I \tag{4}$$

$$\sum_{k=1}^{N} \lambda_k^1 y_{jk} + \ \dots \ + \sum_{k=1}^{N} \lambda_k^N y_{jk} - d_j^+ = TY_j \ , \qquad \forall j \in J \tag{5}$$

$$\sum_k \lambda_k^c = 1, \quad \forall c = \{1,2,\ldots,N\} \tag{6}$$

$$\lambda_k^c \geq 0, \quad \forall k = \{1,2,\ldots,N\}, \ \forall c \tag{7}$$

where:

N: number of DMUs

I: set of inputs

J: set of outputs

x_{ik}: level of input i for DMU k

y_{ik}: level of output j for DMU k

x_i^c, y_i^c: level of input i and output j for DMU c when assessing DMU c

λ_k^c: activity level of DMU k when assessing DMU c

n_i^k, p_i^k: negative and positive deviation variables for input i of DMU k

n_j^k, p_j^k: negative and positive deviation variables for output j of DMU k

d_i^-, d_i^+: negative and positive deviation variables from global targets of input i and output j

P_i^-, P_i^+: user defined preferences over the minimization of positive and negative goal deviations of input i

P_j^-, P_j^+: user defined preferences over the minimization of positive and negative goal deviations of output j

P_i^g, P_j^g: user-defined preferences related to global targets of input i and output j

TX_i, TY_j: global target levels known *a priori* for input i and output j

Model 1 is a goal programming formulation. The model has an objective function and sets of constraints. The first set of constraints (Equations 2 and 3) provides the individual DMU representations and reflects the objective of efficiency. These DEA-like constraints compare the inputs and outputs of the assessed DMU c with the composite units $\sum_k \lambda_k^c x_{ik}$ and $\sum_k \lambda_k^c x_{jk}$ respectively. Each composite unit is basically a convex combination[1] (as in the BCC Model) of all DMUs in the system under study, with a set of activity levels λ_k^c when assessing DMU c. These constraints differ from the conventional DEA constraints due to the introduction of positive and negative goal deviation variables, p_i^c and n_i^c for inputs and p_j^c and n_j^c for outputs, instead of the contraction and expansion factors respectively.

The two-way deviation variables allow under- and over-achievement of the input and output factors and also impact the construction of the efficiency frontier. In conventional DEA the objective function seeks to minimize (maximize) the contraction

[1] Equation 6 is the convexity constraint that ensures variable returns to scale. The proposed model and its variations maintain the convexity property to model variable returns to scale.

(expansion) factor for the inputs (outputs). The objective function thus drives the solution to the problem. The efficient DMUs are evaluated when the contraction (expansion) variable is equal to unity and the distance between the efficient facet and the DMU is minimized via the input excess (output slack) variables. The efficient DMUs then represent points on the efficient frontier. In the reformulated goal programming model presented above the objective function seeks to minimize the two-way deviation variables. The two-way deviation variables represent possible contraction and expansion of both inputs and outputs. The minimization of these variables, thus, drives the solution to the problem. The specific cases of input and output orientations can be obtained by appropriately modifying the objective function. Thus, the efficient frontier constructed in the goal programming formulation may differ from the frontier constructed by conventional DEA.

The second set of constraints (Equations 4 and 5) reflects the objective of effectiveness through the achievement of global input and output targets. In the current reformulation, the aim is to restrict global consumption of each input to less than or equal to the global target and to enable global production of output that is more than or equal to the global target. That is, the decision-maker desires to maximize the negative deviation from the input target and the positive deviation from the output target. Therefore, only deviation variables corresponding to reduction in input usage (d_i^-) and the augmentation of output production (d_j^+) are present in these constraints. Fuzzification of the global target constraints would allow positive input deviation and negative output deviation within pre-specified tolerance limits.

Equation 6 restricts the sum of the activity parameters $\lambda_k\sigma$'s to one and enables variable returns to scale in the formulation. This convexity constraint is used in the same manner as in the BCC model for conventional DEA. Equation 7 imposes the non-negativity condition on the $\lambda_k c$'s.

The objective function of the model (Equation 1) has two parts. The deviation variables are standardized to achieve a standard evaluation system. The first part contains the positive and negative deviation variables associated with the inputs and outputs of individual DMUs. This allows for over- and under-achievement of individual input/output targets for each DMU. The priorities associated with these deviation variables can be interpreted as the extent to which individual DMUs contribute toward achievement of global organizational targets. This feature differentiates Athanassopoulos' (1995) model from conventional DEA, which always assumes input contraction and output expansion for the assessed DMU. Also, by appropriately modifying the signs and magnitudes of the preferences $P_i^-, P_i^+, P_j^+, P_j^-$ different planning scenarios can be implemented (*e.g.,* input contraction and output expansion (conventional DEA), input contraction and output contraction, input expansion and output expansion, etc.). The second part of the objective function contains the deviation variables associated with the global input and output targets. The priorities associated with these deviation variables represent the reward per unit deviation from the global targets. This reformulated version of Athanassopoulos' (1995) model is fuzzified and presented as the Fuzzy GoDEA model in the next section.

5. THE FUZZY GODEA MODEL

A crisp formulation does not allow linguistic specifications such as "essentially satisfied" or "approximately satisfied". The need for such *imprecise* specification of multiple organizational goals with varying relative importance in a hierarchical system motivates the fuzzy model formulation in this research.

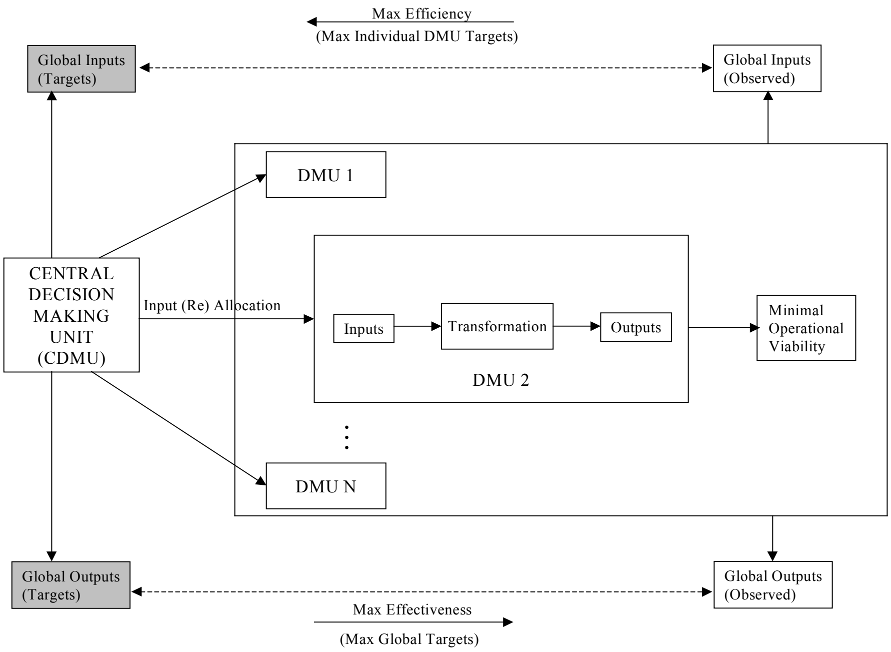

Figure 2: The Conceptual Model

5.1. The Conceptual Model

The model is developed for a hierarchical environment with two levels of decision-making and is represented by Figure 2. At the higher- or super-level is the central decision making unit (CDMU) and at the lower- or sub-level are the individual decision making units (DMUs). The DMUs are under the control of the CDMU insofar as allocation of resources and setting global targets are concerned. The CDMU has a given amount of resources that it wishes to allocate among the DMUs while trying to achieve its global objectives of effectiveness and efficiency. The CDMU specifies global input and output targets for the DMUs based on historical process knowledge and statistical analyses. To achieve these objectives the CDMU could possibly choose to give most importance to meeting global input and output targets through maximal contribution of the DMUs. The CDMU could consider the objective of efficiency for the DMUs to bear secondary importance. The DMUs could assign primary importance to the objective of efficiency and assign secondary importance to the objective of meeting global targets. The model can be solved with different priorities for the fuzzy goals depending on the level of decision-making.

5.2. The Mathematical Formulation

A hierarchical system consists of N DMUs and a coordinating CDMU. The CDMU provides global input and output targets and pre-specifies tolerance limits for the global targets. The individual DMUs specify the tolerance limits for the individual DMU inputs and outputs. Then, the problem is to determine the activity levels that maximally achieve the fuzzy goals of effectiveness (meeting global targets) and efficiency (meeting individual DEA targets). The Fuzzy GoDEA model can thus be written as:

Model 2

$$\text{Find } \lambda_k^c \tag{8}$$

In order to maximally achieve the following fuzzy goals:
DMU Representation:

$$\sum_{k=1}^{N} \lambda_k^c y_{jk} \underset{\approx}{\geq} y_j^c, \quad \forall j = 1,2,...,J \quad \forall c = \{1,2,\ldots,N\} \tag{9}$$

$$\sum_{k=1}^{N} \lambda_k^c x_{ik} \underset{\approx}{\leq} x_i^c, \qquad \forall i = 1,2,...,I \tag{10}$$

Achievement of Global Targets (Effectiveness):

$$\sum_{k=1}^{N} \lambda_k^1 y_{jk} + \ \ldots \ + \sum_{k=1}^{N} \lambda_k^N y_{jk} \ \underset{\approx}{\geq} \ TY_j\,, \quad \forall j = 1,2,..,J \tag{11}$$

$$\sum_{k=1}^{N} \lambda_k^1 x_{ik} + \ \ldots \ + \sum_{k=1}^{N} \lambda_k^N x_{ik} \ \underset{\approx}{\leq} \ TX_i\,, \quad \forall i = 1,2,...,I \tag{12}$$

$$\sum_{k=1}^{N} \lambda_k^c = 1, \quad \forall c = \{1,2,\ldots,N\} \tag{13}$$

$$\lambda_k^c \geq 0, \quad \forall k = \{1,2,\ldots,N\}, \forall c \tag{14}$$

where:
N: number of DMUS
I: set of inputs
J: set of outputs
x_{ik}: level of input i of DMU k
y_{jk}: level of output j for DMU k

x_i^c, y_j^c : level of input i and output j for DMU c when assessing DMU c

λ_k^c : activity level of DMU k when assessing DMU c

"$\underset{\approx}{\leq}$", "$\underset{\approx}{\geq}$": denote fuzzification of the goal or constraint.

The objective function and the constraints of the model are related through the activity levels $\lambda_k c$'s. A set of activity levels is obtained when each DMU is assessed. These activity levels are DMU specific. In other words each DMU when assessed has its own set of activity levels for each input and output for all the DMUs in the data set. The activity levels are free to take on any non-negative value. The convexity constraint models variable returns to scale. Restrictions can be relaxed on the activity levels to incorporate constant returns to scale.

The fuzzy constraints can be treated as fuzzy goals. The fuzzy goals imply that the goals have to be *essentially* met within the specified tolerance limits or bounds. These bounds are pre-specified by the decision-maker based on historical knowledge. Consider the r^{th} fuzzy goal $G_r \underset{\approx}{\geq} g_r$, which signifies that the decision-maker accepts the constraint satisfaction up to a certain tolerance greater than g_r. Consequently, a membership function μ_r for the r^{th} goal $G_r \underset{\approx}{\geq} g_r$ is defined by Zimmermann (1978) as:

$$\mu_r = \begin{cases} 1, & \text{if } G_r \geq g_r \quad (15) \\ \dfrac{G_r - L_r}{g_r - L_r}, & \text{if } L_r < G_r < g_r \quad (16) \\ 0, & \text{if } G_r \leq L_r \quad (17) \end{cases}$$

where L_r is the lower bound or lower tolerance limit for fuzzy goal $G_r \geq g_r$.

Analogously, for the s$^{\text{th}}$ fuzzy goal $G_s \underset{\approx}{\geq} g_s$, which signifies that the decision-maker accepts the constraint satisfaction up to a certain tolerance limit less than g_s, the membership function μ_s is defined as:

$$\mu_s = \begin{cases} 1, & \text{if } G_s \leq g_s \quad (18) \\ \dfrac{U_s - G_s}{U_s - g_s}, & \text{if } g_s < G_s < U_s \quad (19) \\ 0, & \text{if } U_s \leq G_s \quad (20) \end{cases}$$

where U_s is the upper bound or upper tolerance limit for fuzzy goal $G_s \geq g_s$.

The membership functions associated with the fuzzy goals in Model 2 can be expressed based on Zimmermann's (1978) definition of linear membership functions. However, the fuzzy model outlined above cannot be solved in the present form. Therefore, a linear crisp translation is required. A membership function μ_q is associated with each fuzzy goal G_q. There are $(i+j)$ input/output factors and therefore $(i+j)$ DMU representation constraints (DEA type constraints) for every DMU $k = 1,2,...,N$. There are $(i+j)$ global target constraints. Therefore, in total there are $N(i+j)+(i+j)$ fuzzy constraints and consequently, $N(i+j)+(i+j)$ membership functions. Let $(i+j)=m$. Then there are $m(N+1)$ membership functions. The crisp equivalent linear program for the Fuzzy GoDEA model (Model 2) is written as:

Model 3

$$\max \sum_{x_i^c, y_j^c, x_i, y_j} (\mu_{y_j^c} + \mu_{x_i^c} + \mu_{y_j} + \mu_{x_i}) \tag{21}$$

Subject to:
For the Efficiency representations:

$$\mu_{y_j^c} = \frac{\sum_{k=1}^{N} \lambda_k^c y_{jk} - l_j^c}{y_j^c - l_j^c}, \quad \forall j \in J \tag{22}$$

$$\mu_{x_i^c} = \frac{u_i^c - \sum_{k=1}^{N} \lambda_k^c x_{ik}}{u_i^c - x_i^c}, \quad \forall i \in I \tag{23}$$

$\forall \ c \in \{1,2,\ldots,N\}$ (applies to (22) and (23))

For the achievement of Global Targets:

$$\mu_{y_j} = \frac{(\sum_{k=1}^{N} \lambda_k^1 y_{jk} + \ldots + \sum_{k=1}^{N} \lambda_k^N y_{jk}) - L_j}{TY_j - L_j}, \quad \forall j \in J \tag{24}$$

$$\mu_{x_i} = \frac{U_i - (\sum_{k=1}^{N} \lambda_k^1 x_{ik} + \ldots + \sum_{k=1}^{N} \lambda_k^N x_{ik})}{U_j - TX_i}, \quad \forall i \in I \tag{25}$$

$$\sum_{k=1}^{N} \lambda_k^c = 1, \quad \forall c = \{1,2,\ldots,N\} \tag{26}$$

$$0 \le \mu_{y_j^c}, \mu_{x_i^c}, \mu_{y_j}, \mu_{x_i} \le 1, \ \lambda_k^c \ge 0 \tag{27}$$

where:

l_j^c : lower bound on DMU output target y_j^c

u_i^c : upper bound on DMU input target x_i^c and

L_j^c : lower bound on DMU global output target TY_j and $L_j = \sum_k l_j^k$

U_i : upper bound on global input target TX_i and $U_i = \sum_k u_i^k$

In this research U_i and L_j are assumed to be the sum of the individual input/output bounds. The μ_q's represent the degree of satisfaction of the decision-maker.

The Fuzzy GoDEA model 2 has transformed into the equivalent crisp formulation of model 3. This crisp formulation is then solved with suitably developed computer programs in CPLEX. Variations of the base Fuzzy GoDEA model were studied to capture different decision-making scenarios. The variations are summarized by Table 1 and are described in detail by Sheth (1999) and were used to evaluate the newspaper preprint insertion line described by Girod and Triantis (1999).

6. FUZZY GODEA RESULTS FOR THE NEWSPAPER PREPRINT INSERTION PROCESS

Newspaper preprint insertion involves merging incoming newspaper sections and commercial preprints into bundles ready for delivery to newspaper distributors. Major American newspapers are composed of two sections. The first contains the non-news sensitive materials, whereas the second comprises the newspaper head sheet that covers local, national, and international news. Commercial preprints are inserted in the newspaper's non-news sensitive sections and considered jointly, they are labeled packages. Th inclusion and distribution of commercial preprints as part of newspaper packages is a major source of revenue for all large newspaper organizations.

Newspaper firms do not insert preprints every day but a few days per week. The newspaper preprint insertion process analyzed in this research is activated once per week. The main characteristics of this process include the following: 1) variations of the daily work order that impacts production; 2) fluctuations of the manpower requirements; 3) rework; and 4) recycled or wasted newspaper packages. Further, production plans or DMUs were represented by three inputs, i.e., direct labor (DLR), rework (RWK), and raw material (RML) and one output (PCF), i.e., packages adjusted with a complexity factor. For more on the newspaper preprint insertion technology refer to Girod (1996).

Data were accumulated for a forty-eight week time period. Each week represents a single production plan or DMU. As noted by Girod and Triantis (1999), important organizational changes occurred at the midpoint of the study period therefore making it necessary to study the performance of the production line for the first and last twenty-four weeks together and separately. For the sake of brevity in the ensuing discussion, we will primarily focus on the first twenty-four weeks or DMUs. For a detailed discussion of the evaluation of the preprint insertion line for the whole time period using the fuzzy GoDEA model and its variations refer to Sheth (1999). In this section, we first make some comments with respect to the attainment of the efficiency and effectiveness goals across variations. Then we briefly present the results of one of the variations to the base model, i.e., variation nine since it includes a combination of crisp and fuzzy goals and it is solved as a two-stage sequential goal programming model.

6.1. The Efficiency Goal

The efficiency goal was measured using DEA type constraints for the individual DMU representations *i.e.,* each input (DLR, RWK, RML) and output (PCF) for each DMU was measured relative to all the DMUs in the data set. The Fuzzy GoDEA model is not designed to provide an efficiency score for each DMU. The efficiency score can readily be obtained by conventional DEA analysis. However, the achievement of the efficiency goal through the DEA type constraints in the Fuzzy GoDEA formulation provides additional useful insights.

The membership functions associated with the fuzzy efficiency (DEA type) constraints represent the degree of satisfaction of these constraints. The fuzzy efficiency (DEA type) constraints are found in variations 1, 2, 3, 4, 6, and 7. When the membership function for such a constraint equals one or is very close to one it implies that the DEA input or output inequality is satisfied crisply. If any of these membership functions achieves a value less than one then it signifies a relaxation of the DEA structure for that particular DMU. In the present application the DEA structure was maintained crisply for all variations that had fuzzy DEA type efficiency constraints. The activity levels (λ) require additional analysis following the evaluation of the membership functions. In the absence of an efficiency score the activity levels for each DMU reveal whether it is efficient or inefficient. For a DMU to be 100% efficient the activity level associated with it in the composite unit must attain the value one. This implies that such a DMU is its own "reference set" as it is 100% efficient relative to all the members of the data set.

Table 1: Base Model Variations and their Characteristics

Variation	Characteristics
1	The objective function and the membership functions are added. Weights are assigned to the membership function according to the importance that the decision-maker wants to assign to each goal.
2	The goals of effectiveness and efficiency are solved sequentially. Here the achievement of global targets is considered as more important in stage 1 and solved for optimality. The optimal values of the membership functions corresponding to the effectiveness goals obtained from stage 1 are then passed on as constraints for the stage 2 problem. The objective function is then evaluated in stage 2. The solution obtained for stage 2 will thus maintain the solution to stage 1.
3	The goals of effectiveness and efficiency are solved sequentially. Here the achievement of individual decision making unit (DMU) targets i.e. the goal of efficiency is considered as more important in stage 1 and solved for optimality. The optimal values of the membership functions corresponding to the efficiency goals obtained from stage 1 are passed on as constraints for the stage 2 problem. The objective of effectiveness i.e. the achievement of global targets is then evaluated in stage 2. The solution obtained for stage 2 will thus maintain the solution to stage 1.

Table 1 (continued): Base Model Variations and their Characteristics

4	In this case the constraints related to efficiency performance (DMU representation) are fuzzy while the constraints related to achievement of global targets are crisp. The objective function seeks to maximize the sum of the membership functions associated with the efficiency constraints. The decision-maker would typically desire that the global consumption of inputs is maintained below the global target and the global production of output exceeds the global target. The global targets represent the sum of the individual risk-free scenarios for the inputs and outputs. Therefore, the goal would be to improve upon the sum of these bounds at the global level.
5	In this case the constraints related to efficiency (DMU representation) are crisp while the constraints related to achievement of global targets are fuzzy. The objective function seeks to maximize the sum of the membership functions associated with the effectiveness constraints. The crisp DMU representation constraints measure each DMU relative to a composite unit to measure efficiency.
6	Here the stage 1 problem tries to maximize satisfaction of the fuzzy DMU representations or efficiency constraints. The stage 2 problem consists of fuzzy efficiency constraints and crisp effectiveness constraints. The optimal solution values for the efficiency membership functions of stage 1 are passed as a constraint to the stage 2 problem. The objective function in stage 2 tries to minimize the deviations from the global targets. Only negative deviation from the output target and positive deviation from the input target are minimized since positive output deviation and negative input deviation are considered acceptable. Thus the solution to the stage 2 problem will maintain the optimal solution to the stage 1 problem. Consequently, the solution to the deviation variables in stage 2 reveal the extent of satisfaction of the effectiveness constraints given a certain acceptable level of satisfaction of the efficiency constraints.
7	This variation reverses the priority attached to the goals in variation 6. In stage 1 the objective function minimizes the deviations from the global targets. The solution to the deviation variables is then passed as a constraint to the stage 2 problem. In stage 2 the objective is to maximize satisfaction of the fuzzy DMU representations or efficiency constraints. The stage 2 problem consists of fuzzy efficiency constraints and crisp effectiveness constraints. The solution to the stage 2 problem reveals the extent of satisfaction of the efficiency constraints given the least deviation from the effectiveness constraints.
8	In this variation the stage 1 problem is to minimize the deviations from the crisp efficiency targets for each DMU. These crisp DMU representations measure relative efficiency in the conventional DEA sense. The DMUs for which the deviations reach zero are evaluated as efficient. The solutions for the deviations are passed as constraints to the stage 2 problem. The stage 2 objective is to maximize the satisfaction of the fuzzy effectiveness constraints while maintaining the efficiency goal achieved in stage 1.
9	In this variation the priorities associated with the efficiency and effectiveness constraints are the reverse of variation 8. The stage 1 problem is to maximize the satisfaction of the fuzzy effectiveness constraints. The solution μ^*s are passed to the stage 2 problem where the objective is to minimize the deviations from the efficiency targets for the individual DMUs. The solution to the stage 2 problem maintains the satisfaction of the fuzzy effectiveness goals achieved in stage 1.

On the other hand, an inefficient DMU has a reference set (or peers) that consists of *other* DMUs. In conventional DEA this reference set would contain only efficient DMUs. The Fuzzy GoDEA formulation provides a departure from conventional DEA in this regard. The reference set for an inefficient DMU is allowed to have *inefficient* DMUs (as defined by the conventional DEA analysis) in addition to the efficient DMUs. However, in every variation of the Fuzzy GoDEA framework, the reference sets for inefficient DMUs typically include only units evaluated as efficient in *that* variation.

The BCC efficiency scores are used in this research to characterize the behavior of the BCC-inefficient DMUs as peers in the Fuzzy GoDEA model variations. The BCC scores were computed using the observed values for the three inputs and output. Refer to Sheth (1999) as to how the observed values were obtained. Typically, the BCC-inefficient peers for an inefficient DMU will display relatively small amounts of inefficiency *i.e.,* will have high BCC efficiency scores. However, the presence of a large number of BCC-inefficient DMUs in the reference sets cannot be ruled out. A high frequency of BCC-inefficient peer units can be attributed to a large variation in the data.

The peers for the inefficient DMUs for some of the first twenty-four observations along with the BCC efficiency scores and peers are displayed in Table 2 for all Fuzzy GoDEA model variations. The fuzzy formulation applied in this research aims to relax the DEA evaluation and to allow relative efficiency comparison with not only 100% BCC-efficient DMUs but also less than 100% BCC-efficient units. Alternately, the concept of a *crisp efficient frontier* that envelops the data is modified to allow a *thick* frontier. Moreover, the inclusion of DMUs less than 100% BCC-efficient makes a case that it is more realistic for an inefficient unit to attain the input/output levels of *near BCC-efficient* units before trying to achieve the input/output levels of efficient units. The quantification of a *near BCC-efficient unit* is open to subjectivity. The results obtained from the application of this research suggest that the decision-maker would have to make a subjective decision regarding the threshold for near BCC-efficient and BCC-inefficient units. This philosophy is in line with the fuzzy concepts proposed in this research where the decision-maker seeks a compromise that provides a *satisfycing* level of all goals rather than an optimal achievement level of all goals.

The evaluation of efficient DMUs for the packaging line data differs across the Fuzzy GoDEA model variations. Figure 3 shows the number (percentage) of inefficient DMUs by variations for observations 1-24. Table 2 presents the peers for the base case, for model variations 4 through 9 and for the BCC model. DMUs 3, 6 and 7 are efficient across the variations as well as in the BCC evaluation. DMUs 1, 19, 21 and 24 are efficient in the BCC evaluation but are found inefficient in Variation 9. In Variation 9, DMU 19 is found to lie on the same facet of the efficient frontier as DMU 3 while DMUs 21 and 24 are found to lie on the same facet of the efficient frontier as DMU 7. This indicates a higher discerning power of Variation 9 in evaluating efficient DMUs. This, however, limits BCC efficient DMUs 3, 6 and 7 to be the peers in Variation 9. DMUs 4, 13 and 18 are found to be inefficient across the variations as well as in the BCC evaluation.

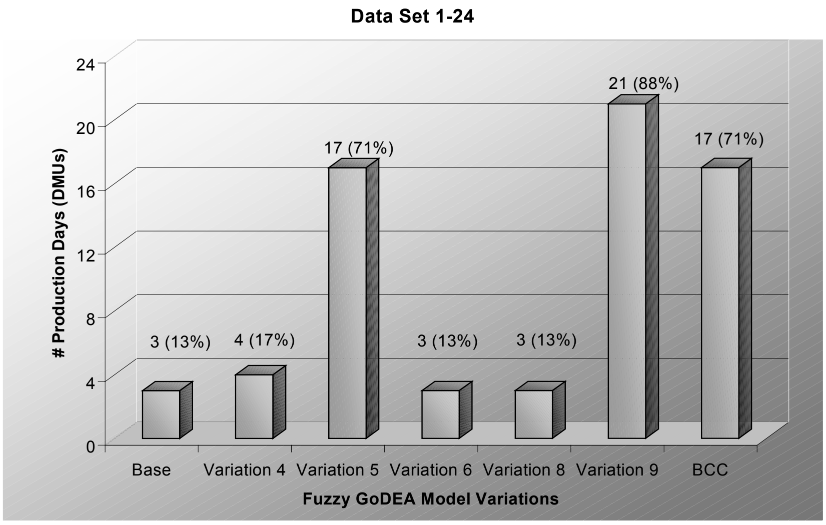

Figure 3: Number (Percentage) of Inefficient Units by Variation

The choice of peers for the inefficient DMUs differs across the variations depending on (i) the solution stage assigned to the efficiency constraints and (ii) the fuzzy or crisp nature. For example, the peers for DMU 4 chosen by Variations 5 and 9 (Stage 1) that have crisp efficiency constraints are a subset of the peers chosen by the BCC model. However, Variations 4 and 6 (Stage 1) with fuzzy efficiency constraints and Variation 8 (Stage 1) with crisp efficiency constraints and positive and negative deviation variables display similarity in their choice of peers but differ significantly from the BCC evaluation.

The Fuzzy GoDEA Base Model and Variations 4, 6 and 7 have fuzzy efficiency (DEA representation) constraints and identify both BCC efficient and BCC-inefficient units as peers for the inefficient units evaluated by each model variation. Variations 5 and 9 have crisp efficiency constraints and identify only BCC-efficient units as peers for the inefficient units.

6.2 The Effectiveness Goal

The effectiveness goal in the Fuzzy GoDEA formulation is measured through the achievement of global targets for the three inputs (DLR, RWK, and RML) and one output (PCF). The efficient contribution of each DMU for each of DLR, RWK, RML, and PCF is aggregated and compared with the respective global target. The decision-maker's goal is to exceed or equal the output target with the aggregate efficient output unit and to be less than or equal the input target with the aggregate efficient input unit.

When the effectiveness constraints are fuzzy the decision-maker allows for a specified tolerance violation of the global targets (variations 1, 2, 3, 5, 8, and 9).

Accordingly, the global targets are so chosen that they *cannot* be met crisply simultaneously. In other words an *ideal* benchmark is chosen for each global target. In the application presented in this research the global targets for the fuzzy effectiveness goals are computed by aggregating the efficient BCC projections for each input (BCC input reducing model) and output (BCC output increasing model). It is intuitive that these aggregate efficient projections can be achieved simultaneously only in the event that *all* DMUs are evaluated as 100% efficient. This, of course, is impossible in relative efficiency measurement unless the values of all observed inputs and outputs are identical across DMUs. The fuzzy effectiveness goals then provide the decision-maker with a measure of the degree of satisfaction related to the achievement of each global target. The membership functions associated with the fuzzy effectiveness constraints reflect this degree of satisfaction. The closer the aggregate contribution to the global target the higher will be the value of the membership function.

In case of crisp effectiveness constraints (variations 4, 6, and 7) the goal programming formulation associates positive and negative deviations with respect to achievement of the global target. However, in this case the global targets have to be redefined due to the unattainable nature of the efficient BCC projections. As in the fuzzy scenario, the decision-maker's goal in the crisp case is to exceed or equal the output target with the aggregate efficient output unit and to stay within or equal the input target with the aggregate efficient input unit. The efficient BCC projections as global targets provide an infeasible region for satisfaction of these crisp constraints. However, the sum of the individual bounds on the inputs and outputs for each DMU provide the decision-maker with one reasonable method of specifying the global targets in the crisp case. The decision-maker's objective would be to ensure at least the satisfaction of these global targets as the sum of the individual bounds can be considered as the *risk-free* global values of inputs and output. Accordingly, the decision-maker would aim to achieve at least these risk free values in the crisp sense.

The achievement of the effectiveness goal or global targets for the inputs and output for the packaging line data differs across the variations of the Fuzzy GoDEA model. Table 3 shows the global target achievements for the Fuzzy GoDEA model variations those with fuzzy and crisp effectiveness constraints. Variations 4 and 6 have crisp effectiveness constraints with the global targets representing the decision-maker's risk-free scenario. Variation 6 has a second level priority attached to the effectiveness constraints. The Base Model and Variations 5, 8 and 9 have fuzzy effectiveness constraints with the global targets representing the decision-maker's ideal benchmarks. The Base Model and Variation 5 have equal priority for the efficiency and effectiveness constraints. Variation 8 has a second level priority while Variation 9 has a first level priority attached to the effectiveness constraints. The Base Model and Variation 8 have identical global target achievement results and show the lowest achievement for all inputs DLR, RWK, and RML and output PCF. For the data set 1-24, achievement for DLR and RWK is maximized at the global targets and achievement for RML is approximately the same for Variations 5 and 9. However, the achievement for PCF is lower in Variation 5 than in Variation 9.

Table 2: Fuzzy Peer Table for Production Days 1-24

Prdn Day	Base	Variation 4	Variation 5	Variation 6	Variation 8	Variation 9	DEA (BCC Input Reducing)	
							Peers	Eff. Score
1	-	-	-	-	-	3,6	-	1
3	-	-	-	-	-	-	-	1
4	1,11,12, 21, 22	1,11,12, 21,22	3,6,7	1,2,9,22	1,2,9,10, 22	3,6,7	1,3, 6,7	0.783
6	-	-	-	-	-	-	-	1
7	-	-	-	-	-	-	-	1
13	1,11,19, 21, 22	1,9,19, 21,22	3,6,7	1,2,11, 19,22	1,2,11, 19,22	3,6,7	1,3, 6,7	0.806
18	1,7,15, 16,20	1,2,12, 15,24	3,7	1,2,14, 16,22	1,2,7, 12,14	3,7	1,7, 24	0.804
19	-	-	-	-	-	3	-	1
21	-	-	-	-	-	7	-	1
24	-	-	-	-	-	7	-	1

Table 3: Output/Input Global Target Achievement for Fuzzy GoDEA Models
GoDEA Models with Fuzzy Effectiveness Goals

Data Set	Output/ Inputs	Global Targets	Global Target Bounds	Base Model	Variation 5	Variation 8	Variation 9
1-24	PCF	3449580	2672290	3097486	3287904	3097483	3342314
	DLR	26437	34748	31450	26437	31448	26437
	RWK	135123	333474	173766	135123	173802	135123
	RML	120013	241243	141141	137228	141107	137713

Table 3 (continued): GoDEA Models with Crisp Effectiveness Goals

Data Set	Output/Inputs	Global Targets	Variation 4	Variation 6
1-24	PCF	2672290	3097486	3097486
	DLR	34748	31450	31450
	RWK	333474	173766	173766
	RML	241243	141141	141141

6.3. Fuzzy GoDEA: Variation 9

The results for Variation 9 for some production observations or DMUs are presented in Table 4. Stage 1 of this variation maximizes the achievement of the fuzzy effectiveness constraints. These membership functions indicate the degree of satisfaction for the decision-maker with respect to the achievement of each effectiveness constraint or global target. The optimal membership function values associated with the fuzzy effectiveness constraints are introduced as constraints for the Stage 2 problem. The objective of the Stage 2 problem is to minimize the positive and negative deviations associated with the achievement of the crisp efficiency constraints.

Table 4: Fuzzy GoDEA Results for Variation 9: Efficiency Goal

Prdn Day	Deviations	Peers	Output/Inputs	Prdn Day	Deviations	Peers
3	0 0 0 0	$\lambda_3^3=1.00$	PCF DLR RWK RML	19	(+)30207.59 (+)295.20 (-)2506.00 (+)1268.25	$\lambda_3^{19}=1.00$
6	0 0 0 0	$\lambda_6^6=1.00$	PCF DLR RWK RML	20	(+)11751.74 (-)332.50 (-)5104.00 (-)2170.97	$\lambda_7^{20}=1.00$
7	0 0 0 0	$\lambda_7^7=1.00$	PCF DLR RWK RML	21	(-)45624.97 (-)287.20 (-)2242.00 (-)4766.64	$\lambda_7^{21}=1.00$
13	(-)16827.89 (-)56.45 0 0	$\lambda_3^{13}=0.42$, $\lambda_6^{13}=0.39$, $\lambda_7^{13}=0.19$	PCF DLR RWK RML	22	(-)12106.46 (-)258.40 (-)6464.00 (-)5846.28	$\lambda_7^{22}=1.00$
9	(+)19934.51 0 (+)1483.04 (+)812.18	$\lambda_3^9=0.34$, $\lambda_6^9=0.66$	PCF DLR RWK RML	24	(-)9757.03 (-)269.50 (-)6906.99 (+)289.13	$\lambda_7^{24}=1.00$

Table 4: Fuzzy GoDEA Results for Variation 9 (continued): Effectiveness Goal

Output/Inputs	Target	Achieved	Slack	Effectiveness (μ)
PCF	3449580.42	3342314.34	107266.08	0.86
DLR	26436.52	26436.52	0.00	1.00
RWK	135123.07	135123.07	0.00	1.00
RML	120013.14	137712.69	-17699.55	0.85

All the deviation variables for DMUs 3, 6 and 7 are equal to zero. Also, the activity level corresponding to the observed input/output values for each of DMUs 3, 6 and 7 is equal to one. Therefore, DMUs 3, 6 and 7 are efficient. DMU 19 has the activity level associated with DMU 3's input/output values equal to one but shows positive deviation for PCF (30,206), positive deviation for DLR (295), negative deviation for RWK (2,506) and positive deviation for RML (1,268). This implies that the efficiency targets were exceeded by amounts equal to the positive deviations and unattained by shortfalls equal to the negative deviations. Physically, the deviation amounts are interpreted as follows. For the output (PCF) a non-zero *positive* deviation for a DMU implies that an increase in output production equal to the deviation amount would render an efficient level of output production. For the inputs (DLR, RWK, and RML) a non-zero *negative* deviation for a DMU implies that a decrease in input consumption equal to the deviation amount would render an efficient level of input usage.

However, DMU 19 does lie on the same facet of the efficient frontier as DMU 3, which appears as its peer. The deviation amounts (see Table 4) for PCF (+30,208) and RWK (-2,506) convey the required increase in PCF production and decrease in RWK quantity that would make DMU 19 as efficient as DMU 3. Similarly DMUs 20, 21 and 22 lie on the same facet of the efficient frontier as DMU 7. When a DMU exhibits *negative* deviation for the output (PCF) or *positive* deviation for an input (DLR, RWK, and RML) it implies better performance than the composite unit for that output/input. For example, DMUs 13, 21, 22 and 24 display negative PCF deviations. All these DMUs have a relatively high BCC efficiency score (81% - 100%). This confirms that they have efficient levels of output production.

Figure 4 graphically displays the results for the achievement of the global targets. The membership functions associated with DLR and RWK are equal to one. This implies that the global targets for DLR and RWK are achieved at the specified target level. The membership function is 0.86 for PCF and 0.85 for RML. The target achievement for PCF is short by 107,266 while the target achievement for RML is exceeded by 17,700 NNSS pieces.

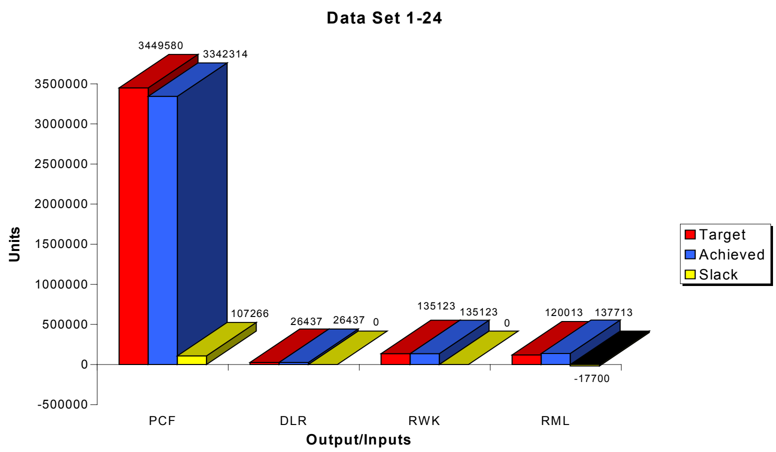

Figure 4: Effectiveness Achievement for Variation 9

6.4. The Fuzzy GoDEA Performance of the Newspaper Preprint Insertion Line

At the process level, on a production day basis, the decision-maker can use the fuzzy peer table (Table 2) to identify inefficient units for different scenarios and make resource reallocation decisions, explore possible efficiency enhancement interventions and further investigate root causes of inefficiency. For example, DMU 4 is inefficient across all variations. The above average RML consumption (6,141 NNSS pieces) and a significantly low PCF production (122,720 packages) explain this inefficiency. The decision-maker can investigate the root causes for such inefficient observations by analyzing the operational records for the packaging line.

Girod (1996) tabulated the symptoms and root causes of inefficiency for severely inefficient DMUs. A similar approach can be adopted to analyze the characteristics of inefficient DMUs. For example, the records showed that for DMU 4 the preprint insertion machine operated at high speeds (greater than 20,000 cycles/hour) causing a high number of NNSS faults (3,000 to 5,000 multiple feeds). Moreover, DMU 4 experienced a high volume of preprint shortage (14,000 NNSS pieces) due to a deficient preprint tracking system. Table 5 summarizes this information and shows symptoms and root causes information for DMU 4.

Table 5: Symptoms and Root Causes for Inefficient DMUs (from Girod (1996))

Production Day (DMU)	Symptoms	Root Cause
4	1. High raw material consumption driven by excessive amount of NNSS faults (3,000 to 5,000 multiple feeds). 2. Significant preprint shortage (14,000 pieces).	1. Preprint Insertion Machine operating at high speed (greater than 20,000 cycles/hour). 2. Preprint Tracking System deficiency.

Thus, the evaluation of inefficient and efficient DMUs by the Fuzzy GoDEA methodology along with the conventional BCC evaluation affords the decision-maker the ability to compare inefficient DMUs with efficient or near-efficient DMUs and implement a best practices approach to improve efficiency

7. CONCLUSIONS AND FUTURE RESEARCH

The fuzzy dimension of this research aims to accommodate imprecise aspiration levels and thus, inherently introduces subjectivity in the analysis and interpretation of the data and the results. The subjectivity component assumes presence in the choice of the membership function, the bounds on the inputs and outputs, the choice of the global targets, and the bounds on the global targets. In the analysis for the results, the decision-maker must use discretion regarding the definition of *near-efficient* units and the numerical value of the membership functions that denote the degree of satisfaction for the efficiency and effectiveness constraints.

The current research can be extended and further investigated with respect to one or more of its components, namely, fuzzy set theory, goal programming and data envelopment analysis. With respect to fuzzy set theory, the suitability of the form of the membership function with respect to the data is an issue of interest. Further, the impact of the form of the membership function on the efficiency and effectiveness results also warrants attention. The physical interpretation of the membership functions for decision-making use constitutes an important part of the decision-making process and requires investigation. Also, formulations to incorporate vagueness in the input and output data as well as imprecision in the aspiration levels for the efficiency and effectiveness goals would be a next step in the realm of fuzzy DEA.

With respect to goal programming, formulations can be developed to model quantifiable preferences of the decision-maker regarding the relative importance of the goals. In the same context, mathematical methods to evaluate these preferences based on operations knowledge can be explored. The relative importance within the effectiveness goals (*i.e.,* relative importance for achievement of global targets between inputs and outputs) also offers research opportunities.

With respect to DEA, the notion of a thick frontier presents great interest in efficiency measurement. This, of course, is an issue that requires DEA to be explored in conjunction with fuzzy set theory. The evaluation of fuzzy efficiency scores is another area of research that arises in efficiency measurement. Finally, some general areas of interest are the construction of bounds for the input and output data, global targets and bounds on the global targets.

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