Measuring Comovements by Regression Quantiles

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ABSTRACT

This article develops an econometric framework to investigate the structure of dependence between random variables and to test whether it changes over time. Our approach is based on the computation—over both a test and a benchmark period—of the conditional probability that a random variable is lower than a given quantile, when another random variable is also lower than its corresponding quantile, for any set of prespecified quantiles. Time-varying conditional quantiles are modeled via regression quantiles. The conditional probability is estimated through a simple OLS regression. We illustrate the methodology by investigating the impact of the crises of the 1990s and 2000s on the major Latin American equity markets returns. Our results document significant increases in equity return comovements during crisis times. (*JEL*: C14, C22, G15)

KEYWORDS: dependence, conditional quantiles

This article proposes an econometric framework to measure dependence between two (possibly heteroscedastic) random variables. The approach is based on the estimation of the probability that a random variable y_t falls below a conditional quantile, given that the other random variable x_t is also falling below its corresponding quantile. Conditional quantiles are estimated via regression quantile (Koenker and Bassett, 1978). In this framework, the stronger the dependence between x_t and y_t, the higher the probability of comovement. We estimate this probability through a simple OLS regression involving quantile coexceedance indicators and derive a test to assess whether comovement likelihoods change over time and across market conditions.[1]

A large body of literature has developed over the years investigating dependence among financial asset returns. Extensive surveys are provided, *inter alia*, by de Bandt and Hartmann (2000), Pericoli and Sbracia (2003), and Dungey et al. (2005). In essence, one can distinguish three different approaches: (i) modeling first and/or second moments of returns (see, for instance, King, Sentana, and Wadhwani, 1994; Forbes and Rigobon, 2002; Ciccarelli and Rebucci, 2007); (ii) estimating the probability of coexceedance (see, among others, Longin and Solnik, 2001; Hartmann, Straetmans, and de Vries, 2004; Bae, Karolyi, and Stulz, 2003); and (iii) methods based on copula theory (see Rodriguez, 2003; Patton, 2004; Garcia and Tsafack, 2011). Each of these methodologies has its own merits and drawbacks. Correlation-based models and Generalized Autoregressive Conditional Heteroscedastic (GARCH)-type approaches are easy to implement, but assume that realizations in the upper and lower tail of the distribution are generated by the same process (see for instance the discussion in Garcia and Tsafack, 2011). Probability models focus on the tails of the distribution, but generally analyze only single points of the support and adopt a two-step estimation procedure, often without correcting the standard errors. Methods based on parametric copulas rely on specific parameterizations of the dependence measure, often using a single parameter to determine the shape of the entire copula.

The approach we propose has several advantages with respect to existing methodologies. First, we show that the coefficients of a simple OLS regression of a quantile coexceedance indicator variable on a constant and economic indicator variables provide consistent estimates of comovement probability. The inclusion of economic variables permits us to test whether they contribute to significantly change the probability of comovement. Second, casting the econometric framework in terms of time-varying regression quantiles permits us to make proper inference, since it is robust to heteroskedasticity which could make correlation measures spurious. In addition, we correct standard errors

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[1]Coexceedance occurs when both random variables x_t and y_t exceed some prespecified thresholds. Baur and Schulze (2005) have proposed an analysis of contagion which is also based on coexceedance and quantile regression. Their approach however is fundamentally different from the one proposed in this article as their coexeceedance is computed assuming zero as threshold.

to take into account that we use a two-step estimation approach. Third, we are able to measure dependence over any subset of the support of the joint distribution, and asymmetries in comovement in the upper and lower tails of the distribution can be tested for. Fourth, since regression quantile is a semi-parametric technique, there is no need to impose any assumptions on the joint or marginal distributions of the variables under investigation. Finally, being based on quantiles, it provides estimates of comovements robust to outliers, as opposed to conventional, average-based measures (Kim and White 2004; White, Kim, and Manganelli, 2008; 2012).[2]

In this article, we show how our methodology can be used to test for changes in comovements during periods of economic stress. We investigate the impact of some of the major financial crises on the main Latin American equity markets during the 1990s. One key unresolved issue is whether the Tequila crisis, the Asian flu, the Russian worm, and more recently the worldwide crisis following the Lehman bankruptcy were episodes characterized by an increase in cross-market linkages. In the finance literature, this concept has been linked to contagion, which is broadly defined as an increase in financial market comovements during periods of financial turbulence. Our results show that comovements in equity markets increase during turbulent periods. Furthermore, the increase tends to be asymmetric, with stronger changes in comovement in the left tail than in the right tail.

The applications of this methodology are not limited to the specific issue of testing for changes in financial comovements. For instance, for strategic-allocation purposes, risk-averse investors could use our methodology to select those asset classes which exhibit lowest comovements especially in the lower tail of the distribution.[3] Economists and policy makers are also interested in measuring cross border dependence and changes thereof among asset returns and economic variables. Our methodology can also be used to assess the impact of major economic events (such as the introduction of the euro) on financial markets, as done for instance in Cappiello, Kadareja, and Manganelli (2010).

This article proceeds as follows. In Section 1 we develop the econometric framework. Section 2 illustrates how our approach can be used to study changes in financial comovements, relating it to the existing empirical contributions. Section 3 describes the data. Section 4 reports the estimated comovements for four Southern American countries. We also compare our approach with estimates based on copula models. Section 5 discusses the empirical evidence about changes in comovements between tranquil and crisis times. Section 6 concludes.

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[2]There is an active area of research in statistics that attempts to extend the notion of quantile to a multivariate framework. We refer the interested reader to the works of Chaudhuri (1996) and Chakraborty (2003).

[3]It is well documented that financial asset comovements increase more during turbulent times than over calm periods (see, for instance, Longin and Solnik, 1995; Karolyi and Stulz, 1996; De Santis and Gérard, 1997; Ang and Bekaert, 2002).

1 ESTIMATING COMOVEMENTS

This section discusses the estimation of the average probability of comovements between two random variables, y_t and x_t.

Let $q_{i,t}^Y$ be the θ_i-quantile at time t of y_t, $0 < \theta_1 < \ldots < \theta_m < 1$, conditional on the available information set Ω_t. Analogously, for x_t, we define $q_{j,t}^X$. Let $F_t(y,x)$ denote the cumulative distribution function for the pair (y_t, x_t). The average probability that both y_t and x_t fall below their respective quantiles over a given time period is given by $\bar{F}_T^{ij} \equiv T^{-1} \sum_{t=1}^T F_t(q_{i,t}^Y, q_{j,t}^X)$.

We propose to estimate $\bar{F}_T^{ij}$ as follows. First, we estimate the conditional univariate quantiles associated with the random variables y_t and x_t. Second, we construct, for each series and for each quantile, indicator variables which are equal to one if the realized random variable is lower than the conditional quantile and zero otherwise. Finally, we regress the product of the indicator variables for series y_t and x_t on a constant. If the interest lies in testing whether the average comovement has changed over different time periods, appropriate dummies can be included in the regression. As we show, the regression coefficients provide a direct estimate of the conditional probabilities of comovements.

In subsection 1.1 we briefly review the estimation of time-varying quantiles, and derive their joint distribution. Next, in subsection 1.2 we discuss the estimation of the joint probabilities and their asymptotic properties.

1.1 Time-varying Regression Quantiles

Let $q_t^Y(\beta_{i,Y})$ denote the empirical specification for the $q_{i,t}^Y$ time-varying quantile conditional on Ω_t, where $\beta_{i,Y}$ denotes the p-vector of parameters to be estimated. Let $\rho_i(\lambda) \equiv [\theta_i - I(\lambda \leq 0)]\lambda$, $\lambda \in \mathbb{R}$, be a piecewise linear "check function," where $I(\cdot)$ denotes an indicator function that takes on value one if the expression in parenthesis is true and zero otherwise. The unknown parameters of the quantile specification can be consistently estimated by solving the following minimization problem (Koenker and Bassett, 1978):

$$\min_{\beta_{i,Y}} T^{-1} \sum_{t=1}^T \rho_i\left(y_t - q_t^Y(\beta_{i,Y})\right) \equiv T^{-1} \sum_{t=1}^T [\theta_i - I(y_t \leq q_t^Y(\beta_{i,Y}))] \cdot [y_t - q_t^Y(\beta_{i,Y})] \quad (1)$$

where T denotes the sample size. Regression quantile is a generalization of the least absolute deviation model: the objective function attaches asymmetric weights to deviations from the quantile, depending on whether the realization y_t falls below or above the quantile. When $\theta_i = 0.5$ the weights become symmetric, and the solution to the problem gives the median estimator. Engle and Manganelli (2004) provide sufficient conditions for consistency and asymptotic normality results.

For the purpose of the present study, we need to derive the joint distribution of the regression quantile estimators of the two time series, y_t and x_t. Define:

$$\underset{p \times p}{D_i^Y} \equiv E[T^{-1} \sum_{t=1}^{T} h_{i,t}^Y(0) \nabla q_t^Y(\beta_{i,Y}^0) \nabla' q_t^Y(\beta_{i,Y}^0)]$$

where $h_{i,t}^Y(0)$ is the value at zero of the density of $\varepsilon_{i,t}^Y \equiv y_t - q_t^Y(\beta_{i,Y}^0)$ and $\nabla q_t^Y(\beta_{i,Y}^0)$ is the gradient of the quantile function evaluated at the true parameter $\beta_{i,Y}^0$. In words, D_i^Y is the average cross-product of the derivative of the quantile function, weighted by conditional density of y_t at its θ_i-quantile. Let $\psi_{i,t}^Y(\beta_{i,Y})$ denote the first derivative of $\rho_i(y_t - q_t^Y(\beta_{i,Y}))$:

$$\underset{p \times 1}{\psi_{i,t}^Y(\beta_{i,Y}^0)} \equiv [\theta_i - I(y_t \le q_t^Y(\beta_{i,Y}^0))] \nabla q_t^Y(\beta_{i,Y}^0).$$

Next, let $\beta_Y \equiv [\beta_{i,Y}]_{i=1}^m$ denote the pm-vector stacking the $\beta_{i,Y}$ regression quantile parameters, $D^Y \equiv diag([D_i^Y]_{i=1}^m)$ the $(pm \times pm)$ block diagonal matrix with the matrices D_i^Y along the main diagonal, $\mathbf{O}_{pm}$ the $(pm \times pm)$ matrix of zeros, and $\psi_t^Y(\beta_Y^0) \equiv [\psi_{i,t}^Y(\beta_{i,Y}^0)]_{i=1}^m$ the pm-vector stacking all the $\psi_{i,t}^Y(\beta_{i,Y}^0)$.[4]

Consider analogous terms for x_t and finally define:

$$\underset{2pm \times 1}{\beta} \equiv [\beta_Y', \beta_X']',$$

$$\underset{2pm \times 2pm}{D} \equiv \begin{bmatrix} D^Y & \mathbf{O}_{pm} \\ \mathbf{O}_{pm} & D^X \end{bmatrix}, \tag{2}$$

$$\underset{2pm \times 1}{\psi_t(\beta^0)} \equiv [\psi_t^Y(\beta_Y^0)', \psi_t^X(\beta_X^0)']'. \tag{3}$$

Note that the block diagonality condition of D^Y, D^X and D is required to ensure that estimation can be carried out independently for each quantile and each random variable. For a more general model where quantiles of one random variables can depend on the quantiles of the same and other variables see White, Kim, and Manganelli (2008, 2012).

The following corollary derives the joint asymptotic distribution of the regression quantile estimators.

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[4]When multiple quantiles are jointly estimated, some estimated quantile functions can cross each other, which is known as the "quantile crossing" problem. If the quantile model is correctly specified, then the population quantile functions are monotonic and the estimated quantile functions will converge to the corresponding population quantile functions. Hence, the quantile crossing problem is simply a finite sample problem in such a case, and should be negligible when the sample size is sufficiently large. If either the quantile model is misspecified or the sample size is not large enough, then the quantile crossing problem can still be of concern. In that case, one can use some recently developed techniques to correct the problem such as the monotonization method by Chernozhukov, Fernandez-Val, and Galichon (2010) or the isotonization method suggested by Mammen (1991).

Corollary 1: *Under assumptions C0–C7 and AN1-AN4 in Appendix A,* $\sqrt{T}A^{-1/2}D(\hat{\beta}-\beta^0)\overset{d}{\to}N(0,I)$, *where* $\hat{\beta}$ *is the vector containing the solutions to* (1) *and* $A\equiv E\left[T^{-1}\sum_{t=1}^{T}\psi_t(\beta^0)\psi_t(\beta^0)'\right]$.

Engle and Manganelli (2004) provide asymptotically consistent estimators of the variance–covariance matrix (see their theorem 3).

1.2 Estimation of the Conditional Probability of Comovement

It is possible to estimate the average probability of comovement between y_t and x_t and test whether it changes across time periods, by running the following regression:[5]

$$I_t^Y(\hat{\beta}_{i,Y})\cdot I_t^X(\hat{\beta}_{j,X})=W_t\alpha_{ij}^0+\varepsilon_t, \qquad i,j=1,...,m, \tag{4}$$

where $I_t^Y(\beta_{i,Y})\equiv I\left(y_t\leq q_t^Y(\beta_{i,Y})\right)$, $I_t^X(\beta_{j,X})$ is defined analogously, $W_t\equiv[1,S_t]$, S_t is an $(s-1)$ row vector of time dummies, and α_{ij}^0 a $(s,1)$ vector of unknown coefficients.[6]

Let $\hat{\alpha}_{ij}$ be the OLS estimator of (4) and denote with $\hat{\alpha}_{l,ij}$ the $(l+1)$-th element of this vector, $l=0,1,...,s-1$. Analogously, let $S_{l,t}$ denote the l-th element of S_t. Let C_l be the number of observations identified by the dummies $\{S_{l,t}=1,S_t^{-l}=\mathbf{0}\}_{t=1}^{T}$, where S_t^{-l} represents the vector S_t without its l-th element and $\mathbf{0}$ is a vector of zeros of appropriate dimension. Define also $\bar{F}_l^{ij}\equiv C_l^{-1}\sum_{t\in\{t:S_{l,t}=1,S_t^{-l}=\mathbf{0}\}}F_t(q_t^Y(\beta_{i,Y}^0),q_t^X(\beta_{j,X}^0)).$[7]

The following theorem shows that $\hat{\alpha}_{ij}$ is a consistent estimator of the average probabilities of comovements in the time periods defined by the dummies.

Theorem 1: *(Consistency)* - *Assume that* $C_l/T\overset{T\to\infty}{\longrightarrow}k_l$, *where* $k_l\in(0,1)$, $l=0,...,s-1$, *is the asymptotic ratio between the number of observations identified by the* l^{th} *dummy* (C_l) *and the total number* (T) *of observations. Under the same assumptions of Corollary 1,*

$$\hat{\alpha}_{0,ij}\overset{p}{\to}plim(\bar{F}_0^{ij}) \qquad i,j=1,...,m,$$

$$\left[\hat{\alpha}_{0,ij}+\hat{\alpha}_{l,ij}\right]\overset{p}{\to}plim(\bar{F}_l^{ij}) \qquad i,j=1,...,m,\ and\ l=1,...,s-1.$$

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[5] Asymptotically, the dependent variable can be constructed with any n-tuple and not just for two variables, as long as T grows at a faster rate than n. In practice, one quickly encounters curse of dimensionality problems, as the number of periods in which all indicator variables are simultaneously equal to one diminishes rapidly as one includes more than two variables. The implication is that the precision of the estimator would deteriorate very quickly.

[6] An interesting extension of this model is to adopt a probit/logit framework with continuous regressors. Such a model could be useful for instance to forecast probabilities of comovement in real time. The interest of the present study, however, is in testing whether (average) comovements have changed over specific periods of time.

[7] We denote with C_0 the number of observations in the benchmark period, i.e., the periods $t\in\{t:S_t=\mathbf{0}\}$. $\bar{F}_0^{ij}$ is correspondingly defined as the average cdf in the benchmark period.

$\hat{\alpha}_{0,ij}$ is the parameter associated with the constant and, as such, it converges to the average probability of comovement in the benchmark period (i.e., the period when all other dummies are equal to zeros, $t \in \{t : S_t = 0\}$). Similarly, since $\hat{\alpha}_{l,ij}$ for $l = 1, ..., s-1$ is the coefficient of the l-th dummy $S_{l,t}$, the sum of $\hat{\alpha}_{0,ij} + \hat{\alpha}_{l,ij}$ converges in probability to the average probability of comovement in the period corresponding to the dummy. According to this theorem, testing for a change in the conditional probability of comovement in the periods identified by the dummy $S_{l,t}$ is equivalent to testing for the null that $\alpha_{l,ij}$ is equal to zero. Indeed, it is only when $\alpha_{l,ij} = 0$ that there is no change in probabilities of comovement relative to the benchmark period. Otherwise, if $\alpha_{l,ij}$ is less than zero, the probability over the l-th dummy period will be lower than the probability during the benchmark period, while if $\alpha_{l,ij}$ is greater than zero, the probability will be higher.

To obtain the asymptotic distribution of this estimator, define first the following terms:

$$\underset{s \times 1}{g_t(\beta_{i,Y}, \beta_{j,X})} \equiv W_t' I_t^Y(\beta_{i,Y}) \cdot I_t^X(\beta_{j,X}) - T^{-1}(W'W)\alpha_{ij}^0,$$

where $W \equiv [W_t]_{t=1}^T$ is a $(T \times s)$ matrix containing all the vectors of dummies from regression (4), and

$$\underset{s \times 2pm}{G_{ij}} \equiv E\left\{ T^{-1} \sum_{t=1}^T W_t' \left[\nabla_\beta' q_t^X(\beta_{j,X}^0) \int_{-\infty}^0 h_{ij,t}(\eta, 0)d\eta + \right.\right.$$
$$\left.\left. + \nabla_\beta' q_t^Y(\beta_{i,Y}^0) \int_{-\infty}^0 h_{ij,t}(0, \upsilon)d\upsilon \right] \right\},$$

where $h_{ij,t}(\eta, \upsilon)$ is the joint pdf of $(y_t - q_t^Y(\beta_{i,Y}^0), x_t - q_t^X(\beta_{j,X}^0))$, and ∇_β denotes the derivative with respect to the $2pm$-vector β. Next let $g_t(\beta^0) \equiv [g_t(\beta_{i,Y}^0, \beta_{j,X}^0)]_{i,j=1}^m$ be the sm^2-vector stacking all the m^2 possible vectors $g_t(\beta_{i,Y}, \beta_{j,X})$, and construct the $(sm^2 \times T)$ matrix $R \equiv [g_t(\beta^0)]_{t=1}^T$. Define also $G \equiv [G_{ij}]_{i,j=1}^m$, an $(sm^2 \times 2pm)$ matrix stacking all the G_{ij} matrices, and $\Psi \equiv [\psi_t(\beta^0)]_{t=1}^T$, a $(2pm \times T)$ matrix where $\psi_t(\beta^0)$ was defined in (3).

Finally, let $\alpha^0 \equiv [\alpha_{11}^{0\prime}, \alpha_{12}^{0\prime}, ..., \alpha_{m(m-1)}^{0\prime}, \alpha_{mm}^{0\prime}]'$ be the sm^2-vector of true unknown parameters to be estimated in (4). Similarly, define $\hat{\alpha} \equiv [\hat{\alpha}_{11}', \hat{\alpha}_{12}', ..., \hat{\alpha}_{m(m-1)}', \hat{\alpha}_{mm}']'$.

Theorem 2: *(Asymptotic Normality) - Under the assumptions of Corollary 1 and AN5 (see Appendix A),*

$$\sqrt{T}M^{-1/2}Q\left(\hat{\alpha} - \alpha^0\right) \xrightarrow{d} N(0, J_{sm^2}),$$

where

$$\underset{sm^2 \times sm^2}{M} \equiv E[T^{-1}(R+GD^{-1}\Psi)(R+GD^{-1}\Psi)'],$$

$$\underset{sm^2 \times sm^2}{Q} \equiv J_{m^2} \otimes (T^{-1}W'W),$$

J_r is the identity matrix of dimension r (where r is a positive integer) and D is defined in (2).

This result is new in the regression quantile literature. Without the correction term $GD^{-1}\Psi$ in the matrix M, we would get the standard OLS variance–covariance matrix. The correction is needed in order to account for the estimated regression quantile parameters that enter the OLS regression. This correction term is similar to the one derived by Engle and Manganelli (2004) for the in-sample Dynamic Quantile test. The main difference is related to the composition of the matrix G. Since two different random variables (x_t and y_t) enter the regression, G contains the terms $\int_{-\infty}^0 h_{ij,t}(\eta,0)d\eta$ and $\int_{-\infty}^0 h_{ij,t}(0,\upsilon)d\upsilon$, which can be interpreted as the bivariate analogue of the height of the density function of the quantile residuals evaluated at zero that typically appears in standard errors of regression quantiles.

The variance–covariance matrix can be consistently estimated using plug-in estimators. The only non-standard term is G, whose estimator is provided by the following theorem.

Theorem 3: *(Variance–Covariance Estimation) - Under the same assumptions of Theorem 2 and assumptions VC1-VC3 in Appendix A, $\hat{G}_{ij} \xrightarrow{p} G_{ij}$, where*

$$\hat{G}_{ij} \equiv (2T\hat{c}_T)^{-1} \sum_{t=1}^T \Big\{ I(|x_t - q_t^X(\hat{\beta}_{j,X})| < \hat{c}_T)I(y_t < q_t^Y(\hat{\beta}_{i,Y}))W_t' \nabla_\beta' q_t^X(\hat{\beta}_{j,X})$$

$$+ I(|y_t - q_t^Y(\hat{\beta}_{i,Y})| < \hat{c}_T)I(x_t < q_t^X(\hat{\beta}_{j,X}))W_t' \nabla_\beta' q_t^Y(\hat{\beta}_{i,Y}) \Big\}$$

and $\hat{c}_T$ is defined in assumption VC1.

1.2.1 Hypothesis testing. Using theorems (2) and (3), a test of linear restrictions on the estimated probability of comovement can be easily constructed.

Corollary 2: *Suppose that α is subject to the u ($\leq sm^2$) linearly independent restrictions $U\alpha^0 = b$, where U is an (u,sm^2) matrix of rank u and b is an u-vector. Under the assumptions of Theorem 3*

$$\sqrt{T}(UQ^{-1}\hat{M}Q^{-1}U')^{-1/2}(U\hat{\alpha} - b) \xrightarrow{d} N(0, J_u),$$

which can be equivalently restated as a Wald test

$$T(U\hat{\alpha} - b)'(UQ^{-1}\hat{M}Q^{-1}U')^{-1}(U\hat{\alpha} - b) \xrightarrow{d} \chi^2(u),$$

where the ˆ indicates estimated quantities.

This result is useful to test for changes in the average probability of comovement. For example, one could be interested in testing whether comovements differ in the upper tail relative to the lower tail, or whether comovements changed in the test period with respect to the benchmark period.

2 MEASURING CHANGES IN FINANCIAL COMOVEMENTS

In this section we show how the econometric model previously developed can be used to test for significant changes in financial comovements.

When $\theta_i = \theta_j$, the comovement between two random variables can be conveniently represented as follows. Let $F_t^-(\theta_i) \equiv \Pr(y_t \leq q_t^Y(\beta_{i,Y}) | x_t \leq q_t^X(\beta_{i,X})) = \theta_i^{-1}\Pr(y_t \leq q_t^Y(\beta_{i,Y}), x_t \leq q_t^X(\beta_{i,X}))$ and $F_t^+(\theta_i) \equiv (1 - \theta_i)^{-1}\Pr(y_t \geq q_t^Y(\beta_{i,Y}), x_t \geq q_t^X(\beta_{i,X}))$. Moreover, define the following conditional probability:[8,9]

$$p_t(\theta_i) \equiv \begin{cases} F_t^-(\theta_i) & \text{if } \theta_i \leq 0.5 \\ F_t^+(\theta_i) & \text{if } \theta_i > 0.5 \end{cases}.$$

Hence, $p_t(\theta_i)$ is the likelihood of a tail event for random variables y, given a tail event occurred for x.

Consider regression (4), where the first dummy variable $S_{1,t}$ denotes crisis times, while the other dummies represent suitably chosen control variables. Using the notation of subsection 1.2, the probability of comovements in crisis times is given by $\bar{p}_1(\theta_i) \equiv C_1^{-1} \sum_{t \in \{t: S_{1,t} = 1, S_t^{-1} = 0\}} p_t(\theta_i)$, while the probability of comovements in tranquil times is $\bar{p}_0(\theta_i) \equiv C_0^{-1} \sum_{t \in \{t: S_t = 0\}} p_t(\theta_i)$, where C_1 and C_0 denote the number of observations during crisis and tranquil times, respectively. We adopt the following working definition of market comovements:

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[8] We could plot both $F_t^-(\theta_i)$ and $F_t^+(\theta_i)$ for the whole range of θ_i between 0 and 1, $0 \leq \theta_i \leq 1$. However, as $\theta_i \to 1$, $F_t^-(\theta_i) \to 1$ and as $\theta_i \to 0$, $F_t^+(\theta_i) \to 1$. The interesting information about the comovements of x_t and y_t can be obtained by plotting $F_t^-(\theta_i)$ for $\theta \leq 0.5$ and by $F_t^+(\theta_i)$ for $\theta > 0.5$.

[9] For hedging purposes, we would be interested in the likelihood that the hedge asset returns are high when the returns on the asset to be hedged are low. We would define $F_t^{+-}(\theta_i, \theta_j) \equiv \Pr(y_t \geq q_t^Y(\beta_{i,Y}) | x_t \leq q_t^X(\beta_{j,X}))$ and $F_t^{-+}(\theta_i, \theta_j) \equiv \Pr(y_t \leq q_t^Y(\beta_{i,Y}) | x_t \geq q_t^X(\beta_{j,X}))$.

In addition, one could include a time dimension to test whether the distribution of returns in one country leads or lags the distribution of returns in another country. This can be easily accommodated in our framework by analyzing the behaviour of $\Pr(y_t \leq q_t^Y(\beta_{i,Y}) | x_{t-k} \leq q_{t-k}^X(\beta_{i,X}))$ with $k > 0$ to analyze spillovers from country X to country Y and with $k < 0$ to analyze spillovers from country Y to country X.

Definition 1: *(Changes in comovements with respect to the information set Ω^S) - Let $\Omega^S \equiv \{S_{2,t}, \ldots, S_{s-1,t}\}_{t=1}^T$ denote the information set represented by the control dummies in regression (4). Comovements with respect to Ω^S will be increasing in a given interval $(\underline{\theta}, \bar{\theta})$ if $\delta(\underline{\theta}, \bar{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}} [\bar{p}_1(\theta) - \bar{p}_0(\theta)] d\theta > 0$.*

Note that $\Omega^S = \{W_t\}_{t=1}^Y \backslash \{1 \cup \{S_{1,t}\}_{t=1}^T\}$, that is it excludes the constant and the first time dummy which identifies the crisis period. $\delta(\underline{\theta}, \bar{\theta})$ measures the area between the average conditional probabilities $\bar{p}_1$ and $\bar{p}_0$ over the interval $(\underline{\theta}, \bar{\theta})$. Unlike correlation-based measures, $\delta(\underline{\theta}, \bar{\theta})$ permits us to analyze changes in dependence over specific quantile ranges of the distribution. For instance, it may occur that $\delta(0, 1)$ is small just because of positive dependence on the left tail of the distribution and negative on the right tail, so that the two values tend to offset each other. Note that the above definition with an empty information set ($\Omega^S = \varnothing$) coincides with the concept of contagion used in the international finance literature (see, for instance, Eichengreen, Rose, and Wyplosz, 1996, or Forbes and Rigobon, 2002).[10]

The characteristics of $p_t(\theta_i)$ can be conveniently analyzed in what we call the "comovement box" (see Figure 1). The comovement box is a square with unit side, where $p_t(\theta_i)$ is plotted against θ_i. The shape of $p_t(\theta_i)$ will generally depend on the characteristics of the joint distribution of the random variables x_t and y_t, and therefore for generic distributions it can be derived only by numerical simulation. There are, however, three important special cases that do not require any simulation: 1) perfect positive correlation, 2) independence, and 3) perfect negative correlation. If two markets are independent, implying $\rho_{YX} = 0$, $p_t(\theta_i)$ will be piece-wise linear, with slope equal to one, for $\theta_i \in (0, 0.5)$, and slope equal to minus one, for $\theta_i \in (0.5, 1)$. When there is perfect positive correlation between x_t and y_t (i.e., $\rho_{YX} = 1$), $p_t(\theta_i)$ is a flat line that takes on unit value. Under this scenario, the two markets essentially reduce to one. The polar case occurs for perfect negative correlation, i.e. $\rho_{YX} = -1$. In this case $p_t(\theta_i)$ is always equal to zero: when the realization of y_t is in the lower tail of its distribution, the realization of x_t is always in the upper tail of its own distribution and conversely.

In general, the higher the comovement between two random variables, the higher $p_t(\theta_i)$.

2.1 Relation with Previous Literature

The estimation of the probability of comovement in equation (4) echoes the literature on performance evaluation of investment advisers (see, for instance, Cumby and Modest, 1987). Cumby and Modest (1987) construct a test similar to

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[10]The World Bank's "very restrictive" definition states that "contagion occurs when cross-country correlations increase during 'crisis' times relative to correlations during 'tranquil' times." See http://www1. worldbank.org/economicpolicy/managing%20volatility/contagion/definitions.html.
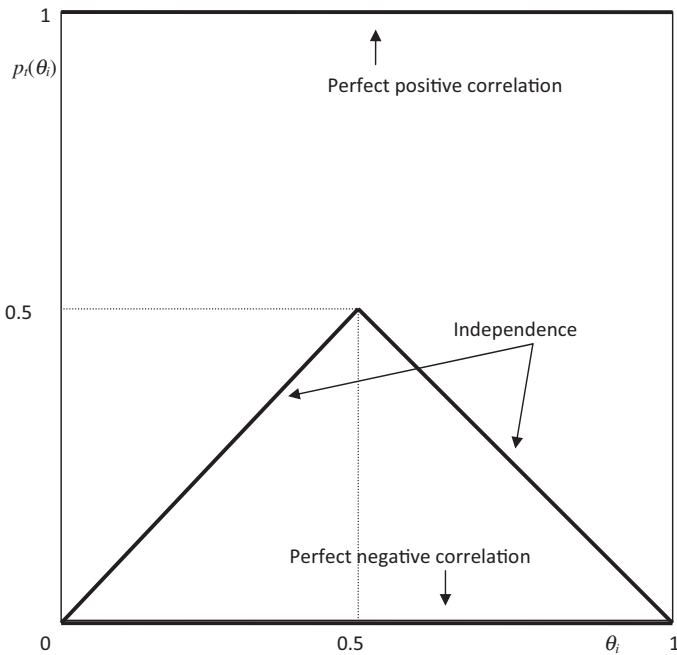
Figure 1 The comovement box. This figure plots the average probability that a random variable y_t falls below (above) its θ_i-quantile conditional on another random variable x_t being below (above) its θ_i-quantile, for $\theta_i < 0.5$ ($\theta_i \geq 0.5$). The case of perfect positive correlation, independence, and perfect negative correlation are represented.

the test carried out in Equation (4) in the sense that they first transform investment recommendations and performance into dichotomous variables and next run regressions where both the dependent and independent variables are indicator functions. If the recommendation has value, it should enter with a positive coefficient in a regression predicting future outperformance. One key difference of our approach is that it is a two-step procedure which defines the indicator variables in terms of exceedances vis-à-vis conditional quantiles estimated in the first step.

Our framework can take into account heteroskedasticity which plagues correlation measures. Previous research (see, for instance, Longin and Solnik, 1995; Karolyi and Stulz, 1996; De Santis and Gérard, 1997; Ang and Bekaert, 2002) suggests that correlation increases when returns are large in absolute value, and in particular over bear markets. However, as pointed out by Longin and Solnik (2001), Forbes and Rigobon (2002), and Ball and Torous (2005), among others, the difference in estimated correlation between volatile and tranquil periods could be spurious and due to heteroskedasticity. By modeling conditional probabilities with regression quantiles, our approach is robust to this time-varying volatility issues (provided the quantile model is well specified).

We can describe existing contributions to the comovement/contagion literature in terms of the comovement box. First, our approach has direct ties with Extreme Value Theory (EVT). Indeed, $\lim_{\theta_i \to 0} p_t(\theta_i)$ is exactly the definition of "tail dependence" for the lower tail used in the EVT literature (similar result holds for the upper tail). Existing contributions (e.g., Longin and Solnik, 2001; Hartmann, Straetmans, and de Vries, 2004) differ from ours on two important aspects. First, they only consider the distribution beyond an (extreme) threshold. Second, in the light of Definition 1, they do not compare this distribution to some benchmark against which contagion can be measured. Moreover, it is not obvious how these approaches can be modified to control for economic variables.

Our methodology is also close to the logit/probit literature (e.g., Eichengreen, Rose, and Wyplosz, 1996; Bae, Karolyi, and Stulz, 2003). The value of $p_t(\theta_i)$ can in principle be estimated through the logit/probit approach. The main issue with this methodology is that, differently from our approach, the probability of contagion is computed with respect to specific points of the distribution support. Moreover, this approach adopts a two-step procedure and it is not obvious how correct inference can be made.

A third strand of the literature uses copula methods to study dependence structure between markets (see, for instance, Rodriguez, 2003; Patton, 2004; Chollette and Heinen, 2006; Garcia and Tsafack, 2011). Loosely speaking, a copula is a multivariate distribution function which relates univariate marginal distributions to form a joint distribution. Empirically, this approach is often implemented using a single parameter to determine the shape of the copula.[11] Furthermore, one can either allow for flexible time variation in the copula parameter to capture changes in the dependence structure while fixing the univariate marginals (Patton, 2004), or one can accommodate volatility regimes while limiting the variation in the copula (Rodriguez, 2003; Chollette and Heinen, 2006).

3 DATA

The empirical analysis is carried out on returns on equity indices for four Latin American countries, Brazil, Mexico, Chile, and Argentina. We choose these equity markets for two reasons. First, they are considered to be emerging markets and therefore believed to be less robust to external shocks than fully developed economies. Second, the four equity markets are open over the same hours during the day. Hence the daily returns we investigate are synchronous, avoiding the confounding effects that nonsynchronous returns can have on the measurement of comovements (see Martens and Poon, 2001; Sander and Kleinmeier, 2003).[12] Equity

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[11] Patton (2006) provides an example of a two-parameter copula. A number of authors use mixtures of copulas, which require the estimation of three or more parameters (see, for instance, Chollete, de la Peña, and Lu, 2005).

[12] Our empirical investigation does not include other Latin American countries such as Colombia and Peru since we choose those economies with the largest market capitalisation value, in U.S. dollars. While Brazil,

returns are continuously compounded and computed from Morgan Stanley Capital International (MSCI) world indices in local currency, which are market-value-weighted and do not include dividends.[13] The data set covers the period from December 31, 1987 to September 3, 2012 for a total of more than 5500 days in which at least one of the markets is open. Although the four equity markets in our sample are almost always open simultaneously, there are instances in which markets are closed in one country and open in the other, as national holidays and administrative closures do not fully coincide. To adjust for these non-simultaneous closures, for each pair of countries, we include only the returns for the days on which both markets were open that day and had been open the day before.[14]

Descriptive statistics for the asset data and the sample characteristics are given in Table 1. In Panel A the overall sample univariate statistics are reported. There is strong evidence of excess skewness and leptokurtosis at 1% significance level, a clear sign of nonnormality. This is confirmed by the Jarque–Bera test for normality. The second part of Panel A reports, for each pair of countries, sample correlations on the first line and sample size on the second line. When considering each market individually (diagonal elements), we have a maximum of 5647 valid daily returns for Mexico and Chile, and a minimum of 5568 returns for Brazil. Bivariate sample sizes instead vary from a maximum of 5647 for Mexico and Chile to a minimum of 5532 for Brazil and Argentina.

We use the definitions of Forbes and Rigobon (2002) to determine the timing of period of financial market stress. In our sample, they cover three subperiods: November 1, 1994 to March 31, 1995 (Tequila crisis); June 2, 1997 to December 31, 1997 (Asian crisis); and August 3, 1998 to December 31, 1998 (Russian crises). In addition, since we have a much longer sample than Forbes and Rigobon (2002), we define four more crisis: March 26, 2001 to May 15, 2001 (Argentinean crisis),[15] February 15,

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Chile, and Mexico exhibit the highest market capitalisation value throughout the sample, Argentina ranks fourth most of the times. Approximately the same outcome holds true when the market capitalisation is expressed in percentage of GDP. In addition to these considerations, Colombia and Peru equity indices are available only since 1993. We do not include in our analysis other emerging market economies, such as those of East Asia, since these markets are not open simultaneously to Latin American markets.

[13]We use equity indices denominated in domestic currencies to remove the impact related to exchange rate changes. For example, if one converts domestic indices into U.S. dollars, the compounded returns will include changes in the exchange rate: $X_t \equiv \ln\left(\frac{P_t E_t}{P_{t-1} E_{t-1}}\right) = (\ln P_t - \ln P_{t-1}) + (\ln E_t - \ln E_{t-1}) = x_t + e_t$, where P_t is the equity index denominated in the domestic currency, E_t is the spot exchange rate, $x_t \equiv (\ln P_t - \ln P_{t-1})$ and $e_t \equiv (\ln E_t - \ln E_{t-1})$. Moreover, we use equity indices without dividends since we are not analyzing a portfolio choice where agents re-invest dividends. In any case, equity returns including dividends track very closely equity returns without dividends.

[14]We also implemented an alternate way to adjust for nonsimultaneous market closures. We retained the returns on the day after the market closure for the market that did close. However, since the return on the day after a market closure is in fact a multi–day return, we adjusted the returns on the market that did not close by cumulating the daily returns over the period the other market closed plus the day it reopened. Lastly we divided the two returns by the number of days of closure plus one. This procedure added between 10 and 25 observations to the different pairs and did not materially affect the results.

[15]These dates coincide respectively with the start and end dates of a series of lowering of Argentina's government debt rating by rating agencies (Moody's and S&P).

Table 1 Descriptive statistics of daily returns on stock market indices

Panel A: Overall sample—December 31, 1987—September 3, 2012

	Mexico	Brazil	Argentina	Chile
Summary statistics				
Mean	0.09	0.33	0.18	0.06
Minimum	−12.69	−21.74	−20.40	−7.36
Maximum	12.14	24.66	39.04	8.60
Std. Dev.	1.57	2.42	3.03	1.13
Autocorrelation	0.14	0.17	0.11	0.23
Skewness	0.06	0.27**	1.35**	−0.09
Kurtosis	8.17**	9.80**	16.30**	7.30**
J-B	6210.26**	10797.35**	43174.21**	4346.55**
Correlations and sample size				
Mexico	1.000	0.427	0.306	0.378
	5647	5568	5622	5647
Brazil		1.000	0.305	0.366
		5568	5532	5562
Argentina			1.000	0.277
			5625	5625
Chile				1.000
				5647

Panel B: tranquil days

Standard deviations, correlations, and sample size				
Mexico	1.457	0.359	0.238	0.313
	5214	5134	5188	5214
Brazil		2.245	0.215	0.266
		5134	5087	5128
Argentina			2.986	0.199
			5177	5177
Chile				1.036
				5214

Panel C: crisis days

Standard deviations, correlations, and sample size				
Mexico	2.491	0.692	0.707	0.609
	433	434	434	433
Brazil		3.904	0.815	0.713
		434	445	434
Argentina			3.473	0.708
			448	448
Chile				2.016
				433

This table reports the summary statistics of daily returns of the four country indices. Data are from MSCI and returns are continuously compounded. The significance level for excess skewness and excess kurtosis is based on test statistics developed by D'Agostino, Belanger, and D'Agostino (1990). The Jarque-Bera (J-B) test for normality combines excess skewness and kurtosis, and is asymptotically distributed as χ^2_m with m=2 degrees of freedom. $*$ and $**$ denote 5% and 1% significance levels, respectively.

2007 to March 30, 2007 (U.S. sub-prime crisis), September 1, 2008 to October 31, 2008 (Lehman bankruptcy),[16] and August 1, 2011 to September 30, 2011 (turbulences in the euro area).[17] For robustness, we have experimented with slightly larger sample sizes for the crisis (extending them by about one month), but results are qualitatively the same. The crisis sample includes almost 450 potential trading days. Excluding market closures and the subsequent day, we have a maximum of 448 valid crisis daily returns for Argentina and a minimum of 433 returns for Mexico and Chile. Panels B and C report univariate sample size and volatilities (diagonal elements) and bivariate sample size and correlations (off-diagonal elements) for both tranquil periods and those of financial market stress. What is striking from Panels B and C is that correlations increase dramatically between tranquil and crisis periods: the average correlation is approximately 0.26 over tranquil days and approximately 0.7 for days of turbulence. Based on this type of evidence traditional tests of correlation would have indicated the presence of contagion. However, the table also documents that for all countries, except Argentina, returns volatility increased dramatically in crisis over tranquil periods. This highlights the heteroskedasticity problem identified by Forbes and Rigobon (2002) and casts doubts on the reliability of the correlation-based evidence.

In the following section we investigate these issues with the comovement box and provide a more robust and nuance answer to the question.

4 AN APPLICATION TO LATIN AMERICA

In this section, we report the results of the methodology to the analysis of comovements across some Latin American equity markets. First, we discuss the conditional quantile estimation procedure (Section 4.1). Second, in Section 4.2 we estimate the probability of comovements over the whole sample period. To illustrate our methodology we compare these probabilities with those obtained from simulations of normal and Student-*t* distributions calibrated to match sample moments. We also compare them to alternative estimations based on copula models.

4.1 Modelling Individual Quantiles

We estimate the time-varying quantiles of the returns with the CAViaR model proposed by Engle and Manganelli (2004). We adopted the following CAViaR specification:

$$q_t^Y(\beta_{i,Y}) = \beta_{0,i,Y} + \beta_{1,i,Y}y_{t-1} + \beta_{2,i,Y}q_{t-1}^Y(\beta_{i,Y}) - \beta_{1,i,Y}\beta_{2,i,Y}y_{t-2} + \beta_{3,i,Y}|y_{t-1}|, \quad (5)$$

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[16]This period saw the collapse of Lehman, and the subsequent bailout of AIG and other major U.S. banks.
[17]During this period Greece (which had received the first bailout package), Portugal and Ireland were downgraded to junk status and markets were scared for Italy and Spain. In September Italy and Spain (government and banks) were downgraded several times.

Table 2 Estimates of the CAViaR specification (5) in the text

Parameters	$\theta = 0.1$		$\theta = 0.25$		$\theta = 0.75$		$\theta = 0.90$	
	Value	*s.e.*	*Value*	*s.e.*	*Value*	*s.e.*	*Value*	*s.e.*
Brazil								
b0	−0.112	0.007	−0.061	0.007	−0.002	0.003	0.036	0.004
b1	0.288	0.006	0.205	0.006	0.090	0.006	0.082	0.006
b2	0.870	0.005	0.877	0.011	0.902	0.004	0.864	0.003
b3	−0.107	0.004	−0.034	0.003	0.099	0.004	0.215	0.005
Argentina								
b0	−0.104	0.005	−0.032	0.004	−0.002	0.003	0.029	0.003
b1	0.183	0.005	0.109	0.005	0.022	0.005	0.022	0.005
b2	0.859	0.004	0.884	0.006	0.883	0.004	0.869	0.002
b3	−0.153	0.004	−0.067	0.003	0.104	0.003	0.200	0.004

where $\beta_{i,Y} \equiv [\beta_{0,i,Y}, \beta_{1,i,Y}, \beta_{2,i,Y}, \beta_{3,i,Y}]'$. A similar model has been estimated for x_t. To illustrate, in table 2 we report the coefficient estimates and the associated standard errors relative to equation (5) for some given θ_i-quantiles for Brazil and Argentina.

The rationale behind this parametrization lies in the autocorrelation (both in levels and squares) exhibited by our sample returns. This CAViaR model would be correctly specified for instance if the true DGP were as follows:

$$y_t = \gamma_0 + \gamma_1 y_{t-1} + \varepsilon_t, \qquad \text{where } \varepsilon_t = \sigma_t \eta_t, \ \eta_t \sim i.i.d.(0,1),$$
$$\sigma_t = \gamma_2 + \gamma_3 |y_{t-1}| + \gamma_4 \sigma_{t-1}.^{18}$$

To check whether the parametrization we propose is sensible, we carry out the in-sample Dynamic Quantile (DQ) test of Engle and Manganelli (2004). The DQ statistic tests the null hypothesis of no autocorrelation in the exceedances of the quantiles as correct specification would require. In theory the quantile of Y could depend on lagged values of X as well. We formally tested for this potential cross-dependence, by implementing the DQ test with 10 lags of the "hit" function for the both Y and X (the "hit" function takes on value $1-\theta$ if the variable exceeds the quantile and $-\theta$ otherwise; see Theorem 4 of Engle and Manganelli, 2004, for details). We report in figures 2A-2B the p-values of the DQ test statistic for the 19 estimated quantiles of Argentinian and Brazilian returns. For comparison, we show in the same picture the DQ test associated to the unconditional quantiles.

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[18]Even if the CAViaR model is misspecified in that it includes an explanatory variable too many to capture the returns autocorrelation in levels, it would still be consistent and the consequent efficiency loss is negligible due to our large data sample.
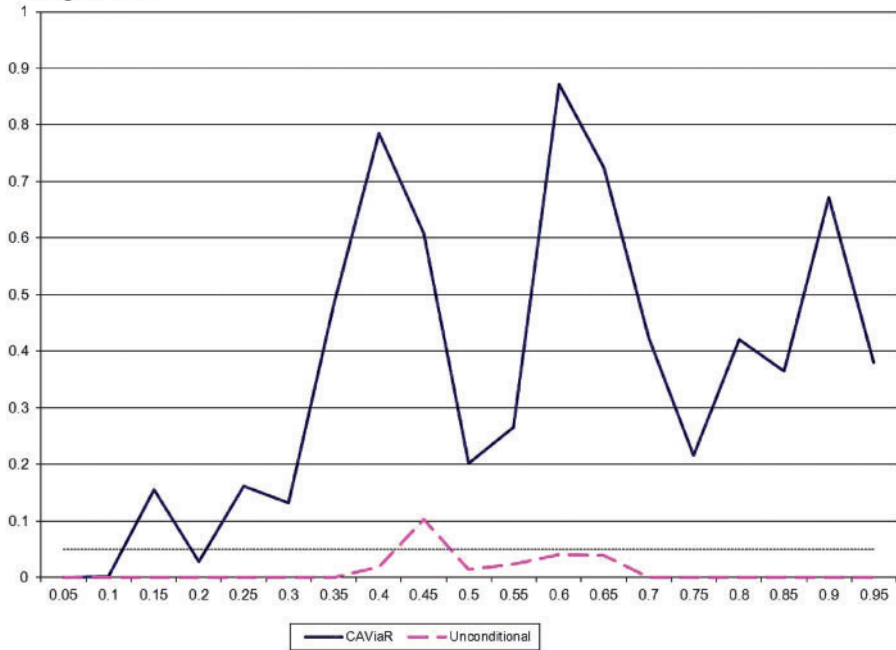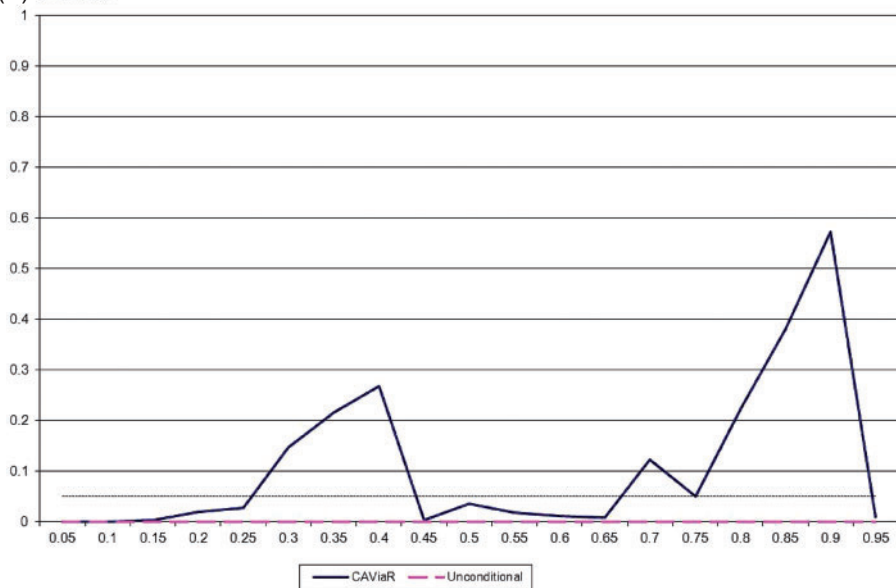
(a) *Argentina*

(b) *Brazil*

Figure 2A–2B P-values of the dynamic quantile test. These figures plot the p-values of the in-sample DQ test statistic of Engle and Manganelli (2004). The DQ statistic tests the null hypothesis of no autocorrelation and no lagged cross-correlation in the exceedances of the quantiles, as the correct specification would require.

Unconditional quantile specifications are rejected most of the times, while CAViaR models are not.[19]

4.2 Estimates of Comovements and Comparison with other Approaches

In this section, we estimate the probabilities of comovements over the whole sample period, i.e., $\bar{p}_T(\theta) \equiv T^{-1} \sum_{t=1}^{T} p_t(\theta)$. For comparison purposes, we simulate probabilities of comovements assuming that returns are distributed according to either a bivariate normal or a Student-t with five degrees of freedom, which accommodates fat tails. The distributions are calibrated using the unconditional correlations and volatilities of the relevant sample returns. We also estimate comovements using copula models with a semiparametric approach. We first constructed the marginal empirical CDF from the returns. In the second stage, the copula parameters are estimated via maximum likelihood. In Figure 3, we compare the estimated conditional probabilities following these different approaches.

Specifically, we estimated the equivalent of equation (4), where only a constant is included:

$$I_t^Y(\hat{\beta}_{i,Y}) \cdot I_t^X(\hat{\beta}_{i,X}) = \alpha_{0,ii} + \varepsilon_t, \qquad i = 1, ..., m. \tag{6}$$

Time-varying quantiles were estimated as described in the previous subsection. When estimating this probability we use the whole sample period, which includes both crisis and tranquil times. More importantly, no assumption about the distribution of returns is needed.

A visual comparison allows one to detect whether estimated probabilities deviate from what would be expected if the true data-generating process followed a normal or a Student$-t$ distribution. Take as an example the country pair Brazil–Argentina displayed in Figure 3A. For $\theta_i \leqslant 0.5$, that is, for returns below the median, the estimated conditional probabilities of comovements are significantly higher than those obtained from the simulation of normal and Student-t bivariate distributions. In particular, they seem to depart from the characteristics of tail independence that both t and normal distributions should exhibit. In contrast, for the right tail, i.e., for $\theta_i > 0.5$, the probability curve obtained with regression quantiles approximately coincides with the comovement probability generated by the simulation. If comovements were analyzed through simple correlation estimates, it would not be possible to detect this asymmetry between right and left tails of a distribution.[20]

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[19] While it is true that even some CAViaR specifications are rejected over some quantile ranges, they do significantly better than unconditional quantiles. It is not obvious how to address the issue of misspecification in this context. While simulation exercises would be able to address only specific forms of misspecification and cannot be easily generalized, they can help to shed some light on the behavior of the estimator proposed in this article. We leave this issue for future research.

[20] See, however, Ang and Chen (2002) and Hong, Tu, and Zhou (2007) for alternative tests of asymmetric correlations which go beyond simple correlation analysis.
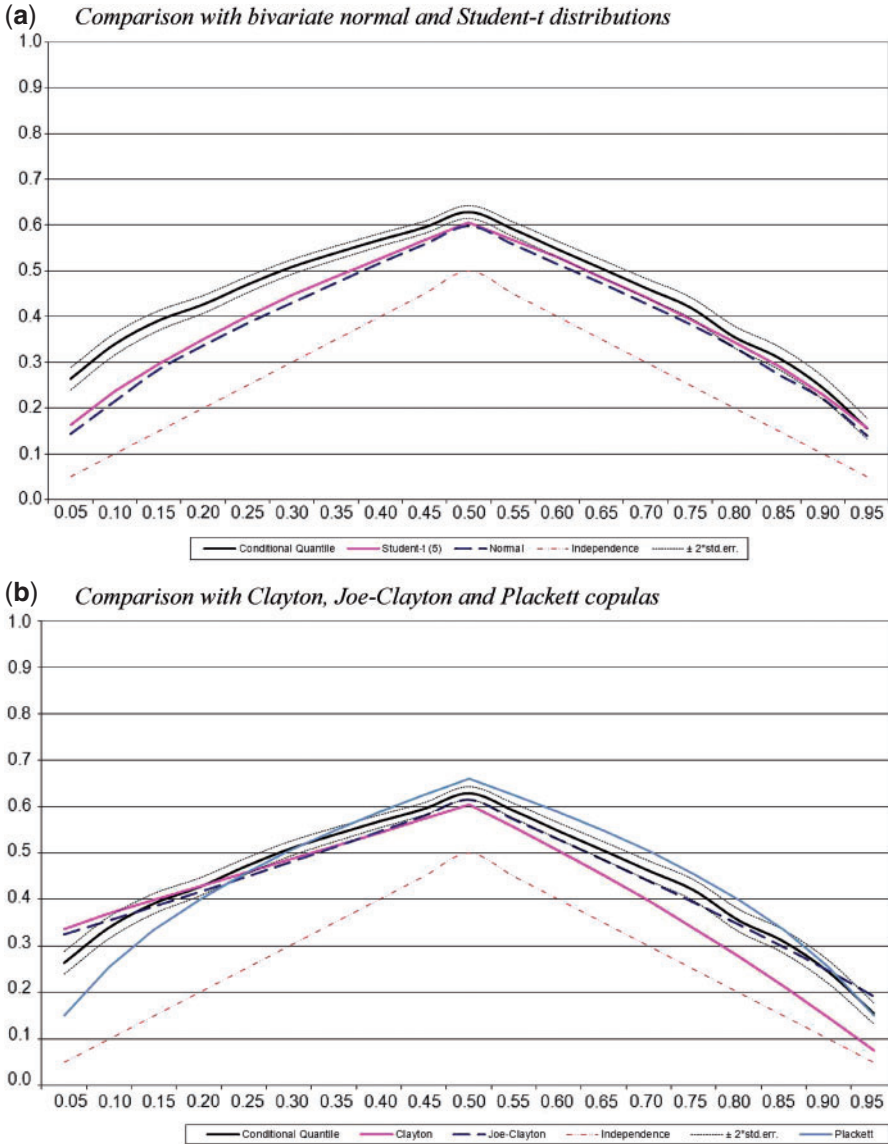
(a) *Comparison with bivariate normal and Student-t distributions*

(b) *Comparison with Clayton, Joe-Clayton and Plackett copulas*

Figure 3 Brazil-Argentina simulated and estimated tail dependence. The figure plots the estimated probability that the second country equity index returns falls below (above) its θ-quantile conditional on the first country index returns being below (above) its θ-quantile, for $\theta \leq 0.5$ ($\theta > 0.5$). The quantiles of each returns series are estimated using conditional quantile regressions. The dashed lines are the two (pointwise) standard error bounds for the estimated comovement. The estimated comovement is compared to a benchmark of independence and to simulated tail dependence based on a bivariate normal or a bivariate Student-t distribution with 5 degrees of freedom Figure 3A and a Clayton, Joe-Clayton, and Plackett copulas Figure 3b. The simulations are calibrated to match the sample volatilities and correlation of the returns series. Daily index returns are from MSCI for the period January 1, 1988 to September 3, 2012 ($n = 5532$).

The comparison with copula models reported in Figure 3B is also instructive. It illustrates how the choice of the copula significantly affects the degree of tail comovement. For instance, while the degree of comovement implied by the Joe–Clayton copula is consistent with that of the conditional quantile, the Clayton copula matches well only the left tail of the distribution, but significantly underestimates the amount of positive comovement. The Plackett copula is symmetric and shows analogous shortcomings to those highlighted with the normal and Student-t simulations.

Next, we briefly discuss the small sample performances of our quantile-based comovement estimator, and we compare our methodology with a multivariate GARCH approach as well as with regime switching models.

Remark 1: Small sample performance of quantile-based comovement estimator—In an application of the methodology developed in this paper, Cappiello, Kadareja, and Manganelli (2010) perform a Monte Carlo experiment to study the finite sample properties of the quantile-based comovement estimator. They find that with a smaller sample size than the one used here, the methodology is powerful enough to detect statistically significant changes in comovements between the test and benchmark periods. We refer the interested reader to that paper for details about the Monte Carlo experiment.

Remark 2: Comparison with multivariate GARCH—Cappiello et al. (2006) compare this methodology with multivariate GARCH estimations. The two methodologies are complementary in the sense that GARCH-based measures provide a short run picture of the correlation evolution, while regression quantile-based measures can be used to analyze changes in long run comovements. One of the strengths of our approach over correlation is that it allows us to detect different comovements between different parts of the distribution. See also the discussion in Garcia and Tsafack (2011) for the shortcomings of GARCH models in detecting asymmetric dependence.

Remark 3: Comparison with regime switching models—Differently from regime switching models, our methodology is not about forecasting periods of high or low comovements. It is rather about understanding the nature of comovement for a given identified period. This could in principle be extended to regime switching models, although we do not pursue this approach in this paper. For instance, let y_t and x_t denote the equity returns on two different markets. In line with Ang and Bekaert (2002) the mean equation for each equity return can be expressed in terms of a two state regime switching model:

$$y_t = \mu^y\left(s_t^y\right) + \sqrt{\sigma^y\left(s_t^y\right)}\varepsilon_t^y,$$

where the mean and variance depend on the regime s_t^y, which follows a two state Markov chain, i.e. $s_t^y = 1, 2$. An analogous equation can be written for x_t. In general

combining s_t^y and s_t^x generates $2^2 = 4$ states s_t. Next, one can compute the probability that returns on, say market Y fall below (or above) a given quantile, provided that returns on market X also fall below (or above) a given quantile and conditional on the states s_t:

$$\Pr\left(y_t \leq q_t^Y(\beta_{i,Y}) \Big| x_t \leq q_t^X(\beta_{i,X}), s_t\right),$$

or, equivalently,

$$\Pr\left(\varepsilon_t^y \leq \frac{q_t^Y(\beta_{i,Y}) - \mu^y}{\sqrt{\sigma^y}} \Bigg| \varepsilon_t^x \leq \frac{q_t^X(\beta_{i,X}) - \mu^x}{\sqrt{\sigma^x}}, s_t\right).$$

Conditional on each state s_t, Markov switching models assume that realizations in the upper and lower tail of the conditional distributions are driven by the same process.

5 IS THERE A CHANGE IN COMOVEMENTS DURING CRISIS TIMES?

In this section, we show how our methodology can be used to investigate if the probability of comovement increases over crisis times relative to tranquil periods for Brazil, Mexico, Chile, and Argentina. First, we estimate the conditional probabilities over tranquil and crisis times and provide tests for the difference in comovement between the two periods. In Section 5.2 we analyze to what extent the differences in comovements can be explained by differences in some control variables.

5.1 Tests of Changes in Comovements

We estimate the probabilities of comovement over crisis and tranquil times, using equation (4) with just the crisis dummies:

$$I_t^Y(\hat{\beta}_{i,Y}) \cdot I_t^X(\hat{\beta}_{i,X}) = \alpha_{0,ii} + \alpha_{1,ii}S_{1,t} + \varepsilon_t, \qquad i = 1, ..., m, \tag{7}$$

where $S_{1,t}$ represents the crisis dummy, constructed as discussed in section 4.

To estimate the individual quantiles underlying the indicator functions, we add the dummy variable $S_{1,t}$ to the CAViaR specification (5) to ensure that we have exactly the same proportion of quantile exceedances in both tranquil and crisis periods:[21]

$$q_t^Y(\beta_{i,Y}) = \beta_{0,i,Y} + \beta_{4,i,Y}S_{1,t} + \beta_{1,i,Y}y_{t-1} + \beta_{2,i,Y}q_{t-1}^Y(\beta_{i,Y})$$
$$- \beta_{1,i,Y}\beta_{2,i,Y}y_{t-2} + \beta_{3,i,Y}|y_{t-1}|.$$

For each market we estimate the model for 19 quantile probabilities ranging from 5% to 95%.

---

[21] Asymptotically, correct specification would imply the same number of exceedances in crisis and tranquil periods. However, in finite samples, this need not be the case. Failure to account for this fact would affect the estimation of the conditional probabilities.

Results are reported in Figure 4A–4F. Our approach permits us to explore how and if the probability of comovements changes for any interval in the support of the distribution. The attractiveness of inspecting all the quantiles lies in the fact that one does not need to arbitrarily specify a large absolute value return as a symptom of a crisis.

In Figure 4A–4F two solid lines are plotted together with the case of independence. The thin line indicates the conditional probability of comovements under the benchmark or, equivalently, over tranquil times. This line is the graphical representation of $\bar{p}_0(\theta)$ in Definition 1. The thick line, instead, shows the conditional probability of comovements during crisis times and plots $\bar{p}_1(\theta)$. The confidence bands associated to plus or minus twice the standard errors are plotted as dotted lines. When the bold line lies above the benchmark, this can be interpreted as evidence for increased comovements. When the two lines approximately coincide, there is no difference in comovements between the two periods. Finally, if the thick line lies below the benchmark, during crises times the comovements between two different markets actually decrease.[22]

The results for Argentina and Brazil (Figure 4A) show striking evidence of increased comovements for most quantiles. For the other country pairs (Figure 4B–F), we find similar significant increases in the probability of comovement in periods of financial turmoil relative to tranquil times, except for the extreme lower and upper parts of the distribution, where standard errors become wider due to the limited number of coexceedances.

It is worthwhile pointing out how, for Brazil and Argentina, the probability of comovement associated to the 10%-quantile jumps from about 30% in tranquil times to about 60% in crisis times. This implies that in quiet periods one should expect Brazilian and Argentinian equity returns to simultaneously fall below their 10%-quantile only one day out of three. In crisis periods, instead, almost two thirds of the days either country equity market experiences a 10% lower tail return, the other one will as well. Similar patterns characterize the other country pairs, although the increases in probabilities are not as large.

The interest may lie in testing whether specific parts of the distribution are subject to increases in comovements. Rigorous joint tests for increases in comovements which follow from the Definition 1 can be constructed as follows:

$$\widehat{\delta}(\underline{\theta}, \bar{\theta}) = (\#\theta)^{-1} \sum_{\theta_i \in [\underline{\theta}, \bar{\theta}]} [\bar{p}_1(\theta_i) - \bar{p}_0(\theta_i)] \tag{8}$$

$$= (\#\theta)^{-1} \sum_{\theta_i \in [\underline{\theta}, \bar{\theta}]} \hat{\alpha}_{1,ii},$$

---

[22]Notice that although the confidence bands are plotted around $\bar{p}_{C_1}(\theta)$, they represent pointwise confidence intervals for the estimated coefficients associated to the crisis dummy, $\hat{\alpha}_{1,ii}$. They could be equivalently plotted around the difference $\bar{p}_{C_1}(\theta) - \bar{p}_{C_0}(\theta)$ and check whether they lie above zero to test for statistical significance.
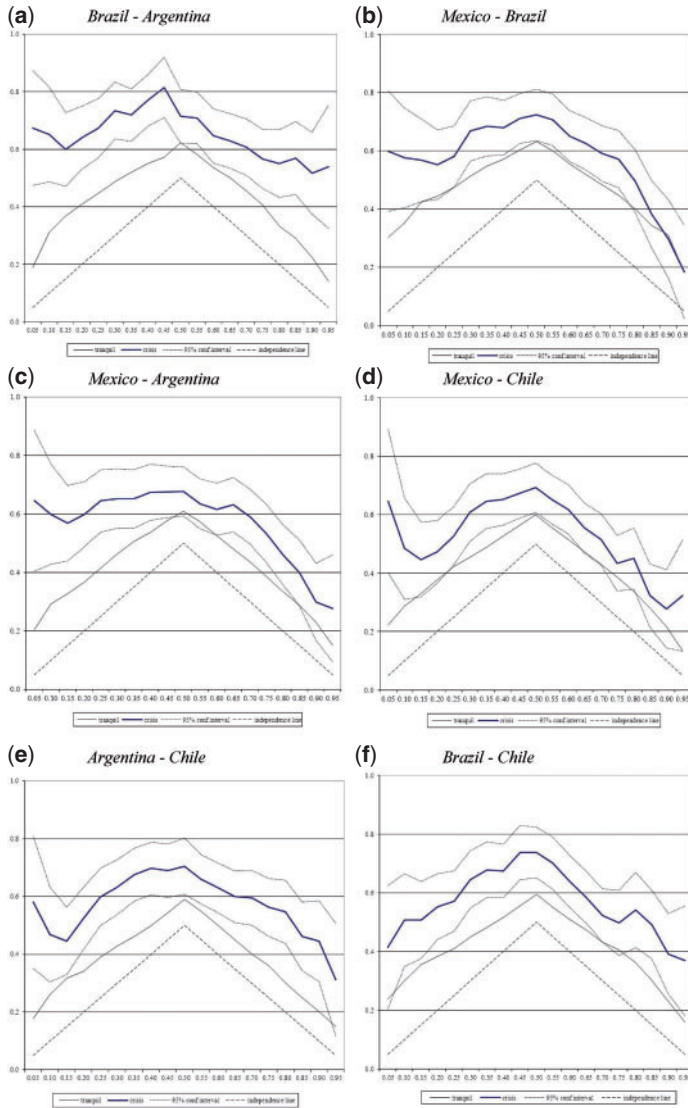
Figure 4 Estimated probability of comovements in crisis vs. tranquil periods. The figures plot the estimated probability that the second country equity index returns falls below (above) its θ-quantile conditional on the first country index returns being below (above) its θ-quantile for $\theta \leq 0.5(\theta > 0.5)$, in crisis and in tranquil periods. The quantiles of each returns series are estimated using conditional quantile regressions. The dashed lines are the two standard error bounds for the estimated co-exceedance likelihood in crisis periods. Daily index returns are from MSCI for the period January 1, 1988 to September 3, 2012 ($n_{Max} = 5647$, Mexico–Chile, $n_{Min} = 5532$, Brazil–Argentina). The crisis sample includes a maximum of 448 (min: 433) observations and cover the sub periods November 1, 1994 to March 31, 1995 (Tequila crisis), June 2, 1997 to December 31, 1997 (Asian crisis), August 3, 1998 to December 31, 1998 (Russian crisis), March 26, 2001 to May 15, 2001 (Argentinean crisis), February 15, 2007 to March 30, 2007 (U.S. subprime crisis), September 1, 2008 to October 31, 2008 (Lehman bankruptcy), and August 1, 2011 to September 30, 2011 (euro crisis).

Table 3 Test of difference in tail coexceedance between crisis and tranquil periods

Country pairs	Lower tail ($\theta_i \leq 0.5$)			Upper tail ($\theta_i \geq 0.5$)			
	$\hat{\delta}(0.05,0.50)$			$\hat{\delta}(0.50,0.95)$			
	Stat.	*s.e.*		*Stat.*	*s.e.*		
Mex.–Bra.	0.148	0.048		**0.076**	0.038		
Mex.–Arg.	0.209	0.050		0.111	0.040		
Mex.–Chi.	0.158	0.048		0.091	0.039		
Bra.–Arg.	0.252	0.046		0.196	0.046		
Bra.–Chi.	0.174	0.045		0.146	0.042		
Arg.–Chi.	0.201	0.046		0.177	0.043		
	$\hat{\delta}(0.05,0.25)$		$\hat{\delta}(0.25,0.50)$		$\hat{\delta}(0.50,0.75)$		$\hat{\delta}(0.75,0.95)$
	Stat.	*s.e.*	*Stat.*	*s.e.*	*Stat.*	*s.e.*	*Stat.*	*s.e.*
Mex.–Bra.	0.175	0.061	0.118	0.044	0.105	0.041	**0.050**	0.046
Mex.–Arg.	0.290	0.065	0.145	0.045	0.112	0.042	0.116	0.050
Mex.–Chi.	0.188	0.065	0.124	0.042	0.084	0.039	**0.091**	0.051
Bra.–Arg.	0.303	0.058	0.204	0.043	0.128	0.043	0.269	0.060
Bra.–Chi.	0.173	0.056	0.173	0.043	0.120	0.041	0.167	0.054
Arg.–Chi.	0.226	0.061	0.181	0.042	0.150	0.042	0.213	0.055

This table reports the average of $\hat{\alpha}_{1,ii}$ over θ_i, i.e. $\hat{\delta}(\underline{\theta},\bar{\theta}) = (\#\theta)^{-1}\sum_{\theta_i \in [\underline{\theta},\bar{\theta}]}\hat{\alpha}_{1,ii}$, as well as the associated standard errors. The resulting t statistics are obtained applying Corollary 2 and provide a joint test for changes in comovements which follows from Definition 1. Statistics indicated in bold are NOT significant at the 5% level.

where $\#\theta$ denotes the number of addends in the sum and $\hat{\alpha}_{1,ii}$ the OLS estimate of (7). Note that according to Definition 1, the information set with respect to which we measure changes in comovements is $\Omega^S = \varnothing$. We can therefore view the above test as a necessary condition for contagion: if $\widehat{\delta}(\underline{\theta},\bar{\theta})$ is not significantly different from zero, we can rule out the presence of contagion. On the other hand, rejection of the null hypothesis $\widehat{\delta}(\underline{\theta},\bar{\theta}) = 0$ signals that comovements do change between tranquil and crisis periods. It still leaves unanswered the question whether these changes in comovement can be explained by economic variables. We will present in the next subsection a test for changes in comovements with a nonempty information set.

For each country pair, Table 3 contains the standard errors associated with the sum of $\hat{\alpha}_{1,ii}$ over all the θ_is, computed using Corollary 2. It reports the test statistics computed over different intervals of θ.[23] For example for Brazil and Argentina, for $\theta_i < 0.5$, the average likelihood of observing a joint coexceedance is 25% higher in crisis times than in tranquil times. For comparison, in Table 4 we also report the correlation matrix computed using the Forbes and Rigobon (2002) heteroskedasticity correction.

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[23]Choosing a finer or coarser grid will result in different values for the test statistic. We have replicated the tests with a finer grid and the results are qualitatively the same.

Table 4 Correlation coefficients with the Forbes–Rigobon (2002) correction

Panel A: Unconditional correlations

	Brazil	Argentina	Chile
Mexico	*0.427*	*0.306*	*0.378*
	0.483	0.652	0.367
Brazil		*0.305*	*0.366*
		0.771	0.463
Argentina			*0.277*
			0.458

Panel B: one sided *t*-tests for Contagion (C) or No contagion (N)

	Brazil	Argentina	Chile
Mexico	N	C	N
	1.52	*12.54*	*−0.26*
Brazil		C	C
		24.18	*2.59*
Argentina			C
			4.85

This table compares the full sample correlation together with the correlation in the turmoil periods, adjusted for differences in heteroscedasticity using the formula suggested by Forbes and Rigobon (2002). Standard full sample correlations are reported in *italics* for comparison. Panel B reports the *t*-test statistics for one-sided *t*-tests examining if the cross-market correlation coefficient during the full period is significantly greater than during the crisis (high volatility) period.

Three interesting points emerge from a close examination of Table 3. First, comovements increase significantly for all country pairs for all parts of the distribution, with two exceptions highlighted in bold. Second, there are instances where one part of the distribution is subject to increases in comovements, while others are not. This is the case for Mexico and Brazil, where the test indicates statistically significant increases in comovements during crises for the lower tail but no increase in comovements in the upper tail. Notice that this analysis could not be carried out with tests based on the estimation of correlation coefficients (Forbes and Rigobon, 2002). Third, the test statistics gets weaker as the range of θ for the tests is selected closer to the right tails.

Overall, our analysis suggests that the distributions are characterized by strong asymmetries, which cannot be detected by simple correlation.

5.2 Controlling for Economic Variables

Our methodology allows the researcher to control for common factors which may drive asset return comovements. Potential control variables could be, *inter alia*, interest rate and bond yield differentials, return volatilities or cross-border

financial flows (an extensive list of potential control variables is given, for instance, in Eichengreen, Rose, and Wyplosz, 1996).

As an illustration, this subsection shows how to control for different levels of volatilities in financial markets. We compute the volatility $\sigma^2_{EWMA,t}$ of the average returns on Argentinian and Brazilian stock markets as an exponentially weighted moving average (EWMA) with decay coefficient equal to 0.97. Next, we identify as periods of high volatility the 10% number of observations with highest EWMA volatility, i.e., we construct the control dummy $S_{2,t} \equiv I\left(\sigma^2_{EWMA,t} > q^{\sigma^2_{EWMA}}_{0.90}\right)$, where

$q^{\sigma^2_{EWMA}}_{0.90}$ is the 90% unconditional quantile of the time series $\{\sigma^2_{EWMA,t}\}^T_{t=1}$. Finally, we estimate the usual Equation (4), where in addition to the crisis dummy we include the volatility dummy:

$$I^Y_t(\hat{\beta}_{i,Y}) \cdot I^X_t(\hat{\beta}_{i,X}) = \alpha_{0,ii} + \alpha_{1,ii}S_{1,t} + \alpha_{2,ii}S_{2,t} + \varepsilon_t, \qquad i = 1, ..., m. \tag{9}$$

To ensure that we have exactly the same proportion of quantile exceedances across all dummy periods, we add both the crisis and volatility dummies to the CAViaR specification of the individual quantiles underlying the indicator functions of the above regression:

$$q^Y_t(\beta_{i,Y}) = \beta_{0,i,Y} + \beta_{4,i,Y}S_{1,t} + \beta_{5,i,Y}S_{2,t} + \beta_{1,i,Y}y_{t-1} + \beta_{2,i,Y}q^Y_{t-1}(\beta_{i,Y})$$
$$- \beta_{1,i,Y}\beta_{2,i,Y}y_{t-2} + \beta_{3,i,Y}|y_{t-1}|.$$

For each market we estimate this model for 19 quantile probabilities ranging from 5% to 95%.

In Table 5, for each country pair, we report the value of the following test statistics for $\hat{\alpha}_{1,ii}$ and $\hat{\alpha}_{2,ii}$:

$$\widehat{\delta}\left(\underline{\theta}, \bar{\theta}\right) = (\#\theta)^{-1} \sum_{\theta_i \in [\underline{\theta}, \bar{\theta}]} \hat{\alpha}_{1,ii} \tag{10}$$

$$\widehat{\xi}\left(\underline{\theta}, \bar{\theta}\right) = (\#\theta)^{-1} \sum_{\theta_i \in [\underline{\theta}, \bar{\theta}]} \hat{\alpha}_{2,ii}, \tag{11}$$

together with the associated standard errors computed over different intervals of θ. By and large, the crisis dummy is still significant even when the volatility dummy is included in the regression (9), although results are sometimes weakened in the right part of the distribution. The volatility dummy is more often than not insignificant, suggesting that comovement likelihoods are not higher in periods of high return volatility. Moreover the test suggests that there is less asymmetry between lower and upper tail coexceedance likelihoods in high volatility periods than in tranquil or crisis times. These results are at odds with the findings of Bae, Karolyi, and Stulz (2003): in our framework periods of high returns volatility do not necessarily coincide with periods of crisis and only in a few cases contribute to increase return comovements.

To check for the robustness of our estimates, we have repeated the exercise using four different specifications. First, we have computed the EWMA volatility

Table 5 Test of difference in tail coexceedance between crisis and tranquil periods when controlling for volatility

Country pairs	Lower tail ($\theta_i \leq 0.5$)			Upper tail ($\theta_i \geq 0.5$)		
$\hat{\delta}(\underline{\theta},\bar{\theta})$	$\hat{\delta}(0.05,0.50)$			$\hat{\delta}(0.50,0.95)$		
	Stat.	s.e.		Stat.	s.e.	
Mex.–Bra.	0.142	0.050		**0.048**	0.042	
Mex.–Arg.	0.230	0.050		0.120	0.039	
Mex.–Chi.	0.123	0.048		**0.043**	0.041	
Bra.–Arg.	0.278	0.045		0.215	0.044	
Bra.–Chi.	0.157	0.044		0.146	0.043	
Arg.–Chi.	0.209	0.045		0.194	0.040	
$\hat{\xi}(\underline{\theta},\bar{\theta})$	$\hat{\xi}(0.05,0.50)$			$\hat{\xi}(0.50,0.95)$		
	Stat.	s.e.		Stat.	s.e.	
Mex.–Bra.	**–0.008**	0.040		**0.067**	0.042	
Mex.–Arg.	**–0.072**	0.036		**–0.047**	0.039	
Mex.–Chi.	**0.066**	0.042		0.121	0.041	
Bra.–Arg.	–0.082	0.036		**–0.050**	0.044	
Bra.–Chi.	**0.050**	0.038		**0.020**	0.043	
Arg.–Chi.	–0.084	0.033		**–0.051**	0.040	

$\hat{\delta}(\underline{\theta},\bar{\theta})$	$\hat{\delta}(0.05,0.25)$		$\hat{\delta}(0.75,0.95)$		$\hat{\delta}(0.25,0.50)$		$\hat{\delta}(0.50,0.75)$	
	Stat.	s.e.	Stat.	s.e.	Stat.	s.e.	Stat.	s.e.
Mex.–Bra.	0.179	0.064	**0.028**	0.049	0.107	0.047	**0.075**	0.048
Mex.–Arg.	0.312	0.066	0.131	0.049	0.159	0.045	0.118	0.040
Mex.–Chi.	0.157	0.065	**0.033**	0.052	0.089	0.043	**0.046**	0.042
Bra.–Arg.	0.332	0.056	0.292	0.056	0.229	0.043	0.147	0.043
Bra.–Chi.	0.156	0.056	0.172	0.052	0.152	0.043	0.115	0.045
Arg.–Chi.	0.227	0.059	0.229	0.050	0.197	0.042	0.166	0.041
$\hat{\xi}(\underline{\theta},\bar{\theta})$	$\hat{\xi}(0.05,0.25)$		$\hat{\xi}(0.75,0.95)$		$\hat{\xi}(0.25,0.50)$		$\hat{\xi}(0.50,0.75)$	
	Stat.	s.e.	Stat.	s.e.	Stat.	s.e.	Stat.	s.e.
Mex.–Bra.	**–0.009**	0.047	0.111	0.049	**–0.005**	0.042	**0.024**	0.048
Mex.–Arg.	**–0.063**	0.046	**–0.041**	0.049	–0.080	0.036	**–0.058**	0.040
Mex.–Chi.	**0.066**	0.055	0.172	0.052	**0.064**	0.040	0.082	0.042
Bra.–Arg.	–0.105	0.046	**–0.051**	0.056	**–0.067**	0.037	**–0.051**	0.043
Bra.–Chi.	0.065	0.049	**0.025**	0.052	**0.044**	0.037	**0.015**	0.045
Arg.–Chi.	–0.098	0.042	**–0.035**	0.050	–0.076	0.032	**–0.065**	0.041

This table reports the average of $\hat{\alpha}_{1,ii}$ and $\hat{\alpha}_{2,ii}$ over θ_i as well as the associated standard errors. The resulting t statistics are obtained applying Corollary 2 and provide a joint test for changes in comovements which follows from Definition 1. Statistics indicated in bold are NOT significant at the 5% level.

by choosing a decay parameter equal to 0.94, instead of 0.97.[24] Second, we have carried out the estimation identifying as periods of high volatility the 5% number of observations with highest EWMA volatility.[25] To do so, we have used a new control dummy, which is defined as $S'_{2,t} \equiv I\left(\sigma^2_{EWMA,t} > q^{\sigma^2_{EWMA}}_{0.95}\right)$. Third, since the volatility of the average return on Argentina and Brazil can go down when correlations are negative, we have assumed that market turbulences are captured by the average volatilities of Argentinian and Brazilian equity returns. To this end, we have constructed another dummy variable $S''_{2,t} \equiv I\left(\sigma^2_{A-EWMA,t} > q^{\sigma^2_{A-EWMA}}_{0.90}\right)$, where $\sigma^2_{A-EWMA,t}$ is the average variance of Argentinian and Brazilian equity returns (still constructed with a decay parameter equal to 0.97), and $q^{\sigma^2_{A-EWMA}}_{0.90}$ is the associated 90% unconditional quantile. Fourth, we control for the strong correlation between the U.S. and the Latin American equity markets. In order to do so, we have computed the EWMA correlation between the average returns on the equity market pair Brazil–Argentina and the S&P500, $\rho_{EWMA,t}$. Next, we have constructed a new control dummy, $S'''_{2,t} \equiv I\left(\rho_{EWMA,t} > q^{\rho_{EWMA}}_{0.90}\right)$, which equals one when $\rho_{EWMA,t}$ is larger than $q^{\rho_{EWMA}}_{0.90}$, i.e., the 90% unconditional quantile of the time series $\{\rho_{EWMA,t}\}^T_{t=1}$.

In the first case, the results are similar to those obtained assuming a decay coefficient equal to 0.97: including the volatility dummy does not affect much the significance of the crisis dummy, which still enters significantly the regression (9). By the same token, when adopting the control dummy variables $S'_{2,t}$ and $S''_{2,t}$ the results do not differ much from the those reported in Table 5. Finally, when we use the control dummy $S'''_{2,t}$, the crisis dummy continues to enter significantly into the regression, even though the control dummy $S'''_{2,t}$ is significantly different from zero most of the times.[26]

6 SUMMARY AND CONCLUSIONS

In this study we propose a new methodology to measure dependence across random variables. Our approach is based on conditional quantiles and permits us to investigate whether dependence across series of interest changes over time or across economic environments. We compute, for all quantiles, the conditional probability that realization of one series fall in the left (or right) tail of their own distribution provided that the realization of the other series have fallen in the same tail of their own distribution. We estimate these conditional probabilities through a simple OLS

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[24]The decay parameter in EWMA processes is arbitrarily chosen. Commonly it is assumed that it takes on values larger than 0.90, so that it captures the high persistence typical of second moments.

[25]In this specification we set the decay parameter equal to 0.97.

[26]To save space, we do not report the results relative to the EWMA volatility computed by choosing a decay parameter equal to 0.94, neither those obtained when using the dummy variables $S'_{2,t}$, $S''_{2,t}$, and $S'''_{2,t}$. Such estimates are available upon request.

regression of quantile coexceedance indicator variables on a constant and economic indicator variables. We derive a simple test of changes in comovements across time periods and market conditions. The full range of conditional dependence is conveniently visualized in what we call "the comovement box."

As an illustration, we use our methodology to investigate whether financial comovements increase during crisis periods across the most important Latin American equity markets. Our results show that, on average, over turbulent times, comovements in equity returns across national markets tend to increase significantly, both in the left and in the right tail of the distributions.

A number of questions can be addressed within the framework we propose. For instance, a persistent issue in the literature is whether the increase in financial markets comovements is due to economic linkages and common macro-economic conditions or to investor behavior unrelated to these fundamental links.[27] A possible strategy to investigate this question would be to define the crisis periods in terms of a set of economic variables and then testing whether the associated coefficient is significantly different from zero. Surprisingly, when we define crisis as periods of high volatility, we find that returns comovements do not change significantly between low and high volatility periods than in times of low volatility.

In finance applications, our approach can be useful for studies of financial stability, as well as for portfolio allocation and risk management. The methodology allows the researcher to estimate the probability of comovements for different ranges of the support of the return distribution and for different market conditions, while taking into account local and global economic forces that may drive returns comovements, and without having to make distributional assumptions.

APPENDIX A – ASSUMPTIONS

When using the generic notation (z_t, Z), we refer to both (y_t, Y) and (x_t, X).

A.1 Consistency Assumptions

C0. (Ω, F, P) is a complete probability space, and $\{y_t, x_t, \omega_t\}$, $t = 1, 2, ...$ are random variables on this space.
C1. The functions $q_t^Z(\beta_{i,Z})$, $i, j = 1, ..., m$, a mapping from B (a compact subset of $\Re^p$) to $\Re$ are measurable with respect to the information set available at time t, Ω_t, and continuous in B, for any given choice of explanatory variables $\{z_{t-1}, \omega_{t-1}, ..., z_1, \omega_1\}$, $\omega_t \in \Omega_t$.
C2. $h_{i,t}^Z(\varepsilon) \equiv h_i^Z(\varepsilon | \Omega_t)$ - the conditional density of $z_t - q_t^Z(\beta_{i,Z}^0)$ - is continuous.
C3. There exists $h > 0$ such that, for all t and for all $i = 1, ..., m$, $h_{i,t}^Z(0) \geq h$.

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[27]See Yuan (2005) for an example of a rational expectation model of contagious crisis unrelated to fundamentals.

C4. $|q_t^Z(\beta_{i,Z})| < K(\Omega_t)$ for all $\beta_{i,Z} \in B$ and for all t, where $K(\Omega_t)$ is some (possibly) stochastic function of variables that belong to Ω_t, such that $E[K(\Omega_t)] \leq K_0 < \infty$.

C5. $E[|z_t|] < \infty$ for all t.

C6. $\{\rho_i(z_t - q_t^Z(\beta_{i,Z}))\}$ obeys the uniform law of large numbers.

C7. For every $\xi > 0$, there exists a $\tau > 0$ such that if $||\beta_{i,Z} - \beta_{i,Z}^0|| \geq \xi$, then $\liminf_{T \to \infty} \sum P[|q_t^Z(\beta_{i,Z}) - q_t^Z(\beta_{i,Z}^0)| > \tau] > 0$.

A.2 Asymptotic Normality Assumptions

AN1. $q_t^Z(\beta_{i,Z})$, is differentiable in B and for all $\beta_{i,Z}$ and $\gamma_{i,Z}$ in a neighborhood υ_0 of $\beta_{i,Z}^0$, such that $||\beta_{i,Z} - \gamma_{i,Z}|| \leq d$ for d sufficiently small and for all t:

(a) $||\nabla q_t^Z(\beta_{i,Z})|| \leq F(\Omega_t)$, where $F(\Omega_t)$ is some (possible) stochastic function of variables that belong to Ω_t and $E[F(\Omega_t)^3] \leq F_0 < \infty$, for some constant F_0.

(b) $||\nabla q_t^Z(\beta_{i,Z}) - \nabla q_t^Z(\gamma_{i,Z})|| \leq M(\Omega_t, \beta_{i,Z}, \gamma_{i,Z}) = O(||\beta_{i,Z} - \gamma_{i,Z}||)$, where $M(\Omega_t, \beta_{i,Z}, \gamma_{i,Z})$ is some function such that $E[M(\Omega_t, \beta_{i,Z}, \gamma_{i,Z})^2] \leq M_0||\beta_{i,Z} - \gamma_{i,Z}|| < \infty$ and $E[M(\Omega_t, \beta_{i,Z}, \gamma_{i,Z})F(\Omega_t)] \leq M_1||\beta_{i,Z} - \gamma_{i,Z}|| < \infty$ for some constants M_0 and M_1.

AN2. (a) $\max\{h_{i,t}^Z(\varepsilon), h_{ij,t}(\eta, \upsilon)\} \leq H < \infty$ $\forall t$, where $h_{ij,t}(\eta, \upsilon)$ is the joint conditional density of $(y_t - q_t^Y(\beta_{i,Y}^0), x_t - q_t^X(\beta_{j,X}^0))$.

(b) $h_{i,t}^Z(\varepsilon)$ satisfies the Lipschitz condition $|h_{i,t}^Z(\lambda_1) - h_{i,t}^Z(\lambda_2)| \leq L|\lambda_1 - \lambda_2|$, $\forall t$, for some constant $L < \infty$.

AN3. The matrices A and D have smallest eigenvalue bounded below by a positive constant for T sufficiently large.

AN4. The sequences $\{T^{-1/2}\sum_{t=1}^T [\theta_i - I(z_t \leq q_t^Z(\beta_{i,Z}^0))]\nabla q_t^Z(\beta_{i,Z}^0)\}$ obey the central limit theorem.

AN5. $T^{-1}\Sigma_{t=1}^T G_{ij,t} \overset{p}{\to} E[T^{-1}\Sigma_{t=1}^T G_{ij,t}]$, where $G_{ij,t} \equiv W_t'[\nabla_\beta' q_t^X(\beta_{j,X}^0)\int_{-\infty}^0 h_{ij,t}(\eta, 0)d\eta + \nabla_\beta' q_t^Y(\beta_{i,Y}^0)\int_{-\infty}^0 h_{ij,t}(0, \upsilon)d\upsilon]$.

The sequences $T^{-1/2}\sum_{t=1}^T [g_t(\beta^0) + (T^{-1}\Sigma_{t=1}^T G_t)D^{-1}\psi_t(\beta^0)]$ obey the central limit theorem.

A.3 Variance–Covariance Matrix Estimation Assumptions

VC1. $\hat{c}_T/c_T \overset{p}{\to} 1$, where the non-stochastic positive sequence c_T satisfies $c_T = o(1)$ and $c_T^{-1} = o(T^{1/2})$.

VC2. $E[F(\Omega_t)^4] \leq F_1 < \infty$, $\forall t$, where $F(\Omega_t)$ was defined in assumption AN1(a).

VC3. (a) $T^{-1}\sum_{t=1}^T \psi_t(\beta^0)\psi_t(\beta^0)' \overset{p}{\to} A$

(b) $T^{-1}\sum_{t=1}^T h_{i,t}^Z(0)\nabla^Z q_t(\beta_{i,Z}^0)\nabla' q_t^Z(\beta_{i,Z}^0) \overset{p}{\to} D_i^Z$

(c) $T^{-1}\sum_{t=1}^T W_t'\left[\nabla_\beta' q_t^X(\beta_{j,X}^0)\int_{-\infty}^0 h_{ij,t}(q_t^X(\beta_{j,X}^0), y)dy + \nabla_\beta' q_t^Y(\beta_{i,Y}^0)\int_{-\infty}^0 h_{ij,t}(x, q_t^Y(\beta_{i,Y}^0))dx\right] \overset{p}{\to} G_{ij}$

APPENDIX B – PROOFS OF THEOREMS IN THE TEXT

Proof of Theorem 1: We denote with $\sum_{C_0}$ the summation over the C_0 periods when all the dummies are zero and with $\sum_{C_l}$ the summation over the C_l periods defined by the dummies $\{S_{l,t}=1, S_t^{-l}=\mathbf{0}\}_{t=1}^T$. Define $\hat{I}_{ij}^{YX} \equiv I_t^Y(\hat{\beta}_{i,Y}) \cdot I_t^X(\hat{\beta}_{j,X})$. The OLS estimators for generic θ_i and θ_j is consistent and therefore:

$$\hat{\alpha}_{0,ij} \xrightarrow{p} \alpha_{0,ij}$$

$$= E[\hat{I}_{ij}^{YX}|S_t=\mathbf{0}]$$

$$= plim \frac{\sum_{C_0}\hat{I}_{ij}^{YX}}{C_0}$$

and

$$\hat{\alpha}_{l,ij} \xrightarrow{p} \alpha_{l,ij}$$

$$= E[\hat{I}_{ij}^{YX}|S_{l,t}=1, S_t^{-l}=\mathbf{0}] - E[\hat{I}_{ij}^{YX}|S_t=\mathbf{0}]$$

$$= plim[\frac{\sum_{C_l}\hat{I}_{ij}^{YX}}{C_l} - \frac{\sum_{C_0}\hat{I}_{ij}^{YX}}{C_0}]$$

We show only that $\sum_{C_0}\hat{I}_{ij}^{YX}/C_0 \xrightarrow{p} \bar{F}_0^{ij}$. The other cases can be obtained similarly. We show first that $C_0^{-1}\{\sum_{C_0}[\hat{I}_{ij}^{YX} - I_{ij}^{YX}]\} = o_p(1)$, where $I_{ij}^{YX} \equiv I_t^Y(\beta_{i,Y}^0) \cdot I_t^X(\beta_{j,X}^0)$. Define $\varepsilon_{i,t}^Z \equiv [z_t - q_t^Z(\beta_{i,Z}^0)]$, $\hat{\varepsilon}_{i,t}^Z \equiv [z_t - q_t^Z(\hat{\beta}_{i,Z})]$ and $\delta_t(\hat{\beta}_{i,Z}) \equiv q_t^Z(\hat{\beta}_{i,Z}) - q_t^Z(\beta_{i,Z}^0)$. Suppose that $\delta_t(\hat{\beta}_{i,Z}) > 0$. The same reasoning goes through for $\delta_t(\hat{\beta}_{i,Z}) < 0$. Then:

$$|\hat{I}_{ij}^{YX} - I_{ij}^{YX}| = |I(\varepsilon_{i,t}^Y \le \delta_t(\hat{\beta}_{i,Y})) \cdot I(\varepsilon_{j,t}^X \le \delta_t(\hat{\beta}_{j,X})) - I(\varepsilon_{i,t}^Y \le 0) \cdot I(\varepsilon_{j,t}^X \le 0)|$$

$$= I(0 \le \varepsilon_{i,t}^Y \le \delta_t(\hat{\beta}_{i,Y})) \cdot I(\varepsilon_{j,t}^X \le 0) + I(\varepsilon_{i,t}^Y \le 0) \cdot I(0 \le \varepsilon_{j,t}^X \le \delta_t(\hat{\beta}_{j,X})) +$$

$$+ I(0 \le \varepsilon_{i,t}^Y \le \delta_t(\hat{\beta}_{i,Y})) \cdot I(0 \le \varepsilon_{j,t}^X \le \delta_t(\hat{\beta}_{j,X}))$$

$$\le 2I(0 \le \varepsilon_{i,t}^Y \le \delta_t(\hat{\beta}_{i,Y})) + I(0 \le \varepsilon_{j,t}^X \le \delta_t(\hat{\beta}_{j,X}))$$

Applying the mean value theorem to the expectation of the first term of the last expression:

$$E[I(0 \le \varepsilon_{i,t}^Y \le \delta_t(\hat{\beta}_{i,Y}))] = E|\int_0^{\delta_t(\hat{\beta}_{i,Y})} h_{i,t}^Y(\varepsilon) d\varepsilon|$$

$$= E|h_{i,t}^Y(\delta_t(\hat{\beta}_{i,Y}))\nabla q_t^Y(\beta_{i,Y}^*)(\hat{\beta}_{i,Y} - \beta_{i,Y}^0)|$$

where $h_{i,t}^Y(\varepsilon)$ is the pdf of $(y_t - q_t^Y(\beta_{i,Y}^0))$ and $\beta_{i,Y}^*$ lies between $\hat{\beta}_{i,Y}$ and $\beta_{i,Y}^0$. Now choose $d > 0$ arbitrarily small and T sufficiently large such that $||\hat{\beta}_{i,Y} - \beta_{i,Y}^0|| < d$.

This, together with assumptions AN1(a) and AN2(a), implies that

$$E[I(0 \leq \varepsilon_{i,t}^{Y} \leq \delta_t(\hat{\beta}_{i,Y}))] \leq E|HdF(\Omega_t)|$$

$$\leq |HdF_0| = O(d)$$

Same reasoning holds for the other term in the inequality. Since d can be chosen arbitrarily small, this result implies that:

$$E\left|C_0^{-1}\left\{\sum_{C_0}[\hat{I}_{ij}^{YX} - I_{ij}^{YX}]\right\}\right| \leq C_0^{-1}\left\{\sum_{C_0}E|\hat{I}_{ij}^{YX} - I_{ij}^{YX}|\right\}$$

$$= O(d) = o_p(1)$$

It remains to show that $C_0^{-1}\sum_{C_0}\left[I_{ij}^{YX} - \Pr[y_t \leq q_t^{Y}(\beta_{i,Y}^0), x_t \leq q_t^{X}(\beta_{j,X}^0)]\right] = o_p(1)$. This term has expectation 0 and variance equal to:

$$C_0^{-2}\sum_{C_0}E[I_{ij}^{YX} - \Pr[y_t \leq q_t^{Y}(\beta_{i,Y}^0), x_t \leq q_t^{X}(\beta_{j,X}^0)]]^2$$

$$\leq C_0^{-1} \overset{T\to\infty}{\to} 0$$

because, by the Law of Iterated Expectations, all the cross products with different time subscript have expectation 0.   Q.E.D.

Proof of Theorem 2: Note that $(\hat{\alpha} - \alpha^0) = [J_{m^2} \otimes (W'W)]^{-1}\sum_{t=1}^{T}g_t(\hat{\beta})$. The proof is obtained by showing that the conditions of theorems 7.2 and 7.3 of Newey and McFadden (1994) hold.

Proof of Theorem 3: The proof is similar to the proof of theorem 3 of Engle and Manganelli (2004).

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