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Measuring Forecasting Performance of Vector Autoregressive and Time Series Regression Models

Taiwo .A .I and Olatayo T.O

Department of Mathematical Sciences, Olabisi Onabanjo University,

Ago-Iwoye, Nigeria.
ab.com1982@yahoo.com and Otimtoy@yahoo.com

ABSTRACT

Correlation and Regression are the traditional approach of determining relationship between two or more variables. When the variables are multiple and the dependent variable is considered having an explanatory variable, then a Vector Autoregressive model is used to determine the structural relationship between the variables. If these variables are co-integrated, *VAR* model is not appropriate, but our focus is on the structural relationship and measuring forecast performance of a *VAR* and Time series regression with Lagged Explanatory Variables. Some Nigerian economic series (Government Revenue and Expenditure, Inflation Rates and Investment) data were analysed and the Root mean Square forecast Error (*RMSFE*) and Mean Absolute Percentage Forecast Error (*MAPFE*) are used as measurement criteria. The VAR model was found to be better than Time series regression with Lagged Explanatory Variables model as indicated by Meta diagnostic tools. The forecast values from the *VAR* model is more realistic and closely reflect the current economic reality in Nigeria indicated by the forecast evaluation tools.

Keywords: Vector Autoregressive, Time Series Regression, Meta Diagnostics, Forecast performance.

INTRODUCTION

For most of the twentieth century, especially since the great depression, most time series macro-economists have looked upon the sharp fluctuations in the economic series as prima facie evidence of major market imperfections (Blanchard and fischer, 1990). In order to investigate the economic fluctuations, multiple time series models was developed as an alternatives to structural econometric models in economic forecasting applications. Many time series econometricians as applied multiple time series models to investigate the fluctuations and these includes for example Aaker *et al* (1982), Adams and Moriarty (1981), Ashley, Granger and Schmalensee (1980) and many others.

One class of multiple time series models which has received much attention recently is the class of Vector Autoregressive *(VAR)* models. These models constitute special case of the more general class of Vector Autoregressive Moving Average *VARMA* models. Although *VAR* models have been used primarily for macro-economic models, they offer an interesting alternative to either structural econometric or univariate (e.g., Box-Jenkins ARIMA or exponential smoothing) models for problems in which simultaneous forecasts are required for a collection of related macro-economic variables. The use of *VAR* models for economic forecasting was proposed by Sims (1980), motivated in part by questions related to the validity of the way in which economic theory is used to provide a prior justification for the inclusion of a restricted subset of variables in the "structural" specification of each dependent variable. Sims (1980) questions the use of the so called "exclusionary and identification restrictions". Such time series models have the appealing property that, in order to forecast the endogenous variables in the system, the modeller is not required to provide forecasts of exogenous explanatory variables; the explanatory variables in an econometric model are typically no less difficult to forecast than the dependent variables.

In addition, the time series models are less costly to construct and to estimate. These imply that *VAR* models offer a parsimonious representation for a multivariate process. In order to determine the parsimonious representation of the *VAR* model, we will compare the forecast performance of *VAR* and Time series regression with Lagged Explanatory Variables. Since in the literature, *VAR* models has been discuss to have a practical forecasting applications because of the relative simplicity of their model identification and parameter estimation procedures, superior forecasting performance and this will be verified by comparing the estimates and forecast evaluations of *VAR* and Time series regression with Lagged Explanatory Variables models.

In this paper, we will compare the vector autoregressive and Time series regression with Lagged Explanatory Variables models. We will make use of some Nigerian macro-economic series for the period of 1970 through 2008: using yearly data. The rationale for the choice of a multiple time series technique is twofold and these are structural estimation and forecasting.

METHODOLOGY

Time series Regression with Lagged Explanatory Variables Model: The time series regression model with lagged explanatory variables is defined as;

$$y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + \cdots + \beta_k X_{t-k} + \varepsilon_t$$

Where y_t is a K-dimensional vector containing observations on the dependent variable, X is a $T \times K$ matrix of explanatory variables, β is a K -vector of coefficients that measures the effect of the explanatory variables on the dependent variable and ε_t is the vector of disturbances. T is the number of observations and K is the lag length. The methods and procedures can be found in the literatures like Reinsel and Sung (1992), Clements *et al*. (1997) and Lütkepohl (2005).

Vector Autoregressive Model *(VAR)*

A process $Y_t = (y_{1t}, y_{2t}, \ldots, y_{nt})'$denote an $(n \times 1)$ vector of time series variables.

The basic $p - lag$ vector autoregressive $(VAR)(p)$ model has the form

$$Y_t = C + \Pi_1 y_{t-1} + \Pi_2 y_{t-2} + \cdots + \Pi_p y_{t-p} + \varepsilon_t \qquad ,t = 1, \ldots, T$$

Where Π_i are $(n \times n)$ coefficient matrices and ε_t is an $(n \times 1)$ unobservable zero mean white noise vector process (serially uncorrelated or independent) with time invariant covariance matrix Σ (Sims 1980). Assuming that the VAR model contains two lagged values of the endogenous variables then a bivariate $(VAR)(2)$ model equation has the form;

$$y_{1t} = C_1 + \pi_{11}^1 y_{1t-1} + \pi_{12}^1 y_{2t-1} + \pi_{11}^2 y_{1t-2} + \pi_{12}^2 y_{2t-2} + \varepsilon_{1t}$$

$$y_{2t} = C_2 + \pi_{21}^1 y_{1t-1} + \pi_{22}^1 y_{2t-1} + \pi_{21}^2 y_{1t-2} + \pi_{22}^2 y_{2t-2} + \varepsilon_{2t}$$

In lag operator notation, the $(VAR)(p)$ is written as

$$\Pi(L)Y = C + \varepsilon_t$$

Where $\Pi(L) = I_n - \Pi_1 L - \cdots - \Pi_p L^p$. The $(VAR)(p)$ is stable if the roots of

$det\left(I_n - \Pi_1 L - \cdots - \Pi_p L^p\right)$ lie outside the complex unit circle (Lutkepohl, 1999).

Data Exploration: The pattern and general behaviour of the series is examined from the time plot. The series was examined for stationarity, outliers and gaussianity. Test for stationarity will be carried out with augmented Dickey –Fuller methods. Details of the test procedures can be found in the literature which include Box and Jenkins (1976); Chatfield 1980); Rao (1994) and Pfaff (2004). The Dickey-Fuller type of test for unit root of integer type will be used in this study. As well, a co-integration test will be carried out using the Johansen co-integration test. Details of the test procedure can be found in Lutkepohl (1991); Watson (1994).

Model Diagnostics: To check for the adequacy of the estimated vector Autoregressive models, the fitted models are subjected to model diagnostics using R^2 and Vector Autoregressive lag order selection criteria will be used to chose the appropriate model order using the AIC and SBIC criteria.

The criterions are specified as

$$AIC = -\frac{2l}{T} + \frac{2n}{T}$$

$$SBIC = -\frac{2l}{T} + \frac{nlogT}{T}$$

Where $l\,(\log - likelihood) = \frac{T}{2}\{k(1 + log2\pi) + log|\hat{\Omega}|\}$, $|\hat{\Omega}| = det\left(\frac{1}{T-P}\sum_t \hat{\varepsilon}_t\, \hat{\varepsilon}_t'\right)$ is the determinant of the residual covariance, P is the number of parameter per equation in the VAR.

The error term is expected to be independently distributed. We check this by testing for the hypothesis of white noise residuals using the

Portmanteau and Residual Normality tests. The Portmanteau test is used to test the hypothesis for autocorrelation, that is

$$H_0: \gamma_1 = \cdots = \gamma_h = 0 \quad vs$$

$$H_1: \gamma_i \neq 0 \; for \; some \; 1 \leq i \leq h \; where \; h > 0$$

Where $\gamma_k = \left(\rho_{ij}(k)\right)$ is the autocorrelation matrix of the residual series with $\rho_{ij}(k)$ the cross autocorrelation of order k of the residuals series $i \; and \; j$. The portmanteau test is the Q-statistic and it is denoted by,

$$Q_h = T \sum_{k=1}^{h} tr(\hat{\gamma}_k^{\iota} \hat{\gamma}_0^{-1} \hat{\gamma}_k \hat{\gamma}_0^{-1})$$

Where $\hat{\gamma}_k = (\hat{\rho}_{ij}(k))$ are the estimated (residual) autocorrelations, and $\hat{\gamma}_0$ and contemporaneous correlations of the residuals (Lutkepohl, 1991).

While for the Time Series Regression, R^2 is used to check the adequacy of the models and Durbin-Watson statistic is used to measure the serial correlation in the residual. Following Johnson and Di Nardo (1997), Dublin Watson statistic is computed as,

$$DW = \sum_{t-2}^{T} (\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2 \Big/ \sum_{t=1}^{T} \hat{\varepsilon}_t{}^2$$

Forecasts of VAR and Time Series Regression models are computed for out-sample values. The optimal forecasts values are then evaluated using the mean squared forecast error (MSFE) defined as,

$$MSFE = \frac{1}{h+1} \sum_{t=s}^{h+s} (\hat{X}_t - X_t)^2$$

the root mean square forecast error (RMSFE) is defined as:

$$RMSFE = \sqrt{\frac{1}{h+1} \sum_{t=s}^{h+s} (\hat{X}_t - X_t)^2}$$

and the mean absolute percentage forecast error MAPFE is given as,

$$MAPFE = \frac{100}{h+s} \sum_{t=s}^{h+s} \left| \frac{\hat{X}_t - X_t}{\hat{X}_t} \right|$$

$where \; t = s, 1+s, \ldots, h+s$ and the actual and predicted values for corresponding t values are denoted by $\hat{X}_t \; and \; X_t$ respectively.

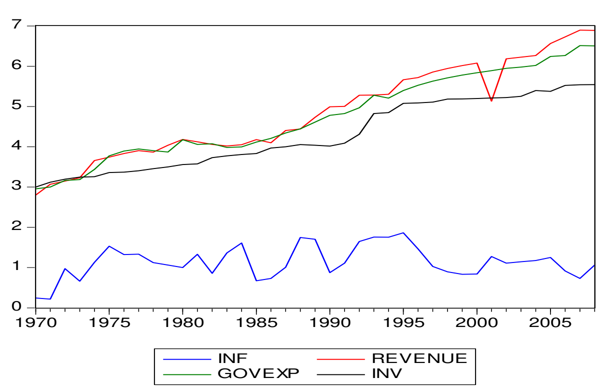
The smaller the values of *RMSFE* and *MAPFE*, the better the forecasting performance of the

model.

RESULTS AND DISCUSSIONS

The examination of the time plot (Figure 1) of the Nigerian macroeconomic series shows that there is some kind of non-stationarity and possibly nonlinearity in the observed data. It is evidence from the time plot that government revenue, government expenditure and investment show a long-term movement in the same direction over the period considered and this shown they are related. But it should be of notice the sharp fall in government revenue in 2001 and this is due to fall in crude oil price in the world market and this lead to budget deficit and increase in price of petroleum product in Nigeria. This sharp fall is referred to as a shift. Only inflation rates have a cyclical movement and a long-term oscillation about the trend every three years. This implies a different cycle of inflation rates is completed every three years.

Figure 1: Time Plot for Government Revenue, Government Expenditure, Inflation Rates and Investment



For the results in *Table* 1, all the macro economic variables tested for stationarity using the Augmented Dickey-Fuller (ADF) test reveals the economic variables are stationary at the first order difference except inflation rates that is stationary at the ordinary level I(0). That is, government revenue, government expenditure and investment are stationary at I(1).

Table 1: Unit Root Test Using Augmented Dickey-Fuller (ADF)

Economic Series	ADF Test Statistic	At 95% Critical Level	Order of Integration
REVENUE	-9.3274	-2.9434	I(1)
GOVEXP	-6.6289	-2.9434	I(1)
INV	-4.7931	-2.9434	I(1)
INF	-4.5341	-2.9434	I(0)

Considering Table 2 below,, none of the deterministic variable is restricted to the co-integration space. Likelihood ratio is the trace test adjusted for degrees of freedom. The critical values are taken from Osterwald- Lenum (1992). The * indicates rejection of likelihood ratio tests at 5% and 1% significance level. Likelihood Ratio test indicates one co-integrating equation at 5% and 1% significance level. In determining the number of co-integrating vectors, we used the degrees of freedom, adjusted version of trace statistic. Then, the test statistic strongly rejects the null hypothesis of no co-integration in favour of one co-integration relationship at 5% and 1% level of significant.

Table 2: Johansen Co-integration Test

Hypothesized No. of CE (s)	Eigen value	Trace Statistic	0.05 Critical Value	0.01 Critical Value	Prob.**
None*	0.616775	56.94743	47.85613	54.68150	0.0056
At most 1	0.406361	24.33691	29.79707	35.45817	0.1866
At most 2	0.173142	6.606430	15.49471	19.93711	0.6238
At most 3	0.004176	0.142264	3.841466	6.634897	0.7060

Models Estimation for the Government Revenue, Government Expenditure, Inflation Rate and Investment

$$\mathrm{INF_t} = \frac{0.524590}{[2.67490]}\mathrm{INF_{t-1}} + \frac{1.046313}{[1.42874]}\mathrm{INV_{t-1}} - \frac{0.990330}{[-1.33618]}\mathrm{INV_{t-2}} + \varepsilon_t$$

$$\mathrm{REVENUE_t} = \frac{0.584411}{[1.10521]}\mathrm{GOVEXP_{t-1}} + \varepsilon_t$$

$$INV_t = \frac{0.067927}{[\,1.26348]}\mathrm{INF_{t-1}} + \frac{0.185018}{[1.94510]}\mathrm{REVENUE_{t-1}} + \frac{1.116582}{[\,5.56184]}\mathrm{INV_{t-1}} - \frac{0.256013}{[-1.26004]}\mathrm{INV_{t-2}} - \frac{0.323372}{[-1.49094]}\mathrm{GOVEXP_{t-1}} + \frac{0.228457}{[\,1.24848\,]}\mathrm{GOVEXP_{t-2}} + \varepsilon_t$$

$$\mathrm{GOVEXP_t} = \frac{0.112079}{[\,1.91962]}\mathrm{INF_{t-1}} - \frac{0.057691}{[-1.01693]}\mathrm{INF_{t-2}} + \frac{0.193554}{[1.87368]}\mathrm{REVENUE_{t-1}} + \frac{0.343143}{[1.55511]}\mathrm{INV_{t-2}} + \frac{0.681178}{[2.89189]}\mathrm{GOVEXP_{t-1}} + \varepsilon_t$$

The four models above were obtained for the VAR Model Estimation and the first model shows that current inflation rates is explained and determined by inflation rates in its first previous lag and investment in the first and second previous lags by observing the values of t-statistics in the parenthesis above. The value of R^2 (0.863832) shows that (86%) of variation in the dependent variable is explained by the independent variables and this indicates a good model fit. The second model shows that current revenue is explained and determined by government expenditure in its first previous lag by observing the values of t-statistics in the parenthesis above and the value of R^2 (0.970179) shows that (97%) variation in the dependent variable is explained by the independent variables and the value of adjusted R^2 squared (0.936229) shows it is good fit. The third model shows that current investment is explained and determined by inflation rates, government revenue, investment and government expenditure in their first previous lags. Investment and government expenditure as well determines the current investment in their second previous lags by observing the values of t-statistics in the parenthesis above and the value of R^2 (0.986910) shows that (99%) variation in the dependent variable is explained by the independent variables and the value of adjusted R^2 squared (0.983245) shows it is good fit. While the last model shows that current government expenditure is explained and determined by inflation rates, government revenue and government expenditure in their first previous lags and inflation rates and investment as well determine current government expenditure in their second previous lag by observing the values of t-statistics in the parenthesis above and the value of R^2 (0.988712) shows that (99%) variation in the dependent variable is explained by the independent variables and the value of adjusted R^2 squared (0.983245) shows it is good fit.

Using *VAR* lag order selection criteria, AIC and SBIC criterion functions were both minima at $lag(1)$ and lag length of order (1) is suggested. Checking for the independent distributed of the residual using Portmanteau test result in table 3 below, the residual passes the white noise test since no autocorrelation is left at $lag(2)$ and on the basis we select $VAR(2)$ as the specification for analysis. Using the Vector Autoregressive Residual Normality test result in table 4 below, the residuals are jointly normal since the joint Skewness, Kurtosis and Jarque-Bera are significant and this implies that residuals are multivariate normal.

Table 3: Residual Autocorrelation (Portmanteau Test)

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	df
1	1.722404	NA*	1.770249	NA*	NA*
2	12.93350	NA*	13.62198	NA*	NA*
3	23.47824	0.1015	25.09713	0.0681	16
4	34.76951	0.3374	37.75705	0.2228	32
5	43.15339	0.6714	47.45090	0.4952	48
6	54.71592	0.7894	61.25134	0.5743	64

Table 4: Vector Autoregressive Residual Normality Test

Component	Skewness	Chi - Square	Df	Prob.
1	1.246508	4.700218	1	0.0295
2	7.302857	28.54331	1	0.0000
3	4.529050	3.604408	1	0.0576
4	1.479899	3.562341	1	0.0591
Joint		40.45028	4	0.0000

Component	Kurtosis	Chi - Square	Df	Prob.
1	0.002944	5.34E-05	1	0.9942
2	-1.592587	15.64073	1	0.0001
3	1.093147	7.368990	1	0.0066
4	0.277066	0.473387	1	0.4914
Joint		23.48316	4	0.0001

Component	Jarque-Bera	df	Prob.
1	4.740271	2	0.9942
2	44.18404	2	0.0001
3	10.97340	2	0.0066
4	4.035728	2	0.4914
Joint	63.93344	8	0.0000

Time Series Regression Analysis: Below are the results obtained from the time series regression with lag explanatory variables analysis.

Regressing the current inflation rates on the present and two past lags of government revenue, government expenditure and investment yields the result in the above table. It can be observed that all p-values are greater than 0.05 and this implies that the present and two past lags of government revenue, government expenditure and investment do not explain and determine the present inflation

Table 5. Dependent Variable (Inflation Rates)

Variable	coefficient	Std.Error	t-Stat.	Prob.
C(1)	0.897943	0.685980	1.308992	0.2016
C(2)	0.500036	0.804837	0.621288	0.5396
C(3)	-0.016037	0.688471	-0.023294	0.9816
C(4)	0.792587	0.546545	1.450178	0.1585
C(5)	0.569595	0.935276	0.609013	0.5476
C(6)	-1.235846	0.825049	-1.497908	0.1458
C(7)	-0.198777	0.328755	-0.604637	0.5505
C(8)	-0.524207	0.335373	-1.563057	0.1297
C(9)	-0.578896	0.360708	-1.604886	0.1202

INF= $C(1)^{*}GOVEXP(1) + C(2)^{*}GOVEXP(-1) + C(3)^{*}GOVEXP(-2) + C(4)^{*}INV(1) + C(5)^{*}INV(-1) + C(6)^{*}INV(-2) + C(7)^{*}REVENUE(1) + REVENUE(-1) + C(9)^{*}REVENUE(-2)$

The value of R^2 (0.819469) shows that (81%) variation in the dependent variable is explained by the explanatory variables and this show a weak predictive power. The value of Durblin-Waston statistic (1.530050) shows no evidence of serial correlation in the residual as DW statistic is around 2.

Table 6. Dependent Variable (Government Revenue)

Variable	coefficient	Std.Error	t-Stat.	Prob.
C(1)	-0.164132	0.101366	-1.619205	0.1170
C(2)	-0.005962	0.108636	-0.054882	0.9566
C(3)	0.073874	0.107186	0.689216	0.4966
C(4)	1.084415	0.294349	3.684119	0.0010
C(5)	-0.140490	0.429525	-0.327082	0.7461
C(6)	0.365418	0.351453	1.039735	0.3077
C(7)	0.020336	0.320819	0.063387	0.9499
C(8)	0.075746	0.505768	0.149765	0.8821
C(9)	-0.409990	0.438081	-0.935878	0.3576

$REVENUE = C(1)^{*}INF(1) + C(2)^{*}INF(-1) + C(3)^{*}INF(-2) + C(4)^{*}GOVEXP(1) + C(5)^{*}GOVEXP(-1) + C(6)^{*}GOVEXP(-2) + C(7)^{*}INV(1) + INV(-1) + C(9)^{*}INV(-2)$

Regressing the current government revenue on the present and two past lags of inflation rates, government expenditure and investment yields the result in the above table. It can be observed from the p-values that government expenditure in its present lag explains and determines the present government revenue.

The value of R^2 (0.971248) shows that (97%) variation in the dependent variable is explained by the explanatory variables. The value of Durblin-Waston statistic (2.441740) shows no evidence of serial correlation in the residual as DW statistic is around 2.

Table 7. Dependent Variable (Investment

Variable	coefficient	Std.Error	t-Stat.	Prob.
C(1)	-0.164132	0.101366	-1.619205	0.1170
C(2)	-0.005962	0.108636	-0.054882	0.9566
C(3)	0.073874	0.107186	0.689216	0.4966
C(4)	1.084415	0.294349	3.684119	0.0010
C(5)	-0.140490	0.429525	-0.327082	0.7461
C(6)	0.365418	0.351453	1.039735	0.3077
C(7)	0.020336	0.320819	0.063387	0.9499
C(8)	0.075746	0.505768	0.149765	0.8821
C(9)	-0.409990	0.438081	-0.935878	0.3576

$INV = C(1)^{*}INF(1) + C(2)^{*}INF(-1) + C(3)^{*}INF(-2) + C(4)^{*}GOVEXP(1) + C(5)^{*}GOVEXP(-1) + C(6)^{*}GOVEXP(-2) + C(7)^{*}REVENUE(1) + REVENUE(-1) + C(9)^{*}REVENUE(-2)$

Regressing the current investment on the present and two past lags of government revenue, inflation rates and government expenditure yields the result in the above table. It can be observed from the p-values that government expenditure in its present lag explains and determines the present investment.

The value of R^2 (0.963713) shows that (96%) variation in the dependent variable is explained by the explanatory variables. The value of Durblin-Waston statistic (0.628702) shows there is evidence of serial correlation in the residual.

Table 8. Dependent Variable (Government Expenditure)

Variable	coefficient	Std.Error	t-Stat.	Prob.
C(1)	0.102465	0.053766	1.905757	0.0674
C(2)	0.088354	0.057142	1.546199	0.1337
C(3)	-0.050658	0.055863	-0.906823	0.3725
C(4)	0.025144	0.157879	0.159264	0.8746
C(5)	-0.149496	0.245917	-0.607914	0.5483
C(6)	0.440922	0.197729	2.229933	0.0343
C(7)	0.163366	0.073477	2.223354	0.0348
C(8)	0.328619	0.086459	3.800883	0.0007
C(9)	0.195908	0.077670	2.522299	0.0179

$$GOVEXP = C(1)^{*}INF(1) + C(2)^{*}INF(-1) + C(3)^{*}INF(-2) + C(4)^{*}INV(1) + C(5)^{*}INV(-1) + C(6)^{*}INV(-2) + C(7)^{*}REVENUE(1) + REVENUE(-1) + C(9)^{*}REVENUE(-2)$$

Regressing the current government expenditure on the present and two past lags of government revenue, inflation rates and investment yields the result in the above table. It can be observed from the p-values that investment in its second previous lag and government revenue in its present and two past lags explain and determine the present government expenditure.

The value of R^2 (0.97248) shows that (97%) variation in the dependent variable is explained by the explanatory variables. The value of Durblin-Waston statistic (2.144710) shows no evidence of serial correlation in the residual as DW statistic is around 2

Vector Autoregressive and Time Series Regression Comparism: Using R^2 to compare the level of variation explained by the series consider as independent variables in *VAR* and Time Series Regression models estimation, it is shown that the variation explained in *VAR* models were slightly higher than the Time Series Regression models. This indicates that the *VAR* models establish a better inter-relationship among the macro-economic series.

Foresting with VAR AND Time Series Regression Models: Vector autoregressive and time series regression models obtained is used to forecast out sample values for all the economic series and static forecast will be used.

Table 9. Forecasting Results Using Vector Autoregressive Models

Year(s)	Government Revenue in Billion(s) naira	Government Expenditure in Billion(s) naira	Inflation Rates in (%)	Investment in Billion(s) naira
2009	6.881106	6.633381	1.105948	5.688482
2010	6.920296	6.687671	1.293117	5.798896
2011	6.999770	6.777695	1.409578	5.925796
2012	7.087328	6.873394	1.502065	6.045665
2013	7.190662	6.980018	1.555559	6.162683

The table above shows forecasting results for government revenue, government expenditure, inflation rates and investment from 2009 - 2013. The forecasting results show that all the economic series are increasing steadily except Inflation rates that are increasing rapidly each year.

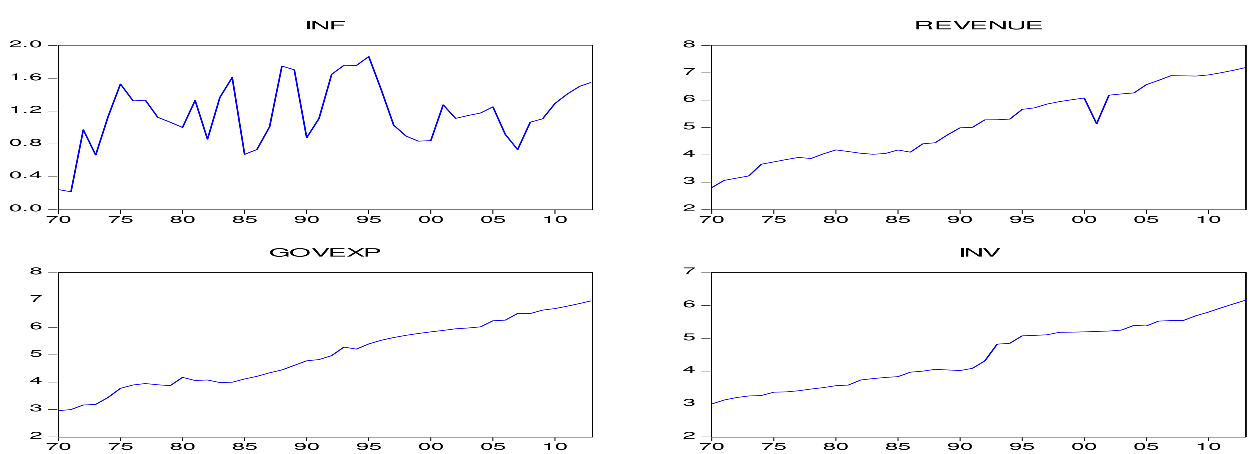


Fig 2. Forecast Graphs Using *VAR Models*

Table 10. Forecasting Results Using Time Series Regression Models

Year(s)	Government Revenue in Billion(s) naira	Government Expenditure in Billion(s) naira	Inflation Rates in (%)	Investment in Billion(s) naira
2009	6.845705	6.633381	1.068430	5.801822
2010	6.939273	6.687671	1.380202	5.873002
2011	7.013139	6.777695	1.440577	6.006178
2012	7.104153	6.873394	1.525205	6.101037
2013	7.110052	6.980018	1.531105	6.110021

The table above shows forecasting results for government revenue, government expenditure, inflation and investment from 2009 - 2013. The forecasting results show that all the economic series are increasing steadily except Inflation rates that are increasing rapidly each year.

Forecast Performance: The forecast performance of the two models will be determined by comparing the forecast evaluation result in table 11

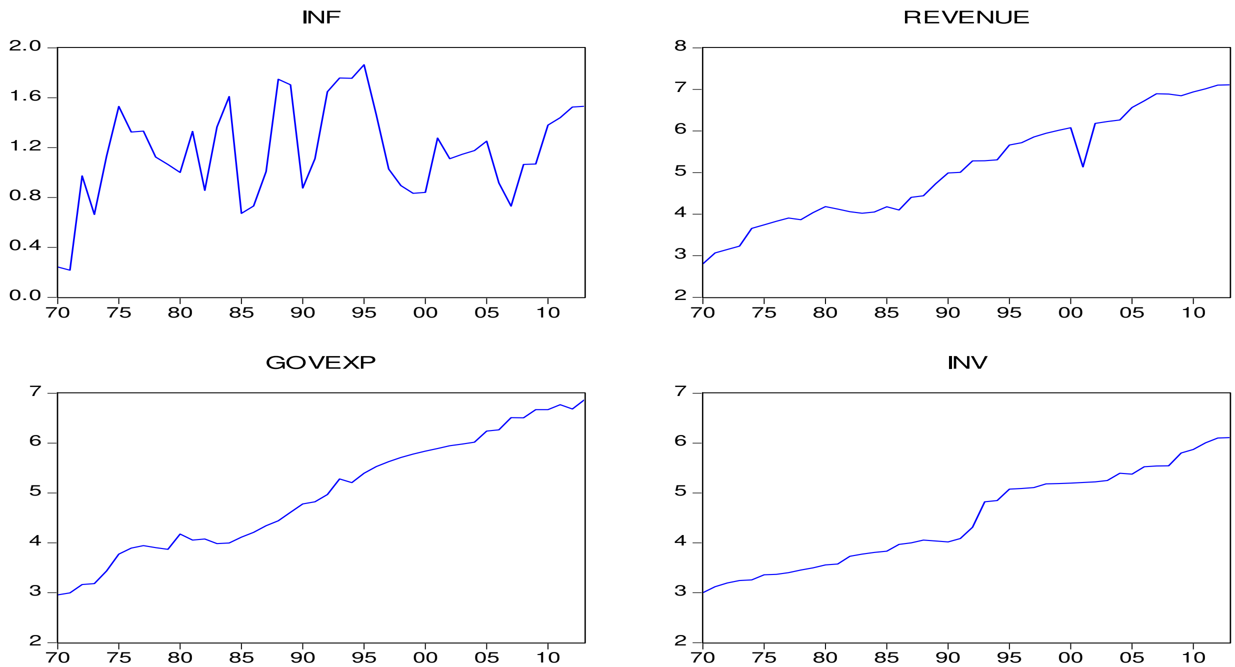


Fig 3. Forecast Graphs Using *Time Series Regression Models*

The results of using *VAR* models to forecast inflation rates, government revenue, government expenditure and investment for the annul period of 2009 – 2013 used for comparison is not intended as a formal model evaluation but simply as a demonstration that the nature of *VAR* model can have important implications for forecast performance. The alternative model considered is time series regression model and this model is very useful as well.

The forecast performance results above shows that using vector autoregressive is better than time series regression for forecasting accurate values for inflation rates, government revenue, government expenditure and investment, since all the values of the forecast evaluations for vector autoregressive models are less than that of time series regression models.

Table 11. Forecast Evaluation (2009 – 2013)

Inflation	Vector Autoregressive	Time Series Regression
Root Mean Square Error	0.291060	0.302457
Mean Absolute Error	0.232815	0.248473
Theil Inequality Coef.	0.110752	0.123405
Bias Proportion	0.003779	0.005437
Variance Proportion	0.003779	0.087037

Government Revenue	Vector Autoregressive	Time Series Regression
Root Mean Square Error	0.172531	0.172975
Mean Absolute Error	0.106115	0.106448
Theil Inequality Coef.	0.016037	0.016355
Bias Proportion	0.000626	0.002305
Variance Proportion	0.030822	0.052282

Investment	Vector Autoregressive	Time Series Regression
Root Mean Square Error	0.087303	0.148090
Mean Absolute Error	0.063060	0.115486
Theil Inequality Coef.	0.009407	0.016131
Bias Proportion	0.002347	0.018518
Variance Proportion	0.035894	0.038640

Government Expenditure	Vector Autoregressive	Time Series Regression
Root Mean Square Error	0.080538	0.093224
Mean Absolute Error	0.063062	0.076598
Theil Inequality Coef.	0.007824	0.009038
Bias Proportion	0.000002	0.011257
Variance Proportion	0.002746	0.007626

CONCLUSION

The time plots showed that each year there is a simultaneously increase in the values of government revenue, government expenditure and investment except a sharp downward shift in government revenue in 2001. While inflation fluctuate from year to year, cyclical in movement and a cycle is completed every three years. The Augmented Dickey Fuller tests shows that other series are stationary at the first differencing, I(1) and only inflation rates is stationary at the ordinary level, I(0). The Johansen co-integration test shows that there is at least one co-integrated series out of four economic series at 5% and 1%.

Vector Autoregressive and Time Series Regression models shows that there exist inter-relationship among all the macroeconomic variables but the relationship were better shown using *VAR* models.

The forecast from the vector autoregressive and time series regression models shows that there is a steady increase in the values of all the macro economic variables except inflation rates with sharp increase for the five years forecasted. While using the forecast performance, vector autoregressive models is better than the time series regression models since the forecast evaluation from the vector autoregressive models is less than that of time series regression models.

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