Measuring Human Development Index: The Old, The New and The Elegant

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(based on a paper with Hippu Salk Kristle Nathan)

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Presentation Format

- Focus of the Study
- The Three Measures
  - Linear average
  - Geometric Mean
  - Displaced Ideal
- The MANUSH Axioms
- Some Propositions
- Class of Measures
- Concluding Remarks

Focus of the study

- NOT rationale behind choosing the indicators
- NOT how the indicators are measured and scaled
- NOT how the indicators are normalized and weighed

INVESTIGATES the appropriateness of
the known two measures of HDI

proposes an alternative measure

Inverse of the Euclidean Distance from Ideal

HDI – *the old (till 2009)*

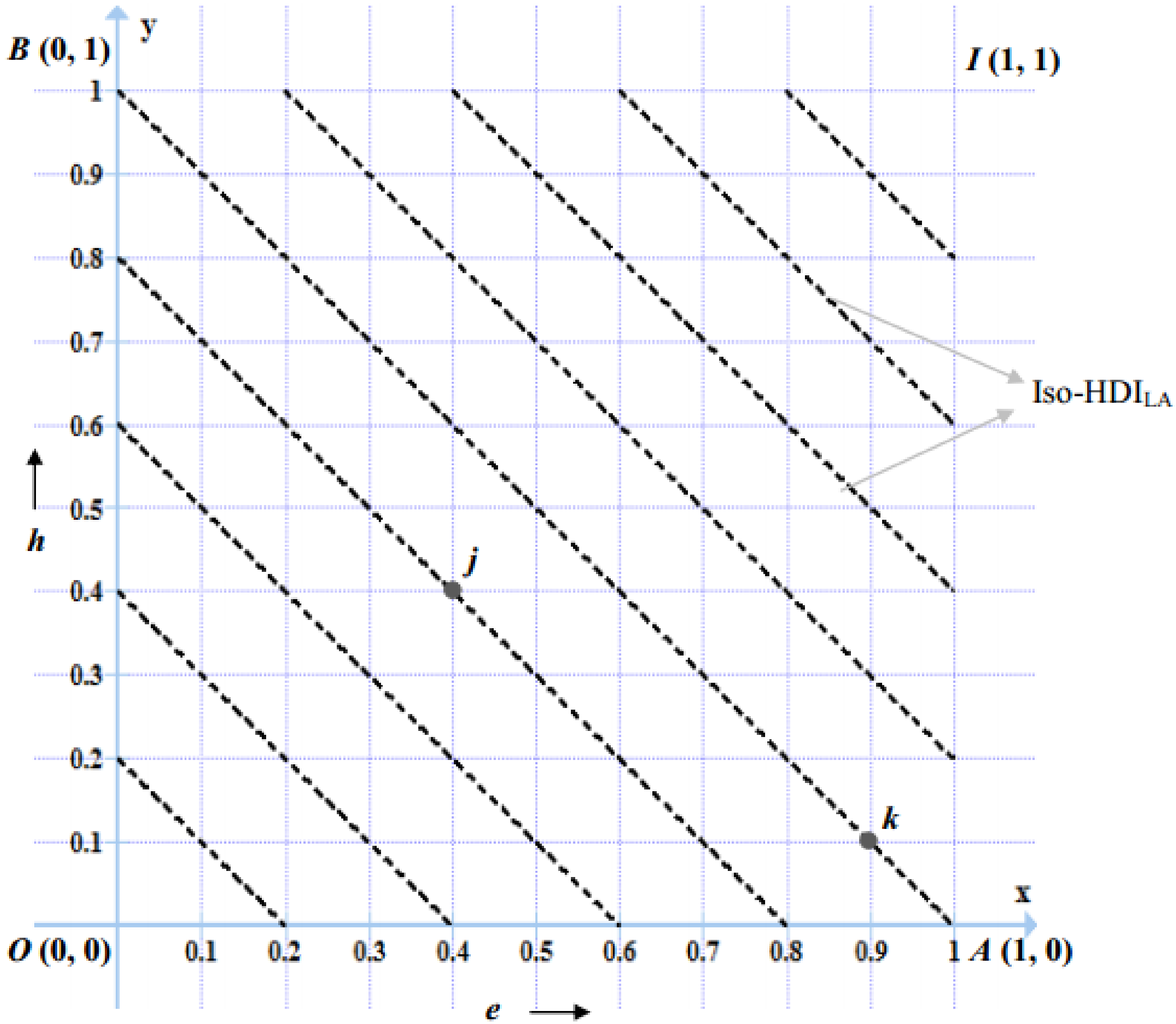
3 dimensions –

1. **A long and healthy life** } **Life Expectancy at birth** → ***h***
2. **Knowledge** } **Adult literary rate (2/3)**, **Gross enrolment ratio (1/3)** } → ***e***
3. **Ability to achieve decent standard living** } **GDP per capita (PPP)** → ***y***

$0 \leq h, e, y \leq 1$

$HDI_{LA} = 1/3\,(h) + 1/3\,(e) + 1/3\,(y)$

Iso-HDI lines – old HDI *(perfect-substitutability)*



HDI – *the new (from 2010)*

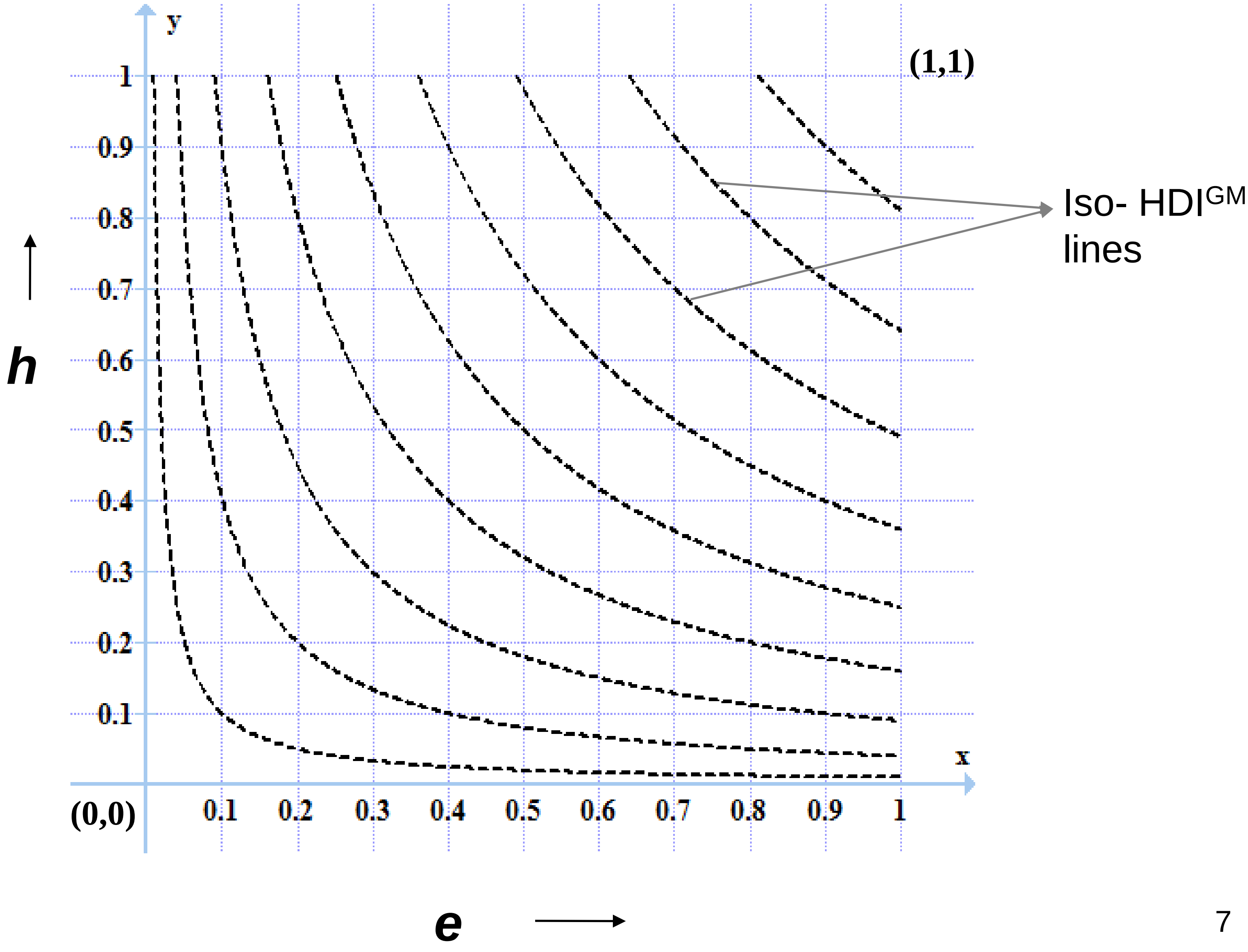
3 dimensions –

1. **A long and healthy life** } **Life Expectancy at birth** → ***h***
2. **Knowledge** } **Mean years of schooling: adults**; **Expected years of schooling: children** → ***e***
3. **Ability to achieve decent standard living** } **GNI per capita (PPP)** → ***y***

$0 \leq h, e, y \leq 1$

$HDI_{GM} = (h * e * y)^{1/3}$

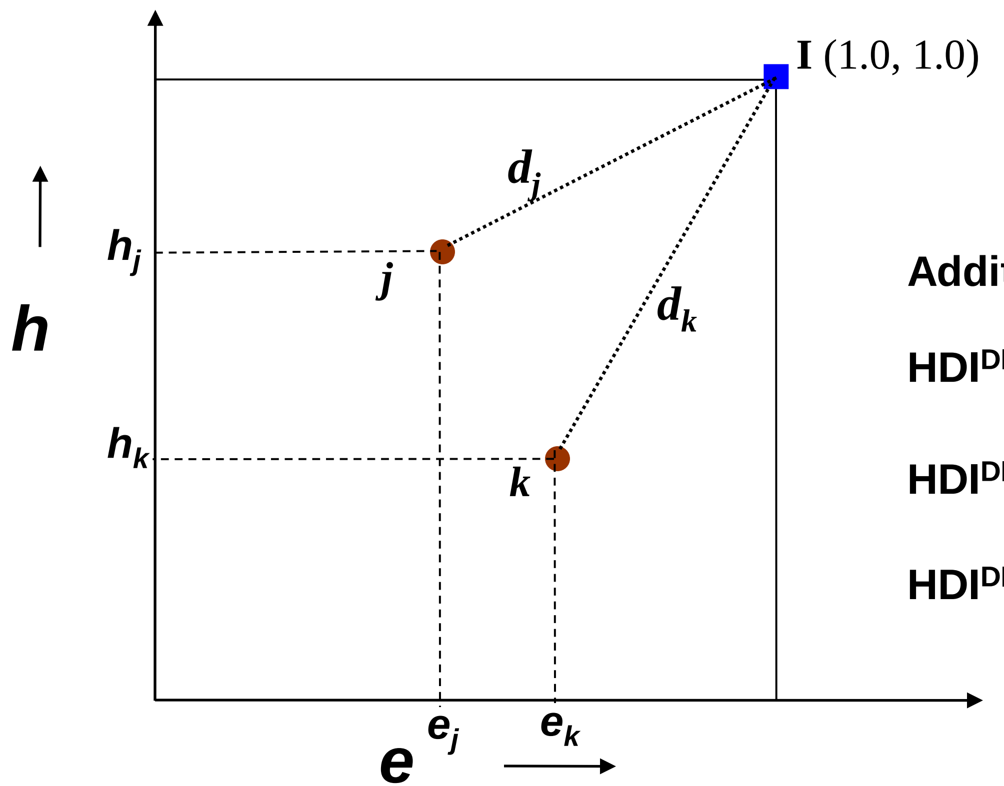
Iso-HDI lines – new method



Displaced Ideal

Zeleny (1974)

better system should have less *distance* from "ideal".



$$d_j = \sqrt{(1-e_j)^2 + (1-h_j)^2}$$

Additive Inverse of distance

$HDI^{DI}_j > HDI^{DI}_k$ *if and only if* $d_j < d_k$

$HDI^{DI}_j = HDI^{DI}_k$ *if and only if* $d_j = d_k$

$HDI^{DI}_j < HDI^{DI}_k$ *if and only if* $d_j > d_k$

HDI – *the elegant (proposed)*

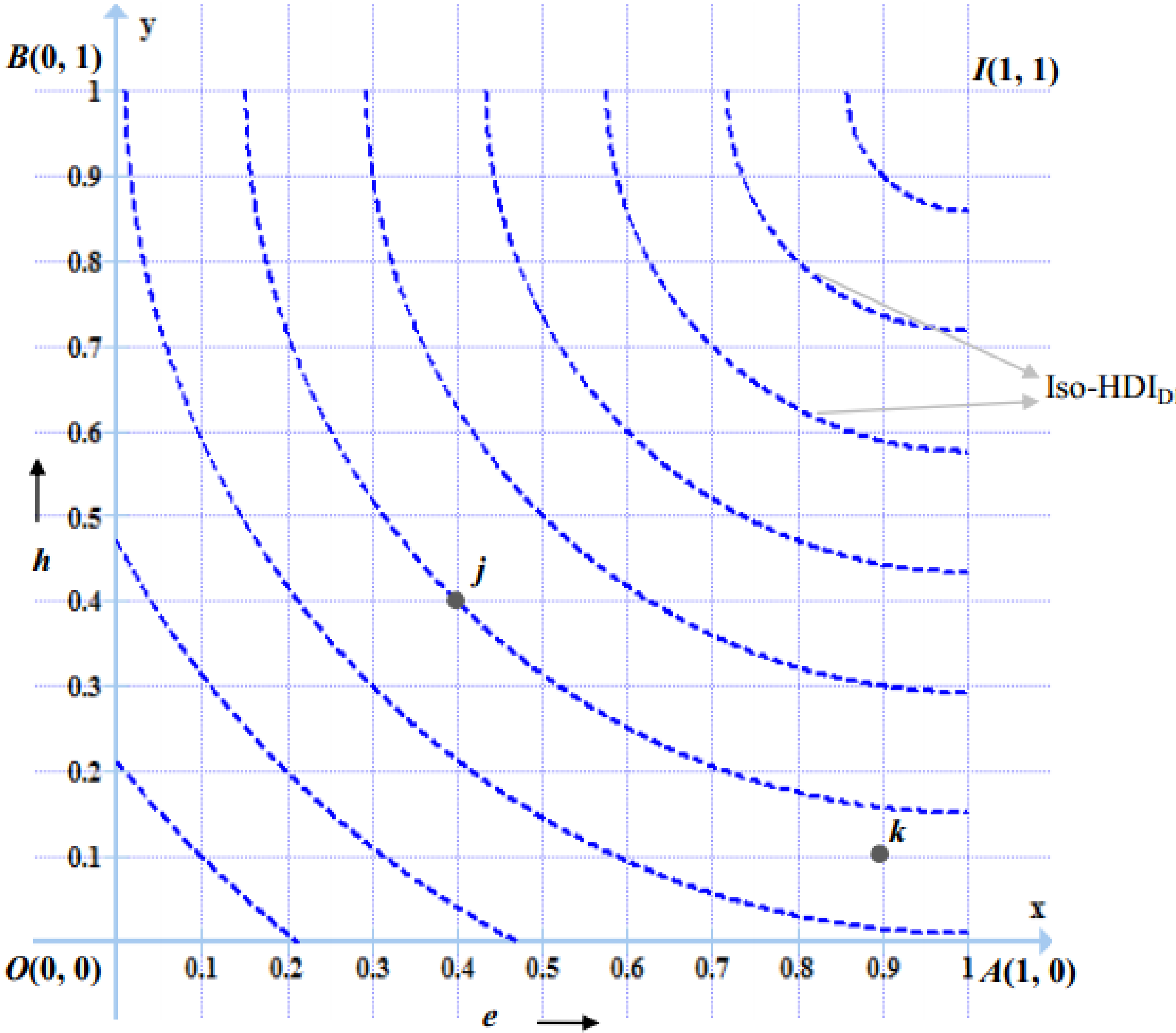
3 dimensions –

1. A long and healthy life } Life Expectancy at birth → ***h***
2. Knowledge } Mean years of schooling: adults; Expected years of schooling: children → ***e***
3. Ability to achieve decent standard living } GNI per capita (PPP) → ***y***

$0 \leq h, e, y \leq 1$

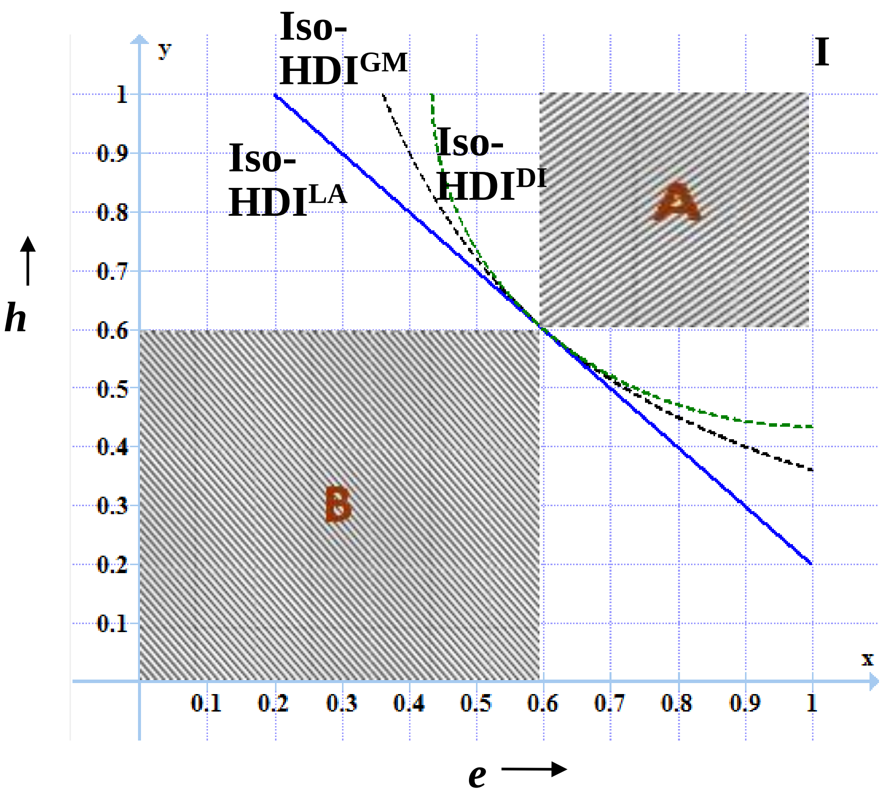
$$HDI_{DI} = 1-(\sqrt{((1-h)^2 + (1-e)^2 + (1-y)^2)}/\sqrt{3})$$

Iso-HDI lines – elegant method



Axiom M: Monotonicity

A measure of HDI should be greater (lower) if the index value in one dimension is greater (lower) with indices value remaining constant in all other dimension.



For any random country *k* in

Zone A $h_k \geq h_j$, $e_k \geq e_j$ (h_k=h_j or e_k=e_j)

LA: $HDI_k > HDI_j$

GM: $HDI_k > HDI_j$

DI: $HDI_k > HDI_j$

Zone B $h_k \leq h_j$, $e_k \leq e_j$ (h_k=h_j or e_k=e_j)

LA: $HDI_k < HDI_j$

LA: $HDI_k < HDI_j$

DI: $HDI_k < HDI_j$

LA, GM* and DI satisfy

* GM fails when any one dimension is zero

Axiom A: Anonymity

A measure of HDI should be indifferent to swapping of values across dimensions.

$$LA: h_j + e_j = h_{j'} + e_{j'}$$

$$GM: h_j * e_j = h_{j'} * e_{j'}$$

$$DI: d_j = d_{j'}$$

LA, GM and DI satisfy Anonymity

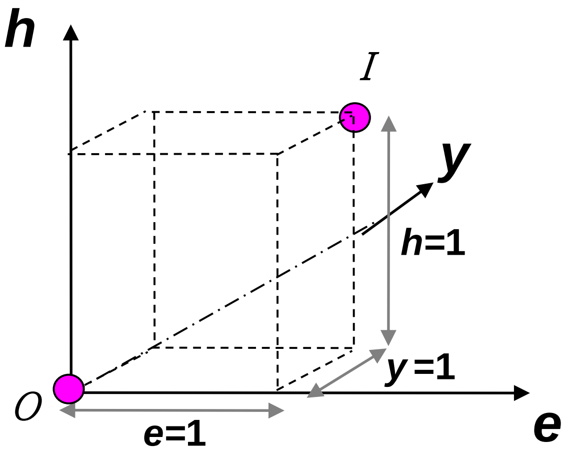
Note that this is a statistical property of symmetry and does not invoke substitution between dimensions

Axiom N: Normalization

A measure of HDI should have a minimum and a maximum i.e. HDI $\in$ (0,1)

HDI = 0: NO development (***h*** **= 0**, ***e*** **= 0**, ***y*** **= 0**) - “Origin”

HDI = 1: COMPLETE development (***h*** **= 1**, ***e*** **= 1**, ***y*** **= 1**) – “Ideal”



LA, GM and DI satisfy this;

for GM the value will be zero if any dimension has no development

Axiom U: Uniformity

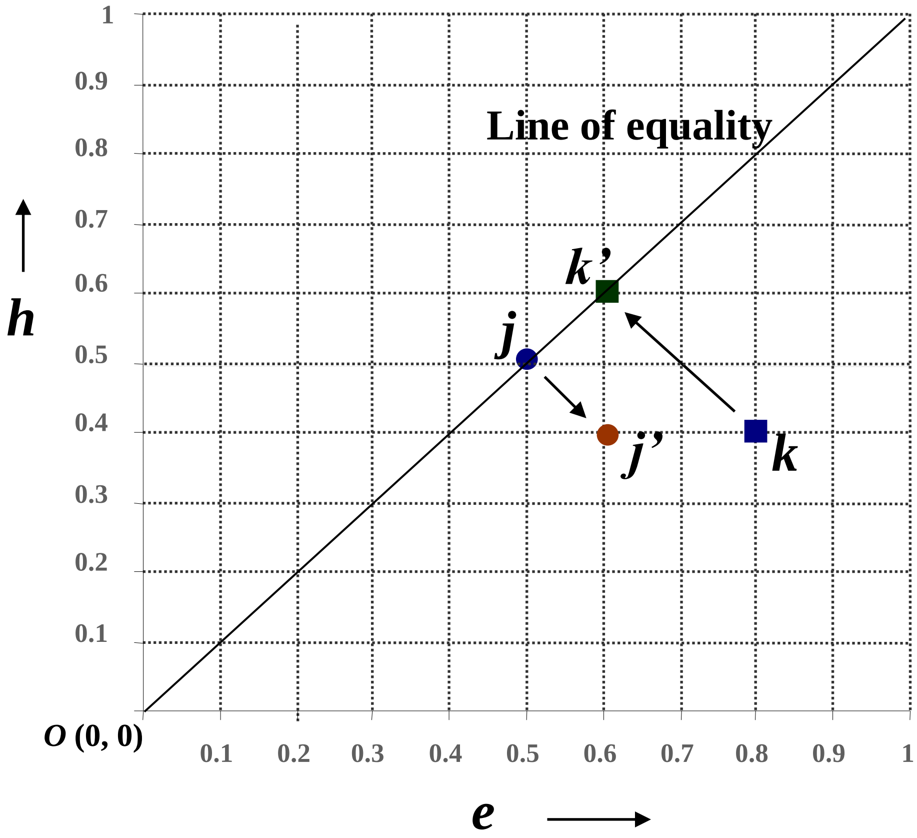


Illustration (1)

Uniform to Non-Uniform

$j(0.5, 0.5)$ $d_j = \sqrt{(0.50)}$ $GM = \sqrt{(0.25)}$

$j'(0.6, 0.4)$ $d_{j'} = \sqrt{(0.52)}$ $GM = \sqrt{(0.24)}$

Change in HDI:

$HDI^{LA}_{j} = HDI^{LA}_{j'}$

$HDI^{GM}_{j} > HDI^{GM}_{j'}$

$HDI^{DI}_{j} > HDI^{DI}_{j'}$

Illustration (2)

Non-Uniform to Uniform

$k(0.8, 0.4)$ $d_k = \sqrt{0.80}$ $GM = \sqrt{(0.32)}$

$k'(0.6, 0.6)$ $d_{k'} = \sqrt{0.72}$ $GM = \sqrt{(0.36)}$

Change in HDI:

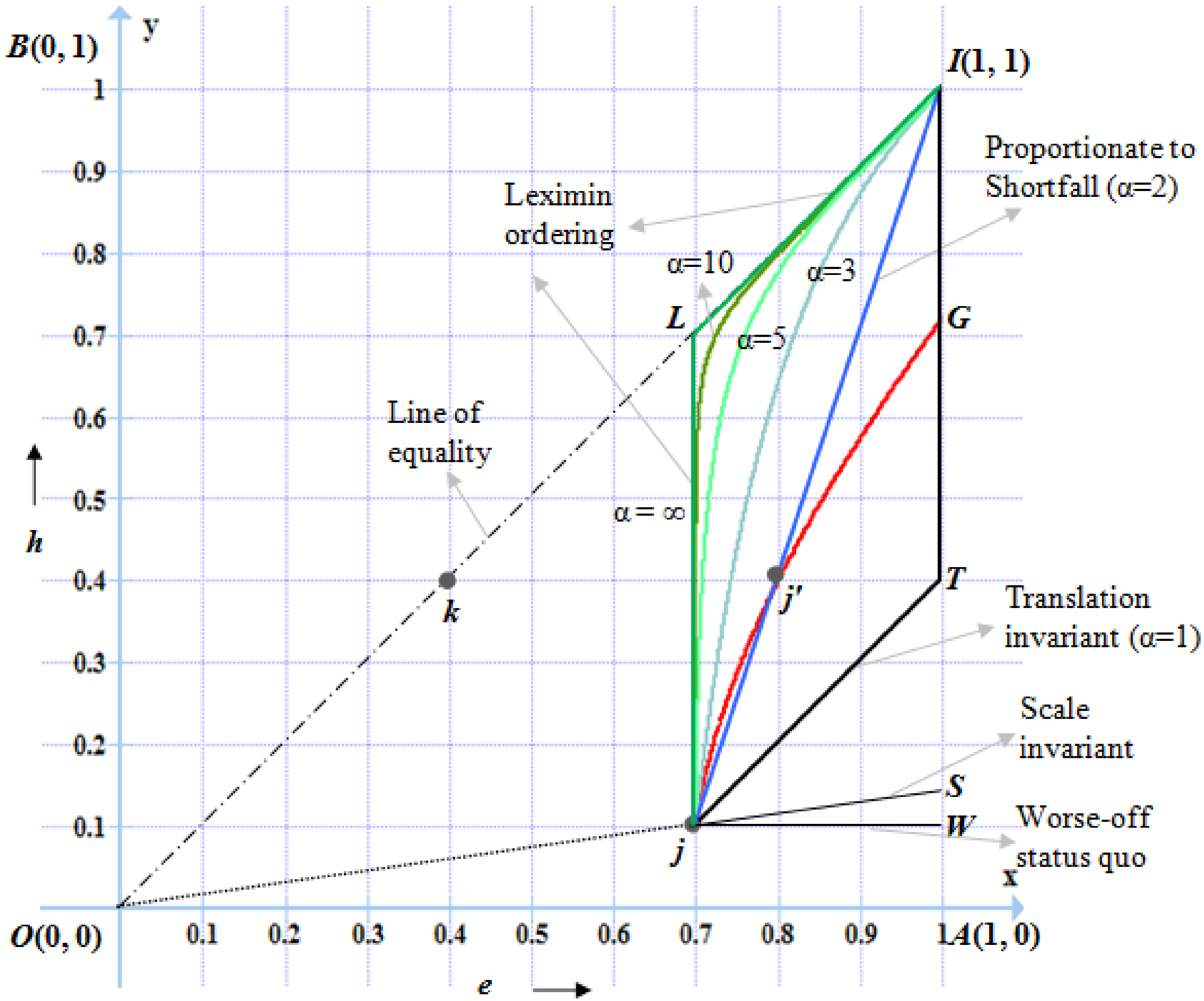
$HDI^{LA}_{k} = HDI^{LA}_{k'}$

$HDI^{GM}_{k} < HDI^{GM}_{k'}$

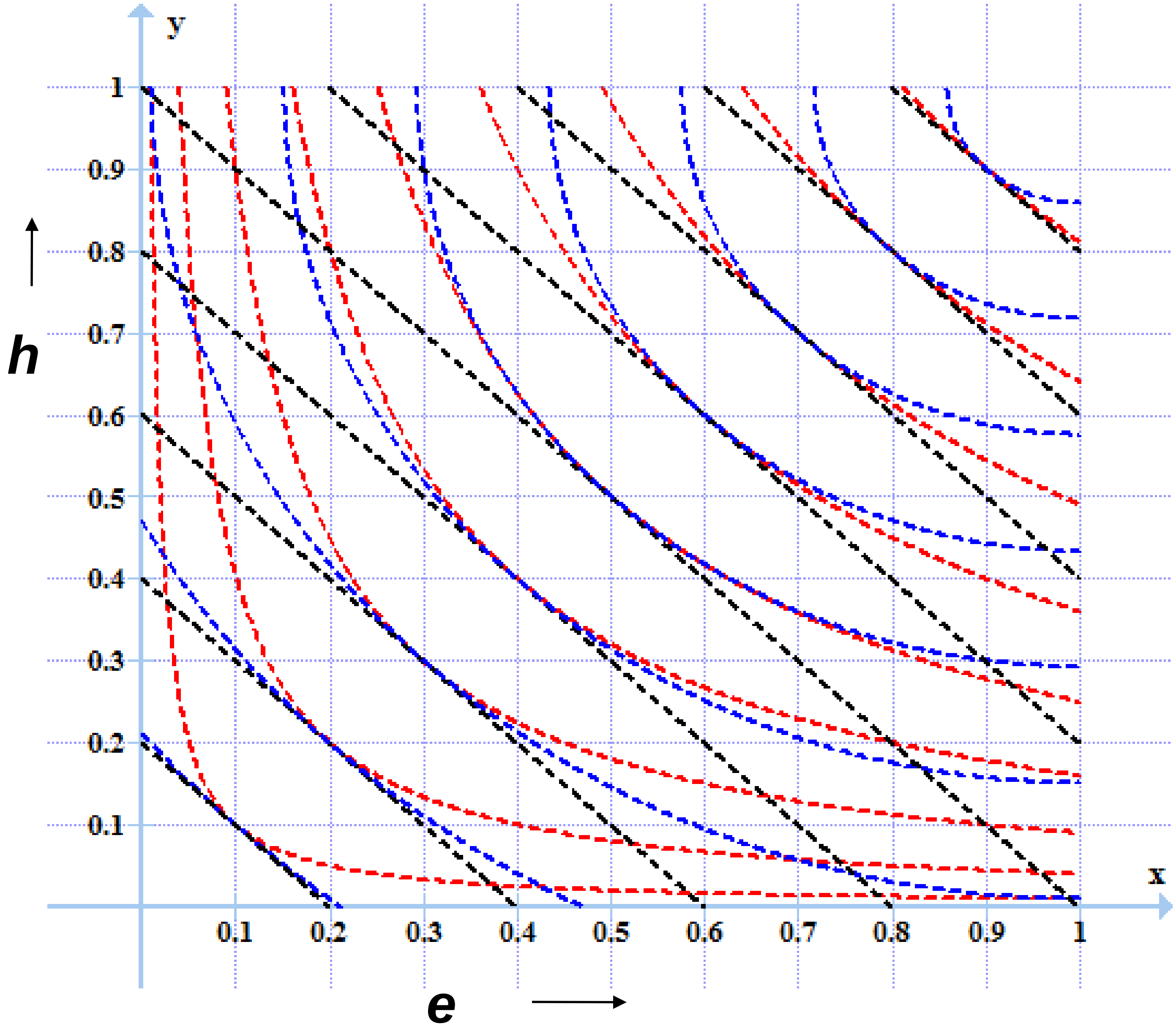
$HDI^{DI}_{k} < HDI^{DI}_{k'}$

LA fails,
GM and DI satisfy

Axiom S: Shortfall Sensitivity



Axiom H: Hiatus sensitivity



Equal gap at higher attainment should be considered worse off

LA and GM fail

DI satisfies

MANUSH Axioms: A Comparison

M: Monotonicity

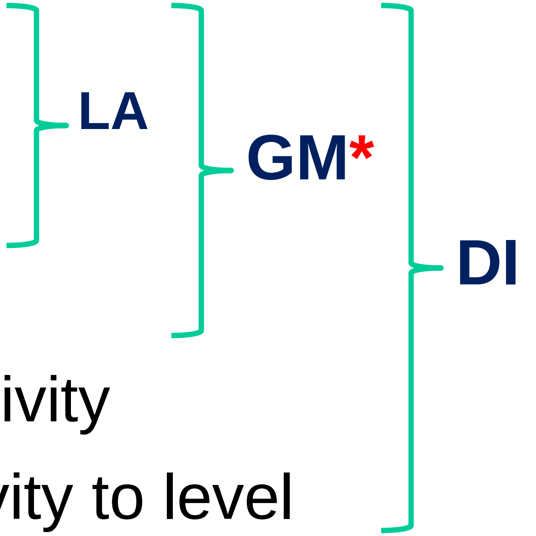
A: Anonimity

N: Normalization

U: Uniformity

S: Shortfall sensitivity

H: Hiatus Sensitivity to level



Perfect Substitution vs Uniformity

- A measure of HDI cannot satisfy perfect substitutability and uniformity simultaneously
- If a measure satisfies perfect substitutability then it will not change for a given mean even if deviation across dimensions change. As against this, uniformity demands that the measure decreases as deviation increases for a given mean.

Hiatus sensitivity to level vs Proportionate deviation

- A measure of HDI cannot satisfy hiatus sensitivity to level and also penalize proportionate deviation of a given gap from uniformity simultaneously.
- The former suggests that the same gap at a higher average attainment should be considered worse off. The latter would imply that at a higher attainment the same absolute deviation would be identified with a lower proportionate deviation, and hence, acceptable.

Class of Measures

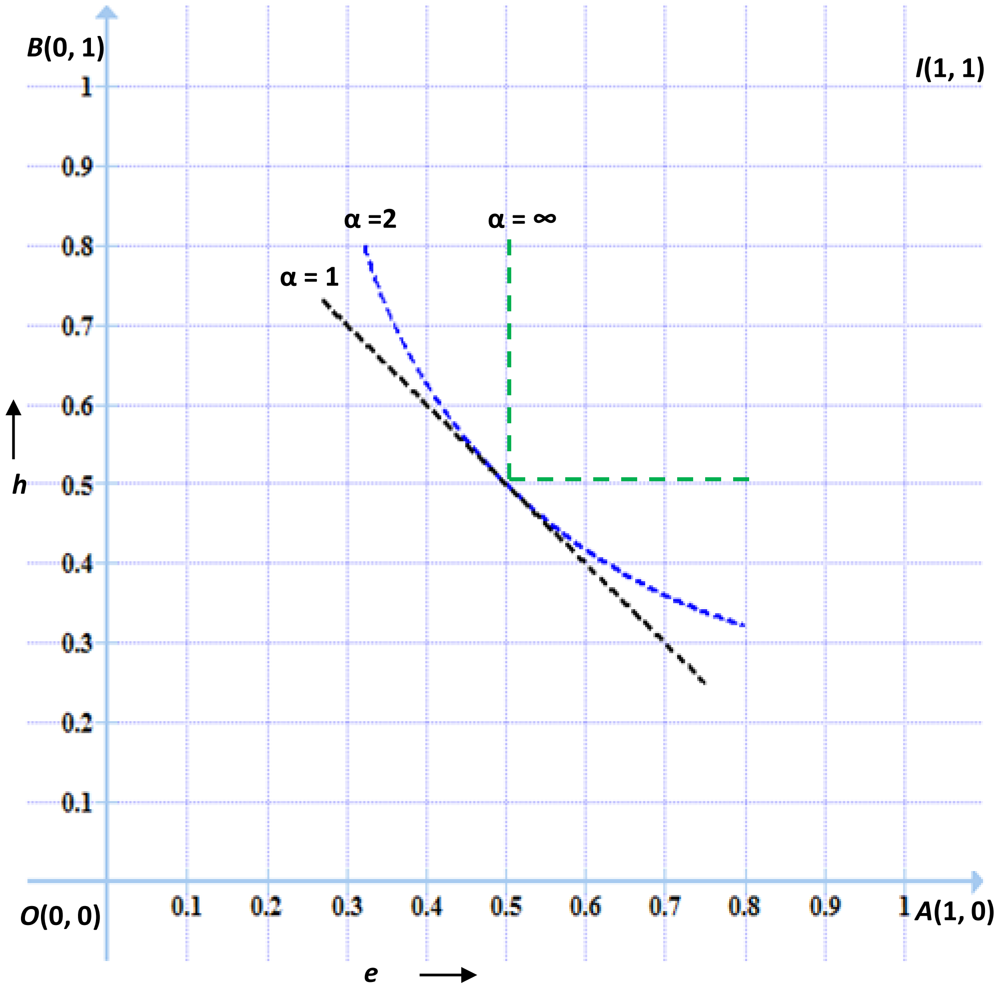
$\mathcal{H}_\alpha = 1-(((1-h)^\alpha+(1-e)^\alpha+(1-y)^\alpha)/3)^{(1/\alpha)}$

$\mathcal{H}_\alpha$ special case at

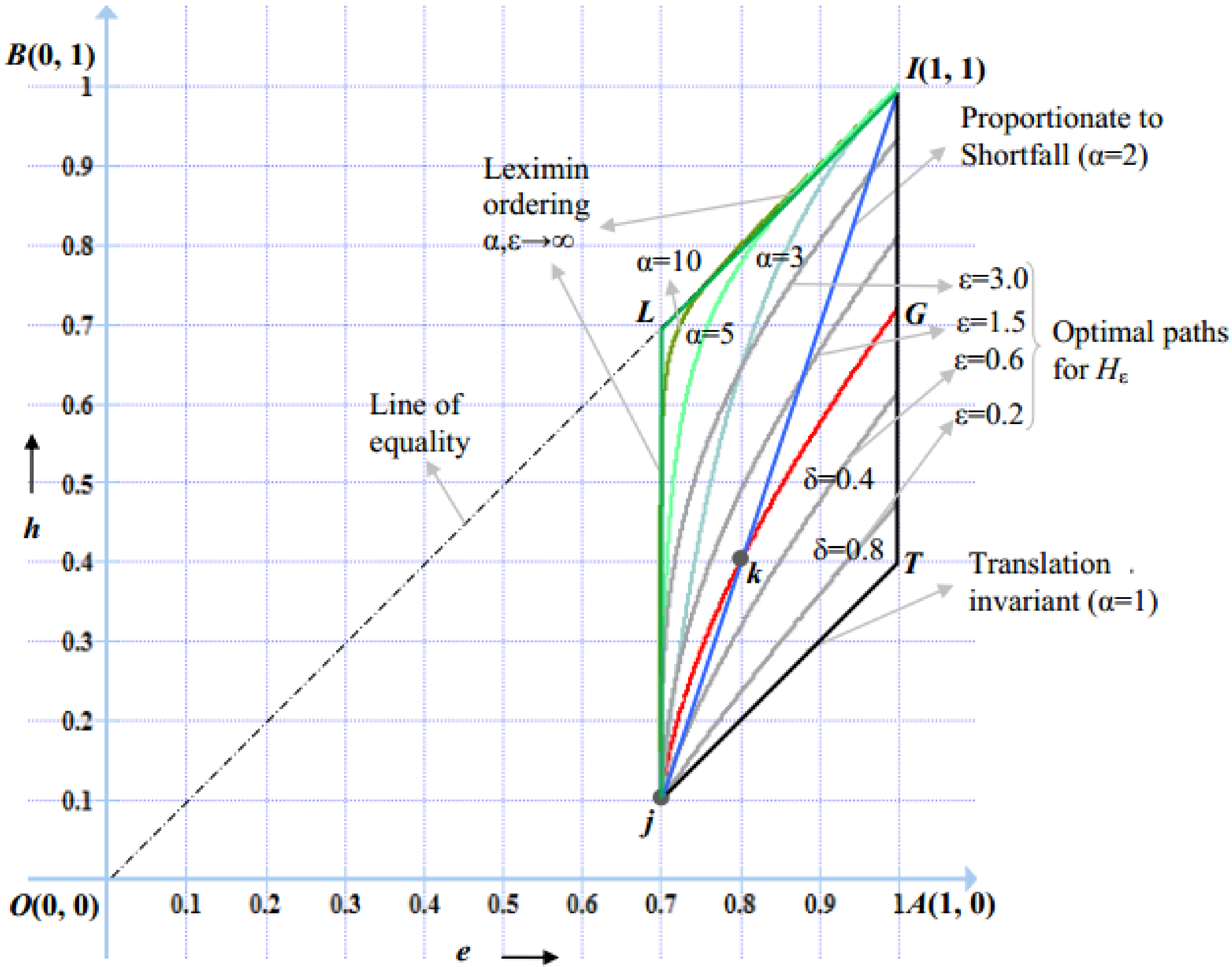
- $\alpha=1$, $\mathcal{H}_\alpha=HDI_{LA}$
- $\alpha=2$, $\mathcal{H}_\alpha=HDI_{DI}$
- $\alpha=\infty$, $\mathcal{H}_\alpha$ is equal to a Rawlsian leximin ordering

MANUSH is necessary and sufficient for $\mathcal{H}_\alpha$; $\alpha\geq2$

Iso-HDI: Class of Measures



Shortfall Sensitivity



Necessary and Sufficient

- MANUSH is necessary and sufficient for $\mathcal{H}_{\alpha}$; α ≥2
  - It is easy to deduce that MANUSH is a necessary condition
  - For MANUSH to be sufficient, we should have an alternative measure, $\mathcal{M}$, or class of measures, that satisfies the axioms. Now, when $\mathcal{M}$ satisfies shortfall sensitivity then the optimal paths are equivalent to that of $\mathcal{H}_{\alpha}$.

Concluding Remarks

- Evaluated three methods of aggregation for measuring HDI
- The proposed displaced ideal method is sensitive to shortfalls across dimensions and imposes greater equity consciousness at higher levels of attainment
- We propose an α-class of measures where the most stringent form of shortfall sensitivity can be identified with the Rawlsian scenario.
- The axioms of MANUSH (its anagram is HUMANS) turns out to be necessary and sufficient for the class of measures when $\alpha \geq 2$.
- The method articulated across dimensions can also be relevant in other contexts – say, across sub-groups.

References

- This paper is based on a working paper available at http://www.igidr.ac.in/pdf/publication/WP-2013-020.pdf.

Some selected references are given below.

- Anand, S and Sen, A (1994) Human Development Index: Methodology and Measurement, Human Development Report 1994.
- Mishra, Srijit and Nathan, Hippu Salk Kristle (2008) On A Class of Human Development Index Measures, WP-2008-020, Indira Gandhi Institute of Development Research, Mumbai.
- Nathan, Hippu Salk Kristle and Mishra, Srijit (2010), Progress in Human Development: Are we On the Right Path? *International Journal of Economic Policy in Emerging Economies*, Vol.3. No. 3, 199-221.
- Nathan, Hippu Salk Kristle, Mishra, Srijit and Reddy, B. Sudhakara (2008), An Alternative Measure of HDI, WP-2008-001, Indira Gandhi Institute of Development Research, Mumbai
- Nathan, Hippu Salk Kristle and Mishra, Srijit (forthcoming), Group Differential for attainment and failure indicators, *Journal of International Development*.
- UNDP (2008), *Human Development Report 2007/08: Fighting Climate Change: Human Solidarity in a Divided World*, Oxford University Press, New Delhi
- UNDP (2010), *Human Development Report 2010: The Real Wealth of Nations; Pathways to Human Development*, Oxford University Press, New Delhi