ESSAY

MEASURING PARTISAN FAIRNESS: HOW WELL DOES THE EFFICIENCY GAP GUARD AGAINST SOPHISTICATED AS WELL AS SIMPLE-MINDED MODES OF PARTISAN DISCRIMINATION?

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The efficiency gap has recently been touted as a general partisan fairness measure with the ability to "neatly slice the Gordian knot the Court has tied for itself, explicitly replying to the Court's 'unanswerable question' of '[h]ow much political . . . effect is too much.'"[1] The measure was endorsed by the district court in *Whitford v. Gill*,[2] and is currently on appeal to the Supreme Court. The plaintiffs in *Whitford*, based on a forty-two-year analysis of state legislative elections, have proposed a "conservative" 7% threshold on the efficiency gap as a standard for judging the constitutionality of partisan gerrymandering.[3] If the Court adopts the efficiency gap as a partisan fairness measure, this decision could have far-reaching implications for redistricting practices and litigation.

We examine the properties of the efficiency gap as a measure of partisan unfairness. With the right to vote, the Court has consistently sought to guard

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1 Nicholas O. Stephanopoulos and Eric M. McGhee, *Partisan Gerrymandering and the Efficiency Gap*, 82 U. CHI. L. REV. 831, 832 (2015).

2 *See* Whitford v. Gill, No. 15-cv-421-bbc, 2016 WL 6837229, at *50 (W.D. Wis. Nov. 21, 2016) (using the efficiency gap to demonstrate that the plaintiff's "representational rights have been burdened").

3 *See* Expert Report and Affidavit at 5-6, Whitford v. Nichol, No. 3:15-cv-00421, 2015 WL 10091020, (W.D. Wis. Jul. 7, 2015) ("My analysis suggests that *EG* greater than .07 in absolute value be used as an actionable threshold.").

against sophisticated as well as simple-minded modes of discrimination.[4] We explore whether the efficiency gap is up to this task by first considering the mathematical properties of the efficiency gap. Does it provide a consistent and stable interpretation across electoral maps (for different states, for different types of elections, e.g. Congress or state houses, and for different time periods), allowing us to compare the value of partisan fairness from one map to another? We then evaluate whether the measure is impervious to the data that is used to compute it. We finally explore its ability to tap the concepts of responsiveness and bias, separate and important facets of partisan fairness.

I. CALCULATING THE EFFICIENCY GAP

The efficiency gap seeks to capture the difference in wasted votes between two parties in an election. If a simple majority is needed to win an election, then every vote cast beyond 50% for the winning candidate is a wasted vote. In addition, every vote cast for the losing candidate is a wasted vote since losing votes also do not translate into an election win and might have been parceled to a different district where they could have been used toward winning a seat.

A hypothetical election scenario provided in prior literature,[5] and shown in Table 1, quickly demonstrates how to calculate the efficiency gap. In this example, we have 10 districts with 100 total voters in each district. Party A receives 70 votes in each of Districts 1–3, 54 votes in each of Districts 4–8, and 35 votes in Districts 9 and 10. Party B receives 30 votes in Districts 1–3, 46 votes in Districts 4–8, and 65 votes in Districts 9 and 10. Since 50 votes are required to win the election in any one district, Party A wastes 70 – 50 = 20 votes in Districts 1–3 and 54 – 50 = 4 votes in Districts 4–8. Because Party A lost the election in Districts 9 and 10, Party A wastes all of its 35 votes in those two districts. Similarly, Party B's wasted votes are calculated and shown in the last column of Table 1. In total, Party A wastes 150 votes while Party B wastes 350 votes. The difference in wasted votes is 350 – 150 = 200. Since there are 1000 total votes, the efficiency gap is 200/1000 = 20%. The substantive interpretation is that Party A won 20% (and, in this case, two) more seats than it would have if the two parties had wasted an equal number of votes.

4 *See* Gomillion v. Lightfoot, 364 U.S., 339, 342 (1960) (quoting Lane v. Wilson, 307 U.S. 268, 275 (1939)) (explaining the purpose of the 15th Amendment).

5 Stephanopoulos and McGhee, *supra* note 1, at 852.

Table 1: Illustration of the Efficiency Gap

District	Total Votes		Wasted Votes	
	Party A	Party B	Party A	Party B
1	70	30	20	30
2	70	30	20	30
3	70	30	20	30
4	54	46	4	46
5	54	46	4	46
6	54	46	4	46
7	54	46	4	46
8	54	46	4	46
9	35	65	35	15
10	35	65	35	15
Total	550	450	150	350

The example is easy to understand, and the measure is simple to calculate. If W_A is the number of wasted votes from Party A, W_B is the number of wasted votes from Party B, T_A is the total number of votes cast by Party A, and T_B is the total number of votes cast by Party B, then the efficiency gap, EG, is simply:

$$EG = \frac{W_A - W_B}{T_A + T_B} \tag{1}$$

If we assume that all districts are equal in population and that there are only two parties, McGhee and Stephanopoulos further simplify the calculation to:

$$EG = S - 2V, \tag{2}$$

where S is the seat share[6] held by a party minus 50%, and V is the vote margin[7] for the same party minus 50%.[8] The concept of the efficiency gap measure is

6 Seat share refers to the percentage of the seats held by one party. If there are ten total seats and two of them are held by Party A, then Party A's seat share is 2/10 or 20%.

7 Vote margin refers to vote percentage held by the same party. If we are calculating a two-party vote, and 65% of the total vote is cast for Party A, and 35% is case for Party B, then the vote margin for Party A is 65%.

8 Eric McGhee, *Measuring Partisan Bias in Single-Member District Electoral Systems*, 39 LEGIS. STUD. Q. 55, 68-69 (2014); Stephanopoulos and McGhee, *supra* note 1, at 853.

simple to articulate and may seem intuitive, but its properties have not been rigorously explored.

II. COMPARING EFFICIENCY GAPS ACROSS JURISDICTIONS

We first examine whether the efficiency gap is able to satisfy a basic and uncontroversial requirement for a partisan fairness measure: the ability to produce a value that is comparable across electoral plans. In a two-party system, one intuitive and effective way of structuring a measure that is comparable across plans is to let the measure take on values in a fixed range, such as $[-x, x]$, where one extreme, $-x$, signals maximum unfairness to one party and the other extreme, x, signals maximum unfairness to the other party. A plan that is symmetrically fair to both parties would then yield a value of zero. If this measure is independent of and not idiosyncratically tied to the underlying population, then it can be used to compare districting plans to each other. That is, if we are comparing one state to another state, the range of possible values in each of the states is the same $[-x, x]$. Rhode Island should not have a different range because its population is smaller than California. In addition, in states where the partisan split favors Republicans (e.g. Wyoming), the range should be the same as in states where the partisan split favors Democrats (e.g. Massachusetts). Finally, the number of districts should not matter—an *actionable* efficiency gap for congressional maps should be the same value as for state legislative maps.

Proponents of the efficiency gap claim that it "is useful for evaluating fairness across a range of plans, even ones in which one party significantly outperforms the other."[9] Through the series of hypothetical districts shown in Table 2 below, we consider how well the efficiency gap fares across a range of plans where the degree to which one party outperforms the other varies. Each scenario has 200 total voters with 100 voters in each of two districts.[10] The distinguishing characteristic between the scenarios is the particular partisan split of voters. In scenario one, there are many more Party A voters than Party B voters. In scenario seven, there is an almost equal number of Party A and Party B voters.

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9 Stephanopoulos and McGhee, *supra* note 1, at 863.

10 In their analysis, Stephanopoulos and McGhee limit their study to states with at least eight congressional districts. *Id.* at 868. As we will see, this reduces the volatility that arises with smaller delegations. A *general* measure of partisan fairness should, however, work for any size delegation. We start with two districts because it is easier to see the properties of the efficiency gap with smaller delegations. Substantively, we also see a compelling case for examining small delegations, since two seats is not unusual for states (24% of the states have two or fewer House seats). Stephanopoulos and McGhee examine only congressional plans with eight or more districts "because redistricting in smaller states has only a minor influence on the national balance of power." *Id.* at 868. While larger states have more districts, restricting the analysis to states with eight or more Members of Congress removes data from twenty-nine (or 58% of) states. If the efficiency gap calculation is not viable for any size delegation, this is indicative of underlying measurement issues.

A. *Limited Efficiency Gap Range*

For any voting scenario, the efficiency gap is a discrete, not a continuous variable in the range [–0.5, 0.5]. Importantly, and unintuitively, it does not take on all possible values in this range. In fact, if more than fifty votes are required to win any one district, then in the first four scenarios, the value of the efficiency gap shown is the *only* possible efficiency gap value. Regardless of redistricting, because of the constraint that there are 200 total voters, with 100 in each district, the value of the efficiency gap is fixed at one single value.

Table 2: Comparing Scenarios

Scenario		Party A Votes	Party B Votes	Party A Wasted Votes	Party B Wasted Votes	Efficiency Gap	Possible Efficiency Gap Values	Party A Winning Efficiency
		Party A near maximum inefficiency						
1.		99	1	49	1	0.48	{0.48}	0.49
		99	1	49	1			
2.		90	10	40	10	0.30	{0.30}	0.40
		90	10	40	10			
3.		80	20	30	20	0.10	{0.10}	0.30
		80	20	30	20			
		optimal efficiency gap fairness						
4.		75	25	25	25	0.00	{0.00}	0.25
		75	25	25	25			
5.		70	30	20	30	-0.10	{-0.10, 0.40}	0.20
		70	30	20	30			
6.		60	40	10	40	-0.30	{-0.30, 0.20}	0.10
		60	40	10	40			
		Party B near maximum inefficiency						
7.		51	49	1	49	-0.48	{-0.48, 0.02}	0.01
		51	49	1	49			

increasingly competitive districts ↓

In scenarios 1–4, Party B does not have enough votes to win any one district, hence, Party B always loses both districts, and Party A always wins both districts. This means that Party B always wastes all of its votes and Party A always wastes ((Total Party A votes) – 50 – 50) votes, since it needs a minimum of 50 votes to win each district and any votes above 50 are excess winning votes. The total number of wasted votes is always exactly half of the total votes. How we redistrict within these partisan scenarios is inconsequential—the value of the efficiency gap does not change.

For scenarios 5–7, the efficiency gap can take on only one of two possible values. For scenario 5, the efficiency gap can be either -0.10 or 0.40. For scenario 6, the efficiency gap can be either -0.30 or 0.20. For scenario 7, the efficiency gap can be either -0.48 or 0.02. Why are there two values for scenarios 5–7 but only one value for scenarios 1–4? The change arises because in scenarios 5–7, Party B has enough votes to win a maximum of one of the districts, while in scenarios 1–4, the only possible outcome is that Party A wins both districts.[11] The first possible efficiency gap value for scenarios 5-7 occurs when Party A wins both districts. The second possible efficiency gap value occurs when Party A and Party B split the districts. Since the efficiency gap treats votes cast by the winning party over 50% and all of the votes cast by the losing party as one in the same, exactly the same number of votes are wasted when the parties split the districts. Again, once the overall partisan and seat split is known, redistricting does not alter the possible efficiency gap values.

B. *Different Efficiency Gap Ranges for Different Partisan Splits*

Because the different scenarios yield different possible values for the efficiency gap, comparing across scenarios is problematic. Redistricting will not affect the efficiency gap in scenario 1. It is always 0.48 (since Party B always wastes two votes, and Party A always wastes ninety-eight votes). Scenario 1 is simply not comparable to scenario 7, where the only possible efficiency gap value is either -0.48 or 0.02. Certainly, within scenario 7, we can say that we would prefer an efficiency gap of 0.02 to a gap of -0.48, but we cannot compare scenario 7's value to scenario 1's value. We can say that both scenarios 1 and 7 are at maximum inefficiency *given* their partisan

11 One can develop the intuition for why the efficiency gap takes on only a small set of values if one simply plays with a few scenarios. This result is not obvious when one is told that the efficiency gap captures wasted votes, but it becomes self-evident with some simple examples. In addition, while the efficiency gap presented in equation (1) does not convey the granularity of the measure, the equivalent expression in equation (2) makes this result obvious, since S and V can take on only limited values.

distribution of voters. However, the values of the maximum efficiency gaps are not identical and not comparable on an absolute or fixed scale.

Since the particular partisan divide restricts the possible values of the efficiency gap, comparisons of efficiency gap values across states, over different years, and whenever the underlying voter population changes, are problematic. That is, we must be wary of comparing, for example, the efficiency gap in California to the efficiency gap in Wisconsin. Because the population base changes due to migration (indeed, this is why we redistrict), which presumably affects the partisan divide, we must also be wary of comparing the efficiency gap in Wisconsin from one year to another.

If one wants to make efficiency gap comparisons, minimally, the measure must be standardized in some way.[12] It can be defensible to prefer that scenario 7 have an efficiency gap of 0.02 rather than 0.48, but it does not follow that we can claim that scenario 6 should also have an efficiency gap of 0.02 to match the level of efficiency in scenario 7. With a partisan divide of 60–40 and two districts, scenario 6 can only have an efficiency gap value of either -0.30 or 0.20. No matter how we redistrict, scenario 6 cannot be made any *more efficient* than 0.20. Thus an efficiency gap of 0.20 for scenario 6 and an efficiency gap of 0.02 for scenario 7 are more similar than implied by the different numerical values, which are on mismatched scales.

A measure of partisan fairness needs to be more nuanced, and cognizant of the partisan context. It must also ensure that the numbers used in any calculation reflect the appropriate underlying entity. For example, in these scenarios, the Winning Efficiency (shown in the last column of Table 2) counts only excess *winning* votes rather than conflating the excess winning votes with losing votes. The numbers for the winning efficiency decline from scenario 1 to 7, as they should, while those for the efficiency gap do not. The winning efficiency measure is not perfect either, but it does fix this one shortcoming of the efficiency gap. In the efficiency gap measure, both excess winning votes and losing vote are considered the same, but waste needs to be defined by its context. Equating unequal waste is mathematically problematic, and outside the spirit of fairness conceived by the courts to protect the rights of voters.

12 Standardization refers to the process of converting values to the same scale. When values are not on the same scale, the numbers are not comparable. For instance, if two values both measure temperature, but one is measured in Celsius while the other is in Fahrenheit, then it may seem that 0 does not equal 32 because the numbers are not the same. However, once these values are standardized or put on the same scale, we can make a proper comparison.

III. COMPARING EFFICIENCY GAPS WITHIN A JURISDICTION

Do comparability issues occur when we only look at a fixed population jurisdiction? To answer this question, we delve further into two issues that we have identified for comparison across jurisdictions. Here, we have two observations.

First, the efficiency gap seeks to identify symmetry, but even within jurisdictions, it still equates excess winning votes with losing votes, which are unequal entities. This can be seen with a hypothetical two-district jurisdiction. Table 3 illustrates different ways in which we can redistrict a fixed population of 120 Party A voters and 80 Party B voters (60-40 split) into two districts. Scenario 1 creates two safe Party A districts while scenarios 2 and 3 create one very safe Party A district and 1 more competitive district that leans toward Party A. In the redistricting world, these are distinct scenarios, with very different political implications. From the perspective of the efficiency gap, however, these scenarios are identical and they are symmetrically or equally fair to the two parties. Similarly, the efficiency gap equates scenarios 4 and 5 even though these scenarios are likely to produce different political outcomes. Scenario 4 has two lopsided elections while scenario 5 has one lopsided election and one competitive election. The core problem is equating excess winning votes with losing votes.

Our second point is that, even for a fixed jurisdiction, the efficiency gap may, but still does not necessarily, span the range [−0.5, 0.5]. Equating excess winning votes with losing votes still limits the range of the efficiency gap. If we let A_i be Party A's votes in District i and B_i be Party B's votes in District i. Then, $A_i + B_i = p$, where p is the population in District i. Because of the one-person, one-vote mandate,[13] p can be regarded as the same across all k districts. Since Party A or Party B wins each election, for two districts, the potential outcomes are {AA, AB, BA, BB}. For the partisan distribution in Table 3, Party B does not have enough votes to win two elections, so outcome BB is not possible. In addition, while wins affect the efficiency gap, the *specific* districts that are won do not affect the efficiency gap. That is, mathematically, outcomes AB and BA are equivalent. Hence, there are only two possible outcomes, either Party A wins both districts (AA) or else the parties split the districts (AB/BA). In the case of AA (scenarios 1–3 in Table 3), Party A *always* wastes 20 votes (the number of votes over 50%) and Party B *always* wastes 80 votes (all of its votes). In the case of AB or BA (scenarios 4–5 in Table 3), Party A *always* wastes 120 − 50 = 70 votes, and Party B *always* wastes 80 − 50

13 *See* Reynolds v. Sims, 377 U.S. 533, 566 (1964) ("Diluting the weight of votes because of place of residence impairs basic constitutional rights under the Fourteenth Amendment....").

= 30 votes. The result is that no matter how redistricting occurs, the efficiency gap cannot take on a value other than –0.30 or 0.20 for this jurisdiction.

Table 3: Fixed Jurisdiction

Scenario	Party A Votes	Party B Votes	Efficiency Gap	Possible Efficiency Gap Values
	Equivalent EG Scenarios			
1.	60	40	-0.30	{-0.30,0.20}
	60	40		
2.	69	31	-0.30	{-0.30,0.20}
	51	49		
3.	55	45	-0.30	{-0.30,0.20}
	65	35		
	Equivalent EG Scenarios			
4.	80	20	0.20	{-0.30,0.20}
	40	60		
5.	49	51	0.20	{-0.30,0.20}
	71	29		

A. *Analyzing a Hypothetical Eight-District Jurisdiction*

The behavior of the efficiency gap measure that we observe with two districts holds regardless of the size of jurisdiction. Consider the eight-district jurisdiction shown in Table 4, where Party A has 55% of the total vote, and Party B has 45% of the total vote.[14] If Party A wins all eight districts, the efficiency gap is -0.40. Party B does not have enough votes to win all eight districts with this distribution of voters. However, if Party B wins seven of the districts, the efficiency gap will be 0.475. It does not matter whether these elections are competitive or not, or which seven elections Party B wins. The

14 We choose an example with eight districts because that is a delegation size that Stephanopoulos and McGhee deem substantively meaningful for redistricting. *See* Stephanopoulos and McGhee, *supra* note 1, at 868 ("[R]edistricting in smaller states has only a minor influence on the national balance of power.").

number of districts won by a party completely determines the efficiency gap value.

Table 4: Possible Efficiency Gap Values for Eight Districts with a 55–45% Partisan Split

Winner Set	Efficiency Gap
AAAAAAAA	-0.400
AAAAAAAB	-0.275
AAAAAABB	-0.150
AAAAABBB	-0.025
AAAABBBB	0.100
AAABBBBB	0.225
AABBBBBB	0.350
ABBBBBBB	0.475
BBBBBBBB	NA

Table 5: Three Different Ways to Redistrict a 55–45% Partisan Split into Eight Districts

Plan X				Plan Y				Plan Z			
Votes		Wasted Votes		Votes		Wasted Votes		Votes		Wasted Votes	
Party A	Party B	Party A	Party B	Party A	Party B	Party A	Party B	Party A	Party B	Party A	Party B
61	39	11	39	65	35	15	35	70	30	20	30
61	39	11	39	65	35	15	35	70	30	20	30
61	39	11	39	65	35	15	35	70	30	20	30
61	39	11	39	65	35	15	35	70	30	20	30
49	51	49	1	45	55	45	5	40	60	40	10
49	51	49	1	45	55	45	5	40	60	40	10
49	51	49	1	45	55	45	5	40	60	40	10
49	51	49	1	45	55	45	5	40	60	40	10
440	360	240	160	440	360	240	160	440	360	240	160
Efficiency Gap = 0.1				Efficiency Gap = 0.1				Efficiency Gap = 0.1			

We can see how redistricting and different levels of packing and cracking[15] are inconsequential to the efficiency gap with the three examples presented in Table 5 (still eight districts and a 55–45 partisan split). In each plan, Party A has the advantage in four districts, and Party B has the advantage in the other four districts. In Plan X, Party A has four safe districts. Party B has the advantage in the other four districts, but these districts are competitive since the margin is slight. In Plan Y, again there is a 4–4 split, but this plan seems more fair since both parties now have the same number of somewhat safe seats, though Party A's seats remain safer. In Plan Z, both Party A and Party B have clearly safe seats. While the competitiveness level differs between Plans X, Y, and Z, the efficiency gap is 0.10 for each plan, because in each scenario the seat split is 4–4. The vote splits in specific districts is inconsequential to the efficiency gap computation.

We have observed the properties of the efficiency gap across various scenarios. It should be no surprise that a general result holds true. For a state with k districts, if the number of Party A votes and the number of Party B votes is fixed, then there are exactly k unique possible values for the efficiency gap.[16]

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15 Packing refers to the practice of placing more voters in a district than is needed to win the election. Cracking refers to the practice of splitting a group of voters into different districts so that they do not have enough voters to win in a single district.

16 We show this result rigorously. Let the k-element set $\{W_1, W_2, \ldots W_k\}$ be the winner set where $W_i = A$ if Party A wins district i, and $W_i = B$ if Party B wins district i. The vote split is defined by d_a–d_b where d_a is the number of districts won by Party A, and d_b is the number of districts won by Party B, and either Party A or Party B wins each district, $d_a + d_b = k$.

Assume, without loss of generality, that Party A has more votes than Party B. In this case, each district can be won by either Party A or Party B, but Party B does not have enough votes to win all of the districts. Since the district split simply reflects the number of districts won, the number of possible district splits is the same as the number of ways to choose k objects with replacement from n distinct objects, where k is the number of districts and n is the number of parties. We need to subtract 1 for the case that is not possible. This leaves us with

$$\frac{(n+k-1)!}{k!\,(n-1)!} - 1 = \frac{(k+1)!}{k!} - 1 \tag{3}$$
$$= k$$

possible values of the efficiency gap.

For each of the k possible district splits, the efficiency gap is the same, regardless of the vote distribution in the individual districts and regardless of which specific districts were won or lost. That is, if for eight districts, the district split is 2–6 (i.e. Party A wins two districts and Party B wins six districts), then whether Party A wins the first two districts, the second two, the first and last, or any of the $\binom{k}{2}$ ways to choose two different winning districts, the value of the efficiency gap for all of these scenarios is identical.

Let T_A = the total number of votes cast for Party A, T_B = the total number of votes cast for Party B, p = the population in each district, W_A = Party A's wasted votes, W_B = Party B's wasted votes. Then the efficiency gap is defined as

IV. THE EFFICIENCY GAP FOR CONGRESSIONAL ELECTIONS

So far we have examined only hypothetical scenarios, which illuminate the properties of the efficiency gap, but are not based on actual political scenarios. We now shift to an examination of the efficiency gap values for congressional elections in each of the fifty states. Recall that the value of the efficiency gap is affected by the partisan split of the population. While we have spoken of *partisan split* as if it is a known entity, its value must be derived from data. Indeed, the Court is well aware of the measurement difficulties here—"a person's politics is rarely as readily discernible—and *never* as permanently discernible—as a person's race. Political affiliation is not an immutable characteristic, but may shift from one election to the next; and even within a given election, not all voters follow the party line."[17]

Plainly, the number of Republicans in a state can be measured in different ways. Two options are the two-party partisan registration and the two-party presidential vote. Neither is a perfect measure of the true underlying voter partisanship. Not all states require partisan registration, and even in states that do, some voters register unaffiliated. Presidential vote is also not a perfect measure of partisanship, and fluctuates from one election to another. There are surely many other possible partisanship measures, for example, the two-party vote "formed by averaging district-level election results . . . in seats won by major party candidates, including uncontested seats."[18] The important point, however, is that all of these measures are *imperfect* estimates of partisanship, an unknown underlying quantity, and the choice of how to estimate this unknown underlying quantity can non-trivially change the efficiency gap value.

$$EG = \frac{W_A - W_B}{T_A + T_B} \tag{4}$$

where

$$W_A = T_A - (0.50\ p)\ d_a \tag{5}$$

$$W_B = T_B - (0.50\ p)\ d_b \tag{6}$$

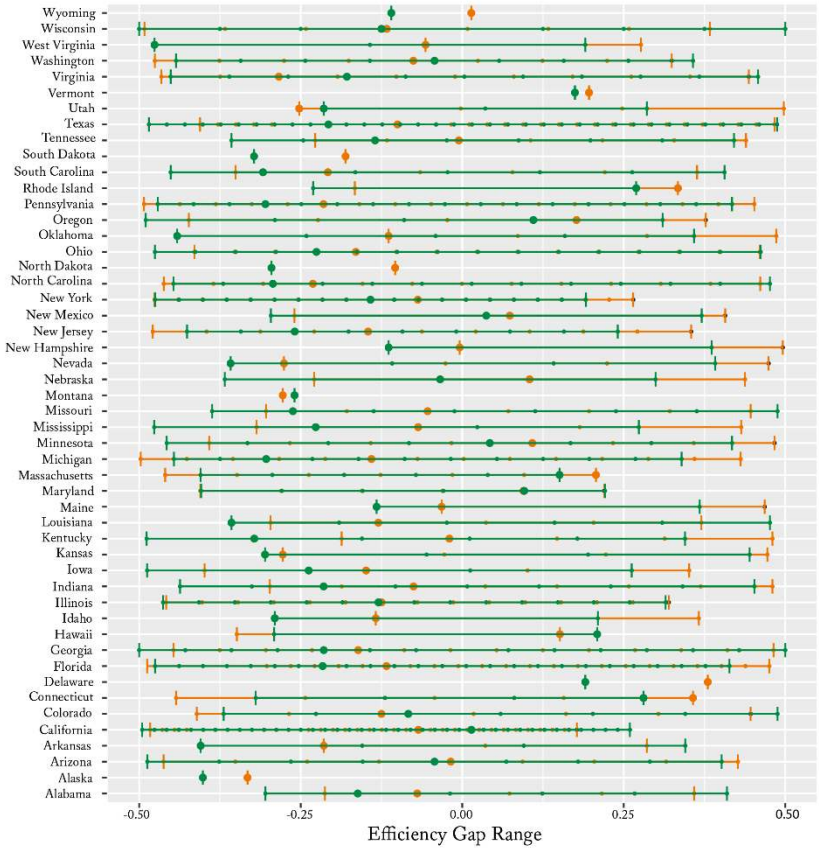
We can see that the wasted votes depend on the vote split, d_a–d_b, but not on the number of votes cast to obtain a win. This becomes obvious when one realizes that the total number of votes is fixed and the number of not-wasted votes is always exactly the number of votes needed to win. The wasted votes are always the rest of the votes whether they are wasted because they are in excess of the minimum number to win or they are wasted because they are cast in a losing election. This result is also implied by McGhee and Stephanopoulos's formulation in Equation (2), $EG = S–2V$, where the efficiency gap depends exclusively on the partisan split and the seat split.

17 Vieth v. Jubelirer, 541 U.S. 267, 287 (2004).

18 Expert Report and Affidavit, *supra* note 3, at 19.

Figure 1 shows possible efficiency gap values for each state given a two-party partisan split. The orange line shows the possible values when partisanship is computed from the 2016 two-party presidential vote.[19] The green line provides the counterpart based on two-party partisan registration from a 2014 survey by the Pew Research Center.[20] The dot indicates the efficiency gap using the current congressional seat split in each state.

Figure 1: Possible Efficiency Gap Values for Congressional Districts across States



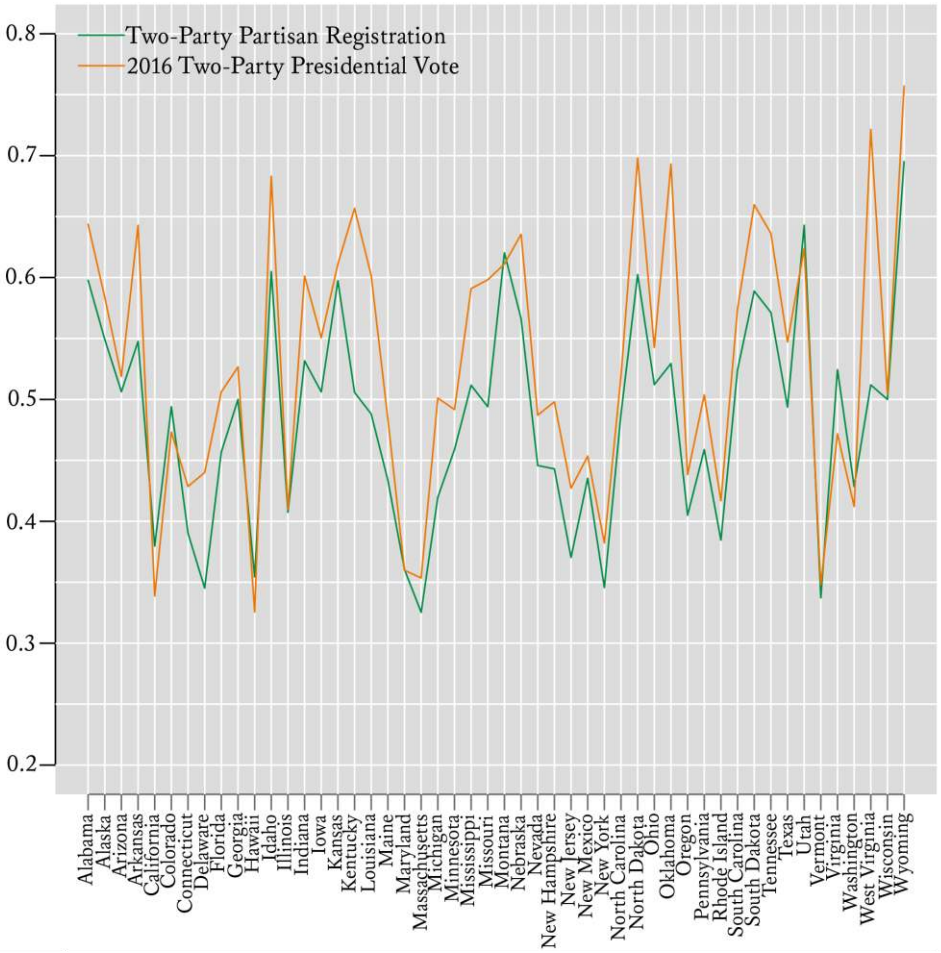
19 THE ALMANAC OF AMERICAN POLITICS (Columbia Books, Inc. 2018).

20 *Religious Landscape Study*, PEW RES. CTR. (2017), http://www.pewforum.org/religious-landscape-study/ [https://perma.cc/DL4P-SAHT].

If we compare orange to green, we can see that how we choose to define party membership is significant. Unfortunately, there is no error-free way of assigning voters to one of two parties. The importance of this choice should be obvious since the number of wasted votes is entirely dependent on the true partisanship of the voters. Sometimes the choice between presidential vote and party registration does not matter much in the efficiency gap calculation. As seen in Figure 1, in Wisconsin and Illinois, the choice of partisanship data is virtually without consequence. In many states, the green and orange dots are not so far from one another. However, in other states, the data choice can significantly change the efficiency gap value. In California and Wyoming the efficiency gap even switches sign. In twenty-two states, the difference in the efficiency gap value is more than 0.10, depending on which definition of partisanship is used. We are not advocating one partisanship measure over another, and the two measures track each other closely ($\rho \approx 0.89$)[21], as we might expect. We are merely pointing out that there are many reasonable measures of partisanship and that choosing a particular measure is a consequential choice to the value of the efficiency gap for a jurisdiction.

21 *See* Figure 2.

Figure 2: Differences between Two-Party Partisan Registration and Two-Party Presidential Vote



In states with smaller congressional delegations, the number of possible efficiency gap values is smaller. We have already proven that the number of possible efficiency gap values is equal to the size of the delegation. As a result, every time a district changes hands, the magnitude of the change in the

efficiency gap value is inversely proportional to the size of the delegation. That is, in South Carolina (with seven congressional seats), if one seat switches parties, the efficiency gap changes by 1/7, or a little more than 0.14. In California (with fifty-three congressional seats), the change in one seat moves the efficiency gap by 1/53, or less than 0.02.

V. THE EFFICIENCY GAP AND PARTISAN FAIRNESS

Mathematically, the efficiency gap exhibits problematic properties. We now switch gears to discuss the philosophical adherence of the efficiency gap with the concept of partisan fairness. Admittedly, the concept of partisan fairness is ambiguous. In the words of Justice Kennedy, "[n]o substantive definition of fairness in districting seems to command general assent."[22] Although the term partisan fairness is not well defined, it is fair to say that there are widely accepted broad contours for the concept of partisan fairness in redistricting maps. These conceptions have fallen into one of two camps.

The first is responsiveness. "State legislatures," the Court explained, "should be bodies which are collectively responsive to the popular will."[23] Accordingly, a measure of partisan fairness should be able to distinguish maps with non-competitive districts that are not responsive to voters from a map that is comprised of competitive districts that are responsive to the voters.

The second camp is bias, sometimes spoken of in terms of partisan symmetry—if a map is biased or unfair to one party, it should be equally or symmetrically unfair to the other party.[24] In determining whether a partisan gerrymandering has occurred, one must be aware of the multi-faceted nature of partisan unfairness, since fairness on one dimension does not preclude unfairness on another dimension. Only a measure imbuing a deeply nuanced understanding of partisan fairness will ensure that we can thwart both "sophisticated as well as simple-minded modes of discrimination."[25]

The two camps are not at odds with one another. They reflect different facets of partisan fairness. It should be easy to agree that unfairness is inherently undesirable in any form. Accordingly, a sensible goal for a measure of partisan fairness is to tap both responsiveness and bias, related, but separate dimensions of the same underlying unfairness phenomenon. At a minimum, we should understand how a measure is consistent or inconsistent

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22 Vieth v. Jubelirer, 541 U.S. 267, 307 (2004) (Kennedy, J., concurring).

23 Reynolds v. Sims, 377 U.S. 533, 565 (1964).

24 *See* LULAC v. Perry, 548 U.S. 399, 420 (2006) ("[T]he measure of a map's bias is the extent to which a majority party would fare better than the minority party, should their respective shares of the vote reverse.").

25 *See* Gomillion v. Lightfoot, 364 US 339, 342 (1960) (quoting Lane v. Wilson, 307 US 268, 275 (1939)).

with these dimensions. If a single measure does not capture the complex and multi-dimensional nature of partisan unfairness well, an understanding of the issues provides a basis for the development of other measures that singly or collectively are able to comprehensively and effectively identify and measure partisan unfairness.

Interestingly, these two facets—responsiveness and bias—are associated with different flavors of partisan gerrymanders. The classic partisan gerrymander involves one party disadvantaging the other party by creating districts that pack excessive numbers of minority party voters into a small number of districts and crack the rest of the voters among many districts to minimize their influence. This tactic wastes some set of the minority votes while more efficiently dispersing the majority voters among districts to ensure victory. These plans exemplify *bias* in favor of one party. Another flavor is the bipartisan gerrymander, where the two parties, majority and minority, join forces to create a sweetheart deal where both parties are protected in safe seats, thereby preserving the status quo via non-competitive elections. Bipartisan gerrymanders, while usually not biasing one party over the other, lack *responsiveness* to the electorate. Both types of gerrymanders minimize the ability of voters to choose their preferred candidate.

A. *Responsiveness Is Not Captured*

Recall scenarios 1–7 from Table 2. We see a surprising incongruence between the efficiency gap and the concept of *responsiveness* and competitive elections. It is clear that as we move from scenario 1 to scenario 7, the level of competitiveness increases. In scenario 7, while both districts are in Party A's favor, the partisan split yields competitive elections. The efficiency gap, however, does not reflect a change in competiveness. The mismatch between the efficiency gap and competitiveness can be traced back to another reason why the efficiency gap is not comparable across jurisdictions. Namely, in scenario 1, the wasted votes accounting for the efficiency gap are almost exclusively votes wasted in winning elections (packed votes). In contrast, in scenario 7, the wasted votes accounting for the efficiency gap are almost exclusively votes wasted in losing elections (cracked votes). In scenario 4, Party A has only excess winning votes while Party B has only excess losing votes. While this is clearly an extreme packing scenario that is the hallmark of partisan gerrymandering, the efficiency gap declares it one of optimal efficiency. It is symmetric in wasted votes, but the symmetry is oddly induced by equating unequal entities. By equating excess winning votes with losing votes, the efficiency gap produces comparability and interpretation issues across jurisdictions.

B. *Bias Is Not Captured*

Further, *bias* and the spirit of packing and cracking is not captured. Notice, for instance, that Party A reaches near maximum inefficiency in scenario 1 where it wins by many more than the required 50%. However, here it is hard to see how, given the vote distribution, Party A could have done better or worse for itself. There is no efficiency to be gained by Party A. In fact, the same can be said for Party B because it also cannot do better or worse for itself. Party B cannot waste votes because it does not have enough votes to gain a majority in any district. *Neither* party can be made better or worse off through redistricting. Although the two parties are *symmetrically* disadvantaged with no bias toward either party, the efficiency gap implies that Party A is much worse off than Party B. The disconnect is that the efficiency gap does not consider the nature or degree of the partisan split. In these cases, it is *efficient* for Party B to seek influence with the available votes rather than attempting to convert votes to seats efficiency, as this is an impossibility.[26] This type of efficiency is intuitive but it is not the type of efficiency captured by the efficiency gap measure. Indeed, whether or not votes are packed is not a function solely of numbers; it critically depends on a strategic placement of voters given the partisan split.

The bias or symmetry assessment is also problematic because the efficiency gap attempts to compare unequal entities to evaluate symmetry. We see this most starkly in scenario 4 of Table 2 where Party A wins both districts with an overwhelming 75–25 margin. Here, Party A wastes fifty excess winning votes while Party B wastes fifty losing votes. Since both waste fifty votes, the efficiency gap is optimal. Stephanopoulos and McGhee cite this case as "technically correct,"[27] but they mean simply that they have computed the wasted votes (defined without distinguishing excess winning votes from losing votes) as they claim to have counted them. While they have indeed counted their definition of wasted votes correctly, their conception of wasted votes is flawed because it conflates unequal types of votes. The problem is in the unit they measure, not in the ability to add and subtract numbers. When the efficiency gap is zero, there is symmetry and no bias of wasted votes as they define waste, but because the symmetry is of unequal entities, the measure fails to capture the type of partisan bias and symmetry at the core of the Court's intuition.

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26 This type of influence is valued by at least some Justices in the Court. *See generally* Holder v. Hall, 512 U.S. 874 (1994).

27 *See* Stephanopoulos and McGhee, *supra* note 1 at 863.

VI. DISCUSSION

The Supreme Court has struggled to identify a manageable standard for partisan fairness.[28] Partisan fairness is not a simple phenomenon and is without a straightforward conceptualization. A specific formulation has not been adopted by the Court. Nonetheless, the Court appears to be amenable to identifying a standard.[29] One proposal before the Court is to use the efficiency gap to measure the extent of partisan fairness.

In the efficiency gap measure, wasted votes are defined as both excess winning votes *and* losing votes, with no distinction between the two. As we have shown, in a variety of situations, equating excess winning votes and losing votes results in unintuitive efficiency gap values that are incongruous with the idea of fairness, either in the form of responsiveness or bias. These odd outcomes signal measurement issues. Stephanopoulos and McGhee acknowledge that "when one party receives more than 75 percent of the statewide vote—the efficiency gap can produce results that at first glance seem strange."[30] They then state that

> "[t]his outcome is technically correct: when a party already holds all the seats, additional votes are wasted since they cannot contribute to more victories. Nonetheless, it fails to capture the idea of fairness at stake in redistricting, since the majority party in this situation could hardly be said to suffer a disadvantage."[31]

Their solution is not to make any modifications to how the efficiency gap is calculated, but rather to say that these are rare occurrences that an analyst should flag ahead of time. Our point is that a general measure should exhibit mathematical properties that ensure it is measuring the quantity of interest, rather than needing to delete cases where its behavior is erratic. Surely, a problematic measure should not be codified by law.

Because they measure a strange unit, we witness a variety of interpretation issues. We also notice other phenomena, like bipartisan gerrymanders, that are consistent with "optimal efficiency" but smack at the notion of fairness to voters. It is always possible to produce a bipartisan gerrymander that has a small efficiency gap. Because the vote difference in each district is inconsequential to the value of the efficiency gap, the two

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28 *See generally* Vieth v. Jubelirer, 541 U.S. 267 (2004).

29 The Court has come within a vote of declaring partisan gerrymandering non-justiciable. However, Justice Kennedy and the four justices in the minority in Vieth, while not yet agreeing on a standard, believe that a standard can be identified. *See generally* Vieth v. Jubelirer, 541 U.S. 267 (2004).

30 *See* Stephanopoulos and McGhee, *supra* note 1, at 863.

31 *Id.*

parties can simply agree to pack Party A voters in their winning districts and pack Party B voters in their winning districts. This creates all safe seats, and as long as the packing is symmetric, the gap in efficiency is minimized. The map is "fair" to both parties, giving them both advantages, while being simultaneously unfair and severely disadvantageous to the electorate. Unfairness to the voters—the original sin of partisan gerrymandering—cannot be overcome by fairness to the parties.

We have also seen that because the efficiency gap is completely defined by two entities, the seat split between two parties and the overall vote split, the outcome is not comparable from jurisdiction to jurisdiction or even within a jurisdiction. Greater volatility is induced for jurisdictions with smaller delegations. Unintuitive results arise when one party has an overwhelming vote advantage. A general measure must carefully and precisely incorporate a deep understanding of the nuances of the underlying concept, and exhibit sound mathematical properties. It does not rely on case-by-case exceptions.

Ensuring the right to vote has proven to be a complex task, involving many intertwined and moving pieces. It would be surprising if the complexity inherent in partisan discrimination could be captured in a single numerical value. Simple-minded modes of discrimination are perhaps good candidates for identification with simple measures. Sophisticated modes of discrimination, especially of complex phenomena like partisan fairness, require nuanced multi-dimensional measures, or perhaps, multiple measures. For ensuring partisan fairness, the efficiency gap is too easily fooled.

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